EARTHQUAKE FREQUENCY

BY

C. G. KNOTT, D. Sc., F. R. S. E.

[Read May 21st 1884.]

Periodicity in natural phenomena is a characteristic which in all departments of Science is the subject of much study. Besides the ordinary periodic phenomena which have been familiar to man since ever he was a reasoning being, there are daily being disclosed to our view others, which though not so obvious are as undoubted. For example, within the present century we have come to recognise the sun-spot cycle of eleven years, and the closely related magnetic variation cycle. The periodicity of meteoric displays, the periodic nature of an electric discharge, and Mendelejeff's arrangement of the chemical elements, may be mentioned as further illustrations. In this paper it is proposed to discuss from the periodic standpoint the question of earthquake frequencies.

According to the accepted theory, earthquakes are the result of the transmission through the earth's crust of a disturbance originating at some definite point underground. The disturbance may be of the nature of an explosion, or simply a rupture or crack in the material of the earth's crust caused by the yielding of the same to stresses impressed upon it. It is generally believed that water plays an important part in the genesis of earthquakes. Percolating down to depths where heated materials are, it may be regarded as accumulating in the spheroidal state until its mass becomes too great to be supported. It is then rapidly volatilised, and expanding forcibly in the form of steam effects a rupture in the walls of the chambers in which it is collecting. Or, it may be supposed to collect in a highly heated condition under considerable pressure. At length the solid boundaries yield to the action of this steadily increasing stress, the pressure is relieved, and the fluid expands suddenly into the roomier region with the same effects as already described. Other causes, however, may be at work to produce the same results. Such are for example the earth's own cooling and shrinking attended by the crumpling and tilting of strata, the centrifugal effect of the earth's rotation, the weights of the continents, tidal stresses due to the direct gravitational action of the sun and moon, or stresses due to meteorological conditions as regards temperature, pressure, wind, rain, snow.

These various causes are conveniently grouped under three headings. Thus, earthquakes may result

(1) from purely terrestrial actions,
(2) from the gravitational action of the sun and moon,
(3) from meteorological conditions.

A few general remarks regarding the first of these will suffice here, inasmuch as from the non-periodic nature of such terrestrial actions they have little concern with the subject as at present treated. There is not the least doubt, however, that the conditions favorable for the genesis of earth tremors depend largely on these purely terrestrial actions. An earthquake always means a yielding to stress, whatever may have been the real source of this stress. Different portions of the earth's crust have different powers of resistance; and the power of resistance of any one region will vary in time. Hence it follows that in certain places and at certain times the earth's crust becomes what may be called seismically sensitive. In the vicinity of great faults, or on the margin of continents and islands, where clearly there has been evinced a comparative weakness—there we should expect earthquakes to take their rise. And there in fact we find the regions of seismic activity. That this seismic sensitiveness is due directly to purely terrestrial causes may safely be assumed; but before yielding can take place and seismic phenomena ensue something more is needed—the necessary stress, namely. Thus earthquake frequency depends on two distinct things—the seismic sensitiveness of the region, and the existence at the proper time of a stress suitably applied.

Now if there is any marked periodicity in earthquake frequency, there must be a corresponding periodicity in either or both of these two independent factors. It is possible of course that there may be a natural period in the growth of the sensitiveness of a given region from one time of yielding to that degree of sensitiveness necessary for the next; but it is difficult to believe that such a periodicity would be generally characteristic of all regions, or of such purely terrestrial causes as have been specified. Since our object is to discuss periodicity as displayed in earthquake frequency over the whole globe, we may confine our attention simply to the causes that fall under the second and third heads given above. In so confining our attention, however, we must not be understood to say that these are the causes of earthquakes, but merely that they are possible determining factors in earthquake frequency.

Having said so much by way of introduction, we now proceed to consider in detail the various cosmic and meteorological phenomena, which may possibly influence seismic activity.

