Determination of the Elements of the Sun's Spin.

by

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Since the discovery of sun-spots by Fabricius and Galileo, the elements of the rotation of the sun, that is, the position of the sun's axis and its period of rotation, have been determined by many astronomers. These results are tabulated in Houzeau's Vade-Mecum de l'Astronomie[1], and differ greatly with the different observers. The various values of the inclination of the sun's equator to the ecliptic lie between 6°½ and 8°, and those of the longitude of the node between 60° and 80°. The two most reliable results were obtained by Mr. Carrington and Dr. Spörer respectively, more than twenty years ago. Recently Dr. Wilsing has determined the elements from the periods of rotation of a single spot. If I is the inclination of the equator of the sun to the ecliptic, and N, the longitude of the ascending node, the results obtained by these three authorities are as follows :—

Authorities.	N.	I.	Epoch.
Carrington[2].	73° 40′	7° 15′	1850
Spörer[3].	74°,523	6° 58′	1861
Wilsing[4].	75°,78	7°,16	1882,0

(1) Houzeau, Vade-Mecum de l'Astronomie. § 162 Rotation pp. 411.

(2) Carrington, Observations of the spots on the sun from 1853 to 1861 pp. 244.

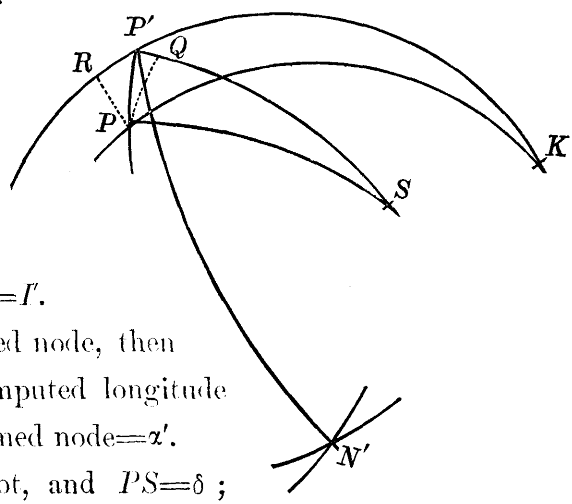
(3) Spörer, Beobachtungen der Sonnenflecken zu Anclam (1874).

(4) Wilsing, Neue Bestimmung der Rotationselemente der Sonne 1884. Astr. Nach. Bd. 107 pp. 277.

The great discrepancies are apparent at a glance. To find the corrections to the assumed values of Prof. Spörer, I took his valuable observations of sun-spots from 1861 to 1884, and selected the spots where motions were pretty regular, since it is clear that for such a determination the duration of the appearance of the spots should be long, and the proper motion regular and small. The number of groups of selected spots amounted to 933, of which the number of regular spots is great for the time of sun-spot maximum.

Method of Reduction.

The formulæ obtained by differentiating the fundamental equations giving the heliographic longitude and latitude of a sun-spot are too complex to be used for the reduction of the materials of many observations. I have adopted instead the method due to Carrington, in which, since the error arising from the assumed position of the pole of the sun is obviously felt chiefly in latitude, and not much in longitude, the variation in latitude only is considered. Consider the celestial sphere, the centre of the sun being the centre of the sphere.



Let K be the pole of the ecliptic.

Let P be the sun's true pole.

Let P' be the sun's assumed pole.

Let $PK=I$ and $P'K=I'$.

Let N' be the assumed node, then the angle $N'P'S$=the computed longitude of the spot from the assumed node=α'.

Let S be a sun-spot, and $PS=\delta$; $P'S=\delta'$.

Then considering the nearly right angled triangle PQP', we have

$$\delta' - \delta = P'Q = PP' \cos (PP'N + N'P'S)$$

$$= PP' \cos PP'N \cos \alpha' - PP' \sin PP'N \sin \alpha'$$

$$\therefore \delta' - \delta = X. \cos \alpha' - Y. \sin \alpha'$$

if we assume

$$X = PP' \cos PP'N, \; Y = PP' \sin PP'N.$$

If X and Y can be found from a series, or from many combined series of observations, then

$$PP' = \sqrt{X^2 + Y^2}, \qquad \tan PP'N = \frac{Y}{X}.$$

But

$$X = PP' \cos PP'N = PR = (N - N)' \sin I$$

$$Y = PP' \sin PP'N = P'R = I - I'$$

Hence we have the following formulæ giving the true elements

$$I = I' + Y$$

$$N = N' + P. \operatorname{cosec} I.$$

The above treatment gives $\delta' - \delta = X \cos \alpha' - Y \sin \alpha'$ as the equations of conditions to find X and Y. It is a tremendous piece of work to apply this equation to each individual set of observations of spots. Hence it is necessary to consider in what way these quantities X and Y can most advantageously be found from the whole accumulated mass of observations. Since each series of observations yields a certain number of observed values of δ' corresponding to observed values of α', we may assume the following series of equations.

$$\delta_a' - \delta = X \cos a - Y \sin a$$

$$\delta_b' - \delta = X \cos b - Y \sin b$$

$$\delta_c' - \delta = X \cos c - Y \sin c$$

....

....

By successive subtractions, we get

$$\delta_b' - \delta'_a = X (\cos b - \cos a) - Y (\sin b - \sin a)$$

$$\delta_c' - \delta'_b = X (\cos c - \cos b) - Y (\sin c - \sin b)$$

....

as the equations of condition to determine X and Y; a, b, c,......... the computed longitudes of the spots from the assumed node being known. Also by interpolating other values of δ' for previously selected values of α' at equal intervals, we simplify the calculation exceedingly. I have found the variations of latitude per 10 degrees of longitude by simply dividing the variation of latitudes by the difference of longitude. For example, for the solar spot No. 44, which appeared in March 1882 (see page 282 of Publicationen des Astrophysikalischen Observatorium zu Potsdam Nr. 17) we find

α'.	$90°-\delta'$.	Variations of $90°-\delta'$ per 10° of α'.
135	−0.32	
		− 5
150	−0.40	
		− 6
163	−0.48	
		+ 7
178	−0.36	
		− 2
192	−0.39	
		+ 1
206	−0.37	
		0
222	−0.37	
		−15
236	−0.58	
		−11
250	−0.75	
		− 4
264	−0.80	
		−10
277	−0.93	

Thus −0°.05, −0.°06,.........are considered to be the variations of latitude between longitudes 140° and 150°, 150° and 160°,......... respectively. This method of approximation will be sufficient for the majority of the observations. Having found the variations of latitude per 10° of longitude, we then tabulate them in the following form.

No.	130°	140°	150°	160°	170°	180°	190°	200°	210°	220° ....
44................		−5	−6	+7	+7	+7	−2	+1	+1	....

I shall not reproduce all the numerical values obtained in this way for the whole interval of nearly 24 years. As an example, I shall give the complete method of reduction for the year 1884. In discussing in this way Spörer's observations, I shall of course adhere to the numbering of the different groups he has given. Table I. contains the number of each Potsdam group and the variation of latitudes per 10° of longitude, the same result being represented in a somewhat altered form in Table II. on separate sheet.

Table I.

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
272	42°	+11°.4	+25		240°	− 9°.2			137°	−17°.4	−13
	58	+11.8	+29		136	− 9.3	0		152	−17.6	− 8
	72	+12.2	− 6		151	− 9.3	− 8		165	−17.7	
	88	+12.1			164	− 9.4	− 6	35*a*	99	+10.7	+ 2
16*a*	109	+12.3	+ 8		180	− 9.5	+ 4		142	+10.8	+26
	122	+12.4	+ 7		193	− 9.45	0		157	+11.2	
	179	+12.8			207	− 9.45	− 2	35*b*	97	+ 9.8	− 2
28	132	− 8.4	0		235	− 9.5	+10		140	+ 9.7	−20
	147	− 8.4	0		264	− 9.2			155	+ 9.4	−25
	175	− 8.4	−24	32*a*	109	+ 6.05	−18		167	+ 9.1	
	192	− 8.8	+14		128	+ 6.4	+21	37	66	+ 9.5	−16
	206	− 8.6	− 7		142	+ 6.7	+13		110	+ 8.8	0
	221	− 8.7			157	+ 6.9	0		124	+ 8.8	−23
(53)	84	− 9.1	+ 3		170	+ 6.9	+ 7		137	+ 8.5	− 5
	113	− 9.0	0		198	+ 7.1	−20		195	+ 8.2	−15
	141	− 9.0	− 7		213	+ 6.8			208	+ 8.0	0
	155	− 9.1	+ 7	33*a*	65	−16.6	0		224	+ 8.0	
	169	− 9.0	0		80	−16.6	−13	40	75	+12.9	+20
	198	− 9.0	−20		107	−16.95	−21		90	+13.2	0
	213	− 9.3	+ 4		123	−17.3	− 7		102	+13.2	− 4

