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On the Effect of Stress on Magnetization and its Reciprocal Relations to the Change of Elastic Constants by Magnetization.

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With 5 plates.

In our previous experiments, we investigated in some detail the change of elastic constants of several ferromagnetic metals and alloys caused by magnetization, with special regard to the order of applying the stress and the field, and found that in some cases, the change is considerably large and moreover that it differs more or less for different orders of applying the stress and the field. In order to find the proper explanation of these facts, it will be necessary to investigate with the same specimens as in the previous experiment, the change of magnetization by stress, with special regard to the order of applying the stress and the field.

Since J. J. Thomson[1] gave his theoretical exposition of the reciprocal relations between magnetism and strain, several theories[2]

1) J. J. Thomson, Application of Dynamics to Physics and Chemistry, Chapter IV.

2) F. Koláček, Ann. d. Phys., **13**, p. 1, 1904; Ann. d. Phys., **14**, p. 177, 1904. A. Heydweiller, Ann. d. Phys., **11**, p. 602, 1903. R. Gans, Ann. d. Phys., **13**, p. 634, 1904. S. Sano, Proc. Tōkyō Math.-Phys. Soc., **2**, p. 175 and 207, 1904.

in the same field have been published, and the present investigation may also afford interesting materials for testing the validity of these theories. In this direction, we have been preceded by Rensing[1)] and Cantone,[2)] in the case of iron and nickel; but, a more extended researches, may not be undesirable. With this view, sets of experiments have been undertaken firstly to investigate the change of magnetization by applying successive stresses under constant fields; and secondly, to investigate the magnetization by applying the magnetizing field under different constant stresses and thence to deduce indirectly the change of magnetization by stresses.

Specimens used had the following dimensions:—

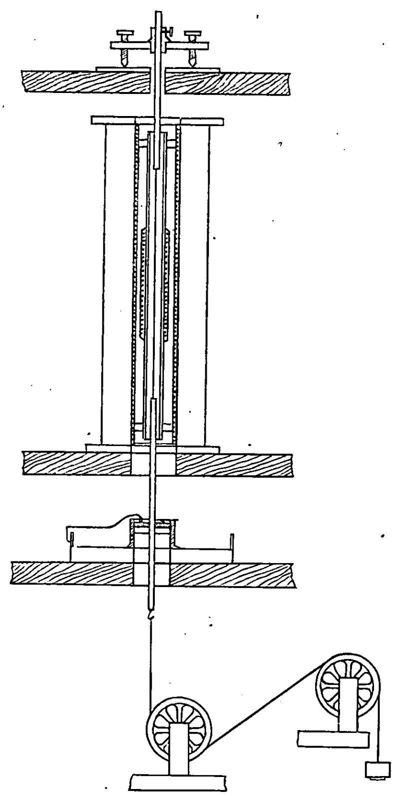
Specimens.	Length.	Diameter.	Demagnetizing factor.
Swedish iron.	21.30 cm.	0.903 mm.	0.00089
Tungsten steel.	26.85	0.885	0.00050
Nickel.	26.87	0.863	0.00056
28.74 % Ni.	26 80	0.964	0.00063
50.72 % Ni.	27.00	0.880	0.00050
70.32 % Ni.	26.86	0.891	0.00068

§ 1. APPARATUS.

The intensity of magnetization was measured by the ballistic method; this method was preferred to the magnetometric one, since in our experiments, fields up to 400 C.G.S. units were required, and consequently the adjustment of a very sensitive magnetometer placed close to a pair of compensating coils traversed by a strong current would be extremely troublesome. The

1) Rensing, Ann. d. Phys., **14**, p. 363, 1904.
2) Cantone, Rend. d. Ist. Lomb., (2) **37**, p. 435, 1904. Ibid. p. 474, 535 and 567.

magnetizing coil which was one that had been used in our previous experiments, was placed in its vertical position. The length and the constant of the coil were respectively 40 cm. and 392.6. A secondary coil was wound on a glass tube (external diameter 1.5 cm.), consisting of 1246 turns of well insulated copper wire (diameter=0.56 mm.) in 6 layers, the length of the coil being 14 cm. This secondary coil was fixed coaxially in the magnetizing coil, so that the former might lie in a uniform field excited by the latter.



To compensate for the induction due to the magnetizing field alone, a similar secondary coil connected in series with the above secondary coil was inserted within another coil equal to and connected in series with the magnetizing coil, so that by sliding the secondary within the primary, the induction could be compensated to any desired degree. These two pairs of coils were placed at a sufficient distance from each other to prevent their mutual action. The ballistic galvanometer for measuring the induced current due to the magnetization of the specimen was drum-shaped, with 0.8 Ω resistance; a mirror with a small magnet was suspended in the center of the coil by a spider thread. Its period of oscillation was about 9 seconds. The galvanometer was connected with the secondary circuit of the system and placed at a distance of about 15 meters from the magnetizing coil to avoid their direct action. The galvanometer was, however, still disturbed, when a strong current was switched on to the magnetizing coil. To prevent this, the compensating primary was so directed that the direct effect of the combined system on the galvanometer was null. To determine the constant of the galvanometer, we should have used the compensating secondary coil, if it had been wound in one layer, so that its effective sectional area could be determined with sufficient accuracy. But, as the ambiguity of the sectional area in the secondary coil of 6 layers was inevitable, another coil was constructed with a thin copper wire wound on a wooden cylinder of 5.04 cm. diameter, in a single layer; the number of turns of the coil was 48. This was always put in series with the secondary circuit and placed at a sufficient distance to be safe from any sensible influence of the magnetizing circuit during the experiments for magnetization. When the constant of the galvanometer was to be determined, the compensating secondary coil was removed from the primary coil and replaced by this coil; then the magnetizing coil for the specimen was shunted off, a weak magnetizing current of known strength switched on to the primary coil, and the consequent deflection of the galvanometer was measured. The constant of the galvanometer was thus determined from the field in the primary coil and the dimensions of the secondary circuit in the usual manner. The resistance of the whole secondary circuit was 10.80 Ω.

The deflection of the galvanometer was read by means of a scale and telescope with a scale distance of 1.527 m. The sensibility of the arrangement was such that one scale division corresponded to a change of 1.42 C.G.S. units of intensity of magnetization. To obtain a smooth motion of the galvanometer mirror due to the induction, it was found necessary that the two primaries as well as the two secondaries should have nearly the same dimensions respectively; the kick, which was observed when the dimensions of these coils were different, was probably due to the self-inductions and the capacities in these coils.

Compensation for the earth field was effected by a special coil of fine copper wire wound on a glass tube in a single layer. This coil fitted closely to the inside of the magnetizing coil and to the outside of the secondary. It was fed by a current from 2 Daniel cells with adjustable resistance in the circuit.

The current in the primary circuit was measured by a Siemens and Halske ammeter with two shunts, $\frac{1}{9}$ and $\frac{1}{99}$. This was occasionally compared with a Kelvin's ampere-balance.

The specimen to be tested was cut to a suitable length (about 27 cm.) so that if placed centrally, it might lie in a nearly uniform field of the magnetizing coil; it was brazed at its both ends to thick rods of brass. The whole was hung vertically in the axial line of the magnetizing coil, and consequently of the secondary coil, the upper rod being firmly clamped to the rigid frame above the coils. To the end of the lower rod, a hook was attached; from this hung a flexible cord which, after passing through a system of two pulleys, was stretched by a weight, without imparting any injurious pendulum motion to the specimen. Near the end of the lower rod, a rigid pin was screwed on perpendicular to the rod. The ends of the pin fitted to the two V-shaped grooves cut lengthwise and diametrically opposite to each other on the inside of a brass cylinder, which could be turned about a fixed vertical axis to any desired angle. The angle of twist was read by means of a graduated circle and the index attached to the torsion cylinder to $\frac{1}{20}$ of a degree. In the experiment of the tension effect, the above arrangement served to prevent any accidental twisting of the specimen without causing a sensible friction to the stretching.

§ 2. METHOD OF EXPERIMENTS.

Our procedure was usually made in the following order: The direct effect of the magnetizing coil on the galvanometer was tested first of all. The specimen was removed, the secondary circuit opened, and the maximum current was passed through the primary. If there were any constant deflection of the galvanometer mirror, the observer signalled to the experimenter who adjusted the orientation of the compensating primary coil, till the deflection was brought to zero, in breaking, making or reversing the current.

Next the secondary circuit was closed, a strong current was passed through the primary, while the observer was watching the galvanometer; the compensating secondary was slid within the primary, till the ballistic deflection was reduced to zero.

Next the compensation for the earth field was effected. For this purpose, the specimen was introduced into the magnetizing coil, clamped firmly and stretched by a suitable tension, care being paid to place the wire co-axially with the coils. The specimen was carefully demagnetized by reversals; the secondary was closed, a weak field excited in the primary, and the consequent deflections noted. After a complete demagnetization, the same magnetizing current was passed in the opposite direction. If the two corresponding deflections of the galvanometer were not equal to each other, the resistance in the compensating circuit was so adjusted, that the reversal of the magnetizing field, if it was repeated two or three times, caused an equal deflection of the galvanometer. This method was found to be very sensitive, a very small change of the current in the compensating system producing a decided inequality of the galvanometer deflections in opposite directions.

The tension effect was first tried. To wipe out any uncertain remanent stress of the specimen, cycles of tensions, from zero to the greatest to be used for the specimen, were passed through before commencing any experiment. As a preliminary to test the working of the arrangement, a series of increasing fields was applied step by step under a constant tension, and the increase of magnetization was observed by the galvanometer. After a complete demagnetization, a weak field was applied and kept constant. While the observer was watching the galvanometer, the experimenter applied a series of increasing tensions step by step; the throw of the galvanometer at each step was recorded. Then the tension was decreased step by step, and the corresponding deflection were sometimes noted. After passing through several cycles of the tensions, the observation was repeated. After a complete demagnetization, the procedure was repeated for another higher field and so on. The number of fields chosen was naturally large for the region where the change of magnetization was considerable, but rare where it was small. The magnetizing current was found to remain nearly constant during an experiment, except in strong fields, where it was sometimes found to vary 2 or 3 per cent. The reading of the ammeter was always observed both before and after the experiment, and the mean was taken.

Instead of increasing the tension step by step, the maximum tension was often applied at once; but it was found that the consequent deflection of the galvanometer was nearly the same as the sum of the deflections obtained by the application of tension in successive steps.

Another series of experiments is possible in this direction. The specimen was demagnetized with the smallest initial tension; it was then magnetized, and then the deflection of the galvanometer due to an additional weight was observed. After several alternate additions and removals of the additional weight, the changes of magnetization due to the addition and removal were observed. Then the demagnetization with the initial and added weights was effected, and the change of magnetization due to a second additional weight was measured, and so on.

Next the magnetization under constant tension was determined. The specimen was first thoroughly demagnetized by reversals, loaded with the empty pan only. A series of successively increasing fields was applied step by step, and the throw of the galvanometer corresponding to each increment of the field was recorded. Demagnetization was again effected, after the specimen had been loaded with an additional tension, and the magnetization tested in the same way; and so on. In this way, the magnetization under different constant tensions was obtained.

The procedure in the experiments on the effect of torsion was similar. The torsion was increased step by step under a constant field and the change of magnetization corresponding to each step was observed. The effect of cyclic twist was also investigated. The effect of the maximum twist applied at once does not differ from the sum of the deflections obtained by a graduated applications of twists, as in the case of tension. The magnetization under constant torsions was next measured. These sets of experiments were repeated for several tensions nearly equal to those used in our previous experiments on the change of rigidity by magnetization.

The standardization of the ballistic galvanometer was made for each set of observations, though the constant remained fairly constant during the whole experiment. Instead of using each time the special coil made for the standardization, we often used the compensating secondary coil for a set of experiments, recording the deflections of the galvanometer corresponding to a series of magnetizing currents, and at the end of a set, the induction of this coil was compared with the standardizing coil. In this way, time and labour were economized, without the risk of introducing any sensible error in the constant of galvanometer.

§ 3. RESULTS OF EXPERIMENTS.

The intensity of magnetization was calculated in the usual manner from the throw of the ballistic galvanometer with a known constant, the numbers of turns of the secondary and the standardizing coil, and the sections of the specimen wire and the standardizing coil. The necessary correction for the reduction to tangent was made for considerable deflections. The magnetizing field was calculated from the reading of the ammeter of known constant, by use of the known turns of the coil. The demagnetizing force, though it was very small, was also taken into account. Tensions were all reduced to weights per square millimeter, and torsions to twists per unit length.

In the following pages, I denotes the intensity of magnetization, H' the external field applied and H the internal or effective field, all expressed in C.G.S. units; T denotes the tension in grams per square millimeter, and τ the twist in minutes of arc per unit of length.

I. SWEDISH IRON and TUNGSTEN STEEL.

The effect[1] of tension or of torsion on the magnetization of iron and steel is so well known that it is superfluous to enter into a detailed description of the effect. Only the general features of the change of magnetization will be given here. It will, however, be noticed that our investigation has one characteristics that several effects of the stress on magnetization were studied on the same specimen with special attention to the order of applying the stress and the field. The specimens were also those on which strains caused by magnetization had been fully studied; hence the numerical results of the present experiment should be of some use to theoreticians, who have either already obtained, or shall attempt to obtain, some reciprocal relations between magnetization and stress, so that they will be given in their proper places.

(*a*) *Change of magnetization by tension under constant field:* $(\delta I, T)_H$.

As will be seen from Figs. 1 and 3, the change of magnetization δI_i due to the initial effect of loading increases up to a moderate field, and then decreases with it. In Swedish iron, curves $(\delta I_i, T)_H$ in weak fields initially bend upward, and after passing through an inflexion point, the curvature changes sign. As the field is increased, the point of inflexion approaches the origin; in strong fields, δI_i is very small, and the curve is nearly straight. In tungsten steel, curve $(\delta I_i, T)_H$ has a slight curvature for all fields.

In weak fields, the effect of removing the suspended weight is very small and slightly increases the magnetization. Subsequent loading causes an increase of magnetization; and unloading,

1) See Wiedemann's Electricität, **3**, chap. 4; Ewing's Magnetic Induction, chap. 9; Winkelmann, Handbuch der Physik, Zweite Auflage, V.1, p. 301–307, 313–319.

a decrease. In strong fields, the initial and the cyclic effect of loading nearly coincide with each other. Curves $(\delta I_c, T)_H$ for cyclic effects are given in Figs. 2 and 4 in magnified scale. Curves $(\delta I_i, H)_T$ as deduced from the initial effect of $(\delta I_i, T)_H$ are given in full lines in Figs. 5, 6 and 7; they rise and then fall steeply in low field, and afterward decrease slowly, cutting the axis of H at the Villari points. The decrease of magnetization reaches a maximum, and then gradually diminishes tending to approach zero, as the field is increased. This maximum decrease had been anticipated from the theory of magnetostriction[1] by Professor Nagaoka and one of us. Curves $(\delta I_i, H)_T$ as deduced from $(\delta I_c, T)_H$ of the cyclic effect rise only slightly in weak fields; but in strong fields, they nearly coincide with curves $(\delta I_i, H)_T$ for the initial effect. The following are the numbers obtained by experiments.

SWEDISH IRON.

Initial $T=152\,gr./mm.^2$; $t=16.5°$ C.

H'	$T=1562$ gr.		$T=3086$ gr.		$T=4648$ gr.		$T=6211$ gr.		$T=7777$ gr.	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
1.25	6.8	0.9	23.2	1.9	56.8	3.2	100.9	4.5	146.3	6.9
2.56	31.3	0.8	94.4	1.5	184.4	2.3	275.8	3.1	343.7	6.1
4.37	106.4	0.1	226.2	−0.2	326.9	−0.6	397.8	−1.0	444.0	−0.8
5.88	115.5	−1.1	197.8	−1.9	251.7	−2.8	285.9	−4.0	306.9	−5.0
7.75	68.7	−1.7	111.1	−3.4	140.9	−5.0	160.7	−6.7	171.6	−8.5
11.90	25.3	−2.6	40.7	−5.2	51.0	−7.9	57.3	−10.5	61.1	−13.2
24.18	1.5	−3.4	1.8	−6.7	1.1	−10.2	−0.1	−13.9	−1.9	−17.4
36.55	−2.7	−3.8	−5.1	−7.8	−7.5	−11.6	−10.3	−15.5	−13.3	−19.6
97.1	−4.0		−7.8		−11.6		−14.7		−18.2	
204.7	−3.8		−7.3		−10.8		−14.4		−18.1	
366.6	−2.7		−5.1		−8.0		−10.6		−13.3	

1) Nagaoka and Honda, Jour. Sc. Coll., **13**, p. 69, 1900; Phil. Mag. **49**, p. 340, 1900.

TUNGSTEN STEEL.

Initial $T=159\,gr./mm.^2$; $t=14.2°\,C.$

H'	$T=1625\,gr./mm.^2$		$T=4837\,gr./mm.^2$		$T=8092\,gr./mm.^2$	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
4.08	0.4	0.2	2.6	0.4	6.5	0.7
10.63	3.2	0.6	13.0	1.8	27.7	3.2
15.70	9.1	1.0	42.1	4.2	98.4	8.1
19.34	40.0	2.6	142.3	6.7	244.9	11.9
23.85	35.3	2.0	98.3	6.1	149.1	10.9
32.16	16.2	1.6	43.6	4.5	67.6	7.9
43.36	6.9	1.2	21.0	3.4	32.8	5.6
98.4	0.4	0.4	1.9	0.4	3.1	0.1
210.0	−0.4	0.0	−1.6	−1.3	−2.6	−2.8
341.0	−0.3	−0.3	−1.5	−1.7	−2.8	−3.4

The effect of applying the maximum tension at once is nearly the same as the effect of the graduated applications of tensions. This will be seen by comparing the following figures with the corresponding ones in the foregoing table.

SWEDISH IRON.

Initial $T=152\,gr./mm.^2$; $t=16.5°\,C.$

$T=1562\,gr./mm.^2$			$T=3086\,gr./mm.^2$			$T=4648\,gr./mm.^2$		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
2.57	33.1	1.0	2.43	84.1	3.3	2.57	194.0	6.2
4.20	101.3	3.2	4.23	206.6	5.3	4.13	323.4	5.4
5.80	116.4	3.2	5.87	198.4	4.0	5.78	259.4	2.8
11.65	27.3	−0.4	11.70	43.7	−2.3	11.57	52.7	−4.9
42.13	−2.8	−3.8	39.62	−4.5	−7.1	39.26	−7.4	−11.0

The change of magnetization by tension under different initial loadings, where the demagnetization was always effected with the initial load, is given in the following table. For an equal increment δT of T, δI_i decreases rapidly, as the initial T increases, whereas in the graduated applications of tension, δI_i increases nearly proportional to δT. The difference between these two values of δI_i is considerable.

SWEDISH IRON.

$t = 16.5°$ C.

$T=152$ gr./mm.^{2}; $\delta T=1562$ gr.			$T=1677$ gr./mm.^{2}; $\delta T=1561$ gr.			$T=3238$ gr./mm.^{2}; $\delta T=1562$ gr.		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
2.57	33.1	1.0	2.49	16.5	−0.9	2.43	9.0	−1.2
4.20	101.3	3.2	4.01	49.5	−0.7	4.08	33.1	−2.3
5.80	116.4	3.2	6.00	59.7	−0.3	5.88	36.2	−2.4
11.65	27.3	−0.4	11.59	13.7	−1.8	11.77	6.4	−2.9
42.13	−2.8	−3.8	39.55	−3.0	−3.7	39.66	−3.0	−4.1

TUNGSTEN STEEL.

