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Seiches in Some Lakes of Japan.

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With 18 plates.

CONTENTS.

§ 1. Introduction.

The observation of *seiches*, or the oscillatory movements of the entire body of water of a lake, was begun in Japan in 1901 by the Earthquake Investigation Committee at the suggestion of Professor Nagaoka. The present paper gives a summary of the results obtained from these observations. Up to 1901 no one had taken the trouble to make such observations except Burton.* He, in 1891, had noticed that the pebbles on the shore of Hakoné lake were regularly covered and uncovered by the water, the length of the period being 55 seconds. This however, as we shall see later on, is by no means one of the principal oscillations in that lake. It must rather have been a secondary undulation peculiar to the particular point of observation. The names of the lakes, where observations have been made, together with the dates and the names of the observers, are given in the following table.

TABLE 1.

Lake.	Date.	Observer.
Biwa	July 29—Aug. 23, 1901.	Nagaoka, Nakamura, Yoshida.
Hakoné	Aug. 28—Sept. 1, 1901.	Nagaoka, Honda, Kuwaki, Yoshida.
Biwa	Sept. 6—30, 1901.	Honda, Kuwaki, Yoshida.
Hamana	Aug. 20—26, 1902.	Nakamura, Honda, Yoshida, Iwamoto, Inouye.
Kawaguchi	Aug. 29—Sept. 1, 1902.	,, ,, ,, ,, ,,
Yamanaka	Sept. 1—4, 1902.	,, ,, ,, ,, ,,
Tōya	Aug. 18—19, 1905.	Honda.
Chūzenji	Aug. 27—28, 1905.	,,
,,	July 6—8, 1906.	,,

As we shall see presently, in some of the lakes like Hakoné and Biwa, the phenomena was very prominent, while in others

* Trans. of the Seismological Society of Japan, Vol. XVI, 1891.

like Hamana it was too insignificant to enable us to determine accurately the length of the periods of oscillation. Besides the cases of the lakes above mentioned, observations were attempted in Kasumigaura, a body of water covering an area of about 18 square kilometers; but no regular oscillation was found, probably owing to excessive shallowness.

1. Instrument.

Before beginning our work in 1901, we secured a Sarasin's portable limnimeter. A full description of this intrument is given by Ebert in the "Zeitschrift für Instrumentenkunde" 1901, pp. 193-201, and accordingly is omitted here. Thinking it desirable, that observations should be made simultaneously in at least two different places, we constructed for the purpose a limnimeter of a simpler design. (Plate I, Fig. 1). In the accompanying figure 1, A is a hollow triangular brass rod one meter long, which while being allowed to move freely is kept upright by two sets of three rollers R and R′, fixed at the two ends of a cylindrical guide tube D. B is a circular brass rod 106 cm. long and one cm. in diameter. To its lower end a float F made of sheet zinc is clamped by T and its upper end is inserted in the hollow of the rod A, to which it is clamped at a suitable length by T. Attached to A is a pen P, which is directed

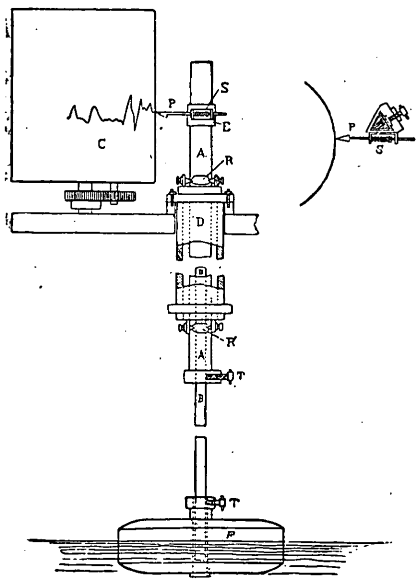


FIG 1.

normally to the face of the recording cylinder C. The position of P can be adjusted by a sliding piece E, and its pressure against the cylinder regulated by a spring S. We shall hereafter designate this limnimeter by the letter "N."

Before beginning our regular work, we endeavored to compare this new limnimeter with Sarasin's at Imazu, one of our stations on lake Biwa; but we soon found that they failed to give concordant results though they were tried in exactly similar conditions i.e. the form and size of the tanks in which the zinc floats swim, and the diameter and the length of the tubes connecting the tank water with that of the lake, were made exactly equal to one another. We found further that the rack and pinion mechanism in the Sarasin limnimeter had a great backlash and we were forced to dispense with this part of the mechanism. When we had modified his instrument by attaching the pen directly to the vertical rod of the swimmer as in our instrument, and also by keeping his recording parts in a vertical position. the records of the two instruments agreed very satisfactorily.

The first series of observations at lake Biwa was made with a Sarasin limnimeter modified as above described, and by a new N limnimeter. The observations at Hakoné lake, and the second series of observations at lake Biwa were made with an N limnimeter, and a Sarasin instrument, which was further modified by entirely dispensing with his recording arragement, for which an ordinary Richard's recording cylinder was substituted. In 1902, four new limnimeters of the N type were constructed and were exclusively used at lakes Yamanaka, Kawaguchi, and Hamana.

In 1905, another portable limnimeter was designed by Honda. We shall distinguish it by the letter "H" (Plate I,

Fig. 2). This new instrument was used at lakes Tōya and Chū-zenji. It consisted of a buoy and a thin wire or string which was attached to the buoy and after passing over a pulley was stretched vertically by means of a counterweight. The buoy was made of sheet zinc and was cylindrical in form, having a diameter of 12 cm. and a height of 10 cm. In order to give the buoy some steadiness, it carried a lead weight on its lower end, or was partially filled with water through a hole made for the purpose and closed by a screw. The pulley had a diameter of 3 cm. and its horizontal axis rested in two agate cups. The counterweight was of lead, and weighed about 300 grams. By means of a pen attached to the wire, the up and down motion of the wire was recorded on a Richard's vertical cylinder which made a complete rotation either once a day or once every two hours. To give steadiness to the pen, and at the same time to diminish the friction as much as possible, the penholder had two horizontal arms, at each end of which a friction wheel was fixed and made to run in V-shaped grooves cut in two vertical guides; and in order to make it easy to adjust the height of the pen, the penholder was attached to the wire in following way. At the point where the penholder was to be attached, the string was divided into two strands, and these two strands were passed through

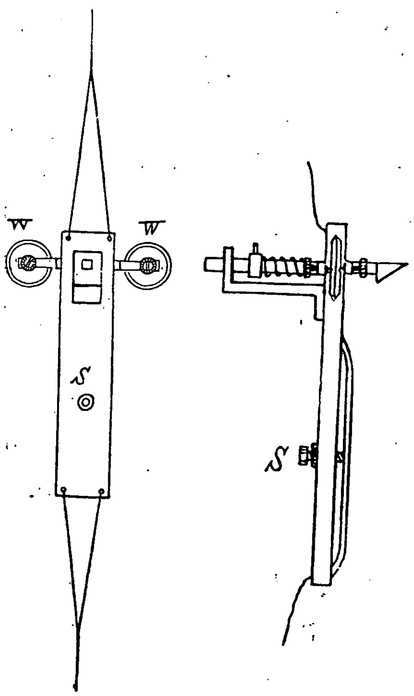


Fig. 2.

holes bored in both sides of the penholder. By this arrangement, the pen could be displaced to any height and fixed there by means of the screw S.

2. Treatment of the records.

It is not an easy task to determine the periods of several component oscillations from the limnograms. Even when there are only two components with different periods, the general aspects of the curve may differ considerably according to circumstances, such as the relative magnitudes of their amplitudes and their relative retardations. The periods of several seiches existing in a given lake do not stand in simple ratios as was formerly thought to be the case. In an interesting paper* entitled "On the hydrodynamical theory of seiches," Chrystal has calculated theoretically the periods and the positions of nodes for seiches in lakes of various forms and has shown that the ratios of the periods of several modes of oscillation may be any whatever. In a rectangular lake of uniform depth, indeed, the periods for uni-nodal, binodal, and trinodal seiches stand in the simple ratios of $1:\frac{1}{2}:\frac{1}{3}\ldots$; but in other lakes they are quite complex, and their ratios may be even incommensurable. The amplitudes and phases of the several components are quite independent of one another. Thus when we have a limnogram before us and wish to determine the periods and phases of the several components, at first we are at a loss how best to proceed. In an expansion of a function in Fourier's series, we know the periods *a priori* and the values of the amplitudes and phases are sought for. Here we know neither the periods, the amplitudes nor the phases. An analytical method of finding the most probable values of the periods of

* Trans. Roy. Soc. Edinburgh XLI. part III, 1905. pp. 599-649.

periodic terms in a given function is not yet known. We are therefore compelled in such cases to use some tentative method. Now when we compare many limnograms of a station with one another and also with those of other stations, we find that it happens very often that as only one kind of oscillation is developed, or as other oscillations, though present, yet have very small amplitudes, the limnograms are extraordinarily simple. Collecting such simple

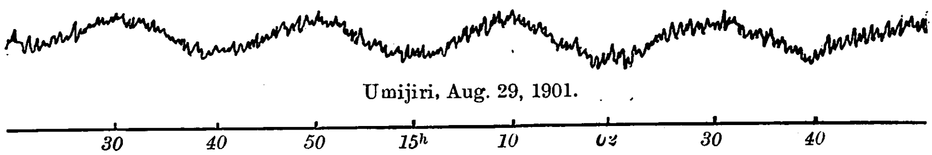


Fig. 3 a.

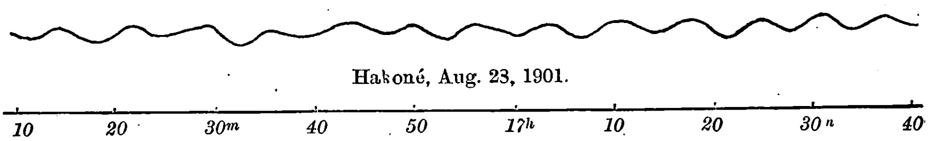


Fig. 3 b.

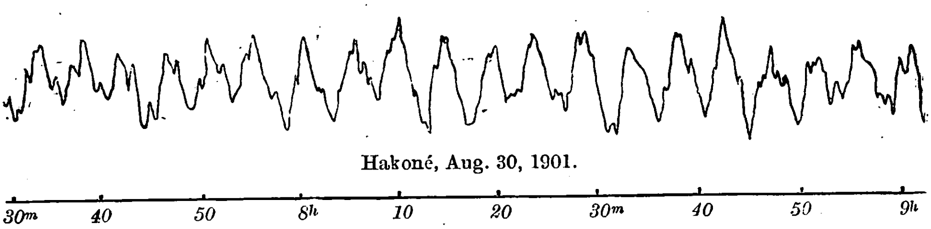


Fig. 3 c.

curves, we can at once deduce pretty accurate values of the several periods. Guided by this knowledge, when we examine other limnograms, where several oscillations are superposed, we can still discern the existence or nonexistence of a certain motion with an approximately known period. An example will make clear what we mean. The accompanying curves are some limnograms taken at Hakoné lake in which *a* is a curve obtained at the Umijiri station with only one well developed oscillation with a period of 15,4 minutes, while Figs. *b* and *c* are curves at the Hakoné station with oscillations of 6,7 and 4,4 minutes, respectively. Now Figs. 5 are curves obtained also at Hakoné. On looking at them we can discern distinctly the existence of the shorter 6,7 minutes period probably accompanied by the longer oscillation of 15,4 minutes.

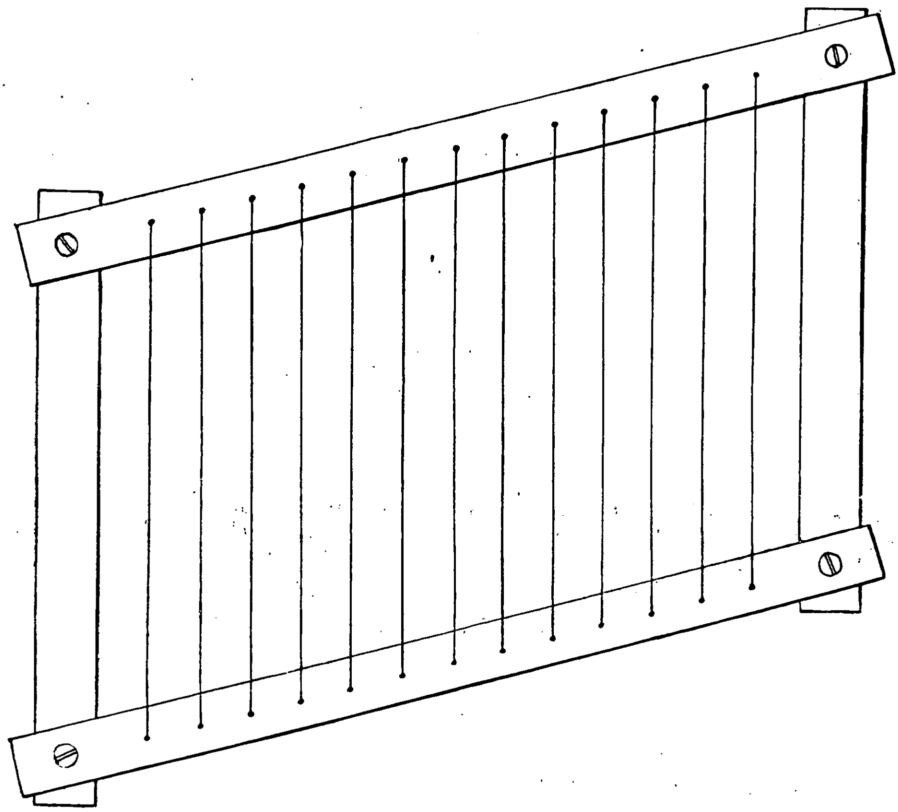


Fig. 4

In order to determine the exact values of the periods of oscillations supposed to be present in a given curve, we made use of a simple device. Four zinc strips about I cm. broad and 30 cm. long, were hinged together at their ends so as to form a deformable parallelogram. In each of the two opposite pieces, a series of equidistant holes was bored, through which a thin thread was passed, as is shown in the accompanying figure, so that we had a number of thread lines which were parallel to one another and the distance between them could be varied at will by deforming the frame work. This frame was put on the limnogram curve in question, and the distance between the lines was so adjusted that the threads best coincided with the maxima and the minima of the sinuous curve. Then by measuring the distance between the threads we could easily deduce the corresponding period from the known rate of the clockwork of the recording cylinder.

Another method was of great help in determining the periods accurately. We shall call this "the method of coincidence." It was simply this. It often happened that a given condition of complex oscillation continued unchanged so that the amplitudes of the component motions did not diminish for a long time, and therefore when we put one limnogram upon another and moved them suitably, we could bring them to coincide closely. Now when the coincidence is perfect, there must be certain integral numbers of component waves during the time, which elapsed between the two curves. These integral numbers can be found from our previous knowledge of the periods. Dividing the time interval above found by these integers we get more reliable values than before. It is needless to say that it is better when possible, to make this interval as long as can be done reasonably. We say reasonably, for when the interval is too long, there will be some uncertainty as to the integer serving as the divisor. The curves Figs. 5, *a* and *b* exemplify the method of coincidence. By placing the curves one upon the other and displacing them horizontally, they will be found to coincide pretty well when the points in equal phases are separated by an interval of 199.8 minutes. Dividing this by 15.4, 6.7, and 4.6, the

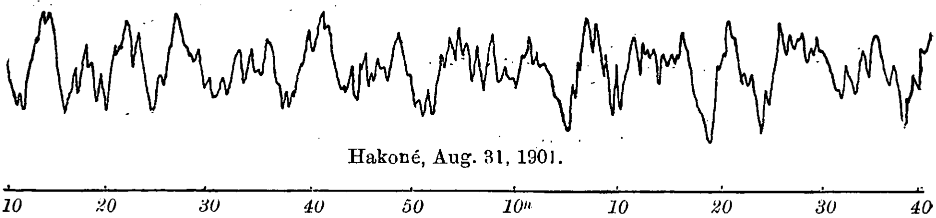


Fig. 5 a.

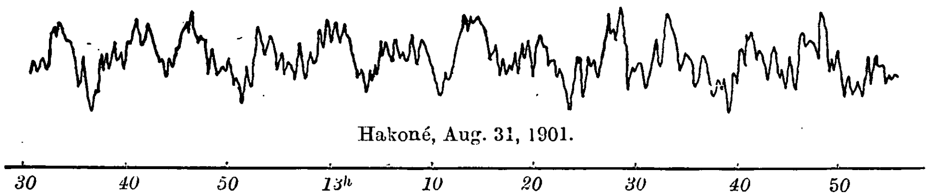


Fig. 5 b,

approximate values of periods, we get the numbers 12.98, 29.8, and 43.4, whence we deduce the periods 15.37, 6.65 and 4.64, minutes.

$$199.8 \div 13 = 15.37.$$

$$\left.\begin{array}{ll} ,, & 31 = 6.43, \\ ,, & 30 = 6.65, \\ ,, & 29 = 6.88. \end{array}\right\}$$

$$\left.\begin{array}{ll} ,, & 42 = 4.75, \\ ,, & 43 = 4.64, \\ ,, & 44 = 4.53. \end{array}\right\}$$

This example shows that the method is adapted for determining longer periods. Of course the actual existence of these periods must be tested in each case. This may be done by means of the framework above mentioned, or by the method which will be explained presently.

The determination of phases of several component oscillations is a difficult task. For this purpose, the several components must be separated from one another. To do this analytically will be too troublesome, if not impracticable. We found the following graphical method, though tentative, to be sufficiently accurate for all practical purposes. It enables us to find the phase and the amplitude of any component. Indeed the method assumes an exact knowledge of the periods. Let T be the period of a motion whose phase we wish to find. We then take from the limnogram a length corresponding to 2^n. T, where n is a certain integer. In order to facilitate the explanation, let us take the particular case of n=3, i. e. take a length corresponding to 8T. We cut this into two equal parts and put one on the other, and draw the mean curve.

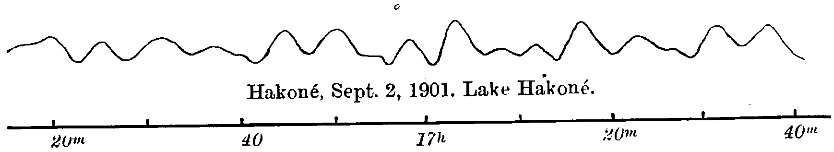


Fig. 6 a.

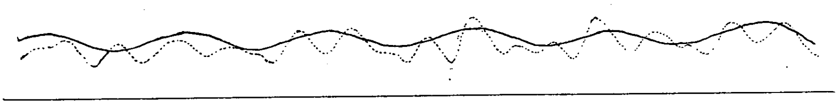


Fig. 6 b.

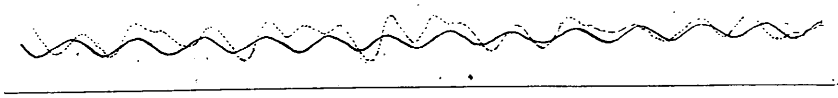
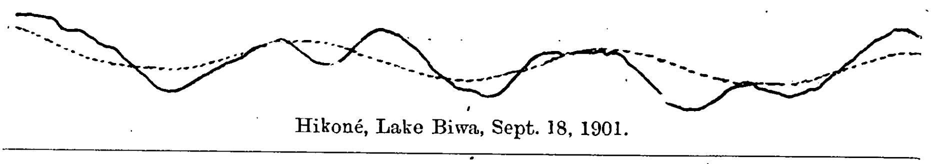


Fig. 6 c.

This mean curve is again halved, and the two halves are superposed and their mean is drawn again. Continuing this process, we get ultimately a mean sinuous curve with a length T. The theory of this method is that, when we consider corresponding points for all these intervals of the length T, the motion with the period T is in the same phase, but other motions are in general in different phases, so that by the process of taking the mean, the motion T only survives, and all other motions destroy themselves. When the intervals taken are large, the result will be a simple sinusoid. Practically we found that eight or sixteen intervals are sufficient for the purpose. The number of intervals need not, of course, be equal to 2^n. T; but the graphical method of drawing the mean curve as above explained is practically more convenient than any analytical method, taking an arbitrary number of intervals. In Fig. 6, the curve 6 *a* is a limnogram obtained at Hakoné, and the curves drawn in full lines in 6 *b* and 6 *c* are the curves obtained by the above method for seiches of periods of 15.4 and 6.7 minutes respectively, by taking for 6 *b* eight, and for 6 *c* sixteen, intervals. The result is simple sinusoids, and we can see their amplitudes



Hikoné, Lake Biwa, Sept. 18, 1901.

Fig. 7 a.
$T = 30.^m3$

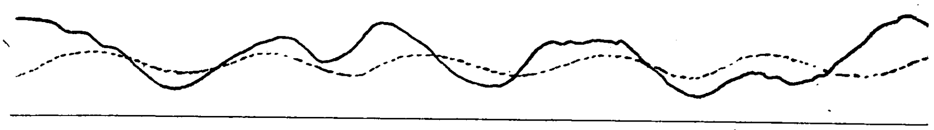


Fig. 7 b.
$T = 16.^m5$

and phases clearly from the figure. The curves in Figs. 7 *a*, 7 *b* are similar one obtained at Hikoné on lake Biwa. This method serves also to verify the existence of any motion in a limnogram, for when the motion in question is not present, the mean curve comes out practically a straight line.

3. Experiments with models.

In order to study several possible modes of oscillation in a given lake and also to determine the periods and the positions of the nodes, we constructed many models, filled them with water, and set them in oscillation. Such experimental study is very instructive, and as we learnt much from it, we shall now devote some space to a description of it.*

The models were made either of wood or of cement. The scale for depth was always made greater than that for length, otherwise the model lake would be too shallow and any motion excited in it would be very much damped and soon die away. To excite an oscillatory motion in the mass of water, it is necessary to give it regularly timed impulses. A method for generating such motion by means of a weighted spiral is described by Chrystal.

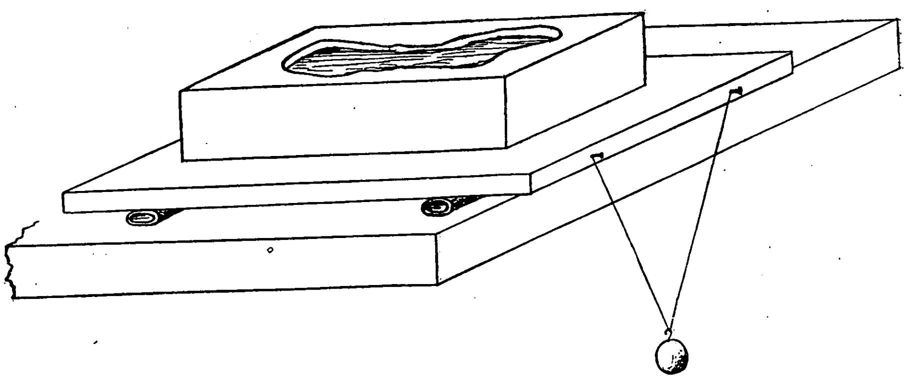


Fig. 8.

But we preferred the method of using a pendulum with a heavy bob. The model was put on a wooden plate placed on a table with two pieces of rubber tubing between the table and the plate to serve as a sort of cushion. To one side of the plate, which

* Honda, Terada, Yoshida and Ishitani.—Journ. of Science College, Tōkyō. Vol. XXIV.

projected a little from the table, two nails were fixed, from which a leaden ball weighing about 2 kilograms was hung. When the ball was made to swing, the impulse was imparted to the water and it was forced to oscillate with the pendulum. The experiment was not so simple however as it appears at first sight. For when the length of the pendulum was so far adjusted that its period came very near to the natural period of the model, and when the water was apparently moving in harmony with the pendulum, it was impossible to say whether the imposed period was the natural period of the model or an enforced period differing slightly from it. One can however decide this point more or less accurately by stopping the ball suddenly and thus removing the force acting on the water; for when the enforced period was equal to the natural period, the movement of the water would go on unchanged and gradually die away, but when the periods did not coincide, the water would soon assume a new phase of oscillation and so come to rest. Thus our experiments with the models were conducted as follows. The natural period was determined approximately by means of a stop watch, when the model was moved by hand and left to itself with no pendulum attached to it. Then the length of the pendulum was adjusted till its period coincided with the approximate period found as above. In general, when any two given oscillating systems are made to move together, their motions are influenced reciprocally, producing an oscillasion with a peried differing from the free periods of the two systems. So here when the model lake and the pendulum are made to move together, the actual period was nearly but not quite equal to the prescribed period. The period was determined afresh, and the motion was attentively watched to see whether the oscillation continued unchanged in its phase and whether the positions of the nodes remained constant; and also, whether, when the pendulum ball was stopped, the water continued to move freely by itself. The length of the pendulum was adjusted till this state was reached, and then the period was determined and the positions of the nodes were noted, In short, trustworthy values of the periods can only be satisfactorily obtained when the water executes *free* oscillations.

