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Considerations on the Problem of Latitude Variation.

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Introduction.

This paper covers some studies on the zenith telescope as an astronomical instrument extensively used in observing the variation of latitude. Herein the writer intends to call attention to the fact that there are, according to his experience, some remarkable phenomena in the behaviour of the zenith telescope, and further, to show the possibility of finding a method of explaining the z-term and closing sum (schlussfehler) in the variation of latitude in connection with these phenomena. The development is made in the following three sections.

I. Singular behaviour of the zenith telescope.

During the years 1906–1911, the writer was in charge of the observations of latitude variation in the Tokyo Astronomical Observatory, under the auspices of the Geodetic Committee. This was in continuation of the series of observations made successively by Professors H. Kimura and K. Hirayama. The instrument used was a zenith telescope of the usual form, made by Wanschaff in Berlin, No. 800, aperture $81^{mm.}$ and focal length $100^{cm.}$.

In order to determine the value of one turn of the micrometer screw, I made observations of polar stars at their greatest elongations every summer and winter. The total number of determinations during the period amounted to 58. When observing the polar stars in their eastern and western elongations, I found a remarkable fact, viz. that the bubbles of the two levels attached to the zenith telescope moved gradually towards the south, both in the eastern and the western positions of the telescope. Moreover, the magnitude of displacement was larger in winter than in summer, and it varied generally with the length of the observation. This can be seen from the following table, in which the first column is the date of observation, the second and third columns the bubble displacements of the first and second levels in units of division, the fourth the observed stars, the fifth the interval of observation, and the sixth the temperature of the observing room.

TABLE I.

Telescope West.

(+ increase, − decrease)

Date			Level I.	Level II.	Star	Interval	Temperature
			d	d		m	o
1906	August	4	+ 0.20	+ 0.05	Polaris	33	22.0
,,	Sept.	5	0.00	+ 0.25	,,	27	21.8
1907	August	17	+ 0.20	+ 0.10	51 H. Cephei	22	23.8
,,	,,	18	+ 0.15	+ 0.10	,,	22	23.4
,,	,,	19	0.00	− 0.05	,,	23	24.7
1908	August	21	+ 0.15	+ 0.05	,,	22	23.0
,,	,,	22	+ 0.05	+ 0.25	,,	23	25.5
,,	,,	26	+ 0.10	+ 0.05	,,	25	22.6
,,	,,	27	+ 0.05	+ 0.50	,,	23	22.6
1909	August	8	+ 0.15	+ 0.25	,,	23	22.8
,,	,,	9	+ 0.35	+ 0.55	,,	23	22.2
1911	Sept.	2	+ 0.20	+ 0.30	,,	23	19.8
,,	,,	3	+ 0.15	+ 0.45	,,	23	21.8
	Mean		(+ 0.13)	(+ 0.18)			

Date			Level I.	Level II.	Star	Interval	Temperature
			d	d		m	o
1909	January	23	+ 0.15	+ 0.20	1 H. Draconis	8	1.0
„	„	29	+ 0.45	+ 0.25	„	8	99.8
„	„	30	+ 0.35	+ 0.25	„	8	99.9
„	„	31	+ 0.25	+ 0.50	„	8	2.0
1910	February	2	+ 0.50	+ 0.30	„	8	0.4
„	„	7	+ 0.05	+ 0.05	„	8	5.0
„	March	5	+ 0.70	+ 0.85	δ Ursae Min.	19	98.1
„	„	7	+ 0.75	+ 0.70	„	19	0.8
„	„	10	+ 0.85	+ 0.75	„	19	99.9
„	„	13	+ 0.40	+ 0.55	„	19	0.6
1911	January	25	+ 0.85	+ 0.40	1 H. Draconis	8	2.2
	Mean		(+ 0.42)	(+ 0.44)			

Telescope East.

Date			Level I.	Level II.	Star	Interval	Temperature
			d	d		m	o
1909	July	24	+ 0.10	0.00	δ Ursae Min.	19	24.0
„	„	25	0.00	+ 0.10	„	20	24.6
„	„	28	− 0.20	− 0.25	„	20	24.1
„	August	2	0.00	0.00	„	20	23.0
1910	October	26	− 0.25	− 0.25	76 Draconis	9	11.9
	Mean		(− 0.07)	(− 0.08)			
1907	February	2	− 0.25	− 0.25	Gr. 750	14	96.8
„	„	5	− 1.00	− 0.75	„	14	99.7
„	„	9	− 0.20	− 0.35	„	14	99.2
„	„	14	− 0.25	− 0.25	„	14	99.8
1908	January	12	− 0.50	− 0.65	Polaris	40	2.7
„	„	16	− 0.60	− 0.65	„	50	2.6
„	„	17	− 0.45	− 0.30	„	45	0.0
„	February	22	− 0.10	− 0.15	Gr. 750	14	7.8

Date			Level I.	Level II.	Star	Interval	Temperature
			d	d		m	o
1909	January	16	– 0.10	– 0.20	Polaris	53	99.2
,,	,,	17	– 0.55	– 0.45	,,	56	98·6
1910	February	3	– 0.45	– 0.50	Gr. 750	14	0.8
,,	,,	7	– 0.10	– 0.10	,,	14	2.8
,,	,,	9	– 0.35	– 0.45	,,	14	2.0
,,	,,	12	– 0.50	– 0.65	,,	14	98.5
,,	,,	15	– 0.45	– 0.45	,,	14	0.7
,,	March	5	– 0.70	– 0.35	51 H. Cephei	23	98.0
,,	,,	7	– 0.20	– 0.35	,,	23	0.2
,,	,,	10	– 0.60	– 0.60	,,	23	99.4
,,	,,	13	– 0.45	– 0.50	,,	23	0.6
1911	February	3	– 0.30	– 0.[illegible]0	Gr. 750	14	0.6
,,	,,	4	– 0.10	0.00	43 H. Cephei	8	2.2
,,	,,	7	– 0.60	– 0.60	Gr. 750	14	0.0
,,	,,	9	– 0.30	– 0.45	,,	14	99.7
,,	,,	10	– 0.50	– 0.35	43 H. Cephei	16	3.3
,,	,,	11	– 0.15	– 0.45	,,	16	6.6
,,	,,	12	– 0.50	– 0.45	,,	16	2.5
,,	,,	21	– 0.30	– 0.45	,,	7	3.8
,,	,,	22	– 0.45	– 0.30	,,	16	5.4
,,	March	2	– 0.55	– 0.50	,,	16	7.8
	Mean		(– 0.40)	(– 0.41)			

1^d of I=1."15 1^d of II=1."15

From this table we see the fact that the displacement in the winter is about four times as large as that in the summer, when reduced to the same interval.

