JOURNAL OF THE COLLEGE OF SCIENCE, IMPERIAL UNIVERSITY, TOKYO, JAPAN
VOL. XXVII., ARTICLE 19.

Studies on the Variables of the Algol-Type.

By

Naozo Ichinohe, *Rigakushi.*

Lecturer in Astronomy, College of Science, Tokyo Imperial University.

INTRODUCTION.

The variation of light of β Persei was found as early as in 1667 or 1669 by Montanari and it was confirmed by Maraldi in 1694, as he sometimes assigned the third magnitude to this star, while it was usually of the second magnitude; and the year 1667 or 1669 is really the very beginning of this branch of variables. But the law governing the peculiar change of light remained unknown more than one century after the discovery. At length, in 1782, Goodricke discovered that the variation of light of this star was periodic, and the change took place only in a small portion of the period, 9¼ hours, being constantly of the second magnitude during the remaining time. Thus, we may consider the year 1782 in which the law was discovered, as the birth-day of this special kind of variable star. Then, a long time went on without any addition of stars in this class of variables till 1848, in which the similar characteristics of light-change were discovered in the stars S Cancri and λ Tauri, respectively by Hind and Baxendell (except ε Aurigae, which was discovered by Fritsch in 1821 as a variable, and more recently the Algol-like nature of the light-change has been suspected by Ludendorff). Eleven years later, the Algol-like variation of light was again detected in δ Librae by Schmidt, in 1859. Thus, although in early times the variables of this class were considered as rare objects, the number of them slowly increased with time until recently it has increased at a great rate. Thus, it became usual to call any one of them simply as an Algol. At the end of the year 1900, the number of Algol-variables was 21 in all, but great has been our surprise when we learn that during the last ten years the number has become quite great and we could count 93 or 94 stars (omitting or including ε Aurigae) in this class of variables at the end of June, this year.

Some comparative studies have already been tried by several astronomers on the variables of this class; but formerly, the number of the stars was very small, as we have just said, so that perhaps it would be dangerous to regard the conclusions as entirely authentic. Even now, the number is yet not sufficiently large to make some important investigations. When we go on a step farther, we shall soon find that for many of them the necessary investigations of the light-curve are not published yet and this is a great obstacle to the attempt to make a comparative study on these variables. As the cause of the variation of light for some of the variables in this class was confirmed to be interpositions of a dark satellite between a bright star and the observer on the earth, as suggested by Goodricke, by the spectroscopical investigations of the stars by Vogel and others, the extention of the spectroscopic observations on all the Algols is greatly to be desired for the studies of these in order to know the general nature of Algols, but unfortunately we have knowledge of this side only for a few stars and most of them are so faint that they are not suitable for the present state of spectrographs. Nevertheless, certain comparative studies on these stars will be profitable, and with this hope, I ventured to do some work on these variables.

This work was begun two years ago for 55 Algols, then available, and the principal results were reported to the Astronomical Club at this observatory (Japan A. H., I, 101 : 1908). The present discussions are based upon the more increased material, but I found the chief results to be the same as those obtained from the previous investigations.

DEFINITIONS OF AN ALGOL.

Strictly speaking, the variables belonging to the type of Algol must be the ones which indicate the similar characteristics, in the mode of variation of light, with those of the star β Persei. At first, the star was known as a star whose light, though it continues a long time as constant, still in a short time, declines in intensity but after it has attained its minimum brightness, it soon begins to increase in light until the the normal brightness is re-attained. Patient studies on β Persei declared that such change of light repeats itself periodically and the light-curve is nearly symmetrical with respect to the minimum. Thus, as the definition of the typical star of the variables belonging to the Algol type, we can enumerate the following peculiarities.

I. The variation of light at first begins with a slow diminishing of light from the normal. Then the rate becomes more rapid and finally losing the rate of variation, it attains to the minimum after the gradual decrease of light. Then, in nearly reversed order, the light increases till the maximum is attained.

II. Such variation of light is periodical and in only a small portion of the period, the light changes gradually and during the remaining interval, it keeps the normal brightness.

But these definitions are not strictly satisfied actually for all the Algols. Thus, when we study Algol stars, it is of the first importance to define these stars under the admissible modifications. In order to so amend the definitions that all the Algols, ordinarily taken in this class, will be included, it will be necessary to consider what are the points that must be changed.

I. The period is not strictly constant but it is subdued by some variations whose laws are unknown or known in part only.

II. The diminution of light takes place generally in only a small part of the period and the ratio (P/d) between the whole period (P) and the duration of the change of light (d) is included between certain limits.

III. In some Algols, the minimum brightness is not constant.

IV. The decreasing and increasing branches are symmetrical in some of them; nearly symmetrical in others; while they differ pretty considerably in others.

V. In some Algols, there is a secondary minimum between the two successive principal minima.

Now, let us suppose that it is permitted to allow these modifications without limit, then the so-called Algols will disappear from the variables as classified. Here it would seem profitable for us to remember the cause of the class of variables. It is well known fact that Algol is the phenomenon produced by a companion around the principal star whose orbital plane passes through the solar system or very near it. Then theoretically considered, eclipsing variables are not confined in Algol types only. Really, for the stars belonging to the β Lyrae type, the cause is pretty fairly established from the spectroscopical considerations. Still, at present, the classifications divides these stars into two different types; the one, being the Algol and the other, the variable of short period.

At the first meeting in 1903 of the South African Association for the advancement of Science, A. W. Roberts presented a paper concerning the eclipsing star and in it, he arrived at the conclusion that S Verolum, one which shows the stationary minimum during some interval; R Arae, an ordinary Algol-type variable; RR Velorum showing a secondary minimum; X Carinae, having an intermediate state between the Algol-type and ζ Geminorum-type as well as RR Centauri of the type of ζ Geminorum or of the type of β Lyrae belong to the eclipsing star.

But if we should classify the variable stars according to the commonly adopted standards, it would be difficult to include the stars like RR Centauri in the Algol-type. And if we do so, we shall have much trouble to distinguish such stars from the variables belonging to the type of the short period. Thus, the cause of the phenomena might not be the foundation of the classification, in the present state of things, although the Algols will possibly be explained by the eclipse theory in all of them. Previously, the Algols were confined for the variables whose period does not exceed over ten days, but recently the length of the period is extending to one year or so and if we take ε Aurigae in this class of variables, the period is extended to twenty seven years and more. Thus, the length of the period will not also be the foundation of the classification. Therefore, we must be content to classify the variables according to the appearance of the light-curve, till the day will arrive as we can distinguish all the eclipsing stars from the other variables. Then, I formulated the following definitions for the Algol-variables.

I. An Algol is a variable star whose light-curve shows the stationary maximum or normal brightness with an eclipsing phenomenon and the duration, in which the normal brightness continues, is comparable with the whole duration in which the eclipse takes place, or far surpasses it; while the period, the time-interval from a minimum to the succeeding one, is practically constant.

II. The eclipse phenomenon begins by diminishing of light from the normal state and reaches a certain minimum brightness. Then the star begins to regain the light through the minimum which continues some time till it reaches the normal.

III. The light-curve of Algol has the decreasing and ascending branches, which are not very far from symmetry with respect to the middle of the minimum, which continues some time.

IV. The normal brightness is, sometimes, interrupted by a secondary minimum without destroying the condition of the stationary maximum.

V. Elements of an Algol are subjected to certain perturbations whose laws are not yet accurately discovered.

LIST OF ALGOLS.

Accepting the above definitions of the Algol, we will now examine the catalogue of variable stars, to pick up the stars whose light-change goes according to these laws. In this way, we compiled a list of Algols but as there are quite a number of variables for which some variation of light is suspected but not yet confirmed, and also some suspected Algols which need more satisfactory observations, the list will be soon changed. The following table contains 93 Algols, and among these stars, there are a few which are classified by some astronomers as the variables of the short period. But these stars were taken up by myself after some deliberations. The table shows the provisional number of Algol, and the year in which each was found, in the order of its discovery, in the first column. The second column represents the number of the list and this is followed by the name of the star. The fourth and fifth represent the position of the star in right ascension and declination, the equinox of 1900 being adopted. The sixth column is the period in the rounded figures and the seventh is the ratio between the period and the duration of the eclipse, the reciprocal being the duration of the eclipse in the term of the period. The eighth, ninth and tenth represent the maximum brightness, range of variation, and the spectrum respectively.

TABLE I.