Of all bodies external to the earth the sun and moon alone can be expected to produce any appreciable effect in virtue of direct gravitational action. The idea that seismic or volcanic phenomena are due to tidal actions produced in the interior parts of the earth is no new one. In the early days of geology, when the earth's solid crust was supposed to enclose a molten interior, such an idea was a very natural one. With this theory as a guide, geologists tried to find some relation between the moon's altitude and the intensity of volcanic eruption, but with small success. In 1839, the theory of the liquid interior enclosed by a thin solid shell received its death blow at the hands of Hopkins, who showed that such a structure was quite incompatible with the astronomical phenomena of precession and nutation. Not only would the tides produced in the fluid interior utterly do away with precession as it exists; but the thin crust would have to be of infinite rigidity to itself resist the deforming tidal action. That precession and nutation might be as they are, the whole crust, supposed rigid, would have to be at least 800 miles thick. In 1862, Sir William Thomson followed up Hopkins' researches by a discussion of the problem of the yielding of the earth to tidal action. That it must yield to some extent is indisputable, since certainly its rigidity is not infinite. But whether the amount of yielding is perceptible to observation is a very different question. The effect of such a yielding, if appreciable, would show itself in the ordinary ocean tides on the earth, diminishing their apparent magnitudes. For the direct solution of this question the British Association, on the motion of Sir W. Thomson, appointed a committee to make careful tide measurements. These, together with the valuable tidal observations published by the Indian Government, have been worked up by G. H. Darwin, whose elaborate papers on the tidal stresses in viscous and elastic globes rank among the classical memoirs of the century. Besides the usual solar and lunar diurnal tides known to every one, there are tides of long periods not generally mentioned in elementary text-books. These are the fortnightly, the monthly, the semi-annual, and annual tides. The first and third arise from changes in the moon's and sun's declinations respectively; the second and fourth from changes in their distances from the earth. Thus, when the moon's declination is zero there is on the whole higher water at the equator and lower water at the poles than when the declination has any other value. Hence there is a small fortnightly oscillation in the values of the semi-diurnal tides as the moon passes from node to node; and a similar effect with a half year period is produced by the sun's motions in declination. In one revolution again the moon's distance from the earth goes through a complete cycle from apogee to apogee. In perigee the tidal effect, which varies inversely as the cube of the distance, is of course at a maximum. And thus arises the monthly tide; and similarly the annual tide. Of these the fortnightly and monthly tides are much the larger; and it is from them that Darwin draws his conclusions. One advantage in taking these long periodic tides from which to calculate is that, if the earth really does yield, it has time to make an adjustment which can be treated on the ordinary equilibrium theory. In all probability the semi-diurnal tidal stresses are too rapid to produce measurable deformation in the solid earth. This question of *time* in producing strain in viscous or solid matter is really a most important one, and seems to have been quite overlooked in earthquake literature. We shall return to it later. Darwin's method, then, consists in expressing by means of suitable formulas the fortnightly and monthly tides. In these there enter certain unknown co-efficients, which however have different definite values or relations to each other, according as the earth yields as a whole or is infinitely rigid. By combining the tidal observations of India, Britain, and France, he deduces the most probable values of these co-efficients; and the final result is that although "there is some evidence of a tidal yielding of the earth's mass, that yielding is certainly small, and that the effective rigidity is at least as great as that of steel."

This conclusion that the earth as a whole is as rigid as an equal sized globe of steel seems at first somewhat startling —especially to one who has no very clear notion of what is meant by the term rigidity, but regards it as synonymous with inflexibility. The geologist indeed has been slow to accept this truth, simply because of the confused meaning which he attaches to the term rigidity. A window pane will rather break than bend; and yet a glass fibre can be used as a thread. The rigidity is the same in both cases, but the inflexibility is much higher in the one case than in the other. The same truth is illustrated in the bahaviour of a steel hairspring and a steel bar. Thus the mere existence of magnificent foldings or rumplings in the hard rocky strata that form the earth's crust can tell us absolutely nothing regarding the rigidity of the material. A sheet of the hardest steel of the same form and size as (say) a stratum of old Red Sandstone would go through very similar transformations under the continued moulding action of the powerful stresses that are known to exist within the earth's substance. To measure rigidity, we must know not only the strain produced but the stress which produces it. No purely geological method can lead us to a knowledge of what this stress is; it can be estimated from the strain only when the rigidity is known. Laboratory experiments, however, are of little use in this quest, since we have access to a very limited part of the earth's substance; and even had we access to the deeper regions of our globe the samples finally experimented on would certainly have quite changed their properties when brought from their warm high pressure depths to the light of day.

Darwin has proved, then, that if there exists any tidal yielding of the earth as a whole it is hardly appreciable, even when searched for by our most refined methods of analysis. Still, although there may be no yielding as a whole, there must be time variations in the tidal stresses at any given point; and these may cause earthquakes if the region chance to be seismically sensitive. No one so far as I know has grouped earthquakes according to lunar or solar declination. Perrey made elaborate comparisons between earthquake frequency and the position of the moon, and obtained apparently definite results. He found that the frequency of earthquakes increased at syzygies, at apogee, and at meridian passage. The last can hardly be accepted as indicating any physical relation, for it is surely impossible for such a short timed periodic change to have any such pronounced effect. Then, according to Chaplin*, the Japan earthquakes have a minimum of frequency at syzygies; and the still more recent discussion by Forel † of the Swiss earthquakes throws strong doubt on any such relation. He finds only 53 per cent. of the total number occurring during the syzygy period. The same percentage is brought out for the earthquakes occurring at the meridian passages of the moon. It might be interesting to tabulate earthquakes according to the moon's declination—though it is extremely doubtful if the result obtained would have a value at all worthy of the labour involved. It is possible, however, that this fortnightly oscillation of the direction of maximum tidal stress in the earth's substance may be a determining factor in their frequency.