Under such circumstances, the question suggested itself, in making the reduction of observations, "Does the bubble displacement correspond exactly to the change of inclination of the telescope ?"

Thereupon I made tentative reductions from two extreme standpoints:

I. The displacement of bubbles is entirely due to some cause in the level itself, and is not due to the variation of inclination of the telescope. The telescope did not move during the observation.

II. The displacement corresponds exactly to the change of inclination of the telescope. This is the assumption usually adopted. In the reduction, the change between two readings was considered to be proportional to the time interval.

With the first assumption, western elongation gave a greater value of the micrometer than eastern elongation; with the second assumption, on the contrary, eastern elongation gave a greater value than western elongation.[1] This fact is easily seen from the series of observations during February-March 1910, in the following table, in which the first column is the date, the second the polar stars, the third the elongations, the fourth the micrometer value with the first assumption, and the fifth the same with the second assumption.

TABLE II.

Date			Star	Elongation	Micrometer Values with the Assumption I.	II.
1906	August	4	Polaris	E	51.″682	51.″701
,,	September	5	,,	E	579	583
1907	February	2	Gr. 750	W	619	595
,,	,,	5	,,	W	712	651
,,	,,	9	,,	W	698	682
,,	,,	14	,,	W	638	622
,,	August	17	51 H. Cephei	E	628	634
,,	,,	18	,,	E	645	653
,,	,,	19	,,	E	654	656
1908	January	12	Polaris	W	706	654
,,	,,	16	,,	W	723	687
,,	,,	17	,,	W	732	698
,,	February	22	Gr. 750	W	638	634

1 Western elongation was observed in the eastern position of the telescope and eastern elongation in the western position.

Date			Star	Elongation	Micrometer Values with the Assumption I.	II.
1908	August	21	51 H. Cephei	E	51″597	51″603
,,	,,	22	,,	E	658	665
,,	,,	26	,,	E	625	630
,,	,,	27	,,	E	651	654
1909	January	16	Polaris	W	687	678
,,	,,	17	,,	W	681	661
,,	,,	23	1 H. Draconis	E	668	678
,,	,,	29	,,	E	656	675
,,	,,	30	,,	E	692	708
,,	,,	31	,,	E	669	690
,,	July	24	δ Ursae Min.	W	662	665
,,	,,	25	,,	W	659	662
,,	,,	28	,,	W	645	630
,,	August	2	,,	W	643	643
,,	,,	8	51 H. Cephei	E	667	679
,,	,,	9	,,	E	619	644
1910	February	2	1 H. Draconis	E	637	660
,,	,,	3	Gr. 750	W	708	683
,,	,,	7	1 H. Draconis	E	675	678
,,	,,	7	Gr. 750	W	689	684
,,	,,	9	,,	W	747	724
,,	,,	12	,,	W	667	651
,,	,,	15	,,	W	687	662
,,	March	5	δ Ursae Min.	E	664	708
,,	,,	5	51 H. Cephei	W	639	629
,,	,,	7	δ Ursae Min.	E	662	705
,,	,,	7	51 H. Cephei	W	658	642
,,	,,	10	δ Ursae Min.	E	692	741
,,	,,	10	51 H. Cephei	W	721	693
,,	,,	13	δ Ursae Min.	E	630	655
,,	,,	13	51 H. Cephei	W	689	664

Date			Star	Elongation	Micrometer Values with the Assumption I.	II.
1910	October	26	76 Draconis	W	51."620	51."606
1911	January	25	1 H. Draconis	E	661	698
„	February	3	Gr. 750	W	652	634
„	„	4	43 H. Cephei	W	707	704
„	„	7	Gr. 750	W	734	700
„	„	9	„	W	663	641
„	„	10	43 H. Cephei	W	717	693
„	„	11	„	W	646	629
„	„	12	„	W	716	689
„	„	21	„	W	674	652
„	„	22	„	W	705	683
„	March	2	„	W	629	598
„	September	2	51 H. Cephei	E	595	609
„	„	3	„	E	583	619

Although systematically different values are obtained from the different elongations on the different assumptions, the mean value from both elongations on the first assumption is nearly equal to that from both elongations on the second assumption. So these mean values may be looked upon as giving approximately the real value of the micrometer.

An assumption, to hold good, must be of such a nature as to yield theoretically the same value from both elongations. From this point of view both of the above assumptions are to be rejected, and we must find an intermediate one which will bring both elongations into conformity.

It is manifest that such a reduction, provided its validity be definitely granted, will need a diminishing factor less than unity. In order to determine the diminishing factor I made a complete reduction of my series of elongation observations, forming the equations of condition in the following form,—

$$A + Bt + C\theta = a + Db$$

where A.........unknown constant
B.........yearly change of the micrometer value
C.........temperature coefficient
D.........diminishing factor
t.........time
θ ....temperature
a.........value of micrometer corresponding to the first assumption.
b.........excess of the micrometer value on the second assumption over that on the first assumption.

For the sake of convenience, I took the year 1910.0 and 7°.0 for the origin of the time and temperature respectively.

