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On the Change of Elastic Constants of Ferromagnetic Substances by Magnetization.

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With plates I–X.

§ 1. INTRODUCTION.

The elastic constants of a substance in a magnetic field is here defined as the ratio of the stress applied to the strain caused thereby, the magnetic force constantly acting on the substance. The change of elasticity is then the difference of this ratio, when the magnetic field is on, and when it is off. In the case of ferromagnetic substances, for which the hysteresis is considerable, the change of elasticity so defined may, or may not coincide with that of the same ratio, when the stress is first applied and then the field. However, little attention seems to have hitherto been paid to this point. Thus, in many* of the experiments hitherto made regarding the change of elastic con-

*) Guillemin, C. R. **22**, 264 and 432, 1846. J. S. Stevens and H. G. Dorsey, Phy. Rev. (2) **9**, 116, 1899. G. Wiedemann, Electricität III, 796. C. Barus, Amer. Jour., **34**, 175, 1887; Phy. Rev. XIII, 257, 1901. Day, Electrician, **39**, 480, 1897. J. S. Stevens, Phy. Rev. (3) **10**, 111, 1900. K. Honda and S. Shimizu, Jour. Sc. Coll., Art. 12 and 13, Vol. XVI, 1902.

stants by magnetization, the right order of properly applying the stress and the field was inverted. In some experiments,* the effect of tension on the magnetic elongation was observed, and the change of the modulus of elasticity was deduced on the assumption that the effect was due to the change of elasticity by magnetization. The acoustical method† used by several experimenters is also an indirect one, and unsuitable for any accurate determinations. In H. Rensing's experiment‡, Kundt's tube was used; but the field in which the bar was placed, was far from being uniform.

Our present experiments were undertaken with the main object of measuring the change of elastic constants of the same specimen by the direct as well as the indirect method and comparing the results thus obtained: the specimen to be tested being placed in a nearly uniform field.

If ξ be the length or the angle of twist of the specimen, S the applied tension or couple respectively, and H the magnetic field, we have the relation

$$\frac{\partial}{\partial H}\left(\frac{\partial \xi}{\partial S}\right) = \frac{\partial}{\partial S}\left(\frac{\partial \xi}{\partial H}\right) \quad \ldots\ldots(1)$$

provided the change is independent of the order of applying the field and the stress. Thence we have

$$\left(\frac{\partial \xi}{\partial S}\right)_H - \left(\frac{\partial \xi}{\partial S}\right)_0 = \frac{\partial(\xi_H - \xi_0)}{\partial H} \quad \ldots\ldots(2)$$

If M be the modulus of elasticity or that of the rigidity respectively,

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*) Bock, Wied. Ann. **54**, 442, 1895; Phil. Mag. (5) 39, 548, 1895. K. Tangl, Ann. der Phys. **6**, 34, 1901. K. Honda and S. Shimizu, loc. cit.

†) Wartmann, Ann. de Chim. et de Phys. **24**, 360, 1848. Trèves, C. R. **67**, 321, 1868; Archives des Soc. Nat. N. S. **33**, 74, 1868. Maurain, C. R., **121**, 248, 1895.

‡) Rensing, Ann. der Phys. **14**, 363, 1904.

$$\frac{\partial\xi}{\partial S} = \frac{c}{M},$$

where c is a constant depending on the dimensions of the specimen. Hence we have finally

$$\frac{1}{M_H} - \frac{1}{M_O} = \frac{1}{c}\frac{\partial(\xi_H - \xi_O)}{\partial S}$$

$$= \frac{\frac{\partial(\xi_H - \xi_O)}{\partial S}}{M_O \frac{\partial \xi_O}{\partial S}},$$

or

$$\frac{M_H - M_O}{M_H} = \frac{\delta M}{M_H} = -\frac{\frac{\partial(\xi_H - \xi_O)}{\partial S}}{\frac{\partial \xi_O}{\partial S}} \quad \ldots\ldots\ldots\ldots(3)$$

Thus the change of elasticity may be calculated from the effect of tension on the magnetic elongation; and that of rigidity, from the change of twist caused by magnetization, provided relation (1) holds.

The validity of (3) depends upon relation (1), which is usually employed in the theory of magnetostriction. It was, therefore, thought, that it would be interesting to test experimentally how far the relation actually holds. Accordingly the present investigation was carried out in the following order:—

I. Experiments on the change of elasticity:
   - (*a*) Measurement of the change of elasticity by the elongation method.
   - (*b*) Measurement of the modulus of elasticity.
   - (*c*) Comparison of the results with those of the flexure method.

II. Experiments on the change of rigidity:
   - (*a*) Measurements of the change of rigidity by the oscillation method.

(*b*) Ditto by Barus's method.

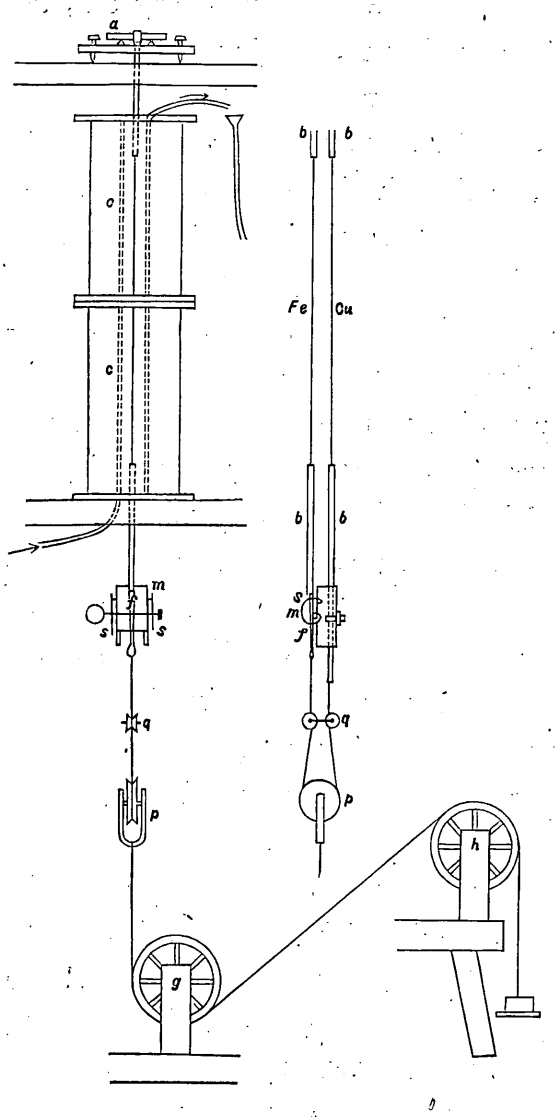
(*c*) Ditto by the ordinary method.

The specimens tested in the present experiment were Swedish iron, tungsten steel, nickel, and nickel steels of different percentages. They were generally tested in the form of wires. For the change of rigidity, the rod was also used in the case of nickel and iron.

§ 2. EXPERIMENTS ON THE CHANGE OF ELASTICITY BY MAGNETIZATION.

(*a*) Measurement of the change of elasticity by the elongation method.

Since the elongation due to the change of elasticity is a small fraction of the total elongation, it was desirable to devise some differential method. For this purpose, the elastic elongation at no field was compensated by that of a non-magnetic wire, and the differential elongation due to magnetic field was observed. As shown in the annexed figure, the magnetic wire to be tested and the compensating copper wire were hung side by side in the vertical magnetizing coil *cc*. To both ends of the wires were brazed brass rods *bb*, whose elongation could be neglected. The upper rods were hung separately on two horizontal brass beams *aa'* resting on pairs of knife-edges, the distance between which could be adjusted at will. To the lower rod attached to the copper wire, a light carriage *m* for a mirror system was clamped. Two fine spiral springs *ss* of german silver attached to the sides of the carriage, supported horizontally the axis of the light mirror with a suitable counterpoising. The lower rod for the magnetic wire carried a flexible band *f* made of a bundle of very



fine copper wires, which pressed the axis of the mirror lightly upon the plane vertical side of the carriage. To the lower ends of these rods, a flexible silk cord was attached; a pulley p hanging in this cord was pulled vertically downward by a weight. The friction-wheel q served to adjust the distance between the two vertical wires. Instead of hanging the weight directly, a system of pulleys gh was used in order to avoid any injurious shock accompanying the loading and unloading of the wires. When the elongations of the two wires due to tension were exactly equal, there was of course no rotation of the axis of the mirror. If, however, the compensation were disturbed in any way, the rotation could be observed in the usual manner by means of a vertical scale and a telescope.

To increase the sensitiveness of the arrangement, a wire about

74 cm. long was used; for want of a sufficiently long coil, two coils of the same dimensions were used, placing the one above another. The length of each coil was 39.90 cm., the internal diameter 5.80 cm., and $4\pi n = 393.5$. The air gap between the two coils was only 1.7 cm., so that the heterogeneity of the field in this space was inconsiderable. The wire to be tested then lay nearly in a uniform field.

The compensating copper wire was chosen of such dimensions as to produce an elongation nearly equal to that of the specimen. The final compensation was always made by properly adjusting the distance between the two knife-edges supporting the horizontal beams, from which the wires were suspended. A certain weight was added to the pan, or removed from it, and the distances of the knife-edges were adjusted, until either the addition or removal produced no rotation of the mirror. Since the elastic elongation was not exactly a linear function of the tension applied, a compensated system for a given initial and additional weight caused a rotation of the mirror for a different initial weight and the same additional weight. Hence the compensation was made anew for every initial weight, 1 kilogram always being used for the applied weight. It was, however, a matter of no great ease to obtain an exact compensation. Hence the slight deflection in no field was always recorded, and taken into account in the calculation of the deflection due to the change of elasticity by magnetization. To prevent shocks in putting on the additional weight, the face of the initial weight was covered by a mat of cotton wool. In this way, the reading of the deflection was greatly facilitated.

In the above arrangement, the rotation of the mirror caused by a change of temperature is very small, as it is affected only

by the relative expansion of the specimen and the compensating wire. But to avoid the effect of heating as much as possible, the coil was waterjacketed.

To increase the sensitiveness of the arrangement, a long scale distance of 7.23 m. was used. The scale was graduated on a ground glass, and illuminated from behind by a mantled gas flame. The telescope was placed in front of the apparatus and near the rotating mirror; consequently the line joining the centers of the scale and the mirror deviated slightly from the line of sight of the telescope. In this way, the scale at such a long distance could be read to a tenth of a millimeter with the observing telescope.

If the specimen undergoes a differential elongation δl, a rotation $\delta\varphi$ of the axis is produced, such that $\delta\varphi = \delta l/2r$, where r is the radius of the axis. If the scale distance be d and the reading on the scale corresponding to the elongation δl be n, we have

$$\delta\varphi = \frac{n}{2d}\,; \quad \text{hence} \quad \delta l = \frac{rn}{d}$$

In our case, $2r = 1.61$ mm., so that a deflection amounting to one division of the scale in the field of the telescope corresponds to an elongation of 4.47×10^{-8} per unit length of the specimen. The sensitiveness of the apparatus was therefore about 10 times greater than that of the apparatus used by many previous investigators.

The magnetizing current supplied by a set of secondary batteries was measured by a Siemens-Halske ammeter, which was occasionally compared with a Kelvin ampere-balance.

The present arrangement also enabled us to measure the magnetic change of length of the specimen under constant tension

by applying a magnetic field to the specimen and observing the deflection of the mirror corresponding to different fields.

Our procedure in the experiment was usually as follows:— The wire to be tested was first annealed for about 4 hours in a charcoal fire, and then gradually cooled. To avoid surface oxidation during the process of annealing, the wire was buried in a fine powder of charcoal, and sealed in an iron tube, which was placed in a furnace. In this way, a small quantity of carbon might enter into the wire, but the oxidation was quite inappreciable. Variation in the diameter of the wire caused by the oxidation was avoided in the above manner.

To either end of the wire thus annealed, was brazed the brass rod before mentioned, and the whole was suspended in position as shown in the foregoing figure. Then the mirror and pulley-systems were placed in position. For avoiding a continuous slight vibration of the mirror, as well as for the rapid damping of the vibration due to loading or unloading the wire, it was necessary that a glass rod fixed to a stand should come lightly in contact with the bifilar cords near the friction-wheel *q*. The addition or removal of a weight from the pan was occasionally observed to cause a lateral displacement of the image in the mirror; this was, however, easily effaced by adjusting the clamp of the carriage to the rod.

The magnetic change of length under constant tension was first measured. Since the hysteresis effect of tension on the length change was found to be considerable, the tension was varied cyclically between zero and its maximum value before beginning the experiment, in order to remove any initial effects.

According to the results of our experiments, the ascending branch in the curve of elongation due to tension, lies a little

above the descending one. In the following tables, the length changes under constant tension are results for the stage of increasing weights, taken after several cycles of loading and unloading. The usual processes of observation were followed. The wire was first loaded with the smallest weight, which was generally 1 kilogram; the demagnetization by reversals was then carefully effected. A series of gradually increasing currents was passed through the magnetizing coil, and the corresponding deflections of the scale were noted. The wire was then completely demagnetized, and the zero position of the scale tested. Another weight of 1 kilogram was added and demagnetized, and the same processes were repeated. In this way, a set of observations corresponding to the different weights was taken.