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	159°	+13°.0			192°	+12°.4	−14		143°	+ 7°.8	+13
38	91	+13.8	+43		206	+12.2	−21		158	+ 8.0	+ 7
	105̇	+14.4	−25		234	+11.6			186	+ 8.2	+21
	117	+14.1	+ 2	47*a*	77	+ 8.2	− 6		200	+ 8.5	+ 7
	{175̇	+14.7			108	+ 8.0	− 7		242	+ 8.8	
	(174	+13.3	−15		122	+ 7.9	−13	58	93	− 6.4	+ 6
	{188	+14.5			137	+ 7.7	− 7		109	− 6.3	−14
	(188	+13.1	0		166	+ 7.5	−11		151	− 6.9	
	{203	+14.5			194	+ 7.2	−13	60	121	+12.6	+48
	(203	+13.1	+10		209	+ 7.0	0		162	+12.8	−16
	232	+14.1			223	+ 7.0			193	+12.3	− 9
41	97	−24.0	+14	47*b*	163	+ 8.0	−10		248	+11.8	
	111	−23.8	− 8		192	+ 8.3	−14	63	130	+10.7	− 7
	123	−23.9	0		206	+ 8.1			160	+10.5	− 2
	178	−23.9	0	49	88	− 9.7	− 8		218	+10.4	− 8
	192	−23.9	− 7		101	− 9.8	−20		230	+10.3	
	207	−24.0	−14		116	−10.1	0	66*a*	140	+10.6	− 8
	234	−24.4			145	−10.1	− 3		152	+10.5	− 2
64	110	−25.7	−27		174	−10.2	+ 7		193	+10.4	+15
	140	−26.5	+ 4		188	−10.1	0		209	+10.6	
	195	−26.3	−13		202	−10.1		72	138	− 7.6	+ 8
	208	−26.5	− 9	55*a*	123	+14.9	+ 6		151	− 7.5	+ 5
	250	−26.9	− 8		139	+15.0	−35		194	− 7.3	+ 7
	263	−22.0			170	+13.9	−31		209	− 7.2	− 8
44	121	+11.8	0		186	+13.4	−32		222	− 7.3	0
	134	+11.8	+20		214	+12.3	+13		237	− 7.3	
	149	+12.1	+10		229	+12.5		73	95	−16.0	+ 2
	178	+12.4	0	56	113	+ 7.9	− 3		139	−15.9	− 8

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	152°	−16°.0	− 7	75*b*	122°	+14°.8	0		179°	+ 7°.9	− 3
	196	−16.3	−19		165	+14.8	− 6		208	+ 7.8	
	212	−16.6	− 8		182	+14.7	0	107	137	+ 8.2	−20
	224	−16.7	−10		193	+14.7			147	+ 8.4	0
	253	−17.0		75	117	+15.3	+ 7		161	+ 8.4	0
98	133	−17.3	0		159	+15.6	−12		219	+ 8.4	−20
	148	−17.3	0		176	+15.4	0		234	+ 8.1	−36
	162	−17.3	−11		188	+15.4	−13		262	+ 8.0	
	176	−17.5	+ 7		(203	+15.0		87	119	− 7.6	−24
	190	−17.4	+29		(202	+15.4	+ 7		148	− 8.3	−14
	204	−17.0	+25		(217	+15.1			177	− 8.8	+ 1
	228	−16.4	−13		(216	+15.5	+ 7		269	− 8.7	
	233	−16.6	+21		(231	+15.9		112	152	− 8.9	0
	247	−16.3			(230	+15.4			165	− 8.9	+ 7
74*b*	189	−14.25	+16	78	141	−13.0	0		180	− 8.8	+ 2
	205	−14.0	0		152	−13.0	+13		208	− 8.75	
	218	−14.0			168	−12.8	0	91*a*	219	− 8.9	+28
74*e*	181	−12.1	− 7		181	−12.8	− 7		233	− 8.5	−19
	196	−12.2	−13		196	−12.9			249	− 8.8	− 7
	208	−12.35	−10	79	116	+12.6	+ 7		264	− 8.9	−36
	223	−12.5	+ 7		131	+12.7	+15		278	− 9.4	−13
	237	−12.4	0		144	+12.9	+ 7		293	− 9.6	
	252	−12.4			159	+13.0	+23	93	141	− 8.7	0
75*a*	122	+13.6	− 7		172	+13.3			155	− 8.7	− 7
	165	+13.3	+ 6	80	107	+ 7.6	− 7		170	− 8.8	+ 7
	182	+13.4	− 8		122	+ 7.5	+15		(184	− 8.6	
	194	+13.3	−10		135	+ 7.7	+10		(185	− 8.7	−14
	223	+13.0			164	+ 8.0	− 7		199	− 8.9	− 7

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	213°	− 9°.0	− 7		221°	−12°.8	0		289°	−13°.8	0
	228	− 9.1	0		231	−12.8	0		303	−13.8	+ 7
	243	− 9.1	−15		246	−12.8	− 9		317	−13.7	− 6
	256	− 9.3	+43		289	−13.2	−15		333	−13.8	+ 7
	270	− 8.7	0		302	−13.4			348	−13.7	
	289	− 8.7		101	182	−19.6	−42	118*a*	219	− 9.3	−21
96*b*	222	−11.1	− 5		196	−20.1	−35		233	− 9.6	−29
	241	−11.2	−60		213	−20.7	−50		247	−10.0	−14
	251	−11.8	−15		223	−21.2	+ 7		278	−10.2	+ 8
	264	−12.0			237	−21.1			290	−10.1	−11
86*c*	220	−11.8	0	113*a*	155	− 9.9	+13		308	−10.3	
	238	−11.8	−55		171	− 9.7	+21	119*b*	225	−14.6	−24
	249	−12.4	+29		199	− 9.1	0		237	−14.89	− 5
	263	−12.0			273	− 9.1	−10		281	−15.1	− 4
97	145	−14.1	−21		313	− 9.5			309	−15.2	+ 7
	159	−14.4	−33	116	208	−13.5	−19		323	−15.1	
	174	−14.9	−21		249	−14.3	0	125*a*	251	+ 8.8	+13
	188	−15.2	+ 7		263	−14.3	−13		266	+ 9.0	0
	202	−15.1	+14		278	−14.5	−10		282	+ 9.0	0
	216	−14.9			309	−14.8	+36		295	+ 9.0	−13
98*b*	186	−19.2	+40		320	−14.4			310	+ 8.8	+ 7
	201	−18.6	+11	138	191	−14.6	+ 7		324	+ 8.9	
	220	−18.4	−20		205	−14.5	+19	125*b*	261	+ 7.5	− 7
	230	−18.6	−43		221	−14.2	0		289	+ 7.3	−14
	244	−19.2			233	−14.2	+14		303	+ 7.1	
99*b*	175	−12.4	0		247	−14.0	+14	127	177	+12.6	−17
	189	−12.4	0		261	−13.8	0		194	+12.3	+18
	203	−12.4	−22		275	−13.8	0		205	+12.5	+14

Table I.—*Continued.*

1884.