Assigning equal weights to all the equations of condition, I treated them by the method of least squares. The solution of the normal equations gave, among others, as the most probable value of the diminishing factor D,

$$D = 0.54 \pm 0.17 \qquad \text{mean error.}$$

This result shows that the regular displacement of the level bubbles is due to two different causes :

I.........A southward displacement of the bubbles due to some cause in the level, independent of the motion of the telescope. This accounts for about half of the total motion.

II.........A regular northward depression of the telescope, corresponding to nearly half of the total displacement of the bubbles.

Owing to these phenomena we should obtain for the micrometer too large a value from western elongation and too small a value from eastern elongation on the first assumption; conversely, too large a value from eastern elongation and too small a value from western elongation on the second assumption.

The first phenomenon may possibly be a disturbance due to the heat of the observer and of the reading lamp. When observing latitude by the Talcott-Horrebow method, the proximity of the observer to the instrument is of short duration; so that the effect would not appear, owing to a reason to be discussed later. And, even if the effect of this were considerable, it would be eliminated from the final result, as the relative position of the observer and the levels does not change in the two positions of the telescope. Therefore I did not try to make any further investigation on this point.

As to the second phenomenon, we can take into consideration the following three causes, —

i. Flexure of the telescope.

ii. Differential change of the telescope stand.

iii. Gradual tilting of the pier and the ground.

A gradual change of the flexure of the telescope tube may be probable, according to K. Hirayama[1], although it does not seem to be very effective, considering the structure of our zenith telescope. And this cannot be the sole cause ; for if it were, the displacement of the bubble must be wholly attributed to a cause in the level itself, as the flexure of the telescope would by no means appear on the level. As this consequence is of course unnatural and also improbable, we are led to seek some other causes. At any rate, as the effect of flexure on latitude from the observation of a star pair, when we begin with the southerly star, has the tendency to cancel that from a star pair, when we begin with the northerly star, the final effect will tend to vanish, if these distinct pairs are impartially contained in a group. Moreover, even if there be a residual, it would be eliminated by the chain method reduction, as it can be looked upon as persistent with the star pairs. So I shall not attempt any further discussion of this subject.

As to the disturbance of the telescope mounting, I can first of all take into consideration the thermal effect of the observer's body and the unsymmetry of the meteorological conditions with respect to the instrument. In winter the wind blows mostly from the north, producing a draught of cold air ; in summer, the south wind prevails, forming a warm air current. These disturbances, combined together, may cause a certain unsymmetrical distribution of heat in the telescope mounting in some way, and therefore a certain change in the inclination of the telescope. As the heating

1) Astronomische Nachrichten, Nr. 4332.

effect varies with the difference of temperature between the body and the instrument, the disturbance would vary inversely with the temperature, i.e., it would be greater in winter than in summer. This sequence agrees well with the observed phenomenon, and we may take this thermal effect as one cause of the behaviour of the zenith telescope.

With respect to the third cause, I may notice the fact that the horizontal pendulum observers in several parts of the world have revealed a remarkable unsteadiness of the ground. The most conspicuous of the regular movements has a period of one solar day, being due to the effect of the solar radiation on the earth's surface.

E. von Rebeur-Paschwitz did pioneer work with the horizontal pendulum of his original form at Karlsruhe (1887), Strassburg (1892–94), and Nicolaiew (1892), to the effect that the pendulum swinging in the prime vertical showed a regular movement in the period of one solar day, and took the southernmost position in the evening and the northernmost position in the morning, as is here shown, —[1]

TABLE III.

Component in the meridian (pendulum swinging in the prime vertical) North +, South −.

Local Mean Time	Karlsruhe	Strassburg	Nicolaiew
0ʰ	+0.″086	+0.″017	+0.″040
2	−0. 065	−0. 016	+0. 024
4	−0. 151	−0. 057	+0. 003
6	−0. 200	−0. 079	−0. 016
8	−0. 184	−0. 064	−0. 030
10	−0. 192	−0. 034	−0. 037
12	−0. 108	−0. 011	−0. 037
14	−0. 016	+0. 015	−0. 029
16	+0. 135	+0. 050	−0. 010

1) Astronomische Nachrichten, Nr. 3148.

Local Mean Time	Karlsruhe	Strassburg	Nicolaiew
18^h	+0.″225	+0.″073	+0.″013
20	+0. 237	+0. 065	+0. 035
22	+0. 204	+0. 041	+0. 045

His result at Strassburg (1892–94) was expressed in the following résumé :[1]

The epoch of daily minimum or southern elongation of the pendulum lies generally between 5^h and 6^h. The daily maximum or northern elongation comes always in the morning between 18^h–20^h, or slightly later in winter. The amplitude varies between 0.″1 and 0.″2.

After the early death of von Rebeur, Ehlert took charge of the observations at Strassburg (1895–96). His result was quite similar to that of his predecessor, as may be seen from the table below.[2]

TABLE IV.

Date	Southern elongation	Northern elongation	Amplitude
Jan. 15	$3.^h0$	$20.^h5$	0.″049
Feb. 15	3. 5	20. 0	0. 067
March 15	5. 5	20. 8	0. 138
May 1	6. 0	18. 0	0. 201
May 23	5. 9	18. 0	0. 138
June 15	5. 7	17. 9	0. 124
July 15	5. 3	17. 9	0. 135
Aug. 15	6. 0	19. 2	0. 187
Sept. 15	5. 5	20. 2	0. 208
Oct. 15	4. 5	20. 5	0. 053
	November and December, indeterminate		
Mean	$5.^h1$	$18.^h9$	

1) Gerlands Beiträge zur Geophysik, Bd. II.
2) Ditto Bd. IV.

He concludes :

Die „tägliche Periode" besteht in einer Schwankung des Erdbodens, welche im Mittel 0.″112 beträgt ; in der Tiefe von 5 m findet das Maximum der Nordablenkung im Mittel um $18^h.9$, derjenigen nach Süd um $5.^h1$ statt. Die Ursache liegt in der Ausdehnung der von der Sonne erwärmten Erdoberfläche, welche sich auch nach der Tiefe hin unter Abschwächung und Verzögerung geltend macht.......Klarer Himmel und grosse Temperaturoscillation verstärken das Phenomen......