The change of elasticity was next measured. We applied an initial load to the pan, demagnetized the wire by reversals, added and removed a weight of 1 kilogram, and adjusted the compensation, until the deflection due to the additional weight was as small as possible. The demagnetization was then carefully effected. The small deflection due to the additional weight for no field was recorded. Then we applied a magnetizing field and observed the deflection due to the additional weight. After the demagnetization, the deflection due to the same load for no field was again tested. The difference between the two deflections for no field and for the magnetic field gives the change of elasticity due to magnetization, provided we know the total elongation caused by the additional weight of 1 kilogram. To avoid any accidental error in observing these deflections, the deflections for several cycles of loading and unloading were, in each case, recorded, and their mean was taken. The initial weight was next increased,

and the same procedure repeated. The compensation was always readjusted for different initial weight.

Our object was partly to test the validity of equation (3), and since the change of elasticity was considerably affected by tension, the initial and the additional tensions were so chosen as to exactly correspond to those in the experiments of the magnetic change of length under constant tension.

In the calculation of the percentage change of elasticity due to magnetization, the values of elasticity were taken from the results of our experiments specially undertaken for this purpose.

The specimens tested and their dimensions were as follows :—

Substances.	Length.	Diameter.
Pure nickel	73.25 cm.	0.928 mm.
Commercial nickel	71.90	0.930
Swedish iron	71.87	0.904
Tungsten steel	73.76	0.886
28.74 % nickel steel	73.80	0.964
50.72 % „ „	73.60	0.880
70.32 % „ „	73.25	0.892

The experimental results are given in the following pages.

NICKEL

(i) Pure Nickel. Figs. 1 and 2.

The change of length $\frac{\delta l}{l}$ by magnetization for different tensions T and external magnetic fields H are given in the following table and Fig. 1.

TABLE I.

$t = 9.°8.$

$T = 1540$ gr./mm.²		$T = 2283$ gr./mm.²		$T = 3021$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
2.1	+ 0.02	4.1	− 0.18	4.3	− 0.11
5.8	− 0.59	7.7	− 0.97	8.5	− 0.73
9.4	− 2.09	11.3	− 2.26	12.6	− 1.67
14.0	− 4.37	15.6	− 3.69	16.6	− 2.81
24.2	− 11.94	24.4	− 7.40	24.4	− 5.27
33.4	− 20.28	33.0	− 12.01	32.8	− 8.25
49.1	− 28.25	49.8	− 23.96	48.4	− 16.11
74.2	− 33.1	74.6	− 32.2	72.7	− 28.96
94.3	− 35.7	93.7	− 35.4	93.4	− 33.9
143.2	− 39.7	142.3	− 40.1	142.2	− 39.7
203	− 42.5	201	− 43.4	201	− 43.5
276	− 44.1	276	− 45.5	275	− 46.0
322	− 44.6	322	− 46.0	322	− 46.8
389	− 44.9	388	− 46.5	388	− 47.3

$T = 3762$ gr./mm.²		$T = 4498$ gr./mm.²		$T = 5240$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
4.1	− 0.07	4.3	− 0.04	3.8	− 0.00
7.8	− 0.42	8.8	− 0.38	8.5	− 0.18
12.5	− 1.30	15.8	− 1.45	13.6	− 0.64
16.8	− 2.28	24.4	− 3.22	18.0	− 1.27
24.4	− 4.27	33.4	− 5.18	24.2	− 2.37
33.6	− 6.76	40.8	− 6.98	34.5	− 4.33
49.6	− 12.23	49.8	− 9.39	49.6	− 7.65
73.6	− 24.23	73.8	− 17.77	73.8	− 13.84
93.6	− 31.7	93.1	− 26.8	93.1	− 21.40
142.2	− 39.3	142.3	− 37.5	142.3	− 35.2
201	− 43.8	201	− 42.9	203	− 41.6
275	− 46.8	275	− 46.5	276	− 45.7
321	− 47.8	321	− 47.7	323	− 47.2
388	− 48.5	387	− 48.7	390	− 48.4

As may be seen from Fig. 1, nickel contracts by magnetization in a way similar to that given by many earlier experimenters; in weak fields, the contraction rapidly increases with the increasing field, but gradually tends to an asymptotic value, as the field attains a higher value. The increased tension decreases the amount of contraction for low fields, while for higher fields, it increases the contraction. Thus there exists a certain field in which the effect of the additional tension is null. This neutral point for two successive tensions, shifts toward the higher field, as the tension increases. In their general course, the curves are quite similar to those already obtained by Mr. S. Shimizu and one of us.*

The change of elasticity of nickel is given in the table below and in Fig. 2.

TABLE II.

$\Delta T = \pm 743$ *gr./mm.*^{2} $t = 10.°0$ *C.*

$T = 1540$ *gr./mm.*^{2}		$T = 3021$ *gr./mm.*^{2}		$T = 4498$ *gr./mm.*^{2}	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
4.5	− 0.05	9.3	− 0.54	11.6	− 0.40
10.4	− 1.59	19.5	− 1.67	25.0	− 1.24
25.0	− 8.41	34.1	− 3.98	51.0	− 3.31
31.3	− 12.15	49.9	− 8.41	70.3	− 6.11
37.7	− 14.40	64.3	− 12.22	99.6	− 9.90
50.3	− 7.10	71.1	− 11.06	106.3	− 8.18
71.5	− 3.10	90.9	− 6.19	129.4	− 5.43
118.2	− 0.42	131.8	− 2.43	191.9	− 3.21
196.7	+ 0.98	210	− 1.01	—	—
287	+ 2.20	288	− 0.61	277	− 1.72
406	+ 2.19	397	+ 0.27	390	− 0.83

*) K. Honda and S. Shimizu, Jour. Sc. Coll., Vol. XVI., Art. 9, 1902.

In the above table, ΔT denotes the weight applied or removed for studying the elongation of the specimen.

The coefficient of elasticity first decreases as the field increases, attains a minimum value, whence it begins to increase, at first rather abruptly and then gradually. The effect of tension is to drive the field corresponding to the minimum elasticity toward the higher field, and also to depress the absolute amount of decrease. For a weak tension, the elasticity increases by magnetization in fields higher than a certain field at which the change is zero. This field is displaced toward the higher field by the increasing tension.

The change of elasticity calculated from the effect of tensions upon the magnetic change of length is as follows:—

TABLE III.

$\Delta T = 743$ *gr./mm.*^{2}

$T = 1540$ *gr./mm.*^{2}		$T = 3021$ *gr./mm.*^{2}		$T = 4498$ *gr./mm.*^{2}	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
20	− 7.4	—	—	—	—
35	−19.1	30	− 3.1	40	− 2.4
40	−17.4	60	−14.3	—	—
60	− 6.5	70	−14.3	80	−12.4
100	− 0.7	90	− 6.1	100	−13.1
200	+ 2.4	150	0.0	130	− 7.6
300	+ 3.6	250	+ 1.2	200	− 3.8
380	+ 3.5	350	+ 2.5	300	− 1.6

Curves are also given in Fig. 2 in dotted lines. Comparing the above results with those directly observed, we notice at once, that the general course of the change of elasticity is quite similar

for both results, the fields in which the change is maximum, coinciding in a remarkable way. The amount of the change is generally greater in the present case than in the previous one.

(ii) Commercial Nickel. Figs. 3 and 4.

The change of length by magnetization is generally similar to that in pure nickel. But the effect of tension is much smaller and the field in which the effect of an additional tension is null does not differ so much for different tensions as in the former specimen. The results are given in Table IV and in Fig. 3.

TABLE IV.

$t = 9.°5$ C.

$T = 1325$ gr./mm.^{2}		$T = 1962$ gr./mm.^{2}		$T = 2600$ gr./mm.^{2}		$T = 3239$ gr./mm.^{2}	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^2$
2.3	− 0.09	2.5	− 0.09	2.6	− 0.09	2.3	− 0.04
6.9	− 0.92	7.8	− 0.78	8.8	− 0.90	8.3	− 0.54
13.7	− 3.04	14.5	− 2.50	16.2	− 2.54	15.4	− 1.90
22.4	− 6.20	22.5	− 5.32	23.1	− 4.63	23.1	− 4.02
38.2	−11.62	38.1	−10.72	39.0	− 9.84	38.9	− 9.02
49.1	−14.62	49.3	−14.02	50.4	−13.22	50.2	−12.32
71.9	−19.16	71.5	−18.76	71.5	−18.44	71.5	−17.60
120.4	−25.20	120.2	−25.44	120.4	−25.40	120.5	−25.46
191.6	−29.46	191.5	−30.3	192.0	−31.0	192.0	−31.4
261	−31.7	258	−32.7	262	−33.3	258	−34.2
328	−32.9	328	−34.0	331	−34.8	330	−35.9
390	−33.7	390	−34.8	392	−35.7	391	−36.8

The change of elasticity is as follows. Curves are also given in Fig. 4 with full lines.

TABLE V.

$\Delta T = \pm 638\ gr./mm.^2$ $t = 8.°4\ C.$

$T = 1326\ gr./mm.^2$		$T = 2600\ gr./mm.^2$		$T = 3874\ gr./mm.^2$	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
7.6	−0.65	6.7	−0.41	6.8	0.00
14.0	−1.24	10.9	−0.70	11.1	−0.53
22.9	−1.60	19.8	−1.34	24.6	−1.33
37.7	−1.90	32.6	−2.36	39.1	−2.26
52.1	−1.96	52.3	−2.87	51.6	−2.92
75.3	−1.48	77.8	−2.29	76.7	−3.26
123.2	−0.40	108.0	−1.72	105.7	−2.66
193.5	+0.45	194.0	−0.14	165.3	−1.33
271	+1.07	262	+0.70	260	−0.09
—	—	330	+0.99	331	+0.49
383	+1.42	397	+1.14	398	+0.78

Here the general aspect of the change is similar to that of the pure nickel. The amount of the change is, however, far less than in the previous specimen. The effect of tension is also to drive the maximum of the change toward higher fields, but not in such a decided way as in the case of pure nickel. Moreover, the increased tension increases the amount of the maximum change, up to the highest tension employed. These remarkable differences between the pure and the commercial nickel are probably due to the imperfect annealing rather than to the impurity

of the latter specimen, as it was slightly annealed with a gas flame.

The change of elasticity calculated from the tension effect on the magnetic change of length somewhat resembles in its aspect that for pure nickel, but the amount of the change is much smaller. The calculated values are given in Table VI and in Fig. 4 with dotted lines.

TABLE VI.

$\Delta T = \pm 638\ gr./mm.^2$

	$T = 1326\ gr./mm.^2$	$T = 2600\ gr./mm.^2$
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
10	−2.23	−0.96
20	−2.97	−1.75
40	−3.26	−2.42
70	−1.19	−2.23
100	+0 44	0
200	+1.78	+1.75
300	+2.97	—

Thus, in pure as well as in commercial nickel, we find a sensible deviation from relation (3).

SWEDISH IRON AND TUNGSTEN STEEL.

(i) Swedish iron. Figs. 5 and 6.

The change of length due to magnetization is given in Table VII and in Fig. 5.

TABLE VII.

$t = 9°.2$ C.

$T = 1627$ gr./mm.²		$T = 2410$ gr./mm.²		$T = 3190$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
3.8	−0.02	3.8	−0.09	3.5	−0.10
8.1	+0.94	8.3	+0.64	8.6	+0.42
12.4	+1.44	12.8	+1.00	14.1	+0.66
17.5	+1.68	16.8	+1.10	17.7	+0.72
23.3	+1.78	22.3	+1.18	23.4	+0.74
35.3	+1.80	33.8	+1.18	35.4	+0.68
54.2	+1.60	52.5	+0.98	53.8	+0.44
78.2	+1.26	74.8	+0.62	78.0	+0.04
117.2	+0.56	113.3	−0.08	113.6	−0.60
185.0	−0.82	186.3	−1.54	187.8	−2.12
250	−2.10	252	−2.80	253	−3.40
318	−3.30	320	−3.94	322	−4.56
384	−4.34	385	−4.92	388	−5.56

$T = 3974$ gr./mm.²		$T = 4754$ gr./mm.²		$T = 5535$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
3.3	−0.08	3.5	−0.10	3.2	−0.06
7.8	+0.20	8.4	+0.04	8.2	−0.09
13.4	+0.38	14.0	+0.12	12.4	−0.05
18.0	+0.42	17.4	+0.13	17.1	−0.05
23.3	+0.42	22.5	+0.11	22.3	−0.08
34.9	+0.34	33.9	+0.06	33.8	−0.21
51.2	+0.12	51.6	−0.18	52.0	−0.52
74.8	−0.29	74.8	−0.64	74.8	−0.94
112.9	−1.08	113.3	−1.44	112.9	−1.74
185.0	−2.54	186.0	−2.94	185.8	−3.24
249	−3.76	251	−4.20	250	−4.48
317	−4.90	317	−5.32	317	−5.62
382	−5.84	382	−6.30	380	−6.58

For small tensions, the change is an elongation in low fields, attains a maximum in a certain field, passes zero, and afterward is changed to a contraction. The maximum elongation decreases with the increasing tension, driving the zero toward the weak field; and beyond a certain tension, the length of the wire steadily contracts with the increase of the field. It seems also that the field corresponding to the maximum elongation slowly moves toward the weaker fields as the tension is increased. These results are in complete agreement with those obtained by former experimenters.