Spot	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	219°	+12°.7	− 7		319°	− 6°.3	0		231°	− 2°.0	−14
	234	+12.6	+ 7		333	− 6.3	0		245	− 2.2	−20
	249	+12.5			347	− 6.3	0		260	− 2.5	− 5
130*a*	232	+ 7.7	−14		351	− 6.3	−18		279	− 2.6	− 9
	246	+ 7.5	− 7		2	− 6.5			290	− 2.7	+ 8
	260	+ 7.4		171	213	− 7.3	0		303	− 2.6	+ 7
130*b*	250	+ 6.4	0		227	− 7.3	− 7		318	− 2.5	
	260	+ 6.4	− 7		241	− 7.4	0	140	188	− 9.2	−23
	275	+ 6.3			255	− 7.4	+ 7		201	− 9.5	+ 7
134	169	− 7.2	−17		269	− 7.3	−10		215	− 9.4	+ 7
	184	− 7.45	− 3		298	− 7.6	0		229	− 9.3	−11
	199	− 7.5	0		312	− 7.6	+ 7		243	− 9.5	0
	212	− 7.5	0		326	− 7.5	+29		257	− 9.5	− 7
	227	− 7.5	− 7		343	− 7.0	+18		271	− 9.6	0
	241	− 7.6	− 7		354	− 6.8	+ 7		285	− 9.6	+ 7
	255	− 7.7	− 4		8	− 6.7	+21		300	− 9.5	−14
	282	− 7.8	+ 7		22	− 6.4			314	− 9.7	+13
	296	− 7.7	−19	137	203	− 1.6	− 7		329	− 9.5	+15
	312	− 8.0	−15		217	− 1.7	0		342	− 9.3	
	325	− 8.2	−14		234	− 1.7	−23	159	196	− 8.0	− 7
	339	− 8.4	− 8		247	− 2.0	−21		210	− 8.1	− 5
	342	− 8.5			261	− 2.3	−29		229	− 8.2	0
152	192	− 7.5	+20		275	− 2.7	−20		240	− 8.2	0
	207	− 7.2	+13		290	− 3.0	−21		254	− 8.2	0
	222	− 7.0	+ 5		304	− 3.3	−14		269	− 8.2	+14
	292	− 6.7	+17		318	− 3.5	+13		343	− 8.0	− 9
	304	− 6.3	0		333	− 3.3			354	− 8.1	−14
	304	− 6.3	0	156	219	− 2.0	0		8	− 8.3	

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
141	179°	+18°.4	−23		260°	−15°.6	+ 7	147	186°	−15.6	+14
	192	+18.1	0		274	−15.5	−14		200	−15.4	−24
	207	+18.1	0		288	−15.7	+20		217	−15.8	− 4
	220	+18.1	0		303	−15.4	−17		242	−15.9	+ 7
	234	+18.1	−13		320	−15.7	− 8		256	−15.8	0
	249	+17.9	−14		345	−15.9			270	−15.8	−12
	263	+17.7	−14	143*a*	186	−11.6	−20		287	−16.0	+13
	277	+17.5	−38		201	−11.9	−14		302	−15.8	
	290	+17.0	0		215	−12.0	0	146	192	+ 8.3	+14
	305	+17.0	+ 6		228	−12.0	+ 6		206	+ 8.5	−24
	322	+17.1			244	−11.9	0		223	+ 8.1	+ 4
142*a*	241	+14.8	− 6		258	−11.9	+ 7		249	+ 8.2	−14
	257	+14.1	+31		273	−11.8	−15		263	+ 8.0	− 7
	273	+14.6	0		286	−12.0	− 7		277	+ 7.9	0
	286	+14.6	−13		301	−12.1	−12		293	+ 7.9	−13
	301	+14.4	0		318	−12.3	0		308	+ 7.7	
	318	+14.4	−14		344	−12.3		166	215	+ 9.3	−20
	344	+14.0		144*a*	179	−24.1	−40		230	+ 9.0	−25
142*b*	237	+15.7	0		194	−24.7	+31		246	+ 8.6	−17
	250	+15.7	+27		207	−24.3	0		258	+ 8.4	
	265	+16.1	+ 8		221	−24.3	−25		277	+ 8.2	0
	278	+16.2	0		237	−24.7			286	+ 8.2	−13
	293	+16.2		144*b*	184	−24.3	+14		301	+ 8.0	0
143	188	−15.2	−20		198	−24.1	+ 8		316	+ 8.0	+ 8
	203	−15.5	+ 7		211	−24.0	−20		329	+ 8.1	+ 7
	217	−15.4	+ 7		226	−24.3	+ 8		343	+ 8.2	
	231	−15.3	−20		239	−24.2	+ 7	148*a*	189	−15.7	− 6
	246	−15.6	0		254	−24.1			207	−15.8	+ 4

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	233°	−15°.7	0		261°	+ 9°.7	± 0		344°	+13°.0	
	247	−15.7	− 7		273	+ 9.7	−13	173	219	+ 8.1	− 8
	262	−15.8	0		289	+ 9.5	−23		231	+ 8.0	+13
	278	−15.8	0		302	+ 9.2			246	+ 8.2	− 8
	293	−15.8		165	213	−12.3	−19		259	+ 8.1	0
155*a*	297	+ 9.7	− 8		229	−12.6	−13		288	+ 8.1	0
	310	+ 9.6	− 6		245	−12.8	+17		302	+ 8.1	
	326	+ 9.5			257	−12.6	−11	174*a*	339	+10.8	+13
155*b*	282	+11.3	0		276	−12.8	−33		354	+11.0	+50
	293	+11.3	−19		285	−13.1	+ 7		8	+11.7	− 7
	309	+11.0			300	−13.0	−13		23	+11.6	
157	216	+ 6.5	+ 9		315	−13.2	− 8	174	244	+10.4	−14
	227	+ 6.6	−13		328	−13.3	− 6		258	+10.2	+19
	242	+ 6.4	−20		344	−13.4	−17		274	+10.5	0
	257	+ 6.1	0		356	−13.6			283	+10.5	
	275	+ 6.1	−27	165*b*	222	−13.5	−12	176	235	+ 8.0	+17
	286	+ 5.8	−14		239	−13.7	−42		247	+ 8.2	+ 7
	300	+ 5.6	+13		251	−14.2	0		262	+ 8.3	− 7
	315	+ 5.8			270	−14.2	−22		277	+ 8.2	−13
160*b*	201	+14.3	+ 7		279	−14.4			292	+ 8.0	0
	215	+14.4	0	168*a*	227	+12.1	−14		306	+ 8.0	+53
	231	+14.4			241	+11.9	+21		321	+ 8.8	
164	333	+ 9.2	+28		260	+12.3	0	177	235	− 5.8	−20
	347	+ 9.6	+43		270	+12.3	−18		250	− 6.1	−11
	1	+10.2	+13		287	+12.0	+53		268	− 6.3	−18
	16	+10.4			302	+12.8	+ 7		279	− 6.5	−14
180	232	+10.2	− 7		316	+12.9	− 7		293	− 6.7	+13
	247	+10.1	−29		331	+12.8	+15		308	− 6.5	−15

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	321°	− 6°.7	+12		324°	+ 5°.2	− 8		343°	+ 4°.8	+ 7
	338	− 6.5	−33		336	+ 5·1	−14		357	+ 4.9	−14
	350	− 6.9	−13		350	+ 4.9	−13		11	+ 4.7	
	5	− 7.1	−13		6	+ 5.1		194*d*	313	+ 6.7	+29
	20	− 7.3	+21	188*a*	284	+ 3.2	− 6		328	+ 7.0	−23
	34	− 7.0			316	+ 3.0	−11		341	+ 6.7	+31
178*a*	300	− 2.7	−14		334	+ 2.8	−25		354	+ 7.1	−36
	314	− 2.9	+12		346	+ 2.5	−29		8	+ 6.6	+39
	331	− 2.7	−14		360	+ 2.1	0		26	+ 7.3	+ 4
	345	− 2.9	−20		16	+ 2.1	0		52	+ 7.4	
	360	− 3.2	−13		45	+ 2.1		196	253	− 6.4	− 7
	16	− 3.4	+14	188*b*	277	+ 3.9	− 7		268	− 6.5	− 7
	30	− 3.2			305	+ 3.7	−11		282	− 6.6	−18
199	264	− 3.0	− 8		323	+ 3.5	0		299	− 6,9	−15
	290	− 3.2	− 6		335	+ 3.5	−29		312	− 7.1	
	307	− 3.1	0		349	+ 3.1	− 6	197	354	− 5.9	0
	317	− 3.1	+ 7		5	+ 3.0	+11		270	− 5.9	+ 8
	331	− 3.0	+ 7		33	+ 3.3			282	− 5.8	−14
	346	− 3.1	0	194*a*	317	+ 4.8	+33		296	− 6.0	+14
	3	− 3.1	− 9		335	+ 5.4	−29		310	− 5.8	− 5
	14	− 3.2	+ 7		349	+ 5.0	+46		329	− 5.9	+ 5
	29	− 3.1	+ 7		2	+ 5.6			351	− 5.8	0
	43	− 3.0	+ 7	194*a*′	18	+ 5.2	− 6		12	− 5.8	+10
	57	− 2.9			36	+ 5.1	+12		22	− 5.7	− 7
186*a*	355	−11.8	+ 7		62	+ 5.4			36	− 5.8	+ 7
	9	−11.7	+18	194*c*	299	+ 5.2	− 7		51	− 5.7	0
	26	−11.4			314	+ 5.1	0		66	− 5.7	
188*c*	307	+ 4.9	+18		330	+ 5.1	−23	203	321	+ 3.5	+13

