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The Systematic Motions of Sun-spots.

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With 3 plates.

The Systematic Motions of Sun-spots.

1. The object of this paper is to call attention to the possible existence of certain systematic motions of the sun-spots, which has not, so far as I know, been hitherto noticed. It was suggested to me by Mr. and Mrs. Maunder's paper "The Solar Rotation Period from Greenwich Sun-spot Measures," (Monthly Notices, vol. LXV, pp. 813–825) in which they summarized various important results relating to the solar rotation period. Among others, the following statement is made: "The rotation periods given by different spots in the same zone of latitude differ more widely than do the mean rotation periods for different zones of latitude." This is based on their Table II, which shows the number of the spot-groups, arranged in zones of latitude 5° wide, giving different synodic rotation periods. Graphical study of that Table led me to think that there is a tendency in the spot-groups to concentrate upon two particular periods. I was also able to find a similar tendency in Carrington's and Spörer's observations of the sun-spots.

2. To begin with the Greenwich sun-spots observations (1879–1901), I simply took the said Table II prepared by Mr. and Mrs. Maunder, and combined the zones of the same latitudes north

Table I.

Greenwich Numbers of Spot-groups, arranged in Zones of Latitudes 5° wide, giving Different Rotation Periods (1879-1901).

Synodic Rotation Period	0° to 5°	5° to 10°	10° to 15°	15° to 20°	20° to 25°	25° to 30°	30° to 35°	>35°
23.4 d			1					
6								
8								
24.0			2	2				
2		3		3				
4	2	2	3	1				
6	3	3	4		1	1		
8	2	4	5	4				
25.0	5	6	5	8	3	1		
2	6	10	11	4	2			
4	3	6	18	8	1	1		
6	4	12	17	12	5	2	1	
8	6	20	25	18	3	1	1	
26.0	12	22	26	15	12	1		
2	6	41	39	23	11	3		
4	16	33	45	29	9	1	1	
6	24	33	49	29	17	5		
8	16	46	61	30	10	6	1	
27.0	14	59	82	48	15	1		
2	5	41	76	63	25	3	1	
4	2	27	52	51	20	6	1	
6	2	5	20	41	26	7	3	1
8	1	8	13	21	24	10	1	
28.0	1	6	8	6	8	11		
2	2	4	5	7	12	4	1	
4	1	2	5	5	10	6		
6		1	4	1	1	2		

Synodic Rotation Period	0° to 5°	5° to 10°	10° to 15°	15° to 20°	20° to 25°	25° to 30°	30° to 35°	> 35°
8			2	6	3	2	1	
29.0			1	1	3	2		
2			1	1		2		
4			1	1	1			
6								1
8			1	1				
30.0			2	1	1			
2								
4								
6					2			
8								
31.0								
2			1					
Mean Rotation Period	d 26.36	d 26.59	d 26.73	d 26.89	d 27.22	d 27.48		

and south. The resulting numbers are given in Table I, which therefore exhibits the distribution of the spot-groups in two ways, the horizontal lines showing the numbers of groups yielding different synodic rotation periods, and the vertical columns the number of groups in each zone of latitude 5° wide. The numbers of Table I are represented graphically in Plate I, a series of curves being drawn, one for each zone. Considering now these curves shown in Plate I, each small circle represents the number of spot-groups corresponding to the given rotation period. They are connected by straight lines, while the heavy continuous lines indicate the smoothed values.

This Plate shows at a glance that, while there are minor differences between the curves for different zones, yet the main features are repeated in a remarkable way in all six series. We may observe that there is a tendency in the spot-groups to concentrate upon one particular period, but each curve is not symmetrical about the ordinate of maximum number. Another important point is the existence of a secondary concentration. At first I thought that if there existed a greater number of recorded observations, then I could get a sort of probability curve, but actually it seems likely that each curve will be a combination of two probability curves. Because of the impossibility of determining rigorously the values of several constants with non-linear equations by the method of least squares, and moreover because of the doubtful nature of the problem, I have endeavoured to determine the positions of the principal and secondary maxima by graphical method. When the maximum is not well pronounced, I have derived the smoothed curves by compounding two symmetrical curves about the directions marked I and II. All doubtful cases (marked ? on the plates) have been excluded. I have thus obtained the following results:

TABLE II.

Angular Velocity corresponding to Maximum I.

Heliographic Latitude	Synodic Rotation Period	Observed Ang. Vel.	Number of Spots in Maximum	Smoothed Ang. Vel. I_G	Maunder's Ang. Vel.
2°.5	26ᵈ.60	14°.52	22	14°.45	14°.44
7.5	27.00	14.32	54	14.38	14.41
12.5	27.075	14.28	68	14.29	14.34
17.5	27.25	14.20	51	14.19	14.25
22.5	27.55	14.05	26	14.05	14.13
27.5	27.90	13.88	11	13.88	13.99

TABLE III.