As the period of a stationary oscillation of a mass of water is proportional to its length and inversely proportional to the square root of its depth, we can easily deduce the period in a real lake from that of the model.

The determinations of the positions of nodes in the model lake are no less difficult than the determination of the periods. Even when the water is oscillating apparently in the most perfect manner, the nodes shift within a small range. For the determination of nodes, we may make use of either of the properties (1) that at a node the vertical motion is zero, so that when the water is rising on one side of it, it is falling on the other, or (2) that at a node the horizontal motion is greatest. To utilize the first property, we watched the surface of the water near a node and observed the motion of the image of a distant object. The mean positions of points where the image remained motionless were noted and taken as the nodes. To make use of the second property, we darkened the water with ink until it was quite opaque, and then we scattered some aluminium powder uniformly over its surface. The model was then illuminated with strong light, and the grains of the aluminium powder seemed like so many bright stars on the dark back ground. Now when the water was in motion, the powder participated in the motion of the water, and described bright lines corresponding to the amplitude of the oscillations. A photographic camera was set up in such a way that its objective directed downward was just above the model. Exposures were made, the durations of which were nearly equal to half the periods of oscillations, and the motions of the powder were photographed. Plates VII, IX &c show pictures obtained in this way. The curved lines drawn normal to the lines traced by the powder at the places where the horizontal motions are greatest, are the nodal lines. The positions of the nodes thus determined are not necessarily accurate, and first method is always to be preferred for the exact determination of nodes; but this method has the advantage that every detail of the oscillation is very clearly brought to light. For example, at the loops where the horizontal motions are least, the images of the powder ought to come out as points; but when the motion was made too violent, we found that the powder at loops described small circles, showing that then the theory of small oscillations could no longer be applied.

4. Seiches and meteorology.

Generally speaking, the limnogram is a smooth curve on a calm day and is wavy when the lake is disturbed by a shower or a wind. But it often happens that the curve is made very irregular by small indentations superposed on it. In such cases the weather is generally dead calm and there is no breeze to agitate the water into ripples, so that the surface of the lake is as smooth as a mirror. The double amplitude of such small indentation is generally two or three millimeters and the period varies from twenty seconds to one minute. Such motions naturally escape our eye; for even the longest wave, which we can follow with our eye, has a period of only a few seconds, and its amplitude is much greater than that of the motion which causes the indentations in the limnogram. Thus in spite of the apparent calmness of the weather, the limnogram betrays the presence of some disturbance in the lake. It is very interesting to note in such cases that some twenty or thirty minutes after the first appearance of the indentation, the weather begins to change. The wind blows with increasing intensity agitating the water into high waves. Soon a shower, sometimes a thunderstorm, arrives and passes over or in front of us. The indentation in the limnogram is therefore a forerunner of the coming meteorological disturbance. The cause of such agitation of the water may be sought for in the rapid barometric pulsation

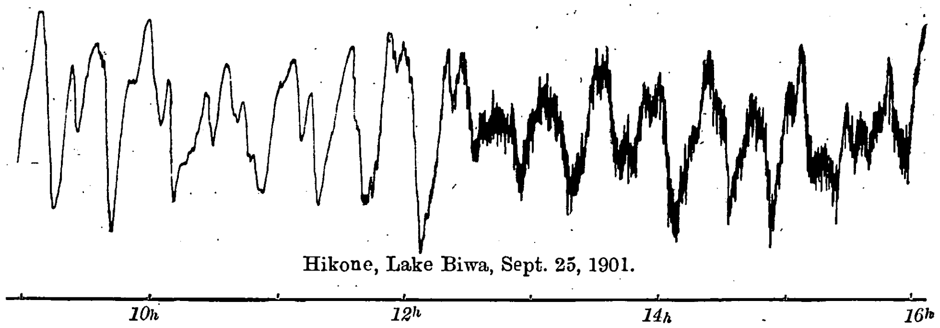


Fig. 9.

at the center of the atmospheric disturbance, which agitates the lake into very irregular motion, and the latter is propagated to us long before the shower itself arrives at the limnimeter. It may happen of course that the shower goes another way and does not come to us at all. Then we have only indentations in the limnogram telling us that a shower passed somewhere over the lake. Fig. 9 shows an example of such indentations. A study of the distribution of barometric pressure and its variations, and particularly a study of the effect of the "Luftwogen" conducted with suitable microbarographs, will throw much light upon this phenomena.

Of the agencies which cause a seiche in a lake, we may mention, after Forel and others, atmospheric electricity, earthquakes, unequal local distribution of atmospheric pressure, strong wind etc.. We had no experience of an earthquake during our observations, but we several times met with pretty heavy thunder storms, and almost always had a very significant seiche after them. This may be due partly to the unequal distribution of atmospheric pressure existing in such cases, but certainly it must be due also to the attraction between the water of the lake and the electrified cloud over it. The water just below the cloud is heaped up and then is let go when the electrification of the cloud is discharged, and as a result the water is set in oscillation. The position of the electrified cloud relative to the lake perhaps determines the mode of oscillation; when it is just above or near a loop of one of the natural modes of motion of the lake, then that particular motion is sure to be excited. Let us now give some examples of such seiches excited after thunderstorms, observed in lake Biwa (see Pl. VI.).

Here we met with many thunderstorms, the first of which occurred on the afternoon of the 13 th. August. There are 14 provincial department offices scattered around the lake, where the conditions of the weather are officially reported once every day. From these reports, we see that the storm was restricted to the southern part of the lake. It began to rain at about 13 h., and continued for about two hours. Though the wind was either light or moderate everywhere blowing mostly in the westerly direction, it caused an anomalous seiche in the lake. The motion was recorded by the limnimeters set at Ōtsu, a station at the south end of the lake, and at Imazu, another station on the northwestern side. The curve traced at Ōtsu is reproduced in

Pl. II. 1,2. The motion there had an extraordinarily long period of 231.2 minutes, though the motion most frequently observed in this lake has a period of about 30 minutes. At this place we heard the sound of distant thunder at about 13 h. 40 m., and at 14 h. 20. m. the shower arrived and then the level of the lake began to rise considerably and afterwards sunk very much. The storm ceased practically at 17 h., and the lake became apparently very calm, but the oscillation continued till the evening of the next day, executing nine oscillations with decreasing amplitudes, and then died away. The readings of high and low water levels from an arbitrary datum line are as follows;

TABLE II.

High water	Low water	Calc. amplitude
148 mm.		78 mm.
	0 mm.	70
122		52
	44	28
97		27
	56	14
79		9
	60	10
78		8
	60	10

From these numbers, the logarithmic decrement was found to be $\lambda=0.1452$, so that the damping factor is $k=1.408$, and the position of the level at equilibrium is 70 mm.. The calculated successive amplitudes are given in the last column of the above table. The curve recorded by the limnimeter at Imazu, where the storm was not observed, shows that there we had from 14 h. the usual indentations on the curve telling us of the presence of a disturbance of the water somewhere, and a seiche of a 30-minute period was excited at 20 h.; but on the whole the motion of the water at Imazu was very irregular but not significant during the time.

On the 17 th. August, another storm passed over the lake. From the reports of the department offices concerning this storm we see that it was restricted to the north part of the lake. Of our limnimeters, only that at Imazu was in action at that time. We heard the rolling of distant thunder at 6 h. 20 m., and at 8 h. 23 m. it began to rain. We had very heavy thunder at 8 h. 36 m., and at 8 h. 42 m.; it ceased to rain at about 9 h.. The sky continued overcast during the rest of the day, and we had no wind during the whole time. Owing to this thunderstorm, a beautiful oscillation was produced with a period of 30 minutes, which as we have stated before, belongs to the seiche predominant in this lake, The motion continued till noon of the next day making about 40 oscillations in all. The first part of the limnograph obtained at Imazu is reproduced in Pl. III. Fig. 1. The record wes unhappily broken between 8 h. 10 m. and 10 h. 40 m. owing to accident to the instrument.

On the afternoon of the next day, i. e. Aug. 18 th., another storm visited the western shore of the northern basin of the lake. The limnimeters were set up on that day at Imazu and Chōmeiji, the latter place lying out of the path of the storm At Imazu, we heard distant thunder first at 14 h., a SW wind began to blow at 14 h. 50 m., and it began to rain at 14 h. 55 m.. After many lightning flashes and low thundering, we had two heavy peals of thunder at 15 h. 6 m. and 15 h. 9 m., and then the storm subsided gradually, and the sky became clear toward evening. As the limnogram at Imazu reproduced in Pl. III. Fig. 2., shows, the level began to rise gradually at 14 h. 30 m. and fell suddenly at 14 h. 52 m., and made many oscillations with a period of 9 minutes, the curve being much indented by shorter irregular oscillations. The 30-minute seiche appeared also at 17 h. 50 m., but it was not well developed. On the whole, this storm caused no significant motion of the water at Imazu. At Chōmeiji, the instrument was set up at 16 h. 50 m. so that the motion of the water just at the time of the thunderstorm was not recorded. From the curve, which is reproduced in Pl. IV, we see that here as in Imazu two motions of 30 m. and 9 m. periods were also excited.

That wind is a cause of the seiche was supposed by many people, though contradicted by some. In Chūzenji lake, it was clearly seen that wind is one of the chief exciting causes of the seiche there. We had such a case also in lake Biwa. The limnogram reproduced in Pl. V. Fig. 2, was obtained at Hikoné on lake Biwa, Sept. 20–21, 1901. On the evening of the 20 th., we had a strong wind accompanied by a light rain on the northern part of the lake, and it caused at Hikoné an anomalous seiche with an amplitude about 9 times larger than that usually appearing there, (Pl. V. Fig. 1). It is worthy of notice that the period of the motion does not change much in either curve.

§ 2. Lake Biwa.

Lake Biwa, the largest lake in our country, is situated in the central part of the main island of Japan, where the land is considerably narrowed by the encroachment of the bay of Suruga on the northwest and of the bay of Isé on the south. It is more than 60 kilometers in length and 16 kilometers broad in its widest part (Pl. VI). Its surface is 86 meters above sea level. The lake has a narrow constriction near Katada (Point No. 11 on the map) which joins the great north basin with shallow southern one of almost negligible size. The lake in fact takes its name from the shape of the *biwa*, a musical instrument somewhat like a guitar, the south basin representing the neck of the biwa. The north basin has two very deep places in it. The deeper one lies off Imazu and has a depth of more than 90 meters, so that the bottom of the lake at this place is below sea level; while the other, about 76 meters deep, lies between Katsuno and Wani. The lake is surrounded on almost all sides by mountains, conspicuous among which are Hiei-zan on the west, and Ibukiyama on the east. The large river Seta flows from its southern extremity, and passing through the cities of Kyōto and Ōsaka, empties into the bay of Ōsaka.

Owing to the lack of necessary equipment, a bathymetric survey of the lake was not carried out at that time, but happily for us Mr. Maeda of the Hikoné meteorological station made it afterwards and published a map, which has been of great service to us. It is reproduced in Plate VI.

Our observations consisted of two series; the first series was begun on the 30 th. of July 1901 and continued to the 23rd of August, this was the first systematic study of seiches ever under-taken in our country. As already stated in the introduction, we had two limnimeters at our disposal, one of the Sarasin type and the other of our own design. We had first to study these instruments, to see how to construct the water tank, and how to connect it with the lake, etc., and to this preliminary work we were obliged to sacrifice the first few days, so that our regular observations must be said to have began on the 7th. of August. The second series extended from the 6 th of September to the last day of that month. The number of stations was 14 in all, for the names and positions of which see the map given in Plate VI.

In the first series of observations we permanently set up at Imazu, a Sarasin portable limnimeter modified however, as has aleady been described, by dispensing with the rack and pinion, and by fixing the recording cylinder vertically. Afterward this latter was replaced by a Richard's cylinder. A portable limnimeter of the "N" type was carried round the lake from one station to another, being set up at each station for from one to three days. In the second series, on the contrary, both instruments were carried about, but the number of stations was limited to five, namely Imazu, Shiotsu, Ōtsu, Nagahama and Hikoné.

We shall now give the results of observations obtained at the several stations.

Chikubushima.

Chikubushima is an island in the northern part of the lake. The observations at this place lasted only 18 hours, and we got the following three different motions. The numbers in the first column are the periods expressed in minutes of time, and those after the sign × are the numbers of waves used in calculating the corresponding periods.

TABLE 3.

Chikubushima.		
minutes.		
61.1	×	5
30.7	×	10
11.0	×	16

As the motions are all very small, the double amplitudes being only a few millimeters, and as the numbers of observations are small, the real existence of these waves is doubtful. But the second motion of 30.7 minutes existed unquestionably.

Chomeiji.

Here the observations were continued for only 13 hours, during which the weather was very calm, and a motion of a 31-minute period was very significant, the double amplitude being more than 3 centimeters. The observed periods are

TABLE 4.

Chōmeiji.		
m.		
32.5	×	3
31.5	×	12
31.0	×	5
9.9	×	9
9.7	×	10
9.4	×	7

We probably have here two motions with periods of 31.8 and 9.7 minutes. For the former, see the curves in Pl IV.

Funaki.

The weather was very windy when we reached Funaki to make the observation, and the lake was so much agitated that we could not recognize any regular oscillation excepting a motion with a period of one minute, which however is decidedly no proper seiche.

Hikoné.

Here we made three series of observations. The first series was continued for a day from the 20th. to the 21st. of August. The periods observed are

TABLE 5.

Hikoné. 1.	
m.	m.
25.8 × 4	14.5 × 16
25.0 × 2	14.1 × 6
24.0 × 16	11.5 × 7
18.8 × 4	10.7 × 12
17.0 × 5	10.0 × 10
16.7 × 6	9.2 × 3
16.4 × 13	9.0 × 8
15.7 × 6	

The second series was continued from the 17th. to the 19th. of September. The weather was generally calm, except at the begining of the observations, when we had a strong N-W wind. The following periods were then observed.

TABLE 6.

Hikoné. II.	
m.	m.
37.8 × 1	20.0 × 2
30.8 × 3	19.9 × 5
30.3 × 3	19.2 × 3
29.9 × 5	18.9 × 7
29.7 × 6	18.8 × 2
24.7 × 4	17.2 × 9
23.6 × 1	16.5 × 5
23.3 × 4	15.4 × 6
22.8 × 17	11.0 × 6
22.6 × 8	8.3 × 8
20.3 × 6	

In the third series of observations extending from the 19th. to the 30th. of September, we observed the following periods.

TABLE 7.

Hikoné. III.	
m.	m.
42.4 × 3	22.0 × 14
32.7 × 7	21.8 × 22
30.6 × 11	21.6 × 11
30.4 × 18	21.5 × 3
30.3 × 20	18.9 × 8
30.2 × 13	17.2 × 13
30.1 × 8	16.8 × 17
29.8 × 7	16.5 × 9

29.4 × 14	16.1 × 7
25.7 × 27	15.9 × 9
25.5 × 12	13.9 × 9
25.3 × 31	12.6 × 10
25.2 × 46	12.3 × 5
25.0 × 39	11.6 × 9
24.9 × 6	10.8 × 14
22.8 × 6	10.0 × 15
22.5 × 14	9.9 × 19
22.3 × 17	8.7 × 5

During this series, we had a heavy storm which threw the lake into a violent motion with an amplitude of about 10 cm., which is about five times larger than its usual value. The period at that time was 16.5 minutes.

Plotting the number of observed waves as ordinates and the corresponding periods as abscissæ on a section paper, we deduce as the most probable seiches at Hikoné the fellowing five motions with periods of 30.3, 25.3, 22.2, 16.5, and 10.3 minutes respectively. For some of the limnograms at this place, see Pl. V Figs. 1, and 2.

Imazu.

Here we made three series of observations. During the first series extending from the 30th. of July to the 7th. of August, we studied and compared our instruments. The second series was continued from the 14th. to the 23rd. of August, and the third series from the 6th. to the 12th. of September. The different motions then observed are tabulated below.

TABLE 8.

Imazu. I.	
m.	m.
63.4 × 9	9.4 × 74
31.0 × 6	9.3 × 25
30.8 × 4	8.7 × 10
30.0 × 42	

TABLE 9.

Imazu. II.		
m.	m.	m.
30.8 × 4	28.5 × 3	8.6 × 21
30.5 × 11	27.1 × 4	8.5 × 28
30.3 × 4	22.0 × 4	8.4 × 4
30.2 × 6	9.8 × 19	8.3 × 30
30.1 × 27	9.6 × 11	8.2 × 53
30.0 × 21	9.5 × 66	8.1 × 15
29.9 × 9	9.4 × 67	7.9 × 5
29.8 × 8	9.3 × 26	6.4 × 12
29.6 × 6	9.2 × 6	5.2 × 14
29.5 × 3	9.0 × 18	5.0 × 6
28.9 × 6	8.8 × 25	4.5 × 10
	8.7 × 41	

TABLE 10.

Imazu. III.		
m.	m.	m.
31.7 × 5	29.5 × 8	17.9 × 7
31.4 × 2	29.3 × 7	17.7 × 7
31.2 × 2	28.9 × 3	17.4 × 6
31.0 × 5	28.8 × 4	10.0 × 13
30.7 × 18	28.7 × 5	9.7 × 27
30.4 × 14	19.1 × 6	9.5 × 18
30.3 × 3	18.7 × 4	9.4 × 8
30.0 × 4	18.1 × 5	8.7 × 8
29.7 × 3	18.0 × 6	8.1 × 10

Plotting these periods on a section paper, we deduce as probable periods of seiches at Imazu, 30.1, 18.1, 9.4 and 8.5 minutes. Of these, the first motion was very conspicuous. Not only did it appear frequently, but it continued sometimes for a long time. Thus early on the morning of the 17th. of August, we had a heavy thunderstorm, which excited this motion very strongly. It appeared first with a double amplitude of about four centimeters and continued till the noon of the next day, making about forty clear oscillations. (Pl. III). This is very good example of a seiche caused by a thunder storm. The presence of the second motion 18.1 was rather doubtful, for it appeared only during the third series of observations under conditions very unfavorable for the determination of a period. The last two motions may, in reality, be one motion, they appeared however pretty often with a double amplitude of about one centimeter, and their presence is quite certain. (Pl. III, Fig. 2).

Kaizu.

The periods observed here during August 3-4 are as follows:

TABLE 11.

Kaizu.		
m.	m.	m.
30.0 × 13	12.3 × 25	11.3 × 49
29.9 × 13	11.7 × 23	

Of these, only the last motion 11.3 was developed to such a magnitude as to enable us to confidently assert its existence.

Katada.

The observations here extended from the 11th. to the. 13th. of August, but the oscillation of the lake was generally very small, which made it difficult to determine the periods. Of the periods

TABLE 12

Katada.		
m.	m.	m.
74.4 × 4	32.1 × 11	10.6 × 13
70.8 × 4	11.9 × 6	10.4 × 21

given here, we may say from a careful examination of the limnograms that there existed at Katada probably twe motions of 32.1 and 10.5 minutes.

Katsuno.

The periods of seiches observed here during August 9–10, are as follows:

TABLE 13.

Katsuno.		
m.	m.	m.
69.5 × 5	11.7 × 9	9.5 × 11
16.5 × 8	11.5 × 4	9.1 × 42

The motions with periods 11.6 and 9.2 were predominating seiches at this place. They were well developed having a double amplitude of about two centimeters.

Nagahama.

The oscillations recorded here during Aug. 21–22 had the periods given in the following table.

TABLE 14.

Nagahama. I.		
m.	m.	m.
35.0 × 2	17.5 × 6	11.9 × 9
34.5 × 3	16.8 × 4	9.5 × 6
32.8 × 2	14.8 × 6	6.7 × 5
18.2 × 7	13.6 × 8	

The most predominant motion among them was 17.5 reaching a double amplitude of 10 cms., but it was much damped and died away very quickly.

During the interval Sept. 6–16. many records were taken with a number of different periods, of which the most prevalent were 31.5, 22.8 and 17.1 having double amplitudes of 2 or 3 cms. The observed values were:

TABLE 15.

Nagahama. II.		
m.	m.	m.
32.1 × 16	24.9 × 16	16.3 × 6
31.8 × 19	24.8 × 20	14.5 × 23
31.5 × 63	23.2 × 4	13.7 × 19
30.8 × 9	22.9 × 21	12.5 × 11
30.6 × 17	22.8 × 35	12.0 × 29
29.5 × 11	22.5 × 12	10.5 × 47
29.4 × 6	17.7 × 14	
28.8 × 14	16.8 × 9	

Okinoshima.

This is a small island in the eastern part of the lake. The periods recorded here during Aug. 19–20 were as follows:

TABLE 16.

Okinoshima.		
m.	m.	m.
10.2 × 11	7.6 × 8	4.9 × 6
9.2 × 9	6.0 × 12	

Of these, the first motion was seen well developed after a thunderstorm. The motion with a period 6.0 was also pretty clear.

Ōtsu.

This place is situated at the south end of the lake, which is very shallow in that part lying south of the neck near Katada. The periods observed during the interval Aug. 13–15 are given in the next table.

TABLE 17.

Ōtsu. I.		
m.	m.	m.
234.5 × 1	202.0 × 2	25.8 × 11
232.5 × 2	72.9 × 1	25.1 × 14
230.0 × 1	71.8 × 10	12.4 × 8
224.5 × 1	65.5 × 3	8.6 × 9
224.2 × 2	32.1 × 7	
209.2 × 2	29.1 × 11	

A motion with a wonderfully long period of about 230 minutes was excited after a very heavy thunder storm. The initial double amplitude was 15 cms., and subsided gradually after making a few oscillations (Pl. II). Other series of observations were made in order to confirm the presence of this slowly oscillating seiche. The clockwork was made to rotate once rapidly to record motions with short periods, and then slowly for motions with long periods. The presence of the motion with the 230-minute period was thus established in ordinary conditions of weather. It is remarkable that this extraordinary motion was nowhere else observed during our stay at the lake. The periods recorded during this second series Sept. 14–19 are given in the next table. The actual presence of some of them is however doubtful owing to their small amplitudes.

TABLE 18.

Ōtsu. II.		
m.	m.	m.
231.3 × 5	26.5 × 3	10.3 × 1
230.1 × 4	25.5 × 6	10.0 × 4
228.0 × 1	24.5 × 1	9.6 × 5
224.0 × 5	23.6 × 22	8.6 × 6
222.3 × 1	22.9 × 3	8.5 × 10
73.7 × 15	21.2 × 2	7.1 × 11
30.1 × 10	20.8 × 10	
26.9 × 4	20.3 × 3	

The probable value of the period for the slowest motion is 231.2 m.

Seta.

Observations extending from the 15th. to the 17th. August gave the following periods, but none of them were quite certain

TABLE 19.

Seta.		
m.	m.	m.
71.1 × 3	32.9 × 9	31.9 × 10
68.9 × 9	32.7 × 10	

Shiotsu.

From two series of observations made during Aug. 8–9 and Sept. 12–13, we have found the periods given below.

TABLE 20.

Shiotsu. I.		
m.	m.	m.
39.2 × 1	30.2 × 3	27.3 × 2
30.5 × 4	29.9 × 3	

TABLE 21.

Shiotsu. II.		
m.	m.	m.
37.7 × 2	30.8 × 3	20.3 × 4
35.4 × 2	30.4 × 7	18.6 × 3
32.5 × 4	28.5 × 6	17.7 × 4
31.7 + 5	20.5 × 3	17.3 × 2

As the most significant motion at Shiotsu we have therefore one, of which the period is 30.4 minutes. This motion appeared very distinctly on the evening of Sept. 12th. and lasted till the morning of the next day.

Wani.

Here we could obtain only two motions 18.3×5 and 8.9×5 during our stay Aug. 10–11.

General results.

If we collect all the values of the periods obtained at the several stations we get about 180 different values; but of course they can not in reality be all different from one another. Plotting the number of times these motions were observed as ordinates and the corresponding periods as abscissae, we get a curve consisting of many maxima much resembling so many probability curves. If we calculate the positions of these maxima, we have the following values, arranged in the order of the number of observed times.

TABLE 22.

(1)	9.4 minutes.	(6)	31.9 minutes.
(2)	30.0	(7)	12.0
(3)	8.5	(8)	10.6
(4)	25.2	(9)	16.7
(5)	22.7	(10)	72.6

These are then the most frequently observed periods in Lake Biwa. We say the most frequently observed periods and not the most frequently occurring periods; because a particular motion is excited only under a condition favorable to it; it may be present one day, but not on the next day. Thus at Hikoné, we had not the 30-minute motion during the first series, but on the contrary only the 10-minute motion was present during it. In order to find out all modes of motion peculiar to a given place, we must sufficiently extend the interval of observation. In many places, our records were taken for only one or two days, and we could not have had the chance of recording some motions proper to the places. We must also bear in mind that a seiche with a few minutes period is easier to detect than a long-period motion executing a complete oscillation in fifty or sixty minutes. Thus, the extraordinary long period of 230 minutes recorded at Ōtsu, of whose real existence we have no doubt, is not at all significant on the curve of frequency just spoken of, for the number of observed times is very small; and therefore we have purposely omitted to include it in the last table in order to do justice to the other motions. In the next table, we give in the first column the periods just determined and in the other columns the periods observed at several stations which are nearly equal to those in the first column the values which had been previously pointed out as certainly existing in a given place being printed in heavy faced type.