Provisional Number.	Year.	No.	Star	α	δ	Period.	P/d	Maximum.	Range.	Spectrum.	Discoverer.
				h m s	° ′	d		m	m		
	1667	1	β Persei	3 1 40	+40 34.2	2.867	7.56	2.1	1.1	B8A	Montanari.
	1848	2	S Cancri	8 38 13	+19 23.6	9.485	10.42	8.2	1.8	A	Hind.
	1848	3	λ Tauri	3 55 8	+12 12.5	3.953	9.38	3.3	0.9	B3A	Baxendell.
	1859	4	δ Libræ	14 55 38	− 8 7.3	2.327	4.03	4.8	1.4	A	Schmidt.
	1869	5	U Coronæ	15 14 7	+32 0.7	3.452	7.73	7.6	1.1	A	Winnecke.
	1871	6	U Ophiuchi	17 11 27	+ 1 19.3	0.839	3.94	6.0	0.7	B8A	Gould.
	1874	7	RS Sagittarii	18 10 59	−34 8.5	2.416	4.57	6.6	1.0	A	Gould.
	1880	8	U Cephei	0 53 24	+81 20.2	2.493	5.42	7.0	2.0	A	Ceraski.
	1886	9	Y Cygni	20 48 4	+34 16.9	1.498	4.54	7.1	0.8	A	Chandler.
	1886	10	V Puppis	7 55 22	−48 58.4	1.454	—	4.1	0.7	B1A	Williams.
	1887	11	R Canis Majoris	7 14 56	−16 12.4	1.136	4.57	5.8	0.6	F	Sawyer.
	1891	12	Z Herculis	17 53 36	+15 8.8	3.993	8.67	7.1	1.2	F	M & K
	1892	13	R Aræ	16 31 26	−56 47.6	4.425	11.55	6.8	1.1	A	Roberts.
	1892	14	X Carinæ	8 29 7	−58 53.2	0.541	1.93	7.9	0.8	A	Roberts.
	1894	15	S Velorum	9 29 27	−44 45.9	5.934	9.59	7.8	1.5	A	Woods.
	1895	16	W Delphini	20 33 7	+17 56.1	4.806	6.68	9.6	2.3	A	Wells.
	1898	17	RX Herculis	18 26 1	+12 32.6	0.889	4.68	7.0	0.6	A	Sawyer.
	1899	18	RR Puppis	7 43 32	−41 7.5	6.430	9.62	9.4	1.3	—	(Cape.)
	1899	19	SY Cygni	19 42 44	+32 27.6	6.006	7.59	11.1	1.9	—	Ceraski.
	1899	20	SW Cygni	20 3 50	+46 0.6	4.573	9.32	9.4	2.1	A	Ceraski.
26	1900	21	Z Vulpeculæ	19 17 32	+25 23.1	2.455	9.84	7.3	1.2	—	Flint.

70	1901	22	V Ursæ Majoris	9 1 12	+51 31.0	201.5	1.94	9.6	1.1	—	Anderson.
73	1901	23	U Scuti	18 48 51	−12 44.0	0.955	5.63	9.1	0.5	A	Ceraski.
78	1901	24	UW Cygni	20 19 38	+42 55.2	3.451	7.87	10.5	2.2	—	Williams.
91	1901	25	RR Velorum	10 17 48	−41 51.3	1.854	13.22	10.1	0.8	—	Innes.
93	1901	26	U Sagittæ	19 14 26	+19 25.7	3.381	6.18	6.9	2.4	A	Schwab.
10	1902	27	UZ Cygni	21 55 13	+43 52.8	31.304	15.65	8.9	2.7	—	Fleming.
13	1902	28	RV Lyræ	19 12 31	+32 14.6	3.599	10.81	11.0	1.8	—	Williams.
14	1902	29	Z Persei	2 33 40	+41 46.1	3.057	6.62	9.4	2.6	—	Williams.
20	1902	30	VV Cygni	21 2 20	+45 22.6	1.477	—	12.1	1.7	—	Ceraski.
4	1903	31	Z Draconis	11 39 50	+72 48.3	1.357	6.79	9.9	3.7	—	Ceraski.
21	1903	32	Y Camelopardalis	7 27 46	+76 17.9	3.305	6.61	9.7	2.1	—	Ceraski.
55	1903	33	VW Cygni	20 11 21	+34 12.5	8.431	10.07	10.3	1.9	—	Williams.
6	1904	34	RX Cassiopeiæ	2 58 49	+67 11.4	32.315	5.38	8.6	0.5	—	Ceraski.
136	1904	35	RV Ophiuchi	17 29 45	+ 7 18.9	3.687	8.78	9.0	2.0	—	Fleming.
154	1904	36	WW Cygni	20 0 37	+41 18.3	3.318	6.76	10.0	2.9	—	Ceraski.
155	1904	37	RT Persei	3 16 45	+46 13.1	0.849	4.99	9.5	2.0	—	Ceraski.
164	1904	38	V Serpentis	18 11 5	−15 33.2	3.453	3.83	9.5	1.6	A	Leavitt.
188	1904	39	RR Draconis	18 40 54	+62 34.5	2.831	7.45	9.9	3.5+	—	Ceraski.
43	1905	40	RU Monocerotis	6 49 22	− 7 28.3	0.896	4.34	9.8	0.7	—	Ceraski.
61	1905	41	RV Persei	4 4 11	+34 0.2	1.974	7.87	10.6	2.2	—	Ceraski.
79	1905	42	RS Cephei	4 48 34	+80 5.9	12.42	—	9.5	2.5	—	Ceraski.
102	1905	43	RW Tauri	3 57 45	+27 51.0	2.769	8.42	7.9	3.7	B5A	Fleming.
103	1905	44	RZ Ophiuchi	18 40 55	+ 7 6.9	261.8	14.54	9.5	0.9	—	Ceraski.
29	1906	45	RW Persei	4 13 20	+42 4.3	13.202	28.78	8.8	2.2	—	Enebo.
30	1906	46	RW Geminorum	5 55 24	+23 8.3	2.865	5.73	9.8	2.3	—	Wolf.
73	1906	47	SS Carinæ	10 54 11	−61 22.9	3.301	5.51	12.3	0.7	—	Leavitt.
77	1906	48	RZ Cassiopeiæ	2 39 54	+69 12.8	1.195	5.17	6.5	1.6	A	Müller.

Provisional Number.	Year.	No.	Star.	α	δ	Period.	P/d	Maximum.	Range.	Spectrum.	Discoverer.
				h m s	° ′	d		m	m		
78	1906	49	SX Sagittarii	18 39 40	−30 35.8	2.077	—	8.7	1.1	—	Fleming.
79	1906	50	RR Delphini	20 38 52	+13 35.1	4.599	9.20	10.5	1.3	—	Ceraski.
120	1906	51	RY Persei	2 38 59	+47 43.3	6.864	7.62	8.0	2.3	—	Ceraski.
121	1906	52	RX Draconis	19 1 8	+58 35.2	1.894	9.07	9.3	0.9	—	Ceraski.
141	1906	53	RZ Centauri	12 55 37	−64 5.4	0.938	1.57	8.5	0.4	A	Leavitt.
149	1906	54	SS Centauri	13 7 9	−63 37.1	2.479	4.96	8.7	1.5	B8A	Leavitt.
158	1906	55	ST Carinæ	10 12 30	−59 42.9	0.902	4.51	9.3	0.9	A	Leavitt.
170	1906	56	SU Centauri	11 6 34	−47 18.0	5.354	5.35	8.7	0.8	A ?	Leavitt.
177	1906	57	SV Centauri	11 43 5	−60 0.5	1.661	2.77	8.8	0.9	A	Leavitt.
182	1906	58	SW Centauri	12 12 28	−49 10.5	5.219	7.45	9.1	2.5	A	Leavitt.
190	1906	59	SY Centauri	13 35 3	−61 15.8	6.631	6.63	9.9	0.8	A	Leavitt.
191	1906	60	SZ Centauri	13 43 50	−58 0.3	2.054	2.05	8.2	0.7	A	Leavitt.
193	1906	61	ZZ Cygni	20 20 41	+46 35.8	0.629	4.08	10.4	1.1	—	Williams.
24	1907	62	RW Monocerotis	6 29 18	+ 8 54.2	1.906	5.96	9.1	1.7	—	Ceraski.
26	1907	63	RZ Draconis	18 21 49	+58 50.1	0.551	2.30	9.5	0.7	—	Ceraski.
27	1907	64	RY Aurigæ	5 11 32	+38 13.1	2.726	9.76	10.8	1.0	—	Ceraski.
29	1907	65	RZ Aurigæ	5 42 53	+31 40.1	3.011	7.92	11.5	2.1	—	Silbernagel.
44	1907	66	RW Ursæ Majoris	11 35 24	+52 33.9	7.33	11.63	9.5	1.0	—	Leavitt.
49	1907	67	RX Geminorum	6 43 38	+33 21.2	12.21	—	8.8	0.8	—	Ceraski.
55	1907	68	W Serpentis	18 4 6	−15 34.0	14.15	—	8.5	1.5	—	Cannon.
56	1907	69	W Scuti	18 18 54	−13 42.5	—	—	9.3	1.1	—	Cannon.