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	336°	+ 3°.7	+14		52°	+17°.3	+ 6	229	213°	− 3°.9	+19
	350	+ 3.9	+ 8		68	+17.4	+58		329	− 3.6	−11
	3	+ 4.0			80	+18.1			357	− 3.9	
207*b*	11	+14.8	−21	217	336	−22.9	−12	223	290	+ 4.7	+ 6
	25	+14.5	+13		2	−23.2	− 8		307	+ 4.8	−18
	40	+14.7	0		15	−23.3	−15		318	+ 4.6	−15
	56	+14.7			28	−23.5	+18		331	+ 4.4	0
207*a*	333	+15.0	+ 5		45	−23.2	+30		347	+ 4.4	−14
	16	+15.2	−21		55	−22.9	− 7		1	+ 4.2	− 7
	30	+14.9	+20		69	−23.0	+21		16	+ 4.3	−14
	45	+15.2	−20		83	−22.7			58	+ 3.7	− 6
	60	+14.9	+ 7	219*a*	352	+13.4	+25		75	+ 3.6	
	74	+15.0			12	+13.9	− 8	228*a*	329	− 8.0	+25
	285	+14.6	+ 7		24	+13.8	−44		349	− 7.5	+32
	299	+14.7	−13		42	+13.0	−17		17	− 6.6	0
	314	+14.5	− 4		54	+12.8	+ 7		34	− 6.6	+12
	359	+14.3	0		68	+12.9	+13		51	− 6.4	−12
	13	+14.3	+19		83	+13.1	+14		63	− 6.9	−15
	39	+14.8	+19		97	+13.3			90	− 7.3	− 7
	55	+15.1	+13	222*a*	289	+ 5.1	0		105	− 7.4	
	71	+15.3	0		316	+ 5.1	0	229	313	− 3.9	+19
	83	+15.3			332	+ 5.1	− 4		329	− 3.6	−11
227*b*	282	+16.2	+ 7		0	+ 5.0	0		357	− 3.9	
	296	+16.3	0		11	+ 5.0	−14	230	307	−12.8	−13
	311	+16.3	+ 5		25	+ 4.8	+ 7		322	−13.0	−29
	353	+16.5	+ 6		40	+ 4.9	− 7		339	−13.5	0
	10	+16.6	+15		54	+ 4.8	0		351	−13.5	−11
	36	+17.0	+19		68	+ 4.8			18	−13.8	0

Table I.—*Continued.*

1884.

Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.	Spot.	α′	Lat.	Diff.
	32°	−13.8°	+7	262*a*	33°	+7.6°	0		94°	+3.3°	+7
	46	−13.7	−13		62	+7.6	−17		108	+3.4	−7
	61	−13.9	−9		92	+7.1	−2		123	+3.3	+7
	105	−14.3			135	+7.0			138	+3.4	+30
231*a*	340	−13.8	+7	262*b*	60	+8.1	+6		148	+3.7	
	10	−13.6	+13		91	+8.3	+2	269	19	−6.3	−3
	26	−13.4	0		133	+8.4			48	−6.4	+7
	41	−13.4	−7	265*a*	92	−15.1	+7		62	−6.3	0
	56	−13.5	+14		106	−15.0	+8		77	−6.3	0
	100	−12.9			119	−14.9	+6		91	−6.3	
238α	4	+14.8	0		136	−14.8	0	270	13	+4.4	−13
	18	+14.8	+6		149	−14.8			28	+4.2	+7
	49	+15.0	−4	266	43	+15.1	−18		42	+4.3	0
	77	+14.9	0		60	+14.8	−47		54	+4.3	+16
	88	+14.9			75	+14.1	+20		97	+5.0	
238β	17	+14.3	−16		90	+14.4	−21	274	24	−12.5	+13
	48	+13.8	+18		119	+13.8	+7		55	−12.1	+6
	76	+14.3	0		134	+13.9	+33		87	−11.9	+6
	87	+14.3			149	+14.4			118	−11.7	+12
253*a*	1	+9.6	+13	266*b*	41	+13.4	−53		135	−11.5	
	17	+9.8	+8		56	+12.6	−50	263	74	−6.8	−7
	30	+9.9	+14		70	+11.9	+7		117	−7.1	0
	44	+10.1	+14		85	+12.0	+10		160	−7.1	
	58	+10.3	+36		114	+12.3	+42	277	56	−7.0	+5
	72	+10.8	+13		128	+12.9	−13		113	−6.7	+8
	102	+11.2	+19		143	+12.7	−6		184	−6.1	
	118	+11.5	+8		159	+12.6					
	130	+11.6		268	65	+3.4	−3				

Table II. gives, as final results, the total sums of the variations of latitude per 10 degrees of longitude and the corresponding total number in the year 1884. In like manner, similar sums were obtained for each year from 1861 to 1884. The results of the whole are tabulated in Table III.

In the last three rows of Table III., the results of 24 years are given, the last row giving the mean value of the variation of latitudes per 10 degrees of longitude. Supposing the weight of each column to be proportional to the total number in the corresponding column, multiply each of these mean values by its weight. Sum the whole, and divide by the sum of the weights. There results a mean excess of —0°.00217, which would imply that on the whole there is an average tendency toward the equator of nearly 8″ in the time during which the sun rotates through 10°. But considering the approximate character of the method of reduction and the probable effects of the errors of observations, we are, I think, justified in altogether neglecting this result.

To annihilate this equal departure on either side of zero, I have deducted this quantity —0°.002 from the mean values in each column.

So that we have finally the following data to find *X* and *Y*.

	0°–10°	10°–20°	20°–30°	30°–40°	40°–50°	50°–60°	60°–70°	70°–80°	80°–90°
Differences.	+ 1.4	+ 2.4	+ 3.7	+ 4.0	+ 4.5	+ 3.5	+ 3.1	+ 4.6	+ 2.8
Weight.	272	261	245	238	236	223	208	201	190

	90°–100°	100°–110°	110°–120°	120°–130°	130°–140°	140°–150°	150°–160°	160°–170°	170°–180°
Differences.	+ 2.2	+ 2.1	− 0.6	− 0.2	+ 3.1	+ 3.7	+ 0.5	− 1.4	+ 0.9
Weight.	182	180	176	187	195	197	203	223	231

	180°	190°	200°	210°	220°	230°	240°	250°	260°	270°
Differences.	− 1.2	− 0.3	− 1.0	− 1.1	− 0.2	− 1.4	− 2.5	− 3.2	− 2.5	
Weight.	256	276	286	294	308	316	318	317	304	

	270°	280°	290°	300°	310°	320°	330°	340°	350°	360°
Differences.	− 6.2	− 4.8	− 3.2	− 2.1	− 0.9	− 1.6	− 1.2	+ 1.3	+ 2.0	
Weight.	305	291	292	286	284	282	278	284	281	

From these we get the following 36 equations of condition,

$$(-015\ X-174\ Y+\ 14=0)\times 2.72 \qquad (+015\ X+174\ Y-12=0)\times 2.56$$
$$(-045\ X-168\ Y+\ 24=0)\times 2.61 \qquad (+045\ X+168\ Y-\ 3=0)\times 2.76$$
$$(-074\ X-158\ Y+\ 37=0)\times 2.45 \qquad (+074\ X+158\ Y-10=0)\times 2.86$$
$$(-100\ X-143\ Y+\ 40=0)\times 2.38 \qquad (+100\ X+143\ Y-11=0)\times 2.94$$
$$(-123\ X-123\ Y+\ 45=0)\times 2.36 \qquad (+123\ X+123\ Y-\ 2=0)\times 3.08$$
$$(-143\ X-100\ Y+\ 35=0)\times 2.23 \qquad (+143\ X+100\ Y-14=0)\times 3.16$$
$$(-158\ X-074\ Y+\ 31=0)\times 2.08 \qquad (+158\ X+074\ Y-25=0)\times 3.18$$
$$(-168\ X-045\ Y+\ 46=0)\times 2.01 \qquad (+168\ X+045\ Y-32=0)\times 3.17$$
$$(-174\ X-015\ Y+\ 28=0)\times 1.90 \qquad (+174\ X+015\ Y-25=0)\times 3.04$$
$$(-174\ X+015\ Y+\ 22=0)\times 1.82 \qquad (+174\ X-015\ Y-62=0)\times 3.05$$
$$(-168\ X+045\ Y+\ 21=0)\times 1.80 \qquad (+168\ X+045\ Y-48=0)\times 2.91$$
$$(-158\ X+074\ Y-\ \ 6=0)\times 1.76 \qquad (+158\ X+074\ Y-32=0)\times 2.92$$
$$(-143\ X+100\ Y-\ \ 2=0)\times 1.87 \qquad (+143\ X+100\ Y-21=0)\times 2.86$$
$$(-123\ X+123\ Y+\ 31=0)\times 1.95 \qquad (+123\ X+123\ Y-\ 9=0)\times 2.84$$
$$(-100\ X+143\ Y+\ 37=0)\times 1.97 \qquad (+100\ X+143\ Y-16=0)\times 2.82$$
$$(-074\ X+158\ Y+\ \ 5=0)\times 2.03 \qquad (+074\ X+158\ Y-12=0)\times 2.78$$
$$(-045\ X+168\ Y-\ 14=0)\times 2.23 \qquad (+045\ X+168\ Y+13=0)\times 2.84$$
$$(-015\ X+174\ Y+\ \ 9=0)\times 2.31 \qquad (+015\ X+174\ Y+20=0)\times 3.81$$

The solution of these 36 equations by means of the least squares gives

$$X= +0^\circ.18051 \quad Y= +0^\circ.03842$$

Hence

$$\delta'-\delta= +0^\circ.185\ cos\ (\alpha+12^\circ.0)$$

Table II.