TABLE 23.

	Chikubushima.	Chōmeiji.	Hikoné.	Imazu.	Kaizu.	Katada.	Katsuno.	Nagahama.	Okinoshima.	Ōtsu.	Seta.	Shiotsu.	Wani.
9.4		**9.7**	9.6	**9.4**			**9.2**	9.5	9.2	9.8			
30.5	**30.7**	31.8	**30.3**	**30.1**	30.0	**32.1**		**31.5**		30.1	**31.9**	**30.4**	
8.5			8.5	**8.5**						**8.6**			**8.9**
25.2			**25.3**					24.9		**25.4**			
22.7			**22.2**	22.0				**22.8**		22.9			
12.0			12.5		12.0	11.9	**11.6**	12.0		**12.4**			
10.6	**11.0**		**10.3**		**11.3**	**10.5**		10.5					
16.7			**16.5**				16.5	**17.1**	**10.2**				
72.6						74.4				72.9	71.1		

The extraordinary period of 231.2 minutes observed only at Ōtsu must be a motion peculiar to the shallow southern basin.

The reason why it was not observed at Katada and Seta, the two other stations on that part of the lake, must be sought for in the fact that the observations at these places were of too short duration. The next long period is 72.6 minutes, which was observed a few times at Ōtsu, Katada and Seta but was not distinct. This might induce us to conclude that this motion is also a characteristic of the southern basin. Our experiments with a model of the lake, however, showed that it is a longitudinal seiche extending over the whole lake. A model of the lake was made in cement on a scale of 90000:1 for length and of 1085:1 for depth. The gravest longitudinal vibration had a period of $t_1=1.58$ sec., which on reduction gave a period of 71.9 minutes for the actual lake. The photographic picture of this mode of vibration is reproduced in Plate VII, Fig 1., which shows that it has two nodes, one across the north basin from a point near Katsuno to a point lying between Hikoné and Chōmeiji, and the other in the south basin so that water meets and recedes from both sides at a place a little north of the narrow neck, or near our station at Wani. On closing up this narrow neck, and setting the water in oscillation, we found that the uninodal motion of the north basin had a nearly equal period, but the south basin could not be set into vibration. Hence it is proper to say that the motion of the seventy-minute period is the uninodal seiche of the north basin, rather than to say that it is a binodal motion of the lake. The forced vibration in the south basin excited by it has however owing to its small depth a greater amplitude than in the north basin, and this is the reason why this particular motion was observed only in the southern stations of Ōtsu, Katada, and Seta. Again from the bathymetric data furnished by Maeda's map we find that the total volume of the lake is 2.762×10^{16} c. cm., which divided by the total surface area

6.861×10^{12} sq. cm. gives as the mean depth 4.025×10^{3} cm.. The length of the lake measured along the deepest line is $L=4.6 \times 10^{6}$ cm., from which it follows that the period of the uninodal longitudinal seiche is 77.2 minutes.

The third motion has a period of 30.5 minutes, and was observed at stations distributed all over the lake, of which it was the predominating oscillation. A glance at the above table shows, however, that at the stations on the northern part of the lake viz., Chikubushima, Hikoné, Imazu, and Shiotsu, the period seems to be always smaller than that at the southern stations as at Chōmeiji, Katada, and Seta, though there is an exception to this rule at Nagahama. It is possible that we have in reality two different motions with nearly equal periods in the two parts of the lake. The fact, that the great north basin has two deep places in it, one off Imazu and the other near Wani and Katsuno may furnish a plausible basis for this conjecture; but we prefer not to enter further into the discussion of this point. Under the supposition that this motion is a transverse seiche of the lake, let us take as the breadth of the lake 2.0×10^{6} cm., and use the depth before calculated, we then find that the transverse seiche should have a period of about 33.6 minutes. It was found with the model lake that it has a stationary motion with a period of $t_2=0.67$ sec., which on reduction corresponds to a period of 30.5 minutes in the actual lake exactly coinciding with the observed value in question. The photographic record of this mode of vibration is reproduced in Plate VII, Fig 2. Careful examination of the change of level in the model showed us that this motion is not so simple as its period might lead us to suppose. The motion is in fact rather transversal in the north part of the north basin, but it has another small node running across its south part, and still another within the south basin.

The nodal lines are drawn in Fig. 2, and show that when the level is rising at Shiotsu and Nagahama, it is falling at Chōmeiji and Imazu. The limnograms obtained at these stations tend to confirm this fact in the actual case, but owing to the presence of other motions it can not be affirmed with certainty. It is proper to call it a binodal seiche of the north basin exciting a uninodal motion in the south basin, rather than to call it a transverse seiche of the lake.

Though other motions with shorter periods, e. g. 9.4 and 8.5, were very prominent, yet they can not be identified with the actual motion.

§ 3. Hakoné Lake.

1. Result of Observations.

The charming lake of Hakoné, a noted summer resort, is situated at 139° 0′E. and 35° 10′N., its surface being about 720 meters above sea level. Its length is more than 6 kilometers, and it is widest at its southern end, where its breadth is about 2 kilometers. It gets very narrow and deep at a distance one third of the whole length from the northern end, where it is about 600 meters broad. The lake is surrounded on almost all sides by pretty steep mountain ridges, except at its northern end, where there is a small meadow. The most prominent peaks are Komaga-také and Kami-yama on the northeastern side, the former rising 630 meters and the latter 720 meters above the level of the lake; while on its southwestern side Mikuni-yama rises about 380 meters above the lake. It is the commoly accepted opinion among our geologists, that Hakoné lake is an atrio lake formed by the choking up of a part of the atrio of the Hakoné volcano. Since the first formation of the atrio, a condsiderable time must have elapsed before the last eruption took place, that caused the formation of the lake. The people say that, when the water is clear, one can still sometimes see the tops of the upright stems of the conifers that have stood immersed in the water from bygone ages. In several places near the lake, stems of trees have been found buried in the ground.

The observations at this lake extended over eight days, The number of the stations were six in all, at one of which, i. e. at Hakoné, the modified Sarasin limnimeter was set up, while an "N"-limnimeter was carried from one station to another in order to compare the phases at different places with that at Hakoné. We also made soundings with a simple plumb line, at 139 points on the lake from Sept. 1st to 3rd., the positions of the points, where the soundings were made, being determined by means of a compass, furnished with a small telescope. The result of our bathymetric survey is given in Plate VIII. The maximum depth is 41 meters, which must be considered to be very great, when we bear in mind that the total surface area is only 6.5 square kilometers. This maximum depth is at the narrowest part of the lake and at a place lying between the stations at Takogawa and Hyakkwan.

The periods of the seiches in this lake as reduced from the limnograms are tabulated in the following table, in which the number of times when a given period was observed is also given after the sign × following the period in question.

TABLE 24.

Station	T_1	T_2	T_3	T_4	T_5
	m.	m.	m.	m.	m.
Hakoné.	15.38× 40.	6.79×620.	4.53×433.	3.90×164.	——
Motohakoné....	15.37× 10.	6.71× 39.	4.82×225.	——	——
Umijiri....	15.36×164.	——	4.54× 28.	——	3.10×102.
Hyakkwan. ...	15.45× 29.	——	——	——	——
Hotokegasaki.	——	6.57× 70.	——	——	3.12×151.
Takogawa. ...	——	6.75× 73.	——	——	——

Thus we got five periods in all, the weighted means of the above numbers being

$T_1=15.38$ minutes,

$T_2=6.76$,,

$T_3=4.63$,,

$T_4 = 3.90$ Minutes.

$T_5 = 3.11$,,

having the ratios

$$T_1 : T_2 : T_3 : T_4 : T_5 = 1.000 : 0.439 : 0.302 : 0.254 : 0.202.$$

The seiche with the longest period of $T_1 = 15,38$ minutes is the uninodal longitudinal seiche. The fact that we did not observe this period at Hotokegasaki and Takogawa, which were the two middle stations, must be interpreted to mean that these places lie on or near the nodal line for this oscillation. This may be shown also by comparing the phases at the two terminal stations, Hakoné and Umijiri. If we write down the times of high water, from the records of the two stations, in which this particular seiche of 15.38 minutes was excellently developed, we have

TABLE 25.

Hakoné.		Umijiri.		Diff.
h.	m.	h.	m.	
17	18	17	11	7 minutes.
	33		25	8
	48		40	8
18	3		55	8
	19	18	11	8

This shows clearly enough the correctness of the above assertion. The curves in Figs. 10 and 11, moreover, show this without any doubt. The dotted curves in there figures were obtained from limnograms of these stations on the 29th, August 1901, by the method before explained of graphically deducing simple curves for 15.38 and 6.76 minutes by previously magnifying them photographically so that they may have the same scale for time in order to facilitate the comparison. The motion in question was clearly in just opposite phases. Further by making use of

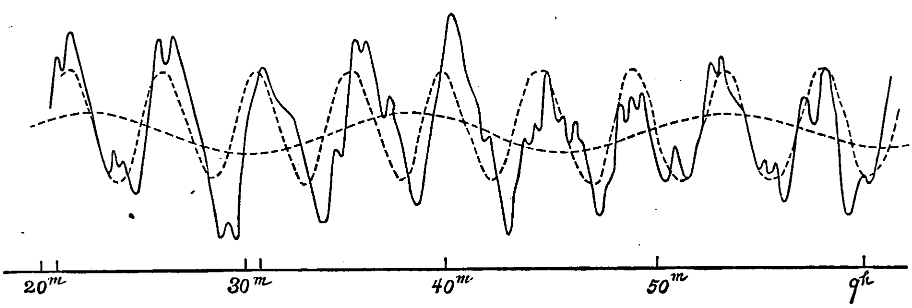


Fig. 10.

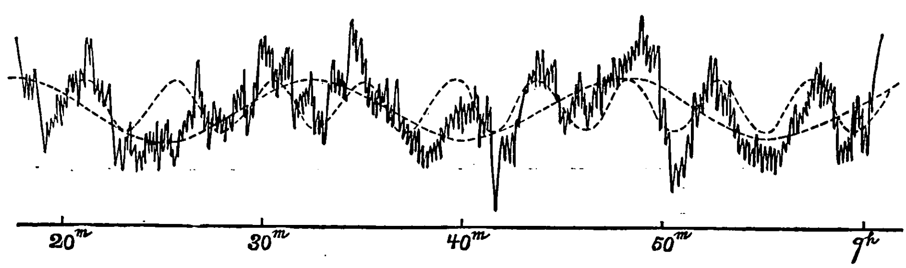


Fig. 11.

bathymetric data and the theoretical formula for the calculation of the period, it will be shown further on, that the period 15.38 belongs to the uninodal seiches.

The second oscillation with a period of $T_2=6.8$ minutes must be the binodal seiche. For, comparing the times of high water for it, we found that Hakoné and Hotokegasaki were in opposite phases, Hakoné and Motohakoné in the same phase, and Hakoné and Takogawa again in opposite phases.

TABLE 26.

Hakoné.		Hotokegasaki.		Diff.
h.	m.	h.	m.	
14	1	14	5	4 minutes.
	8		12	4
	15		18	3
	22		25	3

A pair of dotted curves in Figs. 10 and 11 shows that this motion was in same phase at Umijiri and Hakoné.

Of other oscillations with shorter periods T_3 and T_4, we can not say anything definitely from the limnograms. The experiments conducted with a model of the lake first enabled us to learn some details about them.

The model was made of wood and measured about 33 cm. in length. Three different modes of *free* vibration were found with periods of

$t_1=1.672$ sec.,

$t_2=0.693$,

$t_3=0.439$,

which are in the ratios

$t_1:t_2:t_3=1.000:0.415:0.263$.

The photographs of these three different oscillations are given in Pl. IX., which shows very clearly that the first belongs to the uninodal, and the second to the binodal oscillation. Comparing the ratios $t_1:t_2:t_3$ with the ratios of the periods in the actual lake given in p.43, we see at once that T_1 and T_2 correpond respectively to t_1 and t_2, being the uninodal and the binodal seiches; but of the other shorter oscillations, there remains some uncertainty. We must decide what t_3 is. Plate IX shows that this third oscillation t_3 is quite complex, one of the nodal lines, that lying nearest to the wider end of the lake, being so much curved that we can equally well say that this oscillation has three or four nodes. Indeed, in this motion, we see a place where the figures made by the grains of aluminum powder are points, and this place lies on a loop for the vertical motion; but in reality this is not the case. The figures made by the powder are points, not because they are executing vertical motions like those placed on loops, but because they are actually at rest. We see that a curved nodal line touches the shore at that place, or rather that two nodal lines start in two different directions from that point, which is in fact on a node. We prefer to call the oscillation t_3, not a trinodal, but a quadrinodal seiche, though we were unable to observe in the model any unambiguous trinodal seiche. Now our question is, what oscillation in the actual lake corresponds to t_3? If we take 15,38 minutes as the value of t_1 in the actual oscillation, and calculate t_2 and t_3 by assuming the ratios for $t_1:t_2:t_3$ given above, we get 6.38 minutes for t_2 and 4.06 minutes for t_3, and t_3 seems to correspond rather to T_4 than to T_3. Again if we calculate the actual periods corresponding to t_1, t_2 and t_3 by multiplying them by the scale for length and dividing by the square root of the scale for depth, we have 17.6, 7.3, and 4.6 minutes. Here t_3 is exactly equal to the observed period T_3 and they seem to correspond to each other, but at the same time we see that the values deduced from t_1 and t_2 much exceed T_1 and T_2. If we, however, bear in mind that T_3 was observed at Hakoné, Motohakoné and Umijiri but not at Hyakkwan, Hotokegasaki and Takogawa, while T_4 was observed only at Hakoné, and moreover if we carefully study plate IX we are forced to say that t_3 belongs to T_3 and consequently to assert that the seiche with a period of 4.63 minutes is the quadrinodal seiche in Hakoné lake. We are also inclined to say that T_4 and T_5 are minor seiches developing respectively at the Hakoné and Umijiri ends.

The positions of nodes as determined by studying several motions of the model lake are as follows. For the uninodal seiche, it lies at 0.56 of the total length of the lake from southern Hakoné end; while those for the binodal are at 0.33 and 0.80 from the same end. The nodal lines for the third mode of oscillation are too curved to admit of defining their positions in this manner.

2. Comparison with theories.

Several formulae have been given by different authors such as Mérian, Du Boys et al., for the calculation of the period. For a rectangular lake of a uniform depth h and of a length L, the period T of the uninodal seiches is given by

$$T=\frac{2L}{\sqrt{gh}}$$

When we wish to apply this formula to the case of a lake with a variable depth and breadth, we must take for h its mean value but this gives only an approximate value or values, for we can estimate this mean depth in several different ways. According to Du Boys,* when the lake is symmetrical with respect to its median line, the period for the uninodal seiches is given by

$$T=\frac{2}{\sqrt{g}}\int_0^L\frac{dx}{\sqrt{h}},$$

* Essai théoretique sur les seiches. Archives d. sciences Phys. et Nat. Genève XXV. 1891, p. 628.

integrated along the median line, where h is the depth at a point x. Prof. Nagaoka* has shown that this formula can be applied to a lake whose median line is not straight, provided its curvature is not too great, the integration being performed along the curved median line. In order to see how far the values of T calculated from these formulæ will agree with the observed value, we made the following calculations.

(1) A curved line is drawn along the middle line of the lake and its length L is found to be equal to 6570 meters. The total volume and the total surface area of the lake are measured with a planimeter, and the mean depth h_m is obtained by dividing the total volume by the surface area.

$$h_m = 2668^{cm.}$$
$$\sqrt{h_m} = 51.65$$
$$T = 13.51.^{m.}$$

(2) The curved median line is divided into 64 equidistant segments, and transverse sections are made at each of these segments and plotted on a section paper. We thus have 63 sections together with two end sections of zero area. The areas S of these sections are determined by a planimeter and are divided by the corresponding breadth b of the free surface and the result is taken as the mean depth h for these sections. The values of b in meters, of S in square meters, and of h in meters, together with those of $\sqrt{h}$ and $\frac{1}{\sqrt{h}}$ are given in Table 27.

---

* Proc. Tokyo Phys. Math. Soc, Vol. I. 1902, p. 126.

TABLE 27.

No.	b	S	h	$\sqrt{h}$	$\frac{1}{\sqrt{h}}$
1.	510.	1900.	3.72	1.931	0.5179
2	66	44	6.67	2.583	3871
3	66	68	10.30	3.209	3116
4	48	54	11.25	3.354	2981
5	49	56	11.43	3.380	2958
6	155	242	15.62	3.952	2530
7	170	318	18.71	4.325	2312
8	170	396	23.30	4.827	2072
9	150	384	25.60	5.059	1977
10	196	480	24.50	4.949	2021
11	190	552	29.05	5.389	1856
12	178	544	30.56	5.528	1809
13	168	576	34.30	5.856	1708
14	157	512	32.62	5.711	1751
15	143	482	33.70	5.805	1723
16	137	436	31.80	5.639	1773
17	134	470	35.10	5.924	1688
18	142	476	33.53	5.790	1727
19	148	494	33.40	5.779	1730
20	147	520	35.40	5.949	1681
21	145	490	33.80	5.813	1720
22	124	394	31.77	5.636	1774
23	111	376	33.90	5.822	1718
24	120	412	34.35	5.860	1706
25	126	396	31.35	5.599	1786
26	123	396	32.20	5.674	1762
27	132	382	28.95	5.380	1859
28	132	408	30.90	5.558	1799
29	112	366	32.70	5.718	1749

30	1040.	36200.	34.95	5.911	0.1692
31	95	322	33.90	5.822	1718
32	96	278	28.96	5.381	1858
33	102	258	25.30	5.029	1988
34	97	246	25.35	5.034	1986
35	74	178	24.05	4.904	2039
36	52	114	21.93	4.682	2136
37	49	108	22.05	4.695	2130
38	50	140	28.00	5.291	1890
39	46	126	27.40	5.234	1910
40	53	150	28.30	5.319	1880
41	58	168	28.95	5.380	1859
42	48	144	30.00	5.477	1826
43	40	116	29.30	5.412	1848
44	47	90	19.15	4.376	2285
45	62	90	14.52	3.810	2645
46	70	122	17.44	4.176	2395
47	76	160	21.05	4.588	2179
48	84	160	19.05	4.364	2291
49	76	166	21.95	4.685	2134
50	63	164	26.45	5.142	1945
51	64	152	23.75	4.873	2052
52	65	152	22.75	4.769	2097
53	72	176	24.45	4.944	2023
54	86	214	24.90	4.989	2004
55	114	270	23.70	4.868	2054
56	126	348	27.62	5.255	1903
57	128	338	26.40	5.138	1946
58	110	284	25.82	5.081	1968
59	92	212	23.05	4.801	2083
60	82	142	17.75	4.213	2374
61	68	126	18.54	4.305	2323
62	60	68	11.33	3.366	2971
63	40	24	6.00	2.449	4083

When the mean depth is calculated with

$$h_m = \frac{\Sigma h}{63},$$

we find that

$$h_m = 2499.3\ cm.,$$

so that

$$\sqrt{h_m} = 49.99,$$
$$T = 14.02^m$$

(3) When h is plotted on section paper and h_m is determined by using a planimeter, it is found that

$$h_m = 2414.7\ cm.,$$
$$\sqrt{h_m} = 49.14,$$
$$T = 14.29^m.$$

(4) When the mean value of $\sqrt{h}$ is calculated with

$$(\sqrt{h})_m = \frac{\Sigma(\sqrt{h})}{63},$$

we find that

$$(\sqrt{h})_m = 49.32,$$
$$T = 14.18^m.$$

(5) When $\sqrt{h}$ is plotted, and its mean value is determined with a planimeter, we have

$$(\sqrt{h})_m = 48.60$$
$$T = 14.31^m.$$

(6) When the mean value of the reciprocal of $\sqrt{h}$ is calculated with

$$\left(\frac{1}{\sqrt{h}}\right)_m = \frac{\Sigma \frac{1}{\sqrt{h}}}{63},$$

we obtain

$$\left(\frac{1}{\sqrt{h}}\right)_m = 0{,}02140,$$

$$T \quad = 14.97^m.$$

(7) Let l be the distance between any two consecutive sections and let h' and h'' be the mean depths at these two sections. The mean value of $\sqrt{h}$ between these sections is

$$(\sqrt{h})_m = \frac{1}{l}\int_0^l \sqrt{h} \cdot dx.$$

Let us now assume that the depth h at any place x between these two sections is given by

$$h = h' + \frac{h''-h'}{l} \cdot x,$$

then we have

$$(\sqrt{h}) = \frac{2}{3}\,\frac{h''^{\frac{3}{2}} - h'^{\frac{3}{2}}}{h''-h'}$$

Calculating T by the formula

$$T = \frac{2l}{\sqrt{g}} \cdot \frac{3}{2}\left\{\frac{h_1-h_0}{h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}} + \frac{h_2-h_1}{h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}} + \ldots\ldots\right\}$$

where $l = \frac{L}{64}$ and h_0 h_1 h_2......are the mean depths at several sections, we have

$$T = 15.75^m.$$

(8) Under the same supposition that $h = h' + \frac{h''-h'}{l} \cdot x$, we see that

$$\int_0^l \frac{dx}{\sqrt{h}} = \frac{2l}{\sqrt{h'} + \sqrt{h''}}$$

and calculating T with Du Boys's formula in the following form,

$$T=\frac{2}{\sqrt{g}}\int_{0}^{L}\frac{dx}{\sqrt{g}}=\frac{2}{\sqrt{g}}\int_{0}^{l}\frac{dx}{\sqrt{h}}+\frac{2}{\sqrt{g}}\int_{l}^{2l}\frac{dx}{\sqrt{h}}+\ldots\ldots$$

$$=\frac{4l}{\sqrt{g}}\left\{\frac{1}{\sqrt{h_0}+\sqrt{h_1}}+\frac{1}{\sqrt{h_1}+\sqrt{h_2}}+\ldots\ldots\right\}$$

we find that $T=16.21$ minutes.

Tabulating the results obtained before, we have

TABLE 28.

No.	T.
	m
1	13.51
2	14.02
3	14.29
4	14.18
5	14.31
6	14.97
7	15.75
8	16.21

This shows without any doubt that the observed period of 15.38 minutes must belong to the longitudinal uninodal seiche. When we bear in mind the great variations both in the depth and in the breadth of the lake, it is rather wonderful that the calculated periods differ so little among themselves. We see from the above calculations that if we merely wish to determine which of the observed periods belongs to the uninodal seiche we may use any of the methods given above. Of the calculated periods the seventh value comes nearest to the one observed, though the mode of calculation is not theoretically correct. The last agrees best with the theoretical formula of Du Boys in its mode of calculation, and ought to give the best value. It seems somewhat remarkable that the calculated value 16.2 is the only value that exceeds the observed value noticeably, other modes of calculation giving generally too small values.

If we adopt Du Boys's formula, the positions of the nodes may be determined by the following simple consideration. If the node of the uninodal seiche is situated at $x=\lambda$, it is evident that it must satisfy the condition that

$$\int_0^\lambda \frac{dx}{\sqrt{h}} = \int_\lambda^L \frac{dx}{\sqrt{h}}.$$

Using the data given in Table 27, we find that the node of the uninodal seiche is situated at 0.48 of the whole length of the lake measured from the southern, Hakoné end. This is not in close agreement with the result obtained in the experiment with the model. The positions of the nodes for other motions may be obtained in a similar way.

In a paper entitled "Notes on seiches", Mr. T. Terada,* comparing seiches with the motion of the air column in an organ pipe, has calculated the correction to be applied to the length l (L in our notation) of a lake, when the breadth B (our b) and the area S of the transversal section are not constant; he has also calculated the correction due to the lateral motion of the water; and has shown that

$$\Delta l = \frac{1}{2}\int_0^l cos\frac{2\pi x}{l}\left(\frac{\Delta B}{B_0}+\frac{\Delta S}{S_0}\right)dx + \frac{1}{4}\int_0^l\left(1-cos\frac{2\pi x}{l}\right)\left(\frac{dB}{dx}\right)^2 \cdot dx,$$

where the breadth and the sectional area at point x are put as

* Proc. Tōkyō Math. Phys. Soc. Vol. III. p. 174.

$$B=B_0+\Delta B$$
$$S=S_0+\Delta S$$

respectively. The period is to be calculated by the formula hitherto used, which is in the new notation

$$T=2l\sqrt{\frac{B_0}{gS_0}}.$$

He has also shown a method of determining the positions of nodes. The results arrived at by him in the case of Hakoné lake using our data given in the Table 27, are

(a) The calculated periods are 15.50 minutes for the uninodal seiche and 6.87 minutes for the binodal seiche.