Whatever may be the effect of the moon's motion in declination, that of the sun's will of course be much less. Its distance is greater; and its motion in declination smaller.

---

* Asiatic Society of Japan. Trans. Vol. VI.

† See L'Astronomie (January 1884).

But on the other hand, the period is longer, so that the earth may yield more in proportion to the corresponding variations of stress. One suggestive fact in connection with these motions in declination is that earthquakes abound in tropical and sub-tropical regions—just where the tidal stresses are greatest, and where also the fortnightly and semi-annual variations most tell. Then there is the solar annual tide, due to the periodic change in the sun's distance. The effect of this would be to cause greater earthquake frequency in the half year from September to March, that is in our winter. In the northern hemisphere there is a very marked winter maximum of frequency; but in the southern hemisphere the maximum seems to come between June and September, while in equatorial regions there is no definite annual periodicity at all. We must not, however, conclude hastily that the annual tide has no effect; all we can say is, that there is some other more efficient cause tending to produce earth tremors in the *winter* season, wherever there is a well marked winter season. Still it is well to consider more in detail what features might be expected to accompany a periodicity determined by the semi-annual and annual tides. As already pointed out, the torrid zone with immediately contiguous zones to the north and south, would be the seat of most frequent seismic actions. Then, the semi-annual maximum would occur at localities in higher latitudes during the local summer; while the annual maximum would occur about December or January, to allow for the lagging of strain after stress. At equatorial places, the semi-annual effect would really have a quarter-yearly period; while at points lying in the 10th or 12th latitude line both north and south this effect would vanish. In many of the statistics there are indications of semi-annual periods—as for example in Mallet's general curve of frequency for the whole southern hemisphere.* In searching for such semi-annual periods, we must adopt some means for smoothing off the jaggednesses in our curve of frequency and eliminating the annual period. The method which seems to me to be

---

* British Association Reports (1858)

least open to objection is to take overlapping means of the numbers originally obtained.

Hitherto it must be confessed there has been a good deal of arbitrariness in the treatment of earthquake statistics. As a rule, earthquakes are numbered in monthly groups, which are then combined in seasonal sets. Mailet called January, February, March the winter months—a nomenclature to which strong objection may well be taken—and his example has been generally followed. December, January, February are certainly a truer seasonal combination, thus making Spring begin on the 1st of March, Summer on the 1st of Jüne, and so on. But this grouping in months is purely arbitrary—since our month is a civil and not a natural division of time. Such a grouping may indicate the existence of long periods, but it affords no ready means of distinguishing between the co-existence of several distinct long periods. To do so, we must devise some method of analysing the complex period shown in these monthly sums. For the northern hemisphere there is a well marked annual period in earthquake frequency. Is there a Semi-annual period?—such as might be caused by the changes in the sun's declination.

To answer this question we must do three distinct operations. We must *prepare* our numbers by some strictly accurate mathematical process so as to magnify (first) the annual effect and (second) the semi-annual effect, smoothing away somewhat the effects of the smaller periods. And then by direct comparison of these prepared curves, we must separate out the semi-annual period.

Take for example Professor Milne's catalogue of Japan earthquakes from 1872 to 1880 inclusive, arranged according to months. Form three-Monthly means, tabulating under January the mean of December, January, February; under February, the mean of January, February, March; under March the mean of February, March, April; and so on through the whole twelve months. The effect of this operation is to magnify the annual and semi-annual periods at the expense of the shorter periods; while the quarter-annual period, eighth-annual period, sixteenth-annual period, if there are any, vanish.

Beginning again with the original monthly numbers, take the six-monthly overlapping means, and tabulate them as follows: set the mean for the months from January to June halfway between March and April; the mean from February to July halfway between April and May; and so on. By this process we throw out completely semi-annual, quarter-annual, eighth-annual, etc., periods, and diminish very markedly the effect of all other possible periods.

Now it can be shown* by an application of Fourier's Theorem that from these two curves a third curve may be obtained from which the annual period is completely eliminated. To effect this, the Three-monthly means must be reduced by multiplying by the factor .707. In this way we obtain a prepared semi-annual curve with the annual periodicity involved exactly as it is involved in the annual curve formed from the Six-monthly means. By subtracting the latter from the former, we obtain finally a curve whose longest period is the semi-annual, if such exists. Since the Six-monthly means correspond to the end of the month, and the Three-monthly means to the beginning of the month, it is advisable to interpolate in both series the arithmetic means of the successive numbers taken in pairs; so that to every Three-monthly number there corresponds the mean of two Six-monthly numbers, and to every Six-monthly number the mean of two Three-monthly numbers. The following table will show the method applied to the Japan earthquakes from 1872 to 1880. The monthly and unreduced Three-monthly numbers are spaced out, so as to have room for the interpolated means in the Six-monthly and reduced Three-monthly series.