1884.

Long. / Nos.	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°	190°	200°	210°	220°	230°	240°	250°	260°	270°	280°	290°	300°	310°	320°	330°	340°	350°	360°
272					+25	+25	+29	−6	−6																												
16a											+8	+8	+7	+7	+7	+7	+7	+7																			
28													0	0	0	0	0	−24	−24	+14	+14	−7															
53									+3	+3	+3	0	0	0	−7	−7	+7	0	0	0	+4																
,,															0	−8	−6	−6	+4	0	0	−2	−2	−2	+10	+10											
32a												−18	−18	+21	+13	+13	0	+7	+7	+7	−20																
33a							0	0	−13	−13	−13	−21	−7	−7	−13	−8																					
35a											+2	+2	+2	+2	+26	+26																					
35b											−2	−2	−2	−2	−20	−20	−25																				
37								−16	−16	−16	−16	0	−23	−23	−5	−5	−5	−5	−5	−5	−15	0															
40									+20	0	−4	−4	−4	−4	−4	−4																					
38										+43	−25	−25	+2	+2	+2	+2	+2	−15	−15	0	+10	+10	+10														
41											+14	−8	0	0	0	0	0	0	0	−7	−7	−14	−14														
64												−27	−27	−27	+4	+4	+4	+4	+4	+4	−13	−9	−9	−9	−9	−8											
44													0	+20	+20	+10	+10	+10	0	−14	−14	−21	−21														
47a									−6	−6	−6	−7	−13	−13	−7	−7	−11	−11	−11	−13	−13	0															
47b																	−10	−10	−10	−14	−14																
49										−8	−20	−20	0	0	0	−3	−3	+7	+7	0																	
55a													+6	+6	−35	−35	−35	−31	−31	−32	−32	+13	+13														
56												−3	−3	−3	+13	+13	+7	+7	+7	+21	+7	+7	+7	+7													
58										+6	+6	−14	−14	−14	−14																						
60													+48	+48	+48	+48	−16	−16	−16	−9	−9	−9	−9	−9	−9												
63														−7	−7	−7	−2	−2	−2	−2	−2	−2	−8														
66a															−8	−2	−2	−2	−2	−2	+15																
72															+8	+5	+5	+5	[illegible]	+7	+7	−8	0	0													
73											+2	+2	+2	+2	−8	−7	−7	−7	−7	−7	−19	−8	−10	−10	−10												
98														0	0	0	−14	−14	+7	+29	+25	+25	+24	+21													
74b																				+16	+16	0															
74e																			−7	−7	−13	−10	+7	+7	0												
75a													−7	−7	−7	−7	−7	+6	−8	−10	−10	−10															
75b													±0	0	0	0	−6	−6	0																		
75													+7	+7	+7	+7	−12	−12	0	−13	+7	+7	−7														
78															0	+13	+13	0	−7	−7																	
79													+7	+15	+7	+23																					
80												−7	+15	+10	+10	+10	−7	−7	[illegible]	−3	−3																
107															−20	±0	0	0	0	0	0	0	−20	−36	−36	−36											
87													−24	−24	−24	−14	−14	−14	+1	+1	+1	+1	+1	+1	+1	+1	+1										
112																0	0	+7	+2	+2	+2		+28	−19	−19	−7	−36	−36	−13								91a
93															0	0	−7	+7	−14	−14	−7	−7	−7	0	−15	−15	+43	0	0								
96b																							−5	−5	−60	−15											
90c															−21	−21	−33	−21	−21	+7	+14	+14	0	0	−55	+29											97
98b																				+40	+11	+11	−20	−43													
99b																			0	0	−22	−22	0	0	0	−9	−9	−9	−9	−15							
101																			−42	−42	−35	−50	+7	+7													
113a																+13	+13	+21	+21	+21	0	0	0	0	0	0	0	−10	−10	−10	−10						
116																						−19	−19	−19	−19	0	−13	−13	−10	−10	−10	+36					
138																				+7	+19	+19	0	+14	+14	+14	0	0	0	0	+7	+7	−6	+7	+7		
118a																							−21	−29	−29	−14	−14	−14	+8	−11	−11						
119b																								−24	−5	−5	−5	−5	−4	−4	−4	+7					
125a																										+13	+13	0	0	0	+7	+7					
125b																											−7	−7	−7	−14							
127																			−17	+18	+14	+14	0	−18	0												
130a																								−14	−14	−7											
130b																										0	−7	−7									
134																		−17	−3	−3	0	0	0	−7	−4	−4	−4	−4	+7	+7	−19	−15	−14	−14			
152																				+20	+20	+13	+5	+5	+5	+5	+5	+5	+5	+17	0	0	0	0	0	−18	
171	+7	+21																					0	0	−7	0	0	+7	−10	−10	−10	0	+7	+7	+29	+18	+7	
137																					−7	−7	0	−21	−21	−21	−29	−29	−20	−21	−14	−14	+13				
156																							0	−14	−14	−20	−5	−5	−9	+8	+7	+7					
140																				−23	+7	+7	+7	−14	0	0	−7	0	0	+7	−14	+13	+13	+15			
159	−14																				−7	−5	−5	0	0	0	0	+14	+14	+14	+14	+14	+14	+14	−9	−14	
141																			−23	0	0	0	0	−13	−13	−14	−14	−14	−38	0	0	+6					
142a																									−6	−6	+31	0	0	−13	0	0	−14	−14			
142b																									0	+27	+8	+8	0								
143																				−20	+7	+7	+7	−20	−20	0	+7	−14	−14	+20	−17	−17	−8	−8	−8		
143a																				−20	−14	0	0	+6	0	0	+7	−15	−15	−7	−12	−12	0	0			
144a																			−40	+31	+31	0	0	−25													
144b																			+14	+14	+8	−20	−20	+8	+7												
147																				+14	−24	−24	−4	−4	+7	+7	0	−12	−12	+13							
146																				+14	+14	−24	+4	+4	+4	−14	−7	−7	0	−13	−13						
166																						−20	−20	−25	−25	−17	±0	0	0	−13	−0	0	+8	+7			
148a																				−6	−6	+4	+4	0	0	−7	0	0	0								
155a																															−8	−6	−6				
																													0	−19	−19						
159																							+9	−13	−20	−20	±0	0	−27	−14	+13	+13					
160b	+13	+13																			+7	+7	0											+28	+28	+43	164
180																								−7	−7	−29	0	−13	−13	−23							
165																						−19	−19	−13	−13	+17	−11	−11	−33	+7	−13	−8	−8	−6	−17	−17	
165b																							−12	−12	−42	0	0	−22									
168a																								−14	+21	+21	0	−18	−18	+53	+7	−7	+15				
173																							−8	+13	+13	−8	0	0	0	0							
174a	+50	−7																							−14	−14	+19	0	0						+15	+50	174
176																									+17	+7	−7	−7	−13	0	+53	+53					
177	−13	+21	+21																						−20	−11	−11	−18	−14	+13	+13	−15	+12	+12	−33	−13	
178a	−13	−13	+14																												−14	+12	+12	−14	−20	−20	
199	−9	+7	+7	+7	+7	+7																					−8	−8	−8	+6	+6	0	+7	+7	+7	0	
186a	+7	+18	+18																																	+7	
	−14																															+18	−8	−8	−14	−13	188c
188a	0	0	0	0	0																								−6	−6	−6	−6	−11	−25	−25	−29	
188b	−6	+11	+11																										−7	−7	−7	−11	0	0	−29	−6	
194a			−6	−6	+12	+12																											+33	+33	−29	+46	
184	−14																														−7	0	0	−23	+7	+7	
194d	−36	+39	+39	+4	+4																					−7	−7	−7	−18	−18	−15	+20	+20	−23	+31	−36	
196	0	+10	−7	−7	+7	0	0																					+8	−14	−14	+14	−5	−5	+5	+5	0	197
		−21	−21	+13	0	0																											+13	+13	+8	+8	203c
207	+5	+5	−21	+20	+20	−20	+7																												+5	+5	
	0	+19	+19	+19	+19	+19	+13	0																						+7	−13	−4	−4	−4	−4	−4	
227b	+6	+15	+15	+15	+19	+6	+6	+58																					+7	+7	0	+5	+5	+5	+5	+6	
217	−8	−8	−15	+18	+18	+30	−7	+21																											−12	−12	
219a	+25	−8	−44	−14	−17	+7	+7	+13	+14	+14																										+25	
222a	0	−14	+7	+7	−7	0	0																							0	0	0	0	−4	−4	−4	
229																																+19	+19	−11	−11		
	−7	−7	−14	−14	−14	−14	−6	−6																						+6	+6			0	−14	−14	223
228a	+32	+32	0	+12	+12	−42	−15	−15	−15	−7																						−18	−15	+25	+25	+32	
																																+19	+19	−11	−11	−11	229
	−11	−11	0	+7	−13	−9	−9	−9	−9																							−13	−29	−29	0	−11	230
231	+7	+13	+13	0	−7	−7	+14	+14	+14	+14																									+7	+7	
238a	0	0	+6	+6	+6	−4	−4	−4	0																												
,, β			−16	−16	−16	+18	+18	+18	0																												
253a	+13	+13	+8	+14	+14	+14	+36	+13	+13	+13	+19	+19	+8																								
262a				0	0	0	−17	−17	−17	−2	−2	−2	−2	−2																							
,, b							+6	+6	+6	+2	+2	+2	+2																								
265a										+7	+7	+8	+6	+6	0																						
266					−18	−18	−47	−47	+20	−21	−21	−20	+7	+33	+33																						
266b					−53	−53	−50	+7	+7	+10	+10	+42	+42	−13	−6	−6																					
268							−3	−3	−3	+7	+7	−7	+7	+7	+30																						
269			−3	−3	−3	+7	0	0	0																												
270		−13	−13	−13	+7	0	+16	+16	+16	+16	+16																										
274			+13	+13	+13	+13	+6	+6	+6	+6	+6	+6	+12	+12																							
263								−7	−7	−7	−7	−7	0	0	0	0																					
277							+5	+5	+5	+5	+5	+8	+8	+8	+8	+8	+8	+8																			
Σ	+21	+135	+31	+52	+35	−9	+5	+47	+32	+66	−9	−95	+44	+60	+30	+41	−146	−124	−229	+14	−46	−158	−127	−353	−400	−157	−60	−280	−301	−57	−79	+112	+82	+6	−72	+21	
N	26	25	26	23	26	24	25	24	23	22	24	27	36	36	+41	39	34	34	39	48	49	47	48	45	45	43	39	39	41	39	37	36	32	31	30	+12	