Angular Velocity corresponding to Maximum II.

Heliographic Latitude	Synodic Rotation Period	Observed Angular Velocity	Number of Spots in Maximum	Smoothed Ang. Velocity. IIG
2°.5	?	?	?	(14°.72)
7.5	26ᵈ.225	14°.71	28	14.71
12.5	26.275	14.69	30	14.69
17.5	26.35	14.65	20	14.65
22.5	26.45	14.60	10	14.59
27.5	26.70	14.47	6	14.49

The value in brackets is one obtained by extrapolation.

Angular Velocities of Maxima I and II for Different Zones of Latitude, deduced from Greenwich Observations.

Fig. 1.

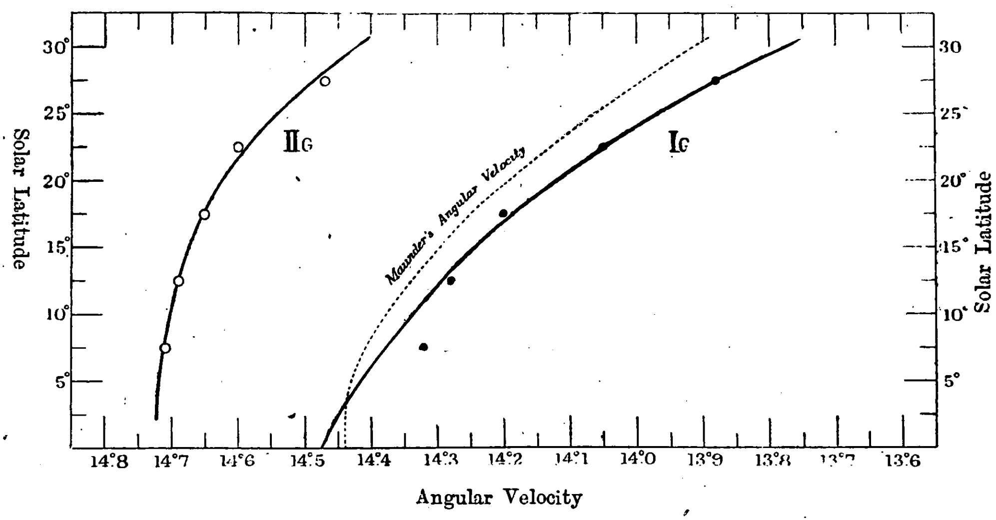


Figure 1 shows diagrammatically the numerical results of Tables II and III.

In the last column of Table II, I have added Maunder's values of daily angular velocity derived from his formula

$\xi = 866'.6 \pm 128' \sin^2\lambda$, where ξ denotes the angular velocity and λ the latitude. These values obtained by his formula are generally greater than my results. That is, they do not correspond exactly to the angular velocities for my Maximum I. As to the second Maximum, I have obtained a pretty good continuous curve by merely connecting the successive positions of Maximum II, corresponding to different zones.

3. Next I pass on to examine Carrington's observations of the spots on the sun so far as they bear on the question of the existence of a secondary maximum. All the materials from which Carrington deduced his formula for the daily motion of the surface of the sun in different solar latitudes are condensed in his "Table of Resulting Diurnal Motions" contained in pp. 213-219 of his work. I have availed myself of the same materials, and deduced Table IV which shows the distribution of the sun-spots observed by him in each zone of latitude 5° wide, with different daily drifts.

TABLE IV.

Carrington's Numbers of Sun-spots, arranged in Zones of Latitudes 5° wide, giving Different Diurnal Motions (1853–1861).

Daily Drift	0° to 5°	6° to 10°	11° to 15°	16° to 20°	21° to 25°	26° to 30°	31° to 35°
From +80′ to +76′	1						
,, +75 ,, +71							
,, +70 ,, +66							
,, +65 ,, +61							
,, +60 ,, +56		1		1			1
,, +55 ,, +51					1		
,, +50 ,, +46		1					
,, +45 ,, +41	1						
,, +40 ,, +36	1	2	2				
,, +35 ,, +31		2	1	2			
,, +30 ,, +26		6	1	1			