62	1907	70	— Scuti	18 43 40	−10 21.0	0.664	2.02	9.3	1.0	—	Cannon.
68	1907	71	ST Persei	2 53 43	+38 47.5	2.648	—	8.5	2.0	—	Leavitt.
78	1907	72	TT Aurigæ	5 2 48	+39 27.4	0.666	1.96	7.8	0.9	—	Leavitt.
121	1907	73	RR Vulpeculæ	20 50 32	+27 32.3	5.051	15.28	9.6	1.4	—	Ceraski.
136	1907	74	SY Andromedæ	0 8 2	+43 9.3	34.93	21.81	9.1	3.9	—	Ceraski.
143	1907	75	TT Andromedæ	23 8 44	+45 36	2.765	6.59	10.5	0.8	—	Ceraski.
165	1907	76	Y Leonis	9 31 5	+26 40.8	1.686	8.43	9.3	1.9	—	Leavitt.
176	1907	77	— Herculis	16 49 54	+17 0.0	20.755	10.38	8.9	0.6	—	Leavitt.
185	1907	78	V Tucanæ	0 48 10	−72 32.6	—	—	8.6	1.7	—	Cannon.
6	1908	79	RY Geminorum	7 21 42	+15 56.7	9.301	—	8.9	1.1+	—	Ceraski.
13	1908	80	RT Lacertæ	21 57 27	+43 24.5	5.073	5.07	9.1	1.4	—	Ceraski.
16	1908	81	— Vulpeculæ	19 13 25	+22 15.7	4.476	6.59	7.0	1.1	—	Astbury.
48	1908	82	SV Tauri	5 45 49	+28 5.1	5.23	—	9.4	1.6	—	Cannon.
50	1908	83	Z Orionis	5 50 11	+13 40.2	—	—	9.7	1.9	—	Cannon.
51	1908	84	— Geminorum	5 54 33	+24 28.1	4.007	—	9.8	1.2+	—	Cannon.
115	1908	85	SW Ophiuchi	16 11 6	− 6 43.8	2.446	—	9.2	0.8	—	Leavitt.
116	1908	86	SX Ophiuchi	16 12 34	− 6 24.9	2.063	—	10.5	0.7	—	Leavitt.
174	1908	87	SZ Herculis	17 35 57	+33 0.8	0.818	4.82	9.5	0.8	—	Ceraski.
21	1909	88	— Andromedæ	23 58 10	+32 17.3	4.115	10.83	9.1	1.7	—	Kopff.
43	1909	89	— Draconis	18 3 4	+58 23.3	5.16	—	9.3	2.2+	—	Whitney.
3	1910	90	— Pegasi	21 59 48	+35 16.5	—	—	—	—	—	Sperra.
28	1910	91	— Cygni	21 9 0	+30 19.6	0.969	4.46	10.8	0.6	—	Williams.
34	1910	92	— Lacertæ	22 40 36	+49 8.2	5.187	—	10.2	1.0	—	Enebo.
37	1910	93	— Herculis	17 9 45	+30 50.6	—	—	9.5	1.5	—	Ceraski.

NOTES.

1. The period is subjected to an inequality whose law is

$$+147^m \, sin \, (0^\circ.024 \; E + 226^\circ) + 22^m \, sin \left(\frac{E}{13} + 216^\circ\right),$$

according to Chandler IV. The symmetry of the light-curve is discussed by various astronomers. But possibly, it is symmetrical.

2. The minimum brightness is usually of the magnitude $10^m.0$, but there was observed by Schmidt on April 14,1882, an excessive darkening of the star, which remained for a whole hour sunk nearly to the twelfth magnitude.

   The period varies, but its law is not yet discovered.

   The light-curve is not symmetrical, and there is a secondary undura-tion near the minimum phase.

3. Period certainly subject to an inequality, whose law is not yet possible to determine and the deviations from the uniform elements sometimes amounts to three hours.

   Plassmann thinks this star as continuously variable, and he also noticed in 1891 a secondary dip in brightness fifty hours after the chief minimum.

   The decrease of light is more rapid than the increase.

4. Schmidt found an inequality of nine years cycle in period, but this has been suspected by Chandler. The light-curve is symmetrical.

5. Both branches deviate from symmetry, although it is very slight, the increase of light being a little faster.

6. The return of light does not proceed uniformly, according to the observations of Sawyer and Chandler. It is arrested about half an hour after minimum, by a "standstill" of some fifteen minutes as in the case of S Cancri.

   The period is affected by an inequality,

$$-\overset{m}{3.0}\left(\frac{E}{1000}\right) + \overset{m}{0.3}\left(\frac{E}{1000}\right)^2,$$

according to Chandler IV.

7. Two kinds of minima appear in a revolution. The principal is of the magnitude $7^m.6$ and lasts $10^h \; 40^m$ in the eclipse, while the secondary minimum is of the magnitude $6^m.9$ and lasts 7 hours. Both curves are symmetrical.

8. Period subjected to an inequality.

$$+95^m \, sin \, (0°.08 \, E + 283°).$$

The minimum is flat, lasting about two hours.

9. Sawyer's light-curve shows the secondary unduration like S Cancri and others. The minimum of the similar type is repeated alternately in the different intervals and the both periods are subjected to the following inequalities.

even min.	$-0^d.0000000255 \, E,^2$
odd min.	$+0^d.0000000190 \, E.^2$

10. This has been admitted into the Algol-type by Hartwig since 1900: but the light-curve by Roberts, which is reproduced in Clerke's "the System of Stars," indicates that this star belongs to β Lyræ-type. I put this star in this list with a question mark.
11. Nearly symmetrical, but the increase of light is a little faster.
12. This star also shows two kinds of light-curves, both similar but in different periods like Y Cygni. The Principal minimum is of the magnitude $8^m.3$, and the secondary is of $7^m.6$, both appearing alternately in the interval 47 and 49 succeeding the other.
13. Roberts considers it not always to be of the same depth in its minimum. The light-curve is symmetrical.
14. The alternate minima are thought by Roberts to be, to a very small extent, unequal, and to succeed each other at slightly different intervals. The light-curve is symmetrical.
15. The light-curve is symmetrical, but it will be classified in the type of U Cephei, showing a flat minimum, lasting more than five hours.
16. Symmetrical light-curve showing a flat minimum.
17. The light-curve is almost symmetrical, but exhibiting a slightly faster increase of light.
19. The light-curve is not symmetrical, the increase of light being faster.
20. The light-curve is almost symmetrical, still the increase of light being faster than the decrease.
22. By Graff, this star is suspected to be an Algol of a quite long period but his further observations made this doubtful.

    Taking all the literature on this star into consideration, I think this is an Algol whose light-curve is not exactly similar in each minimum.

23. According to Blajko, the light-curve is quite symmetrical.
24. The light-curve shows a flat minimum as in U Cephei during one and a half hours. The curve is almost symmetrical, still the increase of light shows a tendency towards a faster variation than the decrease.
25. The variation of light consists of the two similar symmetrical minima of different depth.

Princ. in $10^m.9$ and lasts 6 hours
Secon. in $10^m.1$ and lasts 3 hours.

26. Perfectly symmetrical light-curve. Williams suspected the variation of colour from red to white during the light-change.
27. On March 23 1903, Hartwig observed a secondary minimum of the range $0^m.4$.
28. Nearly symmetrical.
29. Perfectly symmetrical light-curve, of a flat minimum lasting two hours.
31. Perfectly symmetrical.
32. Inequality of the period by Nijland is

$$-0^d.000000104\ E^2$$

33. The light-curve is symmetrical, the one-third of the entire duration being of constant minimum.
35. Constant minimum lasting one and a half hours. Wendell considered there appeared a secondary minimum of a range of only one tenth of the magnitude.
36. The light-curve is symmetrical, showing a flat minimum which lasts a little over an hour.
37. The light-curve is symmetrical and the decrease and increase of light go on uniformly with the time. According to Dugan, a secondary minimum takes place, its range being $0^m.16$.
38. This star shows two kinds of minima and the light-curve tends to that of β Lyræ but I think it would be proper to include this star in the Algols, as the constant light extends over the half of the period. The principal sinks to $10^m.6$ and the secondary, to $9^m.8$.
39. The range of variation is not yet determined as the minimum exceeds 13^m. The light-curve is symmetrical so far as it is observed.
43. Symmetrical light-curve, showing a flat minimum.
44. Symmetrical light-curve showing a flat minimum extending over 10 days and more.

45. Symmetrical light-curve of a flat minimum extending over three hours nearly.
46. Nijland observed a secondary minimum of the magnitude $10^m.3$ but he noted that it ought to be confirmed by the further observations.
47. Symmetrical.
48. The increase of light is much faster than the decrease.
51. A marked difference between both branches of the light-curve, the increase much faster than the other. Nijland observed it to be symmetrical.
53. The light-curve consists from two symmetrical branches gently curved.
55–56. Same as the above.
57. Two kinds of minima appears on the star, the principal sinks till $9^m.6$ and the other to $9^m.1$, both lasting over a half day.
58. Symmetrical light-curve.
59. Symmetrical light-curve of a flat minimum lasting 0.2 day.
60. Symmetrical curve tending to that of the short period variables.
61. Symmetrical.
62. The light-curve is of the type of S Cancri; a marked difference being noticed between both branches. The rise, being faster.
63. Observations showed some difference between each minimum. But the light-curve is symmetrical.
64. Symmetrical.
67. Symmetrical.
70. Symmetrical, tending to the class of the short period variables.
72. Same as the above.
74. Symmetrical.
77. A secondary minimum of the range $0^m.3$ takes place $11^d.06$ after the principal minimum.
80. Principal and secondary both appear.

Princ.	$8^m.8$–$9^m.8$, in range and lasts $0^d.6$	
Secon.	$8^m.8$–$9^m.3$ in range and lasts $0^d.6$.	

The curve is nearly symmetrical.
81. Symmetrical, showing a flat minimum lasting 0.2 day.
87. Unsymmetrical, the decrease of light being faster than the increase. The ascending branch accompanies secondary unduration.
91. Uniform decrease and increase of light with a flat minimum lasting an hour.

DENSITY OF ALGOLS.