---

* See Mathematical Note at end of Paper.

Monthly.	Three-monthly.		Six-monthly.	Differences.	
	unprepared.	prepared.		unreduced.	reduced.
29	35	24.7	35	10.3	1.16
		24.3	35	10.7	1.21
36	34	24	34	10	1.13
		23.7	33	9.3	1.05
38	33	23.3	32.5	9.2	1.04
		22.9	32	9.1	1.03
26	32	22.6	32	9.4	1.06
		21.9	32	10.1	1.14
32	30	21.2	31	9.8	1.11
		21.5	30	8.5	.96
33	31	21.9	27.5	5.6	.63
		20.8	25	4.2	.47
29	28	19.7	25.5	5.8	.66
		17.2	26	8.8	.99
21	21	14.8	26.5	11.7	1.32
		15.2	27	11.8	1.33
13	22	15.6	28	12.4	1.4
		17.3	29	11.7	1.32
32	27	19.1	29	9.9	1.12
		22.3	29	6.7	.76
37	36	25.5	30	4.5	.51
		25.5	31	5.5	.62
41	36	25.5	33	7.5	.85
		25.1	35	9.9	1.12

The last column of reduced differences is obtained from the preceding column by dividing each number by the mean of all the numbers. It shows at a glance where the maximum and minimum points occur; and besides gives a ready means of comparing the characteristics of earthquake frequencies in different parts of the world. By treating all statistics in this way, we eliminate the element of absolute frequency but preserve the relative frequency. It is to be specially noted that the originally derived Six-monthly numbers are half a month later in time than the Three-monthly numbers. Hence the first number entered on the Six-monthly column is one of the interpolated means.

I have treated in this way Mallet's list of the earthquakes of Europe and the adjacent parts of Asia and of Africa from A. D. 306 to 1843;(1) the New Zealand Earthquakes from 1869 to 1879(2); the East Indian Archipelago earthquakes from 1873 to 1881(3); the Chili Earthquakes from 1873 to 1881(4); and the Grecian Archipelago Earthquakes from 1859 to 1873(5). The complete list of monthly, prepared Three-monthly, and Six-monthly, numbers are given in Table A at the end of the paper. Table B contains the reduced difference numbers, arranged for convenience of comparison, of these six regions. Two lists of numbers are given for the New Zealand earthquakes, since in the source from which they were taken there was no indication as to whether certain observations made in different localities on the same *day* really referred to one and the same Earthquake or to different ones. Only a knowledge of the *hour* could determine that; and in no case was this very important datum given. The one series of numbers is obtained by considering all observations of the same date to refer to one Earthquake; the other, by regarding them all as referring to different Earthquakes. This lack of precision renders any argument based upon the New Zealand Earthquakes of very secondary value: although the general similarity of the results obtained from the two sets of numbers seems to show that no great error is committed by either process of treatment.

Corresponding to Table A is Plate I containing the curves for the Six-monthly and Three-monthly means. The curve for the Grecian Archipelago is not given, since in general features it resembles the curves for Europe and Japan. It will be noticed however that the other three sets of curves are quite distinct in their main characteristic. In New Zealand and Chili, the Earthquake frequency has its maximum between the months of June and September, that is, during

---

(1) *British Association Report*, 1858.
(2) *Meteorological Reports*.
(3) *Natuur kundig Tijdschrift voor Nederlandsch-Indie*.
(4) *Observaciones Meteourogicas* (Sautiago) 1884.
(5) Schmidt's *Studien über Erdbeben*, 1879.

the *winter* season in these countries; whereas the curve for the East Indies shows no marked annual period at all.

Corresponding to Table B is Plate II, showing the curves which ought to indicate the semi-annual period if there is such. With the single exception of the East Indies curve, which has *three* pronounced minimums, they all show a distinct semi-annual periodicity. The curves for Europe and New Zealand are very similar to one another; as also the curves for Japan and Chili. This grouping in pairs is the more remarkable, when we consider how dissimilar the annual variation is for the same pairs, as shown in Plate I. The general average curve, figured lowest, has its minimum points about February and July; but it is hardly possible to connect this with any possible periodicity in tidal stresses. The smoothness of the Europe curve as compared with the others almost suggests that this semi-annual characteristic is accidental, and that with a sufficient number of observations it might vanish altogether. And yet it seems too general to be accidental; while the curves for the two Archipelagos, which depart more or less in general characteristics from the others, have a striking similarity to one another. By omitting these in the general average, we should obtain a much more evident semi-annual effect. Further, this can surely be no accidental peculiarity which appears in the results obtained from the Earthquakes of the East Indian and Grecian Archipelagos, since these differ so markedly otherwise, inasmuch as the former has no vestige of an annual period, while the latter has it very pronounced.