Daily Drift	0° to 5°	6° to 10°	11° to 15°	16° to 20°	21° to 25°	26° to 30°	31° to 35°
From +25′ to +21′	3		4	2	1		
„ +20 „ +16	1	6	6				
„ +15 „ +11	1	8	5	4			
„ +10 „ + 6	3	11	7	7	2		
„ + 5 „ + 1	1	15	7			1	
„ 0 „ − 4	2	5	23	6	7	1	
„ − 5 „ − 9		5	9	11	4	2	
„ −10 „ −14		2	6	15	10	2	1
„ −15 „ −19			3	7	7	4	
„ −20 „ −24				5	11	4	
„ −25 „ −29				2	6	2	1
„ −30 „ −34				1	5	10	5
„ −35 „ −39				1	2	4	
„ −40 „ −44					2	4	2
„ −45 „ −49							1
„ −50 „ −54						2	2
„ −55 „ −59						1	
„ −60 „ −64						1	1
„ −65 „ −69							
„ −70 „ −74							
„ −75 „ −79							1
Mean Daily Drift	+17.′7	+10.′8	+3.′3	−3.′9	−1.′6	−29.′1	

The first column of Table IV requires further explanation. It represents Carrington's diurnal motions. According to his words, "The signs prefixed to the diurnal motions are such that + in longitude indicates rotation faster than 14° 11′ per diem (corresponding to the assumed period of 25. 380 days) and − rotation slower than that." The plotted numbers are shown graphically in

Plate II. The method of treatment of the material in Plate II is just the same as before. Here again, as in the previous case, the secondary maximum is clearly brought out in each curve. Graphically determined maxima may be tabulated as follows:

Table V.

Angular Velocity corresponding to Maximum I, deduced from Carrington's Observations.

Heliog. Latitude	Observed Angular Velocity.	Number of Spots in Maximum	Heliog. Latitude	Smoothed Ang. Vel. Ic	Carrington's Ang. Vel.
			2°.5	(14.°36)	14.°42
8°	851′+ 5′=856′=14.°27	13	7.5	14.27	14.35
13	„ − 2 849 14.15	19	12.5	14.16	14.21
18	„ −11 840 14.00	13	17.5	14.02	14.06
23	„ −20 831 13.85	9	22.5	13.86	13.90
28	„ −33 818 13.63	8	27.5	13.67	13.73

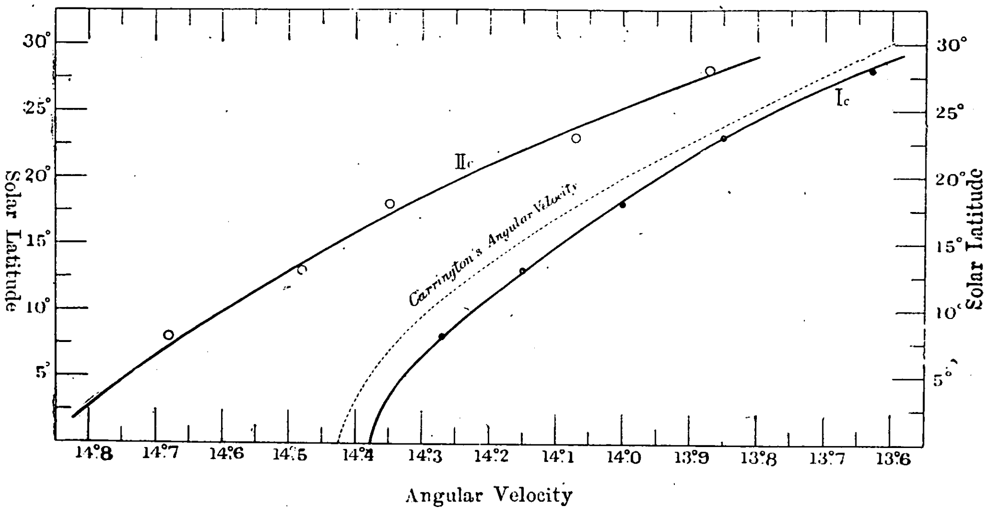
Table VI.

Angular Velocity corresponding to Maximum II, deduced from Carrington's Observations.

Heliog. Latitude	Observed Angular Velocity	Number of Spots in Maximum	Heliog. Latitude	Smoothed Ang. Vel. IIc
			2.°5	(14.°81)
8°	851′+30′=881′=14.°68	4	7.5	14.67
13	„ +18 869 14.48	5	12.5	14.51
18	„ +10 861 14.35	6	17.5	14.34
23	„ − 7 844 14.07	2	22.5	14.13
28	„ −19 832 13.87	4	27.5	13.88

Angular Velocities of Maxima I and II for Different Zones of Latitude, deduced from Carrington's Observations.

Fig. 2.



Carrington's angular velocity given in the last column of Table V is derived from his formula for the angular velocity, $\xi=865'\mp 165'\sin^{\frac{7}{4}}\lambda$.