(b) The node for the uninodal seiche is situated at 0.572 of the total length of the lake measured along the median line from the Southern, Hakoné end; while those for the binodal lie at 0.347 and 0.759 of the total length from the same end.

These are in close agreement with our results above given.

Mr. D. Ishitani* has studied the effect of slight variations in the breadths and the areas of transverse sections of a lake upon the periods of its seiches, and has deduced a formula analogous to that of Mr. Terada. With his formula, he calculated, at our request, the period of the uninodal seiche and the position of the node taking as the path of integration l the middle stream line determined by the experiment on the model above described, and the transverse sections drawn perpendicular to it. The results of his calculations are:

(a) The period of the uninodal seiche is 14.1 minutes.

* Ishitani. Proc. Tōkyō Math. Phys. Soc. Vol. III. p. 170.

(b) The position of the node for the same is situated at 2.9 kilometers from the Umijiri end, i. e. 0.558 of the total length of the lake from the southern, Hakoné end.

Chrystal and Wedderburn have calculated the periods and the positions of the nodes of the Scottish Lochs, Earn and Treig, from bathymetric data according to the theory and formula given by Chrystal in the paper above cited. They found that Chrystal's formula agreed with the observations very satisfactorily, but Du Boys's formula gave values considerably in excess of the observed periods. The following small table gives the periods of the uninodal seiches in minutes as obtained by them.

TABLE 29.

	Observed	Chrystal	Du Boys
Loch Earn	14.55	14.50	17.82
Loch Treig	9.18	9.14	10.20

There must be some reason for this peculiarity, but we are not able to give a satisfactory explanation.

In the following, we shall apply Chrystal's theory to Hakoné lake, so it may not be superfluous here to give here a brief outline of his theory. When the breadth and the form of the transverse section of a lake vary as well as the depth, provided these variations are not too abrupt, he shows that it can be submitted to calculation by introducing two variables, σ and v. σ is the product of the area A (S in our notation) of the transverse section by the breadth b of this section at the surface, while v is the area of the surface of the lake between the trace on the surface of the transverse section corresponding to σ, and any other similar line chosen for reference. Then according to Chrystal to study the oscillation of the lake, it may be looked upon as a lake with a straight median line, uniform breadth, and rectangular cross section, its longitudinal section being the curve with v and σ as abscissa and ordinate of any point of it respectively. This curve is called the *normal curve* of the lake. Thus v is the distance measured along the median line of the reduced lake, and σ is the depth at the point v.

We have calculated the normal curve of Hakoné lake, and found that it is not very simple in form, having one shallow and two

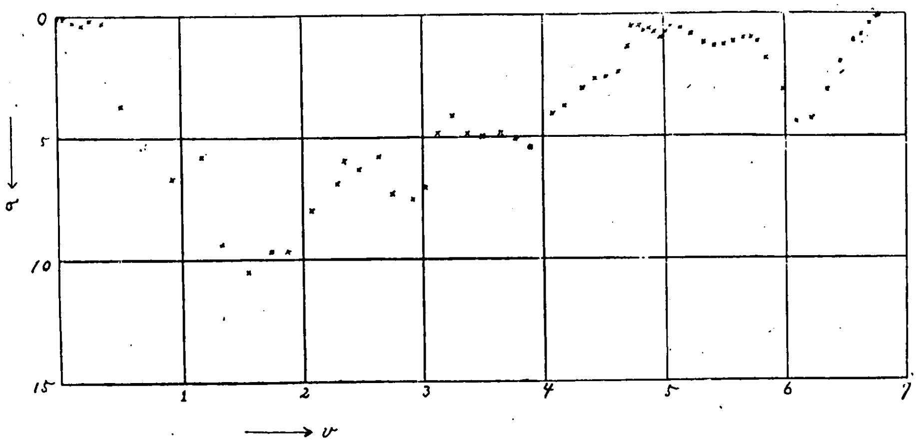


Fig. 12.

maximum depths. Table 30 gives the values of σ and v for the normal curve corresponding to the 63 sections before mentioned. The former is calculated from the data given in Table 27 by multiplying b and S together, and the latter is obtained by means of a planimeter. The unit of length is a centimeter. Fig. 12. shows this normal curve; the shallow in the middle being caused by the great constriction, though at that point it is very deep in the actual lake. As this curve is too complex, we may take as the first approximation for the normal curve four inclined straight lines as shown in Fig. 13. As this is not treated in Chrystal's paper, we

TABLE 30.

No.	σ	v
1	10×10^{11}	0.52×10^{9}
2	29	1.44
3	45	2.16
4	26	2.72
5	27	3.92
6	375	5.20
7	541	7.00
8	673	9.28
9	576	11.56
10	941	13.40
11	1049	15.52
12	968	17.32
13	968	18.84
14	804	20.56
15	689	22.96
16	597	23.52
17	630	24.92
18	576	26.36
19	731	27.52
20	764	29.12
21	711	30.28
22	489	31.44
23	417	32.48
24	494	33.84
25	501	35.04
26	487	36.52
27	509	37.88
28	539	39.04
29	410	40.76
30	375	41.88

31	306	43.12
32	269	44.16
33	263	45.20
34	239	46.28
35	132	47.04
36	53	47.24
37	53	47.92
38	70	48.32
39	58	48.72
40	80	49.16
41	97	49.64
42	69	50.16
43	46	50.56
44	42	50.80
45	56	51.36
46	85	52.28
47	122	53.36
48	134	54.16
49	125	55.08
50	103	55.84
51	97	56.48
52	96	57.24
53	127	57.80
54	184	58.52
55	308	59.60
56	438	60.88
57	432	62.08
58	312	63.48
59	195	64.64
60	114	65.64
61	86	66.28
62	41	67.00
63	10	67.56
64	0	67.76

shall try to work out his theory as applied to this case. Now he shows that in a rectilinear lake where the depth at a point x is given by $h\left(1-\frac{x}{a}\right)$, the horizontal and the vertical displace-

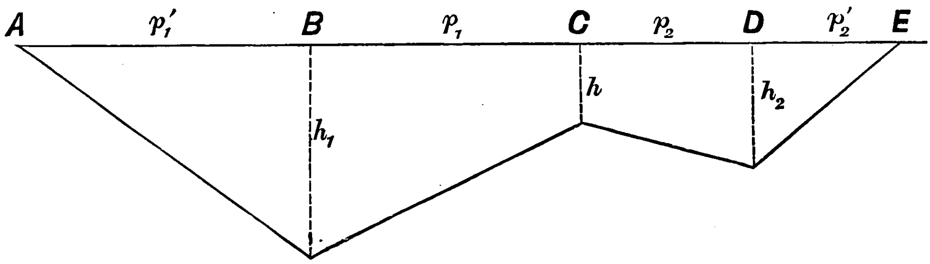


Fig. 13.

ments ξ and ζ of a particle at the free surface will be given by

$$\xi w=\left\{AJ_{1}(w)+BY_{1}(w)\right\}\sin n(t-\tau),$$

$$\zeta=\frac{2a}{h}\left\{AJ_{0}(w)+BY_{0}(w)\right\}\sin n(t-\tau),$$

where

$$n=\frac{2\pi}{T},$$

$$w=\frac{2na}{\sqrt{gh}}\sqrt{1-\frac{x}{a}},$$

A, and B being arbitrary constants, and $J_m(w)$ and $Y_m(w)$ being the Bessel and the Neumann functions of the order m. We shall apply this formula to the normal curve of Hakoné lake.

Let h be the least depth at C, and h_1, h_2 the greatest depths at B and D. Let $AB=p_1'$, $BC=p_1$, $CD=p_2$, $DE=p_2'$; and let the law of depth be $h_1\left(1-\frac{x}{a'_1}\right)$ for AB, $h_1\left(1-\frac{x}{a_1}\right)$ for BC, where x is measured both ways from B and $h_2\left(1-\frac{x}{a_2}\right)$ for CD, $h_2\left(1-\frac{x}{a'_2}\right)$ for

DE, where x is measured both ways from D. Then the depths at A, C, E require

$$p_1'=a_1'$$
$$h=h_1\left(1-\frac{p_1}{a_1}\right)=h_2\left(1-\frac{p_2}{a_2}\right)$$
$$p_2'=a_2',$$

which determine a_1', a_1, a_2, a_2' completely. The displacements for the segment AB are

$$\xi=\frac{1}{w_1'}\left\{A_1'J_1(w_1')+B_1'Y_1(w_1')\right\}\sin n(t-\tau_1')$$
$$\zeta=\frac{2a_1'}{h_1}\left\{A_1'J_0(w_1')+B_1'Y_0(w_1')\right\}\sin n(t-\tau_1')$$
$$w_1'=\frac{2na_1'}{\sqrt{gh_1}}\sqrt{1-\frac{x}{a_1'}}.$$

The boundary condition at A, or at $w_1'=0$, is that $\xi=0$ always

$$\therefore\quad A_1'J_1(0)+B_1'Y_1(0)=0$$

or

$$\frac{B_1'}{A_1'}=-\frac{J_1(0)}{Y_1(0)},$$

but as $\mathrm{Lim}_{w=0}\dfrac{J_1(w)}{Y_1(w)}=0$, we have $B_1'=0$. Therefore for points lying between AB, we have

$$\left.\begin{aligned}\xi&=\frac{A_1'}{w_1'}J_1(w_1')\sin n(t-\tau_1')\\ \zeta&=\frac{2a_1'}{h_1}A_1'J_0(w_1')\sin n(t-\tau_1')\end{aligned}\right\}(1)$$

At B, where

$$w_1'=\frac{2na_1'}{\sqrt{gh_1}}=n\alpha_1' \text{ say,}$$

$$\left.\begin{aligned} \xi &= A_1'\frac{1}{na_1'}J_1(na_1')\sin n(t-\tau_1'), \\ \zeta &= A_1'\frac{2a_1'}{h_1}J_0(na_1')\sin n(t-\tau_1'). \end{aligned}\right\} \quad (2)$$

Similarly for the segment BC, we have

$$\xi = \frac{1}{w_1}\left\{A_1J_1(w_1)+B_1Y_1(w_1)\right\}\sin n(t-\tau_1),$$

$$\zeta = \frac{2a_1}{h_1}\left\{A_1J_0(w_1)+B_1Y_0(w_1)\right\}\sin n(t-\tau_1),$$

$$w_1 = \frac{2na_1}{\sqrt{gh_1}}\sqrt{1-\frac{x}{a_1}}.$$

Putting $w_1=\frac{2na_1}{\sqrt{gh_1}}=na_1$ in these equations, we get the displacements at the point B, and equating them to the values above obtained in (2), we have

$$\tau_1'=\tau_1=\tau \text{ say},$$

$$\frac{1}{a_1}\left\{A_1J_1(na_1)+B_1Y_1(na_1)\right\}=\frac{A_1'}{a_1'}J_1(na_1'),$$

$$a_1\left\{A_1J_0(na_1)+B_1Y_0(na_1)\right\}=a_1'A_1'J_0(na_1').$$

Hence it follows that

$$A_1=\frac{\lambda_1}{\Delta_1}A_1',$$

$$B_1=\frac{\mu_1}{\Delta_1}A_1',$$

where

$$\Delta_1=J_1(na_1)Y_0(na_1)-J_0(na_1)Y_1(na_1),$$

$$\lambda_1=\frac{a_1}{a_1'}J_1(na_1)Y_0(na_1')-\frac{a_1'}{a_1}J_0(na_1')Y_1(na_1),$$

$$\mu_1=\frac{a_1'}{a_1}J_1(na_1)J_0(na_1')-\frac{a_1}{a_1'}J_1(na_1')J_0(na_1).$$

For the segment BC, therefore, we have

$$\left.\begin{aligned} \xi &= \frac{A_1'}{w_1 \Delta_1}\left\{\lambda_1 J_1(w_1) + \mu_1 Y_1(w_1)\right\} \sin n(t-\tau) \\ \zeta &= \frac{2a_1 A_1'}{h_1 \Delta_1}\left\{\lambda_1 J_0(w_1) + \mu_1 Y_0(w_1)\right\} \sin n(t-\tau). \end{aligned}\right\} \quad (3)$$

Putting $w_1 = \dfrac{2na_1}{\sqrt{gh_1}}\sqrt{1-\dfrac{p_1}{a_1}} = n\beta_1$ in these equations, we get the displacements for the point C; they are

$$\left.\begin{aligned} \xi &= \frac{A_1'}{n\beta_1 \Delta_1}\left\{\lambda_1 J_1(n\beta_1) + \mu_1 Y_1(n\beta_1)\right\} \sin n(t-\tau) \\ \zeta &= \frac{2a_1 A_1'}{h_1 \Delta_1}\left\{\lambda_1 J_0(n\beta_1) + \mu_1 Y_0(n\beta_1)\right\} \sin n(t-\tau) \end{aligned}\right\} \quad (4)$$

Proceeding in precisely the same way from the other end of the lake, and equating the values of ξ and ζ at C, we must have

$$\frac{A_1'}{\beta_1 \Delta_1}\left\{\lambda_1 J_1(n\beta_1) + \mu_1 Y_1(n\beta_1)\right\} = \frac{A_2'}{\beta_2 \Delta_2}\left\{\lambda_2 J_1(n\beta_2) + \mu_2 Y_1(n\beta)\right\}$$

$$\frac{a_1 A_1'}{h_1 \Delta_1}\left\{\lambda_1 J_0(n\beta_1) + \mu_1 Y_0(n\beta_1)\right\} = \frac{a_2 A_2'}{h_2 \Delta_2}\left\{\lambda_2 J_0(n\beta_2) + \mu_2 Y_0(n\beta_2)\right\}$$

Eliminating A_1' and A_2', we get the equation for the period, which is

$$\frac{a_1\beta_1 h_2}{a_2\beta_2 h_1} - \frac{\{\lambda_1 J_1(n\beta_1) + \mu_1 Y_1(n\beta_1)\}\{\lambda_2 J_0(n\beta_2) + \mu_2 Y_0(n\beta_2)\}}{\{\lambda_1 J_0(n\beta_1) + \mu_1 Y_0(n\beta_1)\}\{\lambda_2 J_1(n\beta_2) + \mu_2 Y_1(n\beta_2)\}} = 0 \quad (5)$$

Solving this equation for n gives us the period

$$T = \frac{2\pi}{n}. \quad (6)$$

The positions of nodes corresponding to this period will be found by putting $\zeta = 0$ in (1) or (3) or the similar equations for the segments CD and DE, according as we are looking for nodes in the segments AB, BC or CD, DE.

To apply this result to the case of Hakoné, we must proceed as Chrystal and Wedderburn did in the case of Lochs Treig and

Earn, namely instead of trying to solve the equation (5) for n directly, we calculate the left hand side member for some approximate values of n, which we know from our approximate knowledge of T, and then find by interpolation that value of n which satisfies the equation (5).

From the numbers given in table we deduce,

$h_1 = 9.94 \times 10^{13}$,

$h_2 = 4.36$,

$h = 0.38$,

and

$p_1' = 1.644 \times 10^{10}$	$a_1' = 1.644 \times 10^{10}$
$p_1 = 3.728$	$a_1 = 3.877$
$p_2 = 0.848$	$a_2 = 0.929$
$p_2' = 0.596$	$a_2' = 0.596$

so that

$\alpha_1' = 105.3$	$\beta_1 = 48.7$
$\alpha = 248.3$	$\beta_2 = 26.6$
$\alpha_2 = 89.8$	
$\alpha_2' = 57.6$	

The result of our calculations from these data, we are obliged to confess, was not at all satisfactory. While the actually observed value is 15.38 minutes, our calculations gave 22.47 minutes. The reason of this great discrepancy is rather difficult to ascertain, inasmuch as Chrystal and Wedderburn found a very good confirmation of the theory in the cases of Lochs Earn and Treig. The reason must be sought partly at least in the fact that the lakes which they had chosen, satisfy the conditions assumed in the theory almost ideally, while in the present case it is quite otherwise. Chrystal assumed in his theory that there is no component of flow transverse to the average length of the lake.

This assumption is not satisfied in the Hakoné lake owing to the great constriction in the middle, and the experiment with the model shows this point very clearly. (See Pl. IX, Fig 1.) But this is not of course sufficient to explain the great discrepancy of our calculations. The assumption that the normal curve consists of four inclined straight lines must be responsible for it.

In our numerical calculations, we used the tables for J_0 and J_1 given in the appendix of the treatise on Bessel functions by Gray and Mathews, and the small tables for Y_0 and Y_1 of Smith given in the Messenger of Mathematics vol. 26. As the observed period is 15.38 minutes, we took first T=800 and 1000 seconds, hoping to get residuals of opposite signs in the equation (5). The result was however that the residuals were both positive. Hence we tried T=1200 seconds, and found it to be still positive, and proceeded to T=1400 seconds when we got a negative residual. From this we obtained

$$T=1350^{sec.}=22.47^{m.}$$

We have said above that the great discrepancy between the observed and the calculated periods may have been due to the assumption that the normal curve consists of four inclined straight lines. In order to examine this point, we applied the method of calculation adopted under (8) according to Du Boys's formula to the reduced lake given in Table 30, considering it as a lake of uniform breadth but of variable depth σ. According to (8), the required period is

$$T=\frac{4}{\sqrt{g}}\sum\frac{l}{\sqrt{\sigma'}+\sqrt{\sigma''}}$$

where l is the distance between two consecutive transverse sections, with depths σ' and σ''. In the former case, l was constant, but in the present case it is variable, being the difference of two consecutive values of ν in Table 30. The result of the calculation was that

$$T=17.76 \text{ minutes.}$$

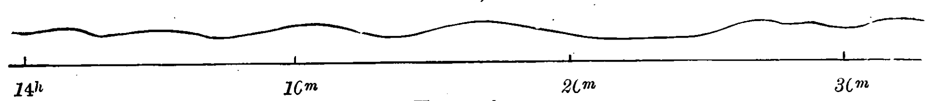
This agrees decidedly better with the observed value than the former value calculated on the assumption of the four straight inclined beds.

§ 4. Yamanaka Lake.

Yamanaka lake lies on the northeastern side of Mt. Fuji at 138°52′ E. and 35°25′ N., and belongs to the chain of lakes that encircles the famous volcanic cone. Its length is about 5 kilometers, its maximum width 2 kilometers, and its surface is about 980 meters above sea level. It is very shallow, the maximum depth, as measured by us, being only 15 meters.

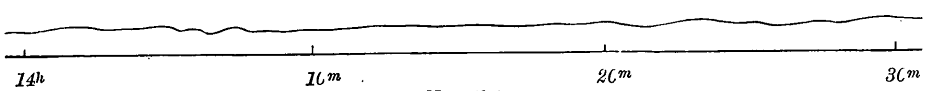
LAKE YAMANAKA.

Sept. 2, 1902.



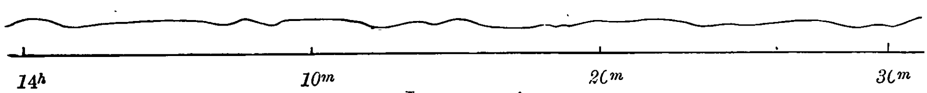
Yamanaka.

Fig 14 a.



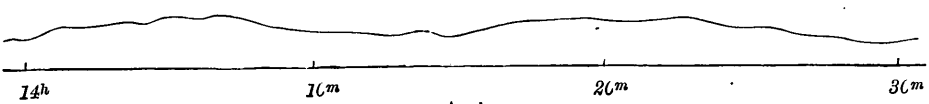
Nagaiké.

Fig. 14 b.



Ipponyanagi.

Fig. 14 c.



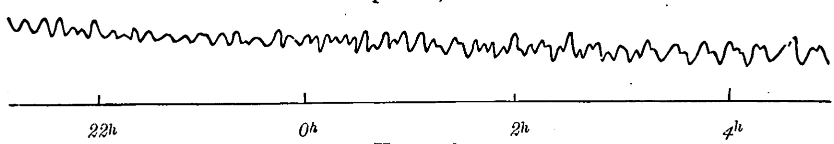
Arai.

Fig. 14 d.

Four limnimeters of the "N" type were set up at the four stations Yamanaka, Nagaiké, Arai and Ipponyanagi, and simultaneous records extending over three days were taken at them. At the same time, we occupied ourselves with taking soundings of the lake at different points, the result of which is given in Plate X.

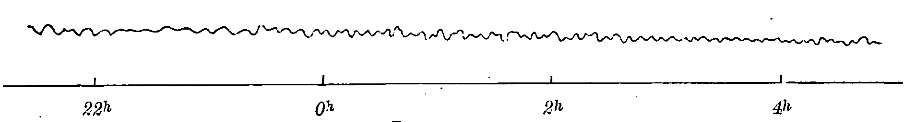
Figs. 14 and 15, give specimens of the curves obtained at the several stations at two different speeds of the clockwork, and as the curves are drawn on the natural scale, they show how the amplitudes were

Sept. 3-4, 1902.



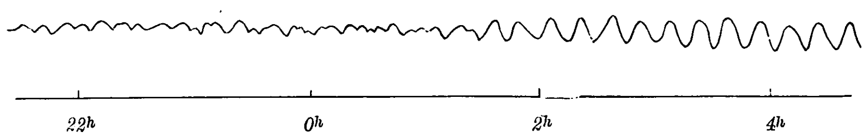
Yamanaka.

Fig. 15 a.



Ipponyanagi.

Fig. 15 b.



Arai.

Fig. 15 c.

everywhere very small, and how difficult it was to determine the periods of the seiches and compare the phases at the several stations. The periods observed are of three kinds:

TABLE 32.

Yamanaka.	15.60 × 5	10.58 × 1	5.50 × 15
Nagaiké.	——	10.90 × 1	5.56 × 1
Arai.	15.63 × 6	——	5.30 × 5
Ipponyanagi.	——	10.40 × 2	5.66 × 1

In this table, the periods are expressed in minutes, followed by the number of times they were observed. The weighted means give us, as the periods of seiches in the lake of Yamanaka,

$T_1 = 15.61$ minutes,

$T_2 = 10.57$,,

$T_3 = 5.46$,,

which are in the ratios of

$$1.00:0.68:0.35.$$

T_1 was observed at the two end stations, and as an examination of the records shows, the two stations were in opposite phases, which indicates that the motion T_1 must be the uninodal longitudinal seiche. T_2 was observed at Ipponyauagi, Nagaiké, and Yamanaka, though only a few times in all. The amplitudes were too small to enable us to compare the phases at these stations and to draw an exact conclusion from them. It is however, without doubt, the binodal longitudinal seiche, as will be seen further on.

The total surface area A and the total volume V of the lake are calculated from the data furnished by our soundings, and found to be

$A = 6.74 \times 10^{10}$ sq. cm.,

$V = 5.53 \times 10^{13}$ c. cm.,

which give us as the mean depth

$h = 820$ cm.

The length and the width measured along the deepest points are 5.42×10^5 and 2.04×10^5 cm. respectively, from which we deduce that the period of the uninodal longitudinal seiche is 20.1 minutes and that the ratio of the periods of the uninodal longitudinal and the transversal seiches is 1.00:0.38. Though the value, 20.1 minutes, very much exceeds the observed value, $T_1 = 15.61$ minutes, yet as the ratio 1.00:0.38 comes so near the ratio $T_1 : T_3$ above obtained, it is very probable that the motion T_3 is the uninodal transversal seiche.

The experiment conducted with a model in cement gave three different motions with the following periods

$t_1=0.90$ sec.,

$t_2=0.50$.

$t_3=0.39$.

Their modes of oscillation were photographed and are reproduced in Plate XI. The existence of other motions could not be determined with certainty, as the periods for them were too small. The ratios between these three periods are

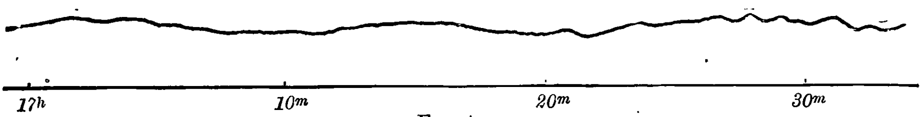
$$t_1:t_2:t_3=1.00:0.67:0.43$$

From the period $t_1=0.90$ sec. of the uninodal oscillation of the model, we deduce a period of 15.9 minutes as that of the actual lake. Again as the ratio $t_1:t_2$ is nearly equal to the ratio $T_1:T_2$, we may safely conclude that $T_1=15.61$ minutes is the uninodal longitudinal seiche, and the $T_2=10.57$ minutes is the binodal longitudinal seiche of the lake. With respect to the third motion, the correpondence of T_3 and t_3 is not so good as in the other motions. The photograph of t_3 shows that it has a nodal line of nearly circular form, with its concavity turned toward the station of Nagaiké, and another nodal line running across the narrow neck in the northeastern end. The motion is therefore mainly transversal, and we may describe t_3 as the transversal oscillation with a secondary synchronous motion in the neck. If we assume that T_3 corresponds to t_3, then the observed motion with a period of 5.46 minutes is the uninodal transversal seiche of Yamanaka lake.

