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TO **PROFESSOR AIKITU TANAKADATE**
ON THE OCCASION COMMEMORATING HIS TWENTY-FIVE YEARS' SERVICE
DEDICATED BY
HIS DEVOTED PUPIL, THE AUTHOR.

On Rapid Periodic Variations of Terrestrial Magnetism.

By

Torahiko TERADA, *Rigakuhakushi.*

PART I.

Introduction.

1. Immediately after the organization of the Imperial Earthquake Investigation Committee, regular magnetographic observations were begun, on the proposal of Prof. Tanakadate, at four stations, Kumamoto, Kyôto, Sendai and Nemuro, with the special purpose in view of detecting any magnetic disturbances which might reveal themselves associated with destructive earthquakes. The instruments used were of the ordinary Mascart type. On the other hand, a general magnetic survey of the empire was undertaken, 1893–96, the result of which has been published in the Journal of the College of Science, Vol. XIV. In discussing therein the local disturbing field due to Mt. Huzi, Prof. Tanakadate came to the conclusion that even if the whole mass of the mountain be suddenly removed, the disturbance in the vicinity would

scarcely amount to 1 γ. Hence it was considered futile to continue the observations with instruments of such low sensibility, and the regular observations were suspended in 1909. At the same time, he devised and constructed, with the able assistance of Dr. H. Kadooka, now expert to the Military Telegraphic Department, a set of extraordinarily sensitive magnetographs, and laid before the Committee the plan of a provisory magnetic observatory equipped with these instruments. The proposal was approved, and the necessary arrangements were promptly made under his supervision. An underground room was excavated for the purpose, in the vicinity of the Marine Biological Laboratory of the Science College, at Misaki. The regular observations were commenced early in 1910. In the summer of 1911, Dr. Kadooka was appointed to his present position, and the author took charge of the Observatory, until April 1914, when the observations were suspended for an indefinite term.

The observers who were successively resident at Misaki and took charge of the instruments were :

Mr. Hideo Momose, formerly Hitotuyanagi, now teacher in the Nagoya Sôdô-syû Third Middle School,
Mr. Takeo Tatiiri, now Assistant in the Meidi Technical School, Tobata, Hukuoka Prefecture,
The late Mr. Kisaburô Matui, at the time of his death teacher in the Sibusi Middle School, Kagosima Prefecture,
Mr. Murato Nakata, now teacher in the Hakodate Commercial School,

to whom the author's sincerest thanks are due for their untiring alertness to their duty, which claimed their utmost patience and attention, to say nothing of the inconveniences of living which had to be endured on account of the lonely situation of the Observatory.

The entire task of examining and studying the magnetographic records in detail and drawing up the report thereon, was entrusted to the author. Though the investigations are as yet by no means completed, it seems now to be the proper time to summarize here the principal results hitherto obtained.

2. OBSERVATORY.—The observatory is situated at the foot of the northern slope (139°37′·5 E, 35°9′·4 N.) bordering the Bay of Aburatubo, Misaki, Province of Sagami, a few minutes walk from the Marine Biological Laboratory belonging to the College of Science, Imperial University of Tôkyô. The station was chosen

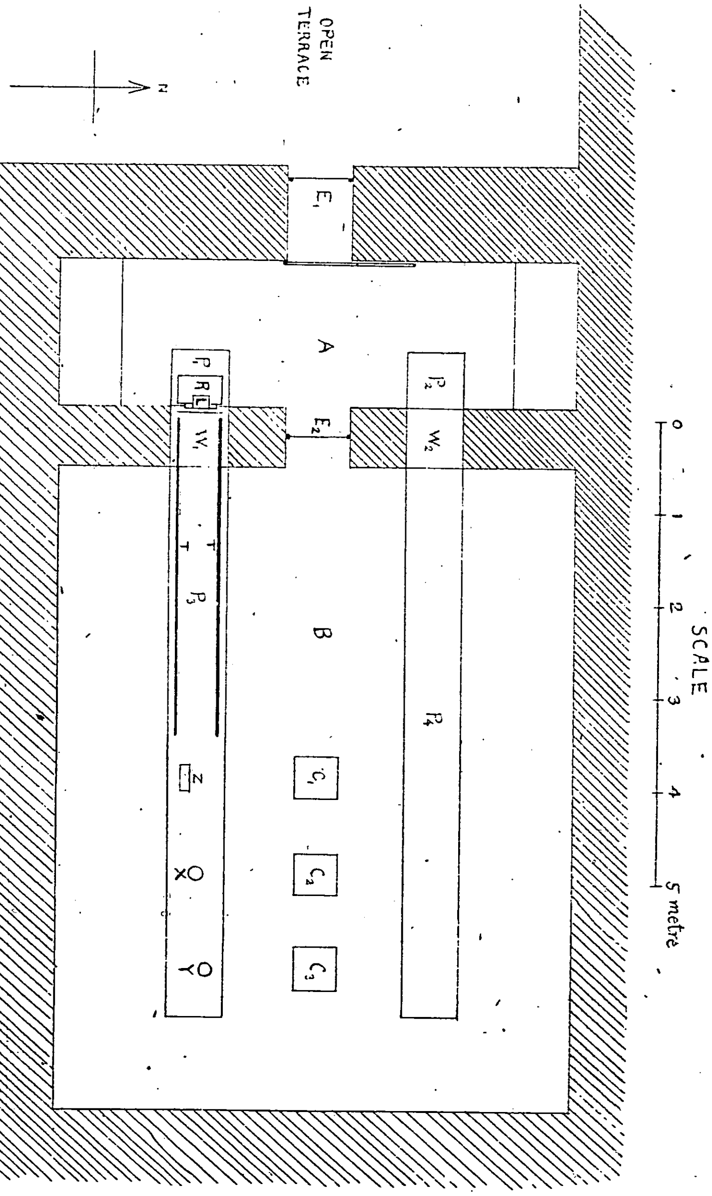

Fig. 1

E_1, entrance; A, antechamber; E_2, doorway; B, interior chamber; P_1 P_2 P_3 P_4, stone pedestals; C_1, C_2, C_3, stone pillars for mounting the deflecting magnet; W_1 W_2, windows; R, photographic recording apparatus; L, lamp; T, T, optical traps; X, Y, Z, magnetic instruments.

on account of its remoteness from any powerful electrical plant[1] and also for the conveniences which its proximity to the Marine Laboratory afforded. The edge of the terrace overlooking the bay was partly cut off to form a vertical cliff. An underground room was then excavated with a narrow entrance (Fig. 1, E_1) opening at the foot of the cliff. Since the rock was entirely of a soft tertiary formation, the excavation was comparatively easy. The approximate size and the arrangements of the room may be seen from Fig. 1. A is the antechamber where the photographic recording apparatus R and the acetylene lamp L used as the source of light, were installed on the pedestal P_1 or P_2. In the interior chamber B, communicating with A by the narrow entrance E_2 and also by the windows W_1 and W_2, two long pedestals P_3 and P_4 are laid for mounting the magnetic instruments. C_1, C_2' and C_2 are small stone pillars on which were fixed the stands for fitting the magnetic bar used for determining the sensibility or constant of the magnetographs.

In the beginning of the observation, two similar sets of instruments were arranged on P_1 P_3 and P_2 P_4 respectively and run simultaneously. The results were almost identical, as was to be expected, and the later part of the observations was almost exclusively made on pedestals P_1 and P_3.

The temperature of the interior chamber was kept tolerably constant, the daily variation amounting to only a fraction of a degree and the annual range scarcely reaching 3°C.

A serious difficulty met with was, however, the extreme dampness of the chamber during the summer months. Not only does the moisture of the external air, saturated at very high temperature, gradually condense in the cooler interior, but humidity is constantly supplied by the percolation of underground water, through numerous fissures in the wall and ceiling, fed by the abundant rain during "*Baiu*," the rainy season on our entire Pacific coast in early summer. This caused so much trouble that during a certain period observations were almost rendered impossible. Though it was not impossible to overcome this difficulty, the funds at our disposal were not sufficient to carry out the

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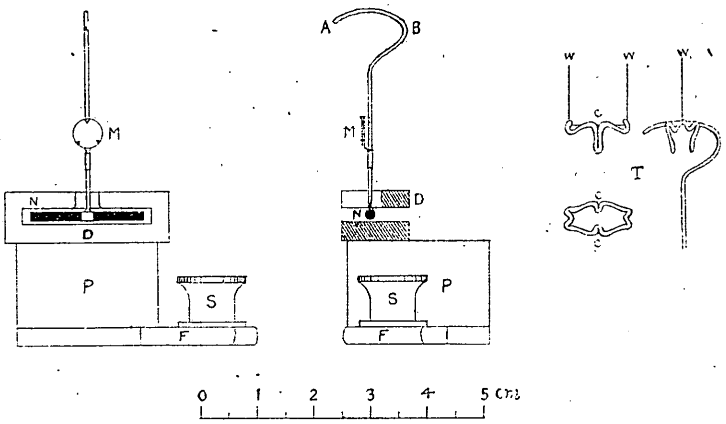
[1] The nearest tramway line is at Kamakura, nearly 20 km. N. from the station.

necessary reconstruction of the room. In any future work of the kind, it will be absolutely neccessary to provide first of all for the removal of this obnoxious humidity.

3. INSTRUMENTS.—Since it was the immediate purpose of the present investigation to detect the most minute disturbances possible, the sensibility of the usual instruments was far from being sufficient. To meet this need, Prof. Tanakadate and Dr. Kadooka devised a specially sensitive set of instruments which will be separately described in the following paragraphs:

a) *West-East- or Y-Component Instrument* (West taken as positive). Maxwell, in his discussion of the theory of bifilar suspension1), suggested the utility for the measurement of the WE-component, of a suspended magnetic needle twisted nearly 180° from its natural direction, which can be made highly sensitive by properly adjusting the breadth of the bifilar suspension. This principle was adopted in the following manner. The magnetic needle (Fig. 2, N) with a length of 20 mm. and a diameter of 1·5 mm., was held by a hook AB, made of aluminium wire of 0·7 mm. diameter, bent in the form as shown in the figure. A kind of light stirrup for supporting the hook with the magnetic needle was made with pieces of thin fused quartz rods, welded together

Fig. 2



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1) Maxwell, Electricity and Magnetism, 3rd. Edition, Vol. II, p. 118.

into shape as shown in Fig. 2, T. This was made to hang on the looped end of the suspension wire *ww*, for which fine Wollaston wire of 5–10 μ diameter was used. The upper arc of the aluminium hook is made to ride on the central V-shaped recess *cc* of the stirrup. M is a light plane mirror of 5 mm. diameter, attached to the hook. The whole system was hung in the metallic case of Mascart's magnetograph made by Carpentier, Paris, from which the usual attachment in the interior was removed. The regulating screw attached at the top of the case for adjusting the breadth of the supension was utilized as such. For damping the natural vibration of the suspension, an electromagnetic damper D made of a copper block was introduced. The damper could be rotated on a cylindrical brass block P, which in its turn could be rotated about the screw S, fixed to the bed plate of the case, passing through the groove cut along the projecting arm F at the foot of L. The damping was very effective, making the vibration completely aperiodic.

To set the suspension in working condition, we proceeded as follows. At first, the stirrup only was hung on the wire, making the breadth of the suspension sufficiently large. After hanging the magnetic needle carefully in its natural direction, twist the torsion head slowly till the needle is turned about 180°, and the luminous image of the slit formed by the reflexion of the mirror M appears within its proper range. Then, after bringing the damper in position gradually narrow the breadth of the bifilar suspension, at the same time adjusting the torsion head so as to keep the luminous spot always within the range assigned to it. Proceeding very carefully in this way, the sensibility can be made extremely great, till at last the suspension attains its unstable position. It must be remarked that near this extreme position any minute change in the elastic property of the wire, the breadth of the bifilar suspension or the weight of the suspended system, may sensibly affect the deflection. Nor is the system independent of the slight inclination and the vertical acceleration of the instrument. In one instance, an inclination of about 4′ produced a displacement of about 2 cm. on the photographic record. Since the latter effect seems to be due chiefly to the rigidity of the suspension wire, it is advisable to use as fine a wire as possible. In any future work, a thorough annealing of the wire after hanging the suspended system is very desirable.

The tripod support of the instrument rested on a thick glass plate, which was provided with a hole and a V-groove for receiving the feet of the levelling screw, and was rigidly fixed to the face of the stone pedestal.

The lens in front of the instrument case was replaced with another with a focal length of 5 m.

The deflecting magnetic rod for determining the sensibility, was fixed to a special holder[1] such as is usually attached to Mascart's magnetograph for a similar purpose, and placed on the support pasted on the pillar C_3 in the same meridian as the instrument. After starting the photographic apparatus, the deflecting magnet was brought in position, *i.e.* horizontally in WE direction, and reversed every 3 or 5 minutes. The magnetic moment of the rod mostly used was $M=77$ c.g.s. at the room temperature. The distance of the deflecting rod from the needle of the instrument was 123 cm., so that with the rod "side on," the change of the WE-component corresponding to the reversal of the deflector was $8{\cdot}3\,\gamma$. The sensibility was so adjusted as to make the corresponding deflection nearly 50 mm.; *i.e.* 1 mm. on the record corresponded to $0{\cdot}17\,\gamma$. The sensibility, though fairly constant, showed occasionally a tendency to decrease slightly after running for 24 hours, though not at all so serious as to affect the general aspect of the results obtained. It was therefore considered preferable to test the sensibility at least once every day and redetermine the constant. This gradual decrease of sensibility is probably due to the influence of a slight elastic time-effect of the suspension wire, made apparent on account of the extreme position of the bifilar system. The presence of a sensible time-effect in the wire used, is also made evident on the photographic record, when an abrupt deviation is effected by means of the deflecting magnet. After an immediate deflection, a slow creeping up of the

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[1] Mascart, Magnétisme Terrestre, p. 195, Fig. 46 c.

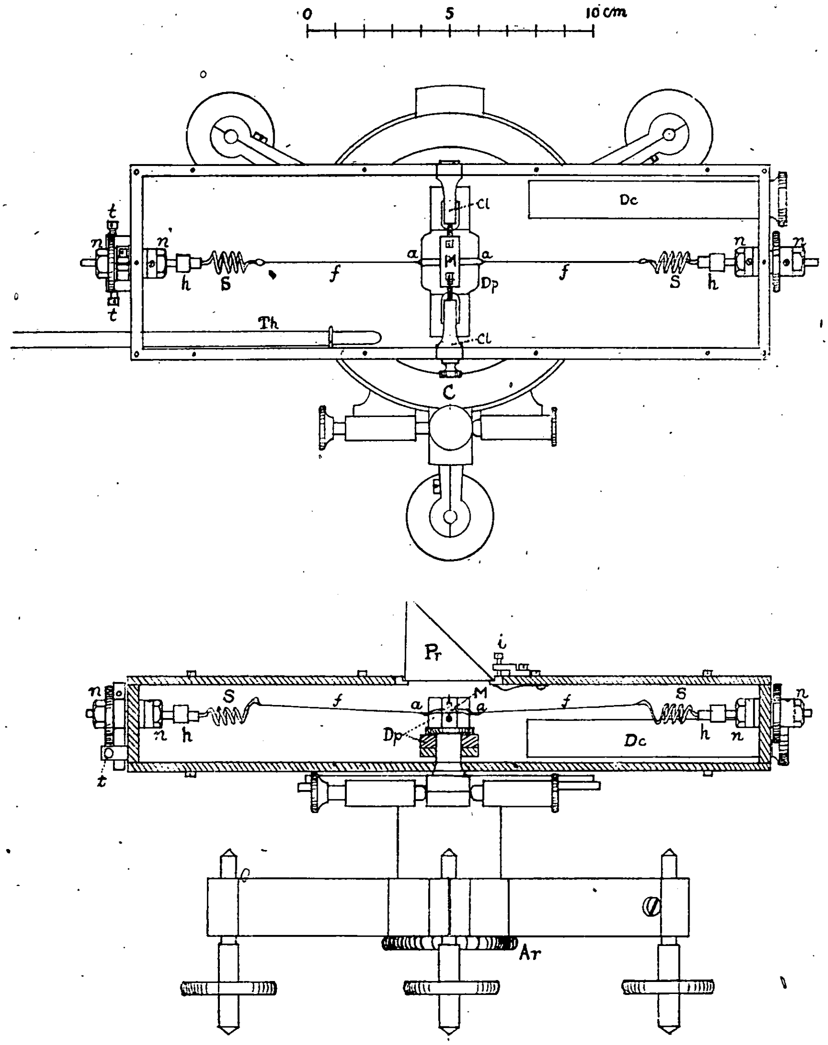
luminous image is always to be traced, which in most cases practically attains the final position after a few minutes (see Pl. I, marked with∗). This effect will of course affect the accuracy of the record, especially in relatively magnifying the disturbances of the longer period compared with the shorter ones. The error may, indeed, amount to several percent in unfavorable cases, if the relative amplitudes of the disturbances with decidedly different periods are to be compared with accuracy. However, the general inferences which will be given in the following communication will not be seriously affected, since here the comparison of amplitudes is either made of the X- and Y-components for waves of the same period, or of the X- and Z-components for waves with the periods usually longer than 1 minute. In the former case, both instruments show after-effects very similar to each other. Even in the latter case, where the Z-instrument is comparatively free from such effect, any serious error will occur only in the case of very short waves. This very disagreeable time-effect could probably be avoided by the use of quartz fibre suspension, directly welded to the quartz stirrup, though in this case we must devise a necessary modification of the suspension head.

b) *North-South- or X-Component Instrument* (North taken as positive). The NS- instrument is essentially the same as that of Mascart's magnetograph. The suspension system is, however, replaced by one quite similar to that of the WE- instrument above described, except that in the present case the plane of the mirror is perpendicular to the axis of the magnetic needle.

Keeping the breadth of the bifilar suspension at first sufficiently wide, the torsion head is slowly twisted, till the luminous spot appears in the assigned range. Then cautiously turn the torsion head, at the same time regulating the breadth of the bifilar suspension, so as to keep the luminous image always within proper range. The unstable position is attained in a certain azimuth of the torsion head, which may be noted down for subsequent readjustment. The determination of the constant is carried out in a way similar to that in the case of the Y-instrument, with the only difference that in this case the deflecting magnet is applied "end on," so that for the same distance of the deflector, the deflection must be made twice as large as in the case of the Y-instrument to insure the same sensibility. The distance of the deflecting magnet from the needle of the instrument was 127 cm., so that 1 mm. on the record corresponds to 0·15 γ.

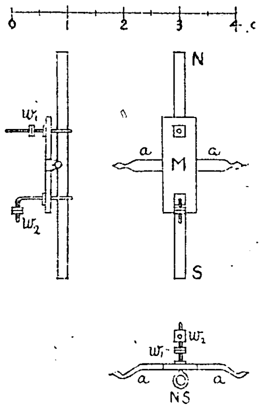
The gradual creeping of the luminous spot is also observed in this case. Hence the same remark applies as to the accuracy of the records.

Fig. 3, A.



c) *Vertical or Z-Component Instrument* (downward taken as positive). Instead of the ordinary Lloyd's balance, a new instrument was designed and constructed by Prof. Tanakadate and Dr. Kadooka, which proved very satisfactory. The magnetic needle (Fig. 3, NS), 4 cm. long and 2 mm. thick, was fixed to the lower face of a rectangular plane mirror M ($6 \times 17 \times 1$ mm.), made of fused quartz plate, platinized on its upper suface by means of cathode discharge. The mirror has a pair of projecting arms *aa* on both sides, to the pointed ends of which are welded quartz fibres *ff* of 0·05–0·08 mm. diameter. The welding may be easily made, after some practice, by pointing the sharp point of a fine oxyhydrogen flame to the spot where the fibre is attached to the arm by slightly wetting the surface. The fibres *ff* are stretched in EW direction by means of helical springs SS1) made of fused quartz rod, of which the one end is welded to the fibre and the other is rigidly fixed to the brass holder *hh* by means of solder applied in the cavity for receiving the quartz rod. By means of the nuts *nn*, the tension could be adjusted without twisting the fibre. By turning the screw *t*, a slight torsion could be given to the fibre, in order to adjust the zero position of the mirror. Damping is effected by means of the copper block D*p*, which serves also for clamping the suspended system when lifted up by means of the screw A*r* at the lowest part of the instrument. P*r* is the reflecting prism originally used in Mascart's vertical component instrument for a similar purpose. The inclination of the prism could be adjusted by means of the screw *i*. D*c* is the desiccator containing calcium chloride and T*h* a thermometer.

Fig. 3, B.



The points of junction of the mirror arms *aa* with the fibre are originally so adjusted that the centre of gravity of the suspended

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1) The spring may be dispensed with, provided that the temperature is kept sufficiently constant, without sensibly impairing the reliability of the instrument, though there is the danger of breaking the fibre by accidental shock given to the instrument during manipulation.

system is very near the line connecting them. The sensibility may be then adjusted by means of a small weight w_1 playing on the fine screw projecting on the upper face of the mirror, while the inclination of the mirror can be regulated by means of another pair of small nuts w_2, running on the horizontal screw as shown in Fig. 3, B.

For determining the sensibility, the same deflecting magnetic rod as used for the preceding two instruments, was placed on the pedestal C_1 in the same meridian as the instrument at a distance of 128·5 cm. In this case the deflector was of course applied and reversed in the vertical position. The sensibility was so adjusted that the reversal of the deflector produces the displacement of about 5 cm. on the record, or 1 mm. corresponds to 0·15 γ.

The instrument worked very satisfactorily. Once carefully adjusted, it remained so constant in every respect that it might have run many years without any further trouble, except for the excessive humidity of the room, which caused gradual rusting of the instrument, especially of the magnetic needle, also a gradual deterioration of the platinized mirror, and necessitated the complete rearrangement of the instrument. The great advantage of quartz fibre is very clearly shown in this instrument compared with the others, since here no sensible creeping up of the luminous image after abrupt deviation is observed, as may be seen from Pl. I. An instrument of a similar construction may be replaced with great advantage for Lloyd's balance, for the usual work of lower sensibility.

d) *Illuminating and Photographic Apparatus.* The source of light used for photographic purposes was an ordinary acetylene burner with two orifices facing each other, which was fed by a capacious tank placed outside the room. The burner was fixed in the interior of the metallic case of the lamp originally attached to Mascart's magnetograph, in front of which a suitable vertical slit and a cylindrical lens was inserted. The lamp was hung over the pedestal P_1 of the antechamber, close by the window W_1, and immediately above the photographic apparatus placed on P_1. The drum carrying the photographic paper was 24 cm. in diameter and 30 cm. in length, and revolved once every 3·76 hours on an average, by means of a suitable clockwork, so that 1 mm. corresponds nearly to 17·8 sec. Closely in front of the drum a fine horizontal slit and a cylindrical lens are introduced, both of the same length as the drum. During the wet season, a desiccator dish containing calcuim chloride was placed under the drum.

For giving time-marks on the record, a clockwork was placed in front of the lamp to interrupt the light for a few mimutes at the end of each hour.

The three magnetic instruments are arranged in succession as shown in Fig. 1. For the Z-instrument an auxiliary lens with the focal length of 2 m. was placed in the path of the incident beam of light, by means of a special holder fixed to the pedestal.

To save the considerable breadth of the photographic paper required, owing to the remarkable sensibility of the instruments, the following device was adopted with success. Two long strips of thick plane glass plate (breadth 10 cm., length 3 m.) were placed along both sides of the long pedestal P_1 (Fig. 1, T). These were held firmly by means of special holders, with their reflecting surfaces vertical and parallel to each other, so that when the luminous beam reflected from the magnetic instrument is deviated just beyond the limit of the horizontal slit of the photographic apparatus, it is caught by one of the glass plates, or "optical traps" as we have called them, and reflected back to the slit. For a still greater deviation the opposite glass caught the beam reflected from the first one, and so on. Except in the extremely damp season, even the third reflection produced a luminous spot intense enough to affect the photographic paper. By properly adjusting the position of the three instruments and also the mean direction of the reflected beams produced by them, an uninterrupted record of the three components could be obtained, even in the case of remarkable magnetic storms.

The mutual magnetic influences of the three instruments were tested by mechanically disturbing each instrument, while the photographic record was being taken. No sensible effect was noticed.

Since the drum carrying the photographic paper revolved once per 3·76 hours, paper of a length of about 4·85 m. was required per day and night. To save the excessive use of paper, it was replaced only twice during 24 hours, *i.e.* usually at 6 o'clock in the morning and evening. Hence the record was run over three times by each component, which caused considerable confusion of the record, on quiet days by the superposition of the trace of the same component, and on disturbed days, by the mutual intersections of the different components either direct or reflected by the traps. The confusion is, however, merely apparent in most cases. The time marks, the difference in the intensity of traces and also slight differences in the optical error of the mirrors of different instruments, causing different shading of the photographic traces, served together as convenient signs for disentangling the intricate record. Ambiguous cases were extremely rare, even if two branches of traces were nearly superposed. After we became familiar in the course of the investigation, with the characteristic behaviours of short period disturbances for different components, the distinction of the different components became still easier.

During the summer months, the pedestal P_3 was covered with large cases of zinc, which consisted of seven pieces and when put together in series, formed a continuous channel extending along the entire length of the pedestal. Five or six dishes with $CaCl_2$ were placed inside the channel. These arrangements were, however, far from being efficient for the prevention of the extraordinary dampness of the chamber during a certain period of the warmest season.

The clock for giving the time mark was occasionally compared with that of the Post Office in Misaki and also checked by means of the sundial placed in the yard of the Marine Laboratory, so that the accuracy of the time was only of the order of a minute at the most. A convenient control could be made by the very frequent occurrence of slight local earthquakes which left distinct marks on the photographic records and could be easily identified with the seismographic record obtained in Tôkyô.1)

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1) The distance between the Observatory and the Tôkyô Central Meteorological Observatory is about 60 km., so that the time taken by the principal phase of seismic waves will never exceed 18 seconds.