4. Similarly I have examined Spörer's observations. The data for our discussion of the spots as observed by him have been taken from the same source as that from which he deduced his expression, $8°.548+5°.798\cos\lambda$, for the daily motion of the sun-spots in different solar latitudes, namely his "Beobachtungen der Sonnenflecken zu Anclam."(1) The observations comprise the period 1861–71, the number of the spots whose angular velocities are computed being 264. To increase this number, I have extended the discussion of his observations(2) up to the end of the year 1878. So the total number of spots employed became 334. Similarly to the others, Table VII and Plate III were prepared. It is unfortunate that none of the curves in Plate III show so striking a secondary maximum as in the previously considered cases, although there is no doubt that each curve

(1) Publicationen der Astronomischen Gesellschaft. XIII 1874 pp. 139—146.

(2) Publicationen des Astro-physikalischen Observatoriums zu Potsdam. Nr. 5. p. 66.

Table VII.

Spörer's Numbers of Sun-spots, arranged in Zones of Latitudes 5° wide, giving Different Diurnal Motions. (1861-1878)

Daily Motion	0° to 5°	5° to 10°	10° to 15°	15° to 20°	20° to 25°	25° to 30°
13.°4						2
13. 5			1			1
13. 6					5	3
13. 7						7
13. 8				7	9	5
13. 9			4	11	12	1
14. 0	2		17	21	8	2
14. 1	1	16	41	24	4	
14. 2	3	30	35	10	5	
14. 3	5	33	20	7	1	
14. 4	11	10	8	3		
14. 5	6	6	2		1	
14. 6	3	4	3			
14. 7	3	3	1	1		
14. 8	2	2				
14. 9	1	1				
15. 0		2				
15. 1			1			
Mean Daily Motion	14°.43	14.°31	14.°18	14.°07	13.°94	13.°71

is not symmetrical about the ordinate corresponding to Maximum I. In fact, the sun-spots selected by Spörer are best suited for finding Maximum I, but not so favourable for finding Maximum II. However, so far as Plate III shows, I can deduce the following results. Better results would perhaps be reached by discussing the angular velocities of all the spots observed by him.

TABLE VIII.

Angular Velocity corresponding to Maximum I.

Heliog. Latitude	Observed Ang. Vel.	Number of Spots in Maximum	Smoothed Ang. Vel. Is	Spörer's Ang. Vel.
2.°5	14.°40	10	14.°36	14.°34
7. 5	14. 25	31	14. 28	14. 30
12. 5	14. 14	38	14. 17	14. 21
17. 5	14. 05	23	14. 03	14. 08
22. 5	13. 89	11	13. 90	13. 90
27. 5			(13. 77)	13. 69

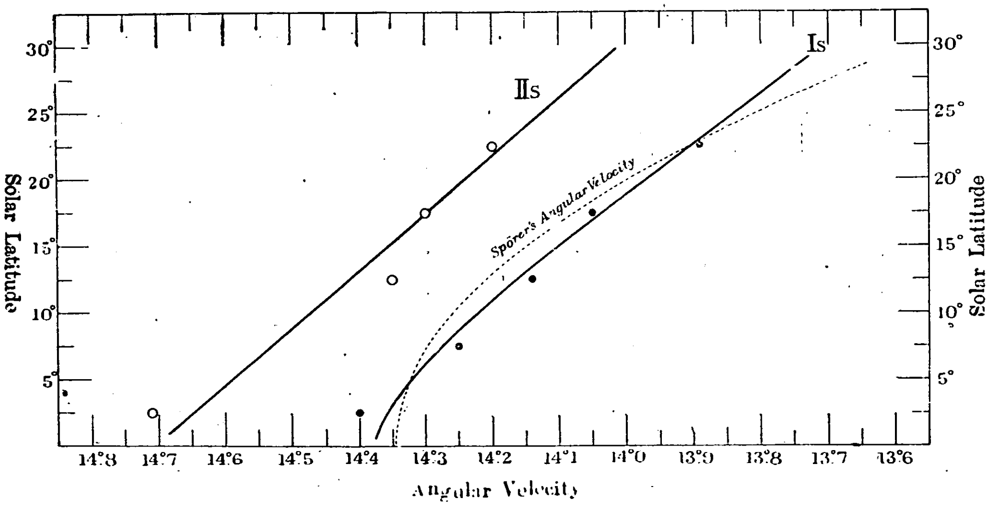
TABLE IX.

Angular Velocity corresponding to Maxmum II.

Heliog. Latitude	Observed Ang. Vel.	Number of Spots in Maximum	Smoothed Ang. Vel. IIs
2.°5	14.°71	1	14.°65
7. 5	?	?	14. 53
12. 5	14. 35	6	14. 42
17. 5	14. 30	7	14. 30
22. 5	14. 20	4	14. 18
27. 5			(14. 07)

Angular Velocities of Maxima I and II for Different Zones of Latitude, deduced from Spörer's Observation.