The change of elasticity for this sample was unexpectedly small, its amount being not very much greater than the limit of experimental error. The values obtained are given in Table VIII and in Fig. 6 with a full line.

TABLE VIII.

$\Delta T = 784\ gr./mm.^2$ $t = 11.°5\ C.$

$T = 1627\ gr./mm.^2$	
H	$\frac{\delta E}{E_H} \times 10^2$
37.1	+0.02
120.4	+0.20
404	+0.28

Thus, $\frac{\delta E}{E}$ is as a whole very small, and increases with the field tending to an asymptotic value.

On the other hand, the calculated values of $\frac{\delta E}{E}$ from the magnetic elongation are decidedly greater than those obtained by our direct method, as may be seen from Table IX, and Fig 6 in dotted lines. The difference is remarkable as well as interesting.

TABLE IX.

$\Delta T = \pm 784\ gr./mm.^2$

	$T = 1627\ gr./mm^2$	$T = 3190\ gr./mm.^2$	$T = 4754\ gr./mm.^2$
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
10	0.9	0.6	0.4
20	1.4	0.8	0.5
40	1.6	0.9	0.8
70	1.7	1.0	0.9
100	1.7	1.1	0.8
200	1.7	1.1	0.8
300	1.4	1.1	0.8
350	1.4	1.0	0.8

(ii) Tungsten Steel. Figs. 7 and 8.

The elongation by magnetization increases at first rapidly, then slowly, and afterward gradually decreases. Tension decreases elongation regularly. The results are tabulated as follows :—

TABLE X.

$t = 12°.5\ C.$

$T = 1693\ gr./mm.^2$		$T = 2509\ gr./mm.^2$		$T = 3322\ gr./mm.^2$	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
10.0	0.03	9.0	0.03	9.5	0.04
13.8	0.09	13.1	0.08	13.5	0.09
19.4	0.60	19.5	0.64	19.5	0.57
24.9	1.39	25.2	1.46	25.2	1.32
42.2	2.72	42.8	2.62	42.7	2.39
64.5	3.26	65.5	3.16	64.9	2.88
96.2	3.56	97.4	3.48	97.0	3.15
167.2	3.72	168.8	3.60	168.0	3.24
246	3.63	246	3.48	245	3.13
330	3.48	333	3.26	331	2.92
399	3.28	402	3.11	400	2.78

$T=4136\ gr./mm.^2$		$T=5762\ gr./mm.^2$	
H	$\frac{\delta l}{l}\times 10^6$	H	$\frac{\delta l}{l}\times 10^6$
8.2	0.04	8.9	0.03
13.6	0.10	13.1	0.07
19.1	0.52	19.3	0.44
24.8	1.15	25.0	1.05
42.2	2.17	42.3	1.92
64.7	2.64	64.0	2.26
97.4	2.87	94.9	2.46
168.6	2.96	166.0	2.51
246	2.82	244	2.34
332	2.61	334	2.10
401	2.47	397	1.93

As in the case of Swedish iron, the change of elasticity is very small, gradually increasing with the field. Its amount does not much exceed the limit of experimental error, as given in table XI :—

TABLE XI.

$\Delta T=\pm 816\ gr./mm.^2$ $\quad t=10°.8\ C.$

$T=1692\ gr./mm.^2$		$T=4947\ gr./mm.^2$	
H	$\frac{\delta E}{E_H}\times 10^2$	H	$\frac{\delta E}{E_H}\times 10^2$
245	0.08	210	0.50
380	0.10	400	0.39

The calculated values of $\frac{\delta E}{E}$ are decidedly greater than those above given, as seen in Table XII and the dotted curves in Fig. 8.

TABLE XII.

$\Delta T = \pm 816$ *gr./mm.*^{2}

	$T = 1692$ *gr./mm.*^{2}	$T = 4947$ *gr./mm.*^{2}
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
50	—	0.4
100	0.2	0.5
200	0.3	0.6
300	0.5	0.6
400	0.5	0.7

Thus, in the case of iron and steel, relation (3) does not hold even approximately, when the tension is small. But as the tension is increased, the discrepancy becomes less.

NICKEL STEELS.

(i) **28.74 %** nickel steel. Figs. 9 and 10.

The specimen first elongates rapidly and then almost proportionally with the field. The effect of tension is generally to depress the change of length. The general feature of the change may be seen in the following table and in Fig. 9.

TABLE XIII.

$t = 10°.3$.

$T = 1427$ gr./mm.^{2}		$T = 2115$ gr./mm.^{2}		$T = 2798$ gr./mm.^{2}	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.2	−0.01	0.3	−0.04	0.2	−0.01
0.6	−0.05	0.6	−0.09	0.6	−0.06
1.2	+0.13	1.1	+0.07	1.3	+0.13
2.0	+0.27	2.1	+0.26	2.2	+0.26
4.1	+0.52	4.2	+0.51	5.4	+0.49
8.9	+0.80	12.8	+0.84	15.3	+0.71
26.1	+1.11	25.4	+1.04	26.0	+0.85
43.9	+1.29	43.3	+1.22	43.4	+1.04
68.3	+1.52	68.1	+1.47	68.5	+1.30
113.6	+1.98	113.6	+1.93	114.1	+1.74
207	+2.87	206	+2.78	208	+2.66
303	+3.77	302	+3.71	305	+3.57
404	+4.67	404	+4.66	405	+4.50

$T = 3487$ gr./mm.^{2}		$T = 4170$ gr./mm.^{2}		$T = 4856$ gr./mm.^{2}	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.4	−0.02	0.2	0.00	0.2	0.00
0.8	0.00	0.6	−0.04	0.5	−0.02
1.5	+0.11	1.3	+0.05	1.3	+0.05
3.5	+0.26	4.2	+0.20	3.0	+0.13
8.5	+0.42	9.3	+0.32	8.0	+0.23
14.7	+0.50	15.2	+0.37	13.9	+0.29
25.6	+0.60	26.2	+0.48	25.8	+0.39
43.0	+0.77	44.2	+0.65	44.1	+0.56
68.1	+1.01	68.3	+0.88	68.7	+0.80
110.5	+1.40	113.9	+1.31	114.1	+1.24
204	+2.28	207	+2.17	207	+2.10
299	+3.15	304	+3.03	305	+3.01
397	+4.03	403	+3.93	405	+3.92

The observed values of $\frac{\delta E}{E}$ are generally small, but are rather greater than those for iron. The elasticity increases by magnetization in a manner similar to that of iron. An unconspicuous maximum is, however, observed in a high fields. Increased tension decreases the amount of the change; this makes the feature of the maximum conspicuous. The following table and the full lines in Fig. 10 show the results of our experiment.

TABLE XIV.

$\Delta T = \pm 688$ *gr./mm.*^{2} $t = 10.°2$.

$T = 1427$ *gr./mm.*^{2}		$T = 2798$ *gr./mm.*^{2}		$T = 4170$ *gr./mm.*^{2}	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
4.1	0.07	5.8	0.07	6.2	0.03
14.4	0.34	14.7	0.25	26.4	0.12
48.3	0.41	44.5	0.31	49.0	0.14
113.7	0.53	113 7	0.41	112.8	0.15
237	0.51	239	0.44	238	0.04
391	0.48	397	0.46	418	0.00

The values of $\frac{\delta E}{E}$ as calculated from the tension effect on the change of length differ from the above values rather considerably for slight tension; but they approach each other for high tension, as may be seen in Table XV and the dotted curves in Fig. 10. Here the change of elongation due to tension being very small, the calculated values of $\frac{\delta E}{E}$ are not very certain.

TABLE XV.

$\Delta T = \pm 688\ gr./mm.^2$

	$T = 1427\ gr./mm.^2$	$T = 2798\ gr./mm.^2$	$T = 4170\ gr./mm.^2$
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
20	0.17	0.56	—
50	—	0.68	0.17
100	0.17	0.73	0.17
200	0.19	0.82	0.15
300	0.22	0.90	0.08
400	0.24	0.97	0.06

(ii) 50.72 % Nickel Steel. Figs. 11, 12, 13 and 14.

The change of length due to magnetization differs widely for different tensions. For slighter tensions, the wire generally elongates by magnetization, except at very weak fields, where it slightly contracts, a fact not observed in previous experiments.* The amount of the elongation increases, rapidly at first, and tends soon to an asymptotic value, as the field is increased. The increased tension depresses the elongation in a remarkable degree, so that beyond a certain tension, the change of length by magnetization changes its sign and becomes a contraction. The effect of tension, however, decreases uniformly with the increase of the tensions. The results are given in Table XVI, and graphically in Figs. 11 and 12.

*) H. Nagaoka and K. Honda, Jour. Sc. Coll. XIX. Art. 11, 1903.
K. Honda and S. Shimizu, Ibid. XX. Art. 6, 1905.

TABLE XVI.

$t = 10°.0$ C.

$T = 890$ gr./mm.^{2}		$T = 1713$ gr./mm.^{2}		$T = 2538$ gr./mm.^{2}		$T = 3360$ gr./mm.^{2}	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.2	− 0.09	0.3	− 0.13	0.2	−0.03	0.2	−0.01
0.6	− 0.44	0.6	− 0.30	0.7	−0.11	0.7	−0.01
1.0	− 0.52	1.1	− 0.13	1.2	+0.13	1.4	+0.12
1.4	− 0.06	1.8	+ 0.50	2.4	+0.83	3.2	+0.43
1.8	+ 0.66	3.4	+ 1.64	4.8	+1.57	—	—
3.8	+ 2.48	7.4	+ 3.67	8.6	+2.38	7.8	0.94
6.6	+ 4.37	14.3	+ 5.60	14.9	+3.19	15.8	1.39
15.6	+ 8.25	22.3	+ 6.90	22.7	+3.76	22.6	1.62
22.6	+ 9.85	35.4	+ 7.96	35.8	+4.23	35.3	1.85
35.6	+11.62	52.0	+ 8.51	52.6	+4.54	52.3	2.01
53.0	+12.49	70.4	+ 8.84	70.8	+4.76	70.4	2.12
70.8	+12.91	108.2	+ 9.43	108.2	+4.96	108.2	2.25
108.2	+13.41	177.4	+ 9.70	178.5	+5.15	178.0	2.36
241	+13.98	240	+ 9.87	242	+5.26	241	2.42
306	+14.10	302	+ 9.97	305	+5.33	304	2.46
390	+14.20	386	+10.08	391	+5.40	387	2.53

$T = 4184$ gr./mm.^{2}		$T = 5003$ gr./mm.^{2}		$T = 5828$ gr./mm.^{2}		$T = 6652$ gr./mm.^{2}	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.2	−0.02	0.2	−0.01	0.2	−0.01	0.2	−0.01
0.3	−0.07	0.7	−0.09	0.6	−0.04	—	—
1.2	−0.02	1.4	−0.11	1.5	−0.10	1.0	−0.07
3.4	+0.11	4.5	−0.18	4.5	−0.20	4.6	−0.29
6.9	+0.26	—	—	8.0	−0.29	—	—
11.9	+0.44	10.8	−0.18	15.8	−0.40	12.7	−0.52
17.8	+0.55	22.6	−0.16	35.7	−0.50	22.6	−0.61
22.8	+0.61	35.9	−0.16	52.7	−0.55	53.1	−0.83
35.8	+0.73	70.8	−0.18	108.7	−0.67	83.5	−0.95
71.2	+0.83	178.0	−0.24	178.0	−0.73	134.6	−1.05
180.0	+0.93	262	−0.25	241	−0.78	224	−1.13
307	+1.00	328	−0.24	306	−0.78	304	−1.14
393	+1.07	390	−0.22	391	−0.77	388	−1.14

The observed values of the change of elasticity are as follows. Curves are also given in Fig. 14 in full lines.

TABLE XVII.

$\Delta T = \pm 823$ *gr./mm.*^{2} $t = 11°.3$ *C.*

$T=890$ *gr./mm.*^{2}		$T=1713$ *gr./mm.*^{2}		$T=3360$ *gr./mm.*^{2}		$T=5003$ *gr./mm.*^{2}	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
0.8	0.09	0.7	0.05	1.6	0.20	1.8	0.00
3.9	0.96	4.1	1.08	6.2	0.64	6.5	0.23
13.2	3.67	13.4	2.82	12.8	0.99	13.1	0.39
26.0	5.55	24.6	3.76	24.5	1.18	24.0	0.56
41.3	6.89	39.2	4.72	40.7	1.42	40.2	0.68
60.5	7.56	67.5	5.19	62.4	1.51	62.4	0.86
93.8	8.05	103.3	5.40	103.3	1.66	104.3	0.99
153.5	8.26	187.6	5.57	159.5	1.76	161.0	1.03
271	8.39	276	5.68	276	1.81	277	1.07
388	8.65	386	5.68	386	1.89	387	1.13

The elasticity increases by magnetization, the increase tending soon to an asymptotic value, as the field increases. The effect of tension is to decrease it decidedly. At $T=5003$ *gr./mm.*^{2}, the change is reduced to about one-sixth of that at $T=890$.

The calculated values of $\frac{\delta E}{E}$ are generally greater than the above values, and the differences becomes smaller, as the tension is increased. With a tension of about 5 kg., this difference almost vanishes. The results are given in Table XVIII and in Fig. 14 with dotted lines.