It is well known fact that the limiting value of the mean density of an Algol-system will easily be found when we have the duration of the eclipse as well as the period determined. In 1899, A. Roberts (A. P. J. X, 308) considered the densities of the four southern Algols observed by himself. Denoting the period of an Algol by P; semi-major axis, by a; the diameters of the components (1) and (2) by pa and qa, respectively and the masses of the components by m_1 and m_2; we have the formulas for the densities of the both components.

$$\text{Density of (1)}=\frac{(0.0092)^3}{p^3\ P^2}\left(\frac{m_1}{m_1+m_2}\right),$$

$$\text{Density of (2)}=\frac{(0.0092)^3}{q^3\ P^2}\left(\frac{m_2}{m_1+m_2}\right),$$

where sidereal year is taken as the unit of time and that of distance is the mean distance of the sun from the earth, namely, the astronomical unit. Thus, if we know the ratios m_1/m_2 and m_2/m_1, the density for each component will be separately known, by the applications of the above formulas, as p and q are the quantities to be determined from the examinations of the elements of Algol-variation. Now, putting

$$\frac{m_1}{m_2}=k, \qquad \frac{m_2}{m_1}=k',$$

the latter factors of the above formulas will be written as follows

$$\frac{1}{1+k}, \qquad \frac{1}{1+k'}.$$

As $\frac{1}{1+k}$ and $\frac{1}{1+k'}$ never exceeds unity, we can calculate the absolute maximum value of the density for both components by

$$\frac{(0.0092)^3}{p^3\ P^2} \text{ and } \frac{(0.0092)^3}{q^3\ P^2}.$$

Under such process, assuming $k=k'=0$, Roberts obtained the absolute maximum values of density for the star X carinæ and other three stars. Finally, assuming k and k' equal, he reached the average density and the mean average density of the four stars was found, in the unit of the density of the sun, 0.13.

The above process is very useful in finding the separate density of the both components, but unless we have previously determined the mass of the components, the results obtained give only a general idea. At the present state of astronomical knowledge, therefore, the limiting values of the mean density of the system may profitably be used.

Now, let us denote

n=mean motion of the satellite around the principal star,

ν=mean motion of the earth around the sun

R=radius of the sun,

Δ=mean distance of the sun from the earth

r=radius of the principal star

δ=density of the Algol under consideration

δ_0=density of the sun

a=mean distance of the satellite.

Then, from the application of Kepler's third law, we have

$$\delta=\delta_0\ \frac{\left(\frac{n}{\nu}\right)\ h^3}{\Delta^3\,(1+\varkappa)^3}\left(\frac{a}{r}\right)^3, \quad\dots\dots(1)$$

where $\varkappa$ is the radius of the satellite, unit being that of the principal star. But we can simplify the above formula by putting Y siderial year expressed in days, and considering R and a expressed in the terms of the astronomical unit and the radius of the principal star respectively. Thus, we obtain

$$\delta = \delta_0 \frac{Y^2 R^3 a^3}{P^2 (1+x^3)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Therefore, for the Algols whose orbits are already determined, as in ten Algols which were observed by Graff and their circular orbits were also determined by him (Mitt. Hamburger Sternwarte, No. 11), the density will easily be obtained, as Graff's orbits were used by Ristenpart (A. N., 178, 31).

But for the stars in which we do not yet know the elements of the orbit, it would be more convenient for us to replace a and x by elements of the light-variation themselves.

As $$1 + x > a \sin \frac{\pi d}{P},$$

we have $$\delta < \frac{\beta}{P^2 \sin^3 \frac{\pi d}{P}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where d represents the duration of the eclipse. As it is easily seen, the value obtained with the formula (3) always exceeds or equals to that given by (2). To determine the limiting value of density according to the formula, it is the first importance for us to determine the constant. H. N. Russell (A. P. J. X, 315) determined this constant, taking the density of water as the standard and assuming the density of the earth 5.53, and also expressing time in hours, as follows:

44.1

André, in his "Traité d'Astronomie Stellaire," (Tome II, 308), deduced the constant from the known density of the Algol, which is previously determined to be 0.23 in the term of the solar density and his constant is, converted into the standard density,

52.2.

Recently Stein gave the constant as follows (M. N. LXIX, 449):

44.0,

which is also converted into the standard density.

So far as I know, the density of the Algols was calculated by the above four astronomers, Roberts on the four stars: Russell on 17 Algols, then available for this purpose, using the data from Chandler's Third Catalogue of Variable Stars and also from other informations. Although André gave somewhat different results for these stars, the data were used exclusively according to Russell and the difference is the result from the different value of the constant. Ristenpart restricted his determination on the stars for which Graff determined the circular orbits. Stein extended his researches on the Algols for which he was able to obtain the necessary data, and the number of the star was 38. My work on this side were begun two years ago before knowledge of Stein's work, adopting the value of constant by Russell. Finally, at the close of the present investigation, I was able to calculate the density for 74 stars, for which torelably known data were collected.

The formula for the calculation was

$$\mathrm{Dm} = \frac{0.07656}{P^2 \sin^3 \frac{\pi d}{P}} \qquad [2.8840]$$

or

$$\mathrm{Dm} = \frac{0.07656}{P^2 \sin^3 \frac{\pi}{r}},$$

where the period is expressed in days and r denotes the ratio between the period and the duration of the eclipse.

In the following table, the results of computation are arranged in the order of increasing period with the data needed for the reduction. The first column is the number of the above list and the second represents the name of star. The third is the period and the next two columns represents the ratio between the period and the duration of the eclipse, and its reciprocal. The last column is devoted to the required density of the Algol-systems. As the constant for the above formula is the same with that of Russell, the results must be coincident to each other for 17 stars when the material is also based on the same catalogue. In fact, as I used different data for some of them, we cannot avoid a little difference between them. At the same time, as the results of 37 stars by Stein are given in the unit of solar density, they will be comparable with mine, when a constant factor 1.41 is multiplied. Actually, as the data for some stars differ from his, the results will not be same even when that factor is taken.

TABLE II.

No.	Star.	Period.	r	$\frac{1}{r}$	Density.
14	X Carinæ	$0^d.541$	1.93	0.518	0.263
63	RZ Draconis	0.551	2.30	0.435	0.269
61	ZZ Cygni	0.629	4.08	0.245	0.573
70	62.1907 Scuti	0.664	2.02	0.495	0.174
72	TT Aurigæ	0.666	1.96	0.510	0.173
87	SZ Herculis	0.818	4.82	0 207	0.512
6	U Ophiuchi	0.839	3.94	0.254	0.297
37	RT Persei	0.849	4.99	0.200	0.521
17	RX Herculis	0.889	4.68	0.214	0.403
40	RU Monocerotis	0.896	4.34	0.230	0.328
55	ST Carinæ	0.902	4.51	0.222	0.356
53	RZ Centauri	0.938	1.57	0.637	0.116
23	U Scuti	0.955	5.63	0.178	0.566
91	28.1910 Cygni	0.969	4.46	0.224	0.300
11	R Canis Majoris	1.136	4.57	0.219	0.232
48	RZ Cassiopeiæ	1.195	5.17	0.194	0.288
31	Z Draconis	1.357	6.79	0.147	0.467
9	Y Cygni	1.498	4.54	0.220	0.131
57	SV Centauri	1.661	2.77	0.361	0.037
76	Y Leonis	1.686	8.43	0.119	0.558
25	RR Verolum	1.854	13.22	0.076	1.708
52	RX Draconis	1.894	9.07	0.110	0.545
62	RW Monocerotis	1.906	5.96	0.168	0.165
41	RV Persei	1.974	7.87	0.127	0.335

No.	Star.	Period.	r	$\frac{1}{r}$	Density.
60	SZ Centauri	2.054 d	2.05	0.488	0.018
4	δ Libræ	2.327	4.03	0.248	0.041
7	RS Sagittarii	2.416	4.57	0.219	0.051
22	Z Vulpeculæ	2.455	9.84	0.102	0.412
54	SS Centauri	2.479	4.96	0.202	0.061
8	U Cephei	2.493	5.42	0.185	0.075
64	RY Aurigæ	2.726	9.76	0.102	0.325
75	TT Andromedæ	2.765	6.59	0.152	0.104
43	RW Tauri	2.769	8.42	0.119	0.206
39	RR Draconis	2.831	7.45	0.134	0.139
46	RW Geminorum	2.865	5.73	0.175	0.066
1	β Persei	2.867	7.56	0.132	0.141
65	RZ Aurigæ	3.011	7.92	0.126	0.146
29	Z Persei	3.057	6.62	0.151	0.086
47	SS Carinæ	3.301	5.51	0.181	0.045
32	Y Camelopardalis	3.305	6.61	0.151	0.073
36	WW Cygni	3.318	6.76	0.148	0.077
26	U Sagittæ	3.381	6.18	0.162	0.058
24	UW Cygni	3.451	7.87	0.127	0.110
5	U Coronæ	3.452	7.73	0.129	0.105
38	V Serpentis	3.453	3.83	0.261	0.016
28	RV Lyræ	3.599	10.81	0.093	0.251
35	RV Ophuichi	3.687	8.78	0.114	0.131
3	λ Tauri	3.953	9.38	0.107	0.138
12	Z Herculis	3.993	8.67	0.115	0.108
88	21.1909 Androm	4.115	10.83	0.093	0.193
13	R Aræ	4.425	11.55	0.087	0.202

No.	Star.	Period.	r	$\frac{1}{r}$	Density.
81	16.1908 Vulpeculæ	4.477 d	6.59	0.152	0.040
20	SW Cygni	4.573	9.32	0.107	0.101
50	RR Delphini	4.599	9.20	0.109	0.096
16	W Delphini	4.806	6.68	0.150	0.036
73	RR Vulpeculæ	5.051	15.28	0.065	0.352
80	RT Lacertæ	5.073	5.07	0.197	0.015
58	SW Centauri	5.219	7.45	0.134	0.041
56	SU Centauri	5.354	5.35	0.187	0.016
15	S Velorum	5.934	9.59	0.104	0.065
19	SY Cygni	6.001	7.59	0.132	0.033
18	RR Puppis	6.430	9.62	0.104	0.056
59	SY Centauri	6.631	6.63	0.151	0.018
51	RY Persei	6.864	7.62	0.131	0.025
66	RW Ursæ Majoris	7.33	11.63	0.086	0.075
33	VW Cygni	8.431	10.07	0.099	0.037
2	S Cancri	9.485	10.42	0.096	0.033
45	RW Persei	13.20	28.78	0.035	0.338
77	— Herculis	20.755	10.38	0.096	0.007
27	UZ Cygni	31.30	15.65	0.064	0.010
34	RX Cassiopeiæ	32.32	5.38	0.186	0.000
74	SY Andromedæ	34.93	21.81	0.046	0.022
22	V Ursæ Majoris	201.5	1.94	0.515	0.000
44	RZ Ophiuchi	261.8	14.54	0.069	0.000
	ε Aurigæ	27.13 y	13.62	0.073	0.000

ALGOLS AND GALAXY.

