Combined Effects of Torsion and Longitudinal Stress on the Magnetization of Nickel.

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With Plates XVI - XIX.

The effect of torsion in altering the induced magnetism of iron has long engaged the attention of many physicists. Among the experimenters in this field of research may be named Wertheim, Wiedemann, Thomson, and Tomlinson (whose latest work on such subjects I have not yet seen.) The experiments of Wiedemann were made by twisting and untwisting the wire, which was placed horizontally in a magnetizing solenoid, till the changes of magnetism became cyclic. But it was Thomson who first investigated the effect of torsion on the magnetism of iron, the wire being at the same time subject to definite longitudinal stresses. In his experiments, the soft iron wire was placed vertically in the earth's field. No one seems to have made similar experiments in different magnetizing fields. Scanty though this kind of investigation has been for iron, it is still more scanty for nickel. Indeed, so far as I am aware, the effect of torsion on the magnetism of nickel wire under various longitudinal stresses has not hitherto been investigated. This accordingly was the problem

I resolved to attack ; and the results that have been obtained will, I believe, be found to contain distinct novelties.

It is well-known from the results of various experimenters that by the application of longitudinal stress, the magnetism of iron increases up to a certain critical load, while that of nickel always diminishes. Thus we should naturally expect that the effect of torsional stress on nickel will be opposite to that on iron. According to Thomson's experiments, the effect of torsion on the magnetism of iron is to increase it at first, but when the twist exceeds a certain angle, it tends to diminish, while on untwisting it increases and attains its former value, and similar things take place when the wire is twisted in the opposite direction. So with nickel, it seemed quite likely that there will be decrease of magnetism till the twist reaches a certain value, and beyond this, the magnetism of nickel will increase, which on untwisting again decreases to its former value. In fact, this is exactly reproduced in one of Wiedemann's experiments,* the curve obtained being just the reverse of one given by Thomson.† But Wiedemann's result was obtained by simply clamping the wire in a horizontal position, so that the wire was subject to a weak longitudinal stress only. On this account, the combined effect of pull and torsion on the magnetism of nickel was still a matter to be determined.

In the following experiments, I have examined these points, and have found that the longitudinal stress produced a singular effect. For weak stresses, the changes of magnetism came out as was to be expected, but when the load exceeds a certain limit, this is no longer the case. The changes of magnetism become gradually altered, and beyond a critical value of the longitudinal stress, one end of the nickel wire acquires the two opposite kinds of magnetism during the

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*Wiedemann's Annalen, Bd. 26, S. 376, 1886.

†Philosophical Transactions, 1879, p. 72.

torsion and detorsion, notwithstanding the absolute constancy of the magnetizing force both in direction and magnitude. This critical value of the load seems to vary with the strength of the magnetizing field, becoming greater as the field is increased. All these points will be described in the following pages.

I must here express my thanks to Dr. C. G. Knott, for his kind suggestions during the course of experiments.

The intensity of magnetization was measured by a direct magnetometric method. The magnetometer consisted of a small mirror hung by a spider thread 11 cms long. This was geometrically fixed in position on a wooden plank according to Thomson's method of the hole, slot, plane. Levelling was effected by three base-screws. In front of the magnetometer a lamp was placed, and the image of the slit was reflected on a circular scale. Its radius was 1 metre, and it was so placed that the magnetometer was just at its centre. The wire to be examined was set vertically due east of the magnetometer. The upper end of the wire was level with the centre of the magnetometer mirror. To each end of the wire, a short stout brass wire was brazed. The lower of these was bent into a hook, so that a pan holding the weight could be hung from it. The upper one was riveted to a strong brass rod projecting from the middle of the side of a table, which rested on stone piers. The nickel wire* was surrounded by a magnetizing coil 45 cms. long. The resistance of the coil was 19.6 Ohms, and the strength of the field for a current of one ampere was 138.4 C. G. S. units. The magnetizing current was sent from 12 Daniell cells, and its strength was adjusted by means of a liquid slide, and measured by a tangent galvanometer.

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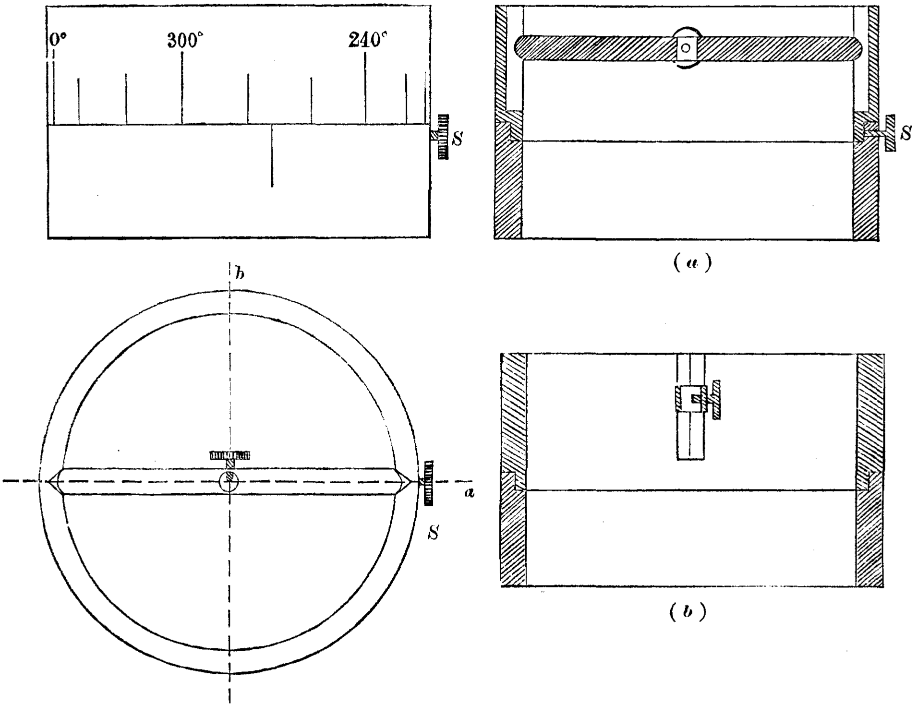
* This wire contained 1.7 per cent. of iron, besides small quantities of carbon as impurities.

The twisting apparatus is shown in the subjoined cut.



(a)

(b)

It consisted of two hollow cylinders of 2.5 cms radius. The lower cylinder was fixed to a tripod stand, while the upper one fitted into it, and was movable. The latter was graduated on its external cylindrical surface at intervals of 20°, and by means of a pointer which was cut on the corresponding surface of the fixed cylinder, the amount of twist was easily read off. The screw *S* attached to the lower cylinder served to fix the upper one at any desired angle of twist. On the inner surface of the movable cylinder, two vertical V grooves were cut opposite each other. On these the two ends of the thick brass diametral rod were made to slide. This rod had a small hole at its centre which was just large enough to allow the passage of the brass wire attached to the lower end of the nickel wire. In order to secure the axial position of this hole with reference to the twisting cylinder, the rod was fixed between the V's and bored on a lathe by turning it together with the cylinder. A small clamping screw served to pin the wire fast against the side of the rod, so that the wire and cylinder rotated together.