$t = 14.2°$ C.

$T=159$ gr./mm.^{2}; $\delta T=1625$ gr.			$T=1784$ gr., $\delta T=1626$ gr.			$T=4996$ gr., $\delta T=1626$ gr.		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
10.63	3.2	—	6.78	1.8	0.3	6.74	1.5	0.3
15.70	9.1	—	14.68	7.7	0.6	14.51	7.9	0.2
19.28	40.0	—	18.45	29.1	2.5	19.44	43.6	1.5
23.85	35.3	—	25.95	22.2	2.2	26.96	14.5	1.5
43.40	6.9	—	38.45	5.6	1.5	38.68	5.8	1.2
98.4	0.4	—	95.3	0.9	−0.9	9.56	0.0	−0.2
210.0	−0.4	—	200.7	−0.4	−0.7	202.5	−0.4	−0.7
341.1	−0.3	—	337.1	−0.4	−0.7	351.5	−0.4	−0.6

(*b*) *Magnetization under constant tensions:* $(I, H)_T$.

The magnetization increases rather rapidly in low fields and gradually approaches saturation. The effect of tension is, in its general features, similar to that obtained from $(\delta I, T)_H$. With low tensions, the increase of magnetization is considerably less than the value of the initial effect obtained from the latter experiment, while with high tensions, the contrary is true. These facts will be seen in the following tables and curves $(\delta I, H)_T$ in dotted lines in Figs. 5 and 7.

SWEDISH IRON.

$t = 16.°5$ *C.*

$T = 152\ gr./mm.^2$		$T = 1714\ gr./mm.^2$		$T = 7929\ gr./mm.^2$	
H	I	H	I	H	I
0.87	18.5	0.88	22.3	0.84	21.3
1.63	52.7	2.17	91.7	1.63	59.8
2.89	161.3	3.12	235.5	3.16	269.8
4.42	509	3.67	441.3	3.86	495.4
6.35	835	4.69	710	4.48	727
8.48	1000	6.44	946	6.57	982
13.60	1141	15.23	1180	13.51	1149
23.72	1229	24.10	1238	23.85	1213
35.60	1275	35.83	1280	35.88	1251
58.39	1328	58.39	1331	54.34	1300
127.3	1416	126.6	1419	124.8	1387
219.4	1493	197.4	1478	195.6	1448
319.2	1542	319.1	1547	296.8	1511
386.5	1569	383.0	1573	381.9	1547

TUNGSTEN STEEL.

$t = 13.7° C.$

$T=159\ gr./mm.^2$		$T=1784\ gr./mm.^2$		$T=4996\ gr./mm.^2$		$T=8251\ gr./mm.^2$	
H	I	H	I	H	I	H	I
2.33	14.4	2.35	14.9	2.77	17.6	2.33	14.2
4.68	31.2	5.14	35.3	5.54	38.5	4.60	29.8
7.90	58.8	8.85	69.1	9.07	71.2	7.99	60.3
12.07	107.5	12.41	114.2	12.87	123.7	12.17	113.6
16.98	202.3	16.76	212.9	16.76	223.6	16.80	241.8
19.29	304.7	19.00	341.2	18.98	381.7	18.99	449.0
22.29	532	22.06	597	21.96	631	22.02	695
27.73	788	27.66	837	27.46	852	27.64	877
33.94	918	33.60	955	33.91	965	33.98	976
41.70	1005	42.33	1041	42.72	1045	42.68	1046
63.2	1115	63.6	1144	63.2	1137	63.2	1133
105.2	1207	105.2	1233	105.2	1223	105.6	1215
158.3	1265	158.4	1291	157.7	1279	158.5	1268
212.6	1308	212.7	1327	210.9	1311	211.6	1304
268.7	1328	269.1	1342	267.1	1339	268.7	1330
345.2	1354	346.3	1380	343.8	1364	343.8	1356

(*c*) *Change of magnetization by twist under different tensions:* $(\delta I, \tau)_{H,T}$.

In Swedish iron and tungsten steel, the curves $(\delta I_i, \tau)_{H,T}$ (Figs. 8, 10 and 12) are similar to those for $(\delta I, T)_H$ for the initial effect. In weak fields where the twisting considerably increases the magnetization, the effect of the first untwisting is very small and slightly increases the magnetization. In Swedish iron, the cyclic effect of twisting (Fig. 9) under low tensions is always to diminish magnetization. In high tensions (Fig. 11), the effect has a singular character : for a small twist, the magnetization increases, but for a large twist, it is diminished. In tungsten steel, the cyclic effect of twist is similar to the cyclic effect of tension in Swedish iron, but in amount it is very small. Curves $(\delta I_i, H)_{\tau,T}$ as deduced from $(\delta I_i, \tau)_{H,T}$ are given in Figs. 13, 14 and 15 in full lines. They greatly resemble those for the tension effect, having points corresponding to the Villari points. With considerable twist, $(\delta I_c, H)_{\tau,T}$ (Figs. 13 and 14) is always negative. The experimental numbers are given in the table below :—

SWEDISH IRON.

$T=152\ gr./mm.^2$; $t=15.°6\ C.$

$\tau=31.9'$			$\tau=42.9'$			$\tau=69.8'$		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
1.34	7.0	−1.0	1.26	18.6	−1.7	1.22	36.8	−4.2
2.43	17.3	−2.5	2.47	39.4	−5.6	2.38	73.2	−11.2
4.12	44.3	−5.4	4.22	90.8	−11.2	4.24	114.7	−20.0
5.86	44.6	−5.6	5.73	84.7	−11.2	5.73	125.0	−21.4
7.79	27.0	−5.3	7.79	43.5	−11.3	7.87	46.7	−22.5
13.56	4.6	−4.4	13.60	2.8	−10.3	13.59	−8.3	−22.5
24.04	−0.7	−3.1	23.78	−4.2	−8.2	23.74	−14.1	−18.3
48.95	−1.3	−1.7	42 01	−3.9	−4.4	39.29	−10.3	−11.5
114.4	−0.3	−0.3	117.5	−1.1	−1.4	116.8	−2.5	−3.0
212.7	−0.0	−0.3	219.0	−0.0	−0.8	215.2	−1.4	−1.4
367.1	−0.1	−0.1	—	—	—	370.6	−0.4	−0.9

$T=3238\ gr./mm.^2,\ t=15.6°\ C.$

$\tau=14.6'$			$\tau=30.2'$			$\tau=58.5'$			$\tau=87.8'$		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
1.21	5.7	0.6	1.22	30.2	3.3	1.22	125.2	3.8	1.21	182.1	−17.1
2.42	22.4	2.1	2.61	116.8	6.4	2.54	296.0	−2.0	2.55	385.0	−32.2
4.02	73.7	3.7	4.05	194.0	5.7	4.17	328.2	−7.0	4.03	401.0	−37.2
5.76	47.0	3.1	5.78	116.2	4.0	5.76	176.4	−6.8	6.00	164.0	−35.6
7.76	27.3	2.4	7.76	56.4	3.0	7.80	86.5	−6.3	7.71	76.8	−32.6
13.63	11.4	1.4	13.57	24.2	1.7	13.63	29.4	−4.7	13.63	16.6	−22.8
22.92	5.8	1.3	21.70	12.4	1.1	22.47	13.8	−3.0	21.42	5.7	−14.4
49.11	1.4	0.9	49.15	3.1	0.6	49.21	2.8	−1.1	48.90	1.4	−5.5
113.5	0.4	0.0	105.8	1.0	0.0	111.2	1.0	−0.1	105.0	−1.0	−1.8
210.2	0.0	0.0	212.7	0.1	0.0	214.1	0.0	−0.0	210.2	−0.3	−0.4

TUNGSTEN STEEL.

$T=3371\ gr./mm.^2,\ t=13.7°\ C.$

$H'=4.01$		$H'=10.72$		$H'=14.53$		$H'=18.39$		$H'=22.40$	
τ	δI_i	τ	δI_i	τ	δI_i	τ	δI_i	τ	δI_i
18.4′	0.9	12.9′	2.2	12.9′	4.0	13.8′	8.9	17.6′	13.4
39.8	2.2	47.5	7.1	39.1	14.4	38.4	32.6	41.4	37.0
69.2	4.9	69.2	15.7	69.2	29.3	69.5	69.9	69.2	63.3
	0.1		0		0		−0.3		−0.3

$H'=30.07$		$H'=46.05$		$H'=98.8$		$H'=209.0$		$H'=364.0$	
τ	δI_i	τ	δI_i	τ	δI_i	τ	δI_i	τ	δI_i
12.9′	3.4	13.4′	1.3	14.5′	0.2	13.0′	0.0	21.4′	0.0
34.8	12.4	48.4	6.6	42.7	1.5	47.6	0.0	—	—
69.2	27.2	69.2	9.1	69.2	2.3	69.2	−0.1	69.2	0.0
	−0.7		−0.8		−1.0		−0.3		−0.2

The numbers in the last row give the cyclic effect of twist corresponding to the maximum twist.

The effect of the graduated application of twist does not sensibly differ from that of applying the maximum twist at once.

(*d*). *Magnetization under constant twist combined with tensions:* $(I,\ H)_{\tau,T}$.

The effect of a constant twist on magnetization is very small; in a small twist, the magnetization slightly increases, but above a moderate twist, it is decreased by twisting. With a moderate twist, curves $(\delta I,\ H)_{\tau,T}$ as deduced from $(I,\ H)_{\tau,T}$ have an opposite sign to those deduced from $(\delta I_i,\ \tau)_{H,T}$; but they have the same sign as curves $(\delta I_c,\ H)_{\tau,T}$ obtained from $(\delta I_c,\ \tau)_{H,T}$. The following tables and the dotted curves in Figs. 13, 14 and 15 will show these changes of magnetization.

SWEDISH IRON.

$T=152\ gr./mm.^2,\ t=15.7°\ C.$

$\tau=0$		$\tau=31.9'$		$\tau=70.3'$	
H	I	H	I	H	I
0.77	16.8	0.68	14.8	0.68	12.5
1.68	47.8	1.29	33.3	1.39	32.5
2.38	95.1	2.29	79.5	2.29	66.0
3.49	256.9	3.31	211.0	2.93	129.3
4.05	406.1	4.21	416.2	3.86	257.7
4.69	571	5.06	632	4.49	410.1
5.91	776	6.51	838	5.74	683
7.58	933	8.33	975	8.68	945
11.45	1094	12.29	1110	12.47	1064
20.51	1215	20.64	1211	20.65	1173
30.81	1266	31.08	1264	31.11	1238
50.90	1320	50.83	1319	50.91	1299
110.8	1408	110.2	1409	111.1	1403
173.5	1466	172.8	1467	173.5	1462
263.5	1524	261.0	1526	263.5	1521
358.1	1568	357.5	1569	359.3	1566

$T = 3238\ gr./mm.^2$, $t = 15.6°\ C.$

$\tau=0$		$\tau=14.2'$		$\tau=29.3'$		$\tau=58.3'$		$\tau=88.3'$	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.66	15.9	0.68	16.8	0.71	17.2	0.19	3.1	0.76	16.5
1.30	38.2	1.29	41.2	1.31	41.8	1.32	37.2	1.54	43.5
2.23	98.0	1.94	79.3	2.19	100.4	2.20	94.3	2.25	79.4
3.15	257.7	2.73	192.8	2.85	192.8	2.95	168.4	2.98	145.9
3.77	450.9	3.51	424.0	3.54	418.4	3.78	351.5	3.82	300.8
4.48	719	4.19	635	4.33	708	4.60	645	4.81	609
5.69	940	5.57	929	5.75	963	6.05	925	6.57	890
6.75	1028	6.67	1023	7.19	1064	7.99	1048	8.25	1005
13.74	1196	15.27	1210	14.75	1219	15.30	1196	12.53	1114
20.98	1250	21.09	1248	21.36	1265	21.21	1242	20.74	1206
31.06	1292	30.99	1288	31.57	1305	30.79	1288	31.00	1265
50.50	1342	51.23	1341	51.65	1356	50.91	1346	50.62	1330
115.7	1436	115.7	1436	113.3	1449	112.4	1444	111.9	1433
180.5	1499	180.5	1499	177.4	1511	175.9	1506	175.1	1496
275.2	1562	274.7	1561	270.8	1573	267.3	1568	266.3	1560
376.2	1610	376.1	1610	370.4	1621	365.6	1616	365.6	1608

TUNGSTEN STEEL.

$T = 3371\ gr./mm.^2$, $t = 13.7°\ C.$

$\tau=0$		$\tau=69.2'$	
H	*I*	*H*	*I*
2.71	16.7	2.22	13.4
4.68	31.7	4.61	29.8
7.98	60.5	10.56	87.0
12.03	109.0	14.72	153.2
16.80	222.6	16.80	209.8
19.03	364.2	18.98	359.2
22.04	630	21.97	622
27.63	844	27.45	840
34.18	958	33.87	958
50.26	1083	42.41	1038
85.9	1185	63.2	1136
132.2	1247	105.6	1223
179.8	1272	158.4	1279
210.5	1307	212.4	1318
267.7	1334	269.1	1341
344.1	1359	343.8	1366

From the results thus far described, it may be concluded that in Swedish iron and tungsten steel, the final magnetization is affected in no inconsiderable degree by the order of magnetizing and straining. This fact stands parallel to the result of our previous experiment that in these metals, the change of elastic constants by magnetization is considerably affected by the order of applying the stress and the magnetizing field.

II. NICKEL.

The effect of stress on the magnetization of nickel has been thoroughly studied by several physicists, so that there remains little to be studied about the effect. Our present experiment has, however, this characteristic that several effects of stress on magnetization were studied with the same specimen over a wide range of the magnetizing field and with special attention to the hysteresis effect.

(*a*) *Change of magnetization by tension under constant field*: $(\delta I, T)_H$.

The initial effect of loading on magnetization in very weak fields is an increase of magnetization by low tension, and a decrease by high tension; but the cyclic effect is always a decrease, unlike the Villari reversal in iron. Above 2 C.G.S. units, however, the initial and the cyclic effects are always a decrease of magnetization.

In low tension, δI_i or δI_c decreases almost proportionally with T; as T is increased, the rate of decrease becomes great, and after passing through an inflexion point, it begins to decrease, as shown in Fig. 16. As the field is increased, the decrease of magnetization passes through its maximum.

Except in weak fields, the cyclic effect $(\delta I_c, T)_H$ (Fig. 16, dotted lines) fairly coincides with the initial effect. Curves $(\delta I_i, H)_T$ deduced from $(\delta I_i, T)_H$ are given in Fig. 17, in full lines. In weak fields, they fall steeply and then gradually rise; as the field is further increased, they slowly tend to approach the axis of the field.

NICKEL.

Initial $T=167$ *gr./mm.*^{2} $t=14.3°$ *C.*

H'	$T=855$ *gr./mm.*^{2}		$T=1711$ *gr.*		$T=3421$ *gr.*		$T=5089$ *gr.*		$T=8513$ *gr.*	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
1.14	+6.6	−13.5	+9.7	−32.2	+3.5	−81.0	−4.1	−99.4	−9.2	−108.6
2.35	−5.8	−18.4	−18.4	−42.8	−62.7	−102.6	−85.5	−128.1	−97.8	−137.9
3.89	−10.9	−20.2	−29.2	−47.7	−83.0	−109.4	−113.8	−143.4	−132.1	−163.6
10.77	−23.4	−28.0	−52.5	−58.7	−119.2	−119.3	−170.2	−181.8	−221.3	−223.2
31.06	−23.3	−24.5	−54.0	−55.1	−122.8	−121.2	−189.2	−185.5	−267.5	−261.1
62.54	−12.4	−12.7	−29.5	−29.5	−76.0	−76.2	−129.6	−129.9	−235.5	−234.6
135.2	−4.8	−4.6	−10.6	−10.7	−25.8	−26.1	−46.4	−47.5	−113.2	−114.4
205.5	−2.0	−2.0	−4.6	−4.9	−11.3	−12.1	−20.5	−21.3	−49.3	−49.9
364.7	−0.3	—	−0.6	—	−2.1	—	−4.1	—	−11.3	—

The effect of applying the maximum tension at once is nearly the same as that of the graduated applications of tension.

The change of magnetization by tension under different initial loadings, where the demagnetization was always effected with the initial load, is given in Fig. 18. and in the following table. The full lines in the figure refer to the initial effect, while the dotted one to the cyclic effect; the initial and the cyclic effect nearly coincide with each other. As the initial tension is increased, the field of the maximum decrease is displaced toward the higher field. For the same increment δT of T, δI is in a marked degree less than that in the last experiment.

NICKEL.

$\delta T = 1022$ *gr./mm.*^{2}, $t = 14.3°$ C.

$T=1877$ *gr./mm*^{2}.			$T=3546$ *gr./mm*^{2}.			$T=5256$ *gr./mm*^{2}.		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
1.08	+3.1	−4.1	1.12	0.0	−1.7	1.17	+0.2	−0.6
2.37	−1.5	−11.0	2.47	−1.6	−3.7	2.50	−0.9	−1.5
4.04	−10.4	−15.6	3.95	−4.1	−6.4	3.89	−2.0	−2.5
10.64	−23.0	−22.8	10.79	−17.0	−17.9	10.65	−6.6	−7.2
30.60	−26.2	−26.0	31.53	−25.4	−25.1	31.51	−18.8	−18.8
62.9	−18.7	−18.4	59.90	−23.0	−23.0	59.2	−24.2	−23.0
132.8	−6.7	−6.4	136.6	−9.2	−8.7	136.3	−11.7	−11.9
227.9	−2.8	−2.5	211.3	−4.1	−4.3	210.7	−5.7	−5.7
349.7	−1.1	−0.6	349.2	−1.4	−1.4	348.5	−1.5	−1.5

(*b*) *Magnetization under constant tensions:* $(I, H)_T$.

The magnetization increases steeply in low fields and after passing through an inflexion point, gradually approaches saturation. The effect of tension is considerably large and always to diminish magnetization; curves $(I, H)_T$ become less steep in weak fields with the increasing tension, and tend to approach each other in strong fields. If we compare δI obtained from $(I, H)_T$ with that from $(\delta I, T)_H$ for the same values of H and T, the former is found to be a little numerically greater than the latter. The dotted line in Fig. 17 represents the values of δI as deduced from $(I, H)_T$.

NICKEL.