Fig. 3.



5. I can now compare the results arrived at from these various observations as follows:

TABLE X.

Angular Velocity corresponding to Maximum I.

Heliog. Latitude	Deduced from Greenwich Observations (1879–1901) I_G	Deduced from Spörer's Observations (1861–1878) I_S	Deduced from Carrington's Observations (1853–1861) I_C	Simple Mean I	Mean Ang. Vel. computed from Maunder's, Spörer's, and Carrington's Formulae
2.°5	14.°45	14.°36	(14.°36)	14.°39	14.°40
7.5	14.38	14.28	14.27	14.31	14.35
12.5	14.29	14.17	14.16	14.21	14.25
17.5	14.19	14.03	14.02	14.08	14.13
22.5	14.05	13.90	13.86	13.94	13.98
27.5	13.88	(13.77)	13.67	13.77	13.80

TABLE XI.

Angular Velocity corresponding to Maximum II.

Heliog. Latitude	Deduced from Greenwich Observations (1879–1901) II_G	Deduced from Spörer's Observations (1861–1878) II_S	Deduced from Carrington's Observations (1853–1861) II_C	Simple Mean II	Mean Synodic Rotation Period
2.°5	(14.°72)	14.°65	(14.°81)	14.°73	26.d19
7.5	14.71	14.53	14.67	14.64	26.37
12.5	14.69	14.42	14.51	14.54	26.56
17.5	14.65	14.30	14.34	14.43	26.78
22.5	14.59	14.18	14.13	14.30	27.04
27.5	14.49	(14.07)	13.88	14.15	27.35

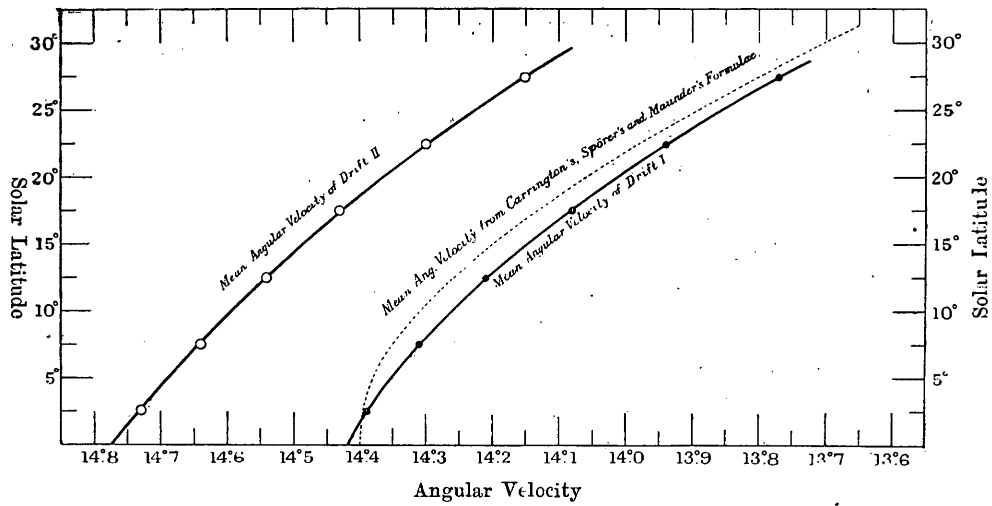
The values in brackets have been obtained by extrapolation.

The mean values corresponding to Maximum I and Maximum II respectively of angular velocities in zones of solar latitude each 5° wide, are shown under the heading 'Simple Mean' of Table

X and Table XI. These velocities are plotted graphically in Fig. 4. The dotted curve in the same figure represents the mean angular velocity, as computed from Maunder's, Spörer's, and Carrington's formulæ. Its numerical values are given in the last column of Table X. For the sake of convenience, the two drifts corresponding to Maximum I and Maximum II will in the remainder of this article be called Drift I and Drift II respectively.

Angular Velocities of Drift I and Drift II.

Fig. 4.



6. On the whole, the angular velocities of Drift I, as represented in Table X, accord very well with the mean values as computed from the formulæ of the three investigators, though there is a small systematic difference of about 0°.04 on the average. In fact, the values obtained by these authors do not exactly represent the angular velocities of spot-groups of maximum occurrence.