PART II.

General Results of the Investigation.

4. The general character of the photographic records obtained by our system of instruments may be seen in Pl. I. and II. The time taken as abscissa increases from left to right. Upward corresponds to the positive direction of each component for the non-reflected trace. Pl. I. is the representative record for a quiet day. The trace of the sensibility determination is to be seen at the beginning of the curves (marked with ∗). Pl. III. is one of the typical records for disturbed days, the parts of the curves reflected from the "traps" are marked with the letters R_1, R_2, the suffix giving the number of reflexions.

Even on the most quiet days, the records shows as a rule numerous trains of more or less regular waves or pulsations, the periods of which range from about 20 sec. to several minutes. Allied phenomena seem to have been studied first by Balfour Stewart[1] who found periods of 30 seconds. Kohlrausch[2] found by direct eye observations a wave of 12 sec. period. Arendt[3] investigated waves with periods of several minutes, frequent in night hours, in connection with his researches on the magnetic disturbances associated with the phenomena of thunderstorms. Eschenhagen[4] found prevalent waves of 30 sec. which appeared most frequently during daytime. Birkeland[5] studied the phenomena for Haldde as well as for Potsdam, and obtained most frequent groups of waves with periods of 10 and a little longer than 30 sec. More recently, van Bemmeln in Batavia[6] studied similar waves with 1–4 minutes periods which he called "pulsa-

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1) Balfour Stewart, Phil. Trans. 1861, p. 425.

2) F. Kohlrausch, Wied. Ann. **60**, p. 336.

3) Th. Arendt, Das Wetter, 1986, p. 241 and 265.

4) Max Eschenhagen, Sitz. Ber. d. preuss. Akad. d. Wiss., Berlin, **32**, 1897, p. 678; Terr. Mag. and Atm. Elec., **2**, 1897, p. 105.

5) Kr. Birkeland, Expédition Norvégienne de 1899—1900 pur l'étude des aurores boréales. Résultats des recherches magnétiques. Christiania, 1901.

6) W. van Bemmeln, Verslag: Konink. Akad. v. Wet. t. Amsterdam, Proc. of the Sec. of, Sc., **2**, 1899—1900, p. 202.

tions" to distingnish them from peculiar disturbances called "spasms," and discovered a remarkable daily period of frequency with a maximum near midnight. Afterwards[2] he compared the materials from Zi-Ka-Wei and Kew, and found for the former station a similar nightly maximum as in the case of Batavia, while for the latter station, the maximum frequency was found in the day time. Recent authorities seem to agree in the opinion that the magnetic waves in question are chiefly due to some fluctuations of the electric current existing in the upper atmosphere, though the actual modes of fluctuation still remain obscure.

Though the original purpose of our investigation was to detect any abnormal disturbances associated with earthquakes, it was in any case necessary to study the characteristic pulsatory waves in some detail, even if these waves should have no direct connection with earthquakes, in order to be able to distinguish which were the normal and which the abnormal disturbances. The present paper is chiefly confined to the study of these characteristic pulsatory disturbances, since unfortunately no positive results have yet been obtained with regard to earthquakes.

In the following, we shall enumerate the most interesting results obtained by the detailed study of the magnetographic records comprising observations extending over four years.

5. *a*) *Generally speaking*, the magnetic waves in question are decidedly more regular in the night than in day time when waves of different periods seem to be very irregularly superposed. In the great majority of cases remarkably long continuous trains of moderately regular waves with period of 30–60 sec. appear at 5^h–7^h in the morning and continue up to 9^h–11^h, with occasional interruptions (Pl. II., III., IV. and V). The number of hours, in the course of which such trains occurred, was noted for successive days, quite regardless of the length or the number of the trains. The number shows occasionally an apparent periodicity of 25–30 days, though generally not so regular as to allow us to deduce any

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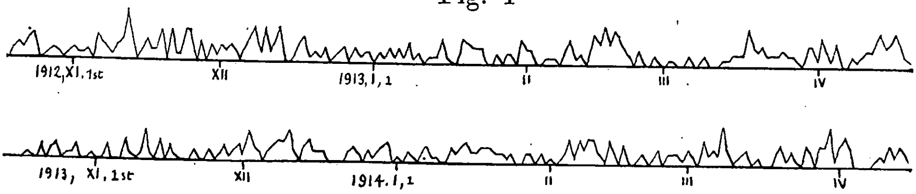
2) W. van Bemmeln, Natuurk. Tijdschr. v. Nederlandsch-Indië, **62**, 1902, p. 71.

definite period from the scanty material at hand. The general aspect will be seen from the annexed figure (Fig. 4).

Fig. 4



Since it was suspected that the number in question may have some relation with the solar activity, it was compared with the "provisory sun-spot number," published in Meteorologische Zeitschrift, but no convincing relation could be found.[1] It will however, be interesting to compare the present number with the occurrence of sun-spots on a definite central area of the solar disc.[2]

b) These *short* period waves appearing simultaneously in the X- and Y-components run remarkably parallel to each other during the morning hours, *viz.* 6^h—8^h say, every detail in one component being repeated by the other with marvellous similarity, no noteworthy phase difference being observed between the two components (Pl. V). Moreover, the amplitudes of the two components are generally of nearly the same order of magnitude. It seems as if these waves were due to the fluctuation of an electric current running from NE to SW, making an angle nearly 45° to the meridian. For these *short* waves, Z-component is comparatively insignificant and may be clearly discerned only when the photographic trace is very fine.

In the later afternoon hours, the parallelism between X- and Y-components becomes imperfect. Some waves are present either in

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[1] W. van Bemmeln compared the frequency of the "pulsations" with the sun-spot number and arrived at a negative result, *loc. cit.*

[2] E. W. Maunder noticed a 27 days period of magnetic disturbances and came to the conclusion that the cause of the disturbance is to be sought on a limited portion of the sun's surface, Monthly Notice of R. Astr. Soc., **65**, 1904—05, p. 2, 538. The paper was criticized by A. Schuster, *ibid*, p. 186. E. Marchand found a connection between the magnetic disturbances and the sun-spots passing the central meridian of the sun's disc, Congr. intern. de meteorolog., Paris, 1900, p. 148.

one or the other component only. Even if a train of waves may be traced in both components, the variation of the disturbing field is generally of irregular rotatory character.

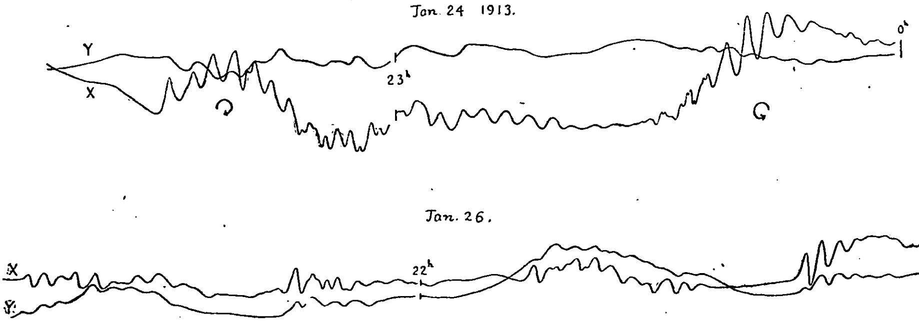
c) Generally, no corresponding regular trains of short waves with such remarkable duration can be seen during the evening hours, nor is the remarkable parallelism between X- and Y-components so conspicuous as in the morning.

d) During the later hours of the evening, the short waves with periods of less than one minute generally disappear, while very regular waves with 2–4 minutes periods appear instead, frequently forming beautiful trains of nearly simple harmonic waves. Another remarkable characteristic of the evening waves is that the Y-component waves are generally inverted with respect to the X-waves, *i.e.* the fluctuation of the horizontal magnetic field is such as could be caused by the periodic variation of a current running from NW to SE. Generally speaking, however, the horizontal components show more or less rotatory character; X- and Y-components showing frequently decided phase difference. This latter point will be fully investigated later.

For these *longer* waves, the Z-component is more conspicuous than for the shorter waves frequent in day time, and runs remarkably similar to the X-component, though lagging behind it by a considerable fraction of the period (see Plates I–IV).

Fig. 5, A (reduced to $\frac{1}{3}$ original size).

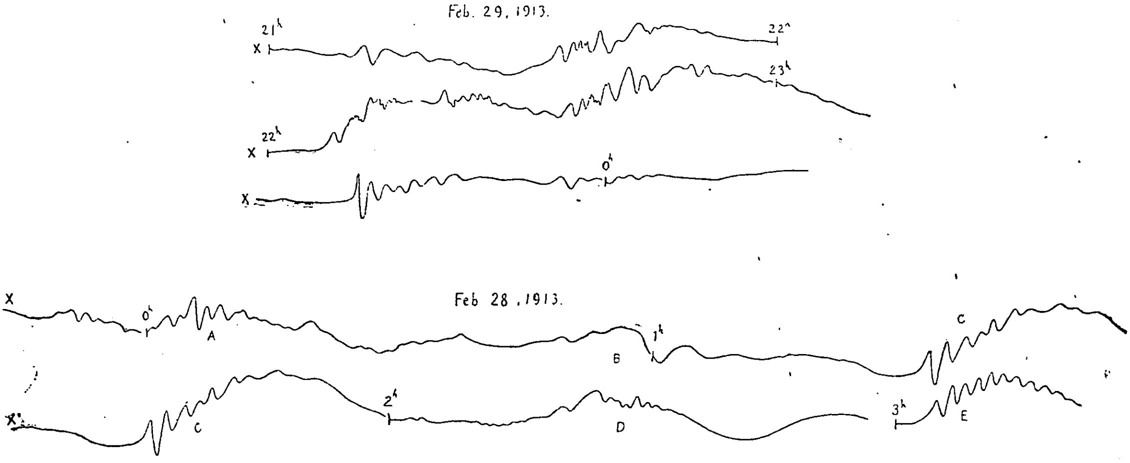
Note the rotatory character of the horizontal component in the upper curves.



e) The beginning of these waves is sometimes gradual, but frequently abrupt, starting quite suddenly after a period of dormant calmness (Fig. 5). The trains with the abrupt beginning are frequently accompanied by an abrupt increase of the average value of the X-component which gradually attains a maximum and then falls slowly (see Figs. 5 and 6). The sudden increase of the X-component is usually accompanied by the simultaneous sudden change of the average value of the Y-component, which is at the outset generally toward E before midnight and toward W after midnight. It seems as if an electric current were suddenly started with rapidly increasing and fluctuating intensity, with its direction NW to SE, or SW to NE according to the hour of occurrence. Disturbances of this kind seem to have been studied by Birkeland who succeeded in tracing a system of whirling currents (tourbillons de courants) extending over the N-hemisphere, fixed relative to the sun. The slow change of the horizontal components gradually attains a rotatory character, the sense of which seems to generally confirm the result obtained by R.B. Sangster,[1] *i.e.* mostly counterclockwise (N-W-S) before, and

Fig. 5, B ($\frac{1}{3}$ original size).

Remark the regularity with which similar disturbances are repeated.



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[1] R. B. Sangster, Proc. Roy. Soc., **84** A. 1911.

clockwise (N-E-S) after a certain hour near midnight. This seems just as if the magnetic disturbing vector, initially inclined to the meridian, rotates in the sense to become parallel to the meridian.

Characteristic disturbances of the above type are met with most frequently just at midnight. In other hours, especially near 18^{h} cases occur not rarely where an abrupt increase of the horizontal component is not accompanied with any conspicuous train of waves.

Fig. 6, A ($\frac{1}{3}$ original size).

Note the parallelism between X and Z.

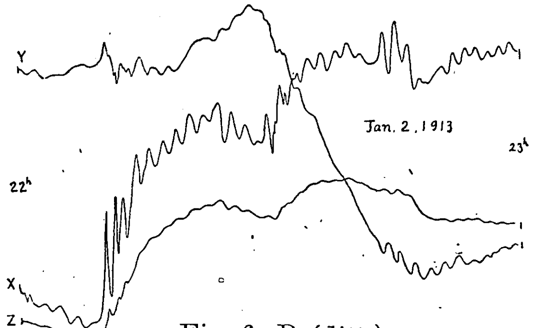


Fig. 6, B (*ditto*).

Remark the apparent "beat" of the waves in the lower curves.

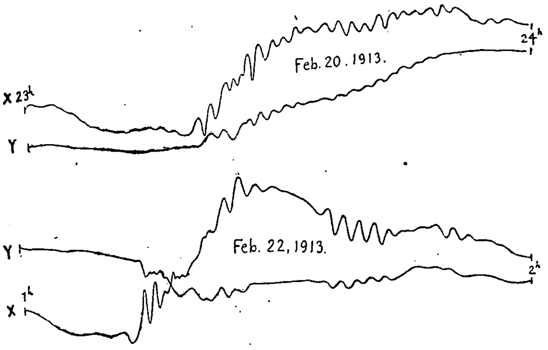
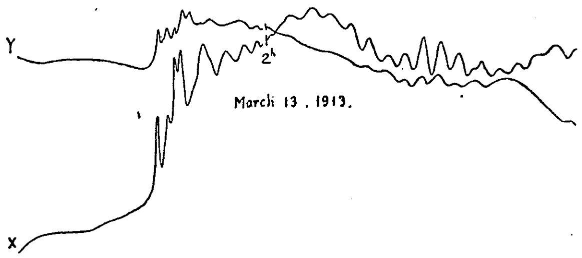


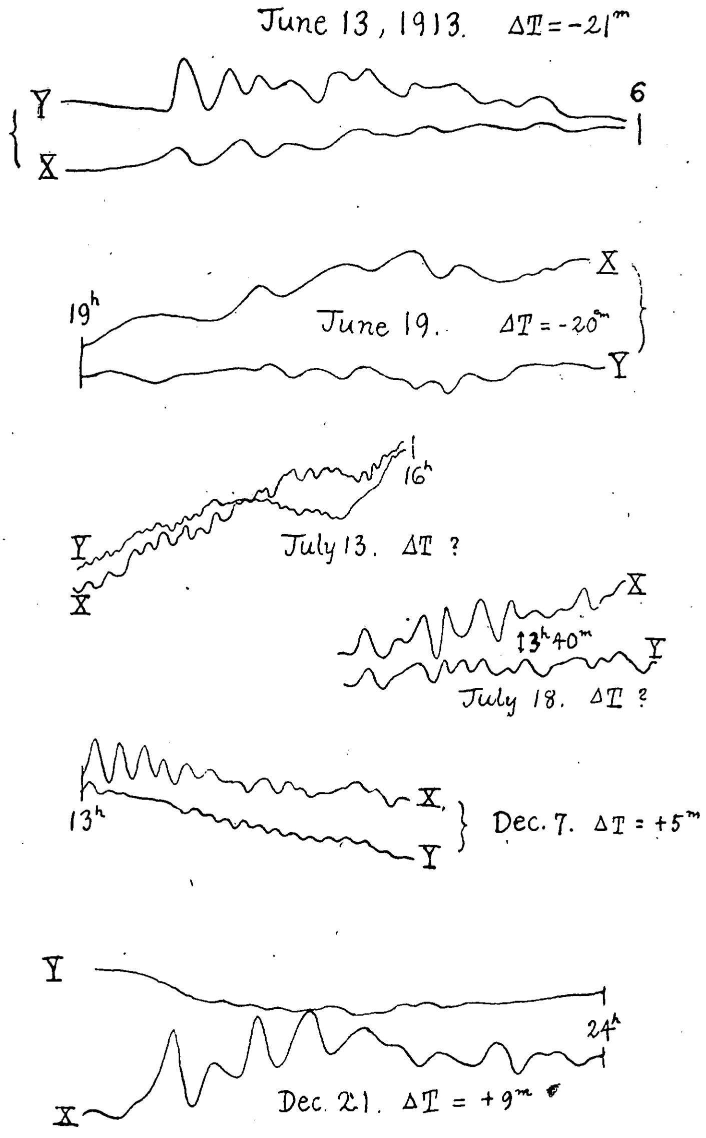
Fig. 6, C (*ditto*).



It is a very remarkable fact that even for these abrupt nonperiodic disturbances of long duration, the Z-component follows the various fluctuations of the X-component with utmost faithfulness, except for a reduced amplitude and a definite retardation. This characteristic behaviour served as a very convenient means for disentangling the chaos of photographic traces in the records of disturbed days.

At the beginning of the abruptly starting train of waves, the amplitude is sometimes very large, in one instance amounting to 5·4 γ with a period of 1·2 minute (see Fig. 6, A). The amplitude is generally damped gradually, with a sensibly constant period. On closer examination, cases were also found where the period of the waves changed gradually in an apparently coherent train. Sometimes,

Fig. 7 ($\frac{3}{4}$ original size).



a very characteristic train is repeated two or three times with nearly equal intervals of 15^m, 40^m *etc.*, as if the same train of waves were recurring to the point of observation with a definite period (Fig. 5, B).

f) It is frequently observed that while a fairly regular train of waves is traced in both the X- and Y-components at the same time, the periods of waves are quite different for the two components (for examples, see Fig. 7). Among 68 conspicous cases chosen, 42 were those in which the periods were longer in X- than in Y-component. The distribution of such cases in different hours of the day may be seen in the following table:

TABLE I.

Hour.	$T_X > T_Y$	$T_X < T_Y$	Hour.	$T_X > T_Y$	$T_X < T_Y$
0– 2	0	2	12–14	8	0
2– 4	4	2	14–16	3	1
4– 6	7	3	16–18	0	1
6– 8	2	2	18–20	2	6
8–10	6	3	20–22	3	0
10–12	4	2	22–24	3	4

A tendency is suspected, though not very apparent, that the cases where the periods of X-waves are greater than those of Y, are most frequent near midnight and noon, while they are comparatively rare in the morning and evening hours.

g) Though the most conspicuous regular trains are chiefly observed with periods ranging from 30^s to 300^s, still longer waves are not at all rare, if we take those trains together in account, with only a small number of maxima and minima. The longest wave traced was of a period nearly amounting to one hour, which may of course be detected in the ordinary magnetographic record of low sensibility, when the amplitude is sufficiently large. The intermediate periods are rather evenly represented in our list of the waves, though there seems to be a maximum of frequency near 15^{m}.

h) Slight local earthquakes, which are very frequent, affect the instruments, leaving a trace similar to that of the Milne seismograph (see for example Pl. V). This is in all probability mainly due to the mechanical shock, to which our instruments, especially X- and Y-instruments, are rather sensitive. These traces, however, served as very convenient and trustworthy time-signals, as already mentioned. Besides, remarkable earthquake waves with long periods, originating in distant regions, are beautifully recorded in our magnetographs (Pl. IV). The most conspicuous periods are of the order 10^{s}–20^{s}, *i.e.* decidedly shorter than the usual magnetic waves. These peculiar waves appear in the vertical component as conspicuous as in the horizontal components,[1] while in the case of the usual magnetic waves of such a short period the vertical component is almost nil in comparison with the horizontal ones. Since our instruments are neither quite free from the influence of inclination nor of acceleration, even in the case of the Z-instrument, we can not be sure whether these waves are not largely due merely to the mechanical effect, though it seems not altogether improbable that an actual magnetic wave is produced by the earthquake. To decide this point, it will be desirable to carry out simultaneous observations with quite different systems of instruments with the same magnetic sensibility, but with considerably different mechanical sensibilities, though it will be difficult to obtain instruments perfectly free from mechanical disturbances.[2]

i) All slight or conspicuous magnetic disturbances which may serve as premonitory signs preceding earthquakes, were carefully sought, throughout the records in hands. At first, a possible connection was suspected between the earthquakes and the characteristic abrupt occurrence of the regular wave trains,

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[1] The vertical-force magnetograph as now constructed may be regarded as a sort of seismograph, inasmuch as the line of suspension does not pass through the centre of the mass of the suspend system.

[2] The trace of the same earthquake on the X-component record of the Eschenhagen magnetograph at Kakioka Magnetic Observatory, was about 6 mm. in maximum ampitude, while the magnetic sensibility was about 5 γ per mm.

described in paragraph *e*). But the trains in question which are most frequent during the night hours, are distributed quite haphazard with respect to the earthquakes, if we either take slight and moderate earthquakes together, or confine our attention to the moderate ones only. In any case, a further study in this direction seems scarcely promising, in so far as the characteristic magnetic waves are of no local phenomena confined within a very limited area of the earth's surface.

On some occasions, slight irregular disturbances of very abnormal type were observed in both components, but usually in one component only, no corresponding simultaneous anomaly being marked in the other. These irregularities are most probably due to some slight accidental shock acting as an impulse to settling down a slight unstable position of the suspended system; for example, a slipping of microscopic magnitude, or the introduction of a minute dust particle at the point where the suspension wire touches the adjusting screw, may cause a quite sensible displacement in such an ultrasensitive state of suspension as adopted in our instruments. To detect premonitory, if any, magnetic disturbances, it will therefore be necessary to protect the instrument more thoroughly from any mechanical shock or other accidental disturbance.

However, during the entire course of the observations, no remarkable *destructive* earthquake occurred in the neighbouring districts, so that we are not yet in a position to draw any definite conclusion as to the value of magnetic observations as a means for predicting strong earthquakes—which was the original purpose of our present installation.

PART III.

Special Statistical Investigations in Detail.

6. In the following, some of the results of the detailed statistical investigations with respect to some of the general results above described, will be given, which are on one hand a confirmation of the qualitative remarks stated in the preceding

paragraphs, and on the other hand constitute the basis of the theoretical considerations to be propounded later on.

It is the pleasant duty of the author to acknowledge here his indebtedness to Messrs. T. Tatiiri, the late H. Suzuki, M. Nakata and S. Hana who successively served as his assistants and computors in examining and making toilsome measurements of the records, and carrying out many tedious calculations, with indefatigable zeal and true scientific candour.

Some parts of the statistical work which necessitated the utmost care in measurements, for example the determination of the phase differences of different components *etc.* were made by the author himself.

7. At first, the photographic record were examined sheet by sheet, the conspicuous waves constituting more or less regular trains were selected and the approximate *mean* periods determined by dividing the time interval by the number of waves contained in the train.[1] It is indeed difficult to decide which trains to take and which to omit. It was therefore decided to take as many trains as possible with different periods, provided that the trains comprise at least three very regular simple waves with sensible amplitudes. When during the course of an hour a very long continuous regular train with nearly constant period occurred, the period was determined only for one or two portions of the train chosen arbitrarily from the most conspicuous parts.

The case is more difficult when the fundamental waves are superposed with different "overtones." Very complicated waves, liable to ambiguities, were omitted, even if it were possible to analyse them into simple components, and the apparently regular trains only were taken into account. The results of the examination were systematically tabulated, separately for each of the three components, according to the successive hours of day. These tables formed the basis of some of the following investigations.

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[1] Strictly speaking, the periods are sometimes variable even in an apparently regular train of simple waves. The inconstancy is too considerable to be explained by the irregularities of motion of the drum carrying the photographic paper.

8. *Hourly distribution of waves of different periods.* The different periods were classified into the following arbitrary groups: $30{\cdot}0^s-49{\cdot}9^s$, $50{\cdot}0^s-69{\cdot}9^s$, $70{\cdot}0^s-89{\cdot}9^s$, $90{\cdot}0^s-129{\cdot}9^s$, 130^s-200^s. The occurrence of waves belonging to the different groups, in different hours of successive days, was plotted down in diagrams similar to those drawn by Bidlingmeier[1] in his "Uebersicht über die Tätigkeit des Erdmagnetismus."

From these diagrams, it could at once be seen that waves with periods shorter than abont 50^s are most frequently met with during the day time, while the longer waves over 90^s periods are generally most frequent during the night. This is naturally true for each of the three components, though in Z-component the shorter waves are generally of almost insignificant amplitudes. To make this remarkable fact more apparent, the following procedure was adopted. Different days of a month, or of a season, were taken together, and the number of those days in the said interval, in which a certain given hour was disturbed by the waves of a certain given period, was counted and then divided by the total number or days in that interval, in which the observation was actually made at the very hour in question; the latter reduction was necessary, since especially in summer, the records were often imperfect. The number thus reduced, multiplied with 100, was called the "frequency" of that given group of waves in that particular hour, for the month or season in question. The following tables give the hourly frequency of the different wave groups in X- and Y-component, for different seasons, all the records available being taken into account. Z-component is omitted, since it is always a reduced facsimile of X-component, as already mentioned.

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[1] Bidlingmeier, Veröffentlichungen d. kais. Observatoriums in Wilhelmshaven.

TABLE II.

The numbers give the frequency of the waves of different periods in %.