$t = 14.6° C.$

$T=167$ *gr./mm.*		$T=1022$ *gr.*		$T=1878$ *gr.*		$T=3588$ *gr.*		$T=5256$ *gr.*		$T=8680$ *gr.*	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.93	9.3	0.92	10.0	0.93	9.1	0.86	5.0	1.66	7.6	2.06	5.8
1.68	82.3	2.02	73.9	1.91	38.9	1.76	11.6	3.50	16.7	4.41	12.9
3.19	147.6	3.36	131.3	3.24	91.1	3.44	26.8	6.64	34.8	7.78	23.4
4.12	170.4	4.02	154.4	4.43	115.8	4.81	42.0	10.76	60.4	13.38	41.7
6.62	216.7	6.65	191.2	6.70	151.6	7.75	73.4	15.96	93.8	26.01	81.5
10.48	266.8	9.66	226.5	10.52	194.2	12.14	115.1	—	—	—	—
16.14	316.6	15.54	279.3	15.72	237.1	17.98	159.3	26.56	144.3	—	—
26.31	373.0	26.30	343.0	26.32	301.2	27.18	214.5	39.31	202.8	39.24	119.3
38.88	413.4	39.09	392.4	39.11	354.6	39.62	270.9	62.9	291.6	59.8	176.2
62.5	466.0	62.9	444.5	63.2	420.8	63.4	352.0	106.3	392.1	105.6	296.8
133.5	490.0	133.9	491.4	134.9	482.8	138.4	451.0	186.1	460.5	185.8	420.0
205.7	506.3	206.1	508.2	206.8	502.7	206.7	479.7	211.7	470.8	227.1	449.5
269.8	511.0	269.9	515.5	270.5	511.6	270.4	491.5	287.0	487.9	284.1	473.0
366.6	514.5	367.9	520.0	368.7	517.6	367.3	499.5	358.9	495.8	354.0	488.7

(c) *Change of magnetization by twist under different tensions :* $(\delta I, \tau)_{H, T}$.

In weak fields, the magnetization is increased by twist, but in strong fields, it is slightly diminished. As shown in Figs. 19 and 20, the curves $(\delta I, \tau)_{H, T}$ bend slightly towards the axis of the twist; the curvatures become less as the tension is increased. Except in weak fields, the initial effect is inconsiderable; it also becomes less as the tension is increased.

Curves $(\delta I_i, H)_\tau$, from $(\delta I_i \tau)_{H, T}$ are drawn in Figs. 21 and 22 in full lines; they have steep positive maxima from which the curves slope down gradually to the higher field, cut the axis of H, become negative and after passing through very inconspicuous negative maxima, very slowly bend towards the axis; the maxima become flatter with the greater tension, and the positions of the maxima as well as the points of intersection with the axis move toward higher fields with the increasing tension. The course of the curves is thus quite similar to that of curves $(\delta I, H)_T$ in the case of iron. The following tables are the numerical data obtained.

NICKEL.

$T=1197\ gr./mm.^2$, $t=14.9°\ C.$

$H'=0.68$				$H'=1.07$				$H'=2.31$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
8.4′	22.1	9.1′	34.2	16.0′	105.3	12.2′	50.9	16.5′	27.8	13.1′	57.2
21.8	72.8	23.6	91.7	26.9	158.3	28.9	109.4	34.4	95.2	30.4	121.6
36.9	130.6	41.8	136.3	40.7	196.6	41.8	137.5	46.3	152.3	43.3	150.9
54.2	188.8	55.2	155.6	54.0	221.2	58.3	161.0	63.1	216.3	58.7	174.9
69.2	221.1	69.2	171.9	69.3	239.5	69.0	172.9	69.4	232.5	69.3	187.1

$H'=3.91$				$H'=10.62$				$H'=23.22$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
10.2′	60.2	8.0′	29.3	10.8′	45.6	7.1′	13.8	8.8′	4.6	14.0′	4.6
23.7	121.0	26.3	91.8	22.6	75.4	22.7	48.0	23.6	20.0	31.7	15.6
36.0	151.9	38.2	117.6	37.2	106.0	39.5	73.2	38.3	30.3	—	—
53.5	179.8	48.7	133.8	50.7	122.5	54.0	88.1	54.9	37.8	50.7	25.0
69.0	198.2	69 2	154.8	69.3	136.7	69.0	97.8	69.3	41.1	69.3	31.1

$H'=50.07$				$H'=99.7$				$H'=168.7$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
11.8′	+7.3	10.1′	+3.0	9.8′	+2.4	15.4′	+0	9.9′	+1.2	11.7′	+0.6
28.8	-3.4	26.8	-7.6	25.8	- 6.6	32.6	-12.5	38.5	-12.2	25.7	-5.2
46.4	-7.6	45.0	-12.2	43.9	-18.7	47.7	-22.0	58.0	-21.4	42.0	-14.8
68.9	-10.8	69.3	-15.7	69.4	-30.7	69.3	-31.8	69.3	-29.4	69.3	-30.1

$H'=241.4$				$H'=358.0$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
13.1′	-0.3	14.2′	0.0	18.8′	0.0	14.2′	0.0
28.9	-4.9	28.9	- 4.6	38.7	- 4.3	22.6	- 2.3
46.6	-13.3	48.0	-13.6	53.5	-10.4	52.0	- 9.6
69.3	-25.6	69.3	-19.3	69.3	-14.2	69.3	-15.8

$T=3546\ gr./mm.^2,\ t=14.7°\ C.$

$H'=0.17.$				$H'=0.96$				$H'=2.40$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
6.1′	0.5	7.6′	10.7	7.4′	4.6	9.6′	14.0	10.0′	20.4	9.7′	31.0
21.1	2.6	24.7	35.5	17.8	11.3	27.5	41.8	23.2	46.8	28.0	89.4
38.7	8.7	34.2	47.3	32.7	20.4	48.0	66.7	36.9	70.2	43.7	129.7
58.2	21.4	52.3	63.6	51.3	32.5	—	—	55.2	104.7	—	—
68.5	31.4	68.5	76.2	68.5	44.4	68.5	85.3	68.5	133.0	68.5	174.6

$H'=4.06$				$H'=10.61$				$H'=24.37$			
τ	δI_i	τ	δI_e	τ	δI_i	τ	δI_e	τ	δI_i	τ	δI_e
8.2′	35.9	7.3′	24.0	7.4′	26.0	9.2′	24.0	6.6′	8.9	8.8′	26.3
22.4	84.9	23.4	78.5	20.8	70.7	24.6	66.4	20.4	36.3	25.0	52.3
37.3	131.4	38.4	121.3	38.5	117.5	42.3	107.3	37.4	64.0	41.5	75.5
53.6	169.1	49.8	147.3	51.0	142.3	—	—	48.8	79.1	55.7	91.2
68.5	196.8	68.5	179.6	68.5	168.2	68.5	146.9	68.5	97.4	68.5	102.0

$H'=49.16$				$H'=94.4$				$H'=168.3$			
τ	δI_i	τ	δI_e	τ	δI_i	τ	δI_e	τ	δI_i	τ	δI_e
7.6′	5.2	10.6′	2.7	8.9′	+3.0	11.2′	−1.1	12.8′	+1.1	10.9′	0.0
24.3	14.3	24.7	9.2	24.0	− 6.4	28.4	−13.3	29.3	−8.4	32.8	−12.2
36.4	20.5	45.6	16.6	38.7	−15.8	45.1	−18.8	49.0	−20.7	51.6	−23.4
50.3	26.0	—	—	51.0	−19.1	—	—	—	—	—	—
68.5	31.8	68.5	23.9	68.5	−22.9	68.5	−24.3	68.5	−31.0	68.5	−31.3

$H'=253.9$				$H'=358.7$			
τ	δI_i	τ	δI_e	τ	δI_i	τ	δI_e
10.1′	−0.3	17.0′	−1.8	12.2′	0.0	12.5′	0.0
27.3	−6.2	35.1	−10.4	28.7	−3.0	29.5	− 4.4
47.0	−17.0	53.7	−19.6	48.0	−9.9	47.1	−11.1
68.5	−28.0	68.5	−27.2	68.3	−18.1	68.5	−19.2

$T = 6286\ gr./mm.^2, \quad t = 14.7°\ C.$

$H' = 2.38$				$H' = 3.99$				$H' = 10.64$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
9.3′	1.8	3.8′	15.4	10.1′	13.9	11.6′	24.5	7.1′	16.9	9.9′	19.4
24.0	5.5	29.8	43.0	24.8	36.3	26.1	55.0	20.4	42.8	26.8	53.5
38.1	11.3	45.4	66.2	39.8	61.9	41.7	88.8	33.8	73.8	42.3	91.8
51.3	20.5	—	—	58.2	95.3	50.6	107.2	51.3	116.5	57.0	121.0
68.8	37.3	68.8	100.8	68.8	116.4	68.8	142.4	68.8	152.5	68.8	141.5

$H' = 24.08$				$H' = 49.02$				$H' = 94.9$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
6.6′	7.9	9.3′	10.4	10.9′	4.7	11.2′	3.0	8.6′	3.5	14.4′	1.5
23.8	36.7	26.9	40.8	23.5	20.7	27.3	21.4	22.9	4.7	31.8	2.6
36.1	63.7	44.1	75.9	40.3	42.0	45.4	44.3	38.7	6.3	—	—
55.4	100.8	56.3	97.6	57.0	61.7	—	—	53.7	8.7	50.3	5.3
68.8	120.4	68.8	115.5	68.8	72.9	68.8	67.7	68.8	10.4	68.8	7.9

$H' = 168.7$				$H' = 254.0$				$H' = 351.2$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
9.9′	+2.1	10.5′	+0.9	7.8′	+1.1	9.9′	+0.9	11.7′	+0.5	17.0′	−0.5
23.2	−5.9	29.2	−9.4	28.9	−6.4	28.5	−5.6	29.8	−4.0	34.6	−6.4
43.5	−15.4	50.3	−19.3	45.4	−14.9	—	—	49.6	−12.1	48.7	−13.1
68.8	−25.8	68.8	−26.0	68.8	−26.4	68.8	−28.1	68.8	−20.9	68.8	−22.2

The effect of the graduated application of twist does not sensibly differ from that obtained by applying the maximum twist at once.

(*d*) *Magnetization under constant twist combined with tensions:* $(I, H)_{\tau, T}$.

The effect of a constant twist on magnetization is to increase the magnetization in weak fields and to diminish it in strong fields. As for curves $(\delta I, H)_{\tau, T}$ (dotted lines in Figs. 21 and 22) deduced from $(I, H)_{\tau, T}$, the general course is quite similar to that of the curves obtained from $(\delta I_i, \tau)_{H, T}$; but quantitatively there is some difference between these two. The difference, however, becomes less with increased tension.

NICKEL.

$T=1197$ *gr./mm.*^{2}, $t=13.7°$ *C.*

$\tau=0$		$\tau=11.4'$		$\tau=34.'5$		$\tau=67.'0$	
H	I	H	I	H	I	H	I
0.87	6.5	0.81	6.1	0.79	4.2	0.79	5.0
1.88	57.0	1.78	47.4	2.00	80.5	1.34	68.8
4.71	151.7	3.07	102.0	3.40	135.3	1.98	133.2
7.68	196.0	4.99	147.1	5.32	211.7	4.04	262.6
10.05	222.0	8.11	195.3	8.36	273.7	8.44	310.5
14.49	262.1	12.48	248.5	11.95	302.6	13.61	328.8
35.74	371.0	34.12	360.9	33.79	368.4	33.80	366.2
60.4	429.9	52.7	406.1	52.85	398.7	52.68	387.7
125.1	484.0	104.1	463.5	105.3	448.7	104.6	425.7
182.2	502.7	169.1	491.0	171.0	478.7	169.3	454.2
218.4	509.2	220.4	501.6	222.5	491.1	220.4	468.0
304.4	518.6	289.9	508.1	282.8	500.4	279.7	479.1
369.1	522.0	356.9	512.7	363.2	507.9	357.3	489.1

$T=3546$ *gr./mm.*^{2}, $t=13.7°$ *C.*

$\tau=0$		$\tau=11.8'$		$\tau=34.6'$		$\tau=66.9'$	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.80	4.8	0.79	4.1	0.80	3.5	0.79	2.7
2.01	13.5	1.98	12.3	1.77	9.4	1.75	14.8
3.80	29.9	3.11	22.1	2.98	24.6	2.84	106.4
6.55	59.5	4.65	37.3	4.53	57.6	4.06	191.5
9.73	91.5	6.86	60.3	6.98	123.0	6.52	248.5
15.03	136.4	11.67	107.3	12.08	195.4	11.22	277.4
23.30	191.4	17.51	156.0	22.53	250.5	22.08	310.6
33.86	245.0	33.48	249.7	33.68	287.2	33.15	333.9
54.1	321.3	57.28	322.6	51.85	332.2	52.87	367.3
113.6	430.2	101.8	414.9	113.3	418.8	112.3	425.6
170.7	462.6	162.3	458.9	171.0	454.3	169.3	456.5
253.7	483.7	275.8	487.4	252.8	479.5	251.6	482.2
354.0	499.1	356.9	494.7	353.7	493.1	350.5	500.1

$T=6286$ *gr./mm.*^{2}, $t=13.7°$ *C.*

$\tau=0$		$\tau=11.8'$		$\tau=34.6'$		$\tau=69.0'$	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*
1.46	5.6	1.17	3.8	1.39	4.0	0.88	2.6
3.03	11.7	2.53	8.4	3.12	11.4	3.14	25.5
5.04	21.9	4.26	16.0	4.89	21.7	5.40	158.4
8.92	43.1	9.89	43.9	7.80	58.8	8.16	211.9
14.29	72.3	15.38	74.3	11.58	119.7	13.27	232.0
23.67	114.8	23.86	119.5	23.35	175.9	23.77	260.3
34.77	156.8	34.93	162.2	34.98	211.0	35.36	285.7
57.33	229.7	56.8	234.7	56.2	268.9	56.58	325.7
98.6	341.0	88.8	321.6	88.1	338.5	88.1	372.5
161.7	429.0	134.3	396.3	147.6	412.3	147.3	427.6
195.5	451.2	190.4	441.9	196.3	444.6	196.2	445.2
272.7	478.7	270.2	471.5	267.5	471.4	267.5	481.3
381.7	465.7	380.5	489.8	375 4	491.3	373.0	503 8

It is to be noticed that in nickel, the initial and the cyclic effects of tension or twist on magnetization nearly coincide with each other except in weak fields, and that the change of magnetization does not much depend on the order of magnetizing and straining. Thus, in nickel, the hysteresis effect is comparatively small except in weak fields; and therefore the agreement between the theory regarding magnetostriction and the experiment might well have been expected. Thus, in our previous experiment, we found that the changes of the modulus of elasticity by magnetization for different orders of magnetizing and straining fairly coincided with each other, while in the case of rigidity, the difference was somewhat greater. In the present experiment also, the tension effect shows a better agreement for different orders of magnetizing and straining than for the torsion effect.

III. NICKEL STEELS containing 28.74, 50.72 and 70.32 per cent of Nickel.

As for nickel steels, experiments on the effect of stress on magnetization have been very few. So far as we know, the effect of tension only was studied by H. Tomlinson[1] with nickel steels of 22, 25 and 30 per cent of nickel, and by Professor H. Nagaoka and one of us[2] with nickel steels of 35 and 45 per cent of nickel. Hence somewhat detailed descriptions of the phenomena will not be unnecessary.

(*a*) *Change of magnetization by tension under constant field:* $(\delta I, T)_H$

The magnetization increases at first rather rapidly, but after-

1) Tomlinson, Proc. Roy. Soc. **56**, p. 103, 1894; Beibl. **18**, 952.
2) Nagaoka and Honda, Jour. Coll. Sci., **16**, Art. 8, 1902.

ward slowly with the tension. The increase in low fields is tolerably large, but in strong fields, it is very small. The initial effect is significant only for weak fields, where the cyclic is remarkably less than the initial. The following tables and Figs. 23, 24 and 25 show these changes of magnetization.

Curves $(\delta I_i, H)_T$ from $(\delta I_i, T)_H$ rise rapidly with the field, attain sharp maxima at low fields, fall at first rapidly and then gradually to asymptotic values, as shown in Figs. 26, 27 and 28 in full lines. The maximum of δI increases with tension. For the same tension, the maximum rapidly increases with the percentage content of nickel.

28.74% NICKEL STEEL.

Initial $T=134$ $gr./mm.^2$ $t=14.0°$ $C.$

H'	$T=1370$ $gr./mm.$		$T=2706$ $gr.$		$T=4077$ $gr.$		$T=6818$ $gr.$	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
0.11	1.1	8.8	—	—	22.2	30.3	36.0	35.3
0.25	20.1	26.4	62.4	57.5	97.5	77.6	130.3	94.1
0.49	46.7	31.8	97.6	64.7	129.2	86.1	155.5	104.0
1.11	43.4	31.7	78.0	59.9	97.6	76.4	113.0	90.0
2.30	31.6	26.8	52.2	45.7	63.3	56.1	72.2	64.5
4.65	18.8	17.2	28.9	27.0	34.0	32.0	38.6	36.3
10.68	5.9	5.3	—	—	11.0	10.0	13.8	12.8
24.32	2.3	2.0	—	—	5.9	5.3	8.8	8.0
55.48	1.5	1.6	—	—	4.9	4.9	7.8	8.0
172.9	1.5	1.3	—	—	4.4	4.1	7.4	7.0
374.3	1.4	1.3	—	—	4.0	3.9	6.6	6.6

50.72% NICKEL STEEL.

Initial $T=160$ *gr./mm.*^{2}, $t=13.7°$ *C.*

H'	$T=1645$ *gr./mm.*^{2}		$T=3249$ *gr.*		$T=4894$ *gr.*		$T=6540$ *gr.*		$T=8185$ *gr.*	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
0.29	103.3	91.0	222.6	138.8	277.8	153.2	300.3	155.1	311.0	153.7
0.70	266.7	140.5	437.9	212.1	503.9	233.3	528.3	235.1	539.7	232.0
1.27	319.8	157.3	476.3	233.4	534.8	254.9	552.3	255.9	557.1	250.0
1.95	253.1	148.7	359.2	215.4	392.0	230.3	397.7	227.5	390.2	219.4
2.76	196.6	136.8	276.3	193.9	298.7	207.3	299.8	204.1	294.0	195.2
5.52	134.5	108.8	186.4	151.3	202.5	162.5	204.0	161.1	199.1	154.4
10.71	84.4	75.4	113.8	102.0	122.5	109.4	123.4	109.4	120.5	106.0
23.77	34.2	32.8	46.1	43.5	49.4	46.6	—	46.6	47.9	45.0
51.44	7.9	7.7	10.7	10.5	11.7	11.0	—	—	10.2	9.2
151.4	0.7	0.4	0.9	0.4	0.9	0.4	—	—	0.2	0.0
360.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—	0.0	0.0

70.32% NICKEL STEEL.

Initial $T=156$ *gr./mm.*^{2}, $t=14.5°$ *C.*

H'	$T=1604$ *gr./mm.*^{2}		$T=3170$ *gr.*		$T=4774$ *gr.*		$T=6379$ *gr.*		$T=7984$ *gr.*	
	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c	δI_i	δI_c
0.13	25.7	32.5	90.0	58.1	111.3	65.1	107.0	66.3	94.5	67.2
0.29	44.4	45.5	94.5	—	109.6	85.2	119.2	—	132.4	91.3
0.70	182.4	143.5	473.3	225.0	582.4	296.3	635.1	—	665.1	339.1
1.08	342.3	152.0	586.5	260.4	666.2	316.6	703.5	—	728.0	364.7
1.49	321.4	149.2	482.5	246.0	546.0	298.3	578.8	—	600.2	344.6
2.66	198.1	131.6	301.5	208.6	352.2	249.8	379.2	—	381.6	287.8
5.05	113.3	93.2	168.5	142.7	199.8	171.1	—	—	228.2	198.4
10.69	46.9	42.5	70.7	65.7	85.7	80.2	—	—	101.7	95.7
24.52	10.7	10.3	17.5	17.5	23.8	22.3	—	—	28.4	28.2
49.70	1.8	2.0	—	—	4.7	4.5	—	—	6.4	6.2
129.3	0.4	—	—	—	0.7	—	—	—	0.8	—
227.0	0.0	—	—	—	0.0	—	—	—	0.1	—
393.2	0.0	—	—	—	0.0	—	—	—	0.0	—

The effect of applying the maximum tension at once does not materially differ from that obtained by graduated application of it.