7. As to Drift II, it will be noticed from Table XI that there is a rather considerable discrepancy in the values derived from the Greenwich observations and from the other two, although there is no great difference between the results deduced from Spörer's and Carrington's observations. It is evident that the values at latitudes 2°.5 and 27°.5 are of low weights, the first being based

upon only a single determination, and the second upon but two. A comparison of Table X and Table XI shows a faster rate of daily angular motion for Drift II than for Drift I. The mean acceleration of the angular velocities is about 0°.35, within the limits of observation, the corresponding acceleration in the rotation period being 0.^{d}71. For the lower latitudes, the rotation periods of the spots belonging to Drift II agree approximately with the recent spectroscopic results obtained by Messrs. Storey and Wilson.(1) Perhaps certain groups of spots by a proper motion of their own come to the same level as the reversing layer and attain its angular velocity.

As to the peculiar motions of sun-spots Prof. Spörer says: "Die Beobachtungen haben ergeben, dass im östlichen Theile einer Gruppe niemals übergrosse Rotationswinkel vorkommen. Man findet daselbst Verkleinerung der Rotationswinkel, aber selten mit bedeutenden Betrage. Uebergrosse Rotationswinkel kommen vor an der Westgrenze der Gruppen und bei neu entstandenen Flecken." He then mentions 66 cases of great angular velocities during the period 1880-84. This excess comes out to be about 0°.5 on the average. Perhaps such spots and those of short duration may greatly contribute to the existence of Drift II.

8. Since the ratio of the amplitudes of the two drifts depends on the choice of spots by each investigator, it is difficult to find the true ratio from the investigations hitherto made. By examining the tables in this article, it will be found that it is actually different for different investigators. It also varies irregularly in different zones. The mean ratio of the amplitudes of the two drifts is about $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ in Maunder's, Carrington's and Spörer's observations respectively. The researches of Mr. and Mrs. Maunder were made on spots in general. They say: "There has been no selection of spots because they seemed to be steady in motion or regular in shape, no rejection because of unsteadiness or irregularity. The only criterion for the inclusion of a group in the discussion has been that it lasted for six consecutive days". This

(1) Storey and Wilson, Spectroscopic Observations of the Sun's Rotation, etc. Monthly Notices LXXI. p. 674.

fact has led me to assume that the sun-spots belonging to Drift II are about $\frac{1}{2}$ of the whole.

9. If we adopt Faye's empirical formula for solar motion, then our results may be expressed as follows:

$$\xi = 14^\circ.37 - 2^\circ.97 \sin\lambda. \quad \text{for Drift I.}$$
$$\xi = 14^\circ.69 - 2^\circ.65 \sin\lambda. \quad \text{for Drift II.}$$

In order to exhibit how these formulæ satisfy the observations, I give the following table.

TABLE XII.

Comparison of Computed with Observed Angular Velocities.

	Drift I.				Drift II.			
λ	ξ Observed	Weight	ξ Computed	O–C	ξ Observed	Weight	ξ Computed	O–C
2.°5	14.°39	$\frac{2}{3}$	14.°36	+0.°03	14.°73	$\frac{1}{3}$	14.°68	+0.°05
7.5	14.31	1	14.32	− 1	14.64	1	14.64	0
12.5	14.21	1	14.23	− 2	14.54	1	14.57	− 3
17.5	14.08	1	14.11	− 3	14.43	1	14.45	− 2
22.5	13.94	1	13.94	0	14.30	1	14.30	0
27.5	13.77	$\frac{2}{3}$	13.75	+ 2	14.15	$\frac{2}{3}$	14.13	+ 2

It will be noticed that the above formula for Drift I nearly coincides with the expression, $\xi = 14^\circ.37 - 3^\circ.10 \sin^2\lambda$ deduced by Faye.

10. By examining all the spots which lasted for more than six consecutive days, Mr. and Mrs. Maunder (loc. cit. p. 818) have deduced another formula, $\xi = 875'.7 \mp 164' \sin^2\lambda$, which nearly corresponds to the mean values in the last line of Table I. This formula may be analysed as follows:

$\frac{1}{4}(\xi$ of Drift I$) + \frac{3}{4}(\xi$ of Drift II$)$

$$= \tfrac{1}{4}(14^\circ.37 - 2^\circ.97 \sin^2\lambda) + \tfrac{3}{4}(14^\circ69 - 2^\circ.65 \sin^2\lambda)$$
$$= 14^\circ.61 - 2^\circ.73 \sin^2\lambda = 876'.6 - 163'.8 \sin^2\lambda$$

That the last expression practically coincides with their formula shows that the latter is greatly influenced by Drift II.