It might at first sight appear that this arrangement might prevent the longitudinal stress being applied uniformly for various angles of twist, because of the friction of the rod against the V's. To test this point, the wire was fixed by the screw, and the longitudinal pull applied by known loads hung on below. The upper end of the wire was fastened to a spring balance, by means of which any variations of stress could at once be detected. With a given load, the wire was twisted though various angles ; but scarcely any sensible variation of longitudinal stress was indicated.

The magnetic experiments were conducted in the following manner. At first, a constant current was made to pass through the magnetizing coil, and the magnetometer zero was determined. The wire was then placed in position, and was first twisted through 180° in what we call the positive direction—although it may as well be stated once for all that the positive direction mean the *first chosen* direction, whether that is, so to speak, with the magnetizing current or against it. Then after a complete revolution in the opposite direction, it was brought back to its original position. This process was repeated till the changes became nearly cyclic. Then at every successive 20° of twist, the deflection of the magnetometer magnet as given by the scale reading was taken and noted, the reading being observed by means of a telescope. Each complete set of experiments was made in a constant magnetic field, while the wire was subjected to gradually increasing longitudinal stresses. The results thus obtained are given in C. G. S. electromagnetic units, though such reduction would have been unnecessary in experiments of this kind. The observed values of the intensity $\mathfrak{J}$ for successive 60° of twist are given in the appendix. In the figures showing the changes of magnetization, the abscissæ denote the amount of twist, while the ordinates represent the intensity of magnetization $\mathfrak{J}$.

The first experiment was made with a nickel wire 40 cms long, and 1 mm. thick. The wire was deprived of its initial magnetism by heating it red hot. It was then placed vertically and so came under the influence of a magnetic field of .34 C. G. S. units. With a steady load of .64 kgs. the wire was subjected to repeated twisting and untwisting. After 6 such operations, the changes became nearly cyclic, and the following readings of deflections were taken at the 7th cycle.

Twist (Positive.)	Readings.	Twist (Negative.)	Readings.
0°	24.2	0°	3.4
20°	35.8	20°	− .2
40°	45.5	40°	− 2.0
60°	55.8	60°	− 2.4
80°	64.4	80°	− 2.6
100°	71.0	100°	− 2.4
120°	76.3	120°	− 1.9
140°	79.8	140°	− 1.2
160°	82.2	160°	− 0.5
180°	83.8	180°	0.0
160°	81.9	160°	0.0
140°	78.8	140°	− .2
120°	73.5	120°	− .3
100°	65.2	100°	− 1.0
80°	51.9	80°	− 1.0
60°	35.9	60°	1.9
40°	22.0	40°	7.8
20°	10.8	20°	17.3
0°	3.4	0°	28.7

The above readings reduced to absolute units are shown plotted in Fig. 1. The amount of load per sq. cm. was 82 kgs., and the twist of 180° corresponded to that of .0785 radian per cm.

Examining the figure we see that the first effect of twisting is to increase the magnetization. The rate of increase is rapid at first, but gradually falls off as the magnetization attains its greatest value at the maximum twist. During the process of untwisting, the magnetization diminishes more rapidly than it increased during twisting, so that for every position of twist, the magnetization during twisting is greater than the magnetization during untwisting. The diminution of magnetization goes on even after the wire has passed the original position from which the twisting was begun, until the apparent magnetization is at length reduced to zero. This happens very soon after the original position of the untwisted wire has been reached, as the process of untwisting is continued as twisting in the negative direction. But now as this negative twisting is continued, the *polarity of the wire changes sign*, a very striking fact indeed. As the twisting is continued on towards −180°, this negative magnetization passes through an arithmetical maximum, becoming finally almost zero. As the wire is being brought back to the position from which it started, the magnetization gradually recovers nearly its original value as shown in the figure.

Now reasoning from analogy, we should expect to obtain by such twisting and untwisting a curve of the form given in Fig. 5, since the behaviour of nickel with regard to the effect of stress in magnetization seems to be just opposite to that of iron (see Sir. W. Thomson's figures for iron, *Philosophical Transactions* 1879). But in this experiment, the curve of magnetization seems to be no resemblance at all to any figured by Sir. W. Thomson; and then there is the very curious fact that the magnetization of nickel in a steady field can be made to change sign by twisting. It first occurred to me that this very extraordinary result must be due to some defect in the arrangement, but careful examination discovered no flaw. The question naturally suggested itself, was this phenomenon a function of the load as well as of the twist? Hence as a next step, the weight was increased by .5 kg., and the experiment was performed in the usual manner.

The result is shewn in Fig. 2. Here the effect is quite similar to the former, the differences being only differences of detail. The march of magnetization with positive twisting is sensibly the same as in the former case; but during negative twisting, the opposite magnetization has increased to more than 10 times its amount in the first experiment. However, the rate of recovery has very much diminished, and even after a twist of 180°, the magnetization is far from reaching the former value.

The same experiment was then performed with the load increased to 5.14 kgs. The result is shown in Fig. 3. There again we find the opposite magnetization still more increased. Indeed, the two kinds of magnetization do not differ much in intensity at the two extreme twists, although the initial or positive magnetization is somewhat predominant. In the two former experiments, there was a distinct tendency towards recovery of positive magnetization as the wire was twisted more and more in the negative direction. Here, however, no such tendency shows itself except in the diminished rate of growth of negative magnetization.

Still increasing the load, we see that the curve (Fig. 4) becomes nearly symmetrical with respect both to the line of zero magnetization, and the line of zero twisting. In this case we find further that the range of the change of magnetization has considerably diminished. The slight excess of the initial magnetization over the other still shows itself, a fact which is probably to be referred to the direction of the magnetizing force.

The general conclusion from these experiments is that in a weak field of .34 units, the increase of load makes the manner of change of magnetization in nickel under the influence of cyclic twisting depart more and more from any slight resemblance which at small loads it seemed to bear to the manner of change for iron. For still smaller load, then it might be possible to obtain the magnetization curve just opposite to that of iron.

In the second series of experiments, the strength of the field was raised to 2.47 units. The load at first applied was only the weight of the brass wire attached to the lower end of the nickel wire, and a brass rod which gripped the wire during the process of twisting. This load was .02 kg., that is, a tension of 2.6 kgs. weight per sq. cm. With this amount of longitudinal stress, the successive twistings and untwistings were performed 7 times, and the following readings of deflection were taken :—

Twist (Positive.)	Readings.	Twist (Negative.)	Readings.
0°	148.7	0	148.0
20°	166.4	20°	167.4
40°	181.5	40°	183.5
60°	193.2	60°	195.3
80°	201.1	80°	202.9
100°	205.9	100°	208.0
120°	209.0	120°	210.0
140°	210.3	140°	211.1
160°	210.9	160°	211.5
180°	211.0	180°	211.6
160°	207.4	160°	207.9
140°	201.8	140°	202.0
120°	192.5	120°	193.4
100°	177.2	100°	178.3
80°	157.3	80°	157.6
60°	136.6	60°	137.5
40°	126.9	40°	127.0
20°	133.0	20°	132.2
0	148.0	0	148.0

The curve thus obtained (see Fig. 5) is nearly perfectly symmetrical with respect to the line of zero twisting. Also, the magnetization remains positive throughout the whole cycle. It is moreover interesting to observe, that the curve is exactly the reverse for that of iron as obtained by Thomson. This shows that the behaviour of nickel twisted in a magnetic field under feeble loads is opposite to that of iron.