TABLE XVIII.

$\Delta T = \pm 823\ gr./mm.^2$

	$T = 890\ gr./mm.^2$	$T = 1713\ gr./mm.^2$	$T = 3360\ gr./mm.^2$	$T = 5003\ gr./mm.^2$
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
10	3.26	3.48	1.31	0.37
30	5.65	6.18	2.10	0.68
100	6.75	7.61	2.52	0.91
200	6.75	8.07	2.67	1.07
300	—	—	—	1.07
400	6.86	8.19	2.80	0.97

(iii) 70.32 % Nickel Steel. Figs. 15, 16, 17 and 18.

The change of length by magnetization resembles that for the former alloy, though the amount of the change is much smaller. Tension always decreases the elongation, the effect being smaller at greater tensions. Though the tension was increased up to $5686\ gr./mm.^2$, we could not observe any indication that greater tensions would change the elongation into contraction. The elongation seems to tend gradually to zero with the increase of the load. These results are seen in Table XIX and Figs. 15, 16 and 17.

TABLE XIX.

$t = 9°.2$.

$T = 806$ gr./mm.²		$T = 1671$ gr./mm.²		$T = 2476$ gr./mm.²		$T = 3277$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.3	−0.18	0.3	−0.18	0.2	−0.06	0.3	−0.09
0.6	−0.48	0.6	−0.45	0.6	−0.28	0.6	−0.21
1.0	−0.80	0.8	−0.60	1.0	−0.20	1.0	−0.11
1.4	+0.06	1.2	−0.09	1.4	+0.15	1.6	+0.11
1.8	+1.23	1.8	+0.61	—	—	3.0	+0.47
2.9	+2.59	3.0	+1.41	2.7	+0.70	4.9	+0.83
6.8	+5.08	4.9	+2.28	6.1	+1.60	10.0	+1.37
14.3	+6.70	17.0	+4.10	14.7	+2.46	17.2	+1.74
33.0	+7.68	34.1	+4.73	33.2	+3.01	33.2	+2.04
48.9	+7.90	44.3	+4.86	49.8	+3.16	49.8	+2.19
71.2	+8.03	66.6	+4.99	71.2	+3.25	71.2	+2.26
109.2	+8.12	107.8	+5.06	109.2	+3.33	108.7	+2.32
193.0	+8.21	185.8	+5.14	193.0	+3.42	191.8	+2.40
287	+8.25	283	+5.20	287	+3.45	286	+2.44
391	+8.35	364	+5.27	392	+3.49	389	+2.48

$T = 4081$ gr./mm.²		$T = 4882$ gr./mm.²		$T = 5686$ gr./mm.²	
H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$	H	$\frac{\delta l}{l} \times 10^6$
0.2	−0.01	0.2	−0.03	0.2	−0.01
0.6	−0.17	0.6	−0.13	0.7	−0.09
1.1	−0.05	0.9	−0.08	1.0	−0.04
1.9	+0.09	1.3	−0.04	1.9	+0.05
2.5	+0.23	2.7	+0.18	3.6	+0.20
4.3	+0.48	7.2	+0.53	7.3	+0.40
7.3	+0.82	16.0	+0.84	16.2	+0.64
15.9	+1.21	33.4	+1.08	33.2	+0.90
33.5	+1.49	49.5	+1.19	49.8	+0.98
71.2	+1.69	71.2	+1.26	71.5	+1.05
109.2	+1.76	108.7	+1.32	109.2	+1.11
192.5	+1.85	192.8	+1.38	192.5	+1.16
288	+1.88	288	+1.42	287	+1.20
392	+1.93	394	+1.44	391	+1.24

The observed change of elasticity is generally similar to that for 50.72 % Ni, but less in its amount, as shown in Table XX and in full lines in Fig. 18.

TABLE XX.

$\Delta T = \pm 805$ *gr./mm.*^{2} $t = 9°.2$ *C.*

$T=806$ *gr./mm.*^{2}		$T=1671$ *gr./mm.*^{2}		$T=3277$ *gr./mm.*^{2}		$T=4882$ *gr./mm.*^{2}	
H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$	H	$\frac{\delta E}{E_H} \times 10^2$
0.7	−0.14	0.7	−0.08	0.7	+0.18	0.6	−0.01
—	—	3.0	+1.17	2.6	0.40	0.8	+0.05
6.6	+2.45	5.3	1.92	4.7	0.53	3.8	0.30
9.7	3.57	9.6	25.2	12.0	1.07	12.0	0.45
19.9	4.13	41.1	3.33	26.1	1.35	25.1	0.60
41.8	4.51	68.9	3.38	53.4	1.42	52.7	0.68
70.8	4.71	136.3	3.45	98.8	1.55	98.8	0.70
177.3	4.92	226	3.72	193.0	1.50	192.6	0.70
260	4.92	309	3.69	285	1.57	284	0.78
386	4.92	388	3.74	393	1.53	393	0.70

The calculated values of $\frac{\delta E}{E}$ are generally greater than the above values, the difference being less for greater tensions, as seen in Table XXI and Fig. 18 with dotted lines. The difference almost vanishes for a tension of 3 kg.

TABLE XXI.

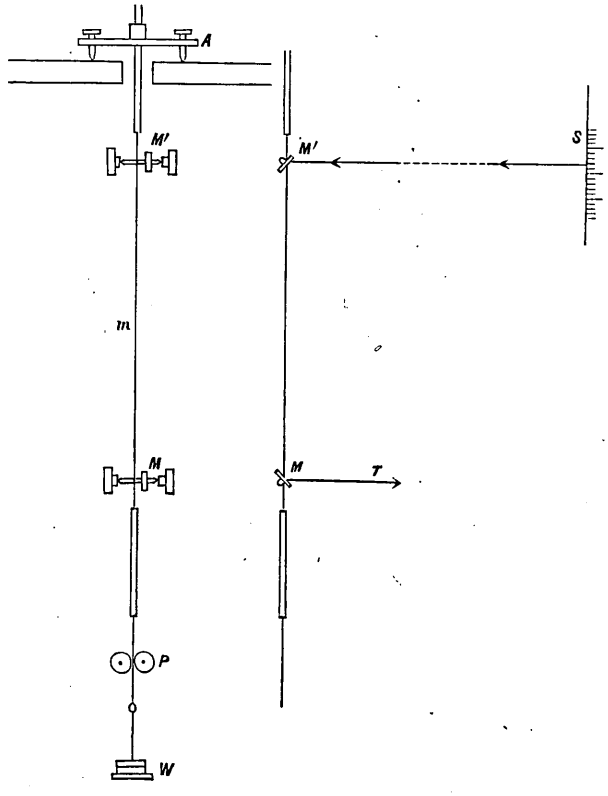
$\Delta T = \pm 805$ *gr./mm.*^{2}

	$T=806$ *gr./mm.*^{2}	$T=1671$ *gr./mm.*^{2}	$T=3277$ *gr./mm.*^{2}	$T=4882$ *gr./mm.*^{6}
H	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$	$\frac{\delta E}{E_H} \times 10^2$
5	4.09	2.40	0.75	0.32
10	6.31	3.12	1.00	0.45
100	7.13	4.25	1.45	0.55
200	7.13	4.28	1.37	0.55
300	7.15	4.29	1.45	0.53

From the above results for nickel steels, we may conclude that for small tensions, the change of elasticity is considerably less than that calculated from the tension effect on the magnetic change of length. But the difference between the two becomes less and less as the tension is increased, almost vanishing for great tensions.

(*b*) Measurement of the Coefficient of Elasticity.

Since it was necessary, in the calculation of $\frac{\delta E}{E}$, to know the coefficient of elasticity in no field, a special arrangement was devised for this purpose.



Referring to the figure, m is the specimen, at the ends of which brass rods of moderate diameter are brazed. The upper rod is clamped to the tripod A resting on a rigid wooden frame. To the lower rod, is attached a flexible cord, which passing through the interspace between the grooves of two friction wheels P fixed

to a stand, carries the scale pan for the weight W. Near the upper and lower ends, the specimen touches the horizontal axes carrying two mirrors M and M' respectively. The axes are so pivoted at their ends, that they rotate with almost no friction. S is a vertical glass scale illuminated from behind; its image formed by successive reflexions in M and M' is observed by a elescope T. If the load on the pan causes the elongations $\delta l'$ and δl respectively at M' and M, the rotations of the axes are given by

$$\varphi'=\frac{\delta l'}{r} \quad \text{and} \quad \varphi=\frac{\delta l}{r},$$

where r is the common thickness of the two axes. Again, if n' and n be the deflections of the scale due to separate rotations φ' and φ respectively, we have

$$\varphi'=\frac{n'}{2d} \quad \text{and} \quad \varphi=\frac{n}{2(d+a)},$$

where $d=M'S$ and $a=MM'$. The actual deflection observed is $s=n-n'$.

Now $$\delta l-\delta l'=r(\varphi-\varphi')$$

$$=\frac{rs}{2d}\left\{1-\frac{a}{d}\left[1+\left(\frac{n'}{s}-\frac{a}{d}\right)\left(1-\frac{a}{d}\right)\right]\right\}.$$

In our experiments,

$$d=624.5 \text{ cm.}, \quad a=65.83 \text{ cm.}, \quad r=0.795 \text{ mm.},$$

and $$\frac{a}{d}=0.1054.$$

Since n' was very small compared with s, and $\frac{n'}{s}$ appears as a small correction, it was sufficient for our purposes to determine n' roughly only once for a specimen throughout the sets of observations. This was easily done by directing the telescope to M' and

reading off the deflexion. Usually n' did not exceed 7 divisions of the scale, while s was greater than 90 divisions.

To prevent shocks by loading, the face of the initial weight was covered with a mat of cotton wool. The wire to be tested was surrounded by a tube of paper to prevent disturbance due to air-current.

Denoting the weight, by which the elongation $\delta l - \delta l'$ is produced, by P, we have for the coefficient of elasticity

$$E = \frac{\frac{P}{A}}{\frac{\delta l - \delta l'}{a}},$$

where A is the section of the wire. P in our experiments was 100 grams, which gave a deflection of more than 90 divisions of the scale. To test the working of the present apparatus, 10, 50 and 100 grams were loaded with the following results:—

TABLE XXII.

$t = 12°$ C.

Weight.	Tungsten Steel.	Nickel.	28.74 % Ni.
10 *gr.*	9.7 *div.*	9.1 *div.*	9.2 *div.*
50	48.1	47.3	46.3
100	96.2	96.0	92.9

Thus the proportionality is nearly satisfied up to 100 grams; this also shows that the friction at different parts of the arrangement is negligibly small.

The coefficient of elasticity differed slightly for different initial tensions, and moreover underwent more or less hysteresis due to the previous load. In our experiments, the weights were

varied from 1 to 7 kilograms, and then again down to 1 kilogram. The results are tabulated in the following table:—

TABLE XXIII.

$t = 12°$ C.

Pure nickel:		Commercial nickel.		Swedish iron.		Tungsten steel.	
$gr./mm.^2$	$E \times 10^{-12}$	$gr./mm.^2$	$E \times 10^{-12}$	$gr./mm.^2$	$E \times 10^{-12}$	$gr./mm.^2$	$E \times 10^{-12}$
1599	1.708	1376	1.818	1689	1.862	1758	1.928
3080	1.760	2651	1.972	3254	1.883	3386	1.938
4557	1.835	3923	2.049	4815	1.916	5011	1.938
6040	1.854	5199	2.125	6380	1.947	6640	1.955
7523	1.867	6477	2.152	7947	1.955	8272	1.949
9003	1.887	7750	2.193	9511	1.962	9900	1.951
10480	1.902	9023	2.212	11700	1.968	11523	1.945
7523	1.888	—	—	7947	1.935	8272	1.937
4557	1.851	5199	2.146	4815	1.919	5011	1.923
1599	1.739	1376	1.851	1689	1.788	1758	1.914

28.74 % Ni.		50.72 % Ni.		70.32 % Ni.	
$gr./mm.^2$	$E \times 10^{-12}$	$gr./mm.^2$	$E \times 10^{-12}$	$gr./mm.^2$	$E \times 10^{-12}$
1480	1.570	1780	1.376	1500	1.866
2850	1.619	—	—	—	—
4220	1.643	5070	1.551	4260	1.982
5600	1.667	—	—	—	—
6970	1.677	8370	1.621	7030	1.996
8340	1.693	—	—	—	—
9710	1.702	11650	1.619	9800	1.992
6970	1.675	8370	1.567	7030	1.986
4220	1.666	5070	1.548	4260	1.961
1480	1.591	1780	1.377	1500	1.864

The mean curves corresponding to these numbers are given in Figs. 19, 20, 21, 22, 23, 24 and 25. They are drawn in a very exaggerated scale. They show distinct hysteresis; in three of them, the ascending branch lies below the descending one, while in the others, the case is reversed. In commercial nickel, the increase of elasticity is the greatest; in tungsten steel, it is the least. In 50.72 % Ni. and 70.32 % Ni., the elasticity seems to attain a vague maximum at a high tension.

In the calculation of $\frac{\delta E}{E}$ for different tensions, the values of E for corresponding tensions were employed. This was absolutely necessary, since for different tensions, E changed more than 10 % in many of the specimens.

(c) Comparison of the results with those obtained by the flexure method.