The mean value of ξ, (column 6 of Table X), as computed from Carrington's, Spörer's, and Maunder's formulæ, is nearly expressed by the formula, $\xi=14^\circ.40-2^\circ.83 \sin^2\lambda$, which may also be analysed as follows:

$$\tfrac{9}{10}(\xi \text{ of Drift I})+\tfrac{1}{10}(\xi \text{ of Drift II})$$
$$=\tfrac{9}{10}(14^\circ.37-2^\circ.97 \sin^2\lambda)+\tfrac{1}{10}(14^\circ.69-2^\circ.65 \sin^2\lambda)$$
$$= 14^\circ.40-2^\circ.94 \sin^2\lambda.$$

11. The present investigation, though cursory, leads me to conclude that there are two apparent drifts in the motions of the sun-spots. The angular velocity of Drift I is represented by

$$\xi=14^\circ.37-2^\circ.97 \sin^2\lambda$$

and that of Drift II by

$$\xi=14^\circ.69-2^\circ.65 \sin^2\lambda,$$

showing a mean rate of about $0^\circ.35$ greater than that indicated by Drift I. The mean ratio of the number of sun-spots in Maximum II to those in Maximum I is 1: 2. This hypothesis of assuming the existence of a secondary drift may be considered a tentative explanation of the phenomena of the distribution of sun-spots with different rotation periods in any particular zone of latitude, and I do not claim that the conclusion I have arrived at do more than approximate to quantitative precision.