According to Hecker's long series of observations at Potsdam in an underground room at the depth of 25 metres (1902–1909)[1], the pendulum occupied the extreme southern position in the evening and northern in the morning, just as in the preceding cases.

	Southern elongation	Northern elongation
March, April, May	6^h	17^h
June, July, August	6^h	18^h
September, October, November	6^h	15^h
December, January, February	6^h	15^h

The horizontal pendulum observations at Kyoto (1910–1911) by Shida[2] gave a similar result, showing a solar daily variation mainly in a SW and NE direction; the pendulum took the extreme south-west position in the evening and the extreme north-east position in the morning. The elongations in the meridian run as follows:—

1) Veröffentlichung des Königl. Preuszischen Geodätischen Institutes, Neue Folge Nr. 32, 49.

2) Memoirs of the College of Science and Engineering, Kyoto Imperial University. Vol. IV.

	Southern elongation	Northern elongation
April - June	8^h	21^h
July - September	10^h	21^h
October - December	6^h	20^h
January - March	6^h	20^h

Observations at Freiburg i. S. made by Hecker at a depth of 190 metres below the surface of the ground (1910–1911) scarcely showed the solar effect. It was almost insensible or negligible, the amplitude not exceeding 0.″001. From these facts we can infer that the solar thermal effect is the more pronounced the shallower the ground work, as might also be guessed from mere conjecture.

While most horizontal pendulum rooms are deep and are kept in as invariable a temperature as possible, our astronomical telescope piers are, on the contrary, rather shallowly founded and are subjected to the diurnal change of temperature in its full-range, from their exposure to the free air. Moreover, astronomical observations are naturally only practicable in clear weather, when the change of temperature of the environment of the pier is great and rapid. Under these circumstances we are led to conclude that the ground and pier, on which the telescope is set up, are undergoing a continuous tilting whilst it is being used in the night, the meridian component of which is in the direction, north downward or south upward, and the magnitude of which is far greater than that experienced in the horizontal pendulum observations. There is, however, a contradiction between the European and Japanese observations. In Europe the amplitude is at its maximum in summer and its minimum in winter; in Japan[1], on the contrary, it is at its maximum in winter and its minimum in summer. This difference may be due to some meteorological causes, depending on the latitude. For low latitudes the latter variation may hold good. It is also

1) Shida, loc. cit.

to be noticed that the rate of the tilting motion increases from evening until midnight.

In this way we are led to the conclusion that the observed phenomenon is to be understood as principally due to the second and third of the above causes, whose effects can be looked upon as generally varying inversely with the temperature during the time of our observations.

The above cited singular behaviour of the zenith telescope was deduced from the regular shift of the level bubbles, as experienced by myself. In order to ascertain whether the same phenomenon is noticed in other observatories, I made enquiry of several latitude observers in various parts of the world respecting this subject. Fortunately, almost all the specialists were kind enough to favour me with detailed answers, for which the writer tenders his most sincere thanks.

Prof. H. Kimura and Dr. M. Hashimoto of the Mizusawa Latitude Observatory recognized the phenomenon in the zenith telescope there used. We can also see the tendency from the results of their elongation observations, as given in "Resultate des Internationalen Breitendienstes," Bd. I., pp. 19-20. In the determination of the value of the micrometer during the period 1899 Dec. 27-1900 March 11, eastern elongations constantly give larger values than western. This corresponds to the case when the reduction was made on assumption II. (*supra*).

I could hear nothing from Tschardui observers. But the series of determinations given in "Resultate," Bd. IV., page 74, makes me suppose that there is also the same tendency there. In the observations during the period 1906 January 19—February 28, the mean value from western elongations (60.″268) is decidedly larger than that from eastern elongations (60.″148). This may be caused by the method of reduction on assumption I. If such is really the the case at Tschardui, we can fairly well account for the fact that the temperature coefficient of the Tschardui micrometer comes out notably larger than those of the other stations. This is because winter observations are principally of western elongation while summer observations are mostly of eastern elongation; so that the winter value comes out too large, but the summer value too small, giving finally too large a temperature coefficient.

Dr. G. Bemporad of the International Latitude Station at Carloforte reported that his instrument showed variations during elongation observations, but that the sense was not systematic. However, Dr. V. Fontana, the director of the station, wrote me afterwards that he had begun some researches on the problem of the systematic shift of the level bubbles, and desired to know my results.

Dr. F. E. Ross of the Gaithersburg Latitude Observatory informed me that he experienced a similar phenomenon, and considered it to be principally due to a temperature effect in the ground, which usually shows a progressive change one way or the other.

Dr. E. I. Yowell of Cincinnati Observatory favoured me with a letter, stating that his telescope shows a similar tendency. He ascribes that effect to the heat radiation from the observer and the reading lamp.

Dr. W. F. Meyer of the Latitude Observatory at Ukiah kindly reported that his own experience was limited, but that his predecessor, Dr. Schlesinger, had observed the same tendency. The following was extracted from the record book, which gives Dr. Schlesinger's opinion in the matter: "there is reason to believe, from an inspection of the results, that these changes in the levels are not wholly due to a real change of inclination of the telescope. No doubt the presence of the observer for so long a period at one side of the levels has an injurious effect upon the levels."

At Union Observatory, Johannesburg (Astronomer, Prof. R. I. Innes), the method of polar star elongations was not employed.

At Kasan Observatory (Dr. M. A. Gratschew) the instrument and method were different.

The following astronomers have kindly let me know that they experienced no systematic movement of the level bubbles.

Prof. E. Doolittle of Flower Astronomical Observatory, University of Pennsylvania. (Warner & Swasey's zenith telescope).

Prof. J. Bonsdorff of Pulkovo Observatory.

Dr. E. Schoenberg of the University Observatory at Jurjew (Dorpat): (transit instrument of broken type made by Repsold).

After all, it may be concluded that the majority of the most careful latitude observers (especially the international observers) experience a systematic shift of the level bubbles, in the same manner as I have already described.