Month. Hours.	X. 30^s—50^s					Y. 30^s—50^s				
	II III VI	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	3·2	2·8	6·6	3·9	4·1	2·0	3·0	3·9	2·3	2·7
1— 2	5·7	5·2	8·7	6·4	6·4	3·3	3·0	7·8	2·3	3·7
2— 3	4·5	6·7	7·8	7·1	6·4	5·3	6·0	3·9	2·8	4·4
3— 4	7·5	8·3	8·4	10·1	8·9	8·2	10·1	7·8	2·0	6·7
4— 5	12·2	9·7	10·3	12·5	11·3	10·3	10·2	11·2	3·6	8·5
5— 6	18·0	14·7	14·1	17·1	16·2	11·7	13·4	6·6	4·4	8·9
6— 7	18·4	14·9	16·0	19·7	17·5	15·4	13·4	16·8	10·9	13·9
7— 8	20·0	13·7	21·4	27·3	20·1	19·8	15·6	25·5	24·2	21·2
8— 9	18·7	14·7	15·4	31·5	20·8	17·8	14·9	16·7	23·5	18·6
9—10	18·6	15·1	14·9	25·9	19·1	15·0	14·5	12·9	19·1	15·8
10—11	13·8	19·6	19·2	21·1	18·4	12·2	13·3	14·5	16·2	14·1
11—12	15·8	20·8	20·6	22·3	19·9	12·5	16·4	7·8	16·9	14·0
12—13	20·7	19·1	20·4	19·5	19·9	12 2	7·2	16·4	16·9	13·5
13—14	19·2	21·8	20·8	16·1	19·3	13·3	6·7	15·9	16·5	13·5
14—15	14·4	21·1	22·2	12·7	17·2	11·7	6·2	14·3	12·6	11·4
15—16	11·6	15·5	10·8	9·9	11·8	4·4	4·7	7·8	8·1	6 3
16—17	10·8	14·2	9·0	7·1	10·1	4·3	2·6	2·8	7·1	4·7
17—18	6·8	10·4	9·2	10·3	9·2	2·9	0·6	4·1	7·0	4·0
18—19	4·2	13·6	6·9	10·8	8·9	3·9	5·1	8·4	5·4	5·4
19—20	5·3	12·8	6·8	6·6	7·8	4·2	6·1	7·0	2·9	4·8
20—21	4·2	6·5	3·4	5·5	5·0	2·7	6·0	1·3	2·2	3·1
21—22	5·5	5·3	3·4	5·6	5·0	2·3	3·7	1·9	3·0	2·7
22—23	1·6	3·1	6·3	3·0	3·4	1·5	2·3	3·8	1·9	2·2
23—24	3·2	4·0	6·3	3·8	4·3	1·6	3·3	5·1	0·7	2·4

Month. Hours.	X. 50^s—70^s					Y. 50^s—70^s				
	II III IV	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	8·6	7·9	12·2	7·4	8·9	3·6	3·5	10·2	3·8	4·8
1— 2	11·1	5·6	5·6	9·4	8·2	7·8	5·0	4·5	4·2	5·5
2— 3	7·5	8·6	10·4	15·2	10·6	9·3	5·5	6·5	5·5	6·4
3— 4	12·8	9·7	9·0	17·6	12·7	6·5	10·6	6·6	9·5	8·4
4— 5	11·4	12·6	7·6	15·2	12·0	11·2	11·7	7·4	8·3	9·8
5— 6	14·6	15·7	13·6	19·9	16·2	9·6	10·7	13·0	10·6	10·7
6— 7	19·2	20·3	16·6	14·7	17·6	11·2	17·2	18·2	6·7	12·5
7— 8	17·5	21·7	22·0	21·6	20·7	18·2	17·1	25·8	16·9	18·9
8— 9	19·6	16·6	16·1	25·5	19·8	16·9	11·8	19·9	23·5	18·3
9—10	18·6	17·4	14·8	18·2	17·3	14·2	7·5	14·0	19·3	14·2
10—11	16·2	18·2	16·1	17·1	17·0	9·4	13·3	11·6	17·6	13·2
11—12	16·0	15·0	16·7	19·8	17·0	6·9	8·5	11·0	20·2	12·4
12—13	15·8	16·4	14·4	19·8	16·4	10·3	5·2	13·2	17·8	12·4
13—14	17·1	12·3	18·3	16·1	16·0	9·5	5·3	11·1	17·1	11·5
14—15	13·5	12·6	15·6	13·9	14·3	9·3	3·4	6·8	14·4	9·4
15—16	10·7	14·1	12·4	11·6	12·1	6·1	2·0	2·7	6·9	4·9
16—17	7·8	9·5	8·0	8·9	8·5	1·7	2·6	3·3	7·0	4·0
17—18	7·2	7·0	11·8	7·5	8·2	1·6	0·6	6·6	6·2	3·8
18—19	6·6	11·4	10·8	9·0	9·3	3·9	6·6	7·0	4·6	5·3
19—20	6·8	8·9	13·6	6·2	8·6	5·0	13·2	10·4	5·1	7·9
20—21	5·0	7·0	12·6	5·9	7·3	2·7	10·6	11·0	5·1	6·8
21—22	4·7	8·9	10·1	7·1	7·7	4·8	6·4	8·5	2·2	4·8
22—23	4·3	6·7	7·8	7·2	6·4	3·1	5·1	6·8	5·9	5·1
23—24	7·2	8·5	9·3	12·0	8·8	2·3	4·2	9·9	2·2	4·1

TABLE II. (*continued*).

Month. / Hours.	X. 70^s—90^s					Y. 70^s—90^s				
	II III IV	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	10·2	9·3	11·1	12·0	10·7	7·2	3·0	6·4	5·0	5·4
1— 2	11·5	6·4	10·8	13·6	10·8	7·3	3·0	7·1	4·3	5·4
2— 3	11·2	9·9	9·4	18·8	12·7	8·9	4·5	7·1	6·7	6·9
3— 4	9·5	8·5	7·9	13·0	9·9	5·7	6·5	3·8	5·6	5·5
4— 5	12·7	11·5	8·7	12·9	11·6	5·0	6·1	5·3	6·4	5·7
5— 6	13·4	12·8	8·1	14·0	12·3	6·2	7·5	4·0	5·2	5·8
6— 7	12·2	9·2	8·3	10·8	10·3	6·2	4·8	6·0	3·6	5·1
7— 8	14·2	12·1	7·9	13·0	12·0	7·4	7·3	7·3	7·4	7·4
8— 9	10·6	8·8	9·6	13·3	10·8	10·1	6·7	6·7	12·1	9·3
9—10	10·5	8·1	5·4	11·1	9·1	6·5	4·5	2·0	8·2	5·8
10—11	8·1	9·6	6·8	9·5	8·6	4·5	5·5	2·2	8·3	5·5
11—12	9·4	10·1	6·4	7·7	8·4	4·7	1·2	1·6	7·7	4·5
12—13	12·0	10·4	10·4	7·6	10·0	5·1	2·6	0·8	10·2	5·6
13—14	8·8	4·7	7·6	9·6	7·8	5·2	2·7	4·8	8·2	5·7
14—15	9·3	9·7	10·9	8·9	9·6	5·6	2·0	3·0	7·5	5·1
15—16	9·0	8·2	8·8	3·8	7·3	3·5	3·3	3·6	4·6	3·9
16—17	3·5	4·9	4·0	2·6	3·7	0·9	1·3	1·4	3·7	2·0
17—18	4·0	4·9	7·7	7·4	6·0	2·4	0·0	4·1	4·0	2·8
18—19	4·6	8·0	9·4	5·4	6·6	5·1	4·1	7·1	4·3	5·0
19—20	7·6	8·9	10·2	10·0	9·1	9·5	8·5	7·0	5·1	7·4
20—21	5·0	6·4	9·3	7·0	6·8	3·8	7·4	11·3	4·8	6·3
21—22	7·1	9·3	9·1	10·4	9·0	4·2	5·5	5·6	5·5	5·2
22—23	7·9	9·8	9·7	9·8	9·3	5·4	4·2	4·5	4·1	4·6
23—24	10·8	6·1	8·8	11·5	9·5	5·9	2·8	3·8	4·1	4·3

Month. / Hours.	X. 90^s—130^s					Y. 90^s—130^s				
	II III IV	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	19·6	9·6	10·6	24·8	16·8	8·8	7·0	8·3	11·5	9·1
1— 2	20·1	9·6	9·2	27·7	17·5	10·6	3·0	7·7	8·2	7·6
2— 3	18·2	9·3	8·8	21·8	15·2	8·9	2·5	7·1	11·1	7·7
3— 4	15·4	9·5	13·2	19·6	14·8	8·6	7·5	8·3	7·2	7·9
4— 5	13·9	10·6	8·0	16·4	12·6	7·4	6·6	8·6	6·8	7·3
5— 6	17·6	9·2	7·1	14·8	12·6	6·0	5·9	3·9	4·8	5·2
6— 7	10·4	7·0	5·0	12·8	9·1	3·8	4·3	4·7	5·3	4·5
7— 8	5·4	7·7	4·2	8·4	6·6	6·2	3·1	3·3	5·8	4·9
8— 9	9·0	6·8	7·6	10·0	8·5	8·1	3·1	6·7	12·5	8·0
9—10	9·7	6·1	5·9	8·5	7·7	6·1	1·5	2·7	8·2	5·1
10—11	5·7	8·1	7·8	12·0	8·5	4·5	0·6	1·5	3·9	2·9
11—12	8·2	6·7	6·4	8·8	7·6	2·6	1·2	2·3	4·2	2·8
12—13	9·1	6·8	6·5	6·4	7·2	1·9	1·3	0·0	5·1	2·5
13—14	7·5	10·2	6·1	10·0	8·5	4·3	2·7	1·6	4·7	3·6
14—15	11·4	7·5	9·4	6·3	8·5	2·8	1·4	0·8	4·8	2·8
15—16	7·7	5·0	8·8	5·0	6·5	2·6	2·0	3·6	3·7	3·1
16—17	5·2	8·5	4·0	5·6	5·8	0·9	2·0	0·7	4·1	2·1
17—18	4·0	3·5	4·6	7·1	5·0	2·8	0·6	1·4	3·3	2·3
18—19	5·4	7·0	11·4	8·9	8·1	7·0	6·6	4·5	5·4	6·0
19—20	7·6	5·0	14·5	14·4	10·4	9·1	6·1	8·3	6·6	7·5
20—21	8·1	8·1	12·2	12·1	10·1	8·0	10·1	10·7	7·8	8·9
21—22	11·0	10·0	11·6	11·1	10·9	9·5	10·1	13·0	7·4	9·7
22—23	12·3	6·4	10·2	19·6	12·5	11·2	8·8	7·7	9·3	9·5
23—24	16·4	6·0	13·1	22·6	15·0	11·8	5·1	5·1	10·1	8·5

TABLE II. (*continued*).

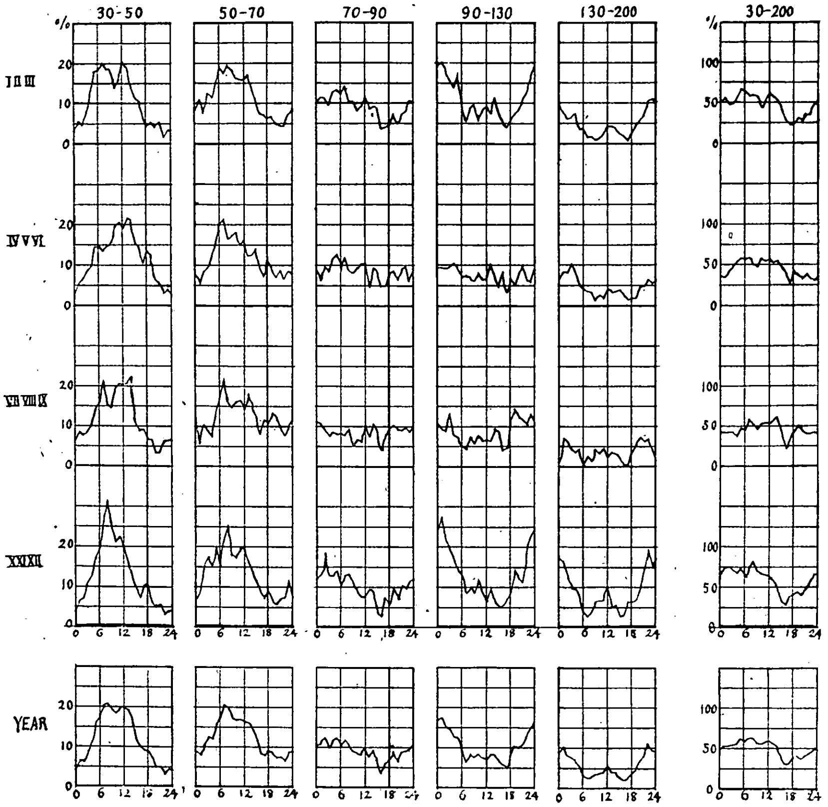
Month. Hours.	X. 130^s—200^s					Y. 130^s—200^s				
	II III IV	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	9·6	6·5	1·5	17·1	9·2	6·8	5·0	1·3	7·3	5·5
1— 2	7·6	8·5	7·2	16·7	10·3	4·9	5·0	3·2	5·8	4·9
2— 3	6·1	8·1	6·2	12·1	8·3	4·0	2·5	1·9	5·0	3·6
3— 4	6·5	10·2	4·7	9·8	7·9	1·2	3·5	3·2	3·6	2·8
4— 5	7·4	8·3	3·2	9·8	7·4	3·7	2·0	3·3	3·6	3·2
5— 6	3·3	5·4	4·4	8·2	5·4	4·6	4·3	3·9	4·8	4·5
6— 7	3·7	4·0	0·6	3·9	3·2	1·7	1·6	1·3	1·6	1·6
7— 8	1·2	3·3	2·6	2·7	2·4	0·8	0·5	0·7	2·3	1·2
8— 9	1·6	3·3	1·5	3·7	2·6	2·8	3·1	0·7	3·8	2·8
9—10	0·8	1·4	4·4	6·3	3·3	2·0	0·5	2·0	4·1	2·3
10—11	1·2	2·7	3·4	6·6	3·6	1·6	0·6	0·7	4·2	2·0
11—12	2·4	1·8	4·4	6·6	3·9	0·4	0·6	0·0	1·9	0·9
12—13	4·1	4·1	2·5	10·2	5·5	0·5	0·0	0·0	2·0	0·8
13—14	4·1	3·2	3·6	5·0	4·0	1·9	2·0	0·8	1·6	1·6
14—15	3·7	3·7	3·1	5·4	4·1	0·5	0·0	0·8	0·4	0·4
15—16	2·5	3·8	2·1	2·7	2·8	0·0	0·7	0·7	1·5	0·8
16—17	1·7	2·8	0·5	3·0	2·1	0·9	0·0	0·7	2·6	1·3
17—18	0·4	1·4	0·5	6·4	2·4	0·8	0·0	0·0	3·7	1·4
18—19	2·7	1·8	3·5	6·5	3·7	4·3	1·5	1·3	7·2	4·1
19—20	4·1	1·8	5·3	6·9	4·6	4·2	4·2	6·4	8·0	5·7
20—21	5·7	4·8	7·3	9·6	6·9	5·3	5·5	7·6	8·5	6·7
21—22	6·9	4·9	6·2	14·6	8·4	7·6	10·6	3·7	5·9	7·1
22—23	10·5	6·7	6·8	19·6	11·3	5·4	8·4	2·6	9·3	6·8
23—24	11·0	5·8	4·4	15·3	9·5	6·6	2·8	5·1	10·1	6·5

Month. Hours.	X. 30^s—200^s					Y. 30^s—200^s				
	II III IV	V VI VII	VIII IX X	XI XII I	Year.	II III IV	V VI VII	VIII IX X	XI XII I	Year.
0— 1	51·2	36·1	42·0	65·2	49·7	28·4	21·5	30·1	29·9	27·5
1— 2	56·0	35·3	41·5	73·8	53·2	33·9	19·0	30·3	24·8	27·1
2— 3	47·5	42·6	42·6	75·0	53·2	36·4	21·0	26·5	31·1	29·0
3— 4	51·7	46·2	43·2	70·1	54·2	30·2	38·2	29·7	27·9	31·3
4— 5	57·6	52·7	37·8	66·8	54·9	37·6	36·6	35·8	28·7	34·5
5— 6	66·9	57·8	47·3	74·0	62·7	38·1	41·8	31·4	29·8	35·1
6— 7	63·9	55·4	46·5	61·9	57·7	38·3	41·3	47·0	28·1	37·6
7— 8	58·3	58·5	58·1	73·0	61·8	52·4	43·6	62·6	56·6	53·6
8— 9	59·5	50·2	50·2	84·0	62·5	55·7	39·6	50·7	75·4	57·0
9—10	58·2	48·1	45·4	70·0	56·5	43·8	28·5	33·6	58·9	43·2
10—11	45·0	58·2	53·3	66·3	56·1	32·2	33·3	30·5	50·2	37·7
11—12	51·8	54·4	54·5	65·2	56·8	27·1	27·9	22·7	50·9	34·6
12—13	61·7	56·8	54·2	63·5	59·0	30·0	16·3	30·4	52·0	34·8
13—14	56·7	52·2	56·4	56·8	55·6	34·2	19·4	34·2	48·1	35·9
14—15	52·3	54·6	61·2	47·2	53·7	29·9	13·0	25·7	39·7	29·1
15—16	41·5	46·6	42·9	33·0	40·5	16·6	12·7	18·4	24·8	19·0
16—17	29·0	39·9	21·5	27·2	30·2	8·7	8·5	8·9	24·5	14·1
17—18	22·4	37·2	33·8	38·7	30·8	10·5	1·8	16·2	24·2	14·3
18—19	23·5	41·8	42·0	40·6	36·6	24·2	23·9	35·4	26·9	25·8
19—20	31·4	37·4	50·4	44·1	40·5	32·0	38·1	39·1	27·7	33·3
20—21	28·0	32·8	44·8	40·1	36·1	22·5	39·6	41·9	23·4	31·8
21—22	35·2	38·4	40·4	48·8	41·0	28·4	36·3	32·7	24·0	29·5
22—23	36·6	32·7	40·8	59·2	42·9	26·6	28·8	25·4	30·5	28·2
23—24	48·6	30·4	41·9	65·2	47·1	28·2	18·2	29·0	27·2	25·8

The results are also plotted in Fig. 8. As will be clearly seen, the waves with periods less than 70^s show decided maximum frequency in the day time, more or less near noon, while those longer than 90^s are more frequent during the night. For 30^s–50^s waves, a tendency is suggested in the X-component that the hour of the maximum frequency is earlier in winter than in summer. Besides, for X- as well as for Y-components a secondary minimum near noon is suspected in spring and autumn. For 50^s–70^s waves, the hours of the day time maxima seem to fall somewhat earlier in the morning than for the shorter waves, and a secondary maximum in the early evening is suggested in some of the Y-diagrams. 70^s–90^s waves form apparently a transition stage to the still longer waves. For 90^s–130^s and 130^s–200^s waves, the

X

Fig. 8, X.



maximun frequencies fall decidedly in the night hours, mostly a little before midnight, while at the same time, a secondary maximum in the day time is suspected in some of the graphs.

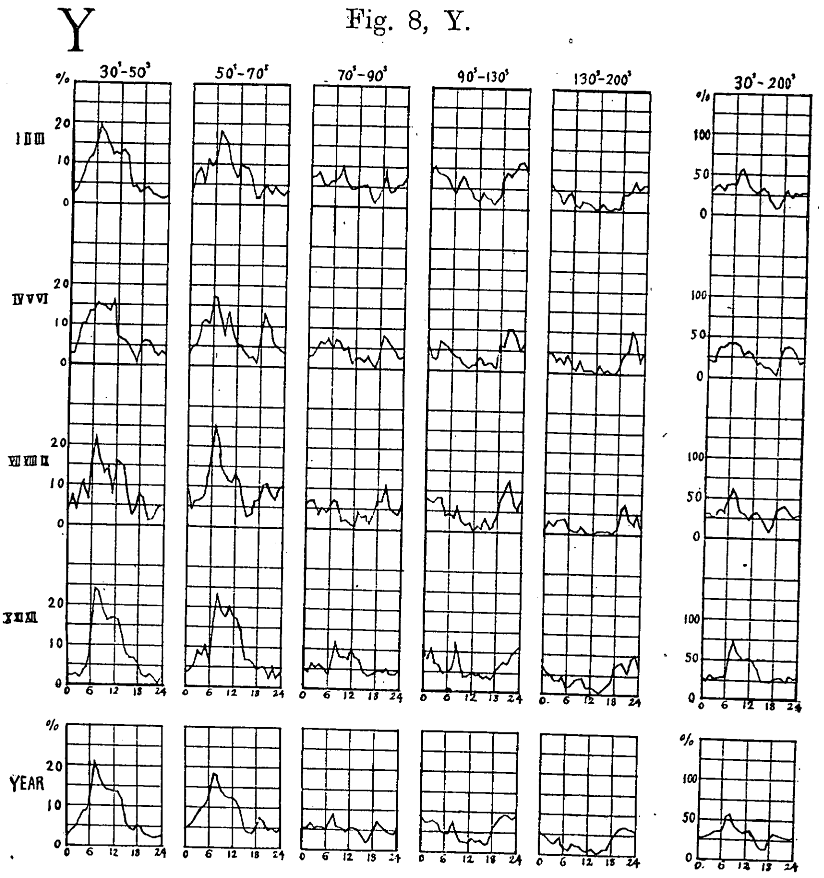


Fig. 8, Y.

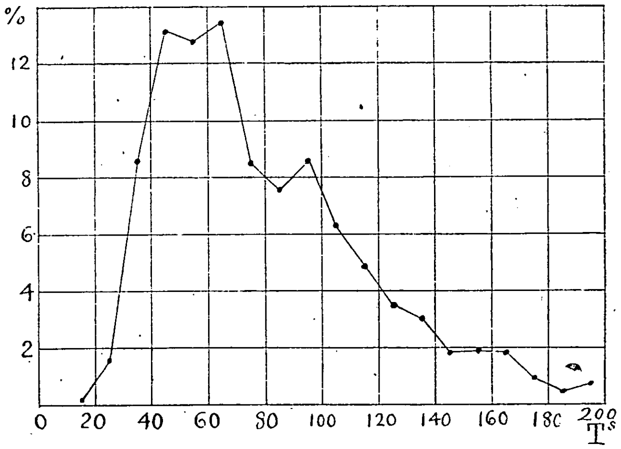
The frequency of all waves, 30s–200s, shows a maximum in the morning and a minimum in the evening.

It is to be remarked that the diurnal variation of the number of "spasms" as well as "pulsations" studied by W. van Bemmeln, and also that of the frequency of the disturbed hours, as could be deduced from the diagrams in Bidlingmeier's paper above cited, show a character quite similar to the variation of the frequency of the "longer waves" here studied, up to the appearance of the secondary daytime maxima.

On closer examination of the diagrams mentioned in p. 25 above, it is frequently found that wave trains with a certain period appear in almost the same isolated hours of two or three successive days, as if to repeat the particular program of the preceding days. This is interesting as one of the facts strongly suggesting that the agent producing these waves is intimately connected with the position of the earth relative to the sun. In some cases, it also happened that the hours of the successive days in which a particular train appears seemed to shift gradually in either direction.

9. *The "spectra" of the magnetic waves.* To find the most frequent period, the observed periods were at first classified second by second and the total number of *trains* (not that of days or hours in which they occurred) belonging to these groups were plotted in a diagram with the periods as abscissa. The diagram obtained showed a great number of maxima and minima with intervals of several seconds. Most of these maxima and minima were, however, found to have no real physical meaning, being either due to chance,[1] or due to some involuntary tendency of particular persons to prefer some particular fractions of the scale division when measuring the wave trains on the records with a millimeter scale.[2] Hence, finally the following groups were taken: 20^s–30^s, 30^s–40^s *etc.* up to 190^s–200^s. The results are given in Table III. and also plotted in Fig. 9. As will be seen, the maximum frequency is generally

Fig. 9.



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[1] T. Terada, Proc. Tôkyô Math.-Phys. Soc., 8, 1916, p. 492.

[2] Note that a tenth of a millimeter corresponds to about 1·8 sec.

at the interval of periods 50^s-60^s. The curve reminds us rather of the energy distribution of the luminous spectra than the usual probability curves. A secondary maximum is seen near the period 90^s. The results for other years are of quite similar character.

TABLE III.

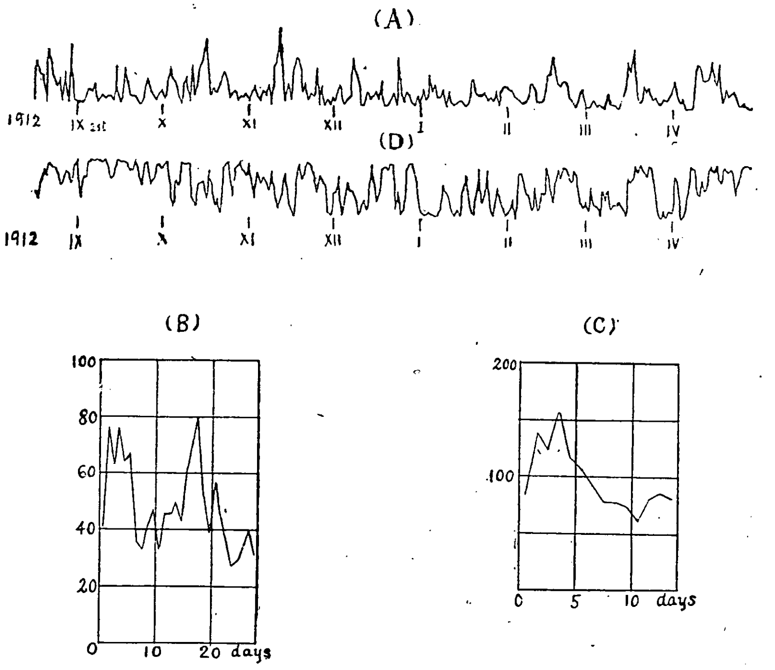
For each season, the first column gives the number of trains and the second the percentage of the total number.