Table III.

	0°–10°	10°–20°	20°–30°	30°–40°	40°–50°	50°–60°	60°–70°	70°–80°	80°–90°	90°–100°	100°–110°	110°–120°	120°–130°	130°–140°	140°–150°	150°–160°	160°–170°	170°–180°
1861 Diff.	+ 56	+ 6	+ 63	+ 14	− 7	− 21	+ 10	− 75	− 6	− 20	− 20	− 13	− 51	− 47	+ 63	− 87	−127	− 47
1861 Nos.	10	7	7	9	8	8	8	7	5	3	2	2	3	3	3	3	7	5
1862 Diff.	+ 52	+ 44	+ 49	+ 25	+ 48	+ 86	+ 97	+101	+ 80	+ 88	− 89	+ 22	− 10	− 6	+ 29	− 4	+ 49	+ 77
1862 Nos.	9	9	10	11	10	10	7	5	4	4	4	3	3	4	3	5	6	8
63,64,65 Diff.	−104	−100	− 6	+ 21	+ 1	− 2	+ 8	+ 87	+ 81	− 3	− 20	− 46	− 41	+ 47	+ 45	+ 33	+ 9	+ 27
63,64,65 Nos.	17	16	13	11	12	12	15	16	15	15	17	15	13	13	13	13	16	19
66,67,68 Diff.	+ 55	+ 69	+ 64	+ 59	+ 85	+ 74	+ 73	− 6	− 44	− 27	+ 17	+ 8	− 20	+ 33	+ 36	+ 26	− 18	+147
66,67,68 Nos.	16	12	12	12	12	12	12	13	10	10	9	9	9	10	8	7	10	12
Sums.	+ 59	+ 19	+170	+119	+ 127	+137	+188	+107	+111	+ 38	+ 66	− 29	−122	+ 27	+173	− 32	− 87	+204
Nos.	52	44	42	43	42	42	42	41	34	32	32	29	28	30	27	28	39	44
1869 Diff.	+ 28	+ 93	+103	+111	+ 112	+ 90	+ 53	+ 40	+ 43	+ 43	+ 45	−100	− 79	− 25	− 20	− 4	+ 15	+ 43
1869 Nos.	18	17	16	15	12	9	8	6	7	8	9	9	11	8	9	9	9	9
1870 Diff.	+ 50	+ 29	+ 47	+ 45	+ 64	+ 47	+ 55	+ 65	+ 13	+ 27	+ 54	− 44	− 82	− 60	− 14	− 33	− 74	+ 1
1870 Nos.	18	17	16	15	11	9	8	7	7	7	8	10	8	9	10	7	9	10
1871 Diff.	+ 6	+ 37	+ 90	+ 67	+ 71	+ 94	+115	+114	+ 78	+ 84	+ 29	+ 17	+ 55	+ 53	+ 25	− 84	− 95	− 6
1871 Nos.	13	14	13	14	14	17	17	16	15	14	15	16	18	22	20	25	23	22
1872 Diff.	− 62	+ 25	+ 68	+ 24	+ 2	+ 12	+ 20	+ 91	− 51	− 43	− 38	− 28	+ 5	+ 12	+ 7	− 22	− 11	− 25
1872 Nos.	12	11	10	11	12	11	11	13	13	11	9	7	7	6	5	4	3	5
1873 Diff.	− 11	− 20	+ 14	+ 57	+ 78	+ 43	+ 54	+ 72	+ 48	+ 50	+ 61	+ 13	+ 17	+ 19	+ 8	− 29	− 5	− 22
1873 Nos.	12	14	12	11	11	10	10	9	7	5	6	5	6	8	9	11	15	12
74,75 Diff.	+ 75	+ 64	+ 65	+ 26	+ 1	+ 14	+ 18	+ 46	+ 22	+ 43	+ 29	+ 14	− 67	− 11	+ 70	− 23	− 95	− 14
74,75 Nos.	9	8	10	9	9	9	6	7	7	8	9	10	10	10	9	14	18	19
76,77,78,79 Diff.	+ 60	+116	+ 94	+ 70	+ 67	+ 52	+ 29	+ 29	+ 35	− 26	− 5	+ 19	+ 38	+ 20	+ 46	− 7	− 19	− 12
76,77,78,79 Nos.	8	9	9	8	6	5	5	5	5	6	5	5	5	5	4	3	3	3
Sums.	+146	+344	+481	+400	+ 395	+352	+344	+457	+188	+178	+175	−109	−113	+ 8	+122	−202	−284	− 35
Nos.	90	90	86	83	75	70	65	63	61	59	61	62	65	68	69	73	80	80
1880 Diff.	− 13	− 47	+ 61	+ 19	+ 31	− 6	− 26	+119	+ 52	+ 28	+ 63	+ 33	+ 39	+120	+ 87	+ 55	+ 71	+112
1880 Nos.	14	15	13	13	16	15	13	13	11	9	8	7	7	9	9	10	10	11
1881 Diff.	+111	− 4	− 65	+121	+ 180	+ 93	+ 90	+100	+ 78	− 39	−165	−113	− 54	+ 99	+126	+189	+ 95	− 64
1881 Nos.	26	24	21	21	21	17	16	14	15	15	14	13	14	14	13	13	14	13
1882 Diff.	− 44	+ 22	+ 48	+ 87	+ 92	+ 52	+ 19	+ 22	+ 9	+ 59	+ 68	+118	+ 11	+ 86	+ 36	+ 39	+ 39	+ 24
1882 Nos.	23	23	19	17	19	18	14	13	15	16	14	13	10	8	15	18	21	24
1883 Diff.	+ 47	+114	+125	+ 94	+ 160	+114	− 25	+ 33	+ 39	+ 41	+144	+ 47	+129	+155	+114	− 34	− 36	+ 46
1883 Nos.	41	40	38	38	37	37	33	33	31	29	27	25	27	30	23	22	25	25
1884 Diff.	+ 21	+135	+ 31	+ 52	+ 35	− 9	+ 5	+ 47	+ 32	+ 66	− 9	− 95	+ 44	+ 60	+ 30	+ 41	−146	−124
1884 Nos.	26	25	26	23	26	24	25	24	23	22	24	27	36	36	41	39	34	34
Sums.	+122	+220	+200	+373	+ 498	+244	+ 63	+221	+210	+155	+101	− 10	+169	+520	+393	+290	+ 23	− 6
Nos.	130	127	117	112	119	111	101	97	95	91	87	85	94	97	101	102	104	107
Total Sums	+327	+583	+851	+892	+1020	+733	+595	+885	+509	+371	+342	−148	− 66	+555	+688	+ 56	−348	+163
Nos.	272	261	245	238	236	223	208	201	190	182	180	176	187	195	197	203	223	231
Means	+ 12	+ 22	+ 35	+ 38	+ 43	+ 33	+ 29	+ 44	+ 26	+ 20	+ 19	− 8	− 4	+ 29	+ 35	+ 3	− 16	+ 7