Thus I have reached the position of being able to make the following résumé, in so far as I have obtained results from the above investigations:—

In most cases the level bubbles of the zenith telescope make a southward shift systematically, when the observer has been near the telescope for a tolerably long time, the magnitude of which varies inversely with the temperature.

This movement may be considered as due, partly to the observer's direct effect on the level, and partly to the tilting of the telescope mounting, owing to the observer's heat disturbance, in conjunction with some meteorological conditions. The regular change in the ground from the solar radiation may also contribute to the latter, although to a comparatively slight degree.

It may not be out of place to remark that we may expect the counter effect to the above phenomenon; that is, when the observer recedes from the instrument after a tolerably long stay near it, the bubble would move back in a northerly direction towards its original position, being freed from the thermal disturbance. This was often experienced by some observers. It seems also to be very desirable from my standpoint that hereafter the elongation observations of polar stars should be more frequently and regularly made and the reduction performed more exhaustively, so as to throw further light upon the subject.

2. A theory of the motion of the level Bubbles.

In using a screw micrometer we are accustomed to eliminate the so-called "dead motion" of the screw by turning it always in the same direction. The same principle is applied in determining the value of a division of the spirit level, always making the bubble come to rest from the same direction. This precaution implicitly recognizes the existence of a failure in the function of the level. It seems to me rather curious that in the practical work of star observation the level is taken as a perfect instrument and the precaution needed in the above case is utterly neglected. Here I intend in the following lines to show that there actually exists such a failure in the function of the spirit level. I will first proceed from the theoretical standpoint and then give an experimental proof.

The motion of a level bubble can be looked upon as analogous to a damped oscillation of a simple pendulum ; the equation of motion accordingly takes the following form, —

$$\frac{d^2\theta}{dt^2} + 2p\frac{d\theta}{dt} + n^2\theta = 0$$

where p and n are constants and θ is the deviation of the bubble from the position of equilibrium. The applicability of this equation was experimentally tested by Bonsdorff[1] and further discussed by Orloff.[2]

The integral of this differential equation comes out in the following three forms, according to the cases, $n>p$, $n=p$, $n<p$ respectively, —

(i) $n>p,\ \theta=e^{-pt}\{a_1\cos\mu t + a_2\sin\mu t\}$

(ii) $n=p,\ \theta=e^{-pt}\{a_3t + a_4\}$

(iii) $n<p,\ \theta=b_1e^{-a_5t} + b_2e^{-a_6t}$

$$\text{where}\begin{cases}\mu^2 = n^2-p^2\\ a_5 = p\left\{1-\sqrt{1-\frac{n^2}{p^2}}\right\}\\ a_6 = p\left\{1+\sqrt{1-\frac{n^2}{p^2}}\right\}\end{cases}$$

Here the quantity n depends on the radius of curvature of the level, and the quantity p is a function of the bubble length and of the viscosity of the liquid.

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1) Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowo, Bd. II, p. 43.

2) Ditto Bd. II, p. 137.

When the radius of curvature is small, and the bubble length fairly long, the case (i) occurs, exhibiting a periodic motion. But in the sensitive level, the radius of curvature is necessarily long, but the bubble length cannot be proportionately long. This class of instrument corresponds to the cases (ii) or (iii), which represent an aperiodic motion. In these two cases the bubble comes to rest asymptotically, that is, after the lapse of an infinite time. In other words, the bubble of our sensitive level does not come to its destination theoretically in a finite time. In practice, however, to wait even a pretty long time is useless, as some other disturbing cause may interfere with it in the meanwhile.

Let t_1 be the time interval, during which we wait for the resting of the bubble (practically, a minute or two), and θ_1 the corresponding value of θ. Thus, —

$$\theta_1 = e^{-pt_1}(a_3 t_1 + a_4) \qquad \text{or} \qquad \theta_1 = b_1 e^{-a_5 t_1} + b_2 e^{-a_6 t_1}$$

Then we are referring to θ_1 instead of 0 as the resting point of the bubble. As this discrepancy can be looked upon as caused by internal friction or viscosity of the liquid, the sense of θ_1 will be naturally opposed, when the direction of motion is opposite, giving $-\theta_1$. Therefore, when the bubble has moved from opposite directions the resting points would differ by the quantity $2\theta_1$. This property would afford a method of determining the magnitude of θ_1, which we may call the resistance of the level.

The occasional lack of parallelism in the two level bubbles of the zenith telescope, commonly experienced by latitude observers, can partly be explained by the above phenomenon, as the parallelism would change by the sum of the resistances of the two levels, when the two bubbles come to rest from opposite directions.

Having thus examined the existence of a defect of the level from the theoretical standpoint, I proceed now to ascertain the order of magnitude of the resistance θ_1 from the experimental side, depending on the principle cited above.

The instrument I made use of in my experiments was a level trier made by Hilldebrand in Freiburg, and the levels subjected to the examination were the following nine pieces :—

No. 1.	The hanging level of the transit instrument, Bamberg No. 7958.
Nos. 2, 3.	The latitude levels of the above.
No. 4.	The hanging level of the transit instrument, Bamberg No. 7959.
Nos. 5, 6.	The latitude levels of the above.
No. 7.	The hanging level of the transit instrument, Bamberg No. 11508.
Nos. 8, 9.	The latitude levels of the above.

First the level was fitted to the trier in such a way that the bubble moved in increasing sense of division in accordance with the increase of the micrometer reading of the trier. Then the micrometer was turned in increasing sense and set, say, to a reading α. The bubble would move increasingly and come to rest at $\beta-\theta_1$, where θ_1 is the resistance. Next, the micrometer was further turned slowly up to, say, division $\alpha+10$. Hereupon it is turned back to $\alpha-1$; then again it is set at the reading α, theoretically bringing the level back to the original position. To this operation the bubble conforms with considerable lag of time, owing to its inertness, and would come to rest in decreasing sense at the point $\beta+\theta_1$. Let the reading of the bubble centre for the first position be S_1 and for the second position be S_2, then we have

$$\theta_1 = \tfrac{1}{2}(S_2-S_1).$$

This process was applied to two such points alternately; for the first point, the motion was in the order increasing-decreasing, for the second point in the order decreasing-increasing. This will eliminate the effect of the gradual change of the pier, not to speak of determining the value of one division of the level. The result of my experiments is given in the following table V, in which the resistance θ_1 is expressed in terms of the unit of division of the levels.