It is a well known fact that the number of stars decreases with the galactic latitude, and the relation between the number of stars and the milky way has been investigated by several astronomers. "Sir John Herschel first called attention to the fact that while a number of the brightest stars in the heavens lie near the course of the galaxy, they symmetrically deviate from it in the direction of a great circle cutting the galaxy at two nodes,.........This relation of the stars in question was fully investigated by Gould." Gould derived the position of the pole of the circle as follows:

$$A = 171^\circ.2$$
$$D = 30^\circ$$

This pole does not exactly coincide to that of the galaxy but nearly same. Recently Newcomb determined various principal circles and found the following results.

Circles		A	D
Galactic Plane (1)		192°.8	27°.2
Galactic Plane (2)		191°.1	26°.8
Stars up to mag.	$2^m.5$	181°.2	17°.4
Stars up to mag.	$3^m.5$	180°.0	21°.5
Wolf-Rayet Stars		190°.9	26°.7

Galactic plane (1) is the result from the stars omitting the branch of the Galaxy between Cygnus and Aquila and (2) is that from the whole stars including the branch.

If such general tendency prevail in all the magnitudes of stars, then we can easily conclude that perhaps this relation will be found in the Algols in their distribution over the heaven, if we can assume that the Algols partake of the common properties of the ordinary stars. This problem was taken up by Prof. Pickering and he found the pole of the circle for which the Algols tends to agglomerate. Although I could not see the original paper, I saw his determination in Clerke's "System of Stars" and it is

$$A=195^\circ$$
$$D=\ 20^\circ$$

This pole somewhat deviates from the pole of the milky way and naturally a question will be raised whether the plane of condensation differs from the galaxy, or the result is only an apparent phenomenon produced from the insufficient material. In fact, Prof. Pickering had very few Algols to dispense with. Thus, if we can suppose that those few Algols were not found uniformly over the entire sky and thus the insufficient material may produce a great error, it would be very important to work on the increased material. With this idea, I took the problem to determine the pole from 93 Algols, enlisted above.

According to the method by Newcomb, used in his "On the Position of the Galactic and other Principal Planes toward which the Stars tend to crowd," a, b, c, are the rectangular coordinates of any one of the given stars, referred to the axes passing through the pole, the equinox of 1900.0 and the point on the equator in 90° of the right ascension: namely, we have

$$a=\cos\delta\ \cos\alpha$$
$$b=\cos\delta\ \sin\alpha$$
$$c=\sin\delta.$$

If we put X, Y, Z the direction-cosines of the required pole referred to the same axes, then the cosine of the distance of the star from the pole, P, becomes

$$P=aX+bY+cZ.$$

Then, squaring it, we have,

$$P^2=a^2X^2+b^2Y^2+c^2Z^2+2caZX+2abZY+2bcYZ.$$

Now putting

$$A=\Sigma a^2 \qquad D-\Sigma ab$$
$$B=\Sigma b^2 \qquad E=\Sigma ac$$
$$C=\Sigma c^2 \qquad F=\Sigma bc$$

for the sums of all the Algols, we have

$$\Sigma P^2=AX^2+BY^2+CZ^2+DXY+EZX+FYZ.$$

Thus, actual computations on the 93 Algols give us

$$A=15.953 \qquad D= \ -4.397$$
$$B=40.896 \qquad E=+13.248$$
$$C=36.138 \qquad F=-\ 1.912.$$

The condition, that ΣP^2 is a minimum, is $d\Sigma P^2=0$, namely

$$(AX+DY+EZ)\,dX+(DX+BY+FZ)\,dY+$$
$$(EX+FY+CZ)\,dZ=0.$$

Besides this, the relation $X^2+Y^2+Z^2=1$ gives a condition

$$XdX+YdY+ZdZ=0.$$

Then, the Lagrangean process of an indeterminate multiplier λ is applied to determine the values of X, Y, Z. For this case, λ will be determined by the solution of the cubic equation

$$\begin{vmatrix} A-\lambda & D & E \\ D & B-\lambda & F \\ E & F & C-\lambda \end{vmatrix}=0$$

Actual calculation of the above equation such that it is in the usual form of the cubic equation, we have

$$\lambda^3-93\lambda^2+2508.4\lambda-15865.3=0.$$

Now solving this equation, we have three real roots,

$$\lambda_1=9.09, \quad \lambda_2=38.15, \quad \lambda_3=45.76$$

Substituting the smallest root in the simultaneous equations,

$$\begin{aligned}(A-\lambda)\,X+\;DY+EZ\;&=0\\ DX\;+(B-\lambda)\,Y+FZ&=0\\ EX\;+FY+(C-\lambda)Z\;&=0,\end{aligned}$$

the solution with respect to the three quantities X, Y, Z will finally determine the position of the pole. But, in order to derive the necessary co-ordinates of the pole, A and D in the right ascension and declination, we have the following three equations

$$\begin{aligned}X&=\cos D\;\cos A\\ Y&=\cos D\;\cos A\\ Z&=\sin D.\end{aligned}$$

Thus, by the final reduction, we arrive to the result

$$\begin{aligned}A&=186^\circ.2\\ D&=\;25^\circ.6.\end{aligned}$$

Comparing this result with that from the galaxy including the branch of it, we see that both the right ascension and the declination do not exactly coincide, but the present result differs very widely from the that obtained by Prof. Pickering from a small number of Algols

$$\begin{aligned}A&=195^\circ\\ D&=\;20^\circ.\end{aligned}$$

Although the southern astronomy developed rapidly, yet we have only 28 Algols in the southern hemisphere against 65 in the northern. Notwithstanding that we can not decide whether this is due to the fact that the southern heaven is not fully explored or not, it would be more reasonable to suppose the former be the case. Thus, I think further exploration will produce some change to the values of the pole, as the present number made some change on the result by Pickering. Thus, so small a deviation of the pole of the Algols from that of the galactic plane might be an accidental error caused by the insufficient material to eliminate the effect of the systematic discovery of the Algols and I think that Algols tend to agglomerate toward the galaxy. Now let us transform the position of the Algols into the galactic longitude and latitude, then we would arrive at the result that the algols increase in number as the position tends to the central line of the milky way.

For the transformation, the pole of the milky way was assumed, according to the result by Prof. Kobold's investigation,

$$A = 191.5$$
$$D = 28.0$$

in order to facilitate the transformation by the use of a graphical table devised by M. Stroobant, in "Annales de l'Observatoire Royal de Belgique, Tome XI.

Counting the number of stars in the order of the galactic longitude in every half quadrant, we have the result.

TABLE III.

Gal. Long.	Number.	Gal. Long.	Number.
0°– 45°	23	180°–225°	1
45 – 90	15	225 –270	10
90 –135	18	270 –315	5
135 –180	13	315 –360	9
Sum	68	Sum	25

Possibly, in the future, we may discover 40 and more Algols in the southern heavens than we have the possibilities of the new discoveries in the northern sky, if we have reason to believe in the uniform distribution of the Algols through the entire sky.

But if we consider the distribution according to the present statistics as real, the number is the maximum in the region contained between the ascending node in the constellation Aquila and the middle of Cygnus and in the region contained between the descending node in Monoceros and the middle of the constellation Argo, the number of Algols is minimum, here it being only one. A graphical representation of Table III may clearly give the general tendency of the distribution of these known stars.

Let us now examine the law of agglomeration of the Algols with respect to the galactic latitude. Suppose the whole celestial sphere is divided into the nine zones, each zone being 20 degrees in its width: thus, the first zone extends from the south pole of the galaxy to the circle of the galactic latitude –70°, the next from –70° to –50° and so on. Then, of course, the fifth zone corresponds to the central part of the milky way, and the last, the region contained between the north pole and the galactic latitude $+70°$. In the following table, the first column represents the galactic zones numbering from the south pole; the second shows the number of Algols contained in the zone; the third, relative area of each zone. The fourth column represents the mean density of distribution of the Algols in each zone and the last indicates the densities of stars in these zones, resulted from Bonner Durchmusterung and investigated by Seeliger.

TABLE IV.