In a previous paper by Messrs. S. Shimizu, S. Kusakabe and one of us, the change of elasticity of magnetic bars, as determined by the flexure method is given. It is highly interesting to compare the results with those obtained by our elongation method.

In the flexure method, we observed a slight change of flexure by magnetization when there was no suspended weight; this perhaps arises from the initial bending of the specimen due to its own weight. In subsequent experiments of the kind, it was thought preferable to subtract this initial change of flexure from that of flexure due to a suspended load. We have therefore recalculated the former results, as shown in Table XXIV and in Figs. 26, 27 and 28.

TABLE XXIV.

SOFT IRON.

$T=500\,gr.$		$T=2540\,gr.$	
H	$\frac{\delta E}{E_H}\times 10^2$	H	$\frac{\delta E}{E_H}\times 10^2$
18.8	−0.03	20.1	+0.26
26.1	+0.56	26.9	+0.55
29.0	+0.77	33.9	+0.76
36.2	+0.89	52.9	+0.91
59.0	+1.01	69.4	+0.97
133.4	+1.04	129.0	+0.99
231	+1.04	212	+1.00
291	+1.04	290	+1.02
383	+1.04	398	+1.07

NICKEL.

$T=183\,gr.$		$T=727\,gr.$	
H	$\frac{\delta E}{E_H}\times 10^2$	H	$\frac{\delta E}{E_H}\times 10^2$
10.5	−0.22	10.0	−0.19
16.9	−0.79	16.7	−0.72
24.8	−0.96	21.9	−0.95
34.5	−1.52	26.4	−0.98
48.1	−0.53	39.6	−0.75
68.2	+0.11	68.2	+0.08
—	—	117.3	+1.35
240	+2.64	218	+2.57
376	+3.01	377	+3.16
496	+3.29	492	+3.25

TUNGSTEN STEEL.

$T=439\,gr.$		$T=1020\,gr.$		$T=1810\,gr.$	
H	$\frac{\delta E}{E_H}\times 10^2$	H	$\frac{\delta E}{E_H}\times 10^2$	H	$\frac{\delta E}{E_H}\times 10^2$
14.0	−0.01	15.1	−0.01	18.2	−0.04
19.4	−0.04	18.2	−0.03	20.5	−0.08
23.1	−0.17	22.4	−0.07	23.0	−0.07
25.4	−0.08	25.6	−0.03	38.4	+1.24
36.6	+1.61	30.2	+0.80	56.5	+1.48
68.1	+1.89	41.1	+1.34	75.7	+1.58
91.9	+2.02	68.5	+1.57	144.3	+1.61
270	+2.12	224	+1.88	230	+1.66
387	+1.99	348	+1.81	361	+1.69
486	+2.10	472	+1.78	460	+1.69

In the former results, the value of $\frac{\delta E}{E}$ depended considerably on the suspended weight; but in our recalculated results, it was only slightly affected by the weight, except in the case of

nickel in weak fields, where its initial decrease was considerably reduced.

For Swedish iron and tungsten steel, the results somewhat resemble those of the present experiment deduced from the magnetic elongation. But they are several times greater as compared with the results of the direct experiment. In nickel, the course of the curve is considerably different from that of the present experiment; its form is, however, very interesting, if we consider it in connection with that of the curve $\frac{\delta K}{K}$ to H. It is here to be noticed that as regards magnetic quality, the specimen in the present experiment was very different from that used in the former experiment, so that the quantitative comparison of the two results is of little value.

§ 3. EXPERIMENTS ON THE CHANGE OF RIGIDITY BY MAGNETIZATION.

(*a*) Measurement of the change of rigidity by the oscillation method.

One of our methods consisted in giving torsional oscillation to the wires magnetized with different fields and calculating the coefficient of rigidity for different fields from the periods of oscillation. The wires tested were the same as those used in the above experiment for the change of elasticity. The wire was hung vertically in the field of the magnetizing coils used in the previous experiment. Its upper end was brazed to a rigid brass rod, which was clamped to the frame above. To the lower end, a similar rod was brazed, to which the oscillating weights were fixed. In order to sufficiently diminish the disturbance due to

the resistance of air, the Foucault current etc., the period of oscillation was made very long by using a weight with considerable moment of inertia. A rectangular brass bar ($42.80 \times 0.895 \times 1.946$ cm.^{3}) was horizontally fixed to the lower end of the lower rod, at its middle point. Two equal cylindrical weights (each 657 gr.) made of lead could be fixed symmetrically on the bar at any desired distance from the middle. Besides, two equal cylindrical weights (1671 and 1640 gr.) of lead could be put on the bar with their axes coinciding with that of the rod. Suitable combinations of these different weights enabled us to adjust the periods of oscillations for different tensions to convenient values. A light mirror was fixed to the lower rod. The image of a horizontal scale placed in front of the arrangement was observed with a telescope in the usual manner.

To start the torsional oscillation of the wire, a lead tube was directed to one end of the horizontal bar. The tube going to the observer was held by his hand; the oscillation was started by blowing through the tube, while observing through the telescope. The amplitude of the oscillation could be increased or diminished at will, by blowing with suitable force on the bar in a suitable phase of its oscillation. In this way, the equality of the amplitude could easily be effected within 1 mm. of the scale. It was usually 5 cm. with a scale distance of 1.46 m., which corresponds to an amplitude of about 1°.

It was necessary to protect the oscillating system from disturbance due to air currents.

Our procedure was as follows:— The wire to be tested was first demagnetized by reversals, the zero of the scale set to the position of equilibrium; a field was applied, and then oscillation started. Care was taken to adjust the amplitude for different fields

so as to make them as equal as possible, in order that in the comparison of periods for different fields, the effects of amplitudes might be neglected. One of the observers signalled at each complete oscillations, while the other recorded the time by a mean time chronometer. According to the usual method of time-observation, 60 consecutive oscillations were availed of for the determination of its period. The first and the last 10 were signalled and recorded; we thus obtained 10 sets of time records for 50 complete oscillations. The mean of these observations gave the period of oscillation, which usually ranged from 10 to 20 seconds. Its value may be considered accurate to $\frac{1}{1000}$ of a second. These processes were repeated for a series of successively increasing fields, the demagnetization being of course effected before each experiment. After a series of observations was taken, the experiment at no field was again repeated, and we usually found the result fairly unchanged.

Since the magnetizing coil was waterjacketed, the heating-effect was inappreciable. The magnetizing current was measured both before and after each experiment, and the mean was taken. The current remained fairly constant during each experiment, except in a few cases.

Though the period of oscillation was long and the oscillating weights were moderately distant from the lower end of the coil, it was necessary to determine experimentally the retarding effect of the Foucault current, which was liable to be produced in the system vibrating in the magnetic field. For this purpose, a copper wire of the same size as the specimen was oscillated in a number of fields and the corresponding periods were determined. We found that for the periods used in our experiments, the effect was negligeably small. Logarithmic decrements for different fields

were also determined; but generally no appreciable change was observed. In some of the magnetic wires, the damping of the oscillation was found to become slightly faster as the field was increased. But, the change of the logarithmic decrement was too small to sensibly affect the period of oscillation, when referred to the equation

$$T = T_0\left(1 + \frac{\varkappa^2 T_0^2}{8\pi^2}\right)$$

where T is the actual period of oscillation, T_0 that for no damping and $\varkappa$ the logarithmic decrement. This agrees with the results* by Ignaz Klemenčič, and H. Tomlinson for iron.

The values of rigidity at no field were calculated from the period of oscillation for the bar only, whose moment of inertia was known from its dimensions.

In the oscillation method, the right order of applying the twist and the field was followed. Since the oscillation was very slow, the value of the rigidity thus determined may properly be considered to be the statical value. The results of experiments are as follows:—

* Ignaz Klemenčič, Wien. Ber. **78**, Nov. 7, 1878, 8.
H. Tomlinson, Proc. Roy. Soc. **40**, 447; Phil. Trans. **179**, 1, 1888.

TABLE XXV. $K_0 = 0.96 \times 10^{12}$

Nickel. Fig. 29. $t = 14°.7$ C.

$T = 1158$ gr./mm.²		$T = 3410$ gr./mm.²		$T = 6215$ gr./mm.²		$T = 9075$ gr./mm.²	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
3.9	−0.21	3.9	−0.10	4.1	−0.00	—	—
10.5	−1.61	10.2	−0.25	12.0	−0.10	11.8	−0.01
23.7	−4.21	24.1	−1.51	24.1	−0.34	24.1	−0.08
46.9	−6.80	48.2	−5.82	47.9	−1.87	47.7	−0.67
78.2	−6.80	77.8	−8.65	78.0	−5.11	77.6	−2.04
117.0	−5.92	116.2	−8.55	116.5	−9.14	123.7	−5.06
190	−3 33	189	−6.58	190	−9.22	196	−8.57
276	−0.43	297	−2.86	276	−6.93	287	−8.59
406	+2.12	390	−0.62	401	−3.21	382	−6.35

Among ferromagnetic metals tested, nickel undergoes the greatest change of rigidity and shows the most remarkable feature as regards the effect of tension. For small tensions, the rigidity at first decreases rapidly and after passing through a minimum, increases slowly with the field. In a certain field, the change is zero, beyond which there is an increase. As the tension is increased, the change becomes less for low fields, but greater for higher fields; the maximum change occurs in a higher field, and its amount increases up to a certain tension, and then slowly decreases. The point of no change shifts toward higher fields, as the tension increases.

Swedish Iron and Tungsten Steel. Figs. 30 and 31.

TABLE XXVI.

SWEDISH IRON; $K_0=0.756\times10^{12}$. $t=14°.6$ C.		**TUNGSTEN STEEL;** $K_0=0.608\times10^{12}$. $t=14°.0$ C.	
$T=3113$ gr./mm.^{2}		$T=3240$ gr./mm.^{2}	
H	$\frac{\delta K}{K_H}\times10^2$	H	$\frac{\delta K}{K_H}\times10^2$
24.5	−0.03	24.2	−0.03
74.4	+0.15	72.1	+0.00
146.2	+0.40	162.7	+0.02
280	+0.28	—	—
385	+0.25	403	+0.12

The rigidity of Swedish iron slightly decreases in weak fields, but generally increases by magnetization. The amount of increase becomes greater and then slowly decreases with the field. The change of rigidity of tungsten steel is very small; it slightly decreases at first, and then steadily increases with the field. Both of these specimens are characterised by the smallness of the change of rigidity as well as that of elasticity. By a similar method, H. Tomlinson* found that for a moderate field, the change of rigidity of an annealed iron wire is insensibly small.

* H. Tomlinson, loc. cit.

Nickel Steels. Figs. 32, 33 and 34.

TABLE XXVII.

28.74 % Ni.; $K_0 = 0.613 \times 10^{12}$. $t = 15°.5$.

$T = 928$ gr./mm.^{2}		$T = 1806$ gr./mm.^{2}		$T = 4096$ gr./mm.^{2}	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
—	—	3.8	0.03	6.2	0.06
12.2	0.29	12.0	0.25	11.8	0.13
48.0	0.43	47.9	0.34	48.1	0.29
117.7	0.53	116.9	0.42	116.3	0.34
206	0.54	204	0.44	204	0.35
303	0.53	301	0.44	300	0.33
409	0.48	406	0.46	403	0.38

50.72 % Ni.; $K_0 = 0.365 \times 10^{12}$. $t = 14°.3$ C.

$T = 1114$ gr./mm.^{2}		$T = 3277$ gr./mm.^{2}		$T = 5974$ gr./mm.^{2}		$T = 8725$ gr./mm.^{2}	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
4.0	−0.60	3.8	−0.23	3.8	−0.37	3.8	−0.22
7.9	−0.23	7.8	+0.28	7.7	−0.28	8.0	−0.59
16.4	+1.24	15.9	+1.48	15.7	+0.46	15.8	+0.51
32.9	+3.17	33.0	+3.03	32.6	+1.41	31.2	+1.05
58.0	+4.73	56.8	+4.14	56.8	+2.31	54.9	+1.75
121.0	+6.01	117.3	+5.44	117.3	+3.46	116.6	+2.58
196	+6.73	205	+6.06	202	+4.07	204	+3.24
283	+7.12	278	+6.57	288	+4.24	290	+3.42
387	+7.16	379	+6.92	386	+4.64	394	+3.60

70.32 % Ni.; $K_0 = 0.640 \times 10^{12}$. $t = 16°.5$ C.

$T = 1086$ gr./mm.^{2}		$T = 3198$ gr./mm.^{2}		$T = 5828$ gr./mm.^{2}		$T = 8510$ gr./mm.^{2}	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
3.3	−1.18	1.9	−0.02	2.4	−0.23	2.3	−0.60
5.2	−0.86	3.3	+0.25	5.9	+0.09	6.0	−0.40
7.8	−0.24	6.2	+0.97	—	—	—	—
15.6	+1.72	12.9	+1.99	11.7	+0.64	11.3	−0.12
24.1	+3.00	—	—	24.2	+1.50	24.5	+0.44
48.0	+4.64	46.3	+4.17	47.7	+2.22	48.6	+1.00
77.6	+5.41	90.5	+4.97	78.4	+2.70	86.5	+1.44
116.2	+5.96	163.2	+5.50	167.0	+3.27	—	—
204	+6.33	238	+5.67	241	+3.49	208	+2.01
300	+6.43	316	+5.79	301	+3.55	306	+2.14
394	+6.55	401	+5.86	410	+3.64	414	+2.08

In general features, the change of rigidity is similar for the three specimens. Magnetization generally increases the rigidity; the change increases rapidly with the field and soon tends to asymptotic values. The effect of tension is invariably to diminish the change of rigidity. The last two specimens, which in weak fields, display a slight decrease of rigidity, resemble each other not only in their general behaviour, but also in the amount of their change. In 28.74 % Ni., the amount of the change is much less than that for the others. It is to be noticed that the general aspect of the change of rigidity resembles that of the change of elasticity.