When the stress was increased to 145 kgs. weight per sq. cm., the magnetization curve lost its symmetry and became as shown in Fig. 6. The intensity of magnetization became greatly diminished, but still remained positive throughout the whole cycle of operations. The features to be noted are that, although suggestive of the symmetrical form given in Fig. 5, the curve is now distinctly one-sided with respect to the line of zero twisting, and indicates much greater magnetization for positive than for negative twists. These peculiarities are still more pronounced when the stress is increased to 209 kgs. as shown in Fig. 7.

This experiment is of special interest, illustrating as it does, the manner in which the magnetization cycle changes character just as the extraordinary phenomenon of change of sign is about to show itself.

As usual the first effect of twisting the wire is to increase the intensity of magnetization, while the effect of untwisting is to decrease it. Then as the untwisting is continued as negative twisting, the magnetization tends to recover its former value. But for the maximum negative twist of two right angles, the magnetization does not nearly recover its former value. Thus it is evident that in such a strength of field, the increase of longitudinal stress tends to make the increase of magnetization during negative twisting gradually less and less until finally for a certain load the increase does not take place at all for the particular range of twist. This is shown in Fig. 8, which is the curve for a tension of 782 kgs. weight per sq. cm.

A study of these from curves (Figs. 5, 6, 7, 8,) shows the character of the changes wrought in the cycles as the load is increased. The symmetry is first lost, the negative loop becoming smaller and smaller. Then, as shown in Fig. 7, it ceases to be a loop, the course of the return curve from greatest negative twist lying above the other, and never cutting it. Then as in Fig. 8, the *upward* course of the curve on the negative side of zero twist vanishes away altogether ; while at the same time the phenomenon of reversal of magnetic polarity shows itself. Thus the double-looped curve for low tensions passes gradually into a single-looped curve as the tension is increased. And after this single-looped curve is obtained, the phenomenon of the reversed polarity begins to appear. The passage from the double looped to the single looped curve betokens a peculiar alteration in the lagging-effect in nickel—an alteration which has no analogue in the case of iron.

To study more carefully the law of *hysteresis* in nickel—to use Professor Ewing's word—the experiments were repeated in stronger fields.

Figs. 9, 10, 11, 12 show the march of events in a field of 4.94 units, the tensions increasing from 400. For smaller tensions the curves are of the approximately symmetrical form shown already in Fig. 5, and do not call for special remark. Here then we see how the already diminished loop for negative twisting, as shown in Fig. 9, has vanished altogether in Fig. 10. At the same time, the curve begins to cross the lines of zero magnetization into the negative region. Fig. 11 is a further development in the same direction. In both of these curves (10 and 11), the quantity $\frac{d\mathfrak{I}}{d\tau}$, the rate of change of magnetization with twist, still changes sign at particular twists.

The negative *tail*—as it might be called—so evident in Fig. 10, has disappeared in Fig. 11 ; while at the same time, the negative magnetization has numerically increased. But now passing to the higher load (see Fig. 12), we see that the various changes discussed above have their final end in a simple single-looped curve, with a large portion in the region of negative magnetization, and with no true minimum points for $\mathfrak{J}$. The twist for which, in the first three cases, the value of $\frac{d\mathfrak{J}}{d\tau}$ changes sign, works towards the left as the load is increased. Thus

for $W = 400$, $d\mathfrak{J}/d\tau = 0$ at $+ 8°$
,, ,, $= 527$, ,, $= 0$ at $- 20°$
,, ,, $= 655$, ,, $= 0$ at $- 60°$

For $W = 1910$, we may regard this critical twist as being too great to be included in the range of greatest twist applied.

It was remarked while discussing the experiment made in the earth's vertical field, that the ratio of the two opposite magnetizations gradually tends to unity as the load is increased ; but this does not seem to be generally the case: There is a certain limit beyond which there is an opposite tendency. The following calculations shows that this must be the case. Let $+ \mathfrak{J}$ and $- \mathfrak{J}$ be the greatest magnetization during positive and negative twists respectively, then in the field $= 4.94$ units, and

$W = 655$; $+ \mathfrak{J} = 188.6$, $- \mathfrak{J} = 49.9$, $+\mathfrak{J}/-\mathfrak{J} = 3.78$
$W = 910$; $+ \mathfrak{J} = 152.1$, $- \mathfrak{J} = 151.3$, $+\mathfrak{J}/-\mathfrak{J} = 1.00$
$W = 1270$; $+ \mathfrak{J} = 119.3$, $- \mathfrak{J} = 113.1$, $+\mathfrak{J}/-\mathfrak{J} = 1.05$
$W = 1910$; $+ \mathfrak{J} = 86.3$, $- \mathfrak{J} = 77.4$, $+\mathfrak{J}/-\mathfrak{J} = 1.11$

The following experiments in field $= 6.71$, show how this ratio depends on the strength of the field. The changes which the magnetization curve undergoes while it is changing its sign are quite analogous to the preceding two cases, as a glance at Figs. 13, 14, and 15 will show. From Figs. 15, 16, 17, however, we see how the opposite magnetization becomes smaller as we increase the field. The ratio $+\mathfrak{I}/(-\mathfrak{I})$ is always a very large quantity and has a minimum value for a particular load. For loads greater than this particular load, the negative magnetization decreases, and at last completely vanishes. Thus for stress $W=1770$ kgs., there is no opposite magnetization therein agreeing with the curves for loads smaller than that for which the negative magnetization first appears. But there is the difference that the curve is a single loop in which $d\mathfrak{I}/\mathfrak{I}_{\tau}$ changes sign during negative twisting and untwisting.

Taking into account all the series of experiments, we see that the greatest value in each series of the ratio $+\mathfrak{I}/-\mathfrak{I}$ tends to increase as the field is increased. The strength of field in which these last experiments were tried seems to be about the critical value for which we can get the two transitions of magnetization—namely the change of sign for particular load and the vanishing of this change for a higher load.