Zone.	Number.	Area.	Algols.	Stars.
I	0	302	0.00	0.38
II	0	869	0.00	0.41
III	1	1330	0.03	0.47
IV	17	1632	0.38	0.77
V	47	1736	1.00	1.00
VI	18	1632	0.41	0.68
VII	8	1330	0.22	0.45
VIII	2	869	0.09	0.37
IX	0	309	0.00	0.35

Looking over this table, we can easily see that the Algols are mostly crowding themselves in the three successive zones IV, V and VI; evidently the zone V is the central part of the milky way and the other two are consecutive ones to the middle zone. Possibly they are parts of the milky way itself: thus, we can safely conclude that the tendency of the crowding of the Algols toward the milky way is pretty remarkable. Besides this, we see that the corresponding zones both in the southern and northern, hemisphere show the different densities of distribution and this fact tends to assert that the southern hemisphere have a scantier number of Algols than the other hemisphere.

But assuming that in both hemisphere the density is uniform and the apparent phenomenon is only the result from the cause of the southern sky unexplored by astronomers as if in the northern, we shall take the mean density of the two corresponding zones in the both hemispheres. The result is in the following small table.

TABLE V.

Zone.	Algol.	Stars.	Ratio.
V	1.00	1.00	1.00
IV & VI	0.40	0.73	0.55
III & VIII	0.13	0.46	0.28
II & VIII	0.05	0.39	0.13
I & IX	0.00	0.37	0.00

The last column, as the ordinate, and the middle of the zone, as the abscissa, represents a curve showing a slight curvature, but the ratios are nearly in geometrical progression: the actual quotients of these values, the preceding divided by the following are 1.82, 1.96 and 2.00. Thus, it may be possible to take these as a constant ratio 1.93. As we have such a simple relation, we can say that the of rate agglomeration is quite rapid in Algols compared with that of the ordinary stars. The same relation was two years ago established by myself from 55 Algols, available for the statistics at that time, but the common ratio came out as 2.10.

Now suppose that the relation between the galactic latitude and the density of distribution of the stars in general can be expressed by an exponential function, then the density of distribution of the Algols over the sky will also be found immediately.

Denote y_s=stellar density of distribution in general.

y_a=density of distribution of the Algols

x =absolute galactic latitude divided by 10,

then we have, from the supposition,

$$y_s = e^{-f(x)}$$

and consequently

$$y_a = (1.93)^{-\frac{x}{2}} e^{-f(x)},$$

assuming that

$$f(x) = ax + bx^2$$

where a and b are constants. By the application of the method of the least squares on the observed data, we obtain the values of a and b, as follows;

$$a = 0.228 \qquad b = -0.013.$$

∴ Therefore, we have

$$y_s = e^{-(0.228x - 0.013x^2)}$$

$$y_a = (1.93)^{-\frac{x}{2}} e^{-(0.228x - 0.013x^2)}$$

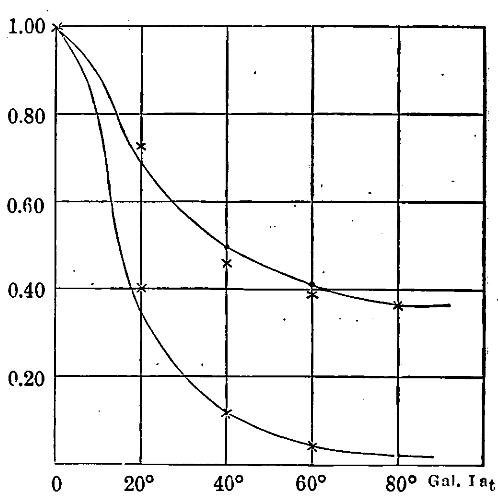
The following table shows the comparizon of the observed values with the computed by these formulas.

TABLE VI.

	Star.			Algol.		
Gal. Lat.	Observed.	Computed.	O–C.	Observed.	Computed.	O–C.
0°	1.00	1.00	0	1.00	1.00	0
20	0.73	0.67	+0.06	0.40	0.34	+0.06
40	0.46	0.50	−0.04	0.12	0.13	−0.01
60	0.39	0.41	−0.02	0.05	0.05	0.00
80	0.37	0.37	0.00	0.00	0.02	−0.02

Fig. I.

Density of Distribution



Upper, stars in general
Lower, Algols only

As the agreement of the computed with the observed is pretty remarkable, the above may be serviceable to give a general idea for the tendency of the distribution of Algols over the heavens. Fig. I, accompanied here, represents graphically the above tables.

STATISTICAL RESULTS OF ALGOLS.

(a) DISCOVERY.

The first star of this class of variable stars was β Persei of the second magnitude, and the two following stars added to the class were not so bright. But generally speaking, the brighter stars were detected in the first period of the history of Algols. This would be a natural consequence of the fact that faint stars did not call us much attentions as now in the early age of the science. Generally, with the development of astronomy, the knowledge of this class of variable stars has also made great progress, since the latter half of the last century; but the last decade especially has been noted for the increase of number. We might enumerate many causes which influenced such progress but the construction of the stellar map containing faint stars till those of the ninth magnitude or even more fainter, as in the Bonner Durchmusterung or the Cape Photographical Durchmusterung and the applications of the photography into the astronomical observation would be the principal causes, combined with the increase of astronomers also.

Perhaps, some considerations on the discovery of the Algol might be interesting as they would show what might expect in the future in general. The table below, VII, shows how the number of Algols increased with time, and in it, from 1845 to the present, each five years have been grouped together. The first column represents the interval of time in which each group extends; the second, the number of Algols added in this interval; the third, the total number of the Algols at the end of each period and the last three columns show the mean period, the mean maximum brightness and the mean range of variation respectively.

TABLE VII.

	Number.	Total.	Period.	Maximum.	Range.
1845–9	2	3	6.8	5.8	1.4
1850–4	0	3	—	—	—
1855–9	1	4	2.3	4.8	1.4
1860–4	0	4	—	—	—
1865–9	1	5	3.5	7.7	1.1
1870–4	2	7	1.6	6.3	0.9
1875–9	0	7	—	—	—
1880–4	1	8	2.5	7.0	2.0
1885–9	3	11	1.4	5.7	0.7
1890–4	4	15	3.7	7.4	1.2
1895–9	5	20	4.5	9.3	1.6
1900–4	19	39	16.4	9.4	2.0
1905–9	50	89	10.5	9.3	1.4
1910	4	93	3.1	10.2	1.0

The examination of the above table shows that the rate of increase greatly accelerated in the last ten years and at the same time, the maximum or normal brightness in its mean value became fainter. Although it is not very significant, the period became longer with the time, or at least, in recent years a few Algols exhibiting comparatively long period have been discovered. Possibly, in the future, we may be able to discover Algols whose range of variation lie less than a half magnitude, but at present, we can not see that the range is decreasing with time, in average. The mean value of ratios of the period by the duration of the eclipse also has not any systematic evidence.

Of course, there might be some brighter stars which really belong to this class of variable stars, while astronomers have as yet failed to detect the nature of the light-variation. But these would be comparatively rare except those having a very small range of variation, and the most fruitful field will be found in the stars whose normal brightness is fainter than the ninth magnitude. Thus, the most part of the remaining region will be cultivated by the photographic explorations.

(b) SPECTRUM OF ALGOLS.

Among 93 or 94 stars of the Algol-type variables, we know their spectrum only for 34 cases of them. Thus, of about two thirds of them, we have not the important knowledge on these stars and it is a great regret for us when we wish to study the properties of this class of variable stars. Classifying their spectrum according to the Draper Catalogue of the Harvard College Observatory, we have the results given in the table below.

TABLE VIII.

Type	B–A	6 stars
,,	A	24 stars
,,	F	3 stars
,,	F–G	1 star
Sum		34 stars.

Thus, if we speak only about the Algols whose spectrum became already known, it would be quite safe to conclude that the Algol, in general, displays the spectrum of the type A. Besides this predominant type of spectrum, we see some stars in the earlier type, but on the contrary, the more advanced type of the spectrum is very rarely met with. This peculiarity may be either connected with the physical properties of the Algol-system or caused accidentally on account of our imperfection to detect the light variation for the advanced stars. Prof. Campbell lately studied spectroscopic binaries statistically and touched on this question and he said as follows (Publ. A. S. P., XXII, 55): "The two members of an Algol-system are in general so near each other and so large in diameter that eclipses occur with great ease: they are observable by those of us who are not situated exactly in the plane of orbit; the eclipses last long time, so that even the unsystematic observations of the past have readily detected variable brightness. As the two members of a system grow further and further apart, corresponding to the longer periods, the number of eclipsing pairs decreases very rapidly,.........Algol stars of the type G–M appear not to have been observed for the reasons that the binary stars of these types have their components relatively far apart; and the eclipses would be of relatively short duration both because they are smaller in diameter, being more condensed. The observer would have to be situated very closely in the plane of the orbit in order to witness an eclipse and his chances for observing an eclipse would be small, as compared with eclipses of close system."

Prof. Schlesinger and Baker expressed the same opinion in their paper "A comparative Study of Spectroscopic Binaries"

(Allegh. Publ., I, 158) as follows; "It is well known fact that the spectra of Algol variables are almost always of very early types. Our explanation for this is as follows: when a binary is in its first stage the two components are of early type, their diameters are large and their separation is small. Such a system would appear as an eclipsing variable from all points situated in a wide zone of the celestial sphere. As the type advances the two stars become smaller and their distance apart increases. Both these circumstances narrow the zone from which an eclipse would now be visible. Consequently in any list of these variables we should expect the number to decrease with the advance of spectral type." This view seems to be acceptable in any way, as it is difficult to think that the star become single as it ages, but we ought not to forget to remark that even when we accept it, still there are points difficult to explain.