(*b*) Measurement of the change of rigidity by Barus's method.

The same specimens, on the other hand, were tested by the differential method used by Barus,* in which the wires were first

* Barus, loc. cit.

twisted and then magnetized. In our experiments, instead of the magnetic wire, a copper one was used for the compensation of the twist at no field, in order to avoid any ambiguity caused by the magnetization of the compensating wire. The specimens were cut from the previous samples into about one-third of their lengths. The copper wire was so chosen that its total twist for a given couple was nearly equal to that of the magnetic specimen to be tested. The two wires were connected in the same line by a copper rod of moderate diameter with a small mirror attached to it, and hung vertically from a torsion circle. The lower end of the specimen was also rigidly connected to another copper rod. To this rod, a rigid pin was perpendicularly fixed, the ends of which slid in the vertical grooves cut on the inside of a flat hollow cylinder at the center of the lower torsion circle. In this way, the tension applied to the pan hanging on the lower end of the lower copper rod was transmitted independently of the twist. The upper and lower ends of this connected system could therefore be twisted to any desired amount by means of the torsion circles, while the tension was constantly acting on the wire. A magnetizing coil, whose length was 30 cm. and $4\pi n = 379.7$, was placed co-axially with the lower wire, which was, in our case, the specimen to be tested. The coil had a small resistance of 0.6Ω, so that the effect of heating was negligibly small, though the water-jacketed arrangement was dispensed with. A long scale distance of 6.797 m. was used to increase the sensitiveness of the arrangement. The sensitiveness was such that a deflection of 1 scale division corresponded to a change of total twist of 15.2″.

To stop the vibration of the mirror, a short brass wire was fixed horizontally to the vertical copper rod a little below the

reflecting mirror and bent downward. Just below it, a small mercury cup was placed, into which the wire dipped.

Our method of observation was as follows :—First the torsion circles were adjusted so that the magnetic field produced no deflection of the mirror. The readings of both circles for this position corresponded to those of the untwisted state of the wire. Next the lower circle was twisted by θ and then the upper circle was twisted in the opposite sense, until the mirror regained its original position for no field. Let the angle through which the upper circle was twisted be denoted by θ'; θ' was nearly equal to θ. After demagnetization by reversals, the mirror usually turned through a small angle; but the amount of the rotation was usually so small that we could leave it out of account. Different fields were successively applied and corresponding deflections were read off. Next the twist was increased and the procedure repeated.

The experiments were made also for different tensions. Before increasing the tension, the wire was untwisted to the original state of no twist, after passing through a cyclic twist of gradually decreasing amplitude about the position of no twist. This procedure was always necessary to restore the wire to the state of no twist, since the simple untwisting usually left a residual deflection.

If the couples required to give unit twist to the two wires be τ and τ' respectively, we have

$$\tau\theta = \tau'\theta';$$

if the angular deflection of the mirror due to magnetization be $\delta\theta$, and $\delta\tau$ the increment of τ, we have

$$\tau_H = \tau + \delta\tau \quad \text{and} \quad (\tau+\delta\tau)(\theta-\delta\theta) = \tau'(\theta'+\delta\theta)$$

or

$$\tau_H(\theta-\delta\theta) = \tau'(\theta'+\delta\theta) \quad \text{and} \quad \delta\tau(\theta-\delta\theta) = \delta\theta(\tau'+\tau),$$

hence by division, we get

$$\frac{\delta\tau}{\tau_H}=\frac{\delta\theta(\tau+\tau')}{\tau'(\theta'+\delta\theta)}=\frac{\delta\theta\left(1+\frac{\tau}{\tau'}\right)}{\theta'+\delta\theta},$$

or,

$$\frac{\delta K}{K_H}=\frac{\delta\theta(\theta+\theta')}{\theta(\theta'+\delta\theta)},$$

where K is the rigidity of the specimen, and δK its change due to magnetization. If $\frac{\delta\theta}{\theta}$ be neglected, we have

$$\frac{\delta K}{K_H}=\frac{\delta\theta}{\theta}\left(1+\frac{\theta}{\theta'}\right).$$

If the change were greater than 1%, the first equation was used, and if it were less, the second equation was availed of.

The results of experiments are as follows:—

Nickel. Fig. 35.

TABLE XXVIII.

$\theta=2.5°$ $t=21.°0$ C.

$T=1022$ gr./mm.^{2}		$T=3096$ gr./mm.^{2}		$T=5465$ gr./mm.^{2}	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
8.3	+ 0.83	4.9	+ 0.84	9.3	+ 0.34
13.9	− 0.24	9.7	+ 1.41	15.0	+ 0.30
23.3	− 2.37	18.7	+ 0.00	21.5	− 0.30
43.7	− 3.11	43.0	− 8.81	26.4	− 1.26
86.8	− 0.41	85.4	− 10.04	42.4	− 7.00
125.6	+ 2.45	123.4	− 6.84	85.3	− 17.33
145.8	+ 3.58	143.2	− 5.37	123.8	− 15.21
207	+ 6.39	204.4	− 1.43	174.2	− 10.45
311	+ 9.17	304	+ 2.53	305	− 2.90
441	+ 11.00	432	+ 5.13	432	+ 0.43

$\theta = 5.0°$

$T = 1022\ gr./mm.^2$		$T = 3096\ gr./mm.^2$		$T = 5465\ gr./mm.^2$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
6.2	+ 0.68	4.0	+0.67	5.6	+ 0.08
11.3	− 0.08	8.8	+1.39	10.1	+ 0.12
19.6	− 1.54	15.3	−0.48	15.4	− 0.31
42.8	− 2.14	26.9	−4.87	27.1	− 3.05
86.3	− 0.00	43.3	−7.94	42.8	− 7.87
124.2	+ 2.18	86.2	−7.33	85.7	−13.51
145.8	+ 3.27	143.6	−3.84	123.8	−11.95
208	+ 5.68	207	−0.70	174.2	− 8.53
310	+ 8.26	309	+2.76	306	− 2.25
439	+10.06	438	+5.38	432	+ 0.76

In nickel, the change of rigidity and the effect of tension are very remarkable. Under small tension, the rigidity first increases slightly, then decreases rapidly as the field is increased, attains a minimum and gradually increases, till in a certain field, it recovers its initial value. As the field is further increased, the rigidity increases by magnetization. If the tension is increased, the field corresponding to the maximum decrease moves towards higher fields, and the amount of decrease is remarkably increased. Tensions here used were so adjusted as to be nearly equal to those in the oscillation method. The effect of the amplitude of twist was to diminish the change of rigidity in its absolute amount.

Comparing the above results with those of the oscillation method, we notice that in weak tensions, the curve of the change of rigidity as given by Barus's method lies considerably above the curve of the change by the oscillation method, and that in greater tensions, the contrary is the case.

Swedish Iron and Tungsten Steel. Figs. 36 and 37.

TABLE XXIX.

SWEDISH IRON.

$T=3271$ *gr./mm.*^{2} $t=20°.0$ *C.*

$\theta=2.5°$		$\theta=5.0°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
4.4	0.32	3.9	0.19
10.5	1.00	10.2	1.01
20.0	1.36	16.5	1.27
42.6	1.88	25.2	1.51
75.5	2.20	42.2	1.81
123.1	2.35	75.5	2.06
174.0	2.51	171.4	2.34
306	2.56	301	2.42
434	2.67	428	2.51

TUNGSTEN STEEL.

$T=3405$ *gr./mm.*^{2} $t=20°.1$ *C.*

$\theta=4.0°$		$\theta=8.0°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
13.6	0.00	12.6	0.00
21.8	0.08	18.3	0.07
27.1	0.14	29.3	0.17
46.0	0.20	44.7	0.21
81.3	0.30	76.4	0.30
134.1	0.38	123.8	0.38
172.9	0.40	172.2	0.41
304	0.45	300	0.49
429	0.51	425	0.52

Thus the rigidity of these metals always increases with the field; the change for Swedish iron is tolerably large; but, for tungsten steel, it is very small. As to the effect of amplitude of twist, it is slightly to diminish the change in Swedish iron, but is almost insensible in tungsten steel. The change of rigidity obtained by Barus's method, is several times greater than that by the oscillation method. Thus, in the case of ferromagnetic metals, relation (3) does not hold even approximately.

28.74 % Nickel Steel. Fig. 38.

TABLE XXX.

$t = 21.°2.$

$T = 948\ gr./mm.^2$		$T = 1768\ gr./mm.^2$		$T = 4240\ gr./mm.^2$	
$\theta = 2.5°$		$\theta = 3.5°$		$\theta = 3.5°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
3.4	0.08	3.8	0.12	3.7	0.11
10.5	0.31	9.5	0.25	10.3	0.23
23.0	0.42	20.5	0.41	19.7	0.30
86.1	0.56	42.7	0.45	42.2	0.34
145.0	0.61	83.5	0.45	86.1	0.40
—	—	142.1	0.45	145.4	0.41
307	0.56	307	0.48	311	0.41
434	0.56	437	0.48	441	0.41
$\theta = 7.5°$		$\theta = 7.5°$		$\theta = 7.5°$	
3.2	0.09	3.8	0.09	3.8	0.12
9.9	0.24	9.3	0.22	10.8	0.21
21.4	0.34	19.0	0.31	20.6	0.28
43.2	0.41	42.9	0.40	23.8	0.31
124.9	0.45	83.5	0.42	84.6	0.35
206	0.45	143.0	0.42	143.9	0.36
310	0.46	307	0.44	309	0.36
436	0.46	436	0.44	436	0.36

50.72 % Nickel Steel. Fig. 39.

TABLE XXXI.

$t = 19°.0$ C.

$T = 1136$ gr./mm.^{2}		$T = 3445$ gr./mm.^{2}		$T = 6084$ gr./mm.^{2}	
$\theta = 2.5°$		$\theta = 2.5°$		$\theta = 2.5°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
3.2	0.30	3.4	0.24	3.0	0.03
7.3	1.32	8.5	1.27	7.7	0.53
14.1	2.93	14.6	2.31	14.1	1.26
22.6	4.35	22.9	3.33	22.9	2.08
35.3	5.87	35.6	4.45	35.3	2.89
49.7	6.95	49.3	5.24	49.9	3.59
69.6	7.90	69.1	6.00	69.6	4.23
94.1	8.66	93.8	6.62	94.1	4.79
132.7	9.42	133.4	7.26	132.7	5.30
202	10.06	203	7.84	203	5.90
302	10.52	303	8.23	303	6.30
430	10.83	429	8.51	429	6.57
$\theta = 5.0°$		$\theta = 5.0°$			
3.1	0.52	3.2	0.23		
7.0	1.35	7.4	1.05		
14.2	2.93	13.9	2.17		
22.3	4.27	22.7	3.28		
34.9	5.70	35.2	4.36		
49.0	6.77	49.5	5.17		
68.9	7.71	69.6	5.97		
93.8	8.51	94.1	6.60		
132.3	9.17	133.4	7.20		
202	9.87	203	7.83		
302	10.30	303	8.27		
428	10.60	430	8.54		

70.32 % Nickel Steel. Fig. 40.

TABLE XXXII.

$t = 19°.2$ C.

$T = 1109$ gr./mm.²		$T = 3361$ gr./mm.²		$T = 5936$ gr./mm.²	
$\theta = 2.75°$		$\theta = 2.65°$		$\theta = 2.55°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
3.8	0.06	3.0	0.26	3.2	0.07
—	—	6.7	0.97	11.1	0.97
11.1	2.20	15.5	2.33	21.1	1.72
23.2	4.44	23.5	3.16	22.9	1.92
35.8	5.70	35.7	3.92	34.7	2.41
—	—	50.3	4.48	48.9	2.92
64.1	6.99	69.2	4.95	68.9	3.32
85.7	7.77	93.8	5.35	93.6	3.64
150.2	8.19	132.1	5.69	131.9	4.00
248	8.60	202	6.03	202	4.28
342	8.69	303	6.25	303	4.54
432	8.83	430	6.34	433	4.61
$\theta = 5.3°$		$\theta = 5.1°$		$\theta = 5.05°$	
4.5	0.36	3.3	0.37	2.7	0.17
—	—	7.8	1.18	9.2	0.82
12.9	2.18	15.8	2.26	18.0	1.50
23.1	3.69	22.8	2.90	23.5	1.83
39.8	5.02	34.2	3.64	34.8	2.32
—	—	48.5	4.23	49.1	2.80
57.9	5.77	68.9	4.70	69.6	3.17
93.8	6.55	93.8	5.06	94.1	3.48
132.0	6.94	131.6	5.39	132.7	3.80
203	7.28	202	5.71	202	4.11
304	7.53	302	5.94	298	4.30
432	7.67	431	6.05	429	4.43

Thus the change of rigidity in these nickel steels is similar, in its general aspect, to that given by the oscillation method. In 50.72 % Ni. and 70.32 % Ni., however, the small initial decrease of rigidity is not observed. Increased twist diminishes the change of rigidity; increased tension affects the change in a similar manner but in a greater degree. The change as given by Barus's method is generally greater than that by the oscillation method. The difference becomes less as the tension is increased. In 28.74 % Ni., the results by the two different methods fairly agree, and the agreement becomes closer with increased tension. Thus, also in these alloys, relation (3) is not generally satisfied, except in 28.74 % Ni. The difference, however, becomes less as the tension is increased.