The change of magnetization by tension under different initial loadings nearly coincides with the above result in the case of

28.74% Ni.; but in 50.72% Ni. and 70.32% Ni., the change is generally greater in the present case than in the former, as may be seen from the following table.

28.74% NI. $t=14.2°$ C. **70.32% NICKEL STEEL.** $t=14.1°$ C.

$T=1504$ gr. $\delta T=1373$ gr.			$T=959$ gr. $\delta T=803$ gr.			$T=1760$ gr. $\delta T=803$ gr.			$T=3326$ gr. $\delta T=1605$ gr.		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
0.18	8.8	2.5	0.13	−2.4	−4.7	0.13	−3.3	−5.8	0.13	−11.6	−12.0
0.29	39.6	15.6	0.30	+4.2	−1.0	0.29	+0.8	−5.7	0.29	+18.4	−11.0
0.68	49.8	25.8	0.68	65.3	+15.6	0.68	74.9	+5.9	0.72	49.7	+ 1.4
1.12	35.6	25.9	1.10	189.5	45.5	1.08	149.1	32.6	1.20	128.2	36.9
1.92	24.6	20.7	1.49	163.5	56.8	1.48	111.2	44.4	1.59	81.2	40.1
3.55	13.8	13.0	2.68	92.8	55.4	2.69	62.4	39.7	2.67	54.2	35.9
7.09	5.7	5.6	5.09	48.6	39.6	5.01	34.1	28.1	5.08	30.3	25.3
14.94	2.2	2.2	10.67	19.5	17.9	10.66	13.8	13.2	10.69	14.7	13.6
38.37	1.6	1.6	25.81	4.2	4.2	25.65	3.5	3.4	25.92	4.2	4.1
115.6	1.5	1.3	49.83	0.9	0.9	52.5	0.6	0.6	49.8	1.2	1.0
212.0	1.3	1.3	106.5	0.3	0.1	106.9	0.1	0.1	107.6	0.0	0.1
343.1	1.2	1.2	205.5	0.0	0.0	208.5	0.0	0.0	208.8	0.0	0.0

50.72% NICKEL STEEL.

$t=14.2°$ C.

$T=983$ gr. $\delta T=823$ gr.			$T=1805$ gr. $\delta T=823$ gr.			$T=3410$ gr. $\delta T=823$ gr.			$T=5054$ gr. $\delta T=823$ gr.		
H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c	H'	δI_i	δI_c
0.26	32.9	14.8	0.29	14.8	−4.0	0.28	3.7	−4.7	0.30	−0.4	−2.1
0.88	159.3	36.1	0.84	95.3	+13.4	0.88	41.6	+1.2	0.84	+17.7	−1.5
1.60	139.6	56.1	1.48	100.5	29.3	1.48	47.3	7.4	1.48	26.7	+0.4
2.07	109.7	55.0	2.05	77.4	33.6	2.07	41.4	10.3	2.17	17.5	1.7
2.93	83.7	55.1	2.87	58.5	33.8	2.86	25.6	10.4	3.86	7.1	1.6
4.29	64.5	48.5	4.07	43.8	30.3	3.90	18.3	9.2	6.66	3.3	1.5
5.65	54.6	43.0	5.55	34.5	26.4	5.57	12.7	7.7	—	—	—
10.67	32.7	28.6	10.73	19.6	16.1	10.75	6.1	4.9	18.66	0.9	0.4
19.24	17.7	15.4	23.97	8.7	7.2	23.88	2.2	2.2	42.53	0.0	0.0
46.49	4.4	3.5	50.44	1.9	1.9	47.83	0.6	1.2	—	—	—
172.4	0.0	0.1	166.1	0.0	0.0	167.2	0.0	0.0	168.0	0.0	0.0
355.0	0.0	0.0	321.0	0.0	0.0	355.0	0.0	0.0	354.0	0.0	0.0

(*b*) *Magnetization under constant tensions:* $(I, H)_T$.

Among other ferromagnetic metals and alloys, nickel steels are characterized by the extraordinary steepness of the curve of magnetization; in a field of 5 C.G.S. units, the magnetization attains a value which is only a little short of its saturation value. The steepness increases with tension, first rapidly and then gradually to an asymptotic value. The enormous values of susceptibility $\varkappa$ are given in the following table and plotted in Fig. 29. It will be noticed that the maximum value increases with the percentage of nickel. In 70.32% Ni., the susceptibility even attains a maximum value of 1015 for $T=4930$ *gr./mm*², which is several times greater than the maximum susceptibility of a well annealed Swedish iron. In very weak fields, the magnetization is considerably increased by tension, but in higher fields comparatively little.

H	28.74% *Ni.*		50.72% *Ni.*		70.32% *Ni.*	
	$\varkappa$		$\varkappa$		$\varkappa$	
	$T=134$ *gr.*	$T=6952$ *gr.*	$T=160$ *gr.*	$T=8344$ *gr.*	$T=156$ *gr.*	$T=4930$ *gr.*
0.20	75	90	55	10	10	33
0.30	110	390	110	25	17	95
0.40	125	530	153	75	28	200
0.60	145	445	243	380	64	470
0.80	150	367	337	710	120	1015
1.00	140	297	410	800	230	860
1.30	124	230	467	685	296	677
1.60	113	190	442	593	288	570
2.00	99	157	397	503	270	470
3.00	73	107	299	370	220	322
5.00	50	64	202	232	160	196
7.00	36	46	150	170	123	140

Curves $(\delta I,\ H)_T$ (Figs. 26, 27 and 28, dotted lines) deduced from $(I,\ H)_T$ take a course quite similar to those deduced from $(\delta I,\ T)_H$ In 28.74% Ni., the maximum δI is generally greater, and the asymptotic value decidedly greater than in $(\delta I_i\ T)_H$ for the same tension and field. On the contrary, δI of 50.72% Ni. is always less than the corresponding value in the last experiment. In 70.32% Ni., there is a fair coincidence between the two values of δI.

28.74% NICKEL STEEL.

$t=14.0°$ *C.*

$T=134$ *gr./mm²*.		$T=1504$ *gr.*		$T=4211$ *gr.*		$T=6952$ *gr.*	
H	I	H	I	H	I	H	I
0.23	20.8	0.22	26.8	0.13	7.9	0.09	3.8
0.72	108.6	0.73	178.4	0.24	63.0	0.24	41.6
0.77	114.0	1.02	204.8	0.33	167.9	0.30	118.9
1.33	162.8	1.41	225.2	0.52	243.3	0.53	260.6
2.00	192.1	2.53	257.4	0.82	272.1	0.77	280.9
3.26	225.5	3.63	274.1	1.21	286.5	1.24	295.5
4.76	249.1	7.31	299.7	4.08	320.3	3.36	317.6
12.11	287.1	12.83	312.0	11.57	339.6	11.28	338.6
19.27	296.0	19.12	318.0	18.96	346.1	18.96	345.6
24.06	299.7	24.44	321.4	24.31	349.1	24.32	349.0
50.1	308.8	50.1	330.2	49.75	357.7	49.9	357.8
123.8	320.3	123.5	341.0	122.4	368.5	122.4	368.7
230.5	329.0	229.0	349.7	227.9	377.2	227.5	377.2
374.8	337.2	372.6	357.8	370.2	385.2	370.9	385.1

50.72% NICKEL STEEL.

$t=13.1°$ *C.*

$T=983$ *gr./mm²*.		$T=3450$ *gr.*		$T=5054$ *gr.*		$T=6699$ *gr.*		$T=8344$ *gr.*	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.30	34.0	0.32	28.6	0.33	24.7	0.31	23.3	0.38	15.4
0.51	96.5	0.48	130.0	0.50	160.4	0.49	113.7	0.49	80.6
0 75	262.6	0.59	314.7	0.62	361.1	0.66	458.6	0.63	285.9
1.48	739	0.85	597	0.89	681	1.08	839	0.80	561.8
1.73	810	1.41	899	1.38	928	1.64	996	0.90	717
2.62	949	2.15	1043	2.12	1062	2.83	1118	1.58	964
4.01	1028	3.83	1143	4.42	1177	4.58	1180	3.05	1106
4.92	1064	5.49	1181	6.87	1215	7.57	1224	7.27	1201
17.01	1210	10.88	1229	11.96	1244	13.79	1253	12.94	1232
23.73	1235	22.62	1260	22.78	1267	23.17	1271	22.95	1253
43 47	1267	43.06	1279	42.80	1283	43.22	1287	42.81	1269
110.9	1290	107.0	1295	105.6	1298	105.3	1302	105.3	1284
234.8	1298	228.0	1302	225.5	1305	224.6	1310	226.1	1292
384.1	1302	371.5	1306	367.9	1309	367.3	1314	366.7	1296

70.32% NICKEL STEEL.

$t=13.5°$ *C.*

$T=156$ *gr./mm²*.		$T=959$ *gr.*		$T=1761$ *gr.*		$T=3366$ *gr.*		$T=4930$ *gr.*	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.25	18.5	0.23	21.9	0.31	26.9	0.26	18.5	0.24	9.5
0.79	94.5	0.75	128.8	0.70	162.2	0.46	132.9	0.43	180.6
0.86	127.9	0.90	327.5	0.85	557.8	0.57	206.5	0.60	289.0
1.13	325.4	1.53	609	1.74	758	0.74	478.8	0.64	347.7
1.52	430.4	1.78	654	2.80	841	0.85	738	0.71	532.0
1.80	503.6	2.77	763	3.83	884	2.20	903	0.76	734.
4.66	769.	5.77	892	6.32	937	3.56	946	1.45	906
8.04	871	10.00	955	11.53	985	6.72	981	3.38	970
11.93	927	19.28	1008	19.67	1016	19.26	1026	6.75	996
26.54	998	24.76	1022	25.14	1028	24.63	1034	24.02	1032
50.9	1022	51.2	1045	52.0	1047	51.1	1049	50.9	1043
104.4	1030	103.2	1052	104.3	1055	102.5	1054	99.0	1048
204.4	1033	204.1	1054	202.3	1057	200.6	1056	201.0	1049
370.0	1034	368.5	1055	365.0	1057	362.6	1057	365.1	1051

(c) Change of magnetization by twist under different tensions: $(\delta I, \tau)_{H, T}$

In very low fields, the magnetization considerably increases with twist; in higher fields, it first increases, but afterward begins to decrease with the twist, and in still higher fields, the magnetization decreases nearly uniformly with the twist, as shown in Figs. 30, 32, 34, 36, 38 and 39. The change of magnetization rapidly increases with the percentage content of nickel.

As for the cyclic effect (Figs. 31, 33, 35, 37, 38 and 40), it coincides fairly with the initial, except in weak fields. In 28.74% Ni. and 70.32% Ni., the increase of magnetization is only observable in very weak fields, and the magnetization generally decreases with twist. In 50.72% Ni, the magnetization first increases with the twist, attains a maximum, and then decreases. As the tension is increased, the change becomes gradually less.

Curves $(\delta I_i, H)_{\tau, T}$ (Figs. 41, 42, 43, 44, 45, 46 and 47 in full lines) obtained from $(\delta I_i, \tau)_{H,T}$ rise and fall steeply in a very low field, cut the axis of H, become negative, and after passing through rather conspicuous negative maxima, slope away gradually toward the axis, with the increasing field. δI is numerically greater for a greater twist. In 50.72% Ni., however, δI for a small twist is always positive, tending to zero, as the field increases.

Curves $(\delta I_c, H)_{\tau, T}$ for cyclic effect shown in the same figures are similar to the above curves, and become coincident with them above a moderate field. The increase of magnetization with small twists becomes less as the tension increases; and for 28.74% Ni. and 70.32% Ni., it almost vanishes at a high tesnion.

28.74% NICKEL STEEL.

$T=959\ gr./mm.^2,\quad t=14.2^\circ\ C.$

$H'=0.10$				$H'=0.21$				$H'=0.48$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.5′	5.4	—	—	13.2′	8.6	13.4′	2.1	14.8′	15.8	17.6′	+1.5
34.5	12.7	36.9′	0.9	38.8	23.3	35.6	2.7	40.2	28.6	41.1	−0.1
68.3	18.9	68.3	1.6	68.2	36.2	68.3	2.0	68.3	31.0	68.3	−2.7

$H'=1.07$				$H'=1.86$				$H'=3.70$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
13.7′	+4.8	15.8′	−2.1	17.8′	−2.4	14.5′	−3.9	15.9′	−5.3	15.4′	−5.4
37.4	+2.3	35.4	−7.2	43.6	−12.2	38.2	−13.9	56.4	−15.9	41.8	−19.8
68.3	−4.2	68.3	−22.0	68.3	−20.9	68.3	−24.6	68.3	−30.7	68.2	−31.7

$H'=6.70$				$H'=14.51$				$H'=30.56$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.8′	−8.9	14.7′	−4.7	13.5′	−1.3	12.4′	−1.1	15.9′	−0.4	18.3′	−0.9
43.4	−18.5	36.7	−15.7	38.7	−7.2	38.7	−7.6	39.4	−2.3	42.1	−3.1
68.2	−29.9	68.3	−30.5	68.3	−15.8	68.2	−16.6	68.2	−5.9	68.2	−6.1

$H'=61.0$				$H'=168.5$				$H'=355.7$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
32.9′	−0.6	30.4′	−0.4	37.9′	0.0	—	—	36.1′	−0.1	—	—
68.2	−2.1	68.2	−2.0	68.2	−0.1	68.2′	−0.1	68.2	−0.1	—	—

$T=1778$ *gr./mm.*^{2}, $t=14.2°$ *C.*

$H'=0.13$				$H'=0.25$				$H'=0.50$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
13.0′	6.1	16.6′	−1.0	12.4′	12.6	16.0′	−0.4	12.0′	14.8	13.8′	−1.0
36.2	18.3	41.5	−1.6	43.5	37.4	41.7	−3.2	40.0	24.2	41.8	−7.6
68.2	26.5	68.2	−1.6	68.2	46.2	68.8	−5.5	68.8	22.6	68.8	−13.7

$H'=1.08$				$H'=1.91$				$H'=3.58$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
16.5′	+1.6	13.5′	−3.7	13.2′	−2.3	16.8′	−6.1	14.0′	−3.7	15.8′	−5.3
38.6	−5.6	39.4	−15.6	40.9	−15.6	37.4	−17.4	40.1	−17.5	41.1	−19.1
68.8	−16.3	68.8	−26.5	68.8	−28.1	68.8	−31.7	68.8	−31.9	68.8	−32.9

$H'=6.81$				$H'=14.61$				$H'=33.61$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.7′	−3.4	14.0′	−3.3	14.1′	−1.2	15.6′	−1.5	—	—	—	—
39.9	−14.7	39.8	−14.9	42.3	−7.3	39.7	−6.6	36.6′	−1.2	32.5′	−1.3
68.8	−27.8	68.8	−28.1	68.8	−14.7	68.8	−14.8	68.8	−4.0	68.8	−4.3

$H'=68.0$				$H'=172.0$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
35.0′	−0.3	26.9′	−0.3	—	—	—	—
68.8	−1.3	68.8	−1.3	68.8′	0.0	—	—

$T=4211$ $gr./mm.^2$, $t=14.2°$ $C.$

$H'=0.13$				$H'=0.24$				$H'=0.48$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.4′	8.6	17.1	0.0	14.3′	16.3	15.4′	−1.3	14.9′	19.7	16.1′	−4.4
41.2	29.7	39.6	0.0	40.6	46.3	40.1	−6.9	39.2	27.4	39.6	−16.7
68.8	41.7	69.0	1.0	69.0	61.0	69.0	−12.2	69.0	22.6	69.0	−29.9

$H'=1.09$				$H'=1.90$				$H'=3.60$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
16.1′	−1.6	15.6′	−5.1	13.6′	−2.6	18.9′	−6.1	14.9′	−2.8	16.5′	−4.0
41.6	−14.8	38.6	−18.7	43.4	−15.2	42.3	−19.2	42.6	−15.3	40.5	−15.2
69.0	−28.9	69.0	−34.6	69.0	−31.2	69.0	−33.2	69.0	−27.7	69.0	−29.1

$H'=6.78$				$H'=14.76$				$H'=35.58$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.2′	−2.3	17.1′	−2.7	14.9′	−0.9	16.6′	−1.2	18.6′	−0.4	20.6′	−0.5
41.7	−11.0	40.5	−10.4	38.8	−4.5	38.8	−4.8	46.0	−1.7	43.5	−1.7
69.0	−21.8	69.0	−21.5	69.0	−11.0	69.0	−11.2	69.0	−3.3	69.0	−3.5

$H'=66.7$				$H'=175.9$				$H'=354.6$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
19.7′	−0.1	26.3′	0.0	15.9′	0.0	—	—	—	—	—	—
69.0	−1.2	69.0	−1.1	69.0	0.0	69.0′	−0.2	69.0′	0.0	69.0′	−0.2

50.72% NICKEL STEEL.