As we saw at the beginning of this article, Algol-type variables are mostly found in the type A of the spectrum and only six stars of the earlier type are included against 24 of the type A. But when we pass to the spectroscopic binaries, the phenomenon is somewhat different: looking over the lists given in the paper by Campbell, above cited, we see that the number of the spectroscopic binaries of the type O and B is 48, but that of the type A is 28 so that, roughly speaking, it is the half of the former. Why can we not discover the Algol character in the earlier types in the equal proportion with the spectroscopic binaries? Will it be not easier to find out the character or to see the eclipse for such earlier stars?

As we saw (pp 28–31), the stars which belong to the type of Algol, tend to agglomerate in the plane of the milky way at an extreme rate. Besides this, the ratio between the period and the duration of the eclipse does not show very wide range and it is limited to 29 in the extreme case, most part of them being included in the limit from three to nine. Such facts make it difficult to accept the above view without further researches.

The mean value of the period of the six stars of the Orion type is 2.4 day, that of the type A is 3.3 day and that of the three stars of the type F is 3.4 day. ε Aurigæ which is a suspected Algol and of the type F–G, has very long period. Therefore, although these values are increasing with the type of spectrum, it is difficult to think the relation well established. The ratio between the period and the duration of eclipse, as well as the density of the Algol-system does not show any relation with the spectrum from such poor material.

(c) PERIOD OF ALGOLS.

The period of the light-variation of Algol-type variables is, in general, short, ranging from a half day to about ten days and only a few exceptions are now known, the extreme case being RZ Ophiuchi of the period 261.8 days and in ε Aurigæ, a suspected Algol, the period is very long as it is 27.13 in years. Thus, if we omit the suspected one, we can say that all of 93 Algols displays a period of less than one year.

Now let us investigate the distribution of the Algols with regard to period; counting the number of Algols in the order of period, we have the following result: here the first column representing the limits of period and the second, the number of stars found in the limits given in the first column.

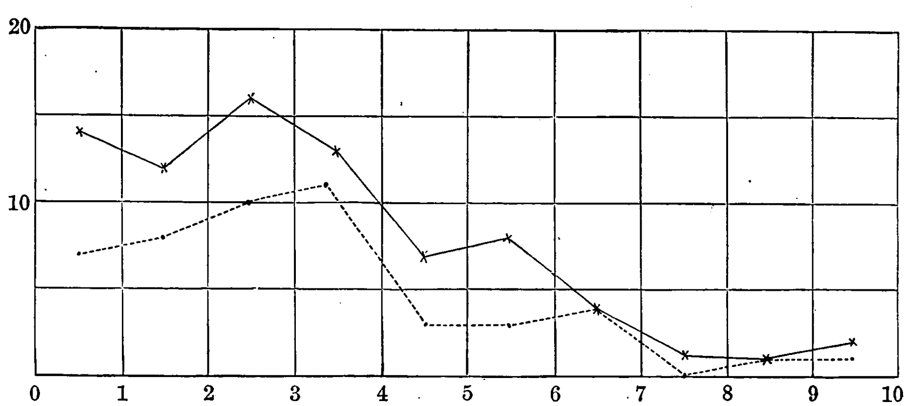
TABLE IX.

Period. (day)	Number.	Period. (day)	Number.
0–1	14	5–6	8
1–2	12	6–7	4
2–3	16	7–8	1
3–4	13	8–9	1
4–5	7	9–10	2

10 Algols have longer periods than 10 days.

In the above table, five stars are omitted because their exact period is yet unknown. In the accompanying figure, the upper curve represents the table graphically.

Fig. II.
Distribution of Periods.



If we speak of the 88 Algols, Algol-type variables have a period from a half day to four full days in 55 cases; from four days to eight days in 20 cases: thus, Algols in the longer period are very rare. The lower curve in the figure is the result from 55 Algols when we made some investigation in 1908; the further addition of Algols does not materially change the result as it is clearly shown in the upper curve. Formerly, it was noted that Algols whose periods lie between two and four days were mostly prevailing, but recently the relation is somewhat changed, as the number of Algols having a period less than one day have increased: still on the whole, the characteristic point that the Algol-type variable in general have a period less than four days is also very remarkable.

Next, let us examine Table II, where the Algols are arranged in the order of their periods, then we shall see there is a general tendency for the ratio to increase with the period. Thus, in order to find the relation between these quantities, I grouped together every nine stars in the order of their periods and obtained eight groups, omitting the last three stars which are of exceptionally longer period. In the following table, the mean values of the period and ratio for each group are given in the first two columns.

TABLE X.

Period.	Ratio. (observed)	Ratio. (computed)	O–C.	Number.
0.716	3.4	3.4	0.0	9
1.094	4.6	4.6	0.0	9
1.975	6.4	6.3	+0.1	9
2.695	7.3	7.3	0.0	9

3.303	6.6	7.9	−1.3	9
4.158	9.5	8.5	+1.0	9
5.611	8.1	9.2	−1.1	9
18.291	13.5	11.1	+2.4	9

The graphical investigation of the above table indicates that the ratio does not increase uniformly with the time, but there is a certain limit to the ratio. In the work done in 1908, I did not proceed further to find a mathematical expression of the relation.

Stein, in his paper on the density of Algols, derived a formula which represents the relation between the period and and the duration of the eclipse. Now modifying it so that we can compare the above values obtained from the observed data with the formula by him, we have

$$r=\frac{1}{0.0797+\frac{0.1875}{P}}.$$

Then, the calculations of the ratios for the values of the period in the first column will give a results which agree pretty well with the second column of Table X. But, as the constants are not very suitable for my case, the following formula is derived from Table X, determining the constants by the application of the method of least squares,

$$r=\frac{1}{0.082+\frac{0.151}{P}}.$$

Now, the last column in the above table is computed with this formula; the agreement between the observed and computed may be satisfactory, but we must remember that when we use the formula, the ratio is limited to the extreme value 12.2, and at the same time, the actual limit is 29 from observation. Whether this relation can be considered as one which has really a physical meaning or is simply illusory, needs further investigation.

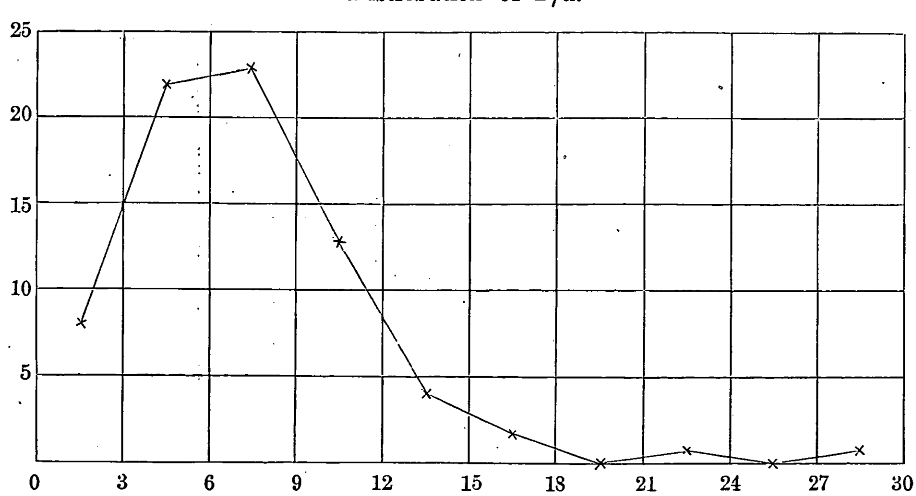
Next, let us consider the distribution of the ratio. The ratios are known in 74 cases among 93 Algols and these are contained among the limits 1.6 and 28.7. Classifying these into several parts, we have the following results:

TABLE XI.

Ratio.	Number.	Ratio.	Number.
0– 3	8	12–15	4
3– 6	22	15-18	2
6– 9	23	18–21	0
9–12	13	21 +	2

This table is again graphically represented in the accompanying figure, Fig. III.

Fig. III.
Distribution of P/d.



The period and the ratio do not show any conclusive relations with the maximum brightness and the range of variation.

(d) DENSITY.

Finding the relation existing between the period and the ratio between the period and the duration of the eclipse, we can pass through to the relation between the period and the density, as we have two equations

$$Dm = \frac{(\bar{2}.8840)}{P^2 \sin^3 \frac{\pi}{r}}$$

and

$$r = \frac{1}{0.082 + \frac{0.151}{P}}.$$

If we can suppose that the latter relation is fully established, D_m will simply be a function of the period only, as the ratio will be expressed by the period. The similar investigation, with the relation between the period and the ratio, of the density from Table II, will give us the first two columns in the next tables.

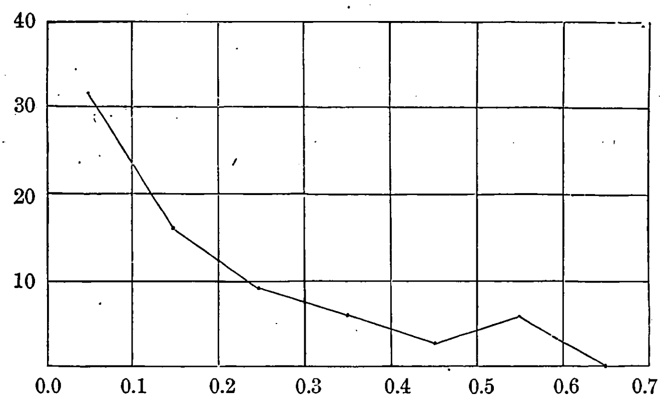
TABLE XII.

