JOURNAL OF THE COLLEGE OF SCIENCE, IMPERIAL UNIVERSITY, TOKYO, JAPAN.

VOL. XXI., ARTICLE 1.

Frequency of After-Shocks
and
Space-Distribution of Seismic Waves.[1]

By

S. Kusakabe, *Rigakushi.*

With one plate.

Contents.

Introduction.

As to the frequency of after-shocks, a hyperbolic formula was empirically obtained by Prof. F. Ōmori[2] about ten years ago. That his formula gives satisfactory results was adequately shown by him in his valuable paper "On the after-shocks of earth-

1) An abstract of this paper may be found in the Proceedings of the Tokyo Physico-Mathematical Society, Vol. II. No. 11. May, 1904. The Pub. of the E. I. C. in F. L. No. 17, 1904.

2) F. Ōmori. This journal, Vol. VII.

quakes," in which the said formula was applied to the three great earthquakes which had then recently occurred in Japan. Lately, another formula in the form of a logarithmic function has been obtained by Mr. M. Enya,* founded on three assumptions. The result of a laborious calculation is given by him to show that the logarithmic formula is equally as good as the hyperbolic.

As to the space-distribution of after shocks, Professor Ōmori proposed the empirical formula

$$y = ab^{-r}$$

where a and b are constants, while r is the distance of an observing place from the seismic focus. As it is reasonable to regard the shocks as proceeding from the seismic focus, the isofrequency curves would take the form of a series of concentrics around the focus, if the earth were a homogeneous solid. The existence of the so-called sympathetic shocks seems a mystery to anyone who adheres to the above view and assumes, without any reason, the surface intensity to be continuous.

As a matter of fact, however, the earthcrust is made up of rocks varying greatly in physical properties, each having its own density and elasticity. To make the variation more discontinuous, rocks of all geological ages have been mingled together as it were, by a series of geological disturbances, and are scattered about through the earthcrust.

Consequently their space-distribution would never be expressed with any approximation to exactness by a formula which is a function of the distance alone.

The first step in a theoretical investigation of the frequency

---

* M. Enya. The Pub. of the E. I. C. (in Japanese) No. 35, 1901.

is to ascertain the cause of the after-shocks. It being a common rule that even phenomena of one and the same category may have different causes, so in the case of earthquakes probably several distinct causes should be recognized, for it is hardly to be supposed that all subterranean disturbances, differing as they do so widely in intensity and in duration, should be referable to any one common mechanical action. To what particular cause or series of causes any earthquake may be due is not, however, a question to be dealt with here.

It is generally accepted, as a matter of course, that an earthquake depends upon a sudden inpulse due to the internal stress of the earth. But the *modus operandi* whereby that internal stress manifests itself in an impulse is a problem, which is by no means of little importance but which unfortunately is neglected in most cases where the wave motions of the earthcrust are discussed. Were the earth a cosmic body of perfect elasticity, as it is generally assumed by clever mathematicians, we might surely expect seismic waves to propagate after certain laws deduced by their subtle analysis.

But, in such a case as the above, how can the initial impulse at the seismic centre be excited by the internal stress itself? *Ut tensio, sic vis,* and consequently no matter whether we adhere to the Humboldt-Naumann volcanistic view, or to a tectonic hypothesis, as Hoernes called it, or to R. Falb's sideric hypothesis,* so very gentle must be the changing of the earthcrust that, though incessant from day to day, it can really be proved only by means of careful observation.

To cause an earthquake, the strain must increase *per saltum*

* Grundzüge zu einer theorie der Erdbeben und Vulkanausbrüche, 1869.

after the accumulated stress has reached the necessary amount at the given point, where the seismic focus is situated. Again, if at the instant when the weak point gave way, all the strata, being released from their overstrained state, were to come to equiliblium at once and there were no residual strain which might recover with the lapse of time, the earthquake at that instant would be the only effect of the accumulated stress.