Thus from a consideration of the possible existence of a periodicity in earthquake frequency due to the semi-annual solar tidal stress, we have been led to search the statistics of earthquakes for the evidence of such a periodicity. Apparently it exists; but it may not be due after all to the cause that suggested its existence. We should indeed rather be inclined to doubt the efficiency of the semi-annual tidal stress, after finding the annual tidal stress so clearly inefficient. There must be some other more potent cause than this latter in producing an annual period, and it is not unlikely that the effect of the semi-annual tide may be similarly masked.

This brings us to the third group of causes specified above.

First as to the direct effect of temperature changes. That there is a greater frequency when the atmospheric temperature is low than when it is high seems an established fact, since that is only another way of saying, the maximum frequency occurs in the *local winter*. But scientifically they are very different statements—the one being simply a statement as to the time of the year in which earthquake frequency reaches its maximum, the other implicitly involving the hypothesis that seismic energy is stronger *because* the temperature is lower. Now it must be borne in mind that the depth to which seasonal changes of temperature appreciably penetrate is at most some ten metres—certainly not more. And this penetration takes place only where there is land. The ocean is practically unaffected*. Again, this penetration takes place so slowly that at a depth of five metres the crust of the annual temperature wave occurs at the winter season, having taken 6 months to reach that depth. Such considerations convince us that to expect a direct effect due to temperature change is sheer folly.

Wind, as regards its possible direct effect at even small depths below the surface, may be mentioned simply to be dismissed from further consideration.

Rain and snow, however, require a closer study. If earthquakes are due largely to the expansion of highly heated water, then it would be natural to expect the rain-fall periodicity to show itself. As it happens, however, the maximum earthquake frequency is in the winter, while as a general rule summer is the rainy season. Hence unless we suppose the rain to take half a year to show itself as a seismic agent, we must look in some other direction for the cause of earthquake periodicity. And such a supposition is hardly to be entertained, when it is remembered under what very different local conditions earthquakes are produced. Snow-fall, however, has quite other characteristics. It is of course greatest in winter. But more than this it may, in virtue of its weight,

---

*See "Challenger" Reports.

influence underground regions in a way that rain cannot. Consider for example the region near Japan.

To the west lies the large continent of Asia; to the East the broad stretch of the North Pacific. Along the line where these meet, earthquakes and volcanoes abound. It is clearly a line of weakness; and this characteristic probably extends to a considerable depth. With such an unequal surface distribution of weight there must be at moderate depths powerful stresses acting unequally in different directions. It is to such æolotropic stresses that materials readily yield. Throughout the whole history of the study into the cause of earth tremors, the attention has I think been fixed too much on the specially disturbed area. We should take a wider view of matters. For exactly as a wood splinter snaps at the centre far removed from the ends where the external stresses are applied,—so in the case of earthquakes the region where they occur is not necessarily where the immediately producing cause acts. Bearing this principle in mind, we easily recognise in the stresses due to unequal distribution of surface matter a real cause for the snappings, crackings, and yieldings generally, which may lead to seismic effects. And further, anything which tends to increase these stresses will tend to increase the frequency of earthquakes. So far as the action of the weight of the continent itself is concerned, there is no reason for the display of any periodicity. But in the accumulated winter snow over the surface of Siberia, there seems to be a periodic phenomenon, which in virtue of its permanence for months at a time may well be expected to influence the rate at which these snappings and crackings occur. The region is always more or less in a condition of strain; in the winter reason, because of the snow accumulated over the dry land, the inequalities of stress are intensified; and hence in the winter season there is a tendency to increased frequency in seismic action. This possible effect of snowfall was suggested to me by Professor Yamagawa of Tokyo University during a conversation on the meteorological influences that might have to do with earthquakes. There are, unfortunately, no statistics of the amount of snowfall obtainable for any wide extent of country except for Canada. The large snowfall on the north west coast of Japan would lead one to expect a very large accumulation over the north east of Siberia; so that, so far as Japan earthquake frequency is concerned, the winter snow is quite a probable determining factor. The same statement applies, though not to such a marked extent, to the western shores of South America, where the lofty contiguous mountain chains become more thickly snow clad in the winter months. In such regions as the East Indian Archipelago or the shores of the Mediterranean, the snow fall must have but a slight if not altogether insignificant effect. To sum up then, it seems not at all unlikely that in earthquake regions where the winter snowfall is excessive, the resulting winter excess of pressure over the land will have an influence on the frequency of seismic action. That it is the only determining periodic cause can scarcely be admitted, since the winter maximum of frequency is evident in regions where it would be difficult to imagine any snowfall at all. The lists and curves already given all show this winter maximum—with the exception of those for the East Indies. In a table given below, the ratio of the winter and summer earthquakes is given for each locality. Winter means the six cold months, summer the six warm months. We shall notice this more in detail later.