§ 5. Kawaguchi Lake.

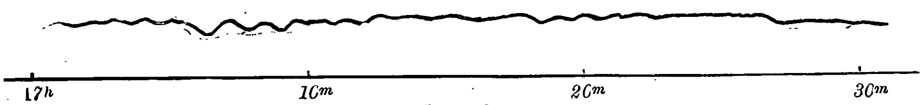
Kawaguchi lake lies to the north of Mount Fuji at 138°45′ E and 35°30′ N. Its length is about 5 kilometers and its width is from 0.5 to 1.5 kilometers, its surface being about 820 meters above sea level (Plate XII). The beautiful island of Unoshima is

Aug. 30, 1902.



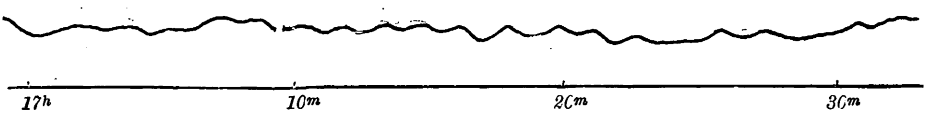
Funatsu.

Fig. 16 a



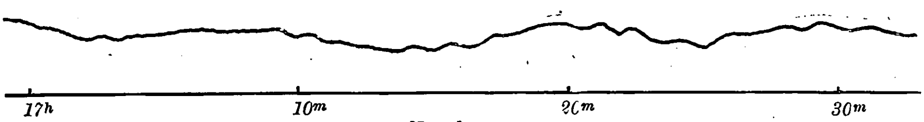
Nagasaki.

Fig. 16 b.



Unoshima.

Fig. 16 c.

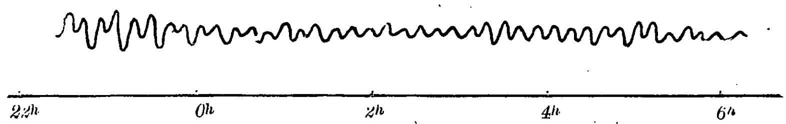


Nagahama.

Fig. 16 d.

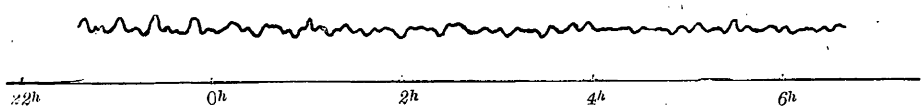
near the narrowest portion of the lake, being a part of a ridge running parallel to the meridian and dividing the lake into two almost independent parts. The deepest point of the lake is just west of this ridge where the depth is 19 meters. Four limnimeters of the "N" type were set up at Funatsu, Nagasaki, Unoshima, and Nagahama, and simultaneous records were taken for four

Aug. 31–Sept. 1, 1902.



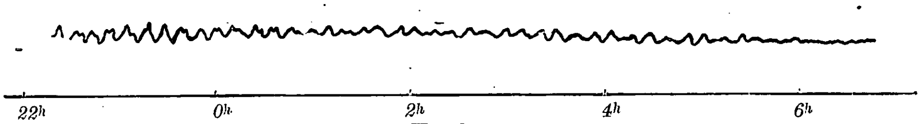
Funatsu.

Fig. 17 a.



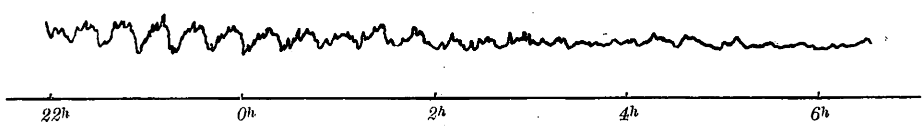
Nagasaki.

Fig. 17 b.



Unoshima.

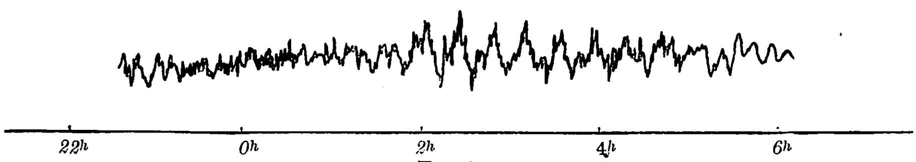
Fig. 17 c.



Nagahama.

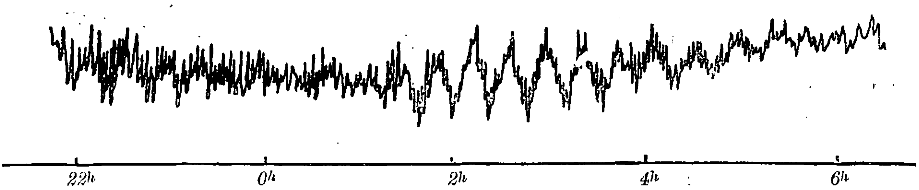
Fig 17 d.

Aug. 30–31, 1902.



Funatsu.

Fig. 18 a.



Nagahama.

Fig 18 b.

successive days. The recorded amplitudes were greater than those in Yamanaka lake, yet as the accompanying curves show, they were not generally large enough to allow us to determine the phases and the periods exactly. As might be expected from the complex form of the lake, a number of differtent oscillations were observed.

TABLE 33.

Funatsu....	22.80 × 10,	11.24 × 17,	10.89 × 11,	8.66 × 10,		
Nagasaki.	23.37 × 3,	11.42 × 3,	10.52 × 7,	8.58 × 3,		6.50 × 4.
Unoshima.		11.46 × 4,	10.59 × 8,	8.27 × 2,	7.82 × 4.	
Nagahama.	23.05 × 9,	11.30 × 1,				6.22 × 2.

The weighted means of these periods are

$T_1=22.98$ minutes,
$T_2=11.50$,,
$T_3=10.66$,,
$T_4=8.58$,,
$T_5=7.82$,,
$T_6=6.36$,,

which stand in the ratios

$$T_1:T_2:T_3:T_4:T_5:T_6=1.00:0.50:0.46:0.37:0.34:0.27.$$

As T_1 was most frequently observed at the two end stations, Funatsu and Nagahama, there is no doubt that it was the uninodal longitudinal seiche. The comparison of the phases shows us also that these stations were in opposite phases (Figs. 18 *a* and *b*). It is a remarkable fact that other oscillations were seldom observed on the western, or Nagahama, side of the lake.

From the total surface area A and the total volume V of the lake, calculated from the data obtained by our soundings, viz.,

$A=5.75\times10^{10}$ sq. cm.,

$V=5.53\times10^{13}$ c. cm.,

we get the mean depth h

$h=963$ cm,

which combined with the length of the lake

$L=5.86\times10^{5}$ cm.

gives, according to Mérian's formula, a period of 20.1 minuts for the uninodal longitudinal motion.

A model of the lake was constructed in cement and from its oscillations (Plate XIII), we got three longitudinal motions with periods

$t_1=1.36$ sec.,

$t_2=0.62$,,

$t_3=0.46$,,

which are in the ratios

$$t_1:t_2:t_3=1.00:0.45:0.35.$$

The first uninodal motion t_1 gives on reduction a period of 23.86 minutes for the actual lake, and evidently corresponds to T_1. The second is a binodal oscillation of the whole lake, but the amplitude is large only on the Funatsu side of the model, the level on the Nagahama side apparently remaining quite still. As the ratio $t_1:t_2$ is very nearly equal to the ratio $T_1:T_3$, and as T_3 was not observed at Nagahama, we may perhaps conclude that the motion $T_3=10.66$ minutes is binodal. The third motion t_3 with a period of 0.46 sec. is trinodal, but as one of its nodal lines is very much curved, and touches the shore, it may be called quadrinodal. From the ratio $t_1:t_3$, it is probable that t_3 corresponds either to T_4 or to T_5, but owing to the shortness of their periods we can not decide which is right. We are inclined however to think that $T_4=8.58$ minutes corresponds to t_3 and is trinodal. As to the motion $T_2=11.50$

minutes, which was frequently observed at Funatsu, we can not say definitely what it is. Since however

$$T_2=\frac{1}{2}\cdot T_1,$$

and since it is nearly equal to T_3, it is very probable that it is the result of the interference of the two motions T_1 and T_3. When a motion T_1 coexists with another T_3 with nearly half the period, then at the moment when their phase relations are such that their high waters coincide, we get a resultant motion like *a* in the accompanying figure; while, on the contrary, when their low waters coincide,

a.

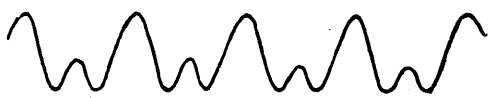


Fig. 19.

b

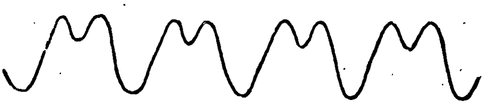


Fig. 19.

the resultant motion is like *b*; and lastly in the intermediate stage, we have apparently a simple wave with a period $\frac{1}{2}\cdot T_1$. The effect of the interference of two such oscillations as T_1 and T_3 is therefore that the resultant motion changes gradually from the state *a* to the state *b* by passing through an intermediate stage of an apparently simple wave with a period $\frac{1}{2}T_1$ and then returns back to the initial state. Such a gradual transition is seen in the beginning of the curve of Funatsu in Fig. 17. We are forced to say that the motion T_2 does not in reality exist.

§ 6. Hamana Lake.

Hamana lake is situated at 138°43′ E. and 34°44′ N. It is connected with the Pacific Ocean by a narrow channel formed by the destructive earthquake which took place on the 20th of September, 1498. Its form is very complex being formed by smaller lakes joined by narrow canals and has a number of small arms (Plate XIV). The main lake is more than 5 kilometers long and from 2 to 3 kilometers wide. By our soundings it was found to be very shallow, being only two or three meters deep in most places. Even at the deepest point, which lies at the entrance of a small canal connecting the main lake with a smaller lake on the north, it is only a little more than 16 meters deep, so that it must be said that it is most unsuitable for observing the phenomena of seiche. The interest, however, lies rather in the opposite direction, that is, in studying whether in such shallow lakes, an oscillatory motion of the whole mass of water can by any means be excited, and also in searching for the effect of the tides in it.

Owing to its excessive shallowness and to the influence of the tides, which produced sometimes a double amplitude of about 10 centimeters, the proper seiches were very difficult to produce, and even when produced, they were soon damped away, so that it was rather hard to detect and pick them out from the records (Plate XV). We set up three limnimeters of the "N" type at Washizu, Horié and Ōsaki, and took curves extending over four days. We succeeded only in isolating the following periods from the records.

TABLE 34.

Washizu.......	37.3×1		11.72×1	9.46×1	
Horié		12.18×9			8.74×1
Ōsaki.		12.64×7		9.64×1	

Thus the only motion apparently having any significance in this lake is the one with a period a little greater than 12 minutes, observed at Ōsaki and Horié. The period of 11.72 minutes once observed at Washizu may also belong to the same motion. Assuming this, the period of this motion is

12.34 minutes.

With a model constructed of cement, we could excite only two motions, viz. a motion with a period of 1.15 sec., and another

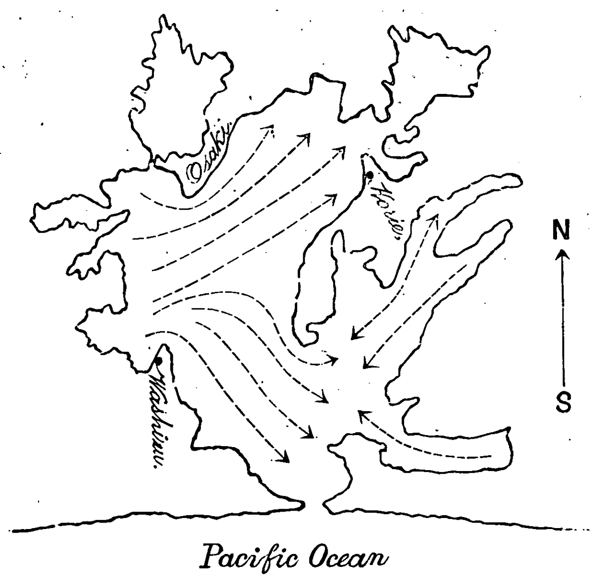


Fig. 20.

with a period a little greater than 0.3 sec., but much smaller than 0.4 sec.. Both were, however, very difficult to excite, and were damped away very soon. The mode of oscillation of the first motion is roughly reproduced in Fig. 20, in which we see a principal motion accompanied with many secondary motions. On reduction, this motion is found to correspond to a motion with a period of 39.3 min. in the actual lake, which is probably identical with the motion 37.3 min. once observed at Washizu. The second motion of the model was principally a motion of the deep basin on the north side of the lake, and is equivalent to a motion with a period of 10–14 min. in the actual lake. It probably corresponds to the observed seiche with a period of 12.34 min.. This view is further supported by the fact that the phases at the stations Horié and Ōsaki were found to be opposite.

As to the effect of the tides on the lake, it produced, as already mentioned, a double amplitude of about 10 cm.. In order to compare the times of high and low water for the diurnal tides within the lake with those in the open sea, we must calculate the latter from the observations at the nearest tide-guage stations. They are situated at Kushimoto and Misaki, each more than a hundred miles distant from Hamana. As it is known that on the Pacific coast of Japan, the co-tidal lines are nearly parallel to the meridians, we may calculate the required times by simple interpolations with respect to longitude. In the following table, the times of high (H) and low (L) water at the two stations are taken from their mareograms.

TABLE 35.

		Kushimoto. (135°45′ E)			Misaki. (139°37′ E)			Hamana. calc. (137°36′ E)	
		h	m		h	m		h	m
L	Aug. 23	2	5	23	0	43	23	1	26
H		8	3		6	22		7	14
L		14	19		12	48		13	35
H		20	16		18	48		19	33
L	24	2	40	24	1	2	24	1	53
H		8	53		7	34		8	15
L		14	56		13	22		14	11
H		20	46		19	26		20	7
L	25	3	40	25	2	15	25	2	59
H		9	40		8	22		9	3
L		15	20		13	47		14	35
H		21	38		20	21		21	1
L	26	4	36	26	3	3	26	3	51
H		71	34		9	55		10	46
L		16	3		14	52		15	29

In the following table, the times of high and low water and the heights of the level (measured from arbitrary datum lines) at the several stations are given.

TABLE 36.

		Washizu.			Horié.			Osaki.			Mean.		
		h	m	cm	h	m	cm	h	m	cm	h	m	cm
L	Aug. 24	18	13	146	18	17	75	18	8	33	18	13	85
H		23	22	235	23	21	147	23	11	119	23	18	167
L	25	7	44	124	7	50	49	7	31	11	7	42	61
H		12	7	168	12	17	97	12	12	52	12	12	106
L		18	24	114	18	31	38	18	50	5	18	35	52
H		——		186	0	34	107	0	14	71	——		121
L	26	8	46	89	8	49	12	8	48	−26	8	48	25

From this table we conclude first of all that the general level of the lake was continuously sinking during the interval here given; and that the amplitude of the tide at Horié seems to be somewhat smaller than those at Washizu and Ōsaki. We can not say definitely what the cause of this sinking of the level was. As the mareograms at Kushimoto indicate a very slow rise of the mean level of the sea on those days, and that at Misaki shows us that the sea-level there had a maximum height on the 25th, it may be conjectured that the mean level of the lake was higher than that of the sea, and that the sinking of the level of the lake was due to the flowing out of water from the lake to the sea.. As the weather was very fine, it may also have been caused by strong evaporation.

In order to compare the relative retardation of high and low water within the lake, we have calculated the differences of the times at Washizu and at the other stations in the next table, in which + denotes the retardation.

TABLE 37.

	Tide at Washizu.				Retardation at	
					Horié.	Osaki.
			h	m	m	m
L	Aug.	24	18	13	+ 4	− 5
H			23	22	− 1	− 11
L		25	7	44	+ 6	− 13
H			12	7	+ 10	+ 5
L			18	24	+ 7	+ 26
H			——		—	—
L		26	8	46	+ 3	+ 2
Mean					+ 5	0

Thus a given phase arrives at Washizu and Ōsaki simultaneously, and at Horié about 5 min. later. This accords well with the fact, that the amplitude of the tide at Horié is smaller than at the other places.

The retardation of the tide within the lake is given in the next table, in which the mean value of the times of high and low water within the lake is compared with that of the open sea as calculated before.

TABLE 38.

	Lake.				Sea.		Retardation.	
			h	m	h	m	h	m
L	Aug.	24	18	13	14	11	4	2
H			23	18	20	7	3	11
L		25	7	42	2	59	4	43
H			12	12	9	3	3	9
L			18	35	14	35	4	0
H			—	—	21	1	—	
L		26	8	48	3	51	4	57

Thus the retardatian for a high tide is remarkably smaller than that for a low tide, the mean value for the former being 3 hours and 10 min., while that for the latter, 4 hours and 25 min.. This dissymmetry in the retardation may be easily accounted for by the general sinking of the level of the lake, before alluded to. Let us assume that the mean levels of the lake and the sea remain constant and that the amplitude of the tide is also constant, so that the changes of the levels can be represented by simple sinuous curves. Further assume that the connection between the lake and the sea is such that it can be diagrammatically represented by Figs. 21_a

and 21_b. Then evidently high and low waters in the lake are represented by the points *a* and *b* respectively, (Fig. 22) and the retardations for them are equal to each other, for the latter are simply the distances between the ordinates at *Aa* and *Bb*.

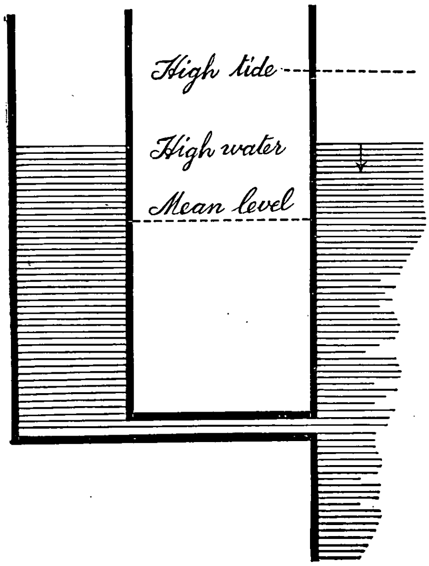


Fig. 21 a.

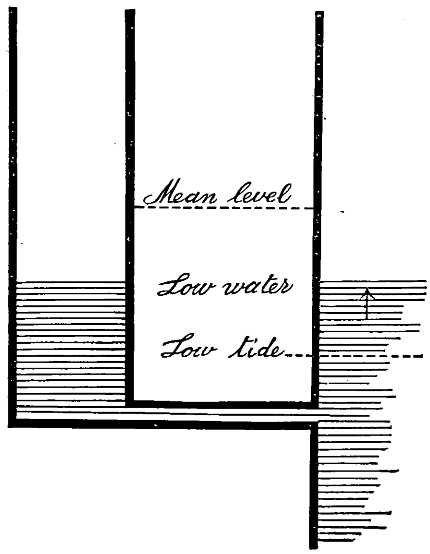


Fig. 21 b.

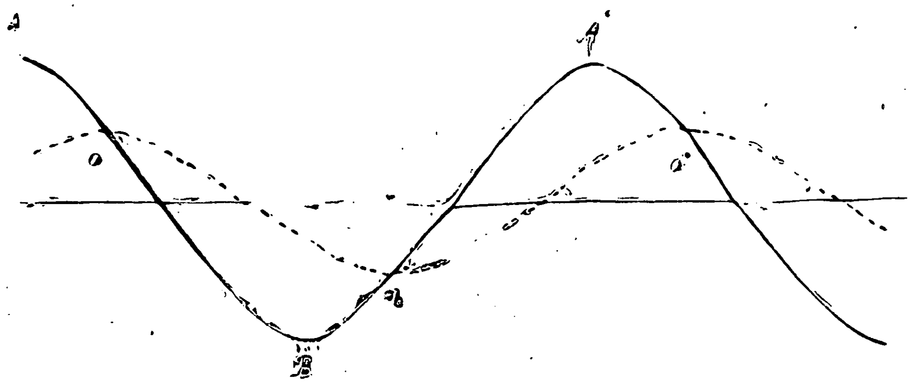


Fig. 22.

But when the mean level of the lake is sinking, then after high water *a*, as the relative velocity is diminished, the rate of the falling water is slower than in the last case, and therefore the inclination of the curve becomes flatter; while on the contrary, the inclination after low water b becomes steeper. The result of this is that the point b is displaced toward the right, and the retardation for high water is made smaller than that for low water

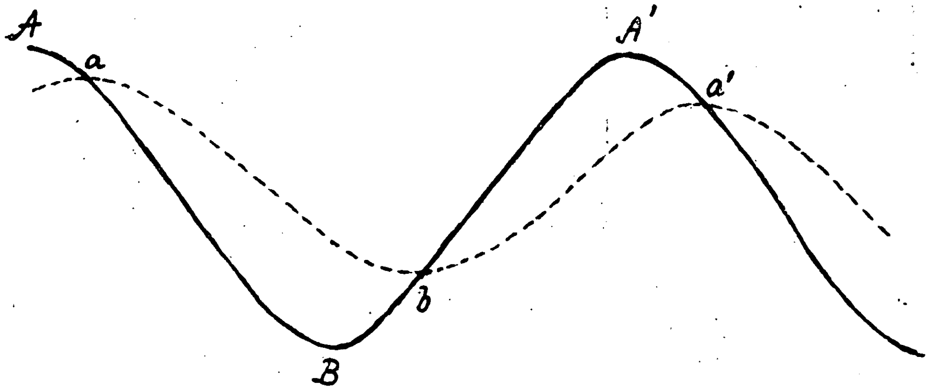


Fig. 23.

(Fig. 23), of which the former gets larger and the latter gets smaller till they are equal to each other, when the mean levels of the lake and the sea are of the same height.

Another point is worthy of attention. As the double amplitude of the tide is about 1.5 metres on the sea shore, at low tide the bed of the canal is higher than the level of the sea. The connection of the lake and the sea is therefore not like that shown in Fig. 21, but rather like that in Figs. 24_a and 24_b.

Fig. 24_a represents the condition when the level in the lake is highest; and Fig 24_b that when the level of the sea is lowest.

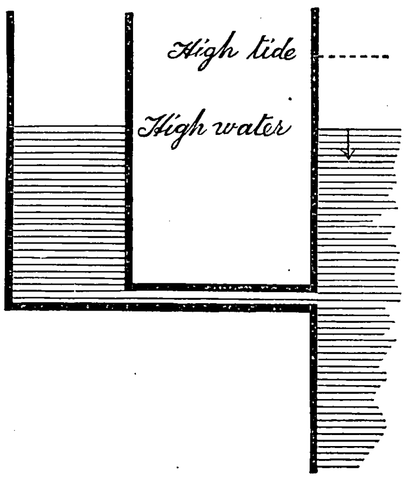


Fig. 24 a.

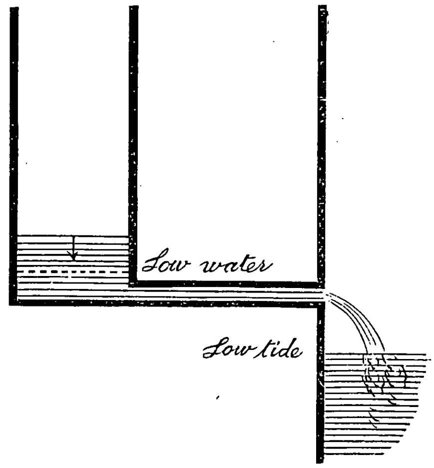


Fig. 24 b.

During a portion of low tide the head of the water flowing outward is determined by the height of the level of the lake above the bed of the channel, and not as in the previous case by the difference of their levels. The rate of the falling of the water in the lake is decreased, and causes a similar dissymmetry in the retardations of high and low waters. The effect of the inconstancy of the cross section of the flowing water probably makes this more prominent.

§ 7. Tōya Lake.

Tōya lake is situated at 42° 36′ N., and 140° 53′ E., in the island of Hokkaidō. and is the deepest lake in Japan, the maximum depth reaching 190 meters (Plate XVI). It is a crater lake of nearly circular form with a diameter of about 10.5 kilometres, and has some small conical islands in it which are very beautiful. The largest, which is just in the centre of the lake, is covered with pine trees.