1912. X.

Season. / Period.	I II III		IV V VI		VII VIII IX		X XI XII		Year.	
10s— 20s	3	0·3	2	0·2	0	0·0	2	0·1	7	0·1
20 — 30	20	1·8	20	2·3	10	0·6	36	2·0	86	1·6
30 — 40	120	10·9	75	8·8	126	7.8	143	7·8	464	8·6
40 — 50	145	13·2	114	13·3	207	12·8	250	13·6	716	13·2
50 — 60	134	12·2	106	12·4	231	14·3	224	12·1	695	12·8
60 — 70	116	10·5	100	11·7	289	17·9	227	12·3	732	13·5
70 — 80	85	7·7	63	7·4	168	10·4	145	7·9	461	8·5
80 — 90	77	7·0	56	6·5	136	8·4	141	7·6	410	7·6
90 — 100	107	9·7	66	7·7	139	8·6	154	8·4	466	8·6
100 — 110	70	6·4	51	6·0	92	5·7	127	6·9	340	6·3
110 — 120	59	5·4	36	4·2	56	3·5	114	6·2	265	4·9
120 — 130	45	4·1	35	4·1	38	2·4	74	4·0	192	3·5
130 — 140	33	3·0	36	4·2	34	2·0	62	3·4	165	3·0
140 — 150	24	2·2	23	2·7	16	1·0	37	2·0	100	1·9
150 — 160	21	1·9	20	2·3	29	1·8	35	1·9	105	1·9
160 — 170	24	2·2	21	2·5	19	1·2	37	2·0	101	1·9
170 — 180	8	0·7	9	1·1	17	1·1	17	0·9	51	0·9
180 — 190	5	0·5	5	0·6	4	0·3	10	0·5	24	0·4
190 — 200	5	0·5	18	2·1	7	0·4	10	0·5	40	0·7
	1101		856		1618		1845		5420	

The result must not be hastily interpreted as showing that the *absolute* maximum of frequency is at the above mentioned interval. If we could multiply the sensibility of the instruments and turn at the same time the recording drum correspondingly faster, the waves most conspicuous in our case might possibly be replaced by others with a decidedly shorter periods. Besides, it must be taken into account that the shorter the period, the greater will be the chance of revealing isolated trains in a given interval of time, even if the mean intensity and number of maxima of the trains be independent of the periods. This partially explains the falling off of the frequency curve toward the longer period in a form of hyperbola. Hence the real meaning of the above result can be simply interpreted as follows: As far as the sensibilities of the present instruments reveal, every period is rather evenly represented as in a continuous spectrum, showing no very sharply defined maximum, except two rather flat relative maxima near 60^s and 90^s. The above holds good when we take the different hours of day and night altogether into account. When the different hours are taken separately, the results are somewhat different as already remarked in §8. This latter point will be referred to again later on, with respect to differently chosen materials. Here it may be remarked only

Fig. 10.



that the secondary maximum of frequency at about 90^s corresponds to the most frequent waves observed during night hours.

No conspicuous group of waves with about 10^s period, observed by Birkeland, could be confirmed in our records, though such waves might have been detected if the amplitudes were somewhat larger than $0{\cdot}2\ \gamma$.

10. *Fluctuation of frequency of regular waves in successive days.* The number of *hours* (not that of trains) disturbed by regular waves was counted for each of the successive days and plotted in a diagram with the days as abscissa. The results showed apparently very irregular fluctuations, either if we take all waves with the periods 30^s–200^s, or only 30^s–70^s waves. Though we must be very cautious in deducing any periodicity from such material, the results show nevertheless a remarkable tendency to suggest the existence of a period, or rather of an "interval" of about 25–30 days. The most conspicuous examples are illustrated by the interval from October, 1912, to April, 1913, especially when the waves with the periods 30^s–70^s alone are taken into account (Fig. 10, A). Fig. 10, B and C were obtained by superposing the successive series of 14 or 28 days[1] respectively. These graphs seem to suggest the existence more of a 14 days period than a 28 days one, the amplitude amounting to the same order of magnitude as the mean value. The last procedure will however, have a definite meaning only when the phenomena are simply and purely periodic, but not when the frequency maxima are repeated with conspicuous amplitudes only a small number of times in a definite period, and then replaced by another of a similar kind, but setting in more or less abruptly with quite accidental time relation to the former. The general aspect of the frequency diagram here in question strongly suggests that the nature of the phenomena concerned is of the type above mentioned. In the above example, we may probably assume the accidental superposition of two independent series with the longer period, say 28 days. The lunar period is to be excluded, since the above fluctuation.

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[1] 28 days instead of 27 days was taken simply because it is an even number, no weight being laid here on the exactitude of the period.

does not keep pace with moon's phases. It will be more plausible to suppose that the period in question is related to the sun's rotation.[1)]

It is to be regretted that our observations were not very adequate for investigations of the kind mentioned above, since in the warmer half of the year the records were occasionally defective. For the same reason, we must unfortunately refrain from carrying out the statistical investigations with respect to the *annual* or *seasonal* variation of the daily frequency.

11. *Probable relation of the frequency of waves with meteorological elements.* Since the phenomena of the magnetic pulsations were suspected of being closely related to some atmospheric phenomena as will be seen later on, it seemed possible that they might have a sensible correlation with some of the meteorological elements. Among the latter elements, cloudiness was considered as a most promising one for investigation, since its correlation with the solar activity is already acknowledged by some authorities,[2)] and moreover, considering the layer of cloud as a conducting sheet covering a considerable part of the earth's surface it may in some way or other play a sensible rôle where electromagnetic waves of not indefinitely long periods are in question. To carry out the comparison effectively, it will however, be necessary to take the cloudiness over a sufficiently wide area of the earth's surface in order to eliminate local irregularities of secondary nature which are very conspicuous in the case of this element. As the only available data in hand, the Monthly Reports of the Central Meteorological Observatory were used. Different groups of the local stations were taken and the average amounts of cloudiness were calculated. The results of comparison were not conclusive, though some correlation was suspected between the frequency of the magnetic pulsations and the average cloudiness for Tôkyô, Tyôsi, Mito and Maebasi,

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[1)] Maunder and Marchand, *loc. cit.* Curiously enough, Schuster, on examining Maunder's data, also noticed a pronounced 14 days period. If the 14 days period was to be invariably found in allied phenomena, then the seat of the direct or indirect cause of magnetic disturbances may be sought in two antipodal portions of the sun's surface.

[2)] Hann, Lehrbuch der Meteorologie, 2nd. Ed., p. 476.

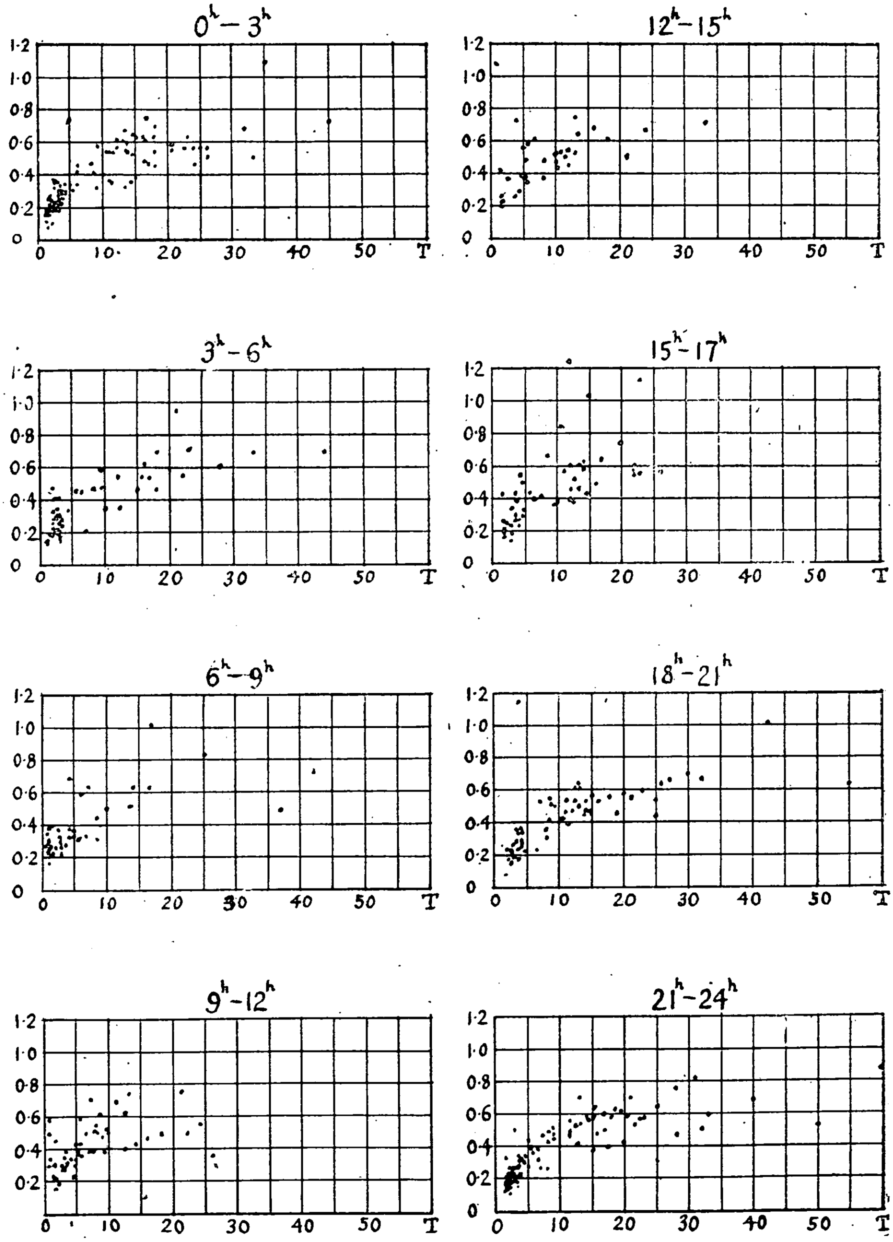
which is shown in Fig. 10, D. This point decidedly deserves further investigation.

12. To carry out a more detailed quantitative investigation regarding the nature of the magnetic pulsations under consideration, the records of 1913 were specially chosen, since in this year the determination of the sensibilities of the instruments was most regularly made and hence most adequate for the quantitative comparison of the different components. It is, however, to be regretted that during the summer months the records were too frequently defective, chiefly due to damage to the photographic paper on account of the extreme dampness notwithstanding the use of the desiccator, and also due to the condensation of the atmospheric humidity in minute drops or mist which was especially dense in the daytime and caused a remarkable absorption of light. It must therefore be remarked that in some statistical studies to be described later, the winter season has a rather overweighing influence on the general results though the statistics extend over a full year. As far as the present investigations are concerned, no serious modification of the general results will, however, be required on that account.

13. *Ratio of amplitudes of X- and Z-components.* As already mentioned, the waves appearing in the Z-component are generally the reduced facsimiles of those in the X-component, except that the former always lags behind the latter in definite amounts depending on the periods, but usually less than a quarter of a period. In other words, the end of the magnetic vector representing the periodic disturbing field revolves on more or less elliptic orbits, with their major axes mostly dipping towards N. To find first a quantitative relation between the amplitudes of the two components, the following reductions were made. Specially regular portions of wave trains were carefully chosen out from among all the records of the year, and for each train the ratio of the mean amplitude of corresponding waves for the two components was calculated, the daily value of the sensibilities being duly taken into account. The results were tabulated together with the corresponding periods and the hours of

occurrence. Plotting the ratios as ordinates with the periods as abscissa in a diagram, the points representing different trains are

Fig. 11.



dispersed rather irregularly as shown in Fig. 11 in which the results are grouped with respect to the hours of occurrence. Next, the results were grouped according to the periods and the mean ratios were calculated for each group of the periods. The results are shown in the following Table and also plotted in Fig. 12 where the mean periods[1] are taken as abscissa.

Fig. 12.

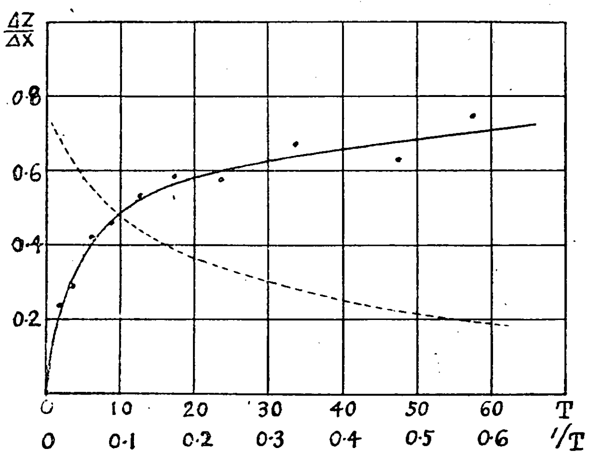


Table IV.

Range of T.	0^{m}–$2^{\mathrm{m}}{\cdot}5$	$2^{\mathrm{m}}{\cdot}6$–$5^{\mathrm{m}}{\cdot}0$	$5^{\mathrm{m}}{\cdot}1$–$7^{\mathrm{m}}{\cdot}5$	$7^{\mathrm{m}}{\cdot}6$–$10^{\mathrm{m}}{\cdot}0$	$10^{\mathrm{m}}{\cdot}1$–$15^{\mathrm{m}}{\cdot}0$	$15^{\mathrm{m}}{\cdot}1$–$20^{\mathrm{m}}{\cdot}0$	$20^{\mathrm{m}}{\cdot}1$–$30^{\mathrm{m}}{\cdot}0$	$30^{\mathrm{m}}{\cdot}1$–$40^{\mathrm{m}}{\cdot}0$	$40^{\mathrm{m}}{\cdot}1$–$50^{\mathrm{m}}{\cdot}0$	$51^{\mathrm{m}}{\cdot}0$–$60^{\mathrm{m}}{\cdot}0$
Mean T.	$1^{\mathrm{m}}{\cdot}72$	$3^{\mathrm{m}}{\cdot}49$	$6^{\mathrm{m}}{\cdot}23$	$8^{\mathrm{m}}{\cdot}90$	$12^{\mathrm{m}}{\cdot}86$	$17^{\mathrm{m}}{\cdot}41$	$23^{\mathrm{m}}{\cdot}76$	$33^{\mathrm{m}}{\cdot}90$	$47^{\mathrm{m}}{\cdot}50$	$57^{\mathrm{m}}{\cdot}50$
$\frac{\Delta Z}{\Delta X}$.	0·235	0·291	0·421	0·459	0·536	0·580	0·574	0·672	0·621	0·745
$\mathrm{tg}^{-1}\frac{\Delta Z}{\Delta X}$.	$13^{\circ}{\cdot}1$	$16^{\circ}{\cdot}1$	$22^{\circ}{\cdot}6$	$24^{\circ}{\cdot}5$	$28^{\circ}{\cdot}0$	$29^{\circ}{\cdot}9$	$30^{\circ}{\cdot}4$	$33^{\circ}{\cdot}4$	$31^{\circ}{\cdot}7$	$36^{\circ}{\cdot}5$

As will be seen, the ratio increases at first rapidly with the periods, then gradually tends to an asymptotic value probably nearly equal to unity. The fact is rather remarkable and must serve as an important basis for explaining the phenomena in

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1) The mean of the actual periods corresponding to the trains taken was calculated, not the simple mean of the interval of the period concerned.

question. For the purpose of a subsequent reference, the value of the ratios are also plotted with $1/T$, *i.e.* the reciprocal of the periods as abscissa (the dotted line in Fig. 12).

The irregular dispersion of the points in Fig. 11 is too remarkable to be considered as due to the inaccuracy of the measurements of the record or to the inconstancy of the instruments, but must be regarded as actually inherent to the nature of the phenomena. Neither is the irregularity at all eliminated, even if we take the ratio of the amplitude of Z to that of the *resultant* horizontal component, $\sqrt{\Delta X^2+\Delta Y^2}$ instead, which was actually calculated for the earlier period of the year.

Though we have not calculated the ratio $\Delta Z/\sqrt{\Delta X^2+\Delta Y^2}$ throughout the year, it will surely be an overestimation of the Y-component if we take $\Delta Z/\sqrt{2}\,\Delta X$ for it, since, as will be seen later in §15, the azimuth of the horizontal component of the disturbing periodic field is generally less than 45° and about 23°·7 on an average (see Fig. 16).

14. *Phase retardation of Z-component with respect to X-component.* Next, the phase relation of X- and Z-components were to be investigated. Since the corresponding waves in the two components appear as a rule widely apart on the photographic records where no special zero-line was recorded, a device was necessary to accurately mark off the corresponding time in each of the components at any part of the record. For this purpose, a kind of sliding T-square, originally constructed by Dr. Kadooka, was found very convenient. For the waves of shorter periods, utmost caution was still needed to avoid serious mistakes, very liable to be committed by the slight inclination of the sliding lineal which was to be kept parallel to the time-mark lines.

The mean retardation of the maxima and minima of ΔZ relative to those of ΔX were calculated in fractions of the periods. The corresponding values for maxima and minima respectively were often sensibly different, especially when the waves are not of simple form, in which case the mean of the two values was simply taken. Plotting the values for different trains with the periods as abscissa, the annexed figure was obtained (Fig. 13, A).

Classifying the period in the same groups as in the case of Fig. 12, we obtain the following Table and also Fig. 13, B.

Fig. 13, A.

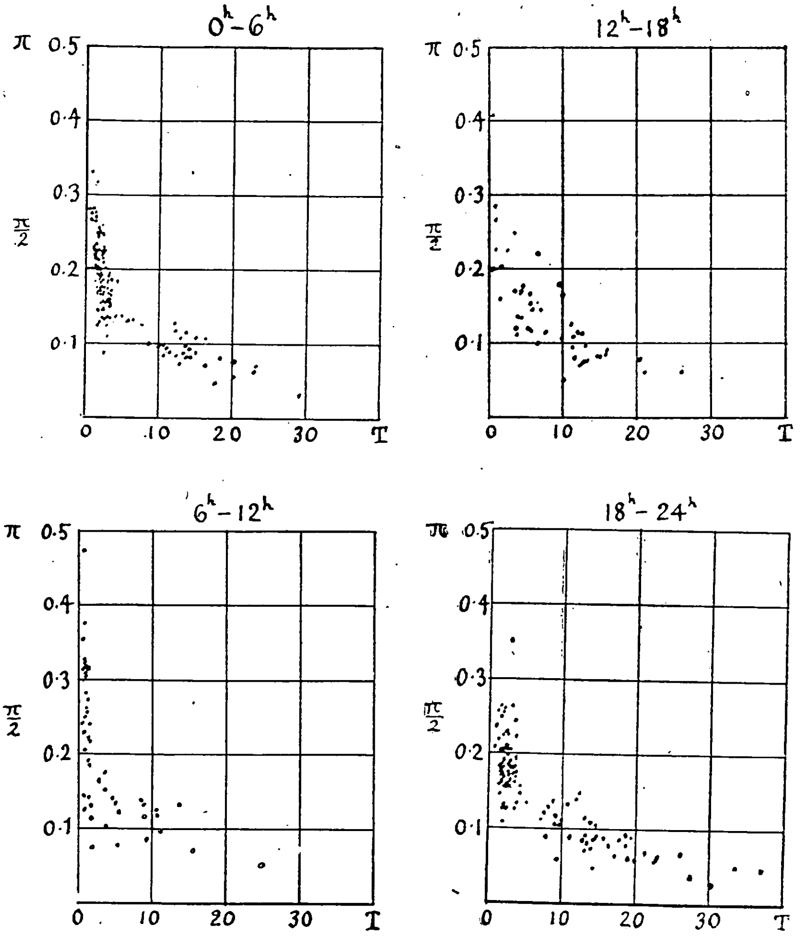
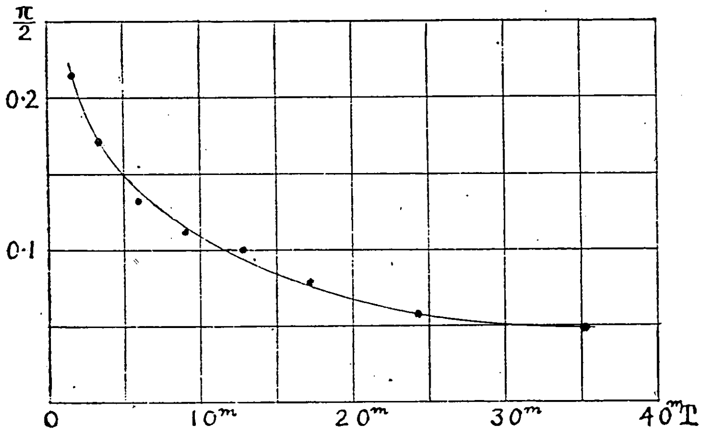


TABLE V.

Range of *T*.	m m 0– 2·5	m m 2·6– 5·0	m m 5·1– 7·5	m m 7·6–10·0	m m 10·1–15·0	m m 15·1–20·0	m m 20·1–30·0	m m 30·1–40·0
Mean *T*.	m 1·59	m 3·34	m 5·95	m 9·11	m 12·80	m 17·22	m 24·45	m 35·15
Phase.	0·215	0·172	0·134	0·112	0·100	0·079	0·057	0·049

As will be seen from Fig. 13, the retardation is remarkable for shorter waves, sometimes amounting to decidedly more than

Fig. 13, B.



a quarter period in the case of 30^s waves and gradually decreasing in a hyperbolic curve apparently tending to zero for longer waves over one hour. Combining the result with that obtained in the preceding article, we may trace the meridional projection of the elliptic orbit described by the end of the periodic disturbing vector of different periods.

15. *Azimuth of periodic disturbing fields, or the relation of amplitudes of X- and Y-components.* While the amplitude ratio between ΔX and ΔZ depends very much on the periods, but not sensibly on the hours of day, the ratio between the amplitudes of X- and Y-waves varies remarkably with the hours of occurrence, but not sensibly with the periods. To investigate the case more closely, the following procedures were adopted.

At first, all available records were examined and the number of regular trains were counted in which the two horizontal components are nearly parallel to each other, *i.e.* of phase difference zero, and also those in which the two are nearly inverted, *i.e.* of phase difference π. The results are given in the Table VI. and also in Fig. 14, regardless of the periods of the waves.

TABLE VI.

The numbers are given in % of all cases for each season.
+ denotes the cases where $\varDelta X$ and $\varDelta Y$ are nearly parallel to each other,
− denotes the cases where $\varDelta X$ and $\varDelta Y$ are nearly inverted to each other.

	III IV V		VI VII VIII		IX X XI		XII I II		Year.	
Hours.	+	−	+	−	+	−	+	−	+	−
0— 1	2·5	1·1	3·5	0·4	1·8	1·0	1·7	1·6	2·2	1·2
1— 2	2·2	1·1	0·7	0·0	2·4	0·0	1·0	1·0	1·6	0·7
2— 3	2·5	1·4	2·1	0·4	2·5	0·4	1·6	1·2	2·1	1·0
3— 4	1·9	1·4	4·6	1·8	1·2	0·0	1·2	0·9	1·8	0·9
4— 5	2·2	1·9	5·6	0·0	2·9	0·2	1·6	0·5	2·5	0·7
5— 6	2·5	1·4	6·7	0·0	1·2	0·4	0·8	0·5	2·1	0·6
6— 7	8·6	0·3	11·6	0·4	4·1	0·2	2·0	0·0	5·5	0·2
7— 8	7·2	1·4	13·4	0·0	14·7	0·0	6·9	0·7	9·5	0·6
8— 9	8·9	1·4	5·6	0·0	15·7	0·6	13·0	1·1	11·5	0·9
9—10	4·7	0·9	5·6	0·4	11·5	1·0	8·5	1·1	7·7	0·9
10—11	2·3	0·3	1·8	0·0	5·7	0·2	6·1	0·6	4·4	0·3
11—12	2·3	0·0	1·8	0·7	6·1	0·0	6·4	0·5	4·7	0·3
12—13	1·4	0·0	1·4	0·4	3·5	0·0	5·4	0·7	3·4	0·3
13—14	1·9	0·2	0·7	0·4	3·1	0·8	4·6	0·6	3·1	0·5
14—15	1·1	0·2	0·7	0·0	1·6	0·2	2·2	0·3	1·6	0·2
15—16	0·6	0·5	1·1	0·0	0·2	0·2	0·7	0·5	0·6	0·3
16—17	0·2	0·3	0·0	0·0	0·2	0·4	0·7	0·9	0·3	0·5
17—18	0·6	0·6	0·4	0·0	0·4	0·6	1·4	1·8	0·8	1·0
18—19	1·6	4·1	1·8	2·1	1·0	2·2	1·1	2·7	1·3	2·9
19—20	1·9	4·7	2·5	4·6	0·4	2·5	0·7	3·3	1·1	3·7
20—21	0·6	5·3	3·2	1·1	0·4	1·8	0·7	2·6	0·9	3·0
21—22	2·2	4·2	3·5	2·8	0·4	2·5	0·7	2·4	1·4	3·0
22—23	1·7	3·1	1·1	3·9	0·2	2·2	0·3	2·3	0·8	2·7
23—24	1·1	1·7	1·1	0·7	0·8	1·0	1·2	2·5	1·1	1·7

It will be seen that the number of cases with the phase difference zero attains a maximum in the early hours of morning, while that with the phase difference π shows a maximum in the evening (Fig. 14).

Secondly, those cases were chosen where either the one, X- or Y-component, is alone conspicuous, while the other is quite insignificant. The distribution of these special cases in different hours is given in Table VII. and plotted in Fig. 15.

As may be seen, the cases where X-waves are alone conspicuous, or in other words, the disturbing field is directed nearly in the meridian, are most frequent near midnight and noon, while the opposite cases, *i.e.* those in which the disturbance occurs nearly in the WE direction, are most frequent in the morning and evening.

Fig. 14.

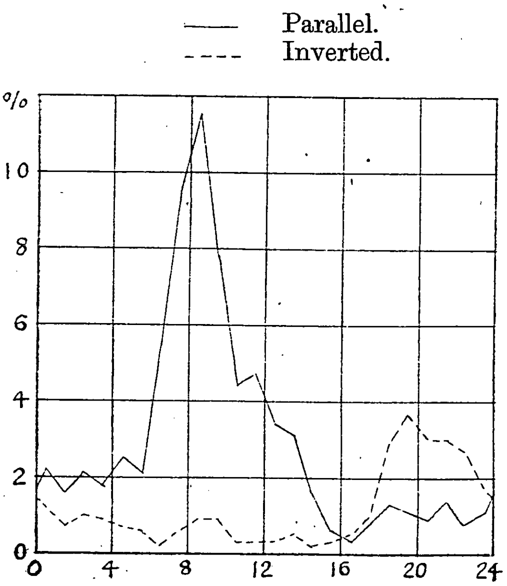


Fig. 15.