The peculiarities, which have just been the subject of discussion, do not however persist at all strength of fields. At still higher fields, a different order of things comes in. Take for example the next series of experiments with a field of 8.06, as shown in Figs. 18, 19, 20, 21, 22. First of all, for the lower tensions, the two-looped curve is more symmetrical than it can be obtained at lower fields (see Fig. 18.) In Fig. 19, for a tension of 782, a symmetry begins to show itself; but the diminution instead of taking place in the left hand or negative loop, takes place in the right hand or positive loop. Figs. 20, 21, show the gradual vanishing away of this right hand loop as the load is increased. Also, exactly as in the former sets of experiments, the curve dips below the zero magnetization line, as the right hand portion loses its loop character. Now if we were to compare Fig. 21 with Figs 17, 12, and 8, 2, and take no account of the intermediate links of development, we should at once regard them as being of essentially opposite character. In the four earlier cases, positive twist increased the magnetization, and negative twist diminished it ; but in the present case, the effects are exactly opposite. In the same way, Fig. 20 present features quite opposite to those presented by Figs. 1, 7, 11, and 16. However, just as in the former sets of experiments, the twist at which $d\mathfrak{J}/d\tau$ vanishes was shown to shift gradually to the negative side as the load was increased; so in the present case, the twist at which $d\mathfrak{J}/d\tau$ vanishes shifts gradually to the right as the load is increased. These very interesting reversals of effects clearly depend on the strength of the field. Then again the particular strength of field at which the reversal of these effects begins to show itself seems to be connected with the fact already discussed that for a particular field the ratio $+\mathfrak{J}/(-\mathfrak{J})$ becomes infinite. In other words, when the strength of the field is such that under sufficient loading the reversal of polarity vanishes away, this seems to be the signal for the new set of conditions to appear. Up to this critical strength of field, it is the left hand loop in the typical symmetrical curve that gradually diminishes under loading. But for strengths of fields higher than this critical value, it is the right hand loop that disappears when the load is great enough. This reversal of effects also seems to be accompanied by a lingering of the magnetization near the zero (see figures 18, and 19.)

It is not necessary to discuss in detail other combinations of field and stress which were experimented upon. There are certain minor differences depending on strength of the field ; but the principal features for fields higher than 8 are the same. The essential characteristics can be gathered from figures, brief explanations of which I shall content myself with giving

Fig. 22 - 23.

These illustrate the changes of magnetization in field 11.9.

Fig. 24 - 27.

These were obtained in field 13.85. The curious transition curve Fig. 30 is specially worthy of note.

Fig. 28 - 31.

These show the gradual changes of magnetization for field 15.78.

Fig. 32 - 34.

were obtained in field 23.50.

Fig. 35 - 40.

were obtained in field 33.54.

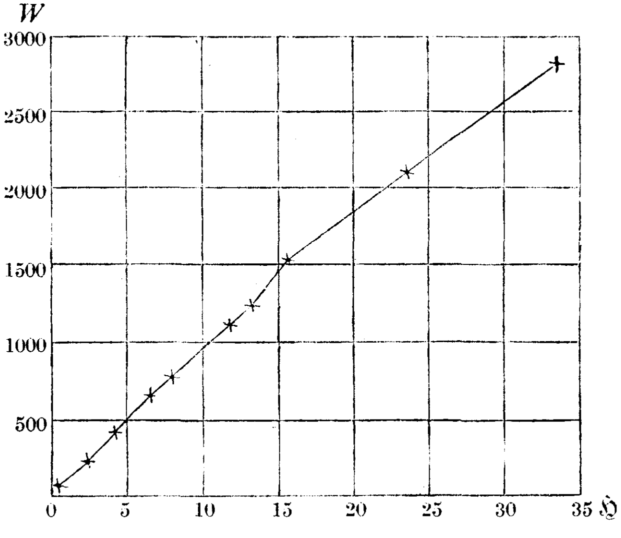
This last was the highest strength of the field at which it was possible to notice the changes of magnetization. For the stronger the field the higher is the load necessary to bring out the curious changes. The critical load for fields higher than 33.54 is greater than the tenacity of the wire.

There is one other point that calls for remark. The range of the change of magnetization under feeble stresses begins gradually to diminish after a certain strength has been reached, so that for a field of 20 or 30, the changes of magnetization by twisting becomes almost inappreciable. This can be accounted for by the fact that nickel wire whether in the normal or twisted condition behaves practically the same as regards magnetization in higher fields. Indeed, nickel is more easily saturated than iron, so that when $\mathfrak{H} = 20$ or 30, it is already far beyond the saturation point. Consequently the difference in the susceptibilities of nickel in the normal and twisted conditions, to which this alteration of magnetic intensities must be ascribed, becomes less and less marked as the strength of the field is increased. Many of the features of magnetization curve are quite simply accounted for by this consideration.

It is very difficult to determine the exact loading at which the reversed phenomenon of reverse polarity makes its appearance in various fields. Assuming, however, that within small ranges of load, the decrease in the intensities of magnetization is proportional to the amount of loading, I obtained the following values for the stress which must be applied so as to effect the reversal of polarity of the particular nickel wire placed in various fields :—

For $\mathfrak{H}$ = .34 , W = 77 kgs. cm^2.
,, ,, = 2.47 , ,, = 217 ,,
,, ,, = 4.94 , ,, = 421 ,,
,, ,, = 6.71 , ,, = 654 ,,
,, ,, = 8.06 , ,, = 783 ,,
,, ,, = 11.92 , ,, = 1120 ,,
,, ,, = 13.85 , ,, = 1220 ,,
,, ,, = 15.78 , ,, = 1530 ,,
,, ,, = 23.50 , ,, = 2110 ,,
,, ,, = 33.54 , ,, = 2830 ,,

Plotting the curve with $\mathfrak{H}$ for abscissa and W for ordinate, we get the annexed figure.



This seems to show that for moderate strengths of field, the load at which the wire begins to show reversed polarity, is nearly directly proportional to the strength of the field. For very weak and strong fields, this rule does not seem to hold. It must be remembered of course that all these peculiarities are for one particular twist only, and that it is possible that quite a different series of effects might exist for other twists.

The general results of these experiments may be thus summarised.

In all magnetic fields with moderate loading the effect of twisting nickel wire is to increase the magnetization. But the increase depends on the strength of the field as well as the longitudinal stress applied. If the field is weak, and the longitudinal stress sufficiently great, the magnetization increases in one direction of twist, and decreases in the other. Eventually for a particular stress which is approximately proportional to the field, the wire begins to show opposite polarity; and the cyclic curve of magnetization passes gradually from a two-looped to a single looped form. For stronger fields similar effects exist. But in fields higher than a critical value, the increase and decrease of magnetization take place for reversed directions of twist, and at the same time, the course of the curve becomes reversed.

In a recent paper (Magnetische Untersuchungen, Wiedemann's Annalen, March 1886) Professor Wiedemann has described certain experiments on the combined effects of magnetization and twist in iron and nickel. He does not seem, however, to have investigated the effect of longitudinal stress in conjunction with these. His chief aim seems to have been to adduce facts in support of his theory of frictionally rotated molecules. Some of the results above described may be expressed in terms of his theory. Thus when the external magnetizing force is great, the magnetic molecules will be held in position more strongly and consequently, the change of magnetization due to twisting will be diminished. This agrees with experiment. But again we saw that by sufficiently loading the wire, we could bring the apparent magnetization down to zero and eventually reverse its sign by mere twisting. Now if this be due to the frictional rotation of molecules, the molecules must, notwithstanding the directive force of 30 units, be rotated through more than a right angle from their first position, while the amount of mechanical twist amounts only to .079 radian per cm. in each direction. Admit it to be so; what effect then may we expect increased loading to produce on the rotation of the molecules? The magnetic molecules in strong fields are acted on only by a greater directive force, and consequently they must tend to remain more in the direction of the magnetizing force; but why they should assume nearly the position of magnetic neutrality when they are subjected to sufficient longitudinal stress, is a question which every supporter of the theory of frictionally rotated molecules is bound to answer.