TABLE V.

Level	Date		Bubble Length	Resistance θ_1	No. of Observations
			d	d	
No. 1.	1915 Feb.	17	24	0.034	58
	„ „	18	24	0.042	30

1 division=1.″32

Level	Date		Bubble Length	Resistance θ_1	No. of Observations
			d	d	
No. 2.	1913 Dec.	31	24	0.022	52
	1914 Jan.	8	24	0.026	42
	„	10	24	0.035	44
	„	12	24	0.042	24
	„	13	24	0.047	32
	„	15	23	0.029	28
	„	31	20	0.049	50
	Feb.	7	20	0.031	26
	„	12	21	0.029	54
	„	28	20	0.029	24

1 division=1.″32

Level	Date		Bubble Length	Resistance θ_1	No. of Observations
			d	d	
No. 3.	1913 Dec.	31	26	0.020	52
	1914 Jan.	8	26	0.012	52
	„	10	26	0.023	44
	„	12	27	0.016	24
	„	13	26	0.024	32
	„	15	26	0.050	28
	„	31	20	0.030	50
	Feb.	7	21	0.031	26
	„	12	21	0.060	54
	„	28	20	0.027	24

1 division=1.″33

Level	Date	Bubble Length	Resistance θ_1	No. of Observations
No. 4	1915 Jan. 6	d 25	d 0.069	24
	„ 16	27	0.025	18
	„ 18	26	0.057	30
	„ 19	26	0.062	34
	„ 21	28	0.043	14
	Feb. 13	37	0.072	21

1 division=1.″22

Level	Date	Bubble Length	Resistance θ_1	No. of Observations
No. 5	1914 Oct. 2	d 26	d 0.587	32
	„ 4	26	0.464	28
	„ 9	25	0.522	32
	1915 Feb. 14	26	0.479	52
	„ 15	27	0.462	16
	„ 16	27	0.441	28

1 division=1.″04

(This level is not fit for use.)

Level	Date	Bubble Length	Resistance θ_1	No. of Observations
No. 6	1914 Oct. 2	d 26	d 0.053	32
	„ 4	26	0.202	28
	„ 9	25	0.080	32
	1915 Feb. 14	26	0.055	52
	„ 15	27	0.047	16
	„ 16	27	0.084	28

1 division=1.″06

Level	Date	Bubble Length	Resistance θ_1	No. of Observations
No. 7.	1915 Feb. 20	d 24	d 0.045	42
	„ 22	24	0.012	68

1 division=1.″06

Level	Date	Bubble Length	Resistance θ_1	No. of Observations
No. 8	1915 Jan. 20	d 20	d 0.050	24
	„ 21	20	0.073	20
	„ 21	32	0.030	10

Level	Date	Bubble Length	Resistance θ_1	No. of observations
		d	d	
No. 8	1915 Jan. 26	33	0.006	42
	„ 27	26	0.040	20
	„ 28	26	0.033	48
	„ 29	26	0.029	52
				1 division=1.″26
		d	d	
No. 9	1915 Jan. 20	19	0.079	24
	„ 21	19	0.060	20
	„ 21	33	0.020	10
	„ 26	34	0.010	42
	„ 27	25	0.040	20
	„ 28	25	0.022	48
	„ 29	25	0.021	52
				1 division=1.″28

These results show that the resistance θ_1 of the level is not of a negligible magnitude, so far as the degree of accuracy required in the zenith telescope observations is concerned, and we can take it as established that this defect is a common property of the spirit level, as we have confirmed its existence both from the theoretical and the experimental standpoints.

Hitherto we have been considering the motion of the bubble in case the level itself is at rest. But as the motion is purely of a relative nature, we can convert the above obtained result to the case when the bubble is at rest and the level moves.

Thus, when the level is put into slight movement from rest, the bubble will accompany it, and will fail to show the real movement of the level. If the movement of the level is less than or equal to the resistance θ_1, as above obtained, the bubble would show no displacement relative to the level.

Moreover, when the level continues to move further, the bubble will follow it to a certain degree. The equation of motion takes then the following form, taking the origin at the highest point of the level,—

$$\frac{d^2\theta}{dt^2}+2p\frac{d\theta}{dt}+n^2\theta=A$$

where the quantity A depends on the quality and movement of the level. Under the same conditions as before, in the case of our sensitive level, we have

$$\theta_{t_1}=\frac{A}{n^2}+\theta_1$$

This equation shows that another perturbing term interferes in this case. Thus it is to be concluded that the indication of the level bubble has an error greater than the resistance θ_1, when the level itself is in motion.

Now I showed in the foregoing part of this essay that there are reasons to believe in an unsteadiness of the telescope and pier, not to speak of the seismic movement of the ground. From the above investigations, we see that the level cannot indicate instantaneously the varying position of the telescope in presence of these disturbances. So it is manifest that the spirit level which we now use is not suited to fulfilling our requirements with the degree of accuracy demanded in modern astronomical measurements. We are faced with the necessity of using some other means in order to realize the present expectations of practical astronomy.

3. Application to the Talcott-Horrebow observations and deduction of effect on the variation of latitude.

In the first section of this essay I discussed certain systematic motions of the levels and zenith telescope, which can be looked upon as due, firstly, to the disturbance from the observer, and secondly, to a terrestrial cause. In the second section I investigated the failure of the function of the level which can be taken as a common defect of the level. Conversely, I can now conjecture how the telescope and level behave under such circumstances. When the observer approaches the telescope, the stand would first suffer a thermal disturbance, and cause the said effect, owing to the unsymmetry of meteorological conditions. To this the level bubble will not respond immediately, because of the defect discussed above. When the disturbance has exceeded a certain limit, the level begins to indicate it to a certain degree. The thermal effect on the level would appear later, as the levels are more distant and better protected. After a fairly long time, the resulting effect would be the observed systematic shift of the level bubbles, in which the regular tilting of the ground partakes to some extent.