Beside the above samples, a number of others including iron and nickel steels of different percentages, were tested with the purpose of studying the different behaviours of the different specimens. The samples were as follows:—

Samples	Diameter	Length
Swedish iron	1.01 mm.	20.18 cm.
23.6 % Ni.	0.71	19.63
26.64 % Ni.	1.01	20.90
24.04 % Ni.	0.96	20.30
35 % Ni.	0.95	19.70
45 % Ni.	0.95	19.58
70.32 % Ni.	1.00	20.12

All these specimens were slightly annealed by a Bunsen flame. In the case of Swedish iron and 70.32 % Ni., if we compare the following results with those obtained above, we can notice the effect of annealing on the change of rigidity. The

irreversible nickel steels were also tested after being cooled in liquid air.

To study the effect of the amplitude of twist, three different twists were usually applied, differing by about 2.5° from each other. Three or four different tensions were also applied for studying the effect of tension.

Swedish Iron. Figs. 41, 42, 43 and 44.

TABLE XXXIII.

$T = 120\ gr./mm.^2$

$\theta = 2.45°$		$\theta = 4.8°$		$\theta = 7.25°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
9.3	0.18	8.5	0.17	9.3	0.13
21.5	0.37	22.2	0.43	21.5	0.36
46.7	0.71	47.0	0.68	46.7	0.59
88.8	0.97	90.9	0.88	88.8	0.81
179.8	1.07	183.5	1.04	182.8	0.90
281	1.12	283	1.09	281	1.02
376	1.14	376	1.14	404	1.07
484	1.23	484	1.20	483	1.07

Thus the imperfect annealing did not change its general behaviour as regards the change of rigidity by magnetization. The amount of the change in the present specimen is, however, about one-half that of the change in the well annealed specimen. The change is almost independent of the tension. The amplitude of twist affects the results in a slight degree; the change increases, in general, at first slightly and then decreases with the twist.

The hysteresis for a cyclic change of the field is given in Fig.

44. The curve was obtained after several cyclic changes of the field.

23.6 %, 26.64 % and 24.04 % Nickel Steels.

Figs. 45, 46, 47, 48, 49, 50 and 51.

TABLE XXXIV.

23.6 % Ni.; $T=242\ gr./mm.^2$

$\theta=2.4°$		$\theta=4.65°$		$\theta=6.9°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
6.7	+0.08	5.9	−0.01	4.4	0.00
24	−0.07	23	−0.02	23	0.00
62	−0.43	88	−0.24	88	−0.12
198	−0.14	197	−0.18	197	−0.12
340	−0.00	335	+0.16	333	−0.00
507	+0.14	501	+0.16	497	+0.09

24.04 % Ni.; $T=133\ gr./mm.^2$

$\theta=2.1°$	
H	$\frac{\delta K}{K_H}\times 10^2$
60	−0.02
107	−0.15
201	−0.08
338	0.00
487	0.00

26.64 % Ni.; $T=120\ gr./mm.^2$

$\theta=2.35°$	
H	$\frac{\delta K}{K_H}\times 10^2$
22	0.00
107	−0.04
209	−0.00
350	+0.02
507	+0.04

The change of rigidity in these irreversible nickel steels is inappreciably small at ordinary temperatures. After cooling them in liquid air, a small change is observed, as shown in the above

table. The change is common for all these specimens; the rigidity first decreases, attains a minimum, and then gradually recovers its initial value, as the field is increased. In still higher fields, a slight increase is observed, which increases with the field.

The fact that the change of rigidity becomes appreciable, only when the specimens are once dipped in liquid air, is to be expected from its magnetic property as regards the cooling in liquid air.*

35 % Nickel Steel. Figs. 52, 53, 54, 55 and 56.

TABLE XXXV.

$T = 135\ gr./mm.^2$

$\theta = 3.25°$		$\theta = 6.3°$		$\theta = 9.35°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
7.4	1.27	8.0	1.06	6.7	0.61
21.5	3.13	21.5	2.63	21.5	2.07
58.1	4.93	57.9	4.36	57.9	3.71
101.1	5.71	101.1	5.05	100.3	4.41
184.6	6.23	185.0	5.60	184.0	4.92
291	6.49	291	5.84	286	5.20
389	6.55	387	5.91	387	5.29
508	6.66	502	6.01	497	5.37

* K. Honda and S. Shimizu, Jour. Sc. Coll. Vol. XX, Art. 6.

$T=1594\ gr./mm.^2$		$T=3060\ gr./mm.^2$		$T=4490\ gr./mm.^2$	
$\theta=3.15°$		$\theta=3.25°$		$\theta=2.85°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
7.1	0.63	6.5	0.31	8.5	0.33
21.5	2.08	21.5	1.65	22.2	1.01
57.8	3.57	57.8	2.89	59.2	1.95
100.2	3.96	100.1	3.43	103.3	2.30
182.8	4.56	183.5	3.90	190.9	2.65
286	4.80	285	4.02	300	2.91
385	4.89	383	4.17	408	2.94
498	5.09	492	4.23	492	2.94

The rigidity considerably increases with the field and tends to an asymptotic value. The effect of tension is regularly to decrease the change of rigidity. The amplitude of twist affects the change similarly as in Swedish iron. The hysteresis-curve also resembles that of the same metal, but is smaller in its area.

45 % Nickel Steel. Figs. 57, 58, 59, 60 and 61.

TABLE XXXVI.

$T=362\ gr./mm.^2$

$\theta=3.25°$		$\theta=6.35°$		$\theta=12.4°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
8.0	−0.49	8.0	−0.42	7.7	−0.23
16.0	−0.41	14.1	−0.33	14.5	−0.22
32.3	+0.32	30.5	+0.23	30.4	+0.15
41.2	+0.63	41.4	+0.61	41.4	+0.52
61.1	+1.40	61.9	+1.29	62.0	+1.08
95.3	+2.21	91.7	+2.25	97.0	+1.91
166.8	+3.10	168.3	+3.24	168.9	+2.95
273	+3.70	273	+3.96	276	+3.65
406	+4.07	406	+4.38	406	+4.05
486	+4.21	486	+4.46	485	+4.14

$T=1390$ gr./mm.^{2}		$T=2623$ gr./mm.^{2}		$T=3895$ gr./mm.^{2}	
$\theta=3.3°$		$\theta=3.1°$		$\theta=3.15°$	
H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$	H	$\frac{\delta K}{K_H}\times 10^2$
4.6	−0.10	4.1	−0.03	4.0	+0.00
8.8	−0.24	8.4	−0.12	9.0	+0.07
14.8	−0.10	—	—	17.7	+0.49
22.3	+0.12	18.4	+0.25	29.5	+1.02
32.4	+0.60	31.1	+0.84	42.6	+1.61
41.4	+1.03	42.8	+1.38	63.4	+2.33
96.0	+2.57	96.7	+2.81	94.2	+2.90
168.0	+3.41	168.7	+3.60	164.5	+3.76
274	+4.01	271	+4.28	269	+4.31
405	+4.17	365	+4.40	391	+4.68
485	+4.29	485	+4.68	474	+4.76

In general features, the change of rigidity is similar to that of the former alloy, but the rate of increase is less for the present alloy than for the former. Under slight tension, we observe, in the alloy, a slight decrease of rigidity in very weak fields. Such an initial decrease is not observable in the former alloy. This decrease of rigidity, however, diminishes with increasing tension, vanishing with a tension of 3895 gr./mm.^{2}

One of the two hysteresis-curves (Fig. 60) refers to the first cycle, while the other (Fig. 61) to a cycle after several cyclic changes of field. The form of the curve closely resembles the inverted form of the hysteresis curve of the magnetic change of length in iron, as discovered by Professor Nagaoka.

70.32 % Nickel Steel. Figs. 62, 63, 64, 65 and 66.

TABLE XXXVII.

$T = 122\ gr./mm.^2$

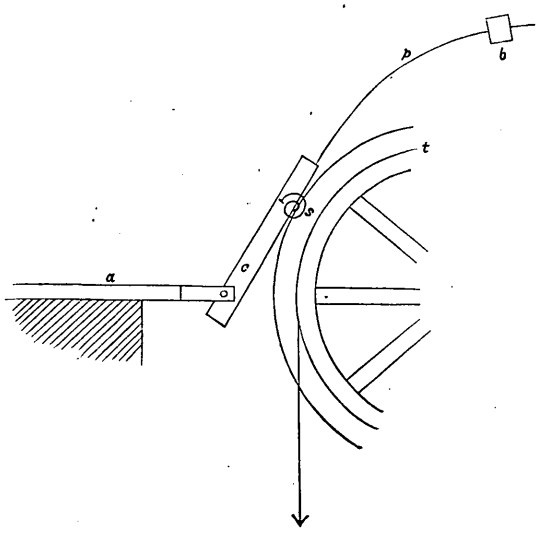
$\theta = 2.95°$		$\theta = 6.35°$		$\theta = 9.4°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
2.6	−0.06	1.9	−0.01	2.2	−0.03
6.3	−0.58	6.7	−0.31	6.3	−0.12
21.6	+0.74	21.1	+0.77	21.1	+0.71
58.5	+2.49	55.7	+2.35	55.7	+2.05
96.5	+3.17	98.0	+3.08	98.0	+2.70
178.0	+3.77	178.3	+3.53	179.4	+3.19
280	+3.95	280	+3.82	280	+3.39
410	+4.08	377	+3.87	378	+3.47
491	+4.10	491	+3.96	492	+3.54

$T = 1440\ gr./mm.^2$		$T = 2760\ gr./mm.^2$	
$\theta = 6.35°$		$\theta = 6.25°$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
2.6	+0.13	3.7	+0.32
6.5	+0.33	6.8	+0.63
21.1	+1.43	21.5	+1.57
55.9	+2.77	56.6	+2.66
98.0	+3.37	99.2	+3.14
179.8	+3.82	181.0	+3.57
279	+4.02	285	+3.82
377	+4.12	387	+3.88
488	+4.22	499	+3.89

In general behaviour, the change of rigidity and its hysteresis resemble those of 45% Ni., except that the amount of the change is a little less than that for this alloy. Comparing the above results with those for well annealed nickel steel of the same percentage, we observe that the effect of annealing remarkably increases the change of rigidity.

(*a*) Measurement of the change of rigidity by ordinary method.

In a paper by Messrs S. Shimizu, S. Kusakabe and one of us,* the change of rigidity of ferromagnetic bars is given, having been determined by first applying the torsion and then the magnetic field. To investigate whether the results are the same or not, if the order of applying the torsion and the field, is reversed, the same arrangement used in the experiment above cited was availed of. The essential parts of the arrangement remained unchanged, except for the mirror system. The rotating cylinder, to which the mirror was fixed, was horizontally supported by means of two very weak spiral springs (*ss*) attached to the sides of a carriage (*c*) similar to that used in our measurement of the change of elasticity. To the carriage, was rigidly fixed a horizontal axis, the conical ends



*) K. Honda, S. Shimizu and S. Kusakabe, loc. cit.

of which fitted to agate cups on the arms of a Y-shaped bar (a). This bar could be adjusted to any desired position, such that the axis of the mirror would be pressed by the plane side of the carriage perpendicularly on a point of the circumference of the torsion wheel (t) rigidly fixed to the specimen. To adjust the pressure properly, a sliding weight (b) was put on a pin (p) protruding from the carriage.

Two vertical scales 30 cm. long were erected at a horizontal distance of 6.45 m. in front of the mirror in the same vertical line; one at the same height as the mirror, and the other about one meter above it. The image of the scales was observed with a telescope at the same height as the mirror. An initial couple was applied and the mirror was so adjusted that the image of the lower scale was observable with the telescope. A suitable weight was then chosen for giving an additional couple, which twisted the rod to such an extent that by the consequent rotation of the mirror, the image of the upper scale just appeared in the field of the telescope. This weight was 500 gr. The readings of the two scales corresponding to the removal and to the addition of the additional weight, were taken. When properly adjusted, their difference was fairly constant for repeated observations; the mean of several observations was always taken as the deflection due to the weight. Next, a field was applied and the procedure repeated; in this way, we obtained the values of deflections corresponding to the different fields, the demagnetization being, of course, carefully made before each experiment. The difference of the deflections in a certain field and in no field gives the change of rigidity.

The sensitiveness of the present arrangement was such that a deflection of 1 mm. of the scale reading corresponded to a twist

of $7.64'' \times 10^{-3}$ per cm. of the specimen. The specimens tested in the present experiment were those used in the previous experiment above quoted, and had the following dimensions:

Metals	Diameter	Length	Rigidity
Soft iron	0.986 cm.	20.43 cm.	7.92×10^{11}
Nickel	1.115	21.42	7.41×10^{11}

On the other hand, the determination of the change of rigidity by the ordinary method of first applying the couple and then the magnetizing field was always made, and the results obtained by the two different methods were compared. The magnetizing coil was the same as that used in the last experiment, so that the heating effect due to the magnetizing current was negligibly small.

