The Constants of a Lens.

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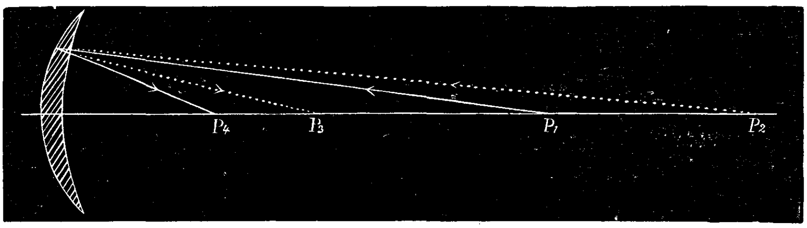
The curvatures of the faces and the index of refraction of the substance of a lens, may be found by the following simple optical method.



In the diagram, let P_1 be the source of light and P_2 the focus due to one refraction, and p_1 p_2 their respective distances from the lens. Also let r_1 and r_2 be the radii of curvature of the front and back faces of the lens respectively. Then

$$\frac{1}{p_1} - \frac{\mu}{p_2} = -\frac{\mu - 1}{r_1} \qquad (1)$$

Considering P_2 as a new source of light we have, for the determination of P_3 the focus due to reflection at the internal back surface of the lens, the equation

$$\frac{1}{p_2} + \frac{1}{p_3} = \frac{2}{r_2} \qquad (2)$$

neglecting the thickness of the lens.

This converging pencil of rays is refracted again at the front face, and the real image is formed at P_4: its focal distance p_4 being given by the equation

$$\frac{1}{p_4} - \frac{\mu}{p_3} = -\frac{\mu - 1}{r_1} \qquad (3)$$

multiplying (2) by μ, and adding (1) (2) (3) we get

$$\frac{1}{p_1}+\frac{1}{p_4}=-\frac{2(\mu-1)}{r_1}+\frac{2\mu}{r_2}$$

This shows that the lens is equivalent to a mirror whose curvature is equivalent to $-(\mu-1)/r_1+\mu/r_2$, and is concave or convex according as $(\mu-1)/r_1 <$ or $> \mu/r_2$. This equivalent mirror becomes a plane when $r_1/r_2=(\mu-1)/\mu$.

Let $\frac{1}{\rho_1}$ be the curvature of this equivalent mirror, or as we shall call it for brevity, the "equivalent curvature," then

$$\frac{1}{\rho_1}=-\frac{\mu-1}{r_1}+\frac{\mu}{r^2} \quad \text{.................................. (4)}$$

Now reverse the lens, that is interchange the reflecting and refracting faces, and let the equivalent curvature be $\frac{1}{\rho_2}$ then

$$\frac{1}{\rho_2}=\frac{\mu-1}{r_2}-\frac{\mu}{r_1} \quad \text{.................................. (5)}$$

But the principal focal length of the lens is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{.......................... (6)}$$

From (4) (5) (6) we get at once *

$$\left.\begin{aligned}-\frac{1}{r_1}&=\frac{1}{\rho_2}+\frac{1}{f}\\ \frac{1}{r_2}&=\frac{1}{\rho_1}+\frac{1}{f}\\ \mu&=1-\frac{\frac{1}{f}}{\frac{1}{\rho_1}+\frac{1}{\rho_2}+\frac{2}{f}}\end{aligned}\right\} \quad \text{...... .... (7)}$$

* The signs of r_1 and r_2 are considered with regard to the first position of the lens.

When the thickness t of the lens is taken into account, equation (2) becomes

$$\frac{1}{p_2+t}+\frac{1}{p_3+t}=\frac{2}{r_2}$$

This combined with (1) and (3) gives the relation

$$\frac{1}{\mu\Big/\left(\frac{1}{p_1}+\frac{\mu-1}{r_1}\right)+t}+\frac{1}{\mu\Big/\left(\frac{1}{p_4}+\frac{\mu-1}{r_1}\right)+t}=\frac{2}{r_2}\text{.........}(8)$$

We have a similar equation for the reversed position of the lens, and also, for direct refraction through the lens, the equation (corresponding to (6) above)

$$1\Big/\left(\frac{1}{q_1}+\frac{\mu-1}{r_1}\right)-1\Big/\left(\frac{1}{q_2}+\frac{\mu-1}{r_2}\right)=\frac{t}{\mu}\quad\text{...............}(9)$$

q_1 and q_2 being the respective distances of the light and its image from the front and back faces of the lens. These equations are strictly rigorous and can be worked out to any desired degree of accuracy.

The most favorable values of p_1 and p_4 for minimizing the errors of experiment, are when $p_1=p_4$ i.e. when the light is placed at the center of curvature of the equivalent mirror. In this case, any ray of the pencil is reflected normally at the back surface of the lens, and returns along the same track. Further this gives at once $p_1=p_4=\rho_1$, and (8) reduces to

$$\frac{1}{\rho_1}=-\frac{\mu-1}{r_1}+\frac{\mu}{r_2-t}$$

$$=-\frac{\mu-1}{r_1}+\frac{\mu}{r_2}+\frac{\mu t}{r_2^2}+\frac{\mu t^2}{r_2^3}+\text{..................}$$

Thus the results in (7) are to be corrected to the first order of small quantities, by diminishing $1/\rho_1$ by $\mu t/r_2^2$ and similarly $1/\rho_2$ by $\mu t/r_1^2$

As for $\frac{1}{f}$, the correction is given by

$$\frac{1}{q_2}-\frac{1}{q_1}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-\frac{t}{\mu}\left(\frac{1}{q_1}+\frac{\mu-1}{r_1}\right)^2+\text{.........}$$

The terms depending upon the thickness t absolutely vanish, when $q_1 = -r_1/(\mu - 1)$ and consequently $q_2 = r_2/(\mu - 1)$,—*i. e.*, when the distances of the object and image from their nearest faces of the lens are proportional to the radii of curvature of those faces. In this case, the pencil of light becomes cylindrical within the substance of the lens. This condition may be experimentally realized by using the approximate values of μ and r as given in (7). There are two cases however in which this correction vanishes without using approximate values:—(1.) When the lens is plano-convex, and the parallel rays of light pass in at the plane surface; (2.) when the lens is double convex with equal curvatures, and the object and image are equally distant from their respective nearest faces of the lens.

The above method fails with meniscus lenses in the case when the "equivalent mirror" becomes convex: but in such instances, the curvature of one face can always be found by direct reflection, and when the lens is reversed the equivalent mirror becomes concave. Thus in general, the curvatures of the faces and index of refraction of the substance of a *convex* lens can be found by a method which is essentially the same as that for finding the focal length of a concave mirror.

The method was tested experimentally and gave for curvatures and index of refraction of a particular lens

$$r_1 = 103.8 \text{ cm.} \quad r_2 = 105.3 \text{ cm.} \quad \mu = 1.515$$

Measured with a spherometer these quantities were found to be

$$r_1 = 104.5 \text{ cm.} \quad r_2 = 105.0 \text{ cm.} \quad \mu = 1.514$$

A clear aperture of 1 cm. diameter is sufficient for applying the method.