During the summer vacation of 1905, one of us set up a limnimeter of the "H" type at Mukōtōya on the northern shore of the lake, and took a series of curves for 22 hours. At the commencement of the observations, the wind was blowing toward the station, and when it gradually subsided, the level fell by a few centimetres, showing that the wind heaped up the water toward the shore; and at the same time a simple oscillation with a period of

$$T=9.29 \text{ minutes}$$

made its appearance (Fig. 25_a). Toward the end of the observation, when the recording cylinder was driven at a slower speed, we had a very beautiful series of beat phenomena (Fig. 25_b).

Aug. 18, 1905.

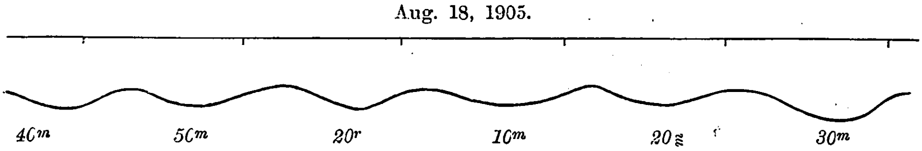


Fig. 25 a.

Aug. 18–19, 1905.

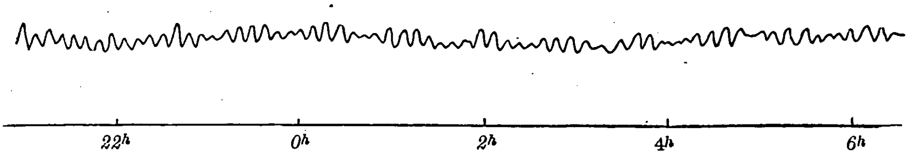


Fig. 25 b.

The soundings of the lake were made by the Naval Hydrographic Office, and the result was placed at our disposal. A model in cement was constructed, and an experiment with it showed us two uninodal oscillations. As the lake is circular, we can not distinguish them by the terms, longitudinal and transversal. In one of them, which we shall denote by t', the nodal line runs E-W, and in the other t'', it runs N-S (Plate XXII). In our model, their periods were found to be

$$t' = 0.67 \text{ sec.}$$
$$t'' = 0.76 \text{ sec.},$$

which on reduction gave

$$t' = 10.3 \text{ minutes}$$
$$t'' = 11.8 \quad ,,$$

as the periods of seiches in the actual lake, and they are both greater than the observed period. We see from this experiment that as these motions have nearly equal periods, when they coexist we must have the phenomena of beat. Since the difference of t' and t'' is 1.5 minutes, we ought to have a maximum or a minimum amplitude every 6, or 7 oscillations, which agrees with the actual observations (Fig. 25). It is to be noted from Plate XXII, that as the lake has a shallow on its south side, though the loops for t' are situated symmetrically with respect to the meridian axis of the lake, the loops for t'' are unsymmetrically situated, i. e. they are much displaced to the south side of the lake,

Calculating the mean depth h of the lake by means of a planimeter, we find

$$h = 9.58 \times 10^3 \text{ cm.},$$

and measuring the diametres of the lake along the directions suggested by the experiment with the model, we get

$$L' = 9.5 \times 10^5 \text{ cm.},$$

$$L'' = 10.2 \times 10^5 \text{ cm.},$$

from which by Mérian's formula the following values for the periods are obtained,

$$t' = 10.3 \text{ minutes}$$
$$t'' = 11.2 \quad ,,$$

These are in close argeement with the values deduced from the model but somewhat exceed the observed periods. As our station Mukōtōya is situated near the nodal line for t'', it is very probable that our observed period 9.29 minutes corresponds to the uninodal seiche t'' with an E-W nodal line.

§ 8. Chūzenji Lake.

Nikkō, a place noted for its beautiful scenery and splendid temples dedicated to the first and third Shōguns, lies within about 100 kilometres north of Tōkyō, and is a favorite summer resort of foreign residents. Among the mountains not far from Nikkō, is the famous and picturesque Chūzenji lake. It is nearly surrounded by steep and thickly wooded mountains, the great Nantai-san grandly towering on its northern side. The lake is about two kilometres wide and six kilometres long, its surface being 1316 meters above sea level. It is one of the deepest lakes in this country, and its greater part has a depth exeeding 100 metres, the maximum depth being 172 metres. The sounding of the lake was carried out by Viscount A. Tanaka, a zealous limnologist, who has very kindly allowed us to make use of his results and also to reproduce them in Plate XVIII.

During the summer vacation of 1905, a limnimeter of the "H" type was set up at Chūzenji, a village near the eastern end of the lake, and a series of limnograms were obtained continuing for 24 hours. The curve is a smooth sine curve, as is shown by the accompaning figure (26) in which *a*, and *b* are curves for two

Aug. 27, 1905.

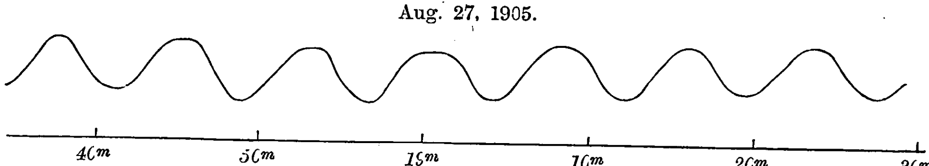


Fig. 26 a.

Aug. 27, 1905.

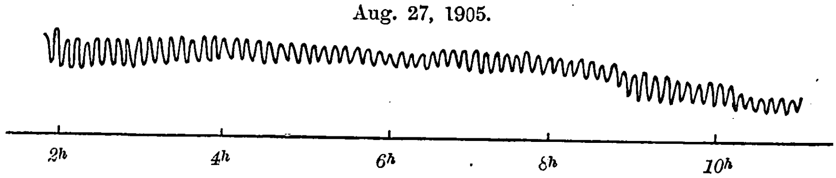


Fig. 26 b.

different speeds of the recording cylinder. Here the motion was so regular and the period so constant that the phase of the seiche after twelve hours could be predicted. The period found in this case was

$$T = 7.70 \text{ minutes.}$$

In the following year we had an opportunity to make further observations on the seiches of this lake. On the 6th of July 1906, a limnimeter of the " H " type was placed at Senju at the western end of the lake and another at Shōbugahama situated near the middle of the northern shore. From simultaneous records (Fig. 27), it was found that the period of the seiche was 7.7 minutes as in the previous year, and that the phases were the same at the two stations, and as the amplitude was very much smaller at Shōbugahama than at Senju, we saw that the nodal line must pass a little east of the former station. On the next day, simultaneous records were taken at Senju and Utagahama at the eastern end of the lake, and, as was expected, it was found that they were in the opposite phases. Leaving the limnimeter at Senju in its place till the end of the observations, the other instrument was carried to Shinnagi on the northern shore, where it recorded a motion of much smaller amplitude but of the same phase as that of Utagahama. As the oscillation was very regular, the phase of the motion at Shinnagi could be predicted from that at Utagahama. These two places are therefore on the same side of the nodal line. On the 8th, the motion of the water at Furunagi was examined with the result that the limnogram was a straight line showing that the nodal line passed through this spot. Simultaneous observations at Asegahama and Chūzenji on both sides of the lake near its eastern end also indicated that the oscillations of the water were in the same phase at these places.

July 6, 1906.

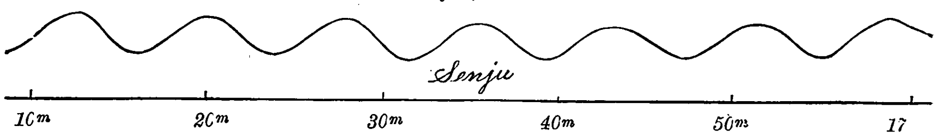


Fig. 27 a.

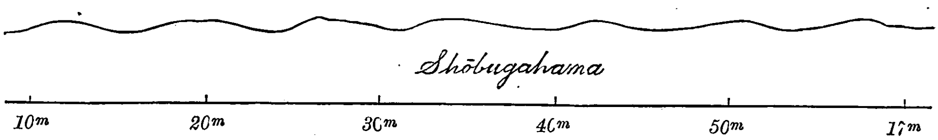


Fig. 27 b.

July 7, 1906.

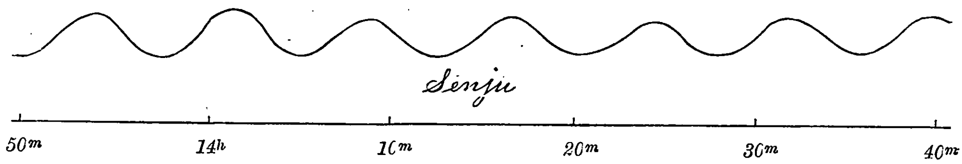


Fig. 27 c.

Fig. 27 d.

Jujy 7, 1906.

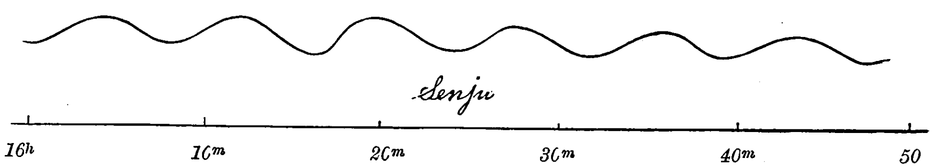


Fig. 27 e.

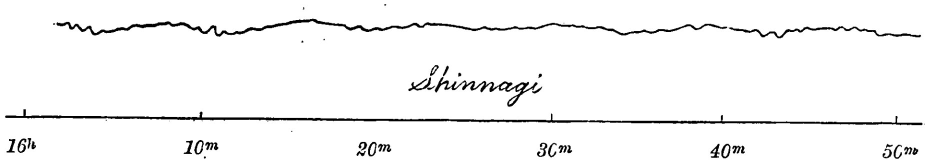


Fig. 27 f.

The periods obtained at the several stations were as fellows;

TABLE 39.

Senju	7.77×80,
Chūzenji	7.81×24,
Utagahama........	7.72×40,
Shōbugahama......	7.94×9,

As the amplitudes in other stations were too small, they are here omitted in determining the period. The weighted mean of the above value is

$$T = 7.77 \text{ minutes.}$$

From the above observations, it can be seen that the seiche of this lake is pure and simple, being a uninodal longitudinal oscillation with a period of 7.77 minutes. Owing to its great depth, a motion once excited continues for a long time without a sensible damping. During the course of the observations, it was also clearly seen that the chief agency agitating the mass of water and exciting the seiche in the lake is the wind.

From the bathymetric data, we have

$$L = 5.87 \times 10^5 \text{ cm.},$$
$$h = 8.54 \times 10^3 \text{ cm.},$$

where h is obtained by dividing the total volume by the total surface area of the lake, and the formula of Mérian gives us 6.72 minutes as the period of the uninodal seiche. The discrepancy between the calculated and the observed periods may perhaps arise from the fact that though the west half of the lake is approximately rectangular in shape, the other half extends considerably towards the south, so that as a whole, the form of the lake can by no means be considered to be rectangular. But as the position of the nodal line was found by observation, we were able to calculate the period from data for the west half of the lake, and obtained 7.78 minutes for the period, taking

$$L = 2 \times 3.20 \times 10^5 \text{ cm.},$$

$$h = 7.70 \times 10^3 \text{ cm.},$$

which agrees better with the observed value.

For the verification of the above result, a model was made in cement on the scale of 1: 18400 and 1: 2000 for the length and the depth respectively. The uninodal oscillation reproduced in Plate VII Fig. 3. could only be found for it with a period

$$t = 1.12 \text{ sec.},$$

which gave on reduction a period of 7.70 minutes for the actual lake.

Mr. Ishitani calculated the period and the position of the node for this lake using his formula above cited with the result that, the period was found to equal 7.0 minutes, and the node to lie at 3.3 kilometres from the west end, both agreeing pretty well with the observed values.

Appendix. Seiches in a Small Pond.

There is a small pond within the grounds of the Imperial University of Tōkyō. During July 1904, one of the authors with Mr. T. Terada succeeded in recording two different motions of the pond. As it is of interest to see that there is a natural seiche in such a small mass of water, the general result then obtained is here given. The pond is irregular in form and has a small island in it, but its general contour is quadrilateral with diagonals about 90 metres long. The sounding were made by us, and the result is given in Fig. 28. The curves recorded on the 14th of July 1904 are also reproduced in Fig. 29.

The periods observed at three stations situated at the northeastern, southeastern, and northeastern corners of the pond were;—

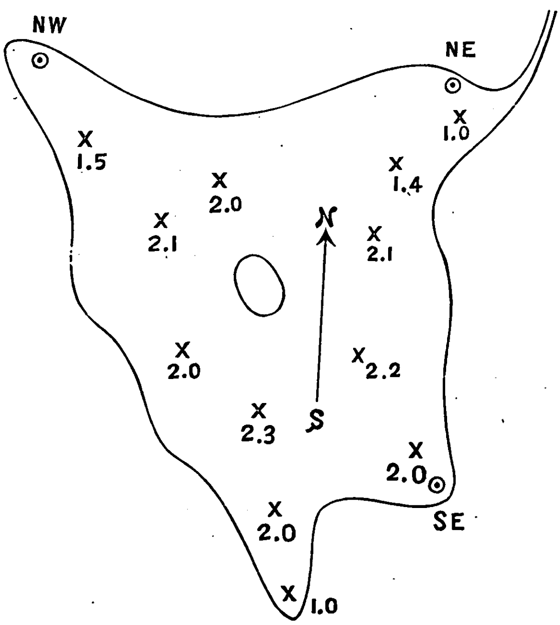


Fig. 28.

July 14, 1904.

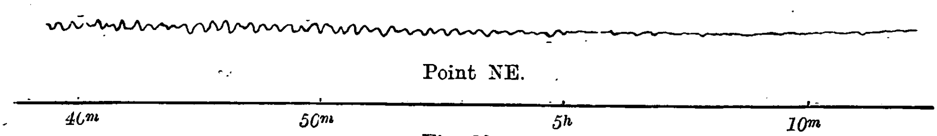


Fig. 29 a.

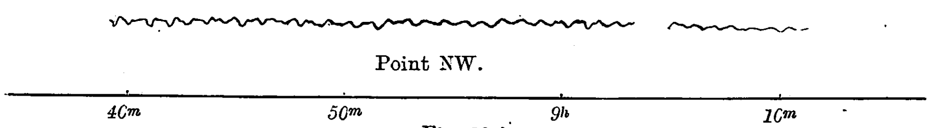


Fig. 29 b.

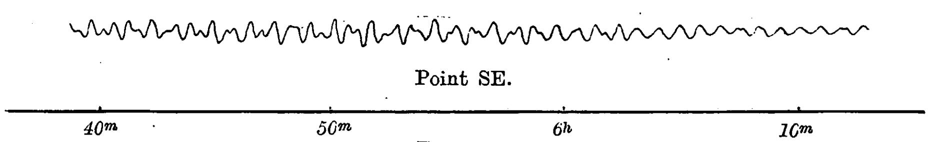


Fig. 29 c.

TABLE 40.

N W	52.3 sec. × 13	31.1 sec. × 18
N E		33.2 × 21 32.4 × 19
S E	51.7 × 22 53.6 × 14 52.0 × 15	32.4 × 20
Mean	52.3 sec.	32.3 sec.

Experiments with a model of the pond showed that the motion with the longer period of 52.3 sec. is the uninodal seiche executing its horizontal motion nearly parallel to the meridian. Taking the mean depth $h=1.5$ metres, and the length $L=98$ metres, the period for it ought to be equal to 51.1 sec., almost coinciding with the observed value.

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SEICHES IN SOME LAKES OF JAPAN.

PLATE I.

PLATE I.

Fig. 1.—N-Limnimeter used in Biwa, Hakonè, Hamana, Kawaguchi and Yamanaka lakes.

Fig. 2.—H-Limnimeter used in Tōya and Chūzenji lakes.

Fig. 1. N—Limnimeter.

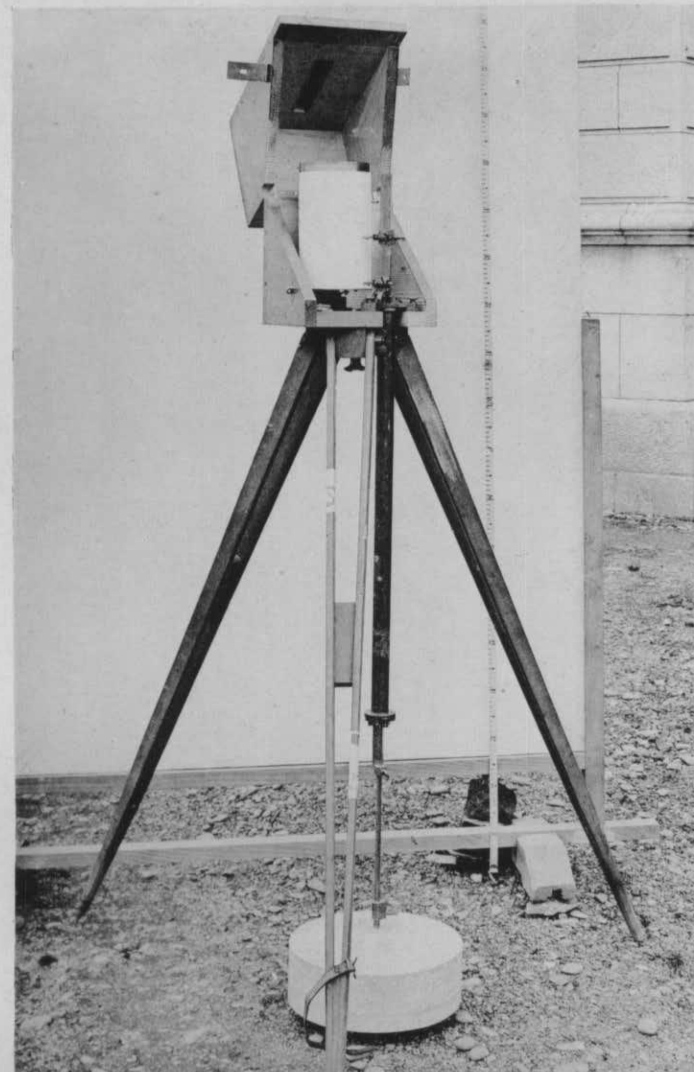
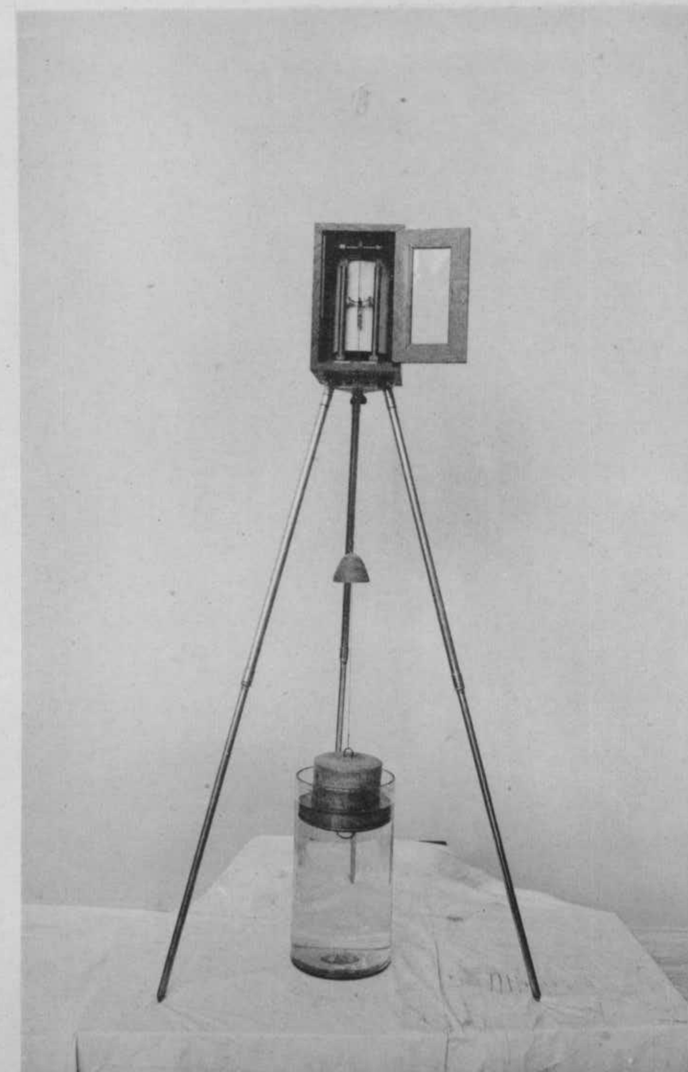


Fig. 2. H—Limnimeter.



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SEICHES IN SOME LAKES OF JAPAN.

PLATE II.

PLATE II.

Limnogram obtained at Ōtsu on Aug. 13–15, 1901, showing an extraordinary seiche caused by a thunderstorm. This and the succeeding limnograms are in natural size, so that the ordinates show the actual vertical motion of the water.


ŌTSU Aug. 13-15, 1901.
Lake Very calm.
11h 12h 13h 14h 15h 16h 17h 18h 19h 20h 21h 22h 23h 0h 1h 2h 3h 4h 5h 6h 7h 8h 9h 10h 11h 12h 13h 14h 15h 16h 17h 18h 19h 20h 21h 22h 23h 24h 1h 2h 3h 4h 5h

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PLATE III.

PLATE III.

Fig. 1.—Limnogram obtained at Imazu on Aug. 17, 1901.

Fig. 2.—Limnogram obtained at Imazu on Aug. 18, 1901.

These are further examples of seiche caused by thunderstorms.

Fig. 1.

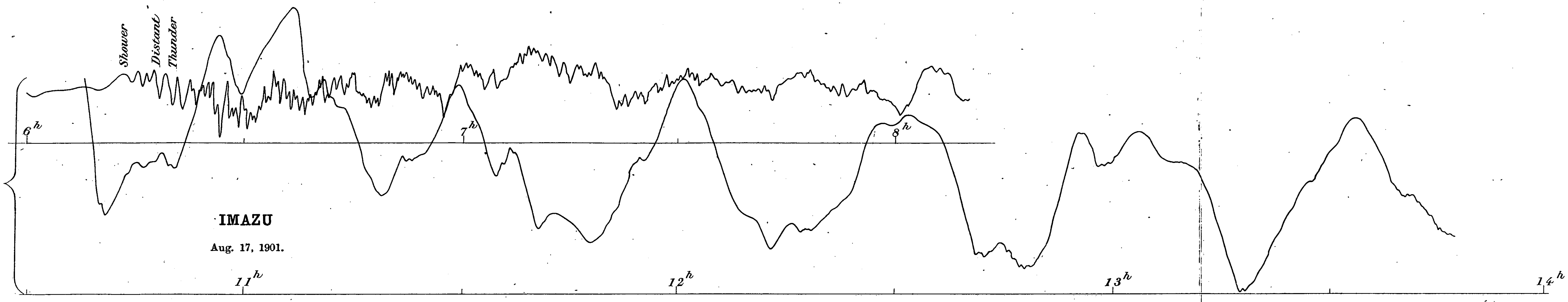
Shower
Distant Thunder
6h
7h
8h
IMAZU
Aug. 17, 1901.
11h
12h
13h
14h

Fig. 2.