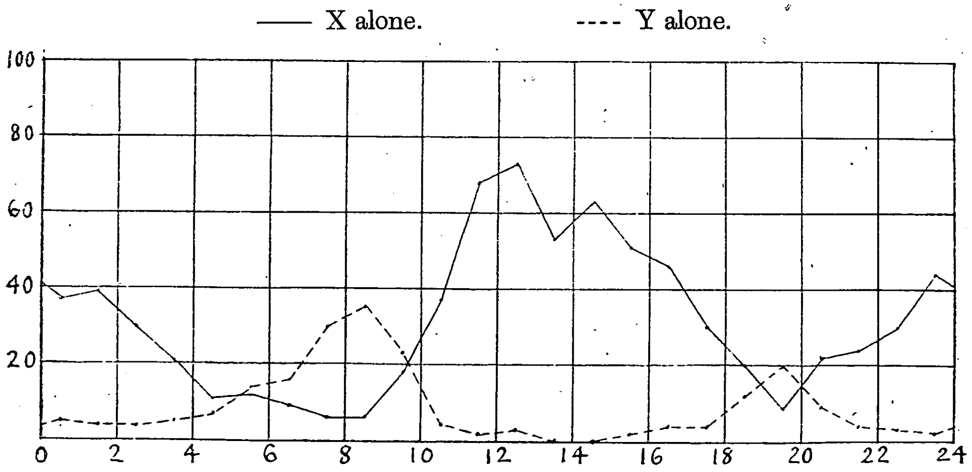
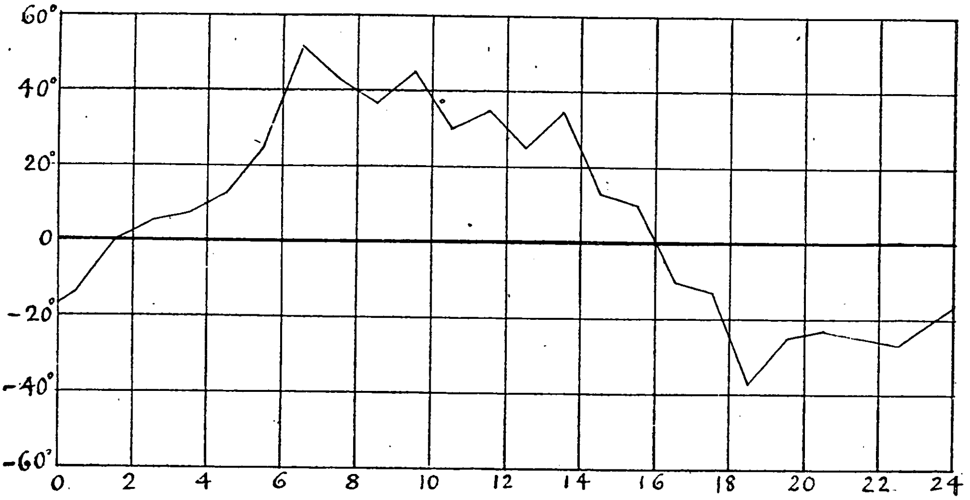


TABLE VII.

Hour.	X only.	Y only.	Hour.	X only.	Y only.	Hour.	X only.	Y only.	Hour.	X only.	Y only.
0—1	37	5	6—7	9	16	12—13	73	3	18—19	20	12
1—2	39	4	7—8	6	30	13—14	53	0	19—20	9	20
2—3	30	4	8—9	6	35	14—15	63	0	20—21	22	9
3—4	21	5	9—10	18	23	15—16	51	2	21—22	24	4
4—5	11	7	10—11	37	4	16—17	46	4	22—23	30	3
5—6	12	14	11—12	68	2	17—18	30	4	23—24	44	2

Thirdly, taking the records of 1913 only, those cases were chosen in which the phase difference of the two horizontal components was either o or nearly π, *i.e.* in which the periodic disturbance is "polarized" in a certain azimuth, and the ratio of the amplitudes was carefully determined which corresponds to the tangent of the azimuthal angle α. The latter angle was counted from N, positive value taken toward W. The results, if plotted in a diagram, with the hours as abscissa, regardless of

Fig. 16.



the periods, show very irregular scattering of the points representing different trains. Nor is the irregularity lessened by choosing the waves of a definite period only.

The most frequent value of the azimuth for each hour was then determined, not by taking the simple mean value, but by plotting graphically the frequency of different azimuthal angles for each hour and taking the maximum point of the frequency curve thus obtained. The results are given in Table VIII. and plotted in Fig. 16, which generally confirms the result to be inferred from Fig. 15. Neither is the present result in contradiction with that shown in Fig. 19, since the maximum and minimum value of $\Delta Y/\Delta X$ must correspond to cases where the direction of the disturbing force makes the largest angle with the meridian, and the zero value of this ratio must correspond to the case where the field is nearly in the meridian. These results show that the azimuthal angle, on an average, undergoes a continous and fairly regular diurnal variation. Though the present data are too defective for drawing conclusive inferences with respect to the seasonal difference of the above relation, they suggest that in summer, the azimuth has a secondary minimum near noon. Besides, it is suspected that the mean azimuthal angle in day time is somewhat less for longer periods than for shorter. These points must, howevér, be postponed for a future investigation.

TABLE VIII.

Hour.	α°	Hour.	α°	Hour.	α°	Hour.	α°
0—1	−13	6—7	52	12—13	25	18—19	—37
1—2	0	7—8	43	13—14	35	19—20	—25
2—3	5	8—9	37	14—15	13	20—21	—23
3—4	7	9—10	45	15—16	10	21—22	—25
4—5	13	10—11	30	16—17	−10	22—23	−27
5—6	25	11—12	35	17—18	−13	23—24	−20

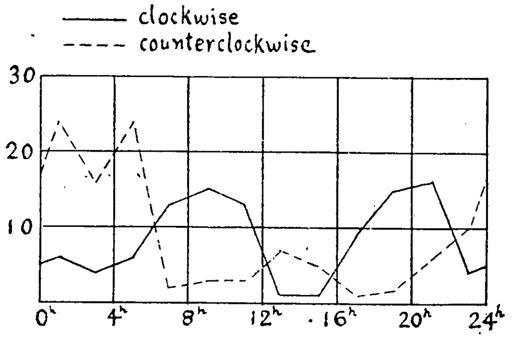
16. Since in the above investigations, the trains chosen from among the records of 1913 were those especially regular and moreover, the periods were determined with special care, the material seems appropriate for studying the frequencies of different periods in different hours of the day, in spite of the relatively small number of trains taken. The results of the examination are tabulated below, which show remarkable predominance of the *longer* waves in the night hours, as already remarked in §9.

TABLE IX. *Number of trains.*

Period in m. / Hour.	0—0·5	0·5—1·0	1·0—1·5	1·5—2·0	2·0—2·5	2·5—3·0	3·0—3·5	3·5—4·0	4·0—4·5
0— 4	0	19	26	**29**	**29**	7	20	6	0
4— 8	4	**66**	39	21	22	10	7	3	1
8—12	4	**93**	49	20	15	7	5	5	3
12—16	1	**45**	40	10	8	7	2	1	1
16—20	0	14	12	19	16	6	3	5	6
20—24	0	6	25	**41**	**31**	24	13	6	8
Sum	9	243	191	140	121	61	50	26	19

17. *Rotatory character of the horizontal components.* As already mentioned, the horizontal components of the periodic disturbing field show as a rule more or less rotatory character. To investigate the relation in detail, regular waves with decidedly rotatory character were chosen from the records of 1913, at the same time with the investigation of the preceding article. The data obtained were classified into two groups, *i.e.* those which showed regular rotation in the clockwise and in the counterclockwise sense respectively.

Fig. 17.



The frequencies of the two groups in different hours were calculated, as shown in Table X. and also in Fig. 17.

TABLE X.

Hour.	Counter-clockwise.	Clockwise.	Hour.	Counter-clockwise.	Clockwise.
0– 2	24	6	12–14	7	1
2– 4	16	4	14–16	5	1
4– 6	24	6	16–18	1	9
6– 8	2	13	18–20	2	15
8–10	3	15	20–22	6	16
10–12	3	13	22–24	10	4

For each group, the frequency shows apparently a *semidiurnal* period. Comparing the result with Fig. 16, it may be noticed that the clockwise rotation predominates in those hours where the direction of the disturbing field is at the maximum deviation from the meridian, while the counterclockwise rotation falls most frequently in the intermediate hours. It must be also remarked that the sense of rotation during night hours is generally opposite to that determined by Sangster[1] for disturbances of decidedly longer durations, and therefore also opposite to the sense in which the abrupt disturbing field described in §5 *e*) tends. In Sangster's case the sense of rotation showed a diurnal period, being of the same sense throughout twelve hours, while in the case of the short waves here in question, it shows a semidiurnal period as given above.

PART IV.

Discussion of the Results.

18. The results of the present investigations so far described seem to throw some light, however faint, on the actual origin

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1) Sangster, *loc. cit.*

of the magnetic pulsations in question, though it is at the present stage rather difficult to draw anything like a conclusive inference, for even if the observations had been carried out for a much longer period with more reliable instruments, the data were in any case confined only to a single station. In the following, we will try, instead of hastening to any premature conclusion, to consider, merely by way of tentative suggestion, different possibilities regarding the probable cause of the phenomena in question. The considerations will inevitably be of a speculative character, but may serve at least as useful hints for projecting future investigations of allied phenomena, especially of the as yet very obscure nature of the electrical as well as mechanical behaviour of the upper atmosphere.

19. A fact strongly impressing us in the first place, is that the occurrence of the magnetic pulsations in question is subjected in more than one respect to a remarkable diurnal variation. This evidence alone is sufficient to infer the important rôle played by the sun, whether its influence be direct or indirect. The position of the sun, not only determines the length of the periods of the most frequent waves in different hours, but also affects the direction in which the periodic magnetic field fluctuates. It seems quite plausible to assume from the outset that the seat of the *primary* cause of the phenomena is chiefly to be sought in our atmosphere subjected to solar radiation of different kinds. The periodic heating of the superficial layer of the earth crust, though it may possibly cause a slow variation of the terrestrial magnetism, may scarcely account for the periodic nature of the disturbances in question. The direct magnetic influence of the sun itself[1] seems also improbable, since if such be the primary cause, the more or less complete screening off of waves shorter than 50^s during night hours must be explained, while 100^s waves are so conspicuous in these hours. Moreover, the unsymmetrical distribution of the characteristic waves at noon is rather difficult to explain on this view. On the other hand, the existence of remarkable electric currents in the upper region

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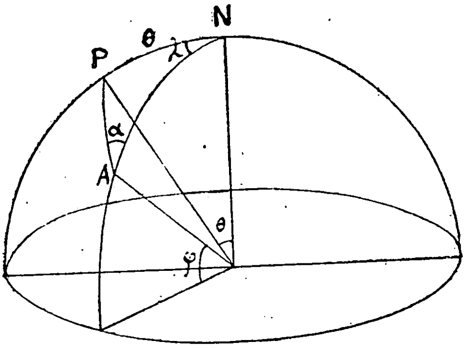
[1] Bosler, Journal de Physique, [6] 2, 1912, p. 877.

of our atmosphere may be regarded as almost an established fact since the classical investigations of Schuster,[1] Birkeland and Störmer.[2] Now, according to Schuster and Bezold, there exists in the upper atmosphere a definite system of currents whose position is nearly fixed relative to the sun, which produces the remarkable diurnal variation of the terrestrial magnetic field. The first suggestion which naturally arises as to the cause of the magnetic pulsations, is the fluctuation of this permanent system of currents.[3] This seems the more plausible, if we remember the very regular diurnal reiteration of the characteristic phenomena. The material nearest in hand for testing this conjecture is the daily variation of the azimuth of the horizontal components of the periodic disturbing field. Examination of Fig. 16 will show that a similar daily variation could be produced, if a nearly circular zonal system[4] of electric currents, fixed with respect to the sun and having its pole situated at a considerable distance from the earth's axis, undergoes some periodic fluctuations in its different parts. Referring to Fig. 18, let N be the earth's astronomical pole, while P is the pole of the zonal current. Denoting the latitude of the point of observation A by φ, and its longitude counted from the meridian containing P, by λ, the azimuthal angle α of the

Fig. 18.



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1) A. Schuster, Philosophical Transactions, **180** A, 1889, p. 467; v. Bezold, Gesammelte Abh., p. 404.

2) Birkeland, *loc. cit.*; C. R., **147**, p. 539. C. Störmer, Archiv for Math. og Naturvidenskab., **31**, 1911, Nr. 11.; Archiv des Sc. phys. et nat., **24**, 1907, pp. 5, 113, 221, 317; numerous papers in C. R.

3) The idea is not at all new, being expressed already by Eschenhagen, Birkeland, van Bemmeln *etc.*, though more or less vaguely.

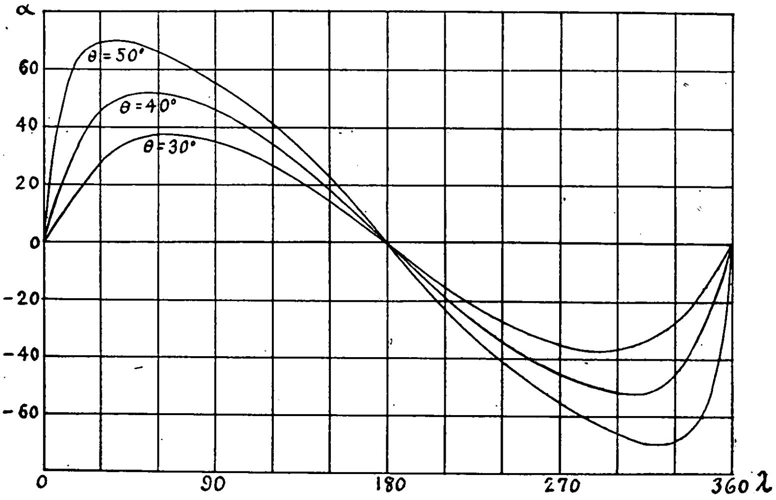
4) We do not necessarily mean a continuous system of current flowing in a given direction at the same time, but only that the directions of the different portions of the current causing pulsations, when put together, form the *portions* of a circular zone with a given pole.

magnetic field due to the current passing near the point A, counted positive toward W from N of A, is given by

$$\sin\alpha = \frac{\sin\theta \sin\lambda}{\sqrt{1-(\cos\theta\sin\varphi+\sin\theta\cos\varphi\cos\lambda)^2}}$$

Putting for example, $\varphi=35°$ and $\theta=30°$, 40° or 50°, the variation of α as the function of λ is shown in the annexed figure (Fig. 19).

Fig. 19.



To compare with Fig. 16, the present figure must be properly shifted along the axis of λ, which is equal to the hours, up to an additive constant. It seems that the meridian corresponding to the pole P, (*i.e.* $\lambda=0$) of the assumed circular current may be placed somewhat in the early hours of the morning. The shapes of the curves, however, do not agree so well as to enable us to determine the most probable value of θ. Indeed, it will be too much to infer at once the existence of such a simple circular current in view of the above consideration, based on the observation at a single station. Still the orientation of the different *portions* of the atmospheric current, of which the fluctuations may produce the magnetic pulsations, must resemble in some measure that of the corresponding portions of the ideal simple system above considered, and must in any case be directed nearly SW–NE or reverse during the day time and NW–SE or reverse during night. The supposed system of currents can not be directly identified with that given by Schuster or Bezold, since in the latter case, a passes the value $\frac{\pi}{2}$ twice during 24 hours, and the corresponding curve of a with the hour as abscissa shows no apparent resemblance either with Fig. 16 or with Fig. 19. Nor is the system of currents deduced by Birkeland[1] from the disturbances of longer durations observed during his memorable auroral expedition similar to the supposed one. Remembering, however, that the daily variation of the terrestrial magnetic field can be represented in the first rough approximation by a system of currents having its axis considerably inclined to the earth's axis, we may regard provisionally the total current system given by Schuster and Bezold consisting of two parts, and that the one part which is the principal and represents a system of zonal currents, shows a more conspicuous regular fluctuation than the remaining secondary part more or less converging toward the pole (see § 28). As a matter of fact, Fig. 19 represents only the *average* distribution of the most frequent azimuth in different hours. In actual cases, the points corresponding to different trains of waves are so remarkably scattered that the adoption of the mean value evaluated in the usual manner seems scarcely justified for seeking the most frequent value. It is very probable that among these widely scattered points, there are many which actually correspond to the fluctuations of the part of the current belonging to the higher harmonics. At any rate, our conjecture seems to be justified in the first approximation, though the position of the pole of the circular current can not be determined with certitude.

20. As to the actual modes of fluctuation of the atmospheric current causing the magnetic waves concerned several possibilities are suggested at the same time. Firstly, we may consider the total intensity as well as the distribution of the current as constant, but oscillating as a whole about its mean position,

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[1] Birkeland, *loc. cit*, Pl. X.

either vertically, horizontally or in a rotatory manner. Secondly, it is also possible that the current itself undergoes a periodic fluctuation, either in the total intensity or in the distribution in its different parts. More probable is the combination of the above two modes. Thirdly, we may suppose a system of parallel currents arranged at nearly equal intervals propagated perpendicular to itself, over the point of observation with a finite velocity, in which case the waves must show a time difference in difference stations. If the results of observations confirm the exact simultaneity of waves in widely distant stations beyond all doubt, the last hypothesis will naturally fall out. Birkeland, indeed, observed the approximate simultaneity of some waves with short periods in two stations so widely apart as Potsdam and Bossekop. The two examples reproduced in his report, however, refer only to nearly the same midnight hour, where the direction of the atmospheric current might well have been approximately parallel to the line joining the two stations. Again, according to the result of the simultaneous observations made at Kyôto, Misaki and Sendai, in April, 1909, by Dr. Kadooka and Prof. Tanakadate, a similar simultaneity is observed within the limit of experimental error; but in this case the distances were not very great. Though these two observations strongly speak against the progressive nature of the periodic disturbances, a further accumulation of evidence will not be considered superfluous for deciding the point beyond all doubt. On the other hand, some disturbances of a longer duration investigated by Ad. Schmidt(1) were actually progressive. Birkeland also attributed a velocity of translation at the rate of 100 km. per minute to some class of perturbations. A possibility of occasional occurrence of progressive waves, if not of regular phenomena, seems to be not yet dispproved. On this view, it will be of some theoretical interest to include the case of progressive waves among the possible cases and see how far the hypothesis is favourable or unfavourable for explaining the different peculiarities of the observed phenomena.

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1) Ad. Schmidt, Met. Zs., **16**, 1899, p. 385.

In the following, we will recapitulate some of the most remarkable results of the statistical investigations and try to review them in the light of the different hypothetical atmospheric currents.

21. First, take the relation between X- and Z-components which show such an intimate connection as regards their amplitudes and phases, as described fully in previous paragraphs. We will proceed to consider the influences of different possible ideal systems of atmospheric currents separately, and compare the results with the observed facts by way of seeking a most plausible explanation.

Since the disturbances with the short periods here in question most probably extend to a limited portion of the earth's surface, as may be judged from their remarkable dependency on the hours of the day, it will be allowed for a first approximation to consider both the surface of the earth and the atmospheric layer carrying the current as a plane.

22. *a*) Consider an infinite linear current i running perpendicular to the meridian at a height h from the earth's surface considered plane, and at a horizontal distance x from the point of observation A. Then the X- und Z-components will be given by

$$\varDelta X=\frac{2ih}{x^2+h^2}\,,\quad \varDelta Z=\frac{2ix}{x^2+h^2}\quad\cdots\cdots\cdots\cdots(1)$$

If the intensity of the current fluctuates in any manner, while its position remains unchanged with respect to the earth, the two components will follow the fluctuation simultaneously, provided that the variation is slow enough for neglecting the effect of the induced current. The ratio of the amplitudes being

$$\frac{\varDelta Z}{\varDelta X}=\frac{x}{h}\,,$$

it may assume any value when x varies from $-\infty$ to $+\infty$. Besides, $\varDelta Z$ will be of opposite phase on both sides of the current.

b) If the above current *moves* perpendicular to itself from $x=-\infty$ to $+\infty$, X-component at A evidently attains a maximum at $x=0$, while $\varDelta Z$ is zero at $x=0$ and has a maximum and minimum at $x=\pm h$ respectively. The end point of the vector describes a circle with the radius i/h.

c) Instead of a linear current, we consider next a current of uniform density i, with a rectangular section having a breadth of $2l$ and a height of $2b$, the middle point lying at a height h and the horizontal distance x from A. The two components are respectively given by

$$\left.\begin{aligned}\frac{\Delta X}{i}&=x\log\frac{(x+l)^2+(h+b)^2}{(x+l)^2+(h-b)^2}\cdot\frac{(x-l)^2+(h-b)^2}{(x-l)^2+(h+b)^2}\\&\quad+l\log\frac{(x+l)^2+(h+b)^2}{(x+l)^2+(h-b)^2}\cdot\frac{(x-l)^2+(h+b)^2}{(x-l)^2+(h-b)^2}\\&\quad+2(h+b)\mathrm{tg}^{-1}\frac{2(h+b)l}{(h+b)^2+x^2-l^2}-2(h-b)\mathrm{tg}^{-1}\frac{2(h-b)l}{(h-b)^2+x^2-l^2},\\\frac{\Delta Z}{i}&=2(x+l)\mathrm{tg}^{-1}\frac{2(x+l)b}{(x+l)^2+h^2-b^2}-2(x-l)\mathrm{tg}^{-1}\frac{2(x-l)b}{(x-l)^2+h^2-b^2}\\&\quad+h\log\frac{(x+l)^2+(h+b)^2}{(x+l)^2+(h-b)^2}\cdot\frac{(x-l)^2+(h-b)^2}{(x-l)^2+(h+b)^2}\\&\quad+b\log\frac{(x+l)^2+(h+b)^2}{(x-l)^2+(h-b)^2}\cdot\frac{(x+l)^2+(h+b)^2}{(x-l)^2+(h+b)^2}.\end{aligned}\right\}\quad(2)$$

When the current degenerates into a current sheet of the horizontal breadth $2l$ and the linear intensity i',

$$\left.\begin{aligned}\Delta X&=2i'\mathrm{tg}^{-1}\frac{2lh}{h^2+x^2-l^2},\\\Delta Z&=i'\log\frac{(x+l)^2+h^2}{(x-l)^2+h^2}.\end{aligned}\right\}\cdots\cdots(3)$$

Again, when the current sheet is vertical with a breadth $2b$,

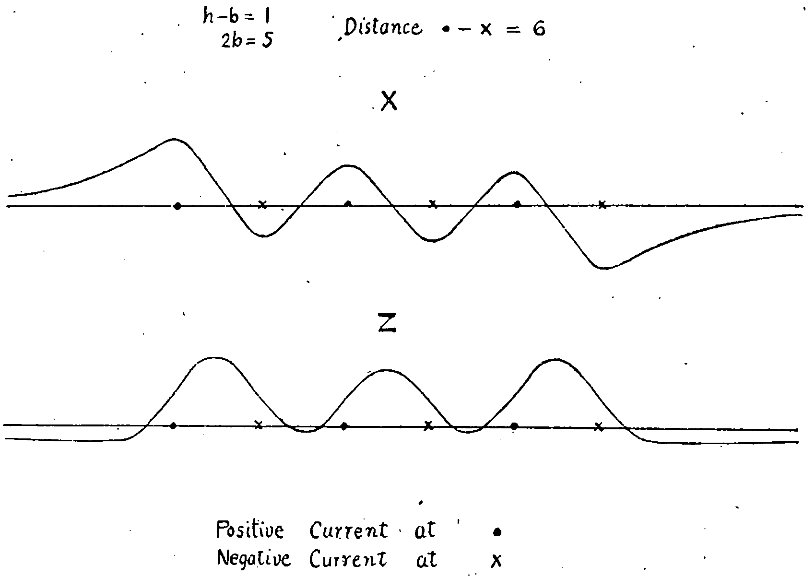
$$\left.\begin{aligned}\Delta X&=i'\log\frac{x^2+(h+b)^2}{x^2+(h-b)^2},\\\Delta Z&=2i'\mathrm{tg}^{-1}\frac{2bx}{x^2+h^2-b^2}.\end{aligned}\right\}\cdots\cdots(4)$$

In the former case (3), the maximum value of ΔZ, ΔZ_m say, which is attained at $x=\pm\sqrt{l^2+h^2}$, will increase indefinitely, while the maximum value of ΔX, or ΔX_m tends to $2\pi i'$, when the ratio b/h increases indefinitely. For $b=0$, the ratio is 1/2. In the latter case (4), ΔZ_m varies from 0 to $\frac{\pi}{2}$ when $2b$ increases from

0 to ∞, h being kept constant, while ΔX_m increases indefinitely with b. The ratio $\Delta Z_m/\Delta X_m$ tends to $1/2$ when b decreases indefinitely, as is already evident from the former results.

d) Let a number of such currents be arranged with the directions alternately positive and negative, and parallel to each other at definite intervals, each moving perpendicular to itself with a uniform velocity. Neglecting the effect of the induced current, the magnetic field at A will undergo a train of periodic variation as illustrated in the annexed figure (Fig. 20) for a special case. As will be seen, the phase retardation of ΔZ is somewhat similar to that in the actual case, though here at the same time with the short waves, a rather remarkable general swelling of the curves appears. The latter may probably be avoided by properly reducing the intensity of the currents toward both ends of the train. In this case, the pulsations must necessarily be *progressive*.