Now the observation of latitude variation is based on the cyclical system of star groups, consisting of pairs of stars selected for the Talcott observations. This method is the so-called chain method. As the result of this procedure, we obtained the polar motion and Kimura's z term. The closing sum is also a product of the chain method. The z term and closing sum form the principal enigmas of present-day practical astronomy.

Now for the first subject numerous causes have been proposed, among which we may mention the following,—

1. Yearly atmospheric refraction.
2. Yearly cosmic refraction.
3. Improper value of the parallax and proper motion of the observed stars.
4. Ditto of nutation and aberration.
5. Actual change in the earth's centre of gravity.
6. Result of computation.
7. Latitude variation of short period.

All these hypotheses may be in a greater or less degree probable, and at the same time no one of them has yet such a firm basis of proof, as to secure our universal assent.

The same may be said of the closing sum, for which the following hypotheses may be mentioned, —

1. Erroneous value of aberration constant.
2. Diurnal atmospheric refraction.
3. Latitude variation of short period.

The first explanation has been universally accepted. But the usually adopted value 20.″47 is quite consistent with the recently determined value of the solar parallax, and does not permit so much increase as to account for the whole closing sum. The second and third are nothing more than conjectures.

Under these circumstances, it does not seem utterly superfluous to make new suggestions, to be further discussed and investigated by a wider circle. With this intention, I venture to declare that the above puzzling subjects may possibly be explained by the phenomenon above discussed, in connection with the defect of the spirit level.

Now, provided that the considerations in the earlier part of this essay on the disturbance of the zenith telescope are applicable to the case when the Talcott-Horrebow observation is made, in which the proximity of the observer is of short duration, the first part of the disturbing effect only may come into play, so that the change of inclination of the telescope, due to the thermal disturbance, would not appear in the position of the level bubbles. Therefore, the level bubbles will be situated too far north of their due position. The result of this is that the corrections depending on the level reading are positively too large or negatively too small, giving finally too large a value of latitude. The principal part of this error varies inversely with the temperature. So the correction to be applied to the latitude is of negative sign and of varying magnitude, depending on the seasons and the hour of the day.

Let the star groups selected for the chain method be from I to XII, as is adopted in the International Latitude Service. And further distinguish these groups by suffixes according to combinations; suffix 1 when combined with the preceding group and suffix 2 when combined with the following group. The period of observation and the corresponding group are as follows, —

Date.	Corresponding Group.
Nov. 2 — Dec. 6	I_2 & II_1
Dec. 7 — Jan. 4	II_2 & III_1
Jan. 5 — Jan. 30	III_2 & IV_1
Jan. 31 — Febr. 24	IV_2 & V_1

Date:	Corresponding Group.
Febr. 25 — March 21	V_2 & VI_1
March 22 — April 15	VI_2 & VII_1
April 16 — May 11	VII_2 & $VIII_1$
May 12 — June 8	$VIII_2$ & IX_1
June 9 — July 9	IX_2 & X_1
July 10 — Aug. 13	X_2 & XI_1
Aug. 14 — Sept. 22	XI_2 & XII_1
Sept. 23 — Nov. 1	XII_2 & I_1

As the temperature continues to fall during the latitude observation in one night, we may tentatively but reasonably assign the following mean corrections to the latitude corresponding to each of the groups, —

I_1	$-0\overset{''}{.}05$	I_2	$-0\overset{''}{.}05$
II_1	-0.07	II_2	-0.07
III_1	-0.08	III_2	-0.07
IV_1	-0.08	IV_2	-0.06
V_1	-0.08	V_2	-0.05
VI_1	-0.07	VI_2	-0.04
VII_1	-0.06	VII_2	-0.03
$VIII_1$	-0.04	$VIII_2$	-0.02
IX_1	-0.03	IX_2	-0.01
X_1	-0.02	X_2	-0.00
XI_1	-0.01	XI_2	-0.01
XII_1	-0.03	XII_2	-0.03

Now in the reduction of the chain method it is assumed that the latitude is constant during the observation of two groups in one night, and under this supposition the star places are reduced to the mean of the whole system. But when the apparent latitude obtained from observation requires the said correction, the reduction to the mean would consequently need a corresponding revision. Thus in order to find how the above corrections enter into the reduction by chain method, i. e., how to reduce them to a true homogeneous system, we must treat these corrections in a manner exactly identical with the chain method reductions.

I form, therefore,—

Correction to	III–IV	=	Difference of corrections to	III_2 &	IV_1	=	+0."01
	IV–V	=	"	IV_2 &	V_1	=	+0.02
	V–VI	=	"	V_2 &	VI_1	=	+0.02
	VI–VII	=	"	VI_2 &	VII_1	=	+0.02
	VII–VIII	=	"	VII_2 &	$VIII_1$	=	+0.01
	VIII–IX	=	"	$VIII_2$ &	IX_1	=	+0.01
	IX–X	=	"	IX_2 &	X_1	=	+0.01
	X–XI	=	"	X_2 &	XI_1	=	+0.01
	XI–XII	=	"	XI_2 &	XII_1	=	+0.02
	XII–I	=	"	XII_2 &	I_1	=	+0.02
	I–II	=	"	I_2 &	II_1	=	+0.02
	II–III	=	"	II_2 &	III_1	=	+0.01

Hence,

the correction to the closing sum, in the usual sense, = +0."18

Further, in order to find the corrections to the reductions to the mean system, I form,—

for the group III ; correction to	IV–III	=	−0."01
"	V–III	=	−0.03
"	VI–III	=	−0.05
"	VII–III	=	−0.07
"	VIII–III	=	−0.08
"	IX–III	=	−0.09
"	X–III	=	−0.10
"	XI–III	=	−0.11
"	XII–III	=	−0.13
"	I–III	=	−0.15
"	II–III	=	−0.17
	Sum	=	−0.99

I apply similar treatment to the other groups and divide the sums by 12.