$t = 14.3° C.$

$H' = 0.28$				$H' = 0.68$				$H' = 1.38$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.1′	22.4	17.2′	8.7	12.8′	51.3	10.4′	6.2	13.0′	55.9	10.5′	9.3
29.4	57.7	35.2	19.8	27.6	118.8	30.7	16.9	26.9	117.9	32.5	23.1
43.2	100.2	53.3	30.3	54.3	254.1	48.9	21.8	47.4	199.0	54.6	11.3
67.9	171.7	68.2	34.7	68.0	305.6	68.1	16.2	67.9	225.0	67.9	−3.9

$H' = 1.83$				$H' = 3.43$				$H' = 5.46$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
10.7′	42.2	14.2′	15.2	14.8′	35.6	12.4′	16.6	10.9′	19.8	13.9′	17.4
26.9	106.8	34.6	27.3	29.3	56.7	27.8	28.7	26.1	38.1	31.4	26.6
46.0	149.6	51.4	18.0	45.7	60.8	49.0	19.7	51.0	33.6	49.6	15.5
67.7	150.9	67.9	−4.7	67.9	37.8	67.7	−9.0	67.7	12.5	67.7	−9.6

$H' = 12.62$				$H' = 24.57$				$H' = 49.61$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
12.0′	13.2	9.8′	8.6	12.0′	8.1	16.6	10.1	15.2′	4.5	17.7′	4.8
38.2	22.8	31.0	18.9	40.6	12.0	39.7	12.0	40.4	5.7	39.0	5.4
52.4	13.8	54.3	6.8	—	—	—	—	—	—	—	—
67.7	−2.7	67.7	−8.3	67.7	−3.6	67.7	−4.7	67.7	−1.8	67.7	−2.7

$H' = 108.8$				$H' = 211.8$				$H' = 377.4$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
16.7′	1.7	15.7′	1.5	15.7′	0.7	15.8′	0.6	10.9′	−0.2	—	—
43.1	1.7	43.6	1.5	45.6	0.7	43.6	0.6	—	—	—	—
67.7	−1.1	67.7	−1.5	67.7	−0.2	67.7	−0.3	67.7	−0.2	—	—

$T=3409\ gr./mm.^2,\quad t=14.3°\ C.$

$H'=0.28$				$H'=0.69$				$H'=1.28$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
12.4′	13.5	11.2′	2.1	12.6′	38.2	11.1′	4.5	12.1′	35.0	13.5′	9.3
29.7	42.4	30.1	4.1	28.9	88.3	27.7	9.0	27.7	83.6	30.7	15.5
52.0	91.7	46.2	5.6	46.0	131.6	46.5	9.0	43.3	122.8	49.6	11.4
67.9	121.5	67.9	5.6	67.9	182.9	67.9	3.0	67.9	153.2	67.9	−2.6

$H'=1.82$				$H'=4.16$				$H'=10.54$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
11.9′	35.6	10.7′	9.5	11.0′	18.0	11.2′	11.7	11.2′	10.1	11.8′	9.0
31.2	81.3	26.9	18.6	27.8	35.0	28.0	22.0	29.1	18.1	38.7	16.3
53.0	108.0	46.5	15.2	46.5	36.1	47.9	15.5	47.6	13.8	50.5	8.9
67.9	108.2	67.9	−4.7	67.9	18.0	67.9	−5.6	67.7	−2.7	67.7	−6.2

$H'=24.62$				$H'=49.35$				$H'=109.0$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
11.3′	5.7	10.9′	4.8	11.8′	3.0	10.7′	2.7	15.2′	1.5	12.0′	1.4
27.7	9.8	28.6	8.9	28.0	4.5	25.4	4.8	41.9	1.5	44.2	1.4
47.3	6.9	48.6	5.7	51.0	2.4	46.7	3.5	—	—	—	—
67.7	−3.3	67.7	−4.2	67.7	−2.4	67.9	−2.3	67.7	−0.6	67.7	−0.5

$H'=212.8$				$H'=376.8$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
17.7′	0.3	13.2′	0.8	23.0	0.0	21.9	0.0
35.1	0.3	37.8	0.8	—	—	—	—
67.7	−0.8	67.7	0.3	67.7	−0.3	67.7	−0.2

70.32% NICKEL.

$T=1123$ *gr./mm.*^{2}, $t=12.7°$ *C.*

$H'=0.29$				$H'=0.69$				$H'=0.89$			
τ	δI_i	τ	δI_c	τ	δI	τ	δI_c	τ	δI_i	τ	δI_c
22.6′	77.0	25.3′	4.4	16.2′	89.0	23.2′	14.1	16.6′	128.2	17.9′	7.9
—	—	—	—	35.5	212.7	—	—	37.3	324.0	42.0	32.7
48.5	118.1	48.9	1.9	54.5	295.0	49.4	34.0	55.7	394.6	—	—
69.3	121.9	69.3	−1.0	69.3	341.0	69.3	42.9	69.3	420.8	69.0	44.5

$H'=1.70$				$H'=3.08$				$H'=4.64$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
14.3′	49.3	25.6′	5.1	22.7′	19.9	18.4′	−3.2	20.1′	4.0	21.0′	−9.0
37.1	98.8	48.8	10.6	48.7	22.8	44.9	−14.8	36.1	−8.1	47.6	−33.6
55.9	110.2	—	—	—	—	—	—	—	—	—	—
69.3	115.0	69.3	13.0	69.4	20.2	69.4	−22.0	69.4	−27.6	69.3	−44.9

$H'=8.96$				$H'=23.23$				$H'=47.04$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
18.5′	−5.8	25.1′	−17.1	25.8′	−10.2	20.1′	−5.2	16.1′	−2.5	21.2′	−2.5
45.8	−44.9	48.5	−49.4	46.5	−35.1	48.0	−36.8	47.5	−17.0	47.9	−17.8
69.3	−67.2	69.1	−69.4	69.3	−60.2	69.3	−60.8	69.3	−32.6	69.3	−33.2

$H'=92.0$				$H'=208.7$				$H'=367.2$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
26.2′	−1.5	—	—	25.3′	0.0	—	—	—	—	—	—
48.7	−6.4	—	—	49.5	−1.5	—	—	—	—	—	—
69.3	−13.2	—	—	69.3	−3.0	—	—	69.3	−1.3	—	—

$T=3366$ *gr./mm.*², $t=12.7°$ *C.*

$H'=0.29$				$H'=0.68$				$H'=0.88$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
23.9′	76.9	23.6′	−3.9	22.6′	81.6	23.6′	−8.0	15.6	111.8	24.3′	−11.4
49.2	97.6	48.8	−14.5	47.6	139.6	47.5	−25.7	36.5	245.8	48.6	−34.1
69.1	100.9	69.3	−20.5	69.2	143.3	69.2	−36.3	69.2	284.9	69.2	−47.4

$H'=1.68$				$H'=3.06$				$H'=5.00$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
19.8′	27.8	22.3′	−12.6	15.8′	0.3	18.2′	−10.4	15.2	−3.8	17.1	−9.7
40.4	21.7	48.5	−43.5	33.5	−16.9	38.3	−37.6	31.0	−33.4	36.1	−36.0
—	—	—	—	52.6	−35.7	59.7	−60.5	51.5	−48.2	54.7	−58.9
69.2	1.6	69.2	−61.2	69.0	−48.5	68.9	−68.9	69.1	−65.3	69.1	−73.1

$H'=10.75$				$H'=23.43$				$H'=47.06$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
15.4′	−4.7	13.3′	−3.1	18.9′	−4.7	14.6′	−2.1	21.8′	−2.6	20.7′	−2.6
35.1	−27.3	31.3	−23.2	34.7	−17.3	35.3	−17.3	46.2	−13.9	41.9	−12.1
55.5	−52.8	55.1	−53.4	55.1	−37.2	51.5	−33.0	—	—	—	—
69.1	−67.5	69.1	−68.0	69.0	−50.6	69.0	−50.2	69.0	−28.4	69.0	−29.3

$H'=104.6$				$H'=190.9$				$H'=348.9$			
τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c	τ	δI_i	τ	δI_c
16.2′	−0.7	15.0′	−0.6	18.9′	−0.1	—	—	18.4′	−0.3	—	—
36.3	−3.3	46.5	−4.4	44.3	−1.5	—	—	44.3	−0.7	—	—
69.0	−11.2	69.0	−10.1	69.0	−4.0	—	—	69.0	−1.7	—	—

The effect of giving a maximum twist at once does not materially differ from that of a graded twisting.

(*d*) *Magnetization under constant twist combined with tensions:* $(I, H)_{\tau, T}$.

The effect of a constant twist on magnetization is comparatively great, especially in high fields. In 28.74% Ni., the magnetization is slightly increased by a small twist, but above a moderate twist, it decreases. In 50.72% Ni., the increase of magnetization by a small twist is not appreciable, but the magnetization always decreases with greater twist. The magnetization of 70.32% Ni., is also decreased by twisting, except in weak fields in which a slight increase is observed. In all cases, the change of magnetization decreases with increasing tension.

In 28.74% Ni., curves $(\delta I, H)_{\tau, T}$ (Fig. 41 in dotted lines) deduced from $(I, H)_{\tau, T}$ show a somewhat different aspect from those deduced from $(\delta I, \tau)_{T, H}$, especially for a small twist. For a small value of twist, δI is always positive and have a faint maximum; for a greater twist, it is first negative and afterward positive; and for a still greater twist, it is always negative, and except in weak fields, it takes a course parallel to the corresponding curve obtained from the last experiment, but the former lies somewhat below the latter.

In 50.72% Ni., curves $(\delta I, H)_{\tau, T}$ (Figs. 42 and 43 in dotted lines) deduced from $(I, H)_{\tau, T}$ have a quite different aspect, i.e., δI is always negative. It rapidly decreases in weak fields, and after passing through a negative maximum, slopes away very slowly toward the axis of H with increasing field. Tension reduces the decrease of magnetization.

In 70.32% Ni., curves $(\delta I, H)_{\tau, T}$ (Figs. 44, 45, 46 and 47 in dotted lines) deduced from $(I, H)_{\tau, T}$ take a course similar to those obtained from the last experiment, but the difference is that in the former, the positive maxima in weak fields are considerably smaller, the points at which δI changes its sign, lie in a lower part of the field, and δI tends more slowly to zero than in the latter. The effect of tension is to push the point of intersections with the axis of H toward the origin.

For these alloys, curves $(\delta I, H)_{\tau, T}$ as deduced from $(I, H)_{\tau, T}$ rather resemble the curves $(\delta I_c, H)_{\tau, T}$ obtained from the last experiment.

28.74% NICKEL STEEL.

$T=959$ *gr./mm.*^{2}, $t=14.6°$ *C.*

$\tau=0$		$\tau=11.4'$		$\tau=34.8'$		$\tau=68.0'$	
H	I	H	I	H	I	H	I
0.29	35.6	0.28	34.9	0.29	28.1	0.30	23.6
0.42	82.8	0.42	85.5	0.39	77.8	0.44	63.0
0.58	121.8	0.52	112.4	0.57	113.8	0.59	96.6
0.90	165.8	0.85	165.3	0.89	162.2	0.92	137.3
1.34	196.7	1.19	192.9	1.41	197.0	1.64	179.5
2.37	235.3	3.15	261.9	2.89	243.3	2.85	211.9
4.54	271.8	5.35	288.9	4.82	272.6	4.68	238.8
6.91	290.3	6.90	299.6	6.75	286.5	6.89	258.3
18.93	314.5	19.04	322.4	19.04	318.7	19.09	297.8
24.15	318.1	24.30	326.0	24.18	323.1	24.35	303.8
49.20	326.8	50.00	334.9	50.0	332.2	50.1	316.4
129.4	337.7	128.7	346.0	127.3	343.5	126.6	329.0
239.8	350.5	238.8	359.6	236.7	357.2	234.2	342.6
348.5	357.0	366.1	367.5	362.6	364.7	341.5	349.4

50.72% NICKEL STEEL.

$T = 1151$ *gr./mm.*², $t = 13.3°$ *C.*

τ=0		τ=11.7′		τ=34.7′		τ=67.3′	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.18	16.6	0.18	15.8	0.16	13.8	0.18	12.3
0.50	87.8	0.46	92.6	0.50	89.3	0.49	72.6
0.65	197.2	0.66	220.7	0.64	217.3	0.65	182.1
0.79	271.8	0.96	528	0.75	399.5	0.91	480.6
1.14	659	1.08	635	1.10	624	1.18	585
1.43	787	1.37	755	1.45	739	2.02	804
2.73	1016	2.41	981	2.05	882	3.34	929
3.90	1085	4.95	1115	3.56	1013	4.60	994
6.04	1155	8.68	1193	6.42	1112	7.84	1084
12.39	1240	15.43	1251	10.83	1183	14.20	1163
22.58	1288	21.61	1279	22.82	1257	22.45	1215
39.94	1319	40.74	1316	40.30	1297	39.97	1268
100.9	1345	99.9	1346	101.3	1332	101.3	1317
216.9	1354	214.5	1350	217.2	1344	217.2	1334
351.6	1358	349.8	1355	353.7	1348	351.1	1340

$T = 3409$ *gr./mm.*², $t = 13.3°$ *C.*

τ=0		τ=11.7′		τ=68.2′	
H	*I*	*H*	*I*	*H*	*I*
0.21	12 6	0.19	14.4	0.20	26.4
0.49	128.9	0.50	115.6	0.50	127.8
0.61	296.9	0.62	309.6	0.62	330.7
0.90	676	0.79	578	1.02	671
1.13	863	1.02	751	1.37	789
2.43	1073	1.52	932	1.64	859
3.16	1125	2.44	1070	3.22	1031
4.47	1169	5.04	1178	4.34	1077
7.60	1220	8.03	1219	8.08	1151
14.56	1256	13.09	1246	13.51	1193
22.08	1273	22.43	1270	22.61	1229
40.81	1291	40.34	1290	40.67	1264
98.1	1308	97.4	1308	99.2	1300
213.0	1317	209.9	1317	213.7	1315
343.8	1321	340.7	1321	345.9	1317

70.32% NICKEL STEEL.

$T = 1123\ gr./mm.^2,\ t = 13.0°\ C.$

τ=0		τ=12.7′		τ=35.5′		τ=68.3′	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.25	23.6	0.20	16.2	0.20	10.0	0.29	17.1
0.74	161.9	0.65	114.4	0.65	144.1	0.63	108.2
0.93	423.4	0.87	302.7	0.91	376.2	0.95	310.2
1.12	575	1.05	555	1.25	672	1.22	689
1.49	688	1.27	635	1.50	701	1.47	713
2.50	809	2.55	790	2.61	774	2.45	764
3.85	890	4.35	883	4.21	834	3.96	809
8.07	988	6.87	939	7.09	888	7.15	858
12.64	1029	10.46	983	11.82	937	11.21	895
17.62	1053	16.66	1020	16.97	970	16.99	930
22.71	1065	21.60	1037	21.86	990	21.93	951
46.73	1090	44.44	1065	44.90	1030	45.18	1005
92.7	1098	87.4	1076	87.1	1047	87.1	1035
181.7	1101	174.6	1080	172.9	1053	172.5	1049
368.1	1104	345.4	1081	342.9	1056	341.2	1055

$T = 3366\ gr./mm.^2,\ t = 13.0°\ C.$

τ=0		τ=12.8′		τ=36.7′		τ=69.0′	
H	*I*	*H*	*I*	*H*	*I*	*H*	*I*
0.21	9.4	0.21	9.0	0.25	31.4	0.22	6.1
0.56	186.2	0.58	193.6	0.64	232.5	0.67	155.9
0.74	442.9	0.73	436.6	0.71	528	0.72	245.1
0.93	784	0.99	824	0.90	768	0.81	400.8
1.33	863	1.34	866	1.45	839	0.94	767
2.40	934	2.18	924	3.25	912	1.23	800
3.64	973	3.70	972	4.64	936	2.51	863
7.83	1016	7.25	1011	8.37	967	3.75	890
11.86	1035	11.44	1033	12.55	989	9.74	941
16.56	1048	16.81	1047	17.65	1006	17.63	976
21.48	1057	21.78	1056	22.78	1018	22.72	991
44.84	1074	44.83	1074	46.43	1044	46.44	1029
86.7	1081	86.0	1082	89.9	1056	89.9	1051
170.8	1084	171.5	1084	172.2	1062	172.2	1062
359.8	1086	361.8	1086	361.7	1065	364.0	1067

Thus in the case of nickel steels, the change of magnetization by tension does not differ much for the different orders of straining and magnetizing. So also in the change of elasticity, we found a fair agreement between the values for different orders, especially at high tensions. On the other hand, the change of magnetization by twist differs sometimes in a considerable degree for the different orders, while in the change of rigidity, the agreement between the values for different orders is generally good, if the tension be large, especially in 28.74 and 70.32 per cent of nickel. In all, alloys, for which the hysteresis effect is small, have also a small difference in the changes of elastic constants by magnetization for the different orders of magnetizing and straining.

Thus far, we have seen that generally the change of magnetization by stresses differs more or less with the different orders of applying the magnetic field and the stress. In some cases, the difference is not only quantitative but also qualitative, as for the effect of twist in Swedish iron or in 50.72% Ni., if the initial effect of twisting under constant field be compared with the results of magnetization under constant twist. On the other hand, there are examples of good coincidence as in the case of the tension effect in nickel and 70.32% nickel steel. Generally speaking, the tension effect shows a better agreement for the different orders of magnetizing and straining, than for the torsion effect; and the discrepancy is remarkable in low magnetic fields, as may be expected from the consideration of the hysteresis effect prominent in that region.

In our preceding paper, we have remarked, that the dependency of the change of elastic constants on the different orders of magnetizing and straining, is probably due to the hysteresis effect accompanying magnetization. This explanation agrees well with the facts brought out by the present experiment.

§ 4. RECIPROCAL RELATIONS.

Among several important reciprocal relations obtained by J.J. Thomson, the two relations, which have the connection with the present experiment, are referred to below.

Let a cylindrical bar of soft iron, whose axis coincides with the axis of x, be magnetized along its axis. Let e, f, g, be the dilatations of the bar parallel to the axes of x, y, z respectively; J.J. Thomson obtained the relation

$$n\frac{\partial e}{\partial I^2}=\tfrac{1}{2}\left\{1-H\left(\frac{\partial \varkappa}{\partial I}\right)_{e,f,g}\right\}\left\{\frac{m}{3m-n}\;\frac{1}{\varkappa I}\left(\frac{\partial I}{\partial e}\right)_{H,f,g}-\frac{m-n}{3m-n}\;\frac{1}{\varkappa I}\left(\frac{\partial I}{\partial f}\right)_{H,e,g}\right\}\text{..................}(1)$$

where $I, H, \varkappa$ have the usual meanings, n represents the coefficient of rigidity, and m is connected with the modulus of compression k by the relation $k=m-n/3$. In his original work, the factor $\frac{1}{2}$ is dropped in the right hand member of the above equation; the error is to be traced back to his equation (41).

Since $dI=\varkappa dH+Hd\varkappa$, we have the relation, supposing the strain to be kept constant,

$$1-H\left(\frac{\partial \varkappa}{\partial I}\right)_{e,f,g}=\frac{\varkappa}{\left(\frac{\partial I}{\partial H}\right)_{e,f,g}}.$$

Hence equation (1) may be written

$$\frac{\partial e}{\partial I}\left(\frac{\partial I}{\partial H}\right)_{e,f,g}=\frac{m}{n(3m-n)}\left(\frac{\partial I}{\partial e}\right)_{H,f,g}-\frac{m-n}{n(3m-n)}\left(\frac{\partial I}{\partial f}\right)_{H,e,g}\text{.........}(2)$$

Again if T is the tension per unit of area, we have

$$\left(\frac{\partial I}{\partial T}\right)_H=\left(\frac{\partial I}{\partial e}\right)_{H,f,g}\left(\frac{\partial e}{\partial T}\right)_H+\left(\frac{\partial I}{\partial f}\right)_{H,e,g}\left(\frac{\partial f}{\partial T}\right)_H+\left(\frac{\partial I}{\partial g}\right)_{H,e,f}\left(\frac{\partial g}{\partial T}\right)_H$$

$$=\left(\frac{\partial I}{\partial e}\right)_{H,f,g}\left(\frac{\partial e}{\partial T}\right)_H + 2\left(\frac{\partial I}{\partial f}\right)_{H,e,g}\left(\frac{\partial f}{\partial T}\right)_H , \quad \text{..............(3)}$$

since we may put $f=g$.