There is still another meteorological consideration to be discussed—namely, the effect of atmospheric pressure. Many investigators have looked to the variation of pressure, and especially to the existence of storm centres, as effective causes of earthquakes—generally with small success. But we must remember what has been already pointed out, that it is not enough to look at what is happening at *the locality where the earthquake is felt*; and that, because of the sluggishness of the earth to yield, a sudden though great variation may not have such an effect as a moderate though steadily applied stress.

Now it is a well known principle in meteorology that in cold weather the pressure of the atmosphere tends to a maximum over land, and in warm weather to a maximum over sea. The physical explanation is obvious and need not be enlarged upon here. What is of chief importance is the fact, which is evident at a glance at any meteorological chart showing the isobaric lines for different seasons. I have prepared two such charts for the whole globe, with which one can at once compare the different atmospheric conditions in July and January (See Plates III, IV). The January chart gives the winter distribution of pressure for the northern hemisphere and the summer distribution for the southern hemisphere; and the July chart gives the corresponding state of things for the northern summer and southern winter. A first glance tells us that in January the isobars are much more crowded in the northern hemisphere and less crowded in the southern hemisphere than in July. That is in *winter* time, the barometric gradients are steepest. The difference between the summer and winter gradients is of course more marked in the northern hemisphere than in the southern, where because of the comparative scarcity of land the climate approximates to a simple oceanic character. All over the northern hemisphere indeed there is a marked annual see-saw of barometric pressure. Take for example the case presented by the region of which Japan occupies the centre. In January a very high barometric pressure exists ever Siberia, and a corresponding low pressure over the North Pacific. The total range is from 778 mm. to 752 mm. with steep gradients over Japan. In July again the distribution is completely altered, the pressure having fallen over land and risen over the sea. The range is now from 766 mm. to 750 mm. and the gradients are much smaller. Here then we have an excessive gradient during January and indeed during the winter months generally. As the warm weather approaches, the distribution begins to change until it reaches the other limit of stability in July. During Spring and Autumn there are probably no steady gradients such as exist in Winter and Summer. Hence if this annual see-saw of pressure has any influence in promoting earth tremors, we should expect a marked maximum in Winter when the barometric gradient over shore countries is at its highest. If we pass to the southern hemisphere we find the same conditions holding—but not so markedly because of the approximately oceanic conditions which exist there. In tropical districts, however, we find no marked difference in the barometric gradients for Winter and Summer. A glance at the isobars in the vicinity of, say, the East Indian Archipelago shows this. Now earthquake frequencies as we have seen just present this very peculiarity. The following table will demonstrate this. The first column gives the name of the region, the second the number of Winter earthquakes, the third the number of Summer earthquakes, and the fourth the ratio of the one to the other. *Winter* and *Summer* are used as throughout this paper in their local seasonal meanings.

	Winter	Summer	Ratio
Europe (Mallet)	1115	846	1·32
Greece Isles (Schmidt)	1973	1605	1·23
Japan (Milne)	213	154	1·38
East Indies (Bergsma)	269	246	1.09
Chili (Vergara)	132	80	1·65
New Zealand	178	166	1·07
	or 321	268	1·20

Two sets of numbers are given as formerly for New Zealand. There is this further peculiarity to be noticed about the New Zealand numbers. As given in the table they are not the maximum and minimum six-month summation numbers; while the corresponding numbers in the other cases are so. Indeed, as may be inferred from the curves already discussed, the New Zealand maximum frequency falls distinctly later than midwinter—so that the ratios given in the table do not bring out to the best advantage the effect of season. It is better however, to treat all the statistics exactly alike; since it is a well known truth that by an arbitrary treatment statistics may be made to yield any wished for result.

What is specially to be noticed in the ratios just given is that in the East Indies the maximum Winter frequency is not strongly marked—indeed, its existence may well be doubted.

From the meteorological reports for New Zealand I find that the barometric gradient seems to attain a somewhat steady maximum in June and a second short-lived maximum in October. For Chili I can get no meteorological results suitable for the purpose—since it is impossible to estimate the gradient from one solitary unreduced observation.

It remains to compare these two possible causes of earthquake frequency—namely, the snow-gradient and the barometric gradient. Both are peculiarly similar as regards their annual variation—being greatest in winter. Both agree in having more pronounced variation in higher latitudes. In these respects both are in remarkable accord with the law of earthquake frequency, which attains a maximum in Winter, the more obvious the higher the latitude. Of the two, the snow gradient will certainly be the more effective, since the pressure differences involved are so much the greater.