According to the intrinsic meaning of the name "After-shock," the nearest cause must be attributed to a residual disturbance in the geotectonic condition after the original shock has ceased. An earthquake not participating in this residual is not an after-shock but an independent earthquake. *To make clear once for all my own standpoint, I must say, that the actual imperfectness of the elastic properties of rocks which compose our planet appears to be the prime cause—all other causes being secondary relative to it—of after-shocks or rather of all earthquakes. Although the magazine of seismic energy is being constantly and steadily replenished by the incessant recovery of the rocks around the seismic centre from their overstrained yielding, the effects are intermittent and manifest themselves as aftershocks.*

As a matter of fact, the folding of rocks and other kindred phenomena pertaining to their manifold changes of shape are found in great abundance within the earthcrust. It may be a question whether such phenomena once occurred with great frequency in a short time under wholly plastic conditions while now-a-days, the crust being permanently set, they occur much less frequently; or whether they are not the results of yieldings, occurring from time to time, wrought by the continuous action of stress, and always ready to recover from the over-strained state.

If the latter were the case, it would not be wholly inconceivable that an overstrained portion of the earthcrust recovers gradually after its stress has been removed by the geological disturbance which caused the original earthquake. In such a case, to be sure, this phenomenon of recovery might be the prime cause of the after-shocks of the original earthquake, since the ultimate result of this phenomenon must be equal to that which may be produced by an oppositely directed stress.

Experimental Researches.

From an investigation by F. D. Adams and J. T. Nicolson,[1] it is evident that even such a comparatively rigid rock as marble may become wholly plastic under suitable conditions. For instance, the diamter of a cylinder increased by 1.388 times its initial, bulged out under endpressure. Another instance where a plate of marble, resting horizontally on four posts at the corners, in the course of about half a century, was considerably bent by its own weight, is reported by T. J. J. See.[2]

As to yielding and recovery in the case of torsion of rocks, the author published some experimental results two years ago,[3] and proposed a logarithmic formula to express the amount of yielding and recovery from it at any instant. To cite some of the results :—(1) The amount of yielding increases proportionally to the logarithm of the time during which the couple

1) An experimental investigation into the flow of marble. Phil. Trans. of the R. S. A. Vol. 195. 1901.

2) The secular bending of a marble slab under its own weight. Nature, Nov. 20, 1901.

3) Pub. of the E. I. C. in F. L. No. 14, 1903. Tokyo. This journal, Vol. XIX, Art. 6. 1903.

acted; (2) the residual surviving the couple after it has been withdrawn increases with the increase of the time during which the couple acted on the specimen; (3) the residual diminishes with the lapse of time and ultimately wholly disappears after an infinite time, i.e. rocks from instant to instant recover from their yielding to overstrain.

In the case of flexure,* though it is not so enormous as in the case of torsion, the phenomena of yielding and of recovery are sufficiently great to be dealt with. To give one instance; in a piece of sandstone which was loaded with 3000 grams-weight, the amount of bending was $\alpha=27.95\times10^{-4}$ radians at the instant of loading. The latter quantity, however, increased to $\alpha=33.86\times10^{-4}$ after $3\frac{2}{3}$ hours and to $\alpha=60.57\times10^{-4}$ after about sixteen hours. Further increase of the flexure could be distinctly observed during about two weeks till at last the yielding, though it was still steadily increasing, was much obliterated by the influence of the temperature-change.

After about two weeks, i.e. 20363 minutes, the specimen was unloaded and the amount of residual bending was observed from instant to instant. As in the case of torsion, it recovered gradually and incessantly. The result of the experiment is given in the following table.

* For the method of measurement and other details the reader is referred to the author's papers: Pub. of the E. I. C. in F. L. No. 17. Tokyo, and this journal, Vol. XX. Art. 9. 1905. An abstract is also given in Proc. of the Tokyo Physico-Mathematical Society, Vol. II. No. 11.