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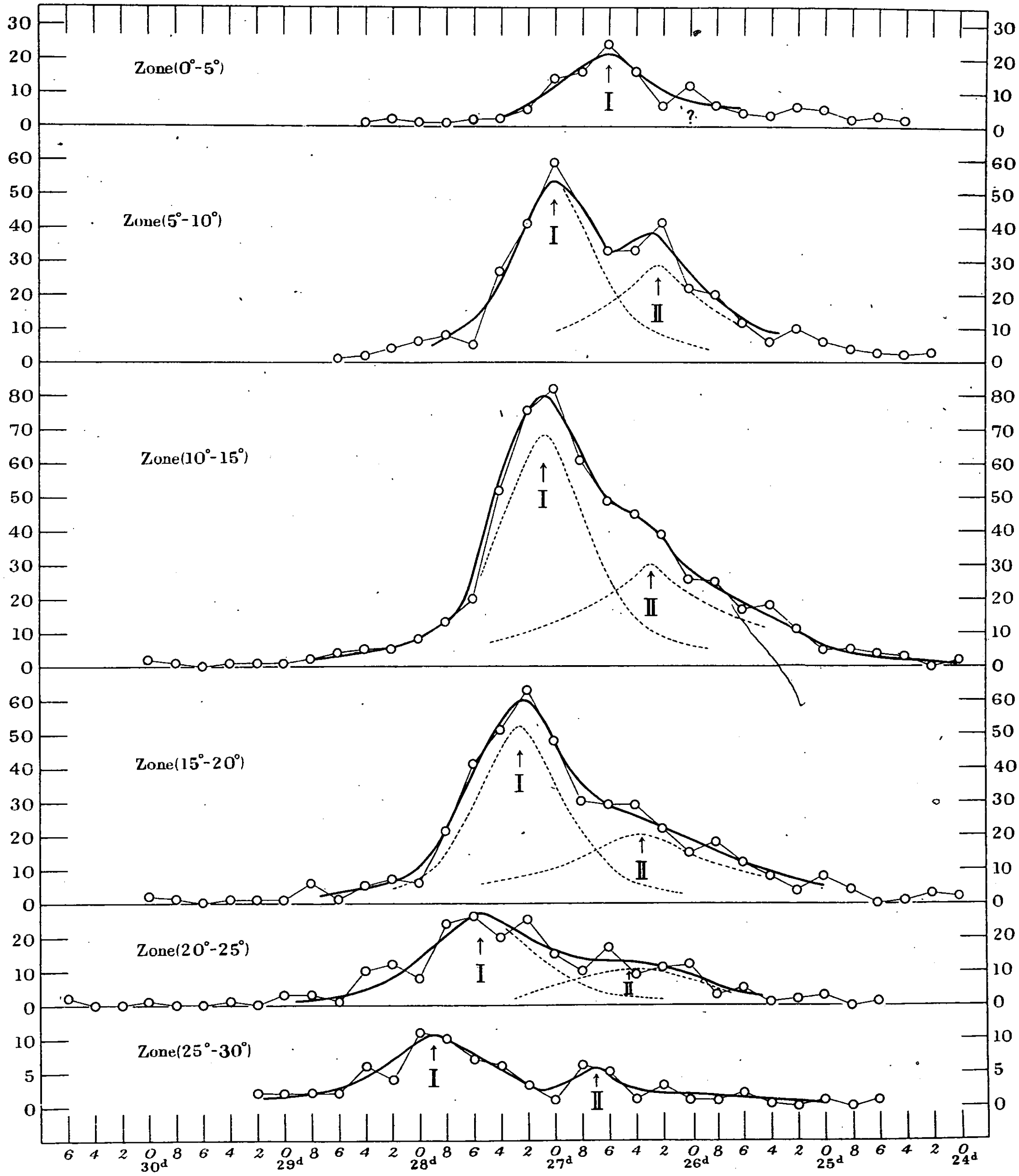
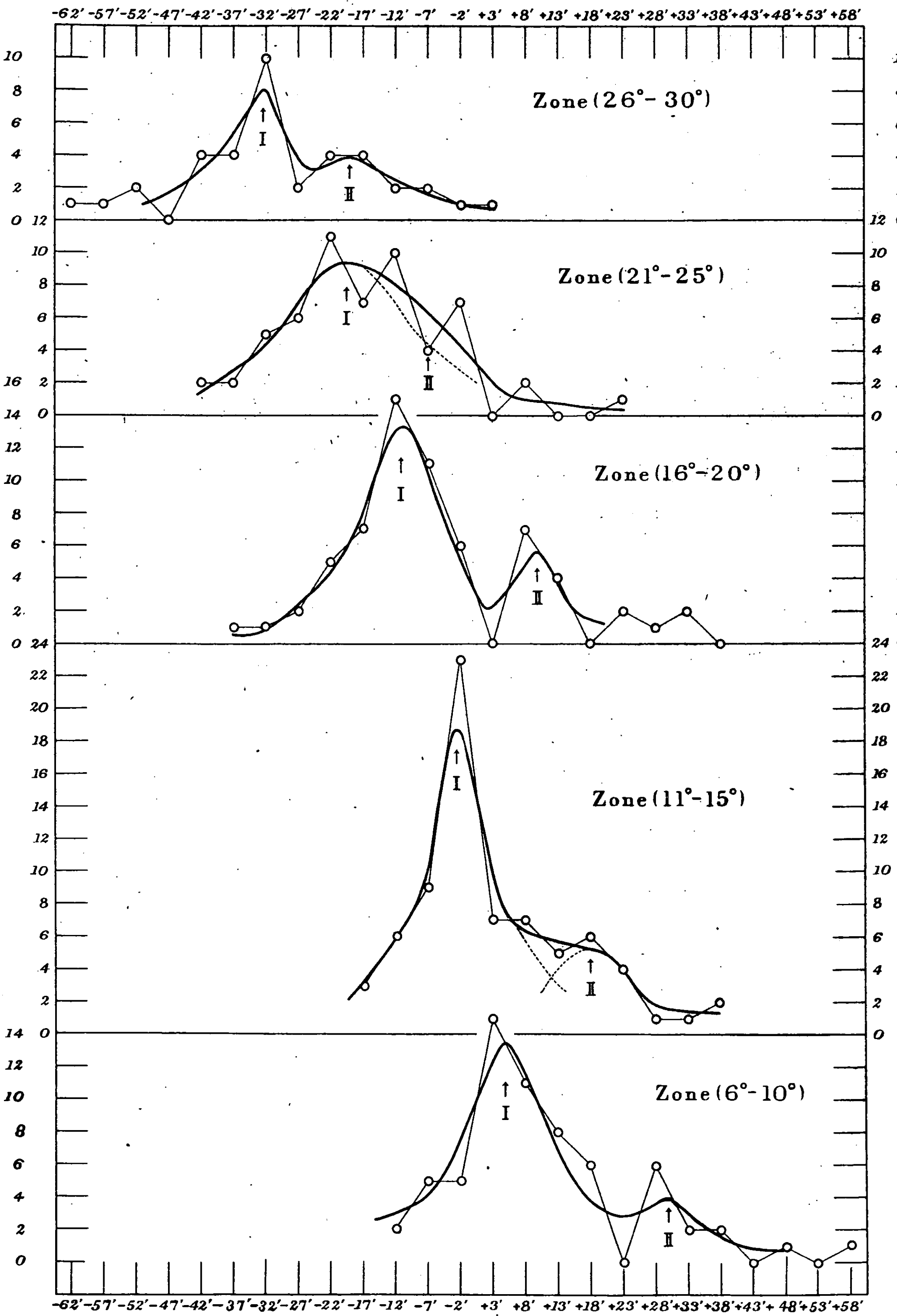
Greenwich Number of Spot-groups with Different Rotation Periods in Zones of Latitude 5° wide.



Carrington's Number of Sun-spots, arranged in Zones of Latitude 5° wide, giving Different Diurnal Motions.



Spörer's Number of Sun-spots with Different Angular Velocities in Zones of Latitude 5° wide.