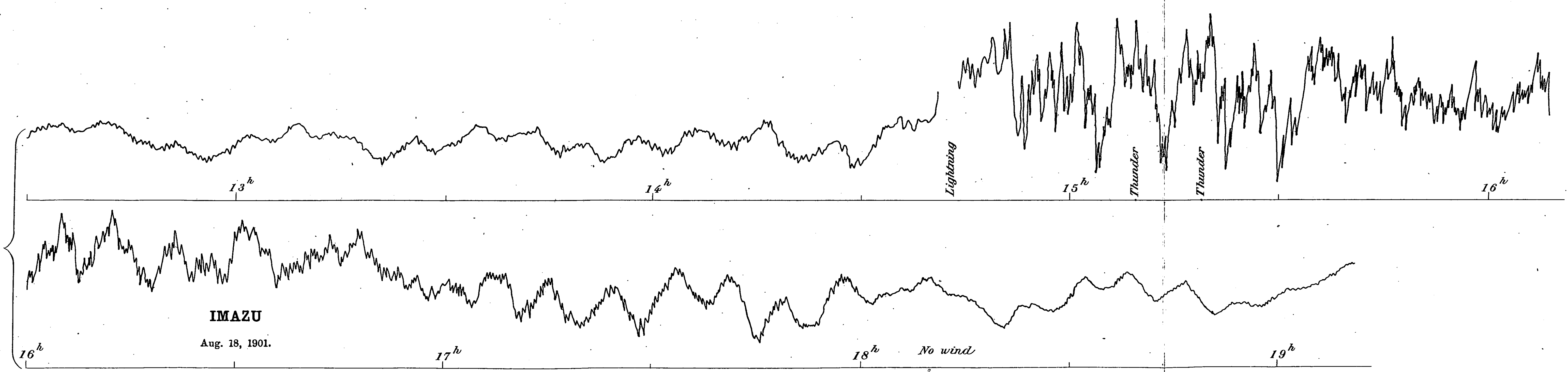
Lightning
Thunder
Thunder
13h
14h
15h
16h
IMAZU
Aug. 18, 1901.
16h
17h
18h
No wind
19h

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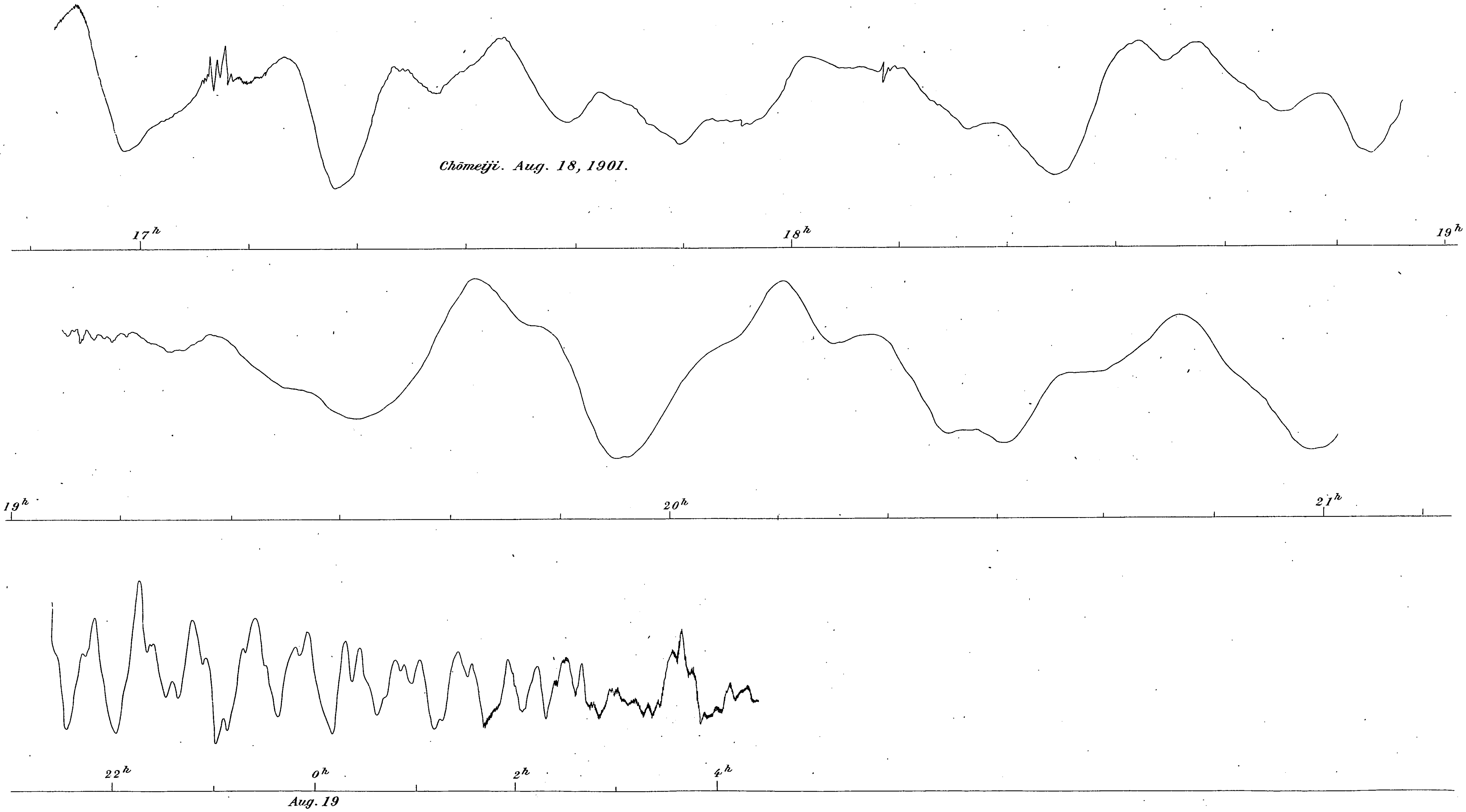
PLATE IV.

PLATE IV.

Limnogram obtained at Chōmeiji on Aug. 18, 1901.

Compare this with Plate III, Fig. 2.


Chōmeiji. Aug. 18, 1901.
17h
18h
19h
19h
20h
21h
22h
0h
2h
4h
Aug. 19

S. NAKAMURA and K. HONDA.

SEICHES IN SOME LAKES OF JAPAN.

PLATE V.

PLATE V.

Limnograms obtained at Hikoné.

Fig. 1.—Ordinary seiche.

Fig. 2.—Effect of a thunderstorm.

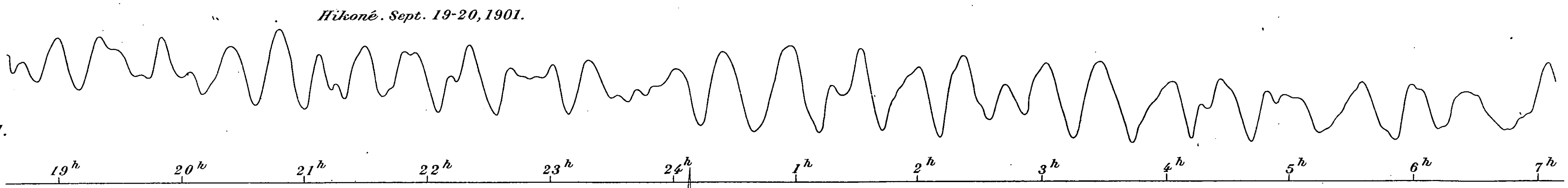

Hikoné. Sept. 19-20, 1901.
19h
20h
21h
22h
23h
24h
1h
2h
3h
4h
5h
6h
7h

Fig. 1.

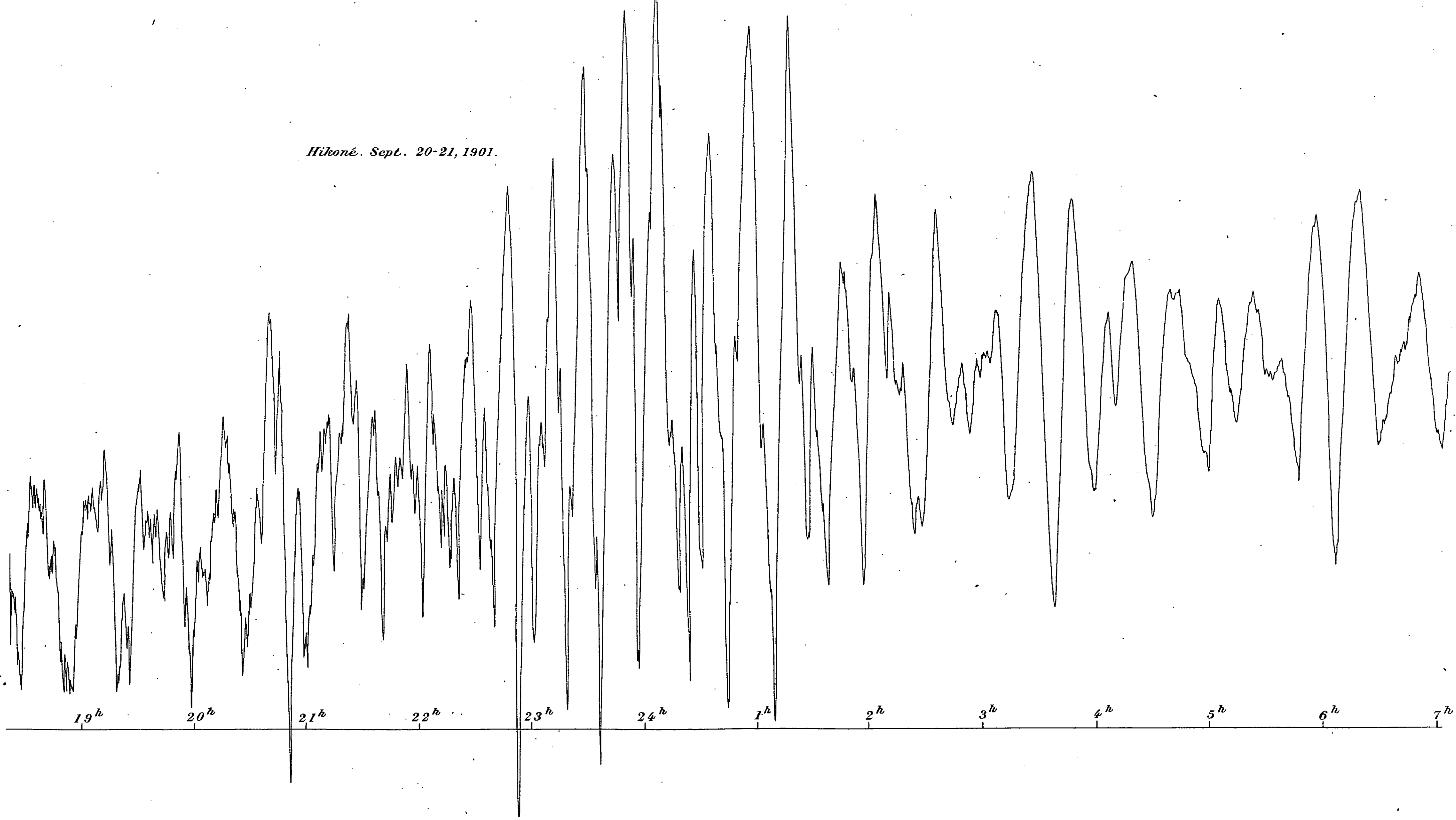

Hikoné. Sept. 20-21, 1901.
19h
20h
21h
22h
23h
24h
1h
2h
3h
4h
5h
6h
7h

Fig. 2.

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SEICHES IN SOME LAKES OF JAPAN.

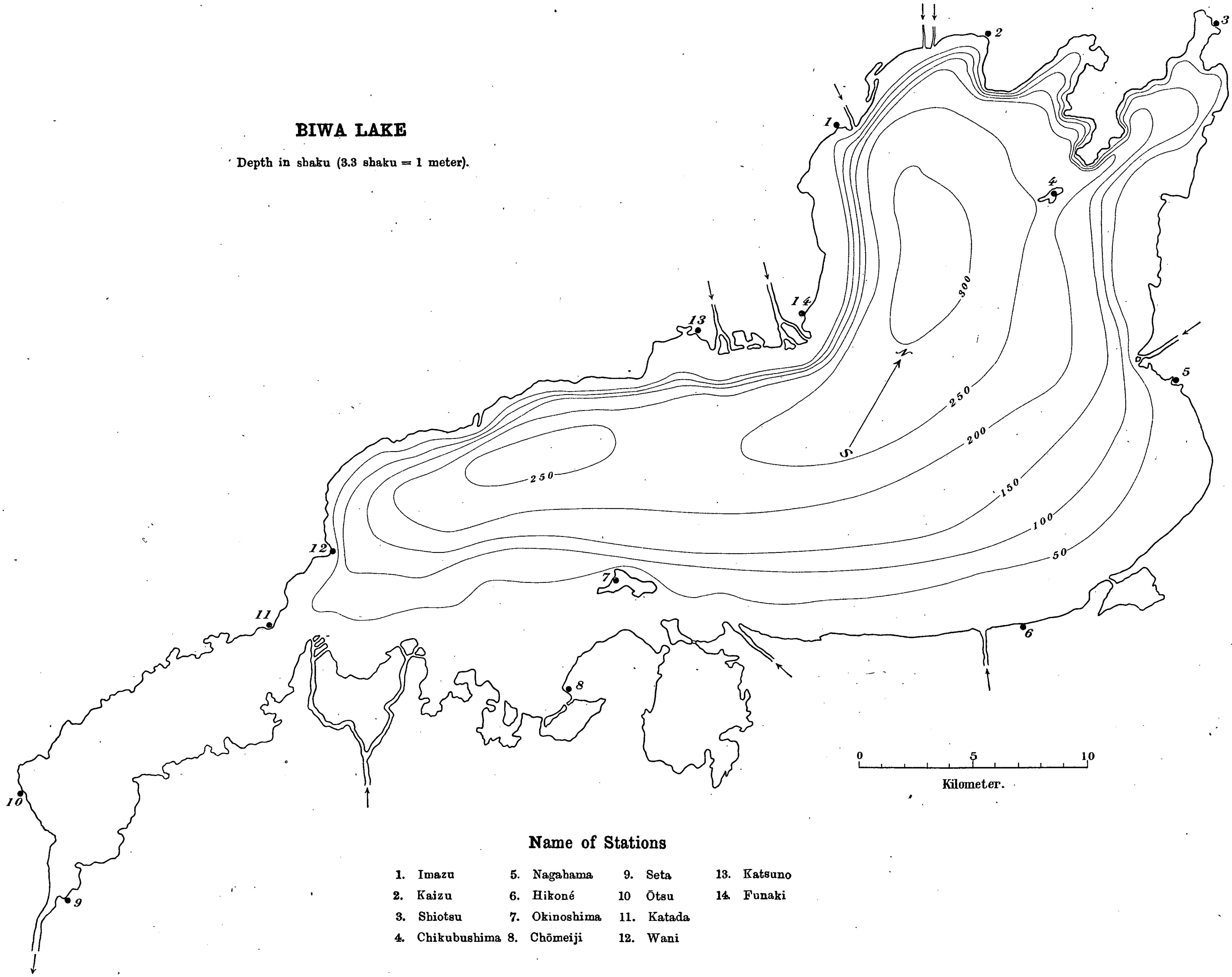
PLATE VI.

PLATE VI.

Map of Lake Biwa with isobathymetric lines, after Mr. Maeda.

BIWA LAKE

Depth in shaku (3.3 shaku = 1 meter).



Name of Stations

1. Imazu
2. Kaizu
3. Shiotsu
4. Chikubushima
5. Nagahama
6. Hikoné
7. Okinoshima
8. Chōmeiji
9. Seta
10. Ōtsu
11. Katada
12. Wani
13. Katsuno
14. Funaki

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PLATE VII.

PLATE VII.

Figs. 1. and 2.—Photographs of the horizontal motion of water in a model of Lake Biwa.

Fig. 3.—Ditto of Chūzenji lake.

Fig. 1. Lake Biwa, $t_1=1.58$ sec.

Fig. 2. Lake Biwa, $t_2=0.67$ sec.

Fig. 3. Chūzenji lake, $t=1.12$ sec.

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PLATE VIII.

PLATE VIII.

Map of Hakoné lake with isobathymetric lines

HAKONÉ LAKE

(Depth in meters).

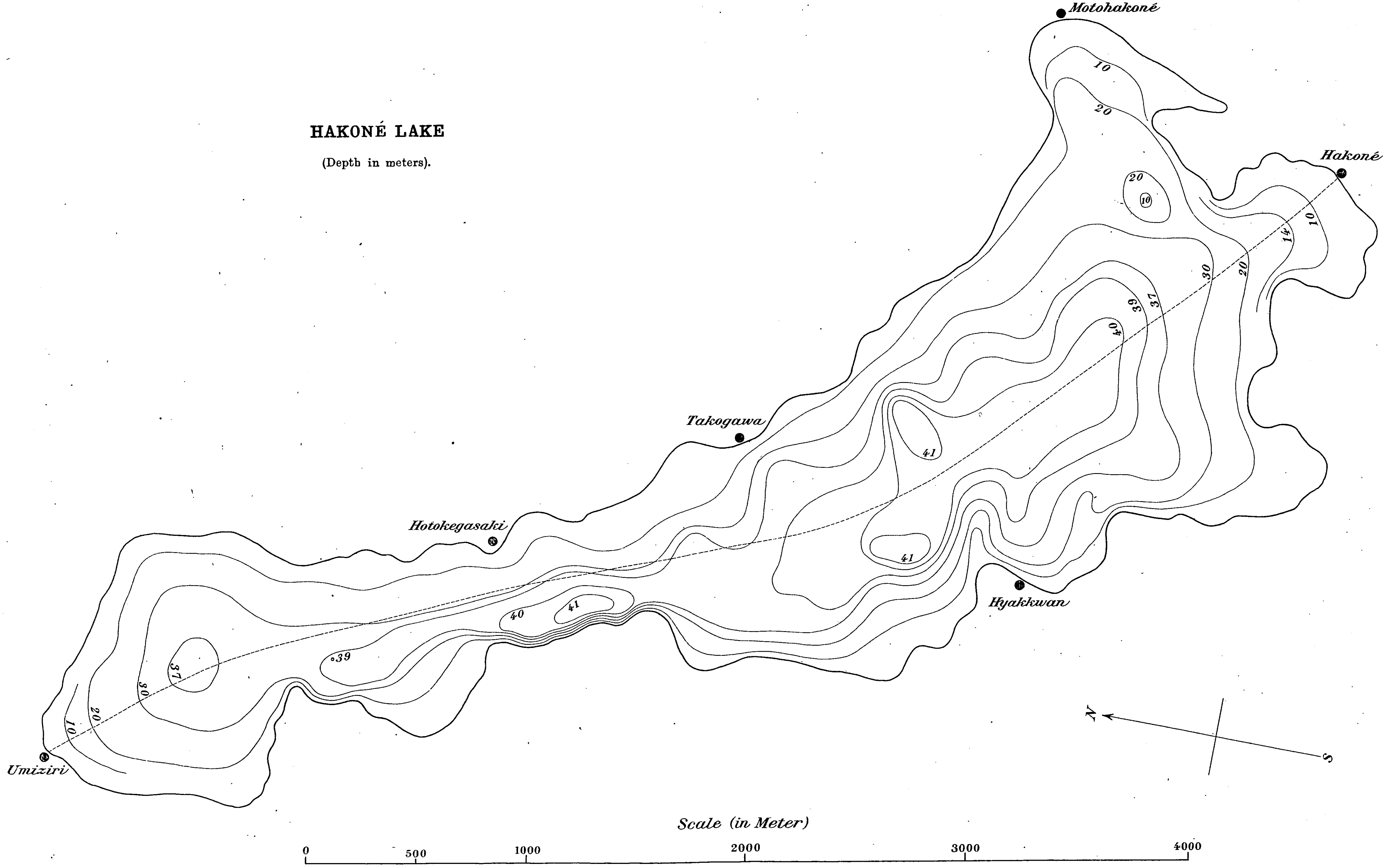

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PLATE IX.

PLATE IX.

Photographs of the horizontal motion of water in a model of Hakoné lake.

Fig. 1. Hakoné lake, $t_1=1.67$ sec.

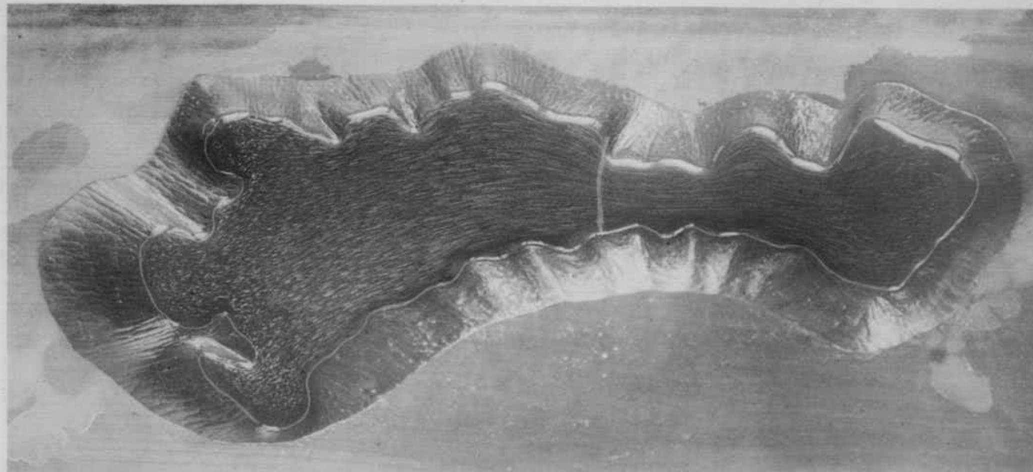

Fig. 2. Hakoné lake, $t_2=0.69$ sec.

Fig. 3. Hakoné lake, $t_3=0.44$ sec.

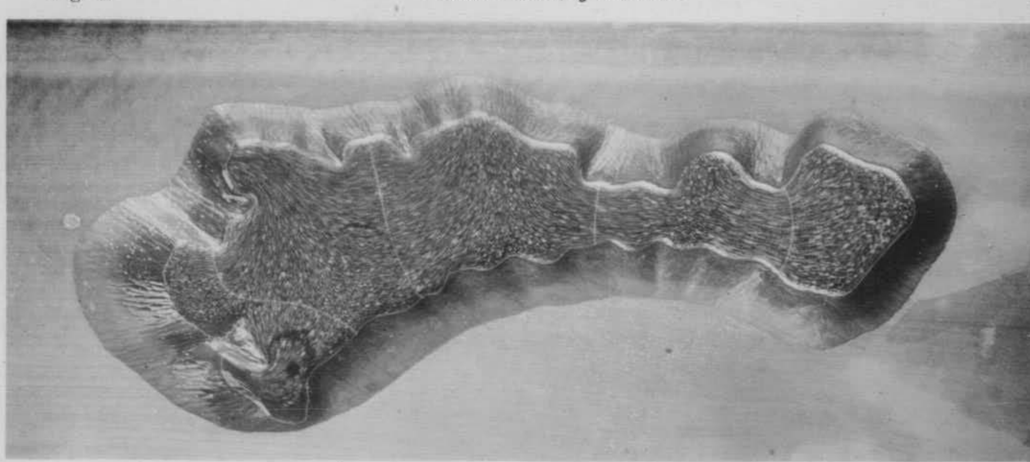

PLATE X.

PLATE X.

Map of Yamanaka lake with isobathymetric lines.

YAMANAKA LAKE

(Depth in meters).

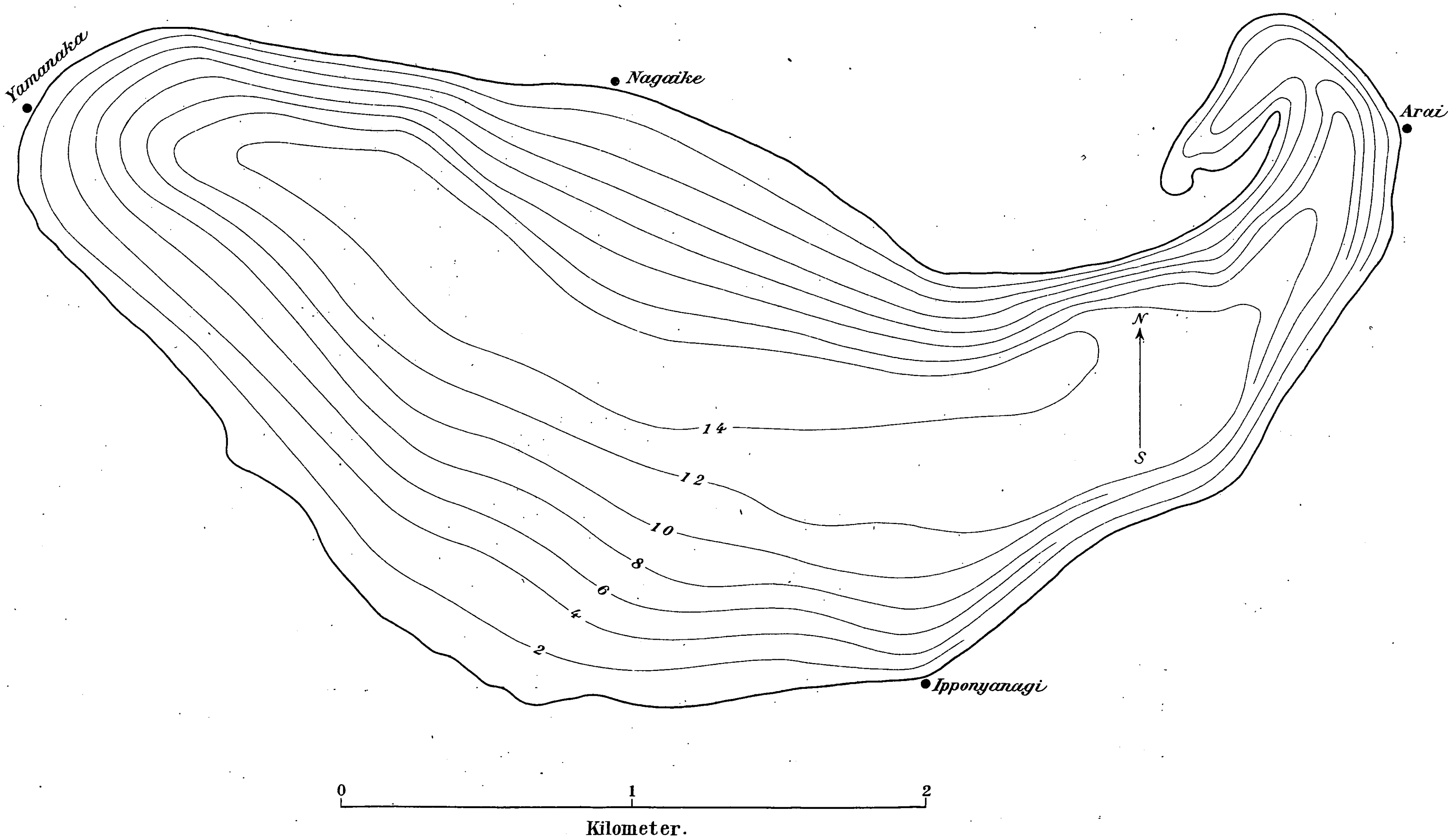

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PLATE XI.