Specimen No. 3_4. Sandstone. $a=1^c153$, $b=1^c120$, $l=10^c0$, $\rho=2.20$
Loaded at $4^h\ 27^m$ P.M. 9^{th} Feb. 1903. $M=3000$ grs. $M_0=3300$ grs.
Unloaded at $7^h\ 50^m$ P.M. 23^{rd}. $M=0$. $M_0=3300$ grs.

Time.	Recovery.	Time.	Recovery.
23^{rd} P.M. $7^h\ 51^m$	20.87×10^{-4} radians.	24^{th} A.M. $8^h\ 3^m$	25.91×10^{-4} radians.
52	21.59	P.M. 1 38	27.50
55	21.87	3 15	28.30
57	22.02	5 34	29.28
8 0	22.30	6 43	31.09
4	22.51	7 14	31.27
6	22.37	8 18	31.47
9	22.59	9 41	31.84
17	22.69	25^{th} A.M. 10 0	34.44
24	22.95	P.M. 6 42	37.63
57	23.32	27^{th} A.M. 8 12	45.27
9 35	24.52	P.M. 5 28	47.38

Thus the amount of recovery increased, in the course of about four days, to more than twice its initial value.

Frequency of After-Shocks.

From what has been explained in the above section we know, as a matter of fact, that a piece of rock yields under the constant action of stress, and also that the residual strain surviving the stress diminishes from instant to instant. This last phenomenon must be the prime cause of the after-shocks. Thus, the first step is to find some formula expressing the rate of recovery, or the rate with which the residual varies with time. One form of such a formula, however, was lately deduced from the logarithmic law of yielding, as it was given in the author's papers above cited. The formula is

$$\rho = k\ log \frac{\Gamma(2p+1)\ [\Gamma(p+t+1)]^2}{[\Gamma(p+1)]^2\, \Gamma(2p+t+1)\, \Gamma(t+1)}$$

where* ρ is the total amount of recovery at the instant t, both ρ and t being reckoned from the instant when the external force is wholly withdrawn, while k and p are constants, of which the former specifies the rock and the latter the time-lapse required by the force to attain its maximum.

Let F be the frequency, then if c is a constant, we have

$$F = c\frac{\Delta \rho}{\Delta t}$$

$$= c\ k\ log \left\{1 + \frac{p^2}{(2p+1)+2(p+1)t+t^2}\right\}.$$

Thus we have a logarithmic form for the frequency of after-shocks. A little consideration of the nature of the constant p will make it reasonable to neglect the term $\frac{t^2}{p^2}$ so long as t is not very large. Then we have, for first approximation,

$$F = k'\ log \left\{1 + \frac{1}{A+B\,t}\right\}$$

which is the same as that of Mr. Enya. Again, expanding the logarithmic function and taking its first term only, we have Prof. Ōmori's formula

$$F = \frac{k''}{h+t}.$$

Though the resulting formulæ for the frequency are tolerably well formed inasmuch as they were tested by Prof. Ōmori and

* The symbol Γ stands for Gamma-function which may be found in any text-book in integral calculus. When p is a positive integer we have the relation Γ(p+1)=1. 2. 3.... ....(p−1). p.

Mr. Enya, the original formula for the recovery is not wholly beyond question. The assumptions under which the formula is deduced are very far from what is actually the case in an earthquake. The force acting on the rock is assumed to increase intermittently, and, what adds to the difficulty, it is assumed to be withdrawn not suddenly but slowly and intermittently. The following may be closer to the actual case.