	180°–190°	190°–200°	200°–210°	210°–220°	220°–230°	230°–240°	240°–250°	250°–260°	260°–270°	270°–280°	280°–290°	290°–300°	300°–310°	310°–320°	320°–330°	330°–340°	340°–350°	350°–360°	
1861 Diff.	+ 32	+ 61	− 65	− 37	+ 3	+ 84	− 13	+ 27	− 15	− 151	− 28	+ 74	+ 18	+109	− 19	− 9	+ 31	+ 49	
1861 Nos.	5	5	7	7	7	7	8	6	4	7	9	9	10	10	8	7	6	8	230
1862 Diff.	− 77	−103	− 94	− 50	− 90	− 72	+ 36	− 226	−102	− 14	− 29	− 50	− 10	− 13	+ 5	− 58	− 21	+ 27	
1862 Nos.	10	11	11	11	11	12	12	12	10	9	8	12	8	10	9	8	7	7	293
63,64,65 Diff.	− 28	− 16	− 46	− 66	− 81	− 27	− 24	− 132	− 25	− 90	− 98	− 104	− 77	− 22	− 4	+ 36	+ 71	+ 38	
63,64,65 Nos.	23	23	24	21	19	23	25	26	24	25	23	22	20	20	17	21	16	18	651
66,67,68 Diff.	+ 79	− 74	+ 5	− 23	+ 99	+109	− 71	+ 21	+ 39	− 86	+ 33	+ 119	+ 54	+ 24	− 95	+ 73	+192	+131	
66,67,68 Nos.	14	14	15	15	18	17	16	15	14	13	10	12	11	13	14	16	15	15	452
Sums.	+ 6	−132	−200	−176	− 69	+ 94	− 72	− 310	−103	− 341	− 122	+ 39	− 15	+ 98	−113	+ 42	+273	+245	
Nos.	52	53	57	54	55	59	61	59	52	54	50	55	49	53	48	52	44	48	1626
1869 Diff.	+ 51	+ 57	− 1	+ 6	− 43	− 59	− 80	− 20	− 68	− 134	− 4	+ 21	+ 10	− 12	− 23	+ 43	+121	+117	
1869 Nos.	10	10	9	11	10	11	10	11	13	12	12	12	12	15	19	18	20	21	425
1870 Diff.	− 35	+ 3	+ 29	+100	+113	+ 72	+ 63	− 14	− 67	− 121	− 132	− 116	−121	− 66	− 2	− 32	+ 10	+ 68	
1870 Nos.	10	11	10	14	15	16	16	15	15	17	18	17	19	15	15	16	17	17	459
1871 Diff.	+ 2	+ 23	− 64	− 21	− 23	−125	−154	− 117	−124	− 60	− 62	− 103	− 65	− 68	− 31	− 31	+ 1	− 37	
1871 Nos.	25	24	27	24	26	26	22	23	23	22	18	16	16	13	10	10	14	14	664
1872 Diff.	− 46	0	+ 24	− 61	−102	−112	−160	− 93	− 83	− 338	− 485	− 335	+111	−269	−162	− 96	+ 34	− 91	
1872 Nos.	6	8	10	12	19	22	24	26	26	26	26	25	26	22	19	16	16	12	493
1873 Diff.	− 32	− 29	− 80	− 16	− 33	− 33	+ 5	− 23	− 22	− 52	− 12	+ 103	− 54	+ 25	+ 25	− 11	+ 60	+ 26	
1873 Nos.	14	14	17	19	18	21	23	23	21	22	20	17	17	15	15	14	15	15	502
74,75 Diff.	− 67	− 41	− 21	− 70	− 70	− 42	−116	− 155	−109	− 58	− 154	− 79	− 58	− 43	− 5	−116	+ 11	+ 46	
74,75 Nos.	21	22	20	20	20	20	21	19	20	19	15	14	11	12	9	10	10	9	473
76,77,78,79 Diff.	− 19	− 19	− 22	− 52	+ 12	− 1	+ 18	+ 5	− 22	+ 3	− 47	− 23	− 13	+ 66	+ 3	− 43	− 47	+ 28	
76,77,78,79 Nos.	4	5	7	6	8	8	8	8	6	7	6	7	9	8	7	7	6	7	226
Sums.	−156	− 6	−135	−114	−146	−300	−424	− 417	−495	− 766	− 896	− 532	−304	−367	−195	−286	+190	+157	
Nos.	90	94	100	109	116	124	124	125	124	125	115	108	110	100	94	91	98	95	3242
1880 Diff.	− 52	− 29	+ 50	+ 32	+ 77	− 28	+136	+ 99	+ 68	− 4	− 17	− 9	− 39	+ 12	− 86	− 62	− 34	− 21	
1880 Nos.	8	12	11	12	12	13	11	8	8	9	10	11	11	14	21	18	16	14	422
1881 Diff.	+ 41	+ 49	+ 40	0	+ 74	+ 54	+ 58	− 27	− 13	− 350	− 143	− 161	− 61	− 91	− 91	−120	− 23	+147	
1881 Nos.	13	14	14	16	19	19	22	25	26	25	24	25	27	24	25	22	25	24	687
1882 Diff.	+ 58	− 39	− 19	+ 5	+ 50	+151	+ 36	− 125	− 73	− 45	+ 32	− 20	− 32	− 8	−154	+ 3	− 11	− 26	
1882 Nos.	28	27	28	28	29	29	26	27	23	20	17	17	15	20	22	25	28	29	738
1883 Diff.	− 42	+ 7	− 35	+ 22	+ 14	−108	−177	− 135	−143	− 159	+ 4	− 237	−132	−166	+ 56	+ 34	+ 2	− 27	
1883 Nos.	26	28	27	28	29	27	29	30	32	33	34	37	37	37	40	39	43	42	1159
1884 Diff.	−229	+ 14	− 46	−158	−127	−353	−400	− 157	− 60	− 280	− 301	− 57	− 79	+112	+ 82	+ 6	− 72	+ 21	
1884 Nos.	39	48	49	47	48	45	45	43	39	39	41	39	37	36	32	31	30	29	1232
Sums.	−224	+ 2	− 10	− 99	+ 88	−284	−347	− 345	−221	− 838	− 425	− 484	−343	−141	−193	−139	−138	+ 94	
Nos.	114	129	129	131	137	133	133	133	128	126	126	129	127	131	140	135	142	138	4238
Total Sums	−364	−136	−345	−389	−127	−490	−843	−1072	−819	−1945	−1443	− 977	−662	−310	−501	−383	+325	+496	−1977
Nos.	256	276	286	294	308	316	318	317	304	305	291	292	286	284	282	278	284	281	9106
Means	− 14	− 5	− 12	− 13	− 4	− 16	− 27	− 34	− 27	− 64	− 50	− 33	− 23	− 11	− 18	− 14	+ 11	+ 18	

Also

$$\left.\begin{aligned} I &= I' + Y = 6^\circ.968 + 0^\circ.038 = 6^\circ.996 = 7^\circ.006 \\ N &= N' + X \operatorname{cosec} I = 74^\circ.523 + 1^\circ.480 = 76^\circ.003 \end{aligned}\right\} \text{for } 1861$$

Here we see that the value of the node is larger than any of the values given at the beginning of the paper. The value of the inclination is nearly the same as that given by Spörer.