Period.	Density. (observed)	Density. (computed)	O–C.	Number.
0.716	0.35	0.30	+0.05	9
1.094	0.31	0.25	+0.06	9
1.975	0.38	0.18	+0.20	9
2.695	0.17	0.14	+0.03	9
3.303	0.08	0.12	−0.04	9
4.158	0.14	0.09	+0.05	9
5.611	0.07	0.06	+0.01	9
18.291	0.07	0.01	+0.06	9

In order to compare the observed values with the hypothetical values, the third column is added here. As the computed densities are systematically less than the observed, we may suspect the correctness of the above formula.

Fig. IV.

Distribution of Densities.



Next, let us observe the distribution of the density. Arranging the Algols in the order of their densities, the actual counting give the following results.

TABLE XIII.

Density.	Number.	Density.	Number.
0–0.1	33	0.4–0.5	3
0.1–0.2	16	0.5–0.6	6
0.2–0.3	9	0.6–0.7	0
0.3–0.4	6	0.7+	1

Here again the graphical representation of these results is given in Fig. IV and as a result of the examination of this, we see that the Algols with a density comparable to that of the sun are very rare and the majority of them are of very low density.

(e) MAXIMUM OR NORMAL BRIGHTNESSS.

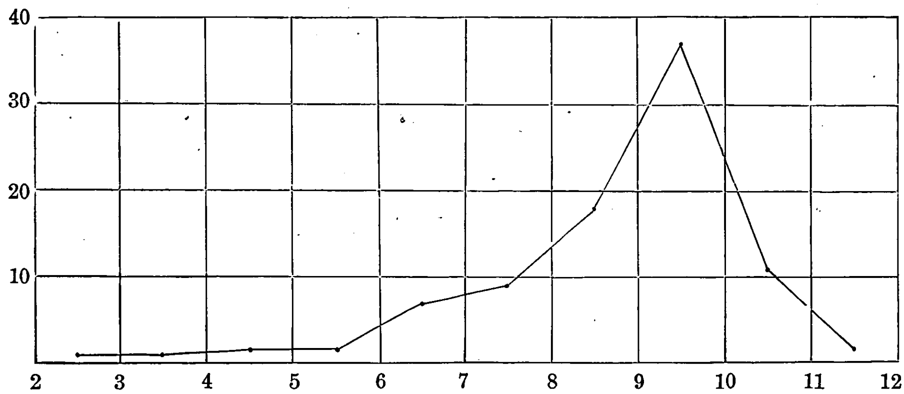
Let us, now, think about the distribution of the maximum brightness and its related problem. Now arrange all the Algols in the order of the maximum, and then, the distribution of the maximum brightness will be found to be as follows;

TABLE XIV.

Maximum.	Number.	Total.	Maximum.	Number.	Total.
0^{m}–1^{m}	0	0	7^{m}–8^{m}	9	22
1–2	0	0	8–9	18	40
2–3	1	1	9–10	37	77
3–4	1	2	10–11	11	88
4–5	2	4	11–12	2	90
5–6	2	6	12–13	2	92
5–7	7	13			

In the above table, I omitted one for which the exact determination of the maximum, i.e., normal brightness is not yet published. We see from this table that the number of Algols increases with the maximum brightness till the tenth magnitude and then the rate of increase is slackened. Possibly, this may be due to the reason that our examination of the stars fainter than the magnitude 10 is not so extended as that of the brighter stars.

Fig. V.
Distribution of Maximum.



So let us turn our attention to the Algols which are brighter than the tenth magnitude only. Now, let us modify the above table so that we can know the number of stars in the first, the second, third magnitude and so on till the tenth magnitude. Here the first magnitude extends from the brightest star to the stars of the magnitude $1^m.4$, the second, from $1^m.5$ to $2^m.4$ and so on.

TABLE XV.

Maximum.	Number.	Total.	Maximum.	Number.	Total.
First	0	0	Sixth	2	6
Second	1	1	Seventh	10	16
Third	1	2	Eighth	8	24
Fourth	1	3	Ninth	33	57
Fifth	1	4	Tenth	24	81

This table serves to find the so-called star-ratio in the Algols, a special part of stars. From the investigation of the stars in general, star-ratio up to the stars of the ninth magnitude was found to be nearly 3.5. The inspection of the above table will at once show that the ratio, if it exists, does not exceed two. Supposing such a ratio to exist and finding the mean value, we have 1.80. The remarkable difference between the stars in general and the Algols only can be explained on either one of the two followed hypotheses:—(1) Algol variables are distributed in space in the same manner as the ordinary stars are. (2) The distribution of the Algols over the heavens is governed by a law different from that of the ordinary stars. If we adopt the first hypothesis, the result above given shows that we have not reached a position where we can claim to have made a uniform discovery of the Algols for all the magnitudes. Possibly, as the Algol becomes fainter in its normal brightness, the chances to discover the variation of light will be less. But if we chose the latter hypothesis, it would be rather doubtful to think that there is a constant star-ratio among the variable stars of this class.

(f) RANGE OF VARIATION.

We have, now, to consider the distribution of the range of variation in the Algols. Theoretically considered, the limit of the range of variation will be extended from infinitely small to infinitely great, i.e., absolutely dark, but practically we are not able to distinguish a very small variation of light in a star, so that the Algols, having such a small range of variation, might remain without the nature of the light-change being detected. Thus, the least value of the range of variation will be the amount of light necessary to cause the sensation to our eyes of an unmistakable change of light, when that amount is reduced from the normal brightness. Therefore, the limit becomes less and less as the method of investigation is developed. At present, the change of light, which is less than half magnitude in its range, is very difficult to distinguish from those caused from accidental origins independent of the real change of light in the star. If the companion of an Algol is absolutely dark, and its dimension equals or exceeds that of the principal star, then the system will show the minimum in the phase of absolute darkness, when the eclipse is central. In fact, the range of variation observed in Algols extends from a half magnitude to four full magnitudes in our stars, excepting RR Draconis whose minimum is not yet determined as it sinks more than $13^m.4$. Arranging the range of variation in the increased sense, the actual counting of the number gives us the distribution of the range of variation as follows:—

TABLE XVI.

Range	1.0	1–2	2–3	3–4
Number	29	40	18	5

Thus, there is a tendency for the number of Algols to decrease with the increase of the range of variation. This may be due to the facts that the central eclipse occurs very seldom, and that the probability of the chances for observing the eclipse phenomena by observers on the earth, increases as the distance of the two components, projected on the celestial sphere at the middle of eclipse, increases and that the companion will be not absolutely dark.

(g) LIGHT-CURVE.

The light-curve of the Algol-variable is, in general, symmetrical with respect to the minimum: and in 58 Algols, of which I was able to find the light-curves; or, at least, some indications of the character of the light-curve, 44 stars show the curves symmetrical; or, at least, very nearly symmetrical, and only in the remaining 14 cases does the light-curve practically deviate from the symmetry. But even in these 14 cases, the most part of them exhibits a very slight degree of asymmetry so that the real existence of the deviation is suspected for these stars. Thus, from the above combined with the facts that follow, we may conclude that the orbit of the Algol-system has, in general, very small eccentricity; or, in the other words, the orbit is generally circular and only in a few cases, it is pretty eccentric.

When we examine the light-curves which are not symmetrical, we shall find again it to be a general tendency that the increase of light is quicker than the decrease, as it is generally the case in the variable of long and short periods.

Among the Algols exhibiting the symmetrical light-curve, we see that 14 cases of them are of flat minimum, a constant light continuing in some interval at the minimum. Such light-curve was at first found in U Cephei, but according to Nijland (A. J. B., XI, 633), a stationary minimum is usual for the Algol. Nevertheless, as the Algols which show a character so pronounced as that of S Verolum, RZ Ophiuchi and VW Cygni, are not very frequent, we may say that the components of the Algol-system are, in general, comparable with each other in their diameters.

The existence of a secondary minimum is possibly sure for nine Algols and besides these, it is suspected for five stars. For a few of the Algols in which the secondary minimum surely takes place, the light-curve of the secondary minimum is almost equal with that of the principal minimum, like X Carinæ, Y Cygni and others. In some cases, the whole light-curve tends to that of a star wnich belongs to the type of β Lyræ. If we include these stars into the type of the Algol, as there is no good reason to divide the variables belonging to the type of β Lyræ from the variables of the type of the Algol, the number of the Algols will be increased 10 or more. But here again a similar difficulty will be met with, i.e., to distinguish these β Lyræ-type variables from those of the short period.

For five stars belonging to the class of Algols, which show the asymmetrical light-curve, the light-curve is very peculiar, having a secondary unduration near the minimum and these can not be fully explained only by the theory of eclipse. The remarkable stars of this class are S Cancri, and RW Monocerotis and besides these, U Ophiuchi, Y Cygni 21.1210 Andromedæ also belong to this class. Some observers found such anomalous deviations in some other stars in certain minima while the mean light-curve shows the ordinary appearence. In such cases, it is rather difficult to decide whether it is only a subjective or atmospheric phenomenon, or whether such variation of light takes place in stars, themselves.

Tokyo Astronomical Observatory, Oct. 20, 1910.