Whatever view may be adopted as to the origin of the seismic energy, it is reasonable to consider the force as increasing constantly with time, i.e.

$$d\,f = k\;dt$$

where k is a constant, and attaining a sufficient amount F it acts suddenly to cause an earthquake at the time T, so that we have

$$F = k\,T.$$

Suppose the logarithmic law of yielding, which was experimentally established in the last series of experiments, to be granted, so that

$$d\eta = K\;d\,f\;log\,\{t+\tau\},$$

where η is the amount of yielding and K a constant specifying the kind of rock, while τ is a constant referring to the choice of origin of time t. Then we have

$$\eta = K\,k\int_0^T log\,(t+\tau)\;dt.$$

If the total force F is suddenly withdrawn at the instant $t=T$ when the original earthquake is supposed to have taken place, it may be easily proved that the residual strain at any instant $t=T+t'$ is given by

$$\sigma = k\,K\,\{T+t'+\tau\}\;log\left\{\frac{T+t'+\tau}{t'+\tau}\right\}.$$

Now, as the frequency is assumed to be proportional to the rate of recovery, we have

$$F=-c\frac{d\sigma}{dt}$$
$$=\frac{c.\,k.\,K.\,T}{t+\tau}-c.\,k.\,K.\,log\left\{1+\frac{T}{t+\tau}\right\},$$

where τ, c, k, K and T are all constants, and t is written for t' whose origin may be any instant, provided the proper value is given to the constant τ.

Here the frequency F may be considered to be composed of two terms F_1, which is hyperbolic and F_2 which is logarithmic, so that h being a constant

$$F_1=\frac{h}{t+\tau},$$
$$F_2=\frac{h}{T}\,log\left\{1+\frac{T}{t+\tau}\right\}$$
$$F=F_1-F_2.$$

As the constant T is, in all probability, very great as compared with the other constants c, k and K, the main term is the first, so that the curve of frequency F is a little different from a hyperbola.

When h is given, the curve F_1 takes a definite form, but the curve F_2 is wholly indefinite so long as T, i.e. the time required by the force to become sufficient to cause the earthquake, is not known. *That is to say, if the time during which the causal agent of the earthquake existed is long, the curve of frequency approaches the hyperbola represented by F_1, but it deviates more and more from the latter curve as the duration T diminishes. For example, the number of after-shocks of an earthquake of an explosive nature is necessarily smaller than that of an earthquake of geotectonic origin, and the frequency curve differs more from* *a hyperbola in the one case than in the other.* It is well known fact that the most characteristic which distinguishes tectonic quakes from volcanic ones is found in very numerous after-shocks. The numerical example given below will show this more clearly.

$h=1000$; $\tau=1$		F_2		F	
t	F_1	$T=100$	$T=1000$	$T=100$	$T=1000$
1	500	39	6	461	494
2	333	35	6	298	327
3	250	33	6	217	244
4	200	30	5	170	195
5	167	29	5	138	162
6	143	27	5	116	138
7	125	26	5	99	120
8	111	25	5	86	106
9	100	24	5	76	95
10	91	23	5	68	86
11	83	22	4	61	79
12	77	22	4	55	73
13	71	21	4	50	67
14	67	20	4	47	63
15	63	20	4	43	59
16	59	19	4	40	55
17	56	19	4	37	52
18	53	18	4	35	49
19	50	18	4	32	46
20	48	18	4	30	44
24	40	16	4	24	36
29	33	15	4	18	29
39	25	12	3	13	22

In the above example, suppose the unit of time to be one month, then the number of after-shocks during the first, second, third, month etc., would be either 461, 298, 217, etc. or 494, 327, 244, etc. respectively, according as the time required by the force to accumulate to the amount sufficient to cause the earthquake was a hundred or a thousand months.

We may remark that the solution of the above formula gives a means of determining the length of time required to generate that earthquake, and this must, at least, elapse before the region to which it refers is again disturbed by a similar catastrophe.

A few words may be inserted here in reference to the situation of the centre of after-shocks. It may appear, *prima facie*, that all of the after-shocks must necessarily proceed from the seismic focus of the original earthquake. But this is not necessarily so, and it does not actually happen, that the centres of the after-shocks and those of the original earthquakes coincide with one another.