Fig. 20.



23. Next, we will consider the effect of the current induced

in the earth, especially for a simple case convenient for mathematical treatment, *i.e.* the case when the current is arranged in an *infinite train of waves*, either *stationary* or *progressive.*[1]

Take the surface of the earth, considered plane as usual, as xy-plane and the positive direction of z downward. For the positive value of z, the space is considered to be filled with a conducting medium with the uniform specific conductivity k and the magnetic permeability μ, while the negative side of z is regarded as a vacuum. Assuming the electric and magnetic force independent of y and denoting their components respectively by $\mathfrak{E}_x$, $\mathfrak{E}_z$, $\mathfrak{H}_x$, $\mathfrak{H}_z$, the usual fundamental equations for the slow variations reduce to

$$\left.\begin{aligned} &\mu\frac{\partial\mathfrak{H}_x}{\partial t}=\frac{\partial\mathfrak{E}_y}{\partial z}, \qquad 4\pi k\mathfrak{E}_y=\frac{\partial\mathfrak{H}_x}{\partial z}-\frac{\partial\mathfrak{H}_z}{\partial x}, \\ -&\mu\frac{\partial\mathfrak{H}_z}{\partial t}=\frac{\partial\mathfrak{E}_y}{\partial x}, \qquad \frac{\partial\mathfrak{H}_x}{\partial x}+\frac{\partial\mathfrak{H}_z}{\partial z}=0. \end{aligned}\right\}\text{.................}(1)$$

since $\mathfrak{E}_x=\mathfrak{E}_z=0$, $\mathfrak{H}_y=0$. Next, assume that the electromagnetic field varies periodically with the frequency $n/2\pi$ and the distribution of the fields represents a two-dimensional wave with the wave length

$$\lambda=\frac{2\pi}{a}.$$

In the case of *stationary* waves, we may assume

$$\mathfrak{E}_y=e^{int-(\beta+i\gamma)z}\sin ax, \text{ ....}(2)$$

where β and γ are considered real. From

$$4\pi\mu k\frac{\partial\mathfrak{E}_y}{\partial t}=\frac{\partial^2\mathfrak{E}_y}{\partial x^2}+\frac{\partial^2\mathfrak{E}_y}{\partial z^2},$$

we obtain

$$4\pi\mu kin=-a^2+(\beta+i\gamma)^2$$

or

$$\beta^2-\gamma^2=a^2, \qquad 2\beta\gamma=4\pi\mu kn,\text{..................................}(4)$$

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[1] The mathematical solution of the case was kindly carried out by Prof. S. Sano, to whom the best thanks of the author are due.

whence

$$\beta=\sqrt{\frac{1}{2}\left(\sqrt{a^4+16\pi^2\mu^2k^2n^2}+a^2\right)},\qquad \gamma=\sqrt{\frac{1}{2}\left(\sqrt{a^4+16\pi^2\mu^2k^2n^2}-a^2\right)}.\quad\dots(5)$$

From the first set of the fundamental equations results

$$\left.\begin{aligned}\mathfrak{H}_x&=\frac{(-\gamma+i\beta)}{\mu n}\,e^{int-(\beta+i\gamma)z}\sin ax,\\ \mathfrak{H}_z&=\frac{ia}{\mu n}\,e^{int-(\beta+i\gamma)z}\cos ax.\end{aligned}\right\}\quad\dots\dots\dots\dots(6)$$

Taking the real parts and putting $z=0$, we obtain at last

$$\left.\begin{aligned}(\mathfrak{E}_y)_{z=0}&=\cos nt\sin ax,\\ (\mathfrak{H}_x)_{z=0}&=\varDelta X=-\frac{(\gamma\cos nt+\beta\sin nt)}{\mu n}\sin ax,\\ (\mathfrak{H}_z)_{z=0}&=\varDelta Z=-\frac{a\sin nt}{\mu n}\cos ax.\end{aligned}\right\}\quad\dots\dots\dots\dots(7)$$

Similarly in the case of *progressive* waves, we may proceed by assuming

$$\mathfrak{E}_y=e^{(nt\pm ax)i-(\beta\pm\gamma i)z}\quad\dots\dots\dots\dots(8)$$

and obtain

$$\left.\begin{aligned}\varDelta X&=-\frac{1}{\mu n}\left\{\gamma\cos(nt\pm ax)+\beta\sin(nt\pm ax)\right\},\\ \varDelta Z&=\mp\frac{a}{\mu n}\cos(nt\pm ax),\end{aligned}\right\}\quad\dots\dots\dots\dots(9)$$

where the values of β and γ are the same as above (5).

Putting

$$\beta=A\cos\varphi,\qquad \gamma=A\sin\varphi,$$

or

$$A=\sqrt{\beta^2+\gamma^2},\quad \mathrm{tg}\,\varphi=\frac{\gamma}{\beta},\quad\dots\dots\dots\dots(10)$$

(7) becomes

$$\left.\begin{aligned}\varDelta X&=-\frac{A}{\mu n}\sin(nt+\varphi)\sin ax,\\ \varDelta Z&=-\frac{a}{\mu n}\sin nt\cos ax.\end{aligned}\right\}\quad\dots\dots\dots\dots(7')$$

The same substitution makes (9), for progressive waves toward N,

$$\left.\begin{aligned} \varDelta X &= \frac{A}{\mu n} \cos\left(nt - ax + \frac{\pi}{2} + \varphi\right), \\ \varDelta Z &= \frac{a}{\mu n} \cos(nt - ax), \end{aligned}\right\} \qquad \text{(9')}$$

and for retrograde waves toward S,

$$\left.\begin{aligned} \varDelta X &= -\frac{A}{\mu n} \sin(nt + ax + \varphi), \\ \varDelta Z &= -\frac{a}{\mu n} \sin\left(nt + ax + \frac{\pi}{2}\right). \end{aligned}\right\} \qquad \text{(9'')}$$

Hence for the case of stationary waves (7'), the ratio of the amplitudes of Z and X is given by

$$\frac{\varDelta Z_m}{\varDelta X_m} = \frac{a}{A} \cot g\, ax \qquad \text{(11)}$$

which can assume any value whatever between $-\infty$ and $+\infty$, as we proceed normal to the wave ridges. Moreover, $\varDelta Z$ is retarded after $\varDelta X$ by φ given by (10).

Again, in the case of progressive and retrograde waves, the ratio of the amplitudes is

$$\frac{\varDelta Z_m}{\varDelta X_m} = \frac{a}{A} = \frac{a}{\sqrt{\beta^2 + \gamma^2}} \qquad \text{(12)}$$

which is always less than, and tends to, unity as the frequency decreases, since

$$A^2 = \beta^2 + \gamma^2 = \sqrt{a^4 + 16\pi^2 \mu^2 k^2 n^2}. \qquad \text{(13)}$$

Besides, $\varDelta Z$ lags behind $\varDelta X$ by $\varphi + \frac{\pi}{2}$ or $\varphi - \frac{\pi}{2}$ according as the waves are progressive or retrograde.

Now, the angle determining the phase is given by

$$\operatorname{tg} \varphi = \frac{\gamma}{\beta} = \sqrt{\frac{\sqrt{a^4 + 16\pi^2 \mu^2 k^2 n^2} - a^2}{\sqrt{a^4 + 16\pi^2 \mu^2 k^2 n^2} + a^2}}$$

which becomes zero for small value of kn and tends to unity for large values. Hence, in the case of stationary waves, the retardation of the vertical component will increase from 0 to $\frac{\pi}{4}$ when kn increases from 0 to ∞. In the case of progressive waves, when kn increases from 0 to ∞, the retardation will vary from $\frac{\pi}{2}$ to $\frac{\pi}{2}+\frac{\pi}{4}$ or $\pi-\frac{\pi}{4}$, while in the case of retrograde waves, it may be considered as negative and varies from $-\frac{\pi}{2}$ to $-\frac{\pi}{4}$.

Now, returning to the actual case, we will try to examine if either of these hypothetical waves could be reconciled with the observed facts regarding the relations between the horizontal and vertical components of magnetic pulsations.

If the stationary waves alone were concerned, we would have to assume that the distance of the observing station from the node of the electric current was in any case greater than $\frac{\lambda}{8}$ but less than $\frac{\lambda}{4}$, in order to explain the fact that the ratio $\varDelta Z_m/\varDelta X_m$ was always less than unity, at least in the case of the slow oscillations where a/A in (11) is nearly equal to unity and cotg ax may become less than unity only when ax or $\frac{2\pi x}{\lambda}$ is greater than $\frac{\pi}{4}$. This will be a rather awkward, though not impossible assumption, if the wave length were small in comparison with the earth's quadrant, since we must then also assume that the location of the current is always limited to a rather narrow range favorable to the above relation. The only plausible hypothesis reconcilable with this assumption is that the wave length of the current producing the slow waves is of the order of magnitude of the earth's meridian, and also that the station lies always not far apart from the loop of the current. In such a case, however, either of our assumption as to the planeness of the earth's surface and the existence of the infinite train of waves, will fail to apply; but the case will rather approach that which was discussed in the preceding paragraph. The present result may be interpreted as merely indicating that the effect of the induced current has the tendency to retard the vertical component with respect to the horizontal one. For such a case, the mathematical calculation given by Lamb[1] for spherical conductors will directly apply, which also shows the retardation tending to $\frac{\pi}{4}$ for the large value of kn. At any rate, the observed

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[1] H. Lamb, Phil. Trans., 1883, p. 526; *ibid.* 180, 1889, p. 513.

waves with periods longer than 10^{m} which show the retardation generally less than $\frac{\pi}{4}$, may easily be explained by a current of rather diffused character subject to fluctuations. Indeed, these longer waves are generally not very regular and rarely form a train with the number of successive maxima greater than three or four,—a decided contrast with the shorter waves of a few minutes periods which form as a rule remarkably regular trains and show occasionally phase retardation of ΔZ greater than $\frac{\pi}{4}$. If the above conjecture be justified in some measure, the current, of which the local fluctuation produces these longer waves, may plausibly be identified with the circular current discussed in § 18 and compared with the principal part of the current causing the diurnal variation of the terrestrial field, since in this way, the variation of the azimuth of the disturbing field may also be explained satisfactorily, at least to the first approximation.

The maximum current intensity may be expected probably near the equator if the phase relation between ΔX and ΔZ is never inverted throughout the N-hemisphere. It will be interesting to see if the relation is actually opposite on the S-hemisphere. v. Bemmeln found $\frac{\Delta Z}{\Delta X}$ in Batavia invariably insignificant.

On the other hand, the most regular waves with periods less than about 4^{m} show as a rule remarkable phase retardation of the vertical component after the horizontal ones, generally greater than $\frac{\pi}{4}$. This seems apparently difficult to explain on the assumption of stationary waves, if we solely rely on the above calculation.

Next, turning for a while to the case of progressive waves, we remark that one fact seems at first sight to be in favour of this assumption. As described repeatedly above, the actual ratio of the amplitudes $\Delta Z_m / \Delta X_m$ never exceeds unity and decreases gradually as the frequency of the wave increases. From (12) and (13), we have indeed

$$\frac{\Delta Z_m}{\Delta X_m}=\frac{a}{\sqrt[4]{a^4+16\pi^2\mu^2k^2n^2}},$$

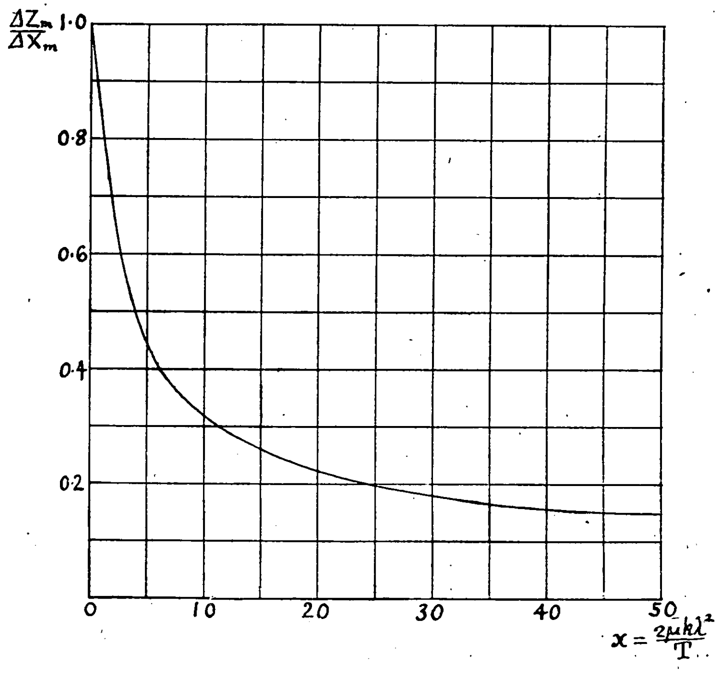
which may be written $=\frac{1}{\sqrt[4]{1+x^2}}$,

if we put $x=\frac{4\pi\mu kn}{a^2}=\frac{2\mu k\lambda^2}{T}$,

denoting the period by T, *i.e.* putting $n=\frac{2\pi}{T}$.

The variation of the calculated ratio $\Delta Z_m/\Delta X_m$ with x as abscissa, may be seen from the annexed figure (Fig. 21). Referring to Figs. 11 and 12, and comparing the present theoretical curve with the curve of the observed ratio with $1/T$ as abscissa, we may observe a striking resemblance, though the ratio of the scales for the two diagrams is not yet known. If we take, for example, the 3^m wave for the purpose of comparison, the observed value of $\Delta Z_m/\Delta X_m$ is nearly 0·28. Making a discount of 10 % on account of the reason discussed at the end of § 13,

Fig. 21.



we obtain 0·25. The corresponding value of x in Fig. 21 is found to be nearly 17. Hence, we must have, assuming for simplicity's sake $\mu=1$,

$$2k\lambda^2=17\times180=3060\,;$$

hence $\lambda^2=\frac{1530}{k}$, or $\frac{2\cdot10^3}{k}$ in round number. Now assuming the conductivity of the earth to be 10^{-13} as is suggested by the result of Schuster's investigation, we obtain

$$\lambda=1\cdot2\times10^8\,\text{cm.} \quad \text{or} \quad 1200\ \text{km., say.}$$

Nearly the same value may be obtained, if we take 10^m waves instead. We have no theoretical ground at hand for assuming that the wave length varies with the periods; still, judging from the remarkable resemblance of the two diagrams above compared, it seems plausible to assume provisionally that the wave length is at least of the same order of magnitude, and the different periods are determined not so much by the difference of the wave length as of the velocity of propagation. In the above example, where $\lambda=1\cdot2\times10^3$ km. for $T=180^s$ the velocity of propagation v will be about $7\frac{\text{km.}}{\text{sec.}}$[1]. The above estimation was solely based on the assumption that the equations (11), (12) *etc.* hold rigorously. Judging from the analogy of the case investigated by Schuster, it is probable that the actual reduction of the vertical component will be decidedly more remarkable than the calculated value. If such be the case, the value of x and accordingly the values of λ and v will become less than those above estimated.

A closer examination of Fig. 12 shows that the asymptotic value of $\Delta Z_m/\Delta X_m$ is not 1, but somewhat near 0·8. If we compare Fig. 12 with a diagram obtained by multiplying the ordinates of Fig. 21 by 0·8, the acceptable value of x become about 10 instead of 17. But this does not alter the order of magnitude of the results.

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1) According to Birkeland, the lateral velocity of the current producing some disturbances was of the order of 100 km. per minute, while that of some auroral bands observed in the polar region was about 300 m. per sec., which is very small in comparison with the above calculated velocity of the hypothetical current.

As for the phase retardation of ΔZ after ΔX, the serious difficulty against the hypothesis of the progressive waves is that, in the theoretical result, the phase angle must be included within the range 90° to 135° or $-45°$ to $-90°$, while in the observed results, it crowds most densely within the limit 45° to 90°. At present we are at a loss to judge whether the difficulty may be evaded by assuming the presence of more conducting layer below the earth's crust, as argued by Schuster to explain a similar discrepancy in the case of the diurnal wave.

On the other hand, if we once admit in the case of the *stationary* wave that cotg ax is somehow of the order of unity, the ratio of amplitude $\Delta Z_m/\Delta X_m$ is just the same as in the case of the progressive waves, and the above calculations generally apply also to this case, except that part concerning the velocity of propagation which is zero in this case. The general mode of dependency of the ratio on the periods is equally favourable for the stationary waves, and we see no strong reason for preferring the theory of progressive waves, as far as the amplitude ratio is concerned. If we take cotg $ax=1$ or 0·8 and $k=10^{-13}$ the wave length estimated seems, however, too small.

At any rate too much weight must not be laid on the numerical results of the above calculations, since they have no claim to accuracy of the quantitative results, if we remember the utmost simplicity of the assumptions on which they are based, even if they may be legitimate in essential features.

24. Next, consider the ideal case where an atmospheric current is subject to a rotatory oscillation remaining parallel to itself. Taking the simplest case of a constant linear current running parallel to the y-axis and oscillating about the mean position, given by its elevation h_0 above the surface of the earth and the horizontal distance x_0 from the point of observation taken as the origin. Let the position of the current i be given by

$$\left.\begin{aligned} x&=x_0+x_1\cos nt, \\ h&=h_0+h_1\cos(nt-\varphi). \end{aligned}\right\} \qquad \text{(1)}$$

We may obtain, neglecting the effect of induction

$$\left.\begin{aligned} \frac{X}{2i} &= \frac{h_0}{x_0^2+h_0^2} + A_x \cos(nt-\psi_x), \\ \frac{Z}{2i} &= \frac{x_0}{x_0^2+h_0^2} + A_z \cos(nt-\psi_z), \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (2)$$

where

$$\left.\begin{aligned} A_x \cos\psi_x &= \frac{h_0}{h_0^2+x_0^2}\left\{-\frac{2x_0x_1}{x_0^2+h_0^2} + \left(\frac{h_1}{h_0} - \frac{2h_0h_1}{h_0^2+x_0^2}\right)\cos\varphi\right\}, \\ A_x \sin\psi_x &= \frac{h_0}{h_0^2+x_0^2}\left\{\frac{h_1}{h_0} - \frac{2h_0h_1}{h_0^2+x_0^2}\right\}\sin\varphi, \\ A_z \cos\psi_z &= \frac{x_0}{h_0^2+x_0^2}\left\{\frac{x_1}{x_0} - \frac{2x_0x_1}{h_0^2+x_0^2} - \frac{2h_0h_1}{h_0^2+x_0^2}\cos\varphi\right\}, \\ A_z \sin\psi_z &= -\frac{x_0}{h_0^2+x_0^2} \cdot \frac{2h_0h_1}{h_0^2+x_0^2}\sin\varphi. \end{aligned}\right\} \quad \cdots\cdots (3)$$

For the special case $\varphi=0$, *i.e.* when the oscillation is linear,

$$\left.\begin{aligned} A_x &= \frac{h_0}{h_0^2+x_0^2}\left\{\frac{h_1}{h_0} - \frac{2(x_0x_1+h_0h_1)}{h_0^2+x_0^2}\right\}, \qquad \psi_x=0, \\ A_z &= \frac{h_0}{h_0^2+x_0^2}\left\{\frac{x_1}{x_0} - \frac{2(x_0x_1+h_0h_1)}{h_0^2+x_0^2}\right\}, \qquad \psi_z=0, \end{aligned}\right\} \quad \cdots\cdots\cdots (4)$$

or

$$\left.\begin{aligned} \frac{A_x}{x_1} &= \frac{1}{1+\xi^2}\left(a - \frac{2(\xi+a)}{1+\xi^2}\right), \qquad \psi_x=0, \\ \frac{A_z}{x^1} &= \frac{1}{1+\xi^2}\left(1 - \frac{2(\xi+a)}{1+\xi^2}\right), \qquad \psi_z=0, \end{aligned}\right\} \cdots\cdots\cdots\cdots (5)$$

if we put $h_0=1,\ x_0=\xi$ and $h_1/x_1=a$.

Hence, $$\frac{A_z}{A_x} = \frac{\Delta Z_m}{\Delta X_m} = \frac{(1+\xi^2)a-2(\xi+a)}{1+\xi^2-2(\xi+a)} \quad \cdots\cdots\cdots\cdots (6)$$

This is greater or less than unity according as $a >$ or < 1. Since $\psi_x=\psi_z=0$, there is no difference of phase, neglecting the induction. If we take the induction into account, Z-component will probably lag behind X, though we are at present at a loss to carry out the calculation.

Again in the special case $\varphi=\frac{\pi}{2}$, *i.e.* when the current oscillates elliptically, with the axes of the ellipse horizontal and vertical respectively, we have

$$\left.\begin{aligned}
\frac{A_x}{x_1}&=\frac{2\xi}{(1+\xi^2)^2}\sqrt{\left(\frac{\xi^2-1}{2\xi}\right)^2\alpha^2+1}\,,\\
\frac{A_z}{x_1}&=\frac{2\xi}{(1+\xi^2)^2}\sqrt{\left(\frac{\xi^2-1}{2\xi}\right)^2+\alpha^2}\,,\\
\operatorname{tg}\psi_x&=\frac{1-\xi^2}{2\xi}\,\alpha\,,\\
\operatorname{tg}\psi_z&=\frac{2\xi}{\xi^2-1}\,\alpha\,.
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(7)$$

It may be seen that

$$\frac{A_z}{A_x}=\frac{\Delta Z_m}{\Delta X_m}=\sqrt{\frac{(\xi^2-1)^2+4^2\xi^2\alpha}{(\xi^2-1)^2\alpha^2+4\xi^2}}\,,\cdots\cdots\cdots\cdots(8)$$

which varies from $1/\alpha$ to α, and then from α to $1/\alpha$, when ξ increases from 0 to 1 and 1 to ∞. The phase difference is given by

$$\operatorname{tg}(\psi_x-\psi_z)=\frac{\alpha}{1-\alpha^2}\cdot\frac{(1+\xi^2)^2}{2\xi(\xi^2-1)}\cdots\cdots\cdots\cdots(9)$$

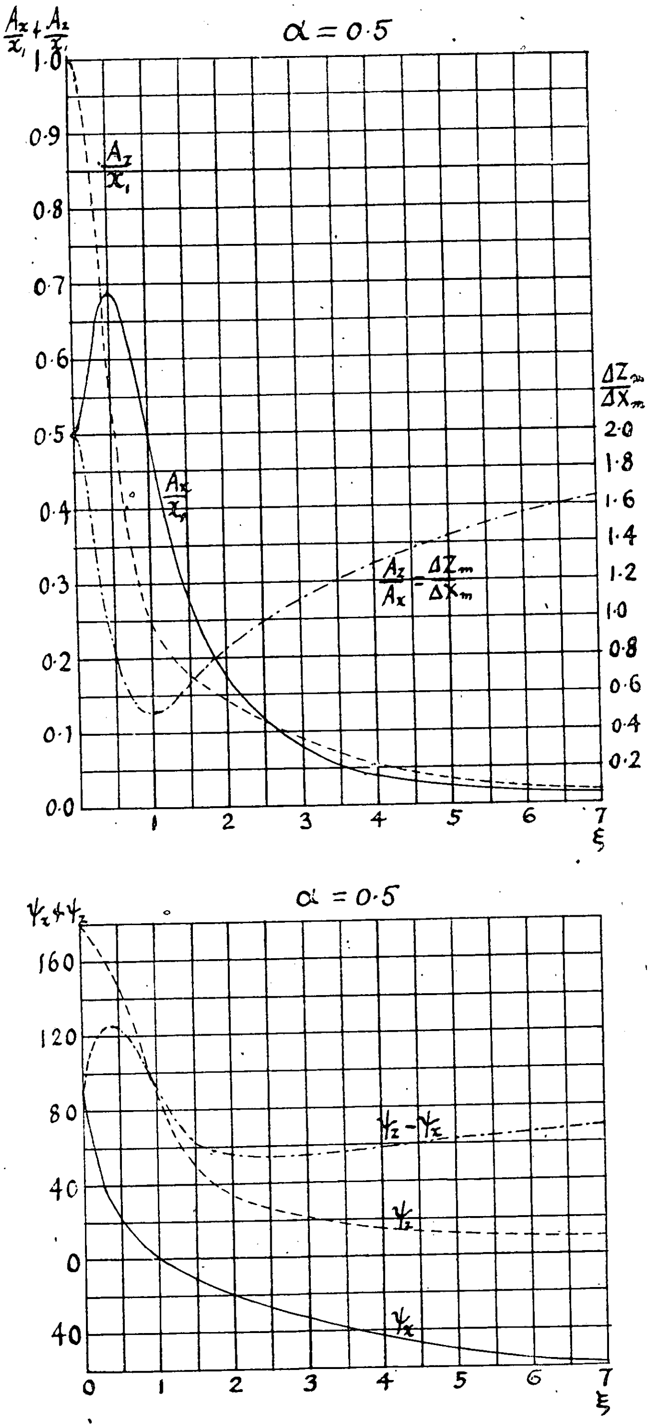
If $\xi<1$, then $\psi_x-\psi_z >$ or $<90^0$ according as $\alpha>$ or <1.

If $\xi<1$, then $\psi -\psi_z <$ or $>90^0$ according as $\alpha>$ or <1.

Thus we see in this case that the phase difference as well as the ratio of the amplitudes may assume different values according to the values of α and ξ. If the effect of the induced current be taken into account, these values will of course be altered considerably. Fig. 22 illustrates the variation of the amplitude ratio and phase difference as functions of ξ for a special case.