If H is zero, we have

$$\frac{\partial e}{\partial T}=\frac{m}{n(3m-n)}, \quad \frac{\partial f}{\partial T}=-\frac{m-n}{2n(3m-n)};$$

hence neglecting the change of elastic constants by magnetization, we have, from (3),

$$\left(\frac{\partial I}{\partial T}\right)_H=\frac{m}{n(3m-n)}\left(\frac{\partial I}{\partial e}\right)_{H,f,g}-\frac{m-n}{n(3m-n)}\left(\frac{\partial I}{\partial f}\right)_{H,e,g} \quad \text{.........(4)}$$

Hence by (2) and (4), we get

$$\frac{\partial e}{\partial I}\left(\frac{\partial I}{\partial H}\right)_{e,f,g}=\left(\frac{\partial I}{\partial T}\right)_H.$$

But $\left(\frac{\partial I}{\partial H}\right)_T=\left(\frac{\partial I}{\partial H}\right)_{e,f,g}+\left(\frac{\partial I}{\partial e}\right)_{H,f,g}\left(\frac{\partial e}{\partial H}\right)_T+\left(\frac{\partial I}{\partial f}\right)_{H,e,g}\left(\frac{\partial f}{\partial H}\right)_T$

$$+\left(\frac{\partial I}{\partial g}\right)_{H,e,f}\left(\frac{\partial g}{\partial H}\right)_T \doteqdot \left(\frac{\partial I}{\partial H}\right)_{e,f,g};$$

hence finally we get

$$\left(\frac{\partial e}{\partial H}\right)_T=\left(\frac{\partial I}{\partial T}\right)_H \quad \text{.....................(5)}$$

As to the twist, J. J. Thomson obtained a relation, which strictly speaking, holds in the case of a thin tube, i. e.

$$n\frac{\partial c}{\partial I^2}=\frac{1}{2\kappa I}\left(\frac{\partial I}{\partial c}\right)_H\left\{1-H\left(\frac{\partial \kappa}{\partial I}\right)_c\right\} \quad \text{............(6)}$$

Here again in his original work, the factor $\frac{1}{2}$ is dropped; c is not the twist τ per unit length, but it is connected with τ by the relation $c=r\tau$, where r is the radius of the thin tube. As in the former case, we have

$$1-H\left(\frac{\partial\varkappa}{\partial I}\right)_c=\frac{\varkappa}{\left(\frac{\partial I}{\partial H}\right)_c} \quad \text{and} \quad \left(\frac{\partial I}{\partial H}\right)_c=\left(\frac{\partial I}{\partial H}\right)_L,$$

in which L is the twisting couple. Equation (6) then becomes

$$\frac{\partial c}{\partial I}\left(\frac{\partial I}{\partial H}\right)_L=\left(\frac{\partial c}{\partial H}\right)_L=\frac{1}{n}\left(\frac{\partial I}{\partial c}\right)_H,$$

or, very nearly $\quad r^2\left(\frac{\partial\tau}{\partial H}\right)_L=\frac{1}{n}\left(\frac{\partial I}{\partial\tau}\right)_H.$

The last equation, if it be integrated over the cross-section of the wire of radius R gives

$$\frac{\pi}{2}R^4\left(\frac{\partial\tau}{\partial H}\right)_L=\frac{1}{n}\left(\frac{\partial}{\partial\tau}\int 2\pi rIdr\right)_H=\frac{\pi R^2}{n}\left(\frac{\partial I_m}{\partial\tau}\right)_H,$$

where I_m is the mean intensity of magnetization. Hence

finally $\quad \left(\frac{\partial\tau}{\partial H}\right)_L=\frac{2}{nR^2}\left(\frac{\partial I_m}{\partial\tau}\right)_H. \quad \text{.......................(7)}$

From a thermodynamical consideration, *A.* Heydweiller obtained two relations, neglecting small quantities,

$$\frac{\partial e}{\partial H}=\frac{\partial I}{\partial T}+\frac{I(1-2\sigma)}{E}, \quad \text{......................(8)}$$

$$\frac{1}{E^2}\frac{\partial E}{\partial H}=-\frac{\partial^2 I}{\partial T^2}-\frac{1-2\sigma}{E}\frac{\partial I}{\partial T}, \quad \text{............(9)}$$

where E is the modulus of elasticity, and σ the Poisson ratio. In his original paper, σ was put equal to $\frac{1}{3}$. Equation (9) was obtained by differentiating equation (8), considering σ and E to be constant. But in magnetic fields, both σ and E vary considerably with tension, as is shown by our previous experiment, so that if we retain the second term in the right-hand side of equation (9), the term $I\frac{\partial}{\partial T}\left(\frac{1-2\sigma}{E}\right)$ must be subtracted from it. But these terms being small compared with the first term, they may be neglected without causing any considerable error. The second term in equation (8) is also very small.

On another occasion,* Heydweiller gave relations, which are very nearly equal to the last two with the second terms suppressed, and remarked that they are correct. Heydweiller's equation (8) differs from that given by J. J. Thomson by a term of second importance.

Rensing experimentally tested relation (9) in the case of iron and nickel, and showed a fair agreement between the theory and the experiment.

R. Gans criticized Heydweiller's equations and proposed his own, i. e.,

$$\frac{\partial e}{\partial H}=\frac{1}{4\pi}\frac{\partial B}{\partial T}+\frac{(\mu-\mu_0)}{4\pi E}H\left(\frac{1}{\mu_0}\frac{\partial B}{\partial H}-2\sigma\right).$$

If the medium surrounding the magnet be air, we may put $\mu_0=1$; hence

$$\frac{\partial e}{\partial H}=\frac{\partial I}{\partial T}+\frac{I(1-2\sigma)}{E}+\frac{2\pi}{E}\frac{\partial I^2}{\partial H} \quad \text{..............}(10)$$

Thus, Gans's equation differs from that of Heydweiller by the term $\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$, which generally is not very small, but in weak fields, it sometimes overweighs the first term. As in the case of Heydweiller, differentiating the above relation with respect to T, Gans obtained an expression for the change of elasticity, which differs from that of Heydweiller by the term $-\frac{2\pi}{E}\frac{\partial^2 I^2}{\partial T\partial H}$. Here again, it was assumed that σ and E are independent of T, a supposition not admissible in a magnetized wire.

*) Rensing, loc. cit. p. 377.

By a similar consideration as Heydweiller, A. Kolâcêk obtained equation (8), and also a relation between magnetism and twist, i. e.,

$$\frac{\partial \tau}{\partial H} = s\frac{\partial I}{\partial L}, \quad \text{......................(11)}$$

where s is the cross section of the wire. Since

$$L = \frac{\pi}{2}R^4\tau n,$$

the above equation becomes

$$\frac{\partial \tau}{\partial H} = \frac{2}{nR^2}\frac{\partial I}{\partial \tau}, \quad \text{...............(12)}$$

which coincides with equation (9).

M. Cantone obtained two relations by equating the change of magnetic energy due to a tension or a twist to the change of elastic energy caused by magnetization, i. e.,

$$e_m = \frac{\partial}{\partial T}\int_o^I H dI \text{ and } \tau_m = s\frac{\partial}{\partial L}\int_o^I H dI,$$

where e_m and τ_m are the magnetic strains. By differentiating the above equations with respect to H, we have

$$\frac{\partial e}{\partial H} = \frac{\partial}{\partial T}\left(H\frac{\partial I}{\partial H}\right), \quad \text{...............(13)}$$

$$\frac{\partial \tau}{\partial H} = s\frac{\partial}{\partial L}\left(H\frac{\partial I}{\partial H}\right). \quad \text{............(14)}$$

Cantone tested the second relation by experiment and found a satisfactory agreement in iron and nickel. For the first relation, he also made a comparison between theory and experiment, but the data he used were taken from experiments by different physicists, so that they do not refer to the same specimen. Though the comparison shows a satisfactory agreement, it is not certain, whether it was by chance or not.

By a direct method, Dr. S. Sano obtained the relation

$$\frac{\partial e}{\partial H}=\frac{\partial I}{\partial T}+\frac{I(1-2\sigma)}{E}+\frac{2\pi}{E}\frac{\partial(\varkappa_0^2H^2)}{\partial H}\text{ , }\quad\text{.........(15)}$$

where $\varkappa_0$ is a term in the expression of susceptibility, which is independent of the strain. Since $I \doteqdot \varkappa_0 H$, Sano's equation practically coincides with Gans's. For the change of elasticity, Dr. Sano obtained

$$\frac{1}{E^2}\frac{\partial E}{\partial H}=-\frac{\partial^2 I}{\partial T^2}\text{ , }\quad\text{...............(16)}$$

which is practically the same as Heydweiller's equation, but different from Gans's by a term not negligibly small in weak fields. The above equation was obtained independently of the relation for $\frac{\partial e}{\partial H}$. As to the effect of twist, Dr. Sano obtained an equation, which can be transformed into (12).

Thus far, the relations for $\frac{\partial e}{\partial H}$ given by Heydweiller, Gans, Kolâcêk and Sano all agree with one another in the first important term $\frac{\partial I}{\partial T}$. Relation (1) given by J.J. Thomson does not differ in reality from others. Relation (13) given by Cantone also coincides with others in the first term, provided $\varkappa$ is independent of H. The second term $I(1-2\sigma)/E$ in (8) and (10) may be neglected for the first approximation; the third term in relation (10), which becomes important in weak fields is properly to be added.

The relations for $\frac{\partial E}{\partial H}$ given by Heydweiller, Gans and Sano also agree with each other in the first term $-\frac{\partial^2 I}{\partial T^2}$. Gans's differs principally from the others by a term not generally small in weak fields.

As regards the relation for $\frac{\partial \tau}{\partial H}$, Kolâcêk's and Sano's coincide with each other. Thomson's relation (6) also does not differ from the others. If $\varkappa$ be independent of H, Cantone's formula coincides with the others.

Thus, the chief relations to be tested by experiments are as follows:—

$$\frac{\partial e}{\partial H}=\frac{\partial I}{\partial T}+\frac{2\pi}{E}\frac{\partial I^2}{\partial H},$$

$$\frac{1}{E^2}\frac{\partial E}{\partial H}=-\frac{\partial^2 I}{\partial T^2},$$

$$\frac{\partial \tau}{\partial H}=\frac{2}{nR^2}\frac{\partial I}{\partial \tau}=s\frac{\partial I}{\partial L}.$$

Our present experiments combined with the previous investigations on the change of elastic property due to magnetization furnish us with good materials for the testing of these relations.

Our results taken as a whole, give for the effect of tension as well as of twist, two different sets of experimental data corresponding to the different orders of applying the magnetic field and the stress. The mutual correspondence of the results in the previous and the present experiments in this respect is tabulated below:

Change of Strains by Magnetization.	*Change of Magnetization by Strain.*
1. Magnetic elongation under constant tension.	1.′ Magnetization under constant tension.
Elongation by tension under constant field.	Change of magnetization by tension under constant field.
2. Magnetic twisting under constant couple (Barus's method).	2.′ Magnetization under constant twist.
Change of rigidity under constant field (oscillation method).	Change of magnetization by twist under constant field.

The theoretical relations to be tested were, however, deduced on the supposition that the magnetization is independent of the order of magnetizing and straining, so that in comparing the theory and the experiment, too much weight is not to be placed on the above correspondency.

The values of $\frac{\partial e}{\partial H}$, $\frac{\partial E}{\partial H}$ and $\frac{\partial \tau}{\partial H}$ were deduced from our previous experiment, while the corresponding values for $\frac{\partial I}{\partial T}$, $\frac{\partial^2 I}{\partial T^2}$ and $\frac{\partial I}{\partial L}$ were obtained from the present experiment. In the following tables, $\left[\frac{\partial I}{\partial T}\right]_H$ and $\left[\frac{\partial^2 I}{\partial T^2}\right]_H$ are the values of these differential coefficients obtained from curves $(\delta I, T)_H$; while $\left[\frac{\partial I}{\partial T}\right]_T$ and $\left[\frac{\partial^2 I}{\partial T^2}\right]_T$ are the values of the coefficients from curves $(I, H)_T$. $\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$ is the value obtained from the results of the elongation method; while $\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$ is that obtained from the tension effect on magnetic elongation. $\frac{1}{\tau}\left[\frac{\partial \tau}{\partial H}\right]_H$ is the value of the coefficient obtained from the result of the oscillation method, and $\frac{1}{\tau}\left[\frac{\partial \tau}{\partial H}\right]_\tau$ that obtained from the Barus's method; while $\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$ and $\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$ are the values obtained from curves $(\delta I, \tau)_H$ and $(I, H)_\tau$ respectively. To find the values of these differential coefficients, corresponding curves were carefully drawn on section papers; a fine straight line drawn on a thin glass plate was brought into contact with the curve at the required point and the trigonometrical tangent of the inclination of this straight line was evaluated.

SWEDISH IRON.

H	$T=1627$ *gr./mm.*^{2}				$T=2410$ *gr./mm.*^{2}			$T=5535$ *gr./mm.*^{2}		
	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-8}$									
5.9	+18	+72	+63	+33	+16	+57	+46	+1.0	+20	+0.0
11.9	+14	+13	+12	+14	+8.0	+11	+10	+0.7	+2.0	+0.0
24.2	+0.8	+0.6	+3.3	+4.7	+0.5	+0.2	+3.1	−1.0	−0.8	−7.0
36.6	−0.7	−1.7	+2.0	+2.7	−0.8	−1.7	+0.0	−1.7	−1.9	−6.2
97.1	−1.8	−2.4	—	+1.2	−1.8	−2.4	+0.0	−2.1	−2.0	−5.0
207	−2.0	−2.2	—	+0.7	−2.0	−2.2	+0.0	−1.9	−2.3	−8.0
367	−1.4	−1.7	—	+0.4	−1.4	−1.7	+0.0	−1.4	−1.9	—

H	$T=1627$ *gr./mm.*^{2}				$T=3190$ *gr./mm.*^{2}			$T=4754$ *gr./mm.*^{2}		
	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-17}$									
5.9	—	−54	—	—	−30	—	—	−19	—	—
11.9	0.0	−28	−29	−6	−21	−29	−20	−19	−29	−41
24.2	−0.39	−11	−7.1	—	−6.4	−4.5	−21	−8.8	−2.8	—
36.6	−0.61	−4.6	—	−22	−3.3	−1.8	−31	−3.3	−1.0	−12
97.1	−1.23	+0.03	—	—	−0.53	+1.0	−13	+0.05	—	—
207	−0.20	+0.84	—	—	+0.03	+0.28	−18	+0.03	−0.6	−34
367	−0.06	+0.64	—	—	+0.21	−0.28	—	+0.00	−0.3	—

$T=3255$ *gr./mm.*^{2}

H		$\tau=6.3'$	$\tau=6.3'$	$\tau=6'.3$	$\tau=58'.0$
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$	
	Measured in $10.^{-4}$				
4	——	−11	+128	+64	−16
6	——	−13	+92	−8.0	−12
8	——	−11	+48	−4.8	−8.6
14	+0.23	−3.6	+24	−1.6	−6.2
22	−0.15	−2.7	+12	——	−4.8
50	−0.40	−1.1	+3.0	——	−0.96
110	−0.40	−0.33	+0.7	——	−0.48
200	+0.12	−0.11	+0.0	——	−0.48
300	+0.07	−0.07	——	——	−0.24

TUNGSTEN STEEL.

H	$T=1693\ gr./mm.^2$				$T=3322\ gr./mm.^2$			$T=5762\ gr./mm.^2$		
	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-8}$									
10.6	+0.9	+3.3	+0.0	+0.65	+0.9	+3.3	+0.0	+0.7	+4.6	+0.0
23.9	+1.2	+23.0	+24	+20.6	+1.1	+22	+17	+0.5	+18	+13
43.4	+3.4	+4.3	+0.0	+4.8	+3.2	+5.4	+0.0	+5.5	+4.4	+0.0
98.4	+0.57	+0.38	−2.9	+1.2	+0.62	+0.40	−2.9	+0.35	+0.40	−5.0
210	−0.10	−0.32	−3.0	+0.50	−0.17	−0.37	−4.2	−0.25	−0.38	−3.6
341	−0.25	−0.32	−3.1	+0.23	−0.21	−0.38	−3.1	−0.29	−0.36	−3.6

H	$T=1692\ gr./mm.^2$				$T=4947\ gr./mm.^2$			
	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-17}$							
23.9	−0.10	−0.81	−3.2	−52	−3.6	−4.1	−18	−43
43.4	−0.10	−0.90	+8.3	0.0	−2.1	−3.5	−6.2	0.0
98.4	−0.16	−1.20	0.0	0.0	−1.8	−0.83	−0.0	+1.2
210	−0.26	−0.67	0.0	0.0	−0.0	−0.32	−0.0	+4.2
341	−0.05	−0.0	0.0	0.0	+0.41	−0.17	−0.0	+0.0

$T=3340\ gr./mm.^2$

H		$\tau=20.0'$	$\tau=20.0'$	$\tau=37.6'$
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
	Measured in $10.^{-4}$			
10.7	+0.23	——	+1.5	——
18.4	+0.10	−1.0	+7.9	+1.0
30.1	−0.04	−0.53	+3.2	−0.35
98.8	−0.03	−0.15	+0.3	−0.57
364	−0.03	−0.03	−0.04	−0.52

NICKEL.

H	$T=1540\ gr./mm.^2$				$T=2283\ gr./mm.^2$			$T=5240\ gr./mm.^2$		
	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-7}$									
10.8	−5.3	−3.7	−5.1	+0.12	−4.0	−3.8	−5.1	−0.40	−2.2	−3.9
31.1	−7.7	−3.7	−4.9	+0.11	−5.4	−4.2	−5.4	−1.8	−3.8	−3.2
62.5	−2.2	−2.2	−2.8	+0.06	−3.1	−2.6	−4.3	−3.1	−3.5	−3.1
135	−0.68	−0.71	−0.66	+0.02	−0.74	−0.78	−1.7	−2.0	−1.6	−1.8
206	−0.32	−0.32	−0.38	+0.01	−0.28	−0.38	−1.1	−0.80	−0.72	−1.1
365	−0.05	−0.05	−0.05	+0.00	−0.11	−0.05	−0.64	−0.17	−0.17	−0.3

H	$T=3021\ gr./mm.^2$				$T=4498\ gr./mm.^2$			
	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-16}$							
10.8	+4.7	+4.7	+1.0	+10.6	+2.3	+2.3	+2.0	+0.0
31.7	+10.0	+10.1	+0.79	—	+3.7	+4.2	+8.9	+15.3
62.5	+12.2	+14.0	−4.0	+3.1	+7.6	+13.7	−2.6	+7.3
135	−2.7	−3.0	−2.1	−5.1	−3.3	−5.2	−3.6	+4.1
206	−0.67	−0.7	−1.6	−2.3	−1.2	−1.3	−0.85	+3.4
365	−0.28	−0.3	−0.41	−1.6	−0.44	−0.55	−0.39	+5.1

T	$T_m=1143\ gr./mm.^2$				$T_m=3400\ gr./mm.^2$			
	1158 *gr.*	1022 *gr.*	1197 *gr.*	1197 *gr.*	3410 *gr.*	3096 *gr.*	3546 *gr.*	3546 *gr.*
		$\tau'=12.6'$				$\tau=12.6'$		
H	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_T$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial T}\right]_H$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_T$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial T}\right]_H$	$\frac{s}{\tau}\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-3}$							
2.4	+0.80	−0.72	+7.4	—	+0.20	−1.4	+1.9	—
4.0	+1.56	−3.2	+6.1	+0.87	+0.27	−3.4	+3.6	+0.30
10.6	+1.80	+2.3	+3.5	+2.0	+0.50	+2.5	+3.5	+3.5
23.8	+1.60	+0.84	+1.1	+0.52	+1.28	+3.5	+1.9	+1.8
49.6	+0.14	−0.37	−0.44	−0.57	+1.24	+0.4	+0.7	+0.96
97.0	−0.22	−0.60	−0.33	−0.78	−0.02	−0.48	−0.78	−0.39
168	−0.34	−0.40	−0.17	−0.70	−0.26	−0.50	−0.17	−0.26
358	−0.18	−0.17	−0.00	−0.52	−0.20	−0.20	−0.05	—

28.74% NICKEL STEEL.