	$\mathfrak{H} = .34$				$\mathfrak{H} = 2.47$				$\mathfrak{H} = 4.94$			
τ	$\mathfrak{J}$ $W=82$ (*Fig.* 1.)	$\mathfrak{J}$ $W=273.$	$\mathfrak{J}$ $W=655$ (*Fig.* 3.)	$\mathfrak{J}$ $W=1280.$	$\mathfrak{J}$ $W=2.5$ (*Fig.* 5.)	$\mathfrak{J}$ $W=145$ (*Fig.* 6.)	$\mathfrak{J}$ $W=273.$	$\mathfrak{J}$ $W=782$ (*Fig.* 8.)	$\mathfrak{J}$ $W=82.$	$\mathfrak{J}$ $W=400$ (*Fig.* 9.)	$\mathfrak{J}$ $W=655$ (*Fig.* 11.)	$\mathfrak{J}$ $W=1270.$
+ 0	32	54	51	33	195	59	112	109	207	36	115	49
+ 60	73	99	87	61	253	155	196	159	250	143	171	90
+120	100	112	100	77	275	209	215	178	274	197	186	113
+180	110	115	103	83	276	220	220	182	280	200	189	119
+120	96	93	71	50	252	194	185	121	259	160	142	65
+ 60	47	24	8.4	7.1	179	107	53	9.4	171	57	43	− 1
− 0	4.5	− 52	− 54	− 29	195	51	− 29	− 82	208	77	− 33	− 51
− 60	− 3.1	− 81	− 85	− 53	277	88	− 69	−130	257	103	− 50	− 87
−120	− 2.4	− 87	− 95	− 68	275	125	− 79	−150	280	169	− 38	−108
−180	00	− 86	− 97	− 73	256	136	− 77	−154	282	193	− 16	−113
−120	− .4	− 66	− 65	− 41	253	118	− 64	− 95	251	155	− 0.1	− 63
− 60	2.5	− 15	− 8.6	− 1.3	180	55	− 16	11	176	68	35	− 1.3
− 0	38	54	51	34	194	59	110	107	209	35	110	49
	$W=145$ (*Fig.* 2.)	$W=400.$	$W=910.$	$W=1910$ (*Fig.* 4.)	$W=82.$	$W=209$ (*Fig.* 7.)	$W=527.$		$W=273.$	$W=527$ (*Fig.* 10.)	$W=910.$	$W=1910$ (*Fig.* 12.)
+ 0	34	57	41	23	107	81	126		92	73	79	26
+ 60	78	97	75	43	191	168	181		173	159	131	55
+120	99	110	92	59	240	203	200		210	190	150	78
+180	105	114	97	65	251	209	204		221	193	152	86
+120	91	85	61	38	224	176	159		184	149	69	49
+ 60	39	16	4	7.6	139	77	44		85	51	− 15	6.8
− 0	21	− 60	− 46	− 16	110	11	− 78		87	− 19	− 85	− 27
− 60	− 37	− 90	− 73	− 35	180	23	−140		186	− 2.8	−134	− 53
−120	− 37	− 98	− 84	− 49	222	40	−163		236	61	−150	− 71
−180	− 36	− 99	− 86	− 56	235	48	−167		257	96	−151	− 77
−120	− 29	− 62	− 51	− 30	210	42	−125		216	83	− 98	− 44
− 60	− 7.0	3.0	− 3.7	− 1.1	128	26	− 17		112	47	− 7.2	4.2
− 0	38	61	42	24	108	81	−122		95	64	81	28

τ	ℌ = 6.71			ℌ = 8.06			ℌ = 11.92			ℌ = 13.85	
	𝔍 $W=464$ (*Fig.* 13.)	𝔍 $W=719$ (*Fig.* 15.)	𝔍 $W=1770$ (*Fig.* 17.)	𝔍 $W=655.$	𝔍 $W=1040$ (*Fig.* 19.)	𝔍 $W=2410$ (*Fig.* 21.)	𝔍 $W=2.5$ (*Fig.* 22.)	𝔍 $W=1110.$	𝔍 $W=2070$ (*Fig.* 13.)	𝔍 $W=846$ (*Fig.* 24.)	𝔍 $W=1290$ (*Fig.* 26.)
+ 0°	45	30	12	58	0	− 1.8	271	26	− 1.8	40	0
+ 60	104	59	21	155	32	− 7.9	281	134	0.0	163	33
+120	148	95	32	195	76	− 8.1	283	176	9.6	196	131
+180	166	116	39	204	97	− 7.4	281	185	18	203	156
+120	140	95	28	126	62	3.3	243	115	17	152	103
+ 60	79	49	13	31	15	7.3	249	27	12	65	36
− 0	17	1.7	4	54	18	17	273	15	15	40	0
− 60	42	7.6	3	152	66	30	281.7	110	28	142	31
−120	86	38	8	194	112	37	282	143	42	182	131
−180	104	56	11	205	131	41	281	151	49	199	157
−120	85	44	8	126	78	24	244	90	28	149	107
− 60	45	24	7	32	17	8.4	247	19	7.6	50	38
− 0	40	31	11	55	5	− 1.5	271	24	− 1.7	39	.5
	$W=591$ (*Fig.* 14.)	$W=1110$ (*Fig.* 16.)		$W=782$ (*Fig.* 18.)	$W=1630$ (*Fig.* 20.)		$W=971.$	$W=1170.$		$W=1110$ (*Fig.* 25.)	$W=1830$ (*Fig.* 27.)
+ 0°	33	22		10	4.5		83	− 1.7		0.0	− 1.0
+ 60	74	41		89	− 1.3		168	93		93	− 2.1
+120	118	61		159	2.6		192	159		163	14
+180	137	74		179	6.4		197	175		179	27
+120	116	58		107	8.9		117	111		125	24
+ 60	63	27		19	10		12	22		43	21
− 0	7	0		11	25		81	− 12		2	32
− 60	19	0		95	48		144	73		87	66
−120	58	14		163	63		162	127		161	97
−180	76	25		182	70		166	139		179	109
−120	61	19		112	41		95	83		125	67
− 60	32.5	14		23	15		24	14		42	26
− 0	33.3	22		9	3		86	− 2		9	− .5