From the above theory of yielding, however, it comes to be the common rule rather than an exception that they do not generally coincide. The seismic focus of the original earthquake is, of course, a region where the stratum giving way to the internal stress, was crushed or dislocated, e.g., in the case of a tectonic earthquake. All the after-shocks, however, are the result of recovery from the yielding so that they take place most frequently where the rate of recovery is the greatest. No doubt, a region once dislocated can never recover so as to cause any after-shocks. *Of the neighbouring regions, that part which consists of rocks most capable of yielding and recovery is most likely to become the centre of after-shocks. Hence, though the after-shocks are the residual effects of the original earthquake, the* *centre of them will be transferred elsewhere to some neighbouring weaker region.*

To give an example, in the case of the Mino-Owari earthquake,[1] which was caused by a sudden falling of the Palæozoic strata on the right wing along the line of the 'fault of Neo,' accompanied by lateral shifting toward the north-west,[2] the centre of the after-shocks was transferred considerably southward[3] to the lowland of Mino and Owari, which is believed to have been recently formed out of the sediments of the confluent streams, the Kiso and the Nagara.[4]

Iso-Frequency Curves of After-Shocks.

The discussion in the last section refers to the frequency of after-shocks in the very centre of the disturbance. Here a hint is given to show how the geological distribution of rocks plays the greater part in diversifying the form of iso-frequency curves, which would all be similar to each other if the earth were a homogeneous isotropic body.

It is a matter of course that a seismic wave propagating through a medium having a greater hysteresis fades more rapidly than one propagating through another medium having a less hysteresis. That is to say, the distance between two successive iso-frequency curves should be dependent on the geological distribution of the rocks in the region. *Not only is the frequency*

---

1) A short description is given in Professors B. Kotō's and F. Ōmori's papers cited below.

2) B. Kotō. On the cause of the Great Earthquake in Central Japan, 1891. This journal, Vol. V. p. 353.

3) F. Ōmori. On the After-shocks of Earthquakes. This journal, Vol. VII.

4) B. Kotō. Loc. cited. p. 307.

dependent on the geological nature of the region under considera-tion, but it must be greatly affected by the geological distribution of the rocks lying between the seismic focus and the said region.

In other words, the seismic-wave-conductivity, if we may be allowed to employ such a term from some analogies in heat and electricity, may be different for different rocks, so that in one direction an earthquake may be propagated with comparatively smaller fading than in another direction.

Now, as was experimentally shown by the author himself,* the amount of hysteresis generally decreases with an increase of the elastic constants, while the latter increase with their age of formation. Although nothing, as yet, can be said about any numerical relation between hysteresis and the age of formation of different rocks, yet in the rocks so far examined, a certain relation seems to exist.

To illustrate this fact, the hysteresis curves for a few specimens of rocks are shown in Figs. 2 and 3. If there were no hysteresis, the curve would, of course, shrink to a single line, so that the amount of hysteresis may be conventionally compared by the area enclosed by the curve. It is, then, a matter of fact that the area is enormously great for new rocks such as sandstone and rhyolite, and gradually diminishes as rocks of older ages are examined, until it becomes very small for the oldest rocks of the Archæan age. Thus it would not be a wild conjecture to say that the amount of hysteresis gradually diminishes from Cainozoic, through Mesozoic and Palæozoic, to Archæan rocks, in a definite, though not yet ascertained, ratio.

An inference to be drawn from the above is that the seismic

---

* This journal, Vol. XIX, Art. 6, and Vol. XX., Art. 9. The Pub. of the E. I. C. in F. L. Nos. 14 and 17. Tokyo.

wave conductivity is least for Cainozoic rocks, and increasing step by step from Mesozoic to Palæozoic, it becomes several times greater for Archæan rocks. Hence, provided the frequency of after-shocks at the seismic focus be given, the frequency at any place having a given distance from the seismic focus increases with the geological age of the rocks forming the path of the wave between the focus and the place.