In the above discussion, i was considered constant. If it varies at the same time, the results will be more complicated, in some cases giving rise to a higher harmonic oscillation of the magnetic field. In the case treated in § 22, the external source of the disturbance was such as could be represented by the wave-like distribution of current in a *fixed* horizontal layer. If such current layer is disturbed into a wave motion simutaneous with the fluctuation of the intensity, each elementary current will be subjected to motion similar to that expressed by (1) of this paragraph. This motion will greatly modify the relations between the X- and Z-component waves discussed in the preceding paragraph as is suggested by the above simple example. It seems plausible

Fig. 22.


α = 0.5
$\frac{A_x}{x_1}$ + $\frac{A_z}{z_1}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\frac{A_z}{x_1}$
$\frac{A_x}{x_1}$
$\frac{A_z}{A_x} = \frac{\Delta Z_m}{\Delta X_m}$
$\frac{\Delta Z_m}{\Delta X_m}$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
ξ
α = 0.5
ψ_x + ψ_z
160
120
80
40
0
40
$\psi_z - \psi_x$
ψ_z
ψ_x
0
1
2
3
4
5
6
7
ξ

to consider that the difficulties met with in the preceding paragraph with regard to the phase relation may partly be accounted for by a similar *combination* of the motion and the variation in the intensity of the current. Such a combination is quite probable in view of the reciprocal action of the magnetic field on the conducting layer carrying the current. Though these points seem to suggest many interesting problems worth investigation, it will at present be rather too far fetched to introduce any further hypothesis.[1]

25. The fact described in § 5, *g*) that two trains of waves with different periods sometimes appear simultaneously in the X- and Y-components respectively, may probably be explained by the combination of fluctuations in the direction as well as in the intensity of the current. Take the simplest case, when a linear current at a distance d from the point of observation A in its mean position, is fluctuating in its intensity as given by

$$i=i_0 \cos pt,$$

while it is oscillating stationarily about the position of equilibrium, in such a manner that it were always tangential to a string vibrating in a horizontal plane with its node at the foot of the perpendicular from A to its mean position. If the angle made by the current to its mean position at the time t be α, then tg α will vary as $C\cos(qt+\varphi)$, where C is the tangent of the maximum angle of inclination. If the mean position of the current be perpendicular to the meridian, the periodic variation of X and Y will be given by

$$\varDelta X=\frac{i_0}{d} \cos pt,$$

$$\varDelta Y=\frac{i_0 C}{d} \cos pt \cos(qt+\varphi).$$

Thus a "combination tone" will arise in the component parallel to the mean direction of the current. The effect of the induced current will probably enhance the "summation tone" in comparison with the "difference tone." If the oscillation is the

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[1] These points will be touched upon again later in § 28.

neccessary result or cause of the fluctuation of the intensity, p must be equal to q, in which case the period of ΔY will be octave to that of ΔX. This latter case was frequently observed, though not always.

The above consideration seems to explain the fact that the cases where the period of the X-waves is longer than that of Y are decidedly more frequent than the opposite cases, if we remember that the hypothetical current is generally inclined to the circle of latitude by an angle decidedly less than 45.° It is however strange to observe that the hourly distribution given in § 5 is rather conformable to the supposition of the "difference tone" being more conspicuous. This latter point requires further investigation.

Different facts remain still to be considered in the light of the above hypotheses, *i.e.* the dependency of the prevalent periods on the hours of occurrence, the rotatory character of the horizontal components, *etc.* We shall consider these points later, after having tried to ascertain the nature of the supposed atmospheric electric currents.

26. Thus far, we have tried to find some plausible explanations of the observed phenomena on the assumption of a system of fluctuating currents existing in the upper atmosphere, neither alluding to the possibility of such, nor giving hint as to the cause of the supposed fluctuations. It will be by far the more interesting physical problem to inquire into the origin of these remarkable periodic variations. Though we are not yet in a position to answer the question in any definite sense, it will not be quite out of place to say a few words on the subject, merely by way of suggesting many interesting problems regarding the electrical behaviour of the upper atmosphere.

The existense of a permanent system of atmospheric currents producing the remarkable diurnal variation of the terrestrial magnetism, first effectively elucidated by Schuster, may in these days be regarded as a universally accepted hypothesis and requires scarcely any further comment. It is the fluctuation of this current that we are here chiefly concerned with, and the question is, why and in what manner these variations are produced. Schuster attributed the origin of the current to the induction caused by the daily oscillation of the upper atmosphere, which may be regarded as a conducting layer enveloping the entire earth. The first suggestion strongly appealing to us is that this diurnal wave may occasionally be accompanied by numerous trains of secondary waves with decidedly shorter periods compared with the daily or semidiurnal periods. An analogy is afforded in the case of the secondary undulations of oceanic tides.¹⁾ Whatever may be the cause, any mechanical wave motion produced in the upper atmosphere will produce corresponding waves of the induced current which will very much resemble the case discussed in § 23. Supposing such to be actually the case, let us see how far the assumption may be justified.

Taking first the case of the progressive waves, a serious difficulty confronting the assumption is to explain the enormous velocity of propagation as obtained in the preceding calculation which is far greater than the sound velocity, even if the upper atmosphere be entirely of hydrogen at the temperature of the stratosphere. The case would become more favorable only, if by any modification in the method of estimation, we could bring down the calculated velocity to a plausible range.

Another alternative hypothesis of the stationary wave, where the wave length is of the order of magnitude of the earth's meridian, must then be considered. If the atmospheric oscillation be purely mechanical and be one of the modes of the natural *tangential* vibration of the *whole* atmosphere, the period corresponding to such harmonics of low order can scarcely be of the order of a few minutes, as is most frequently observed, in so far as we can judge from the results of investigation²⁾ on the phenomena of the daily variation of atmospheric pressure.

These considerations alone seem to lead us rather to

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1) K. Honda, T. Terada, Y. Yoshida and D. Isitani, Secondary Undulations of Oceanic Tides, Journal of the Coll. of Sc., Imp. Univ., Tôkyô, **24**, 1908.

2) Rayleigh, Phil. Mag., [5] **29**, 1890, p. 173; M. Margules, Sitz. ber. d. kön. Akad. d. Wiss. z. Wien, **99**, 1890, p. 204, *etc.*; H. Lamb, Proc. Roy. Soc., A **84**, 1911, p. 551.

conviction, that if the fluctuations of the current be either of the kinds here considered, it is not merely due to the mechanical wave motion of the upper atmosphere. This leads the question towards the weakest side of our knowledge.

The natural period of the usual electrical oscillations cannot in any case be as long as several minutes, even if we take into account the effect of the upper atmospheric shell; nor can it be of such indefinite periods as actually observed.

Though it seems *so far* difficult to say anything definite about the physical cause which may produce the fluctuations in question, the existence of the fluctuations above considered seems to be suggested, on the other hand, by the phenomena of the aurora polaris. Firstly, the frequently observed auroral arc vividly reminds us of our hypothesis of the diffused zonal current considered in § 23. According to the numerous descriptions of the phenomena, the luminosity seems to undergo different modes of variation, though we could find no record definitely stating a periodicity of a few minutes. Such a slow variation, even if present, would probably have eluded discovery by ordinary simple eye-observations, not equipped with special means for that purpose. A special investigation in this direction seems in any case desirable; for example a minute photometrical study of cinematographical records as obtained by Störmer.[1] Again, among the numerous records of the auroral displays,[2] we frequently meet with descriptions of peculiar bands of luminosity, running nearly perpendicular to the meridian and showing wave-like propagation mostly toward S. It is very probable that similar phenomena are of quite frequent, or rather of almost daily occurrence, although usually the intensity may be so weak that it rarely shows a luminousity strong enough to attract the attention of the naked eye. Indeed, it is well known that the characteristic "auroral lines" of spectra are almost invariably visible in different latitudes. If such bands of

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1) C. Störmer, Videnskapsselskabets Skrifter, Math.-Naturv. Klasse, 1911, No. 17; Astrophys. Jour., **38**, 1913, p. 311; Bull. Soc. Astr. France, 1913, Nov.

2) E. E. Barnard, Observations of Aurora made at the Yerkes Obs., 1897–1902, Astrophys. Jour. **14**, 1902, p. 135. M. Brendel, Ueber das Nordlicht von 30, Juni, Met. Zs., 1908, p. 552.

luminosity correspond to the traces of the *horizontal* electric current, as Birkeland supposed,[1] they remind us strongly of the hypothetical parallel currents considered above. It must be noticed that according to the calculation cited, the progressive motion must be S to N instead of N to S, in order to explain the observed phase relation of X- and Z-components. It might, however, be surmised that the motion of the current in the upper atmosphere is in many respects of opposite directions in the lower and higher latitudes, as is suggested by Schuster-Bezold's system of currents. A similar observation in a considerably high latitude will be in any case very desirable, especially with a reliable instrument for the vertical component.

27. As to the remarkable dependency of the periods of the most frequent waves on the hours of day, we may suggest first of all that it probably has some relation with the difference in the temperature as well as in the state of ionization of the upper atmosphere. According to Prof. Nagaoka,[2] the thickness of the conducting layer in the atmosphere must also be decidedly greater during the day than in the night.

If the cause of the periodic fluctuation of the atmospheric current is to be sought in the purely *horizontal* oscillation of the atmosphere; the dependency of its periods on the hours of day is rather difficult to explain, even if we suppose the atmosphere consisting of different layers with remarkably different composition and temperatures.

Another, and probably the last possibility is that in the upper atmosphere there may occur a *vertical longitudinal* wave of limited extent, the period of which may sensibly depend on the effects of the solar radiations. That the vertical vibration of the atmosphere is possible, and may have a definite *natural period* depending on the sound velocity, has been fully

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[1] According to the recent investigations of Störmer, the auroral band is produced by the *vertical* inflow of *positive* corpuscles. If this be actually the case, the idea must be abandoned.

[2] H. Nagaoka, Proc. Tôkyô Math.-Phys. Soc., **7**, 1914, p. 403; Revue générale des Sciences pures et appliquées, **26**, p. 570.

investigated by H. Lamb and also by Prof. S. Sano.[1] Both authorities agree in the result that the period must be roughly of the order of *5 minutes*, if we assume a plane earth and take the ordinary value of the velocity of sound, which is assumed to be uniform throughout the atmosphere. According to S. Sano, the period is given by

$$T=\frac{4\pi c}{\gamma g}$$

where c is the velocity of sound and $\gamma=1{\cdot}41$. Since $c=\sqrt{\gamma R\theta}$, *i. e.* the period increases with the temperature, we must assume a lower temperature on the side of the upper atmosphere facing the sun than on the opposite side. This seems at first sight strange, but is in accordance with the fact that the diurnal variation of temperature in the upper layer shows a tendency to become opposite in phase compared with the lower layer[2]; though the amplitude is of course small, a tendency is suspected, that it increases with height. The difficulty is to explain the actual *amount* of the difference of the characteristic periods, the night-wave being nearly twice as long as the day-waves. The absolute temperature must then be assumed nearly 4 times higher on the night side, if the difference of the periods is to be exclusively attributed to the difference of θ. If the corpuscular radiation from the sun chiefly frequents the night side of our atmosphere, as is supported by different phenomena pertaining to aurora and magnetic storms, the effective temperature of that side of the atmosphere may be raised considerably by the presence of the additional kinetic energy of the free electrons. As to the increase of c due to the ionizations, no conclusive experimental evidence is yet afforded,[3] especially for the highly rarefied state

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[1] H. Lamb, Proc. London Math. Soc., [2] **7**, 1907, p. 122. S. Sano, Bull. of the Central Meteor. Obs., Japan, **2**, No. 2, 1913.

[2] T. Reger, Arbeiten d. kön. preuss. aeronautischen Observatoriums bei Lindenberg, 8, p. 229.

[3] W. Küpper found a sensible increase of c for gases ionized by radiations of different kinds, Dissertation, Marburg, 1912; Ann. d. Phys. [4] **43**, 1914, p. 905. W. H. Westphal obtained, however, a negative result, Verh. d. deutsch. phys. Ges, 1914, p. 613. These results refer to gases at ordinary pressure.

of gases. In this respect, many interesting physical problems may be suggested which seem worth special consideration.

On the other hand, if a more or less limited portion of the atmosphere be subjected to the oscillation of vertical type, the period will surely depend on the extent of the area disturbed. According to a personal communication by Prof. Sano, such a mode is actually possible and the period will decrease with the area disturbed. At least, the periods shorter than five minutes may partly be explained in this way. The period of vertical vibration may, however, also depend on the modes of laminar structure of the atmosphere in the distribution of the temperatures and the wind velocities of the different layers. The inquiry in these directions involves many intricated problems for mathematical physicists, and may better be left for the specialists in these lines.

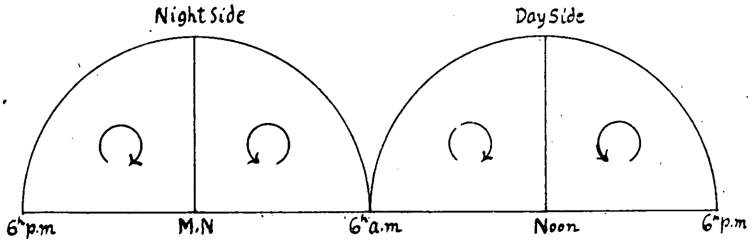
If the vertical vibration of the atmosphere be the actual cause of the magnetic pulsation as appears most probable, the investigations of the latter phenomena will in any case afford very valuable material for studying the actual physical conditions of the upper atmosphere, and may offer probably an unexpectedly wide field for new researches in different directions.

According to Prof. Sano, mechanical disturbances of any kind must gradually subside into a regular vertical natural vibration, which is comparatively slowly dissipated. This theoretical result is in harmony with the observed phenomena, *viz.*, the subsidence of an abrupt change of the terrestrial field into a train of regular damped waves, and also the peculiar persistent character of some trains. These points will be touched on once more in § 28.

28. Finally, let us consider the last crucial test of the different hypotheses, the peculiar behaviour of the *rotational* oscillation of the horizontal components. If we assume the observed sense of rotation for different hours, as generally applicable for the entire northern hemisphere, the distribution of the different senses of rotation in different hours will be roughly represented by the annexed figure (Fig. 23), where the semicircles

represent the hemispheres for day and night respectively. The distribution, expressed in other words, is such that when the periodic magnetic field is directed toward due south, the WE-component, just passing zero, is increasing toward the direction

Fig. 23.

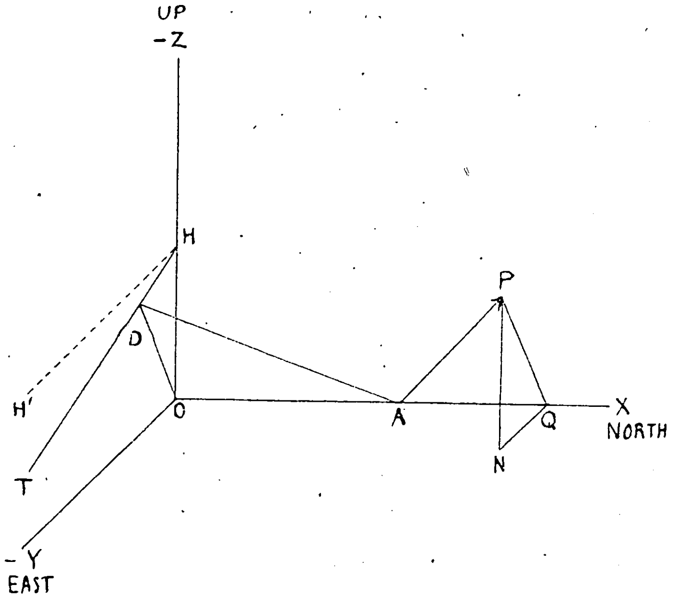


of the meridian at 6^h a.m., or 6^h p.m., for all points of a.m., or p.m. region respectively. A similar variation might have been produced, if there existed two sets of *horizontal* atmospheric oscillations corresponding to the zonal harmonics of the second order, the one having its axis at the earth's axis and the other in a direction passing through the equator at the meridians corresponding to 6^h p.m. and 6^h p.m.; the two component vibrations having a proper phase difference. Purely mechanical vibration of such a type must, however, have a more or less definite, and certainly much longer period, and is scarcely apt to explain the observed phenomena in the case of the most frequent short waves.

Another suggestion is that along the atmospheric current of more or less linear character, running near the place of observation, a sinuous motion is propagated, in which case the periodic disturbance may become rotatory and the sense of rotation will be opposite on both sides of the current. It seems to suffice, therefore, to assume a belt of current making no considerable angle with the meridian at 0^h or 12^h on which a sinuous motion is propagating with a proper velocity. The assumption is so far not utterly contradictory to the assumption of the circular current mentioned earlier, since the latter has the pole in the morning hour, decidedly deviating from the earth's pole. But in this case the phase relation of ΔZ an ΔX must be opposite on both sides of the current which is never actually the case. If the above be the case, the magnetic waves will naturally be of progressive character.

Again, to explain the phenomena in question on the assumption of a progressive wave of the sort considered in § 23, it may probably suffice theoretically to consider two intersecting systems of waves propagated with proper velocities, though as a matter of fact, we have neither an evidence nor a physical ground for assuming the existence of such a complicated system of currents. The wave length of such waves must, however, be of the order of the earth circumference, in order to explain Fig. 23; hence the case is essentially similar to that considered at the outset of this paragraph.

Fig. 24.



Finally, we may suppose that the atmospheric electric current is *not purely horizontal* as was generally assumed in the above discussions, and also that the vertical component current is subjected to a periodic fluctuation with a proper phase relation to that of the horizontal one; or less probably, to a rotatory oscillation of the kind considered in § 24, but in a horizontal elliptical orbit instead of the vertical one. These will probably remain the only admissible hypotheses, if the progressive nature of the magnetic waves be ultimately denied by the observations.

The simplest ideal case conceivable of this kind is that in which a linear current is subjected to a *see-saw motion in the vertical plane*, about its horizontal mean position. Referring to the annexed figure,[1] where xy-plane corresponds to the earth's surface, let HH′ be the mean position of the current i, at the height h, and let it be vibrating about its mean position in the vertical yz-plane, as if it were a tangent to a string subjected to a stationary vibration, at its node at H. Denote the frequency of vibration by $\frac{p}{2\pi}$. A is the point of observation at a distance x from the origin, and AD the perpendicular from it to the current HT at time t. If AP represents the magnetic field at A due to the current at t, it is perpendicular to AD in the plane perpendicular to HT and of the magnitude

$$\frac{2i}{AD}=\frac{2i}{\sqrt{x^2+h^2\cos^2\theta}},$$

if we denote by θ the inclination of HT to HH′. Let Q and N be the feet of the perpendiculars from P to Ox and xy-plane respectively. Then, denoting the angle ∠OAD by ψ,

$$AQ=\ X\ =AP\sin\psi,$$
$$QN=-Y=PN\sin\theta,$$
$$PQ=-Z=PN\cos\theta,$$

since the angle ∠NPQ$=\theta$. But $PN=AP\cos\psi$ and $\operatorname{tg}\psi=\frac{h\cos\theta}{x}$. Hence, we have for the fields of the three components:

$$X=\frac{2ih\cos\theta}{x^2+h^2\cos^2\theta}\qquad\text{for N-component,}$$

$$-Y=\frac{2ix\sin\theta}{x^2+h^2\cos^2\theta}\qquad\text{for E-component,}$$

$$-Z=\frac{2ix\cos\theta}{x^2+h^2\cos^2\theta}\qquad\text{for Upward-component.}$$

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[1] The coordinates axes are chosen lefthanded to correspond to our initial convention.

According to our assumption, θ is given by

$$\operatorname{tg}\theta = C\cos(pt+\varphi).$$

If C is small and we neglect the small quantities of the second order against unity, we may put $\cos\theta \doteq 1$ and $\sin\theta \doteq C\cos(pt+\varphi)$. Hence

$$X=\frac{2ih}{x^2+h^2}\,;\quad Y=-\frac{2ix}{x^2+h^2}C\cos(pt+\varphi)\,;\quad Z=-\frac{2ix}{x^2+h^2}\,.$$

Thus, the Y-component only will be sensibly affected while the others remain constant, if i be considered constant. If i be variable and given by

$$i=i_0+i_1\cos qt\,,$$

where i_0 and i_1 are constants and i_1 is small compared with i_0, the periodic parts of the three components will be given by

$$\Delta X=\frac{2i_1h}{x^2+h^2}\cos qt,$$

$$\Delta Y=-\frac{2i_0xC}{x^2+h^2}\cos(pt+\varphi),$$

$$\Delta Z=-\frac{2i_1x}{x^2+h^2}\cos qt,$$

neglecting the small quantities of the second order in ΔY. If $p=q$, as may be the case when the see-saw motion is the direct cause or result of the fluctuation of the current and moreover if $\varphi\neq 0$, we will have a rotatory motion of the kind desired. Besides, we notice that the sense of rotation is opposite on both sides of the yz-plane. The relation of ΔX and ΔY is just the same as in § 22 a), as is evident. ΔZ must be of the opposite sign on both sides of the current. The results of observations show, however, that the phase relation between ΔX and ΔZ is generally the same all through, ΔZ never being ahead of ΔX. Hence the maximum intensity of the current in question can not be running largely in the meridional direction, but probably more or less near the equatorial zone, since otherwise we should expect that in some hours ΔZ may run in advance of ΔX. To reconcile the present case with the result shown in Fig. 23, it will be plausible to assume that in passing along the four quadrants 0^h–6^h, 6^h–12^h, 12^h–18^h, 18^h–24^h, the phase angle differs successively by π, if i_0 be of the same sign throughout; or i_0 changes the sign alternatively, if φ remains constant. It we are to identify i_0 with the equatorial portion of the circular current assumed in § 19, then it will suffice to suppose that a *portion* of the circular belt or ring carrying the current is subjected to a *radial* vibration, as if it were a portion of the ring carrying out a *dilatatory* vibration in such a way that the amplitude is given by $\cos^2\lambda$ say, where λ is the longitude counted from noon. The fluctuation of the intensity must be considered as keeping pace with the vibration and the period must be different for day and night.

Though the above solution is by no means unique, yet it may at least serve as a provisional explanation as far as the phenomena of rotatory pulsation are concerned. It is, moreover, not contradictory to the principal facts of observation already enumerated: *viz.*, not irreconcilable with the mode of the daily variation of the azimuth of the horizontal components, since the circular current assumed may be considered to have its maximum intensity always near the equator; and again not inconsistent with the observed amplitude ratio and phase relation of ΔZ and ΔX, since the see-saw motion chiefly affects ΔY, and the discussions on these relations given earlier hold good here also without any essential modification.

According to van Bemmeln, the ratio $\Delta Z/\Delta X$ at Batavia seems to be invariably of insensible magnitudes. This fact is in harmony with the above assumption that the maximum current is near the equator. On a theoretical ground, this seems also admissible if we may attribute the origin of the disturbing current to the vertical motion of the atmosphere, since general planetary circulation of the whole atmosphere must be in any case vertical in these regions, where the horizontal intensity of the magnetic field is also greatest.

29. The above taken as granted, it seems plausible to assume next that the *impulse* causing the vertical vibration, if not solely mechanical, is to be attributed to the reciprocal action

of the magnetic field on the change of the current,[1] assuming the current already caused by other agents. In § 19 we have suggested that the zonal part of the atmospheric current seems to be more liable to the fluctuations than the part running in the meridional direction. This is in some measure favourable to the hypothesis.

If the mechanical vibration of the atmosphere be once started, the periodic fluctuation of current naturally results on account of the induction due to the general terrestrial field. The intensity of the induced current will be greatest at the equator for a given amplitude of the oscillation.

The cause of the initial impulsive increase of the current may on one hand be considered as the result of the motion of the atmosphere brought about in a purely mechanical way, for example, an abrupt advent of the vertical flow of the upper atmosphere caused by the release of some instability of equilibrium; or we may conceive a limited portion of the atmosphere excited to a vigorous vertical current, as in the centre of a cyclone, caused as a secondary disturbance accompanying the daily exchange of air between the day and night hemispheres.

On the other hand, the abrupt increase of current may also be attributed to an external agent, for example, a sudden increase of the conductivity of air caused by the inflow of the corpuscular radiation from the sun.[2]

Among the above two possibilities, the latter is more plausible, being in accordance with the commonly accepted view as regards the origin of magnetic disturbances, which is strongly supported by the intimate relation existing between the solar activity and the disturbances in general. Fig. 10 is also favourable to this hypothesis. The former is rather doubtful,

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[1] In view of this consideration, an interesting physical problem presents itself: Consider a current passing through highly rarefied gases, acted upon by an external magnetic field; will the gases really flow *as a whole* as Fleming's rule specifies for a solid conductor? As far as we are aware, the answer is not yet given either experimentally or theoretically, especially for such a case as is analogous to the atmosperic currents.

[2] A. Schuster concluded on the basis of energy consideration that the E.M.F. of the current causing magnetic disturbances is to be attributed to the motion of the atmosphere, the sun's radiation serving only as an agent increasing the conductivity.

since it is an open question whether in the higher part of our atmosphere there may occur any remarkable vertical *convection*, as is the case in the troposphere. At any rate, the fact that an abrupt increase of the general field is most frequently accompanied by a train of remarkable pulsations, is in harmony with the above idea that the increase of the current is associated with the increase of the vertical impulsive motion of the atmosphere which subsides into a vertical natural vibration. The characteristic disturbance of this type occurs as a rule near midnight. On the other hand, examples are by no means rare, especially in the evening, where an abrupt increase of the horizontal component occurs, but not at all accompanied by a train of waves. How is the fact to be explained ? It may at first be suggested that in this case the abrupt increase of the current takes place in the direction of the general terrestrial field; but as a matter of fact, the increment of the Y-component in such a case is not generally of a different order of magnitude compared with that of the X- component. Another possible altervative is that in the hours which are near the boundary between the day and night hemispheres, the regular vertical vibration of the atmosphere is not so easy as at midnight. The latter assumption seems in some measure plausible, if we consider very probable heterogeneity of the atmosphere in that region forming the transition stage of the illuminated and non-illuminated halves. The minimum of the frequency of pulsations of all periods falls actually in the evening hours (Fig. 8). The morning hours are not necessarily in the same condition as the evening hours. In the former, the atmosphere is being rapidly heated up and ionized, but on the night side of the boundary a dormant homogeneity of the night-atmosphere may probably prevail; whereas in the latter, the gases are cooling down and the ions recombining, and gradually passing into the nightly condition. In the morning, the transition may therefore be abrupt and the area of the heterogeneous atmosphere comparatively small, while in the evening the heterogencity may extend to a considerable area of the earth's surface.