The correction to the closing sum is to be distributed equally among the groups, so that the quantity,—

$$\frac{0''.18}{12}\times 11\div 2=0''.083$$

is to be added to the above. The resulting quantities constitute the reductions of the star places to a true homogeneous system,—

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0″.000	+0″.005	0″.000	−0″.005	0″.000	+0″.005	+0″.010	+0″.005	0″.000	−0″.005	−0″.010	−0″.005

Combining these with the said corrections to the latitude observation, we apply the usual process of reduction and form differences with the mean of all of them. Arranging them with respect to seasons, we have, —

Mean Date	Corrections	Mean	Resulting Corrections
Jan. 18	−0″.070, −0″.085	−0″.078	−0″.033
Febr. 12	−0.065, −0.080	−0.073	−0.028
Mar. 10	−0.050, −0.065	−0.053	−0.013
Apr. 4	−0.035, −0.050	−0.043	+0.002
Apr. 29	−0.020, −0.035	−0.028	+0.017
May 26	−0.015, −0.030	−0.023	+0.022
June 25	−0.010. −0.025	−0.018	+0.027
July 27	−0.005, −0.020	−0.013	+0.032
Sept. 3	−0.020, −0.035	−0.028	+0.017
Oct. 13	−0.035, −0.050	−0.043	+0.002
Nov. 19	−0.050, −0.065	−0.058	−0.013
Dec. 21	−0.065, −0.080	−0.073	−0.028
	Mean	−0.045	

These are the corrections to the latitude variation which may be looked upon as common to all stations. Therefore naturally it is the correction to that term of latitude variation which is independent of polar motion, or z term.

Now, according to Ross[1], the mean value of z term during the years 1900–1905, can be put in the following analytical form, —

$$+0.''027 \sin (\odot + 170^\circ) \qquad (\odot = \text{sun's longitude})$$

when the effects of stellar parallax, Oppolzer's term in latitude variation and nutation are taken into account and excluded. Hence it follows that the result obtained through my argument is practically sufficient to account for the closing sum and z term.

Now, as the phenomenon discussed by me is considered to arise from some unsymmetry of the meteorological conditions and also from the solar radiation on the ground, its sense should be inverted for the southern hemisphere and is to be considered as an odd function of latitude, vanishing at the equator. So it can be looked upon as varying with $\sin \varphi$. Under such a conception, the effect on the southern observations should be opposite to that on the northern, the correction to the latitude being of positive sign and the correction to the closing sum negative.

The correction to be applied to the latitude variation independent of longitude would be

positively small for January, and

positively large for July.

Therefore, the correction to be applied to the yearly term in the variation of latitude, or z term, is

negative for January, and

positive for July.

The sign is the same as that of the northern observations, and is sufficient to interpret the result obtained from the southern observations.

As to the amplitude of the z term, it depends jointly on the said phenomenon and the seasonal variation, both of which can be looked upon as varying with $\sin \varphi$; so we can consider the amplitude of the correction of z term as dependent on $\sin^2 \varphi$. This signifies that the amplitude of the z term increases from the value

1) Astronomische Nachrichten, Nr. 4593.

zero at the equator, towards north and south latitudes symmetrically. To ascertain whether this is actually the case or not, observations at the equatorial zone are very desirable.

The numerical results so far obtained I do not intend to regard as definitive. But they may serve as a clue for the yet unsolved problems, or an indication of the direction in which we must proceed in order to account for them.

Now, as my argument has been advanced from two causes, viz.

1. some regular movements of the zenith telescope;
2. a defect of the spirit level,

I may also state with diffidence the results to which my considerations have led me, as follows:—

> If we assume a regular change of inclination of the zenith telescope, varying with the seasons and the hour of the day, the three phenomena, that is, z term, closing sum, and the regular shift of the spirit level, can all become explicable together. Therein the defect of the level makes the error enter into the latitude value and gives rise to the z term and closing sum.

My idea, therefore, leads to the conclusion that these enigmatic subjects in the latitude problems would disappear, if the level were really a perfect instrument, or if we were to use another and an ideal means instead of this insufficient instrument.

The photographic observations made with the Cookson floating zenith telescope at the Greenwich Observatory by Mr. Jones[1] are very important in regard to the present problem. The latitude variation obtained from this series was compared with the results of the International Latitude Service. Herein the agreements was particularly improved, when the z term was subtracted from the latter series. As to the aberration constant, the value

$$20.''467 \pm 0.''006$$

was obtained. In other words, this novel series of observations was practically free from the z term and closing sum. Arguing from

1) Monthly Notices of the R.A.S., Vol. LXXV, No. 7.

my standpoint, this is to be looked upon as a natural and necessary consequence.

The notion that the level is not suited to the most delicate measurements is not novel. Already as early as in the year 1893, E. von Rebeur-Paschwitz[1] insisted on the inadequacy of the spirit level to meet the disturbance in precise astronomical measurements from pulsatory or microseismic movements of the ground. Also G. H. Darwin is said to have declared to the same effect,—"I venture to predict that at some future time practical astronomers will no longer be content to eliminate variations of level merely by taking means of results, but will regard corrections derived from a special instrument as necessary to each astronomical observation."

Although the motive of my idea is not strictly the same as that of these authorities, our resulting conceptions are convergent, and I am pleased to conclude my paper with the expectation that Darwin's prediction will be promptly actualized in order to meet the degree of accuracy required in up-to-date astronomical observation.

Lastly, the writer desires to express his sincere thanks to the several astronomers and observers referred to above, by whose efforts and information he has been enabled to accomplish this work.

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1) Astronomische Nachrichten, Nr. 3177.