In support of the above statement, I may cite the case of the Mino-Owari earthquake. Seismologists have never enjoyed so good an opportunity as that afforded them by the convulsion in the Mino-Owari plain, of observing an enormous number of after-shocks at a multitude of stations well distributed around the seismic centre. After the catastrophe of October 28th, 1891, the after-shocks were extraordinarily frequent or almost incessant: indeed, 1503 of them occurred during the remaining two months of that year, and during the next year 867 were recorded in the Meteorological Station of Gifu. The number of observing stations for these seismic waves, on the other hand, amounted to thirty-three in all, i.e. 14 in Mino, 9 in Owari and 10 in Mikawa Province.

Four curves of iso-frequency carefully drawn—irrespective, of course, of the geology of the regions—by Prof. F. Ōmori* are shown, with the corresponding geological distributions of rocks supplied by the author himself, in Fig. 1. The iso-frequency curve for $F=500$ lies wholly within Quaternary rocks and is in an elongated form extending nearly north and south between Gifu and Nagoya. The central region of the after-shocks may be in a similar form. The succeeding curves, however, so far

* F. Ōmori. Loc. cit.

from being similar to the first, are in quadrantal forms. In the western part, indeed, where the curves lie within Quaternary rocks, they are all parallel to each other; but in the other three directions they shrink in or swell out with all possible irregularities.

These irregularities, however, become regular when the geological distribution of rocks in the corresponding regions is taken into account. To express this in the form of a simple rule, the curve swells out where Palæozoic, or better Archæan rocks, predominate, and shrinks in where Cainozoic rocks prevail. This simple law is sufficiently satisfied up to very minute portions, as the figure proves most clearly.

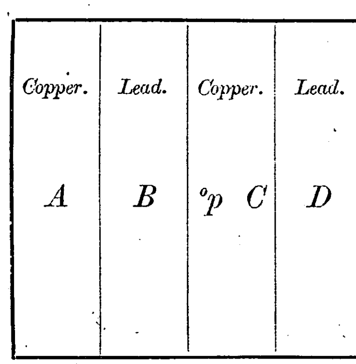
As a corollary, since the geological map indicates only the surface distribution of rocks, we may conclude that the seismic wave is mainly transmitted through the earth's surface, or more probably, seismic action is mainly due to surface waves discussed by Lord Rayliegh, and recently propounded by Lamb for isotropic media. Any further discussion, however, as to the seismic wave conductivity of different rocks requires more precise quantitative investigation of the amount of hysteresis for these rocks, which may possibly prove a life-long problem.

Seismic Frequency and Degree of Damage in a Given Region.

It will not be entirely out of place to insert here discussions on a topic relating to seismic frequency and the degree of damage in a given region, in order that above statements may not be misunderstood. Of the above conclusions, relating to frequency, the one concerns the frequency of after-shocks at the central region of a given earthquake, while the other relates to the case where the seismic centre is outside the region which we are considering. Any one who has any knowledge of astronomy knows that the ratio of the numbers of eclipses of the moon and of the sun theoretically observable at a given observatory differs from the actual number of eclipses known to occur.

Similarly, though of a wholly different nature, the actual frequency at a given station may be different from that indicated in the above sections of this paper. A certain region, notwithstanding the scarcity of the quakes originating in it, may be frequently visited by seismic waves proceeding from the surrounding districts. Again, one region may be shaken so slightly as to escape attention, while the other weaker region is violently damaged by the same seismic energy.

A complicated case such as the above, may be most clearly illustrated by analogy taking the case of heat. Let four thick pieces of copper (A, C) and lead (B, D) be arranged side by side as in the annexed figure. Suppose we heat the system at a point p to a high temperature, sufficient to melt the greater part of the lead piece B, but little of the copper piece C. Then, taking into account the heat conductivity and melting point of the two metals, we may easily so adjust the breadths of the pieces B and C, that the lead piece D also melts in the part where it is in contact with the copper piece C, though the latter does not participate in the melting. The remaining piece A, though it is nearly as distant from the source of heat as the



lead piece D, may be only slightly heated by virtue of the bad conductivity of the lead piece B.