τ	$\mathfrak{H} = 15.78$				$\mathfrak{H} = 23.5$			$\mathfrak{H} = 33.54$			
	$\mathfrak{J}$ $W=527$	$\mathfrak{J}$ $W=1040$	$\mathfrak{J}$ $W=2010$ (*Fig.* 30.)	$\mathfrak{J}$ $W=2800$ (*Fig.* 31.)	$\mathfrak{J}$ $W=2.5$ (*Fig.* 32.)	$\mathfrak{J}$ $W=1070$	$\mathfrak{J}$ $W=2140$ (*Fig* 34.)	$\mathfrak{J}$ $W=2.5$ (*Fig.* 35.)	$\mathfrak{J}$ $W=1070$	$\mathfrak{J}$ $W=2140$	$\mathfrak{J}$ $W=2820$ (*Fig.* 39.)
+ 0°	200	35	3.5	— 8.2	268	97	.4	279.8	88	19	7.9
+ 60	244	153	1.4	— 22	274	160	12	281.1	147	60	5.1
+120	259	200	16	— 24	276	186	74	280.2	176	95	32
+180	262	212	30	— 22	275	195	102	279.8	188	107	46
+120	197	141	30	— 4	263	131	66	276.7	148	70	36
+ 60	115	51	29	14	255	53	23	277.7	84	30	23
— 0	199	31	46	36	269	99	— .4	280.5	90	18	17
— 60	244	132	189	60	273	161	12	281.4	149	67	28
—120	261	204	127	...	275	187	73	281.1	177	107	66
—180	266	216	141	83	273	196	106	279.8	188	120	82
—120	204	148	99	52	261	131	67	277.0	148	79	57
— 60	111	55	40	18	252	54	24	276.7	104	34	21
— 0	203	32	4.9	— 7.7	268	97	— .5	279.8	88	16	7.6
	$W=782$ (*Fig.* 28.)	$W=1420$ (*Fig.* 29.)	$W=2260$		$W=527$	$W=1630$ (*Fig.* 33.)		$W=527$ (*Fig.* 36.)	$W=1630$ (*Fig.* 37.)	$W=2690$ (*Fig.* 38.)	$W=3210$ (*Fig.* 40.)
+ 0	123	11	— 3.5		174	18		194	47	5.6	1.1
+ 60	224	56	— 18		209	102		228	103	18	— 5.3
+120	249	152	— 17		229	141		242	139	40	— .4
+180	256	172	— 7.9		236	150		247	152	49	5.3
+120	187	107	.7		194	86		218	106	34	8.8
+ 60	81	38	11		124	26		171	49	19	12
— 0	131	14	25		175	18		188	48	16	19
— 60	225	63	52		210	99		225	104	42	34
—120	253	164	102		230	143		241	142	73	49
—180	260	184	122		236	153		246	154	83	57
—120	194	121	79		177	90		218	109	53	37
— 60	97	44	31		127	29		169	51	23	16
— 0	128	13	— 3.3		174	18		193	47	3.9	.8