H	$T=1427$ *gr./mm.*^{2}				$T=2798$ *gr./mm.*^{2}			$T=4856$ *gr./mm.*^{2}		
	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-8}$									
4.7	8.0	11	25	2.1	4.4	4.0	14	2.2	2.4	4.1
10.6	3.0	2.2	16	0.4	1.8	1.3	12	1.2	1.6	2.9
24.3	1.3	1.6	17	0.2	1.1	1.3	12	1.0	1.2	2.6
55.5	1.0	1.3	16	0.04	0.9	1.2	12	1.0	1.1	2.6
173	1.0	1.2	15	0.03	1.0	1.1	11	0.9	1.1	3.1
374	1.0	1.0	15	0.00	1.0	1.0	11	0.9	1.0	3.4

H	$T=1427$ *gr./mm.*^{2}				$T=4170$ *gr./mm.*^{2}			
	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-17}$							
4.7	−6.1	−13	−111	—	−4.1	−3.0	−1.5	—
10.6	−4.0	−8.9	−7.9	−26	−3.0	−2.6	—	−51
24.3	−2.0	−4.8	−2.6	—	−1.6	−1.6	−0.3	—
55.5	−0.4	−1.8	−1.8	—	−0.2	−0.3	−0.1	—
374	−0.0	+0.1	−0.0	−17	−0.2	+0.1	−0.0	−51

T	$T_m=950$ *gr./mm.*^{2}							
	928 *gr.*	948 *gr.*	948 *gr.*	948 *gr.*	959 *gr.*	959 *gr.*	959 *gr.*	959 *gr.*
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$			$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$			$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
H		$\tau=6.4'$	$\tau=12.7'$	$\tau=19.1'$	$\tau=6.4'$	$\tau=12.7'$	$\tau=19.1'$	$\tau=38.2'$
	Measured in $10.^{-4}$							
1.9	—	−2.5	−3.5	−3.0	−2.5	−2.4	−2.8	−2.7
6.7	−1.5	−3.4	−2.8	−2.4	−6.1	−4.6	−3.6	−3.2
14.5	−0.6	−1.0	−1.0	−0.9	−2.3	−2.0	−1.7	−2.4
30.5	−0.5	−0.35	−0.28	−0.38	−0.62	−0.8	−0.54	−2.1
61	−0.2	−0.23	−0.04	−0.07	−0.20	−0.20	−0.20	−2.1
300	+0.03	+0.02	−0.00	−0.00	−0.00	−0.0	−0.00	−1.7

T	$T_m=4200$ *gr./mm.*^{2}				
	4096 *gr.*	4240 *gr.*	4240 *gr.*	4211 *gr.*	4211 *gr.*
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$	
H		$\tau=8.9'$	$\tau=19.1'$	$\tau=8.9'$	$\tau=19.1'$
	Measured in $10.^{-4}$				
3.6	−1.2	−3.0	−2.9	−4.5	−3.9
6.8	−1.3	−2.2	−2.0	−3.2	−2.0
14.8	−0.95	−0.50	−0.65	−1.2	−0.96
35.6	−0.25	−0.20	−0.17	−0.44	−0.31
66.7	−0.15	−0.10	−0.03	−0.00	−0.00
300	−0.00	−0.00	−0.00	−0.00	−0.00

50.72% NICKEL STEEL.

	$T=890$ *gr./mm.*^{2}				$T=2538$ *gr./mm.*^{2}			$T^*=6652$ *gr./mm.*^{2}		
H	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-7}$									
1.9	11.9	20	9.0	11.2	5.3	4.8	4.7	−0.62	−0.16	−0.78
5.5	6.3	8.2	7.1	2.2	2.4	2.7	3.6	−0.83	−0.16	−0.42
10.7	3.7	5.7	4.0	1.0	1.3	1.5	2.4	−0.25	−0.08	−0.57
23.8	1.7	2.2	1.9	0.1	0.48	0.56	1.3	−0.09	−0.05	−0.51
51.4	0.34	0.61	0.70	0.07	0.13	0.15	0.73	−0.05	−0.02	−0.51
151	0.04	0.03	0.32	0.01	0.02	0.02	0.42	−0.01	−0.01	−0.51
360	0.03	0.00	0.25	0.01	0.01	0.00	0.28	−0.00	−0.00	−0.36

	$T=890$ *gr./mm.*^{2}				$T=5003$ *gr./mm.*^{2}			
H	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-7}$							
5.5	−1.5	−1.5	−7.5	−1.9	−0.19	−0.20	−0.51	−0.66
10.7	−1.9	−1.9	−3.4	−0.62	−0.13	−0.15	−0.10	−0.56
23.8	−0.58	−0.79	−1.4	−0.21	−0.10	−0.09	−0.10	−0.36
51.4	−0.08	−0.27	−0.41	−0.03	−0.04	−0.03	−0.00	−0.41
151.0	−0.02	−0.02	−0.03	−0.01	−0.01	−0.01	−0.00	−0.34
360	−0.01	−0.01	−0.00	+0.00	−0.00	−0.00	−0.00	−0.23

* Under a high tension, this specimen slightly contracts by magnetization, which is also to be expected from the effect of tension on magnetization.

T	$T_m=1140$ gr./mm.²					
	1114 gr.	1136 gr.	1136 gr.	1151 gr.	1151 gr.	1151 gr.
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
H		$\tau=6.2'$	$\tau=18.6'$	$\tau=6.2'$	$\tau'=52.3'$	$\tau=18.6'$
	Measured in $10.^{-3}$					
5.5	−0.0	−2.4	−2.4	+8.7	−1.5	−2.5
12.6	−1.8	−2.2	−1.9	+5.1	−1.3	−2.3
24.5	−1.2	−1.2	−1.2	+3.3	−0.86	−1.6
49.6	−0.60	−0.65	−0.64	+1.4	−0.41	−0.72
109	−0.17	−0.20	−0.20	+0.54	−0.09	−0.45
212	−0.07	−0.06	−0.06	+0.22	−0.07	−0.36
377	−0.02	−0.05	−0.05	+0.00	−0.00	−0.00

T	$T_m=3400$ gr./mm.²					
	3277 gr.	3445 gr.	3445 gr.	3409 gr.	3409 gr.	3409 gr.
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
H		$\tau=6.2'$	$\tau=18.6'$	$\tau=6.2'$	$\tau=52.3'$	$\tau=17.4'$
	Measured in $10.^{-3}$					
4.2	−1.5	−1.8	−1.8	+9.1	−1.2	−1.6
10.5	−1.8	−1.7	−1.4	+5.0	−1.1	−1.5
24.5	−0.95	−1.0	−0.90	+2.4	−0.69	−0.73
49.6	−0.42	−0.47	−0.47	+1.3	−0.35	−0.44
109.0	−0.17	−0.17	−0.17	+0.48	−0.13	−0.12
212	−0.07	−0.05	−0.05	+0.09	−0.04	−0.00
377	−0.02	−0.02	−0.02	+0.00	−0.00	−0.00

70.32% NICKEL STEEL.

	$T=806$ gr./mm.²				$T=1671$ gr./mm.²			$T=3277$ gr./mm.²		
H	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]_H$	$\left[\frac{\partial I}{\partial T}\right]_T$	$\frac{\partial e}{\partial H}$	$\left[\frac{\partial I}{\partial T}\right]$	$\left[\frac{\partial I}{\partial T}\right]_T$
	Measured in $10.^{-7}$									
1.5	19	27	20	11	12	13	14	3.8	5.1	4.9
2.7	11	14	12	3.7	6.0	8.7	9.4	2.3	4.6	3.7
5.1	6.2	7.1	9.2	1.9	4.1	5.1	4.3	1.6	2.4	2.6
10.7	1.7	3.6	4.1	0.56	2.2	3.1	2.3	0.89	1.0	1.0
24.5	0.4	0.6	3.1	0.13	0.33	0.6	0.5	0.17	0.3	0.0
49.8	0.09	0.20	2.6	0.03	0.07	0.2	0.0	0.08	0.1	0.0
129	0.01	0.03	2.0	0.00	0.01	0.03	0.0	0.01	0.00	0.0
300	0.01	0.00	2.0	0.00	0.01	0.00	0.0	0.00	0.00	0.0

H	$T=1671$ *gr./mm.*^{2}				$T=3277$ *gr./mm.*^{2}			
	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_H$	$-\frac{1}{E^2}\left[\frac{\partial E}{\partial H}\right]_T$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_H$	$\left[\frac{\partial^2 I}{\partial T^2}\right]_T$
	Measured in $10.^{-16}$							
5.1	−10.4	−28	−19	−26	−7.2	−6.7	−14.6	−6.1
10.7	−3.0	−3.9	−8.7	−14	−2.3	−1.6	−15.3	−6.3
21.5	−0.93	−1.2	−0.61	−7.6	−0.77	−0.70	−0.5	−1.6
49.8	−0.30	−0.39	−0.00	−0.6	−0.08	−0.10	−0.0	−0.0
129	−0.10	−0.03	−0.00	−0.0	−0.02	−0.02	−0.0	—
300	−0.03	−0.01	−0.00	−0.0	−0.00	−0.00	−0.0	—

T	$T_m=1112$ *gr./mm.*^{2}					
	1086 *gr.*	1109 *gr.*	1109 *gr.*	1123 *gr.*	1123 *gr.*	1123 *gr.*
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
H		$\tau=12.2'$	$\tau=18.2'$	$\tau=12.2'$	$\tau=18.2'$	$\tau=46.0'$
	Measured in $10.^{-3}$					
3.1	−1.4	−1.1	−1.1	+1.3	+0.65	+1.1
4.6	−2.0	−1.9	−1.4	+0.13	+0.19	+0.42
8.9	−2.6	−2.3	−1.8	−0.30	−0.87	−0.14
23.2	−1.4	−1.2	−1.0	−0.63	−0.51	−0.36
47.2	−0.50	−0.44	−0.41	−0.53	−0.31	−0.42
92.0	−0.16	−0.19	−0.16	−0.07	−0.09	−0.42
299	−0.04	−0.03	−0.04	−0.00	−0.00	−0.28
367	−0.02	−0.03	−0.02	−0.00	−0.00	−0.07

T	$T_m=3336$ *gr./mm.*^{2}					
	3198 *gr.*	3361 *gr.*	3361 *gr.*	3366 *gr.*	3366 *gr.*	3366 *gr.*
	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_H$	$\frac{1}{\tau}\left[\frac{\partial\tau}{\partial H}\right]_\tau$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$		$\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_\tau$
H		$\tau=11.8'$	$\tau=18.0'$	$\tau=11.8'$	$\tau=18.0'$	$\tau=34.5'$
	Measured in $10.^{-3}$					
3.1	−2.4	−1.8	−1.4	−0.35	−0.70	−1.1
5.0	−2.8	−2.0	−1.2	−0.83	−1.4	−1.0
10.7	−1.3	−1.5	−1.2	−0.76	−0.79	−0.95
23.4	−0.76	−0.84	−0.72	−0.52	−0.48	−0.67
47.2	−0.32	−0.33	−0.32	−0.17	−0.18	−0.56
105	−0.12	−0.10	−0.10	−0.04	−0.07	−0.53
191	−0.04	−0.03	−0.02	−0.00	−0.02	−0.53
349	−0.01	−0.01	−0.01	−0.00	−0.00	−0.53

Among all the specimens tested, nickel affords the best evidence in favour of the theories above tested. The discrepancies due to the difference of the orders of applying the stress and the field, are generally small, when compared with those in the case of other specimens. The agreement is especially good in the case of tension effect, if the term $\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$ be suppressed; the difference between the values $\frac{\partial e}{\partial H}$ and $\frac{\partial I}{\partial T}$ is of such orders of magnitude that they may be explained by the errors introduced in estimating these values from the corresponding diagrams. For the rest of the specimens, the agreement is tolerably good in many cases, except a few cases in which it completely fails. Generally speaking, the tension effect shows a better agreement between theory and experiment, if $\frac{\partial e}{\partial H}$ be compared with $\left[\frac{\partial I}{\partial T}\right]_H$; while $\left[\frac{\partial I}{\partial T}\right]_T$ deduced from the $(I, H)_T$ curve is often of a different order of magnitude, as in the case of 28.74% Ni. As for the comparison between $-\frac{1}{E^2}\frac{\partial E}{\partial H}$ and $\left[\frac{\partial^2 I}{\partial T^2}\right]_H$, the agreement is less remarkable, but the discrepancies may in many cases be due to the errors introduced in estimating the curvature of the curves for obtaining $\left[\frac{\partial^2 I}{\partial T^2}\right]_H$. $\left[\frac{\partial^2 I}{\partial T^2}\right]_T$ deduced from $(I, H)_T$ curve is often of a different order of magnitude, as in the case of Swedish iron. For torsion effect, things are much more complicated, except in the case of nickel and nickel steels of 28.74 and 70.32 per cent of nickel, in which the agreement is fairly good. For the last two specimens, $\frac{s}{\tau}\left[\frac{\partial I}{\partial L}\right]_H$ generally agrees with $\frac{1}{\tau}\left[\frac{\partial \tau}{\partial H}\right]_H$ or $\frac{1}{\tau}\left[\frac{\partial \tau}{\partial H}\right]_\tau$, while in the case of 50.72% Ni., the former is of a different sign from the latter for small twist. In all cases, the discrepancies become less in high fields.

As to the term $\frac{2\pi}{E}\frac{\partial I^2}{\partial H}$ obtained by Gans and Sano, it may be noticed that its introduction makes the agreement between theory and experiment rather worse. The origin of this term is, however, to be traced to the fundamental assumption that at the ends of the specimen wire, the lines of induction issue normally from its end faces — an assumption far from being realized in our actual experiments. Hence the importance of this term must be reduced, when applied to the case usually subject to experiments.

Thus far, the agreement between theory and experiment is in general to be considered as fairly good, if we consider the difficulties encountered in measuring the minute strains caused by magnetization, and also the considerable dependence of the magnetization upon the order of magnetizing and straining. Since the theories, which are based upon quite different considerations, all agree with one another in the first important term, it may be concluded that for the first approximation, they are all verified by the present experiment. It seems however impossible to decide experimentally the correctness of the terms of second importance for ferromagnetic substances, in which the hysteresis effect appears in no inconsiderable amount.

In conclusion, we wish to express our best thanks to Dr. S. SANO for useful suggestions in working out the theoretical part of the present investigation.

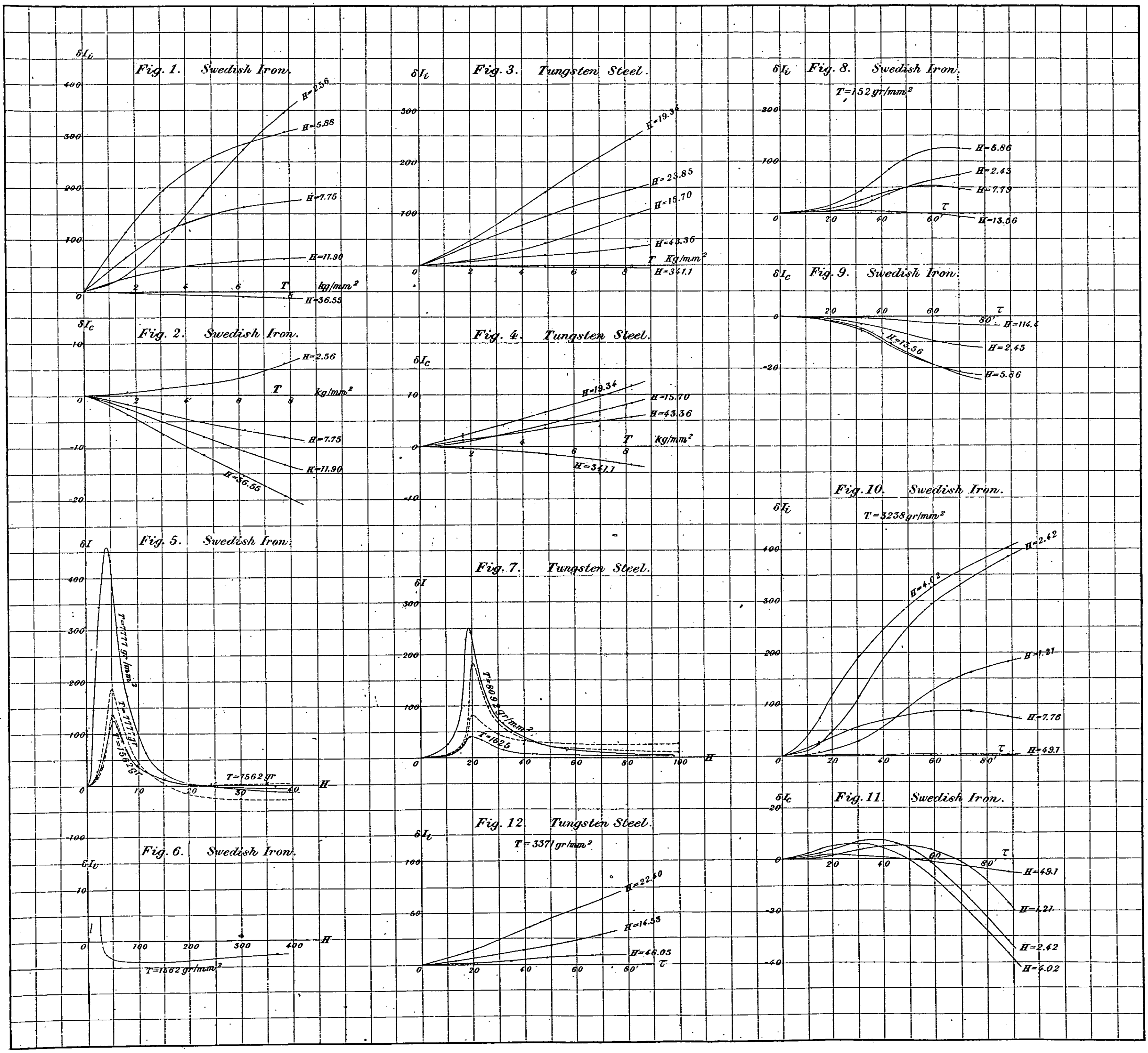


Fig. 1. Swedish Iron.
δIi
H=2.56
H=5.88
H=7.75
H=11.90
H=36.55
T kg/mm²
Fig. 2. Swedish Iron.
δIc
H=2.56
H=7.75
H=11.90
H=36.55
T kg/mm²
Fig. 3. Tungsten Steel.
δIi
H=19.34
H=23.85
H=15.70
H=43.36
H=341.1
T Kg/mm²
Fig. 4. Tungsten Steel.
δIc
H=19.34
H=15.70
H=43.36
H=341.1
T kg/mm²
Fig. 5. Swedish Iron.
δI
T=1777 gr/mm²
T=1562 gr
H
Fig. 6. Swedish Iron.
δIv
T=1562 gr/mm²
H
Fig. 7. Tungsten Steel.
δI
T=809 gr/mm²
T=1625
H
Fig. 8. Swedish Iron.
δIi
T=152 gr/mm²
H=5.86
H=2.43
H=7.79
H=13.56
τ
Fig. 9. Swedish Iron.
δIc
H=114.4
H=2.43
H=13.56
H=5.86
τ
Fig. 10. Swedish Iron.
δIi
T=3238 gr/mm²
H=2.42
H=4.02
H=1.21
H=7.76
H=49.1
τ
Fig. 11. Swedish Iron.
δIc
H=49.1
H=1.21
H=2.42
H=4.02
τ
Fig. 12. Tungsten Steel.
δIt
T=3371 gr/mm²
H=22.40
H=14.53
H=46.05
τ