The high degree of heat conductivity in copper, is analogous to the small hysteresis of old rocks*, while the low melting point of lead corresponds to the inferior elasticity of new rocks. To say that, copper being a good conductor, may be easily heated even when the source of heat is remote, is wholly different from saying that it may be often melted under the same conditions. Although lead melts very easily, it does not conduct heat very well, so that even the portion comparatively near to the source of the heat may remain solid. But it is so only when the part intermediate between it and the source is also lead. If the intervening metal is copper, the case is reversed.

All these complicated phenomena find their analogy in the case of seismic waves. *Whether a region is frequently visited by seismic waves or is not can never be determined by the data relating to that region alone. A severe damage does not necessarily indicate that the epicentre of the earthquake lies near by. However great the destruction is, it is nothing more than the superficial effect of the earthquake which is wholly controlled by surrounding conditions. The main factor which determines the degree of disturbance is the geological distribution of the rocks in the whole domain.*

The existence of the so-called sympathetic shock or *Relaisbeben* is also due to the reason just stated. In the above illustration, one who knew nothing of the thermal properties of the metals might call the melting of the lead piece D sympathetic.

* This is simply an analogy, conventionally adopted for sake of illustration. The general rule that an analogy, however perfect, does not explain all the facts connected with it, is true in this case.

The Mino-Owari earthquake gives a concrete instance which explains this phenomenon also. From B. Kotō's* valuable paper we may cite the geology of the domain.

The extensive and populous plain of Mino and Owari is on three sides bounded by mountains mainly of Palæozoic formation. Granite and schistose rocks make up the main blocks of the range. Beyond the mountain-ridge, in the north, lies a plain of considerable extent, where the Mesozoic formation is extensively developed, and later on has been intruded into in places by masses of Tertiary eruptions. The city of Fukui lies in the basin of the River Kudzuryu draining this plain.

Similarly, in the west, there lies a plain of recent geological era at the eastern border of Lake Biwa, the city of Hikone being the most populous one in the region. It was very remarkable that these two regions were severely shaken and greatly damaged by the earthquake, whereas the stretches of land between these and the Mino-Owari plain suffered very little.

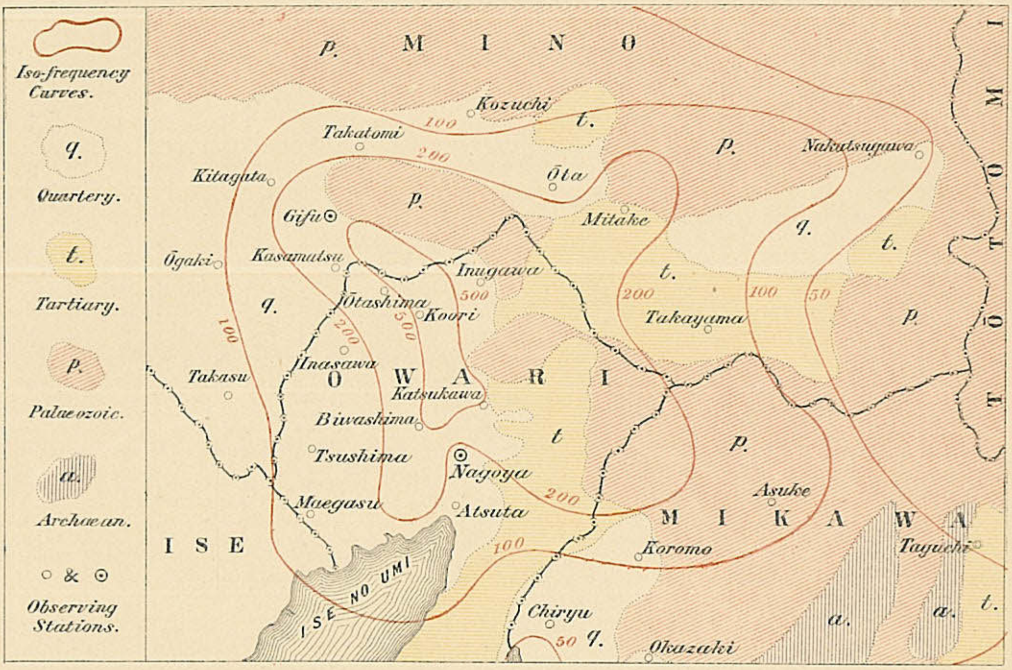
Under the point of view in question, it means simply that the Palæozoic rocks, having less hysteresis, conducted the seismic wave very well but were not damaged by virtue of their high elasticity, while the Cainozoic rocks in contact with them, wholly absorbing the seismic energy, were severely shaken in consequence of their inferior elasticity and large hysteresis.