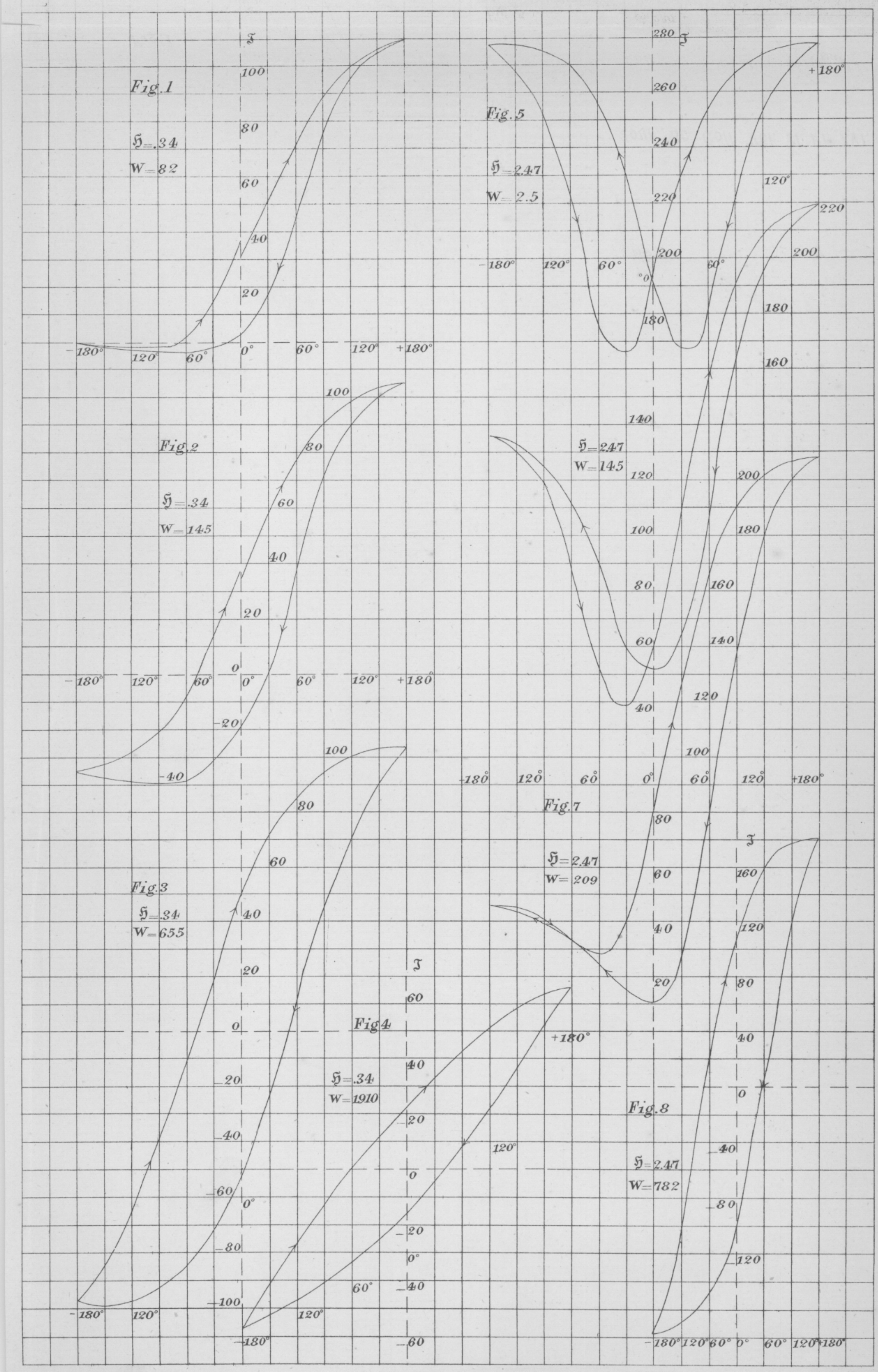


Fig.1
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Fig.2
ℌ=.34
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Fig.3
ℌ=.34
W=655
Fig4
ℌ=.34
W=1910
Fig.5
ℌ=2.47
W=2.5
ℌ=2.47
W=145
Fig.7
ℌ=2.47
W=209
Fig.8
ℌ=2.47
W=782
ℑ
-180°
120°
60°
0°
60°
120°
+180°
20
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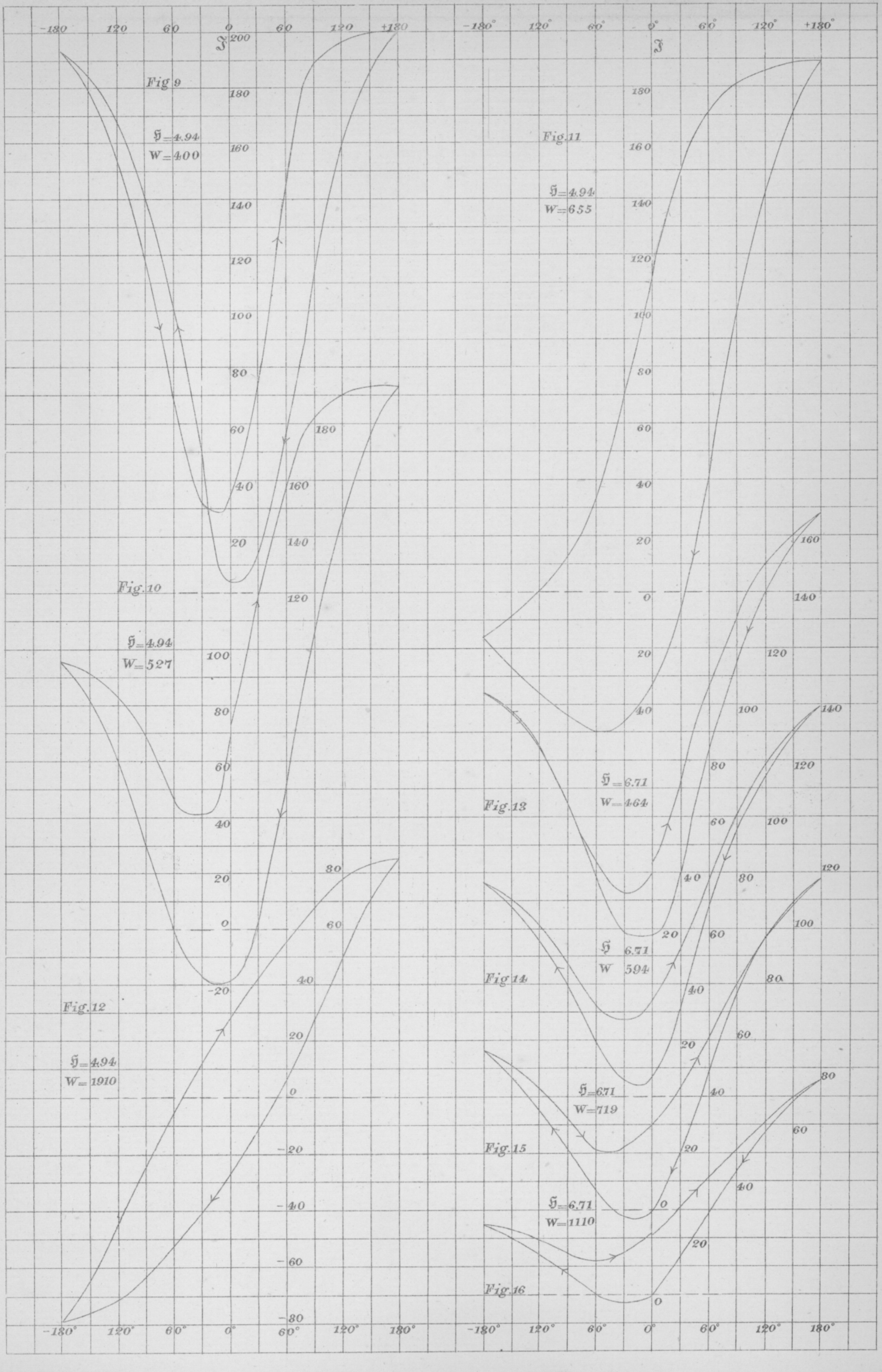
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ℑ
ℑ
Fig 9
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W=400
Fig. 10
ℌ=4.94
W=527
Fig. 12
ℌ=4.94
W=1910
Fig. 11
ℌ=4.94
W=655
Fig. 13
ℌ=6.71
W=464
Fig 14
ℌ 6.71
W 594
Fig. 15
ℌ=6.71
W=719
Fig. 16
ℌ=6.71
W=1110
200 180 160 140 120 100 80 60 40 20
180 160 140 120 100 80 60 40 20 0 -20
80 60 40 20 0 -20 -40 -60 -80
180 160 140 120 100 80 60 40 20 0 20 40
160 140 120 100 80 60 40 20
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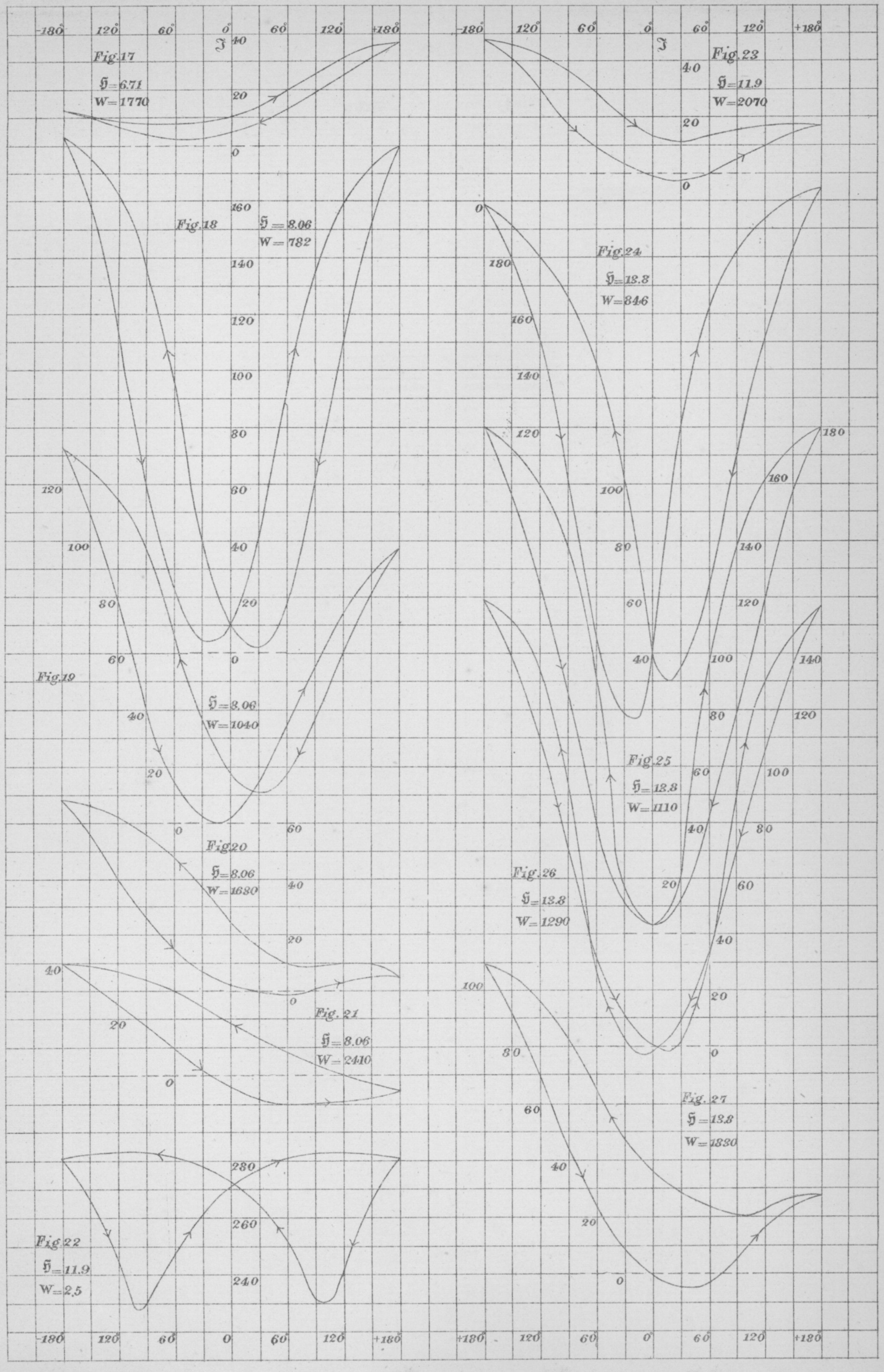
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120°
+180°
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Fig.17
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W=1770
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0
Fig.18
ℌ=8.06
W=782
160
140
120
100
80
60
40
20
0
Fig.19
ℌ=8.06
W=1040
120
100
80
60
40
20
0
Fig.20
ℌ=8.06
W=1630
60
40
20
0
Fig. 21
ℌ=8.06
W=2410
40
20
0
Fig.22
ℌ=11.9
W=2.5
280
260
240
Fig.23
ℌ=11.9
W=2070
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20
0
Fig.24
ℌ=13.8
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160
140
120
100
80
60
40
Fig.25
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W=1110
180
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120
100
80
60
40
20
Fig.26
ℌ=13.8
W=1290
140
120
100
80
60
40
20
0
Fig. 27
ℌ=13.8
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+180°
120°
60°
0°
60°
120°
+180°