PLATE XI.

Photographs of the horizontal motion of water in a model of Yamanaka lake.

Fig. 1. Yamanaka lake, t_1=0.90 sec.

Fig. 2. Yamanaka lake, t_2=0.50 sec.

Fig. 3. Yamanaka lake, t_3=0.39 sec.

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PLATE XII.

PLATE XII.

Map of Kawaguchi lake with isobathymetric lines.

KAWAGUCHI LAKE

(Depth in meters)

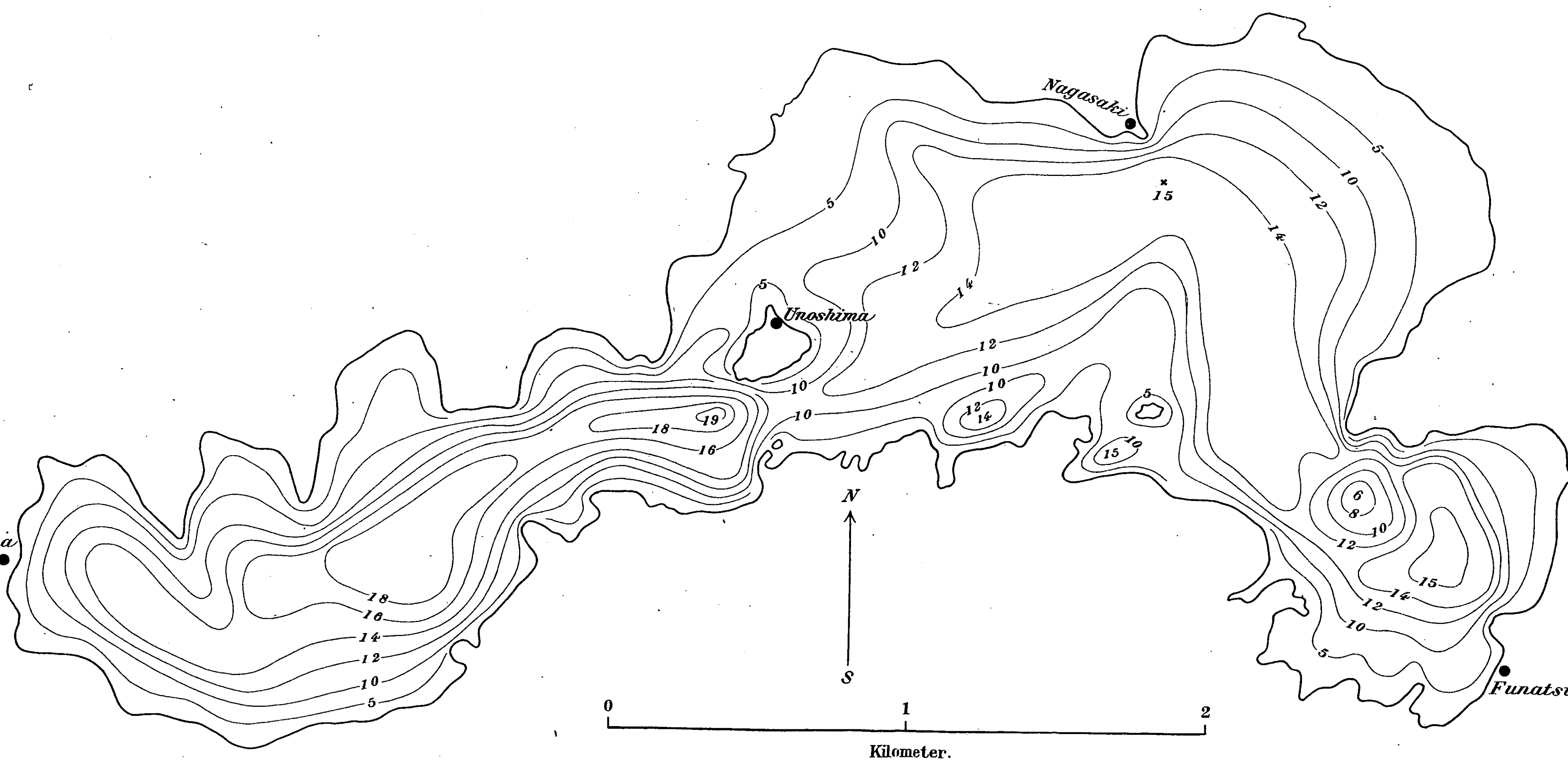

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PLATE XIII.

PLATE XIII.

Photographs of the horizontal motion of water in a model of Kawaguchi lake.

Fig. 1. Kawaguchi lake, $t_1=1.36$ sec.

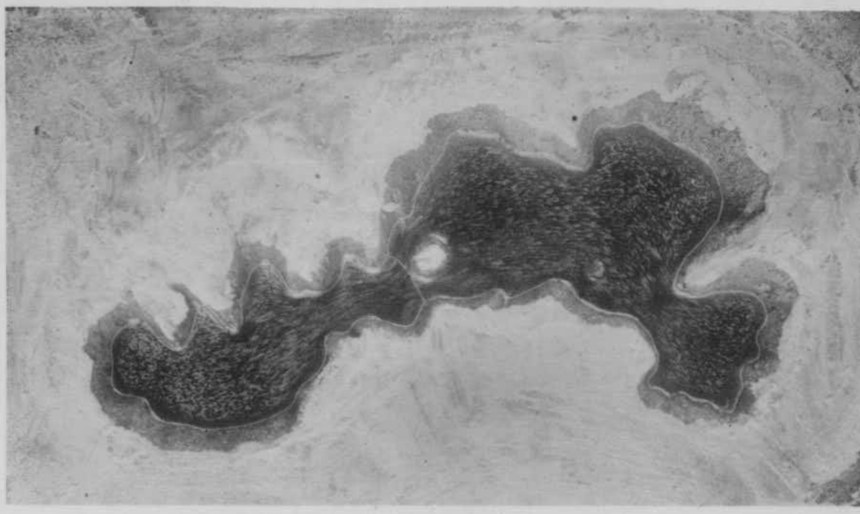

Fig. 2. Kawaguchi lake, $t_2=0.62$ sec.

Fig. 3. Kawaguchi lake, $t_3=0.46$ sec.

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PLATE XIV.

PLATE XIV.

Map of Hamana lake with isobathymetric lines.

HAMANA LAKE

(Depth in meters).

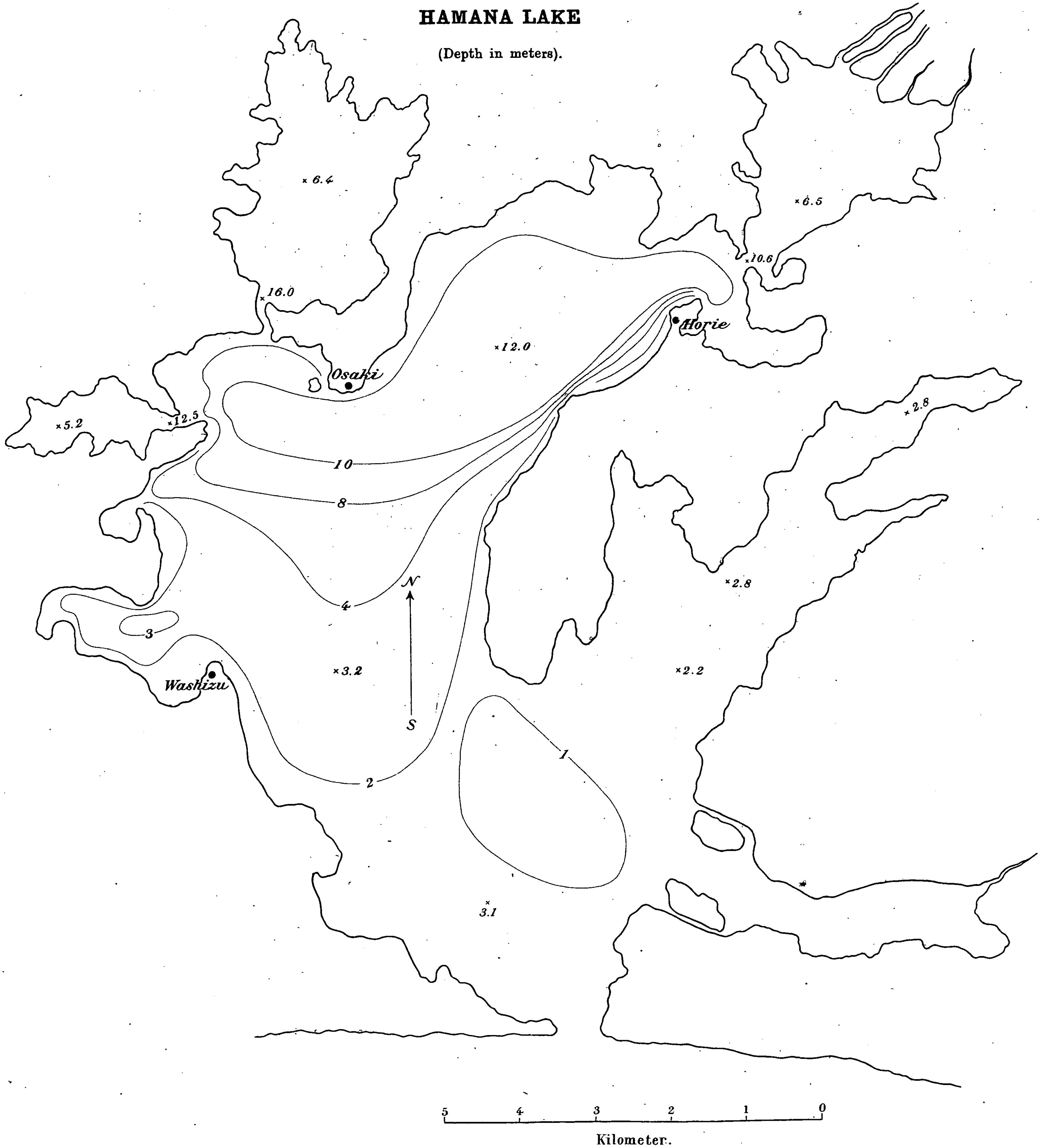

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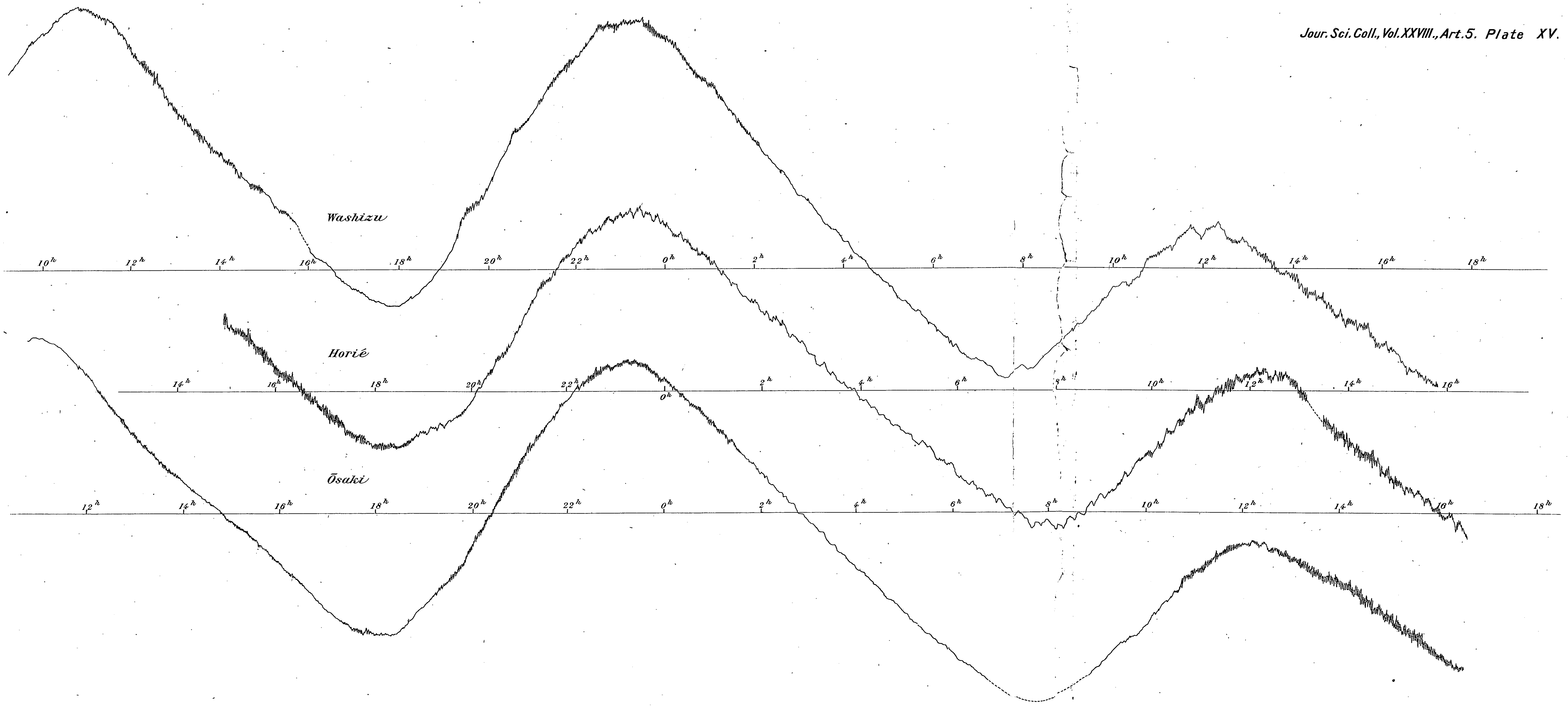
PLATE XV.

PLATE XV.

Limonograms obtained at Washizu, Horié and Ōsaki on Aug. 24–25, 1902.



Washizu
10h
12h
14h
16h
18h
20h
22h
0h
2h
4h
6h
8h
10h
12h
14h
16h
18h
Horié
14h
16h
18h
20h
22h
0h
2h
4h
6h
8h
10h
12h
14h
16h
Ōsaki
12h
14h
16h
18h
20h
22h
0h
2h
4h
6h
8h
10h
12h
14h
16h
18h

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PLATE XVI.

PLATE XVI.

Map of Tōya lake with isobathymetric lines drawn by the Naval **Hydrographic Office.**

TŌYA LAKE

(Depth in meters)

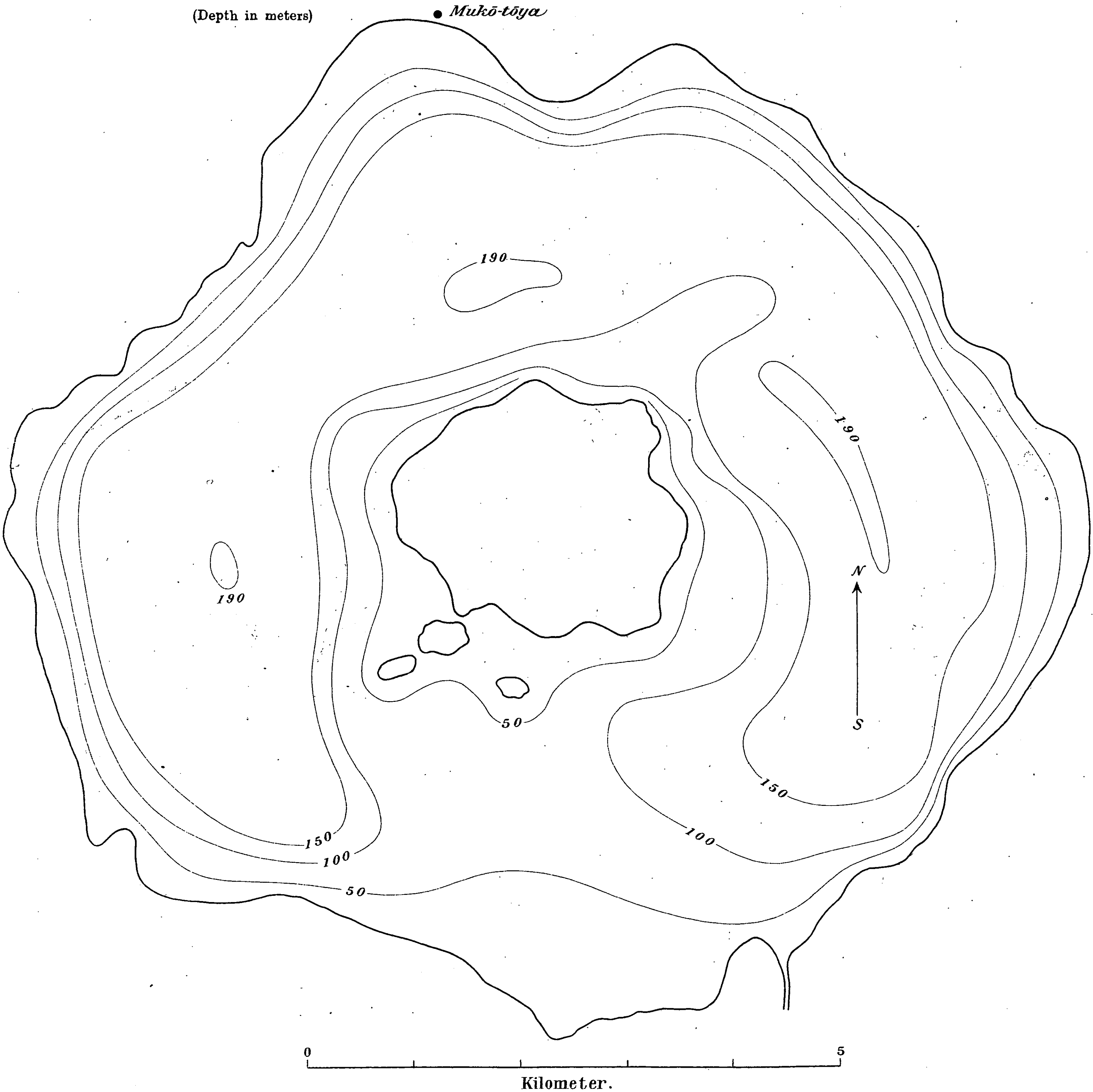

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PLATE XVII.

PLATE XVII.

Photographs of the horizontal motion of water in a model of Tōya lake.

Fig. 1. Tōya lake, $t_1=0.67$ sec.

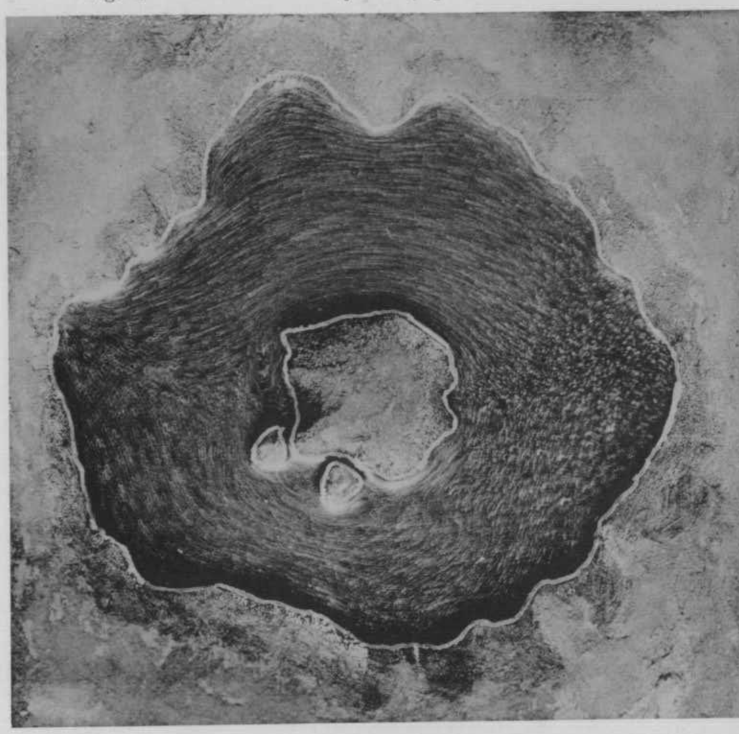

Fig. 2. Tōya lake, $t_2=0.76$ sec.

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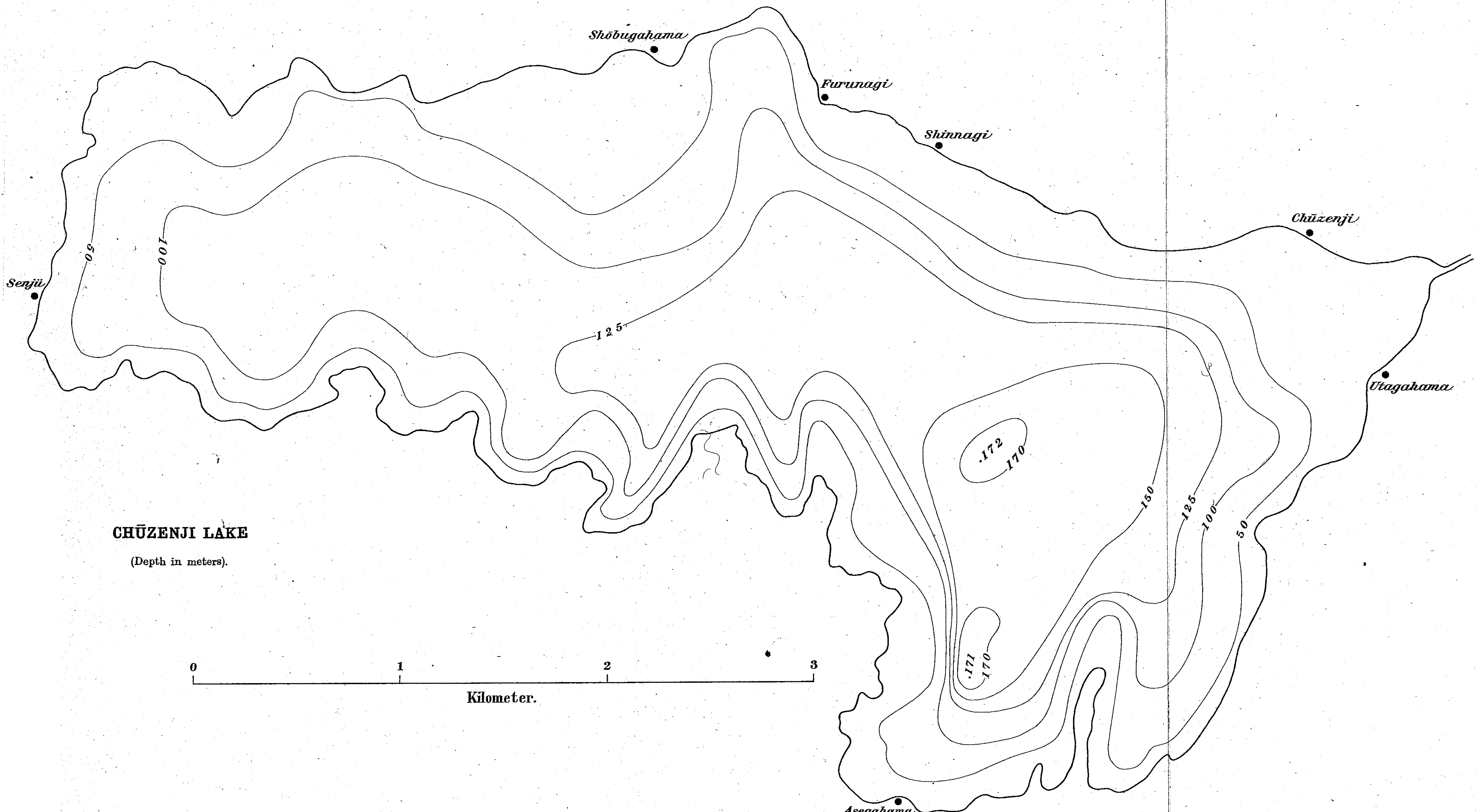
PLATE XVIII.

PLATE XVIII.

Map of Chūzenji lake with isobathymetric lines after Viscount Tanaka.



CHŪZENJI LAKE

(Depth in meters).