The present investigation certainly indicates a semi-annual period; but an accumulation of observations is needed to establish it as more than conjectural. The annual period in earthquake frequency is, however, a fact well established; and the discussion in the present paper indicates two possible determining factors. Let us briefly summarise the argument.

The cause of earthquakes is probably to be referred to the earth's heterogeneity of structure or to the inequality of stress due to the irregularities of its surface. Rupturing or yielding is not determined by the amount of stress only: it depends in great measure upon how the stress is applied. For rupture to take place the stress must be different in different directions; and the difference between the greatest and the least stresses is an important datum in estimating the tendency to break. So far as can be judged, the only periodic stresses which exist of period long enough to tell upon the earth's substance are the fortnightly, monthly, semi-annual, and annual tides, the annual variation of snowfall, and the steady annual and perhaps semi-annual oscillation of barometric pressure over the earth's surface. Inasmuch as the earthquake frequency reaches its maximum in Winter wherever there is a marked Winter season, we must pass from the tidal stresses due to the sun as of little account. We seem, however, to find in the accumulations of Winter snow and in the long period oscillations of the atmospheric pressure two possible determining factors in earthquake frequency.

The steeper the gradient, the more frequent the earthquakes.

The argument in support of this view is of a cumulative nature. First we notice that earthquakes occur chiefly in littoral countries, just where the greatest stress due to inequality of surface distribution may be expected to exist, and where also the snow and barometric gradients are steepest. Secondly, earthquake frequency has a distinct annual period in those regions where these gradients have a distinct annual period—maximum corresponding to maximum. In equatorial regions there is no marked maximum for either earthquakes or gradients. Thirdly, we have evidence of a semi-annual period in earthquakes; and in reality the barometric gradient has a semi-annual period, whose Summer maximum however is small compared to the Winter maximum.

It may be urged as an objection that the difference of pressure over contiguous areas is surely far too small to have any appreciable effect at a depth of several Kilometres, where no doubt most earthquakes originate. But in dynamics we have many applications of the well known maxim that it is the last straw which breaks the camel's back. If the earth's crust has at any instant a strong tendency to yield to accumulated stress, a very slight additional stress will be enough to to hasten the rupturing; and, after all, a difference of pressure of 6 millimetres of mercury between the ends of the arc joining Tokyo and Nagasaki is no despicable stress. And yet this steady average gradient lasts for three months of each year.

The conclusion then is, that the annual periodicity in earthquake frequency, when it does exist, finds a possible explanation in the annual periodicity of two well known meteorological phenomena—namely, snow accumulations over continental areas, and barometric gradients; at least no other cause that can be named or imagined fulfils all the conditions.

MATHEMATICAL NOTE

Any periodic function of a single variable can be expanded, according to Fourier's Theorem, in terms of a series of simple harmonic functions of that variable. Let f be the function, and ϑ a particular multiple of the variable. Then we may write

$$f = a_0 + a_1 \cos(\vartheta + \alpha_1) + a_2 \cos(2\vartheta + \alpha_2) + \dots + a_n \cos(n\vartheta + \alpha_n) + \dots$$

Here f goes through all its values as the value of ϑ changes by 2π. The successive terms are obviously simple harmonic functions of periods equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of the original period.

Integrate this expression between limits $\vartheta - \psi$, $\vartheta + \psi$; and divide by 2ϕ. This is equivalent to taking *means* over the interval 2ϕ. We find

$$\frac{1}{2\phi}\int_{\vartheta-\phi}^{\vartheta+\phi} f d\vartheta = a_0 - \frac{a_1 \sin\phi}{\phi}\cos(\vartheta + \alpha_1) - \frac{a_2 \sin 2\phi}{2\phi}\cos(2\vartheta + \alpha_2) - \dots$$

$$- \frac{a_n \sin n\phi}{n\phi}\cos(n\vartheta + \alpha_n) - \dots$$

Consider now the two cases corresponding to the three-monthly and six-monthly means of earthquake frequencies, namely

$$2\phi = \pi, \quad 2\phi = \frac{\pi}{2}$$

The corresponding expressions become

$$x_1 = a_0 - \frac{2a_1}{\pi}\cos(\vartheta + \alpha_1) \qquad + \frac{2a_3}{3\pi}\cos(3\vartheta + \alpha_3) - \dots$$

$$x_2 = a_0 - \frac{4a_1 \sin\frac{\pi}{2}}{\pi}\cos(\vartheta + \alpha_1) - \frac{2a_2}{\pi}\cos(2\vartheta + \alpha_2)$$

$$- \frac{4a_3 \sin\frac{\pi}{2}}{3\pi}\cos(3\vartheta + \alpha_2) + \dots$$

The terms containing *even* multiples of ϑ vanish in the former, and those containing *even* multiples of 2ϑ vanish in the latter; so that the next term of both is the term in 5ϑ.