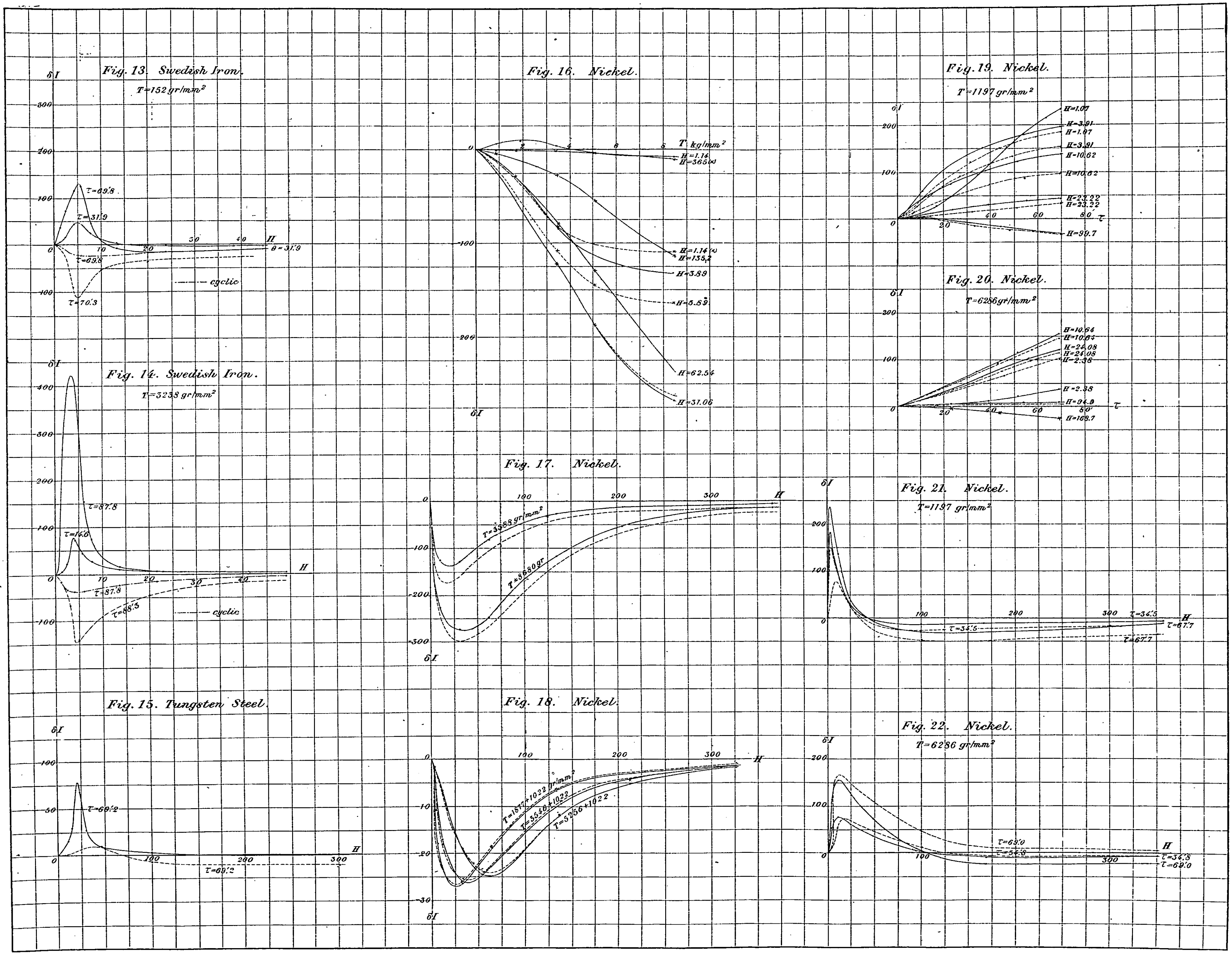


Fig. 13. Swedish Iron.
T=152 gr/mm²
Fig. 14. Swedish Iron.
T=3238 gr/mm²
Fig. 15. Tungsten Steel.
Fig. 16. Nickel.
T kg/mm²
Fig. 17. Nickel.
T=3588 gr/mm²
T=8680 gr
Fig. 18. Nickel.
T=1677+1022 gr/mm²
T=3546+1022
T=5256+1022
Fig. 19. Nickel.
T=1197 gr/mm²
Fig. 20. Nickel.
T=6286 gr/mm²
Fig. 21. Nickel.
T=1197 gr/mm²
Fig. 22. Nickel.
T=6286 gr/mm²
cyclic
δI
H
τ

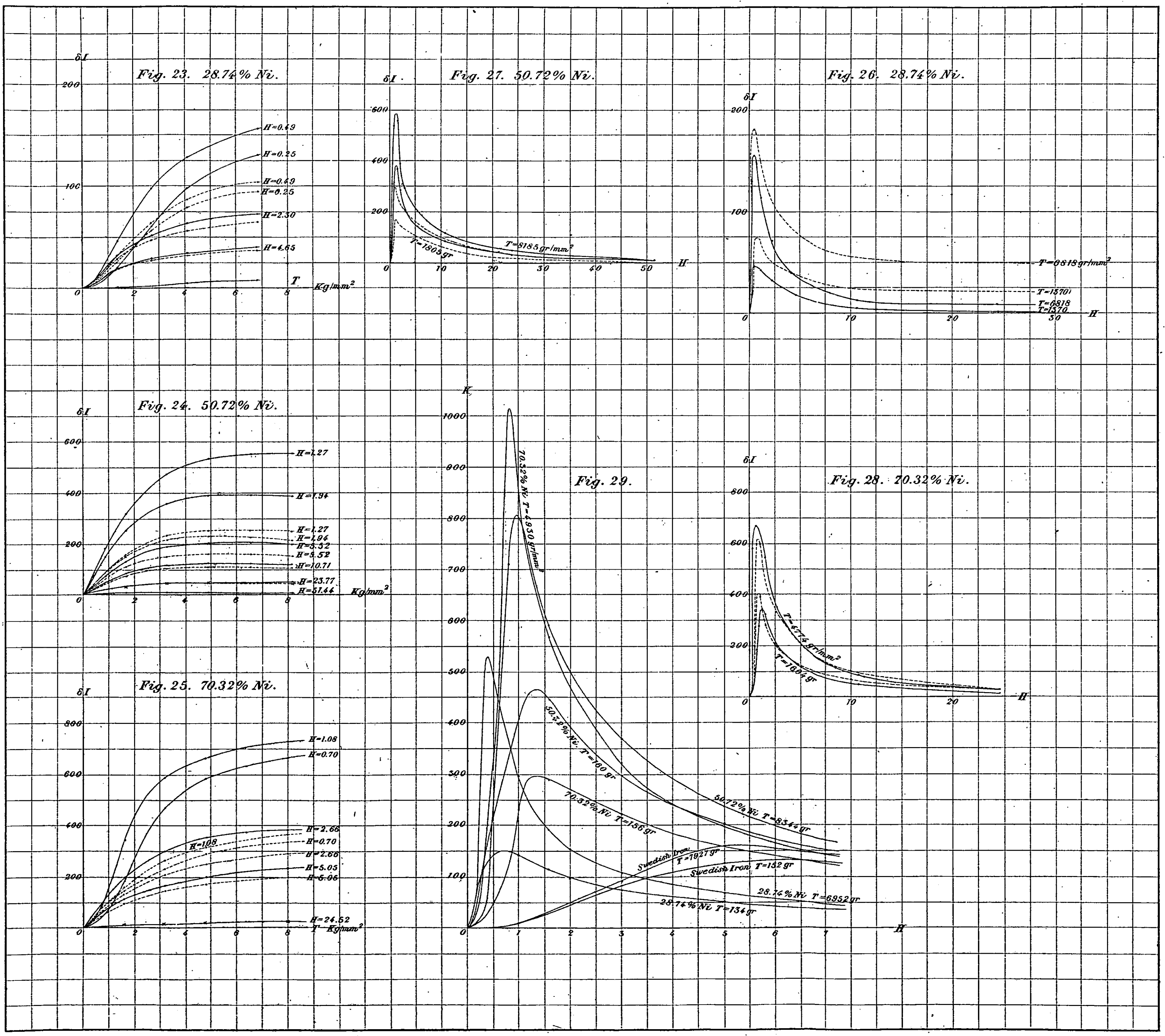


Fig. 23. 28.74% Ni.
δI
H=0.49
H=0.25
H=0.49
H=0.25
H=2.30
H=4.65
T Kg/mm²
Fig. 27. 50.72% Ni.
T=8185 gr/mm²
T=1805 gr
H
Fig. 26. 28.74% Ni.
T=8818 gr/mm²
T=15701
T=6818
T=1370
H
Fig. 24. 50.72% Ni.
H=1.27
H=1.94
H=1.27
H=1.94
H=3.52
H=3.52
H=10.71
H=23.77
H=51.44
Kg/mm²
Fig. 25. 70.32% Ni.
H=1.08
H=0.70
H=2.66
H=1.08
H=0.70
H=2.66
H=5.05
H=6.06
H=24.52
T Kg/mm²
Fig. 29.
K
70.32% Ni T=6930 gr/mm²
50.72% Ni T=160 gr
70.32% Ni T=156 gr
50.72% Ni T=8344 gr
Swedish Iron T=7927 gr
Swedish Iron T=132 gr
28.74% Ni T=6952 gr
28.74% Ni T=134 gr
H
Fig. 28. 70.32% Ni.
T=6776 gr/mm²
T=1864 gr
H

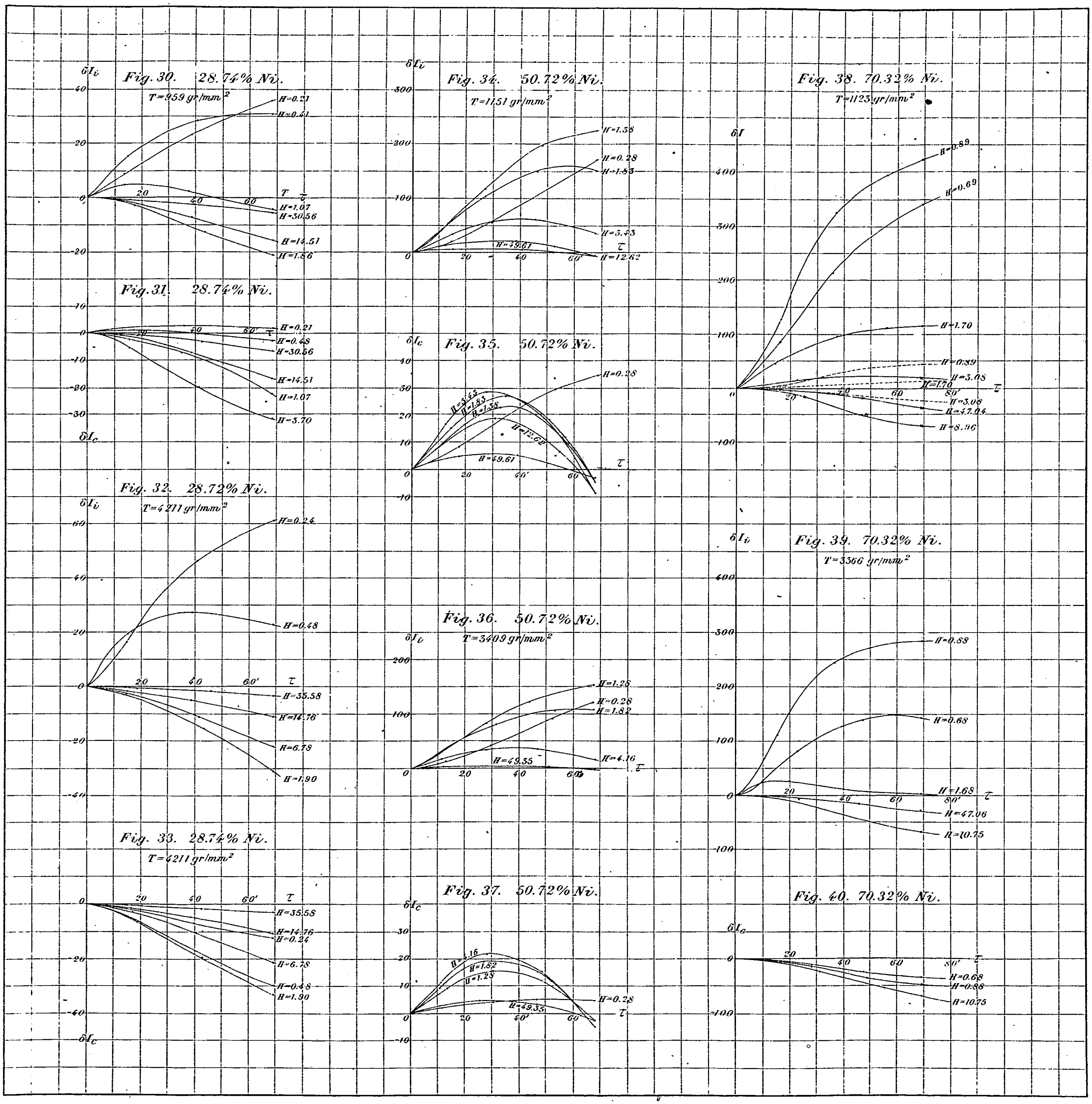


Fig. 30. 28.74% Ni.
T=959 gr/mm²
Fig. 31. 28.74% Ni.
Fig. 32. 28.72% Ni.
T=4211 gr/mm²
Fig. 33. 28.74% Ni.
T=4211 gr/mm²
Fig. 34. 50.72% Ni.
T=1151 gr/mm²
Fig. 35. 50.72% Ni.
Fig. 36. 50.72% Ni.
T=3409 gr/mm²
Fig. 37. 50.72% Ni.
Fig. 38. 70.32% Ni.
T=1123 gr/mm²
Fig. 39. 70.32% Ni.
T=3366 gr/mm²
Fig. 40. 70.32% Ni.

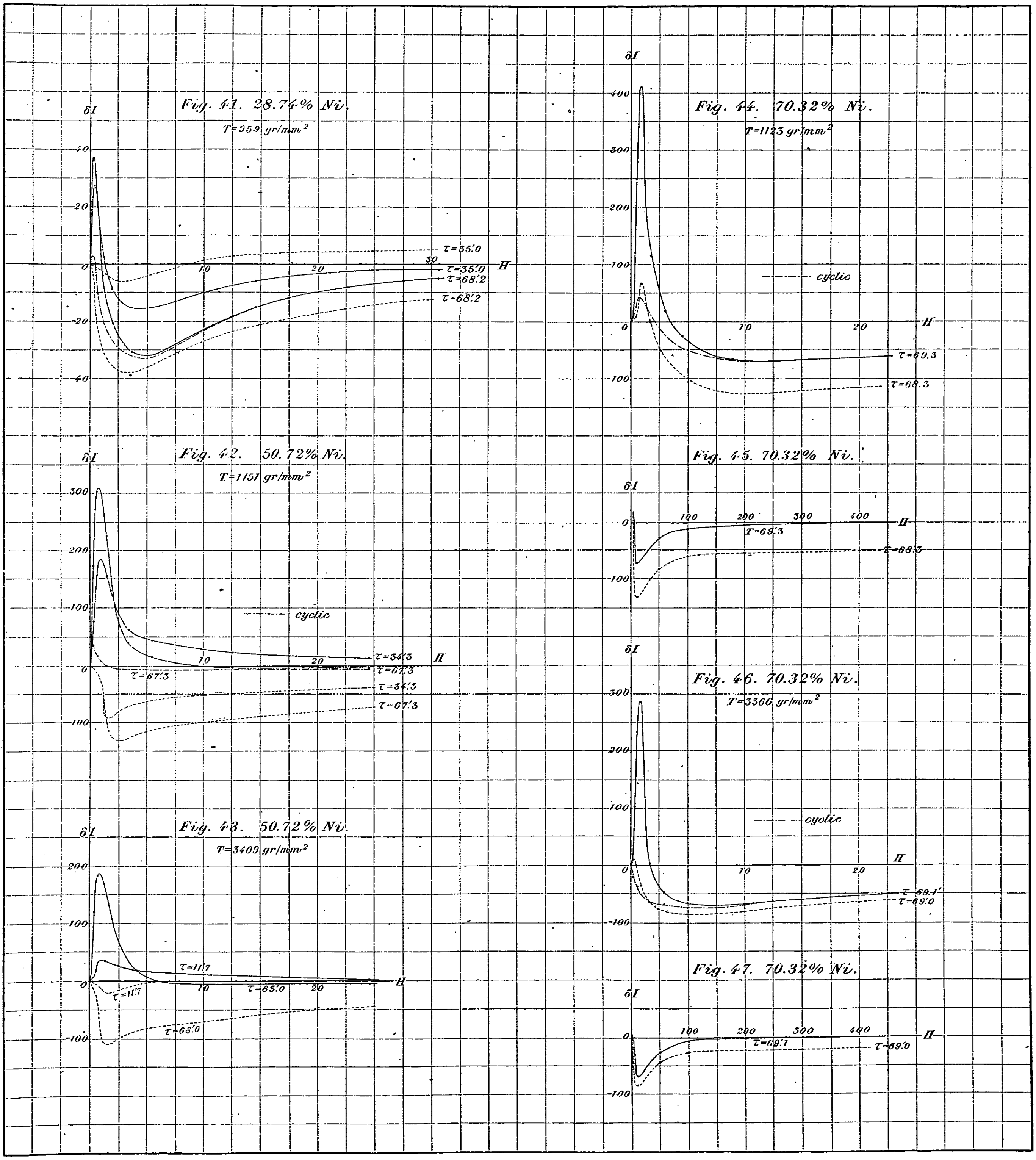


Fig. 41. 28.74% Ni.
T=959 gr/mm²
δI
τ=35.0
τ=35.0
τ=68.2
τ=68.2
H
Fig. 42. 50.72% Ni.
T=1151 gr/mm²
δI
cyclic
τ=34.3
τ=67.3
τ=67.3
τ=34.3
τ=67.3
H
Fig. 43. 50.72% Ni.
T=3409 gr/mm²
δI
τ=11.7
τ=11.7
τ=68.0
τ=68.0
H
Fig. 44. 70.32% Ni.
T=1123 gr/mm²
δI
cyclic
τ=69.3
τ=68.3
H
Fig. 45. 70.32% Ni.
δI
T=69.3
τ=68.3
H
Fig. 46. 70.32% Ni.
T=3366 gr/mm²
δI
cyclic
τ=69.1
τ=69.0
H
Fig. 47. 70.32% Ni.
δI
τ=69.1
τ=69.0
H