As a check on this result, I calculated the variations of latitudes at equal intervals of 30° degrees of longitude, by summing the three successive columns of the Table III. and dividing the results by the average number of spot occurrences. Also to see if there is a noticeable secular change in both node and inclination, I divided the interval of 24 years into three separate periods; first, from 1861 to 1868; second, from 1869 to 1879; third, from 1880 to 1884. Separate reductions were made for these three distinct periods. The resulting differences of latitude and the number of spots are shown in Table IV.

Table IV.

	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
(A) 1861...1868	Diff.	+ 248	+ 383	+ 406	+ 75	+ 78	+ 85	− 326	− 151	− 485	− 424	− 30	+ 560
	Nos.	46	42	39	31	28	37	54	56	57	53	50	48
	Means.	+ 5,4	+ 9,1	+ 10,4	+ 2,4	+ 2,8	+ 2,3	− 6,0	− 2,7	− 8,5	− 8,0	− 6,0	+ 11,7
		+ 4,6	+ 8,3	+ 9,6	+ 1,6	+ 2,0	+ 1,5	− 6,8	− 3,5	− 9,3	− 8,8	− 1,4	+ 10,9
(B) 1869...1879	Diff.	+ 971	+1147	+ 989	+ 244	+ 17	− 521	− 287	− 560	−1336	−2194	− 766	+ 61
	Nos.	89	76	63	61	67	78	95	116	124	116	101	95
	Means	+ 10,9	+ 15,1	+ 15,7	+ 4,0	+ 0,3	− 6,7	− 3,0	− 4,8	− 10,8	− 18,9	− 7,6	+ 0,6
		+ 13,0	+ 17,2	+ 17,8	+ 6,1	+ 2,4	− 4,6	− 0,9	− 2,7	− 8,7	− 16,8	− 5,5	+ 2,7
(C) 1880...1884	Diff.	+ 542	+1115	+ 594	+ 246	+1082	+ 307	− 232	− 295	− 913	−1747	− 677	− 183
	Nos.	125	114	98	88	97	104	124	134	131	127	133	138
	Means.	+ 4,3	+ 9,8	+ 6,1	+ 2,8	+ 11,2	+ 3,0	− 1,9	− 2,2	− 7,0	− 13,8	− 5,1	− 1,3
		+ 4,4	+ 9,9	+ 6,2	+ 2,9	+ 11,3	+ 3,1	− 1,8	− 2,1	− 6,9	− 13,7	− 5,0	− 1,2
(D) 1861...1884	Diff.	+1761	+2645	+1989	+ 565	+1177	− 129	− 845	−1006	−2734	−4365	−1473	+ 438
	Nos.	259	232	200	179	193	220	273	306	313	296	284	281
	Means.	+ 6,8	+ 11,4	+ 9,9	+ 3,2	+ 6,1	− 0,6	− 3,1	− 3,3	− 8,7	− 14,7	− 5,2	+ 1,6
		+ 7,5	+ 12,1	+ 10,6	+ 3,9	+ 6,8	+ 0,1	− 2,4	− 2,6	− 8,0	− 14,0	− 4,5	+ 2,3

Table IV. gives the following four sets of equations of conditions deduced respectively from A, B, C, and D, of that table. These four sets of equations I shall call A', B', C', and D' respectively.

A'.

$(-134\,X-500\,Y+45=0)$ 1.15
$(-366\,X-366\,Y+83=0)$ 1.05
$(-500\,X-134\,Y+95=0)$ 0.98
$(-500\,X+134\,Y+16=0)$ 0.78
$(-366\,X+366\,Y+20=0)$ 0.70
$(-134\,X+500\,Y+15=0)$ 0.93
$(+134\,X+500\,Y-68=0)$ 1.35
$(+366\,X+366\,Y-35=0)$ 1.40
$(+500\,X+134\,Y-93=0)$ 1.43
$(+500\,X-134\,Y-88=0)$ 1.33
$(+366\,X-366\,Y-14=0)$ 1.25
$(+134\,X-500\,Y+109=0)$ 1.20

B'.

$(-134\,X-500\,Y+130=0)$ 1.11
$(-366\,X-366\,Y+172=0)$ 0.95
$(-500\,X-134\,Y+178=0)$ 0.79
$(-500\,X+134\,Y+61=0)$ 0.76
$(-366\,X+366\,Y+24=0)$ 0.84
$(-134\,X+500\,Y-46=0)$ 0.88
$(+134\,X+500\,Y-9=0)$ 1.19
$(+366\,X+366\,Y-27=0)$ 1.45
$(+500\,X+134\,Y-87=0)$ 1.55
$(+500\,X-134\,Y-168=0)$ 1.45
$(+366\,X-366\,Y-55=0)$ 1.26
$(+134\,X-500\,Y+27=0)$ 1.19

C'.

$(-134\,X-500\,Y+44=0)$ 1.25
$(-366\,X-366\,Y+99=0)$ 1.14
$(-500\,X-134\,Y+62=0)$ .98
$(-500\,X+134\,Y+29=0)$ .88
$(-366\,X+366\,Y+113=0)$ .97
$(-134\,X+500\,Y+31=0)$ 1.04
$(+134\,X+500\,Y-18=0)$ 1.24
$(+366\,X+366\,Y+21=0)$ 1.34
$(+500\,X+134\,Y-69=0)$ 1.31
$(+500\,X-134\,Y-137=0)$ 1.27
$(+366\,X-366\,Y-50=0)$ 1.33
$(+134\,X-500\,Y-12=0)$ 1.38

D'.

$(-134\,X-500\,Y+75=0)$ 1.30
$(-366\,X-366\,Y+121=0)$ 1.16
$(-500\,X-134\,Y+106=0)$ 1.00
$(-500\,X+134\,Y+39=0)$ 0.90
$(-366\,X+366\,Y+68=0)$ 0.97
$(-134\,X+500\,Y+1=0)$ 1.10
$(+134\,X+500\,Y-24=0)$ 1.37
$(+366\,X+366\,Y-26=0)$ 1.53
$(+500\,X+134\,Y-80=0)$ 1.57
$(+500\,X-134\,Y-140=0)$ 1.48
$(+366\,X-366\,Y-45=0)$ 1.42
$(+134\,X-500\,Y+23=0)$ 1.41

The set of equations (A') gives $X=+0^\circ.1323$ $Y=+0^\circ.077$
" " " " (B') " $X=+0^\circ.2134$ $Y=+0^\circ.060$
" " " " (C') " $X=+0^\circ.1692$ $Y=+0^\circ.004$
" " " " (D') " $X=+0^\circ.1792$ $Y=+0^\circ.038$

These give the following results :—

from	A'	$N=75°.602$	$I=7°.045$
,,	B'	$N=76°.379$	$I=7°.028$
,,	C'	$N=76°.184$	$I=6°.964$
,,	D'	$N=75°.992$	$I=7°.006$

It should be noticed that the values of the ascending node obtained from D' serves to check the previous determination. The difference amounts to almost forty seconds of arc which I do not consider too much for this kind of determination. A comparison of the values deduced from A', B', and C', seems to hint at a certain increase in the values of node in time. This was assumed by Spörer, but I do not insist on this point firmly. I also here notice that in his reduction of spot observations, Spörer has since 1880 applied a correction for refraction due to the sun's atmosphere ; so that the heliographic latitudes deduced are somewhat different. I have, however, used the deduced values of longitudes and latitudes without alteration, throughout the long series of 24 years.

The principal conclusion to be drawn from the present calculations is that the values of the ascending node of the sun's equator given by Carrington and Spörer are distinctly smaller than the values now deduced. Wilsing's values, however, resembles mine pretty closely. Finally I venture to propose the following elements for 1861.

$$\left.\begin{array}{l} I=7°.006 \\ N=76°.003 \end{array}\right\} \text{for } 1861$$

In conclusion I desire to acknowledge my indebtedness to Prof. Terao, Director of the Tōkyō Astronomical Observatory for his kind and valuable suggestions.