If we now form the expression

$$x_1 - \frac{x_2}{2\sin\frac{\pi}{2}}$$

we shall obtain a series in which the term in $\cos(\vartheta + \alpha_1)$ no longer exists, that is, a function whose most important term has a period exactly half that of the original function.

TABLE A.

Monthly, Three-Monthly (reduced) and Six Monthly Means of Earthquake Frequencies.

MONTHS		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Japan 1872-80	M.	29	36	38	26	32	33	29	21	13	32	37	41
	T.	24.7	24	23.3	22.6	21.2	21.9	19.7	14.9	15.5	19.1	25.5	25.5
	S.	35	33	32	32	30	25	26	27	29	29	31	35
Europe 306-1843	M.	228	189	172	147	126	131	148	147	147	176	148	202
	T.	147.7	138.6	119.4	104.6	94.7	95.4	100.4	101.8	110.2	110.9	123.7	136.4
	S.	181	176	165	152	145	141	146	149	161	173	183	186
New Zealand 1869-79	M.	31	27	37	23	22	31	28	36	38	22	26	23
	T.	19.1	22.6	20.5	19.1	17.7	19.1	22.6	24	22.6	20.5	17	19.1
	S.	28	27	29	28	30	30	30	30	29	29	28	28
	M.	41	41	49	47	30	50	60	59	71	46	44	47
	T.	29.4	31.1	31.8	29.7	29.7	33.9	41	46	41.7	38.2	31.8	31.1
	S.	45	42	43	47	50	53	53	56	55	51	48	45
E. I. Archipelago 1873-81	M.	47	41	51	39	31	52	36	52	36	54	36	40
	T.	30.4	32.5	31.1	30.4	31.1	30.4	33.2	29	33.2	29.7	30.4	29
	S.	42	42	44	42	44	41	44	44	42	44	44	45
Chili 1873-81	M.	13	6	7	13	18	20	26	25	23	20	24	17
	T.	8.5	6.4	6.4	9.2	12	14.8	17	17.7	16.3	15.6	14.1	12.7
	S.	13	12	13	15	18	21	22	23	23	20	17	15
Grecian Archipelago 1859-81	M.	240	382	325	280	219	253	226	348	279	467	298	261
	T.	184.5	223.4	232.6	194.4	177.5	164.7	195.1	177.5	258.1	244.6	241.8	188.1
	S.	298	285	283	281	275	268	299	312	313	316	321	329

TABLE B.

REDUCED DIFFERENCES OF ROWS T AND S OF TABLE A, WITH GENERAL AVERAGE.

Months	Japan	Europe	New Zealand		E.I. Archi.	Chili	Greek Archi.	Average
I	1.16	.75	1.05	1.08	1.06	1.08	1.41	1.08
	1.21	.79	.85	1.02	.86	.9	1.02	.95
II	1.13	.83	.58	.86	.77	1.19	.75	.87
	1.05	.98	.65	.74	.83	1.10	.62	.85
III	1.04	1.07	.88	.74	.96	1.19	.53	.93
	1.03	1.11	1.08	.85	1.08	1.02	.76	.99
IV	1.06	1.13	1.10	1.06	1.02	.94	.96	1.04
	1.14	1.10	1.13	1.2	.92	.86	1.04	1.06
V	1.11	1.13	1.33	1.31	.96	.88	1.10	1.12
	.96	1.05	1.36	1.26	1.08	.90	1.14	1.11
VI	.63	1.	1.28	1.22	.98	.92	1.17	1.03
	.47	.90	1.08	1.08	.76	1.	.91	.89
VII	.66	.90	.87	.83	.76	.88	.97	.84
	.99	.94	.79	.66	1.05	.92	1.23	.94
VIII	1.32	.96	.70	.59	1.22	.94	1.40	1.02
	1.33	.9	.79	.85	1.05	1.17	1.03	1.02
IX	1.4	.94	.81	.96	.80	1.31	.6	.97
	1.32	1.03	.88	1.05	.86	1.37	.68	1.03
X	1.12	1.17	1.	1.03	1.08	1.15	.77	1.05
	.76	1.17	1.21	1.11	1.13	1.02	.8	1.03
XI	.51	1.14	1.23	1.23	1.10	.86	.96	1.02
	.62	1.1	1.16	1.15	1.16	.90	1.16	1.04
XII	.85	1.08	1.05	1.07	1.26	.65	1.49	1.06
	1.12	.92	1.05	1.03	1.24	.86	1.56	1.11