Again, cases are quite common where remarkable trains of waves occur without any general swelling of the horizontal component. Among such trains, we may distinguish two types, *viz.*, those with the abrupt beginning and those gradually increasing in amplitudes. The former, which is most frequent during night, may probably be explained by a transient increase of the atmospheric current with a duration comparable with, or shorter than the natural period of the atmospheric oscillation. Indeed, the first wave of the train of this type is often sensibly shorter than the following regular waves. The latter class, which is generally the case with the shorter day-waves and also sometime with the longer night-waves, may be caused either by the mechanical disturbance of the atmosphere propagated from a remote region, or by some different agents. Among other conceivable causes of the vertical vibration of the upper atmosphere, we may cite the instability of the discontinuous horizontal motion of the higher layers. That the upper layer has a considerable angular velocity with respect to the earth's surface is well known from the observation of the "illuminated night cloud." It is then probable that a favorable vertical distribution of density may cause a remarkable wave motion of different types. The gravitational wave, as occurs in the case of an incompressible fluid, will not answer our purpose. But it is more than probable that a radial or vertical expansional wave which is the most persistent type,, is excited in this way—a case somewhat analogous to the excitation of the organ pipe by the stream of air running along the loop of the vibrating air column. If the upper atmosphere be arranged in different layers with different temperatures and different general wind velocities, as is the case in the troposphere, different periods may occur at the same time. The irregularities of the pulsations during day time might well have been produced in this way, if we consider that the laminar structure may be enhanced in some way or other, by the influence of solar radiation. Beside the thermal effect, light pressure may also play some sensible rôle in this respect, since it acts in differentiating the gases with different absorbing powers and molecular sizes.[1]

Again, if there be present any sensible fluctuation of the solar radiation[2] with a short duration, the thermal effect, and also the radiation pressure in some measure, may contribute to the initiation, if not the direct excitation of the vertical motion of the upper atmosphere. The remarkable irregularities of the day-waves may partly be accounted for in this way.[3]

As to the origin of the magnetic waves with the periods longer than 10^m, we may probably suggest the slow atmospheric waves possible in the case when two layers with different temperatures are superposed.[4]

The fact described in § 17, that the hourly distribution of the senses of rotation of the disturbing magnetic vectors are quite different for the short waves and the long undulations, requires an explanation. It may only be suggested for the present that the difference is in any case due to the difference in the modes of atmospheric free vibration. Any further discussion must be postponed till the nature of the vibrations of the atmosphere has been more fully investigated from the theoretical side, especially for a *spherical* atmosphere.

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[1] Adopting Debye's approximate formula for an *absorbing* sphere (Ann. d. Phys., [4] **30**, 1909, p. 117), the pressure of the light wave with the wave length λ on a sphere of radius a may be put

$$1{\cdot}83 \cdot \frac{2\pi a}{\lambda} \cdot \pi a^2 \frac{S}{c},$$

where S is the energy of the radiation per sec. per cm.^{2} Assuming $a = 10^{-8}$, $\lambda = 5 \times 10^{-5}$ and $S = \frac{3}{60} \times 4{\cdot}10^7$, we obtain a pressure $0{\cdot}48 \times 10^{-22}$ dynes. Taking the mass of the sphere as $1{\cdot}6 \times 10^{-24}$gr. for a hydrogen atom, we obtain an acceleration of 30 cm/sec. The mean free path at the pressure of 0·007 mm. being about 1·76 cm. the mean velocity may be estimated to be of the order of 5·1 cm/sec. The path traversed is therefore only 184 m. per hour. This is probably too small to answer our purpose. But if we take a "Dipole" instead of the absorbing sphere, the value of the pressure may in some cases be estimated to be decidedly greater. Near 200 km. from the earth's surface, the pressure will vary rapidly with the height; in the higher layer, the path traversed by molecules or atoms may be enormously greater than the above estimation.

[2] Together with the back-radiation from the earth.

[3] The *corpuscular* radiation being deviated by the terrestrial field, will not be confined to the day hemisphere, or rather prefers the night side. The fluctuation of this radiation may be considered to be of the type considered in p. 79.

[4] H. Lamb, Proc. R.S., A **84**, 1911, p. 551.

30. From all we have discussed at length in the preceding paragraphs, we can as yet draw no convincing conclusion regarding the actual mechanism producing the magnetic pulsations in question. Many things, indeed, turn on the crucial observation regarding the universal *simultaneity* of the periodic phenomena. If it be *proved* ultimately beyond doubt as seems probable, there remain only those hypotheses at disposal, which lead to the simultaneous occurrence of the phenomena for an area of considerable extent. It that case, the magnetic waves may probably be explained by the *combination of periodic fluctuations, in intensity as well as in position and inclination of the atmospheric current*, which is then to be considered of rather diffused, but not of linear character, having the maximum intensity near the equator. The origin of these fluctuations might then very probably be sought in the *vertical stationary oscillation* of limited portions of the upper atmosphere accompanying the diurnal oscillation of the entire atmosphere.[1]

At any rate, it must be admitted that the present results of observation refer to a single station, and the observations are far from being complete. A further study of the allied phenomena, especially the simultaneous observations in at least *three* stations, sufficiently apart from each other, will be desirable. The results will not fail to advance our knowledge on the nature of the atmospheric current of which we have at present only a very vague idea. The interest attached to the problem at hand is by no means confined to the limited subject of terrestrial magnetism. The phenomena have a much wider bearing than at first sight appears, on various interesting problems in different branches of physics, for examples those regarding solar physics, meteorology and also especially those regarding the electrical and mechanical behaviours of highly rarefied gases under the action of different kinds of radiation.

In conclusion, the author wishes to express his sincerest

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[1] In this case, however, the phenomena of the propagating auroral bands accompanied with no corresponding magnetic maves, become rather incomprehensible and throw some doubt on Birkeland's original conception pertaining to the nature of the luminous band.

thanks to Prof. A. Tanakadate, under whose supervision the entire work was carried out, for his kind guidance throughout the course of the investigations. Most cordial thanks are also due to Prof. H. Nagaoka and to Prof. S. Sano, for the interest shown in the investigations and for many valuable suggestions and instructions given. Last, but not least, we feel very much indebted to Prof. I. Ijima, the Director of the Marine Biological Laboratory, for his kindness and generosity in affording lodging and many other conveniences to the observers resident in Misaki, for which the best thanks of all the participators in the present work are due.

Summary of the Results of Investgations.

1. The period of the magnetic pulsations has no sharply defined value, varying from about 20 seconds to nearly 1 hour. Nor is it exactly constant even in a coherent train.

2. During the day time, waves of 0·5–1 minute periods predominate, whereas during the night hours longer periods 1·5–2·5 minutes are most frequent.

3. A periodicity of 25–30 days is suspected in the daily frequency of the pulsations.

4. The vertical component of the waves is a reduced reproduction of the NS-component except the phase retardation. The shorter the period, the more remarkable is the reduction of the amplitude as well as the phase retardation.

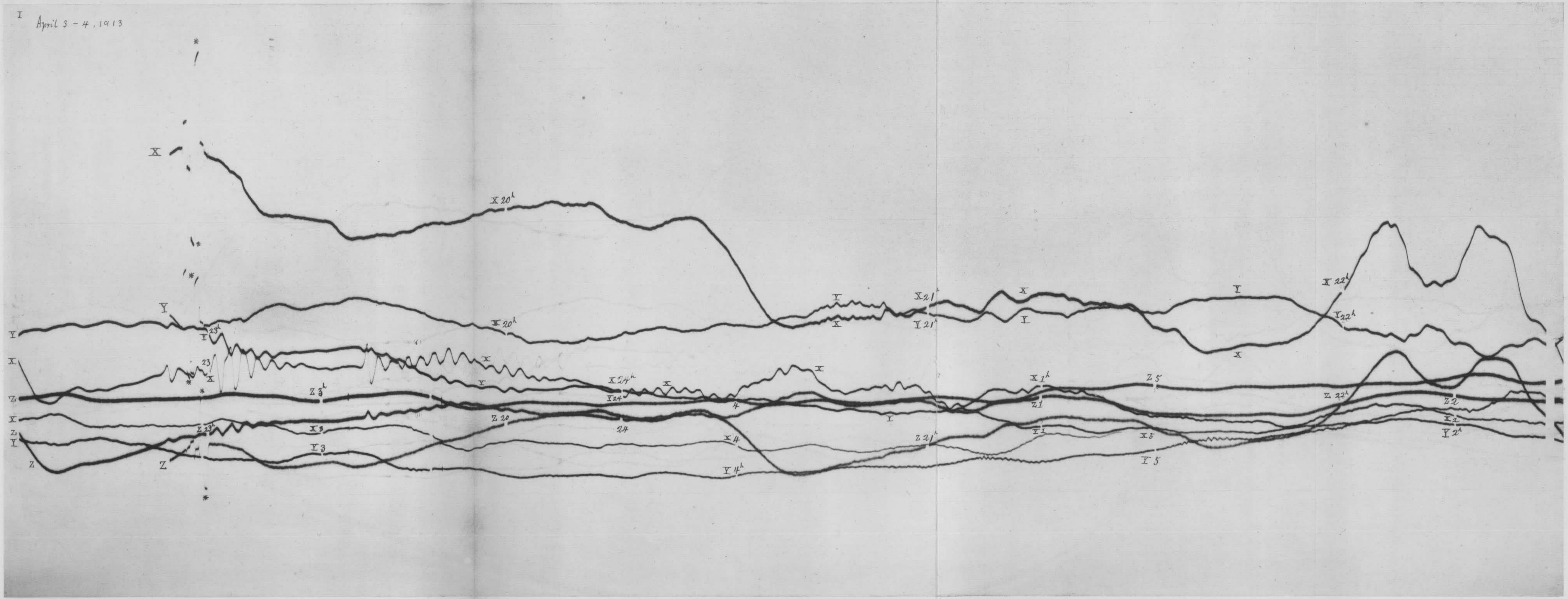
5. The azimuth of the linearly pulsating magnetic field undergoes a remarkable diurnal variation, showing maximum deviations from the meridian a few hours before noon and midnight.

6. The disturbing field is generally more or less rotatory. The sense of rotation shows a semidiurnal variation. The clockwise rotation is most frequent during the hours between sunrise and noon, as well as between sunset and midnight, while the opposite rotation falls most frequently in the remaining hours.

7. The observed results may probably be explained by the fluctuations of the horizontal electric current existing in the upper atmosphere, and causing the diurnal variation of the terrestrial magnetism.

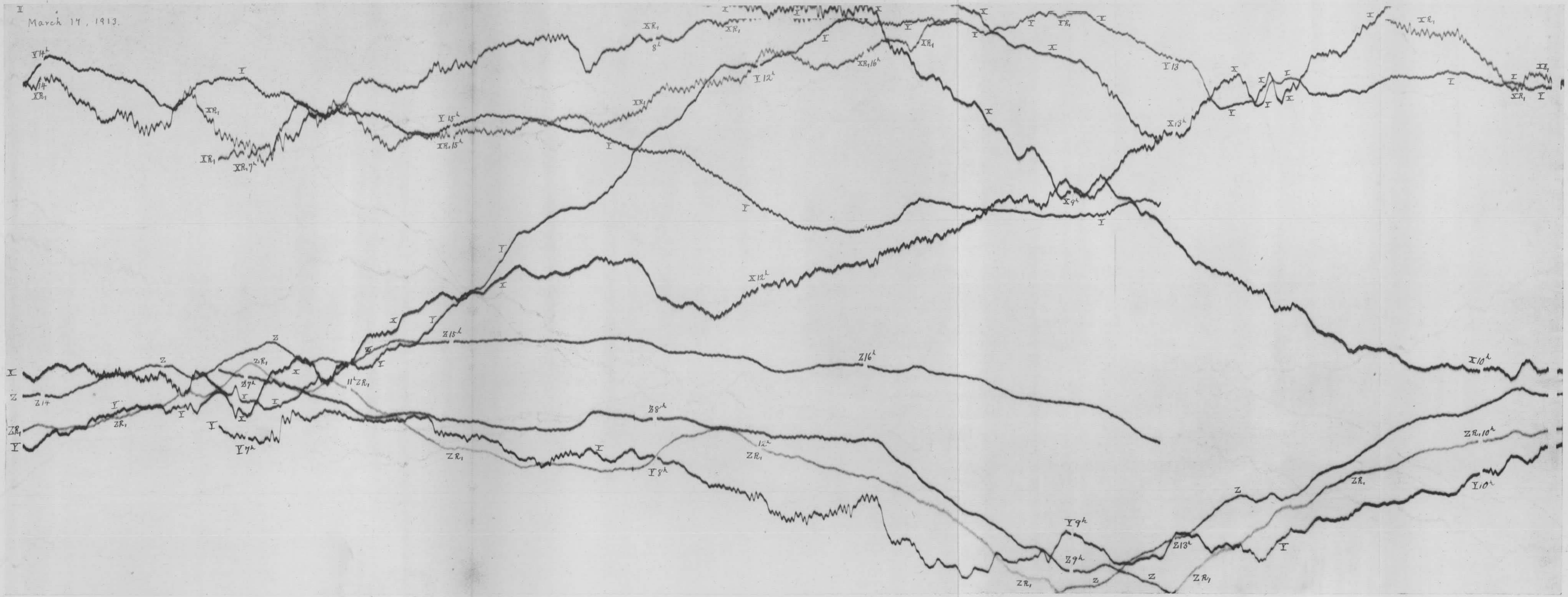
8. Two diverging lines of theoretical considerations intended for the explanation of the phenomena in question, are given; the one based on the assumption of the *simultaneity* of the phenomena in a wide area, and the other on the assumption of *a progressive nature* of the pulsations, though some evidences at hand seems to speak rather strongly against the latter assumption. The results of the discussions turn out rather favourable for the hypothesis of simultaneous disturbances than for that of progressive waves. If the simultaneity be universally established, the phenomena may probably be accounted for by the fluctuation of the atmospheric current, in its *intensity* as well as in its *location.* The fluctuation must then very probably be attributed to the more or less *vertical oscillation* of limited portions of the upper atmosphere. If such be actually the case, we have in the phenomena of the magnetic pulsations a very valuable clue for studying the physical conditions of the upper atmosphere unattainable by the usual means, and then, it may be hoped, for following the hourly or daily changes occurring in the remotest part of our atmosphere.

Published May 25th, 1917.

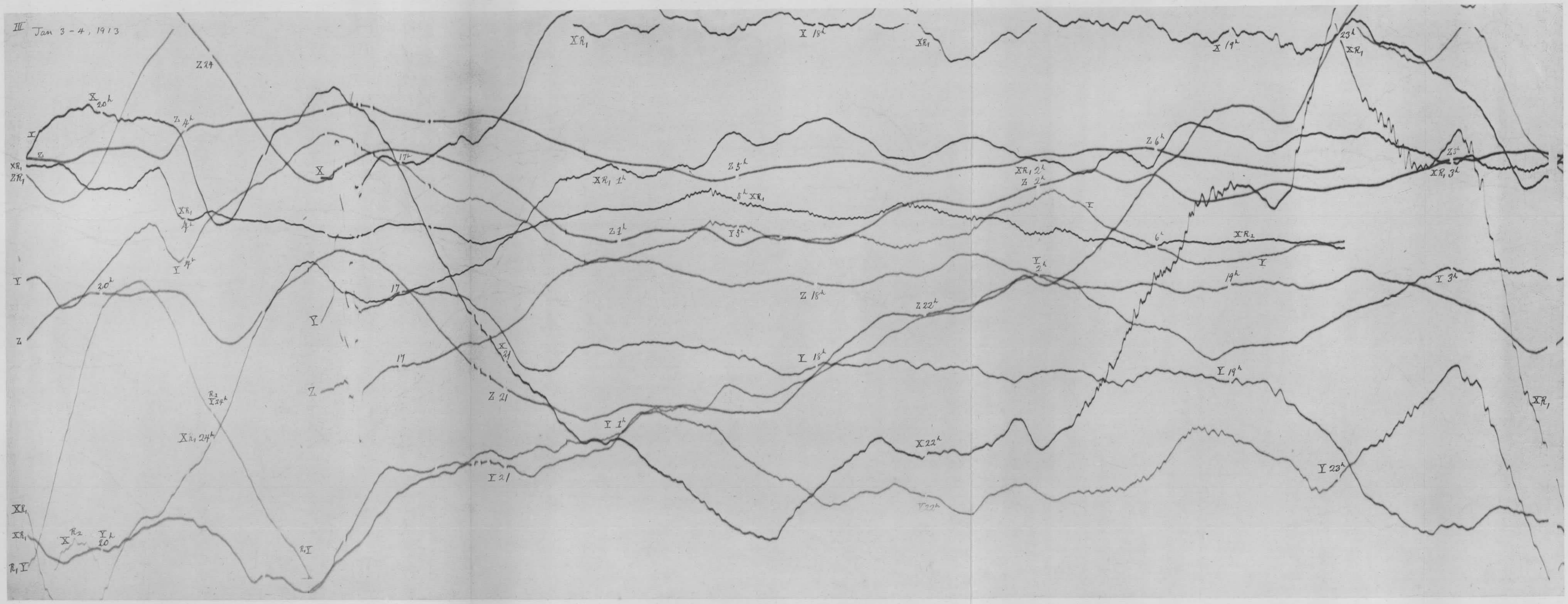


Night record on a comparatively calm day, with characteristic long waves in midnight and short waves in the morning. April 3–4, 1913. (Reduced to $\frac{2}{3}$ original size).

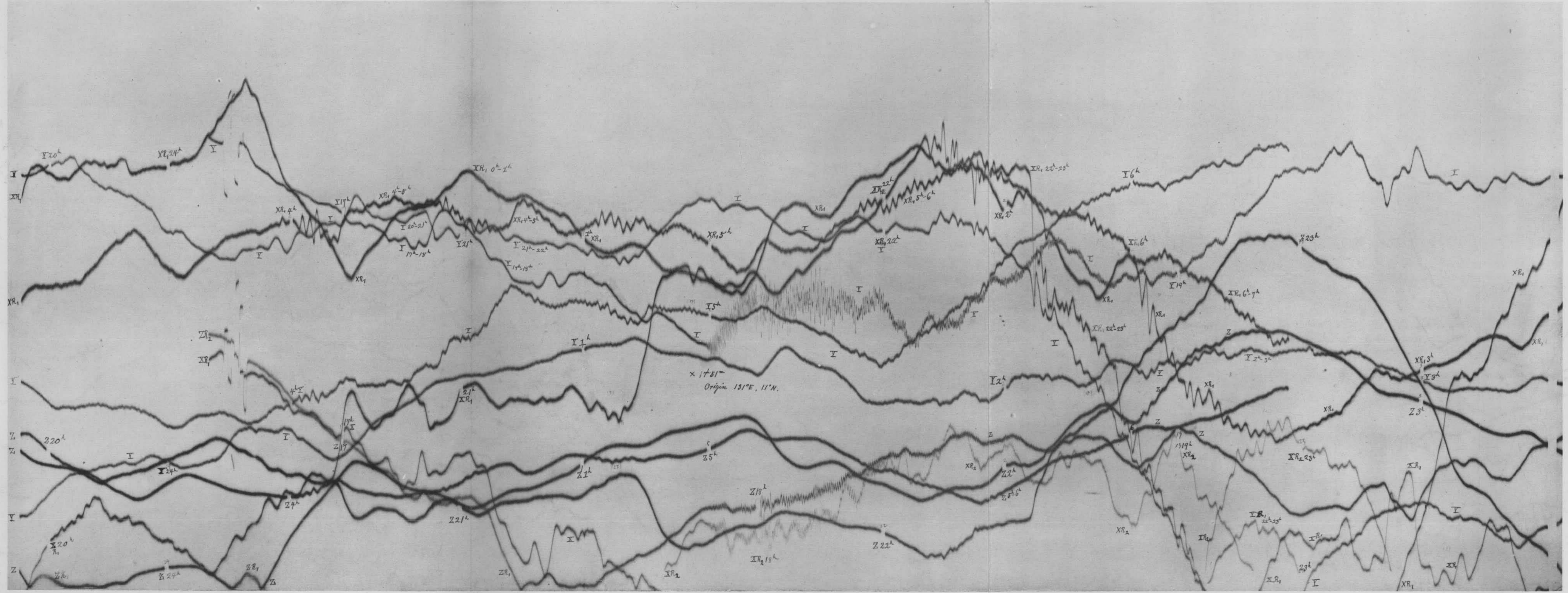
T. Terada. On Rapid Periodic Variations of Terrestrial Magnetism.



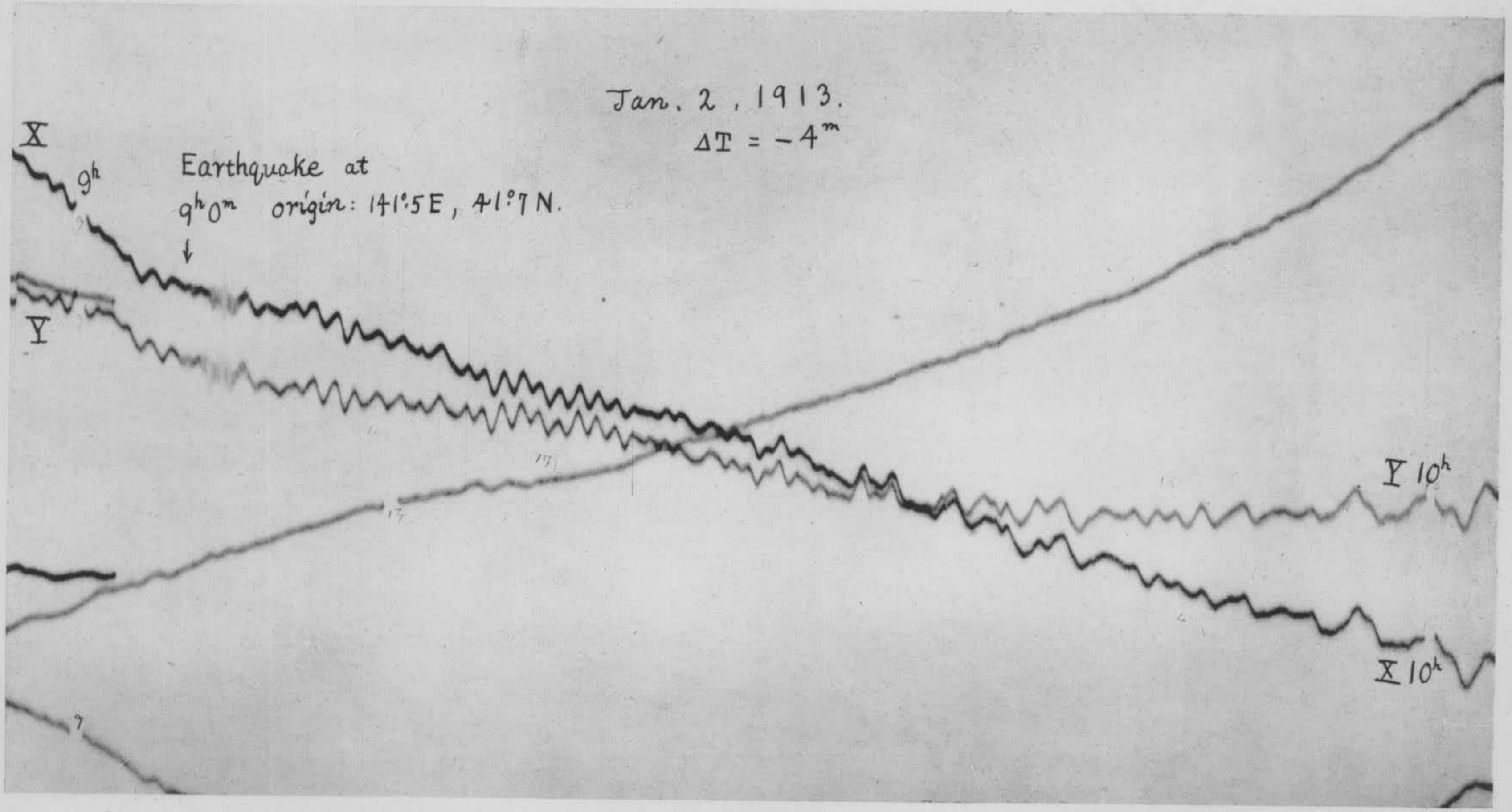
Characteristic day record. Note the short waves in the morning. In this example similar waves recur in the evening. March 17, 1913. (Reduced to $\frac{2}{3}$ original size).



Disturbed day. Regular long waves characteristic to midnight not conspicuous, in spite of the remarkable disturbance of the general field. Note the morning trains of short waves, similar trains not occurring in the evening. Jan. 3–4, 1913. (Reduced to ⅔ original size).



Record of a distant earthquake, originated at a southern part of North Pacific. March 14–15, 1913. (Reduced to $\frac{2}{3}$ original size).



Note the approximate parallelism between X and Y. Record of a local earthquake indicated at 9 a.m. (Natural size).

T. Terada. On Rapid Periodic Variations of Terrestrial Magnetism.