In conclusion, I wish to express my best thanks to Prof. H. Nagaoka for his kind guidance throughout the whole of this investigation.

---

* B. Kotō. Loc. cit. Geology and Topography of Mino and Owari.

Fig. 1.



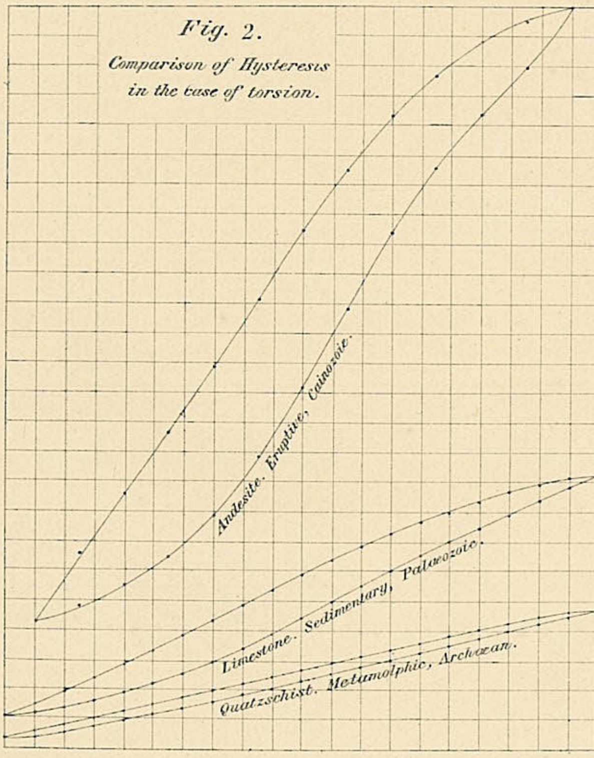


Fig. 2.
Comparison of Hysteresis in the case of torsion.

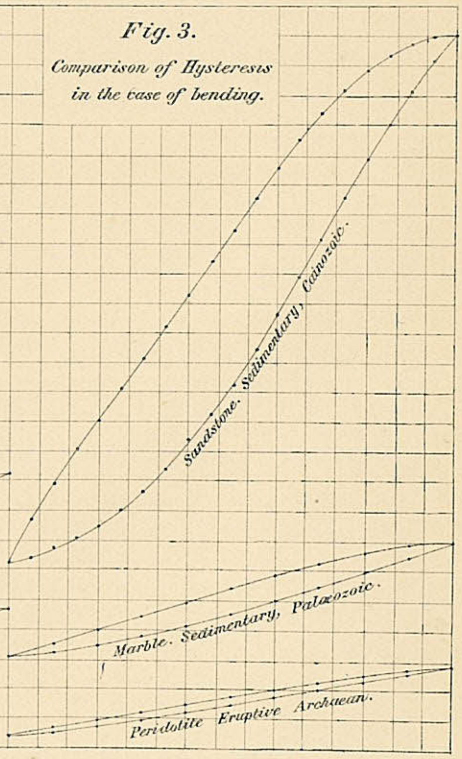


Fig. 3.
Comparison of Hysteresis in the case of bending.