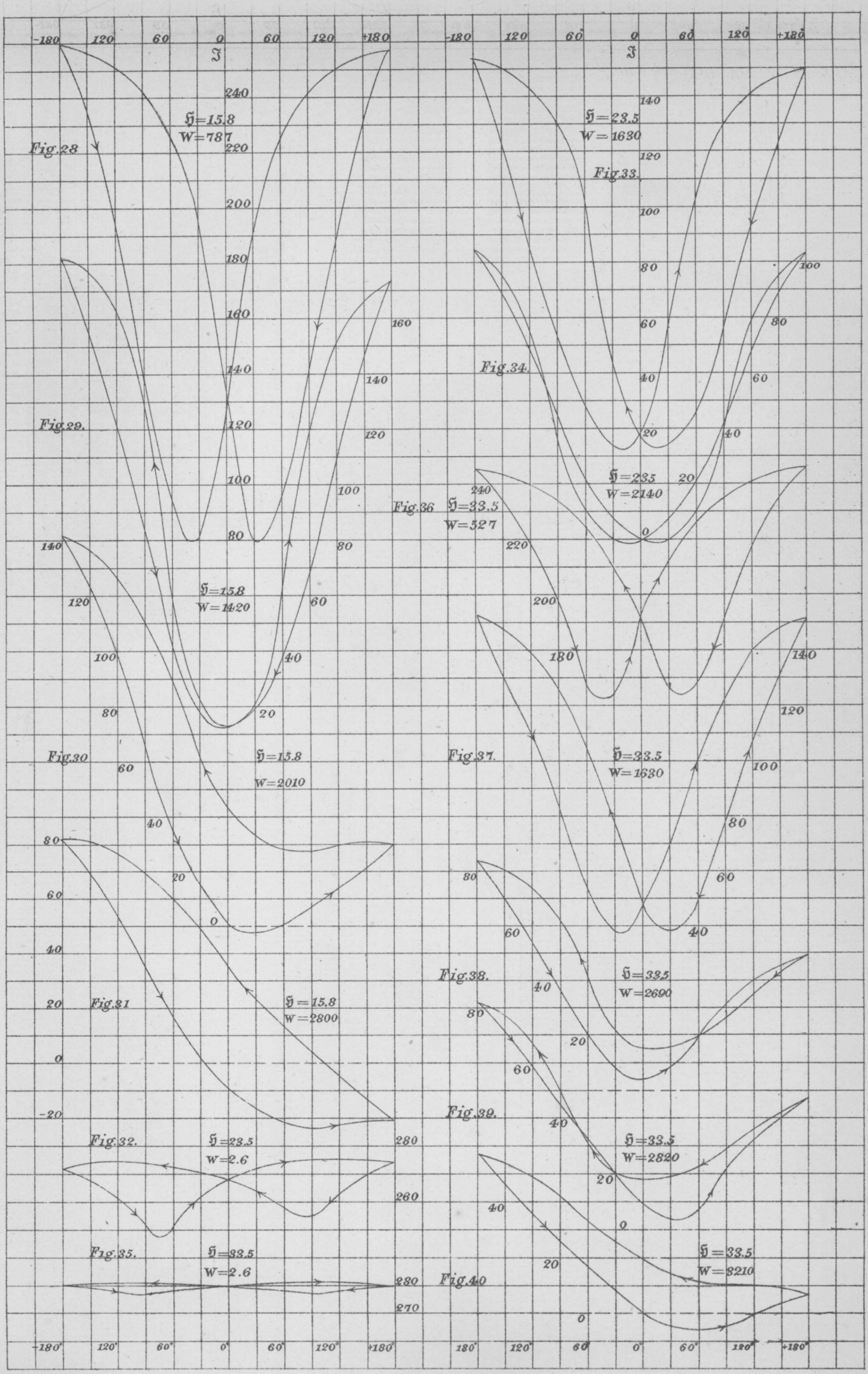
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Fig.28
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W=787
Fig.29
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Fig.30
ℌ=15.8
W=2010
Fig.31
ℌ=15.8
W=2800
Fig.32
ℌ=23.5
W=2.6
Fig.35
ℌ=33.5
W=2.6
Fig.33
ℌ=23.5
W=1630
Fig.34
ℌ=23.5
W=2140
Fig.36
ℌ=33.5
W=527
Fig.37
ℌ=33.5
W=1630
Fig.38
ℌ=33.5
W=2690
Fig.39
ℌ=33.5
W=2820
Fig.40
ℌ=33.5
W=3210
-180°
120°
60°
0°
60°
120°
+180°