In using the present arrangement, all necessary precautions were taken, which were referred to in the paper above quoted. The results are given in the following:—

Nickel. Figs. 67 and 68.

TABLE XXXVIII.

$t = 14°.7$ C.

$N=7.00\times10^6$ $\theta=12.83''$		$N=14.09\times10^6$ $\theta=25.83''$		$N=21.27\times10^6$ $\theta=38.98''$	
H	$\frac{\delta K'}{K_H}\times10^2$	H	$\frac{\delta K'}{K_H}\times10^2$	H	$\frac{\delta K'}{K_H}\times10^2$
6.3	+0.05	6.3	+0.07	5.6	+0.05
14.5	+0.12	13.5	+0.12	13.4	+0.11
20.5	−0.02	20.5	−0.01	22.0	−0.07
28.4	−0.34	26.7	−0.31	26.1	−0.24
34.9	−0.65	35.0	−0.68	33.8	−0.60
56.0	−1.37	55.7	−1.37	55.7	−1.24
82.8	−1.35	83.5	−1.33	81.0	−1.20
114.0	−0.60	115.8	−0.59	114.0	−0.50
164.8	+1.15	167.0	+1.16	164.8	+1.14
236	+3.40	238	+3.39	236	+3.22
288	+4.59	292	+4.61	289	+4.34
359	+5.81	365	+5.87	362	+5.57
407	+6.38	412	+6.48	410	+6.17
519	+7.39	523	+7.53	518	+7.21

$\Delta N = \pm 3.50 \times 10^6$, $t = 16°.7$ C.

$N = 7.00 \times 10^6$ $\theta = 12.83''$		$N = 14.09 \times 10^6$ $\theta = 25.83''$	
H	$\frac{\delta K}{K_H} \times 10^2$	H	$\frac{\delta K}{K_H} \times 10^2$
—	—	68	−0.08
15.1	−0.19	15.4	−0.31
22.2	−0.55	22.2	−0.57
48.2	−1.65	48.5	−1.60
82.4	−2.35	82.4	−2.29
127.1	−2.56	127.5	−2.38
192.5	−1.94	193.2	−1.64
288	−0.67	286	−0.34
398	+0.59	386	+0.59
508	+1.37	495	+1.54

In the above tables, N is the moment of force applied in C.G.S. units, and θ the corresponding twist of the specimen as calculated from the modulus of rigidity. The change of rigidity obtained by our direct method is denoted by $\frac{\delta K}{K_H}$, while that obtained by the indirect method, by $\frac{\delta K'}{K_H}$. The values of $\frac{\delta K'}{K_H}$ fairly coincide with those obtained by the previous experiment; but it is to be noticed that the field here given is not the effective one, but the external field applied. The difference between $\frac{\delta K}{K_H}$ and $\frac{\delta K'}{K_H}$ is remarkable; its amount is of the same order of magnitude as the change itself. The increase of the angle of twist slightly affects the change of rigidity, always diminishing it in absolute amount. The general feature of $\frac{\delta K}{K_H}$ is quite similar to that for nickel wire with weak tension, as obtained by the oscillation method.

It is curious to observe that the residual twist is always positive and uniformly increases with the field, as shown in Fig. 68.

It may be noticed that the course of the curve of $\frac{\delta K'}{K_H}$ close-

ly resembles that of $\frac{\delta E}{E}$ as given by the flexure method. In both cases, the right order of applying the stress and the field is inverted.

Soft Iron. Figs. 69 and 70.

TABLE XXXIX.

$t = 19°.0$ C.

$N = 7.00 \times 10^6$ $\theta = 19.65''$		$N = 14.09 \times 10^6$ $\theta = 39.56''$		$N = 21.27 \times 10^6$ $\theta = 59.70''$	
H	$\frac{\delta K'}{K_H} \times 10^2$	H	$\frac{\delta K'}{K_H} \times 10^2$	H	$\frac{\delta K'}{K_H} \times 10^2$
7.3	0.01	8.4	0.02	9.3	0.01
22.0	0.07	22.0	0.07	22.0	0.06
40 0	0.11	39.6	0.14	39.6	0.12
73.3	0.23	73.3	0.27	72.6	0.25
111.3	0.63	111.0	0.68	110.6	0.65
160.5	0.95	160.5	1.03	160.5	1.02
252	1.34	252	1.42	252	1.43
322	1.51	315	1.58	315	1.60
396	1.66	395	1.73	396	1.75
511	1.79	508	1.87	509	1.87

Since the time of the previous experiment, the specimen had been once annealed at a temperature of about 1100° C, so that the change of rigidity $\frac{\delta K'}{K_H}$ was somewhat increased in the present experiment. The effect of the amplitude of twist on the change of rigidity is very small; the increase of the twist slightly augments the change of rigidity up to the range of the twist used. The residual twist is always positive as in the case of nickel. $\frac{\delta K}{K_H}$ is always inappreciable in the present arrangement; if it exist at all, it can not be greater than 0.05 %, for the maximum field used.

Thus, in nickel and iron rods, $\frac{\delta K'}{K_H}$ is always far greater than

$\frac{\delta K}{K_H}$, so that relation (3) can not be satisfied even approximately.

In concluding the descriptions of the results of our experiments, the following remarks may be added. Since we measured the changes of elasticity and of rigidity for the same wires of ferromagnetic metals and alloys, it will be now interesting to calculate the change of Poisson ratio by magnetization. Since the change of elastic constants is considerably affected by tension, it is necessary to calculate the change of the ratio from the changes of elasticity and rigidity for the same tension.

If σ be the Poisson ratio, we have

$$\frac{\delta\sigma}{\sigma} = \frac{\frac{E}{K}}{\frac{E}{K} - 2}\left(\frac{\delta E}{E} - \frac{\delta K}{K}\right).$$

The value of $\frac{E}{K}$ used in the calculation of the following table is that for no field; and the values of $\frac{\delta E}{E}$ and $\frac{\delta K}{K}$ are those obtained by the tension effect of the magnetic elongation and by the Barus's method respectively, for in these two sets of experiments, the stress was first applied and then the magnetizing field.

TABLE XL.

Specimen.	Nickel.	Swedish iron.	Tungsten steel.	28.74% Ni.	50.72% Ni.	70.32% Ni.
Tension.	3020 *gr.*	3270 *gr.*	3320 *gr.*	1770 *gr.*	3350 *gr.*	3280 *gr.*
H	$\frac{\delta\sigma}{\sigma} \times 10^2$	$\frac{\delta\sigma}{\sigma} \times 10^2$	$\frac{\delta\sigma}{\sigma} \times 10^2$	$\frac{\delta\sigma}{\sigma} \times 10^2$	$\frac{\delta\sigma}{\sigma} \times 10_2$	$\frac{\delta\sigma}{\sigma} \times 10^2$
10		− 2.2			− 0.3	− 1.6
20		− 3.8		−1.0		
30	− 6.1		0.0		− 1.9	
100	+37	− 7.4	0.0	−1.3	− 8.3	−14.5
150	+50					
200	+23	− 9.0	0.1	−1.3	−10.1	−13.5
250	+ 7					
300			0.3	−1.1		−13.8
350		−10.2			−11.1	

From the above table, it will be seen that the Poisson ratio is generally diminished by magnetization. In nickel, it is very large amounting to even 50 per cent; but in tungsten steel, it is almost zero, indicating a tendency to increase. The change of the Poisson ratio also considerably varies with tension, and the above table shows only an example of the change.

§ 4. CONCLUDING REMARKS.

The results obtained in the present investigation may be summarised as follows:—

(i) In Swedish iron and tungsten steel (Figs. 71 and 72), the change of elastic constants is generally positive, but its amount is extremely small, less than 0.5%. The change by the indirect method is several times greater than that by the direct method. It is interesting to notice that the change of elasticity and that of rigidity almost coincide with each other, not only in their general aspects, but also in their quantitative relations.

(ii) In nickel (Fig. 73), the change of elastic constants is remarkably large, amounting to about 15% in the change of elasticity and 7% in the case of rigidity. The elastic constants first decrease and then increase, as the field becomes greater. The elastic constants by the indirect method are numerically greater than those by the direct method.

(iii) In 28.74% Ni. (Fig. 74), magnetization increases the elastic constants by a small amount. The changes given by the two different methods nearly coincide with each other for a moderate tension. In 50.72 % Ni. and 70.32 % Ni. (Figs. 75 and 76), the increase of the elastic constants is remarkably large, approaching in amount the change of rigidity in nickel. Except with a

very weak tension, the change by the indirect method is greater than that by the direct; but this difference becomes less as the tension is increased. With a tension of 3 or 4 kg. per square millimeter, it nearly vanishes.

(iv) In Fig. 77, curves showing the relation between the change of rigidity (by indirect method) and the percentage content of nickel in nickel steels are given. These curves correspond to the change of rigidity in a slightly annealed state. They show a marked maximum at about 50% Ni., and a minimum at about 24.40 % Ni.

From the results above given, it is evident that there are some cases in which, relation (3) given in the earlier part of this paper, does not hold even approximately, so that equation (1) can not be freely used in any quantitative discussion.

There are many analogous cases in the problem of magnetostriction. As will be seen from our subsequent paper to be published presently, the change of magnetization by stretching a magnetized wire does not agree with the change deduced from the results obtained by magnetizing the wire in the unstrained as well as the stretched state. Similar phenomena are also observable in the change of magnetism caused by the twist. Next, we may cite the case of the Wiedemann effect. It is well known that in iron and nickel, the twist produced by magnetizing the wire traversed by an electric current is generally greater than the twist caused by passing the current through the magnetized wire. The difference is remarkable; in some cases,* the former is several times greater than the latter. In nickel steels,† the difference is, however, very small. Again, take the case of magnetizing a

*) and †) K. Honda and S. Shimizu, Jour. Sc. Coll., **16**, Art. 14.

wire traversed by an electric current.* The change of magnetization by the longitudinal current is considerably greater, in the case of magnetizing the wire traversed by the current, than in the case of passing the current through the magnetized wire. All these phenomena may perhaps arise from the hysteresis effect of magnetization; i.e. the final states attained by a magnetic substance differ according to the order of applying the field and the stress, or the field and the longitudinal current.

In conclusion, it may be remarked that, since the change of the elastic constants by magnetization is not so small as is generally believed, in any theory of magnetostriction aiming at the quantitative agreement between the theory and the experiment, these changes must necessarily be taken into account. In addition to this, an equality such as

$$\frac{\partial^2 Q}{\partial x \partial y} = \frac{\partial^2 Q}{\partial y \partial x},$$

in which Q is a quantity which depends upon two apparently independent variables x and y, can not be used without experimental verification. These facts make the development of the theory very difficult.

In passing, the following remarks may be added. If we consider, in the above equation, Q as the length of a specimen, x the temperature and y the magnetic force, we have

$$\frac{\partial}{\partial H}\left(\frac{\partial l}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial l}{\partial H}\right),$$

provided l is independent of the order of applying the field and the temperature. Integrating, we get

$$\left(\frac{\partial l}{\partial t}\right)_H - \left(\frac{\partial l}{\partial t}\right)_0 = \frac{\partial (l_H - l_0)}{\partial t}.$$

*) K. Honda, Ibid. **11**, p. 284, 1899.

If α be the coefficient of thermal expansion,

$$\alpha_H - \alpha_0 = \delta\alpha = \frac{\partial(l_H - l_0)}{l_0 \partial t}.$$

Thus the change of the coefficient of thermal expansion by magnetization is equal to the temperature coefficient of the magnetic elongation. As the latter coefficient is known from the experiment* by Mr. S. Shimizu and one of us, the values of $\delta\alpha$ are calculated and graphically drawn in Figs. 78, 79, 80, 81 and 82.

By referring to the figures, we see that the change of the coefficients of thermal expansion by magnetization depends considerably upon temperature. Ordinates of the curves represent the change of the mean coefficient of expansion between two temperatures belonging to each curve.

The change of the mean coefficient of expansion in nickel (Fig. 78) between the ordinary and liquid air temperatures first decreases, attains a minimum, and then gradually increases, as the field becomes greater, till it is greater than its initial value. At a temperature higher than the ordinary, the change of the coefficient of expansion steadily increases, soon approaching an asymptotic value. In a given field, its value increases with temperature, and after passing through a maximum, slightly decreases. The maximum amount of the change is of the order of 1% of the coefficient itself.

In soft iron and tungsten steel (Figs. 79 and 80), the change of expansion is very small. Up to a moderate temperature, the coefficient of expansion increases steadily with the field, except in weak fields, in which a small decrease is observed. At higher temperatures, the change becomes negative for all fields. In iron, a maximum decrease is observed.

*) K. Honda and S. Shimizu, Jour. Sc. Coll., XX, Art. 10, 1903.

In cast cobalt (Fig. 81), the change of the coefficient of expansion at low temperature increases with the field, reaches a maximum, and then decreases. As the temperature becomes higher, the amount of the maximum gradually lessens; the position of the maximum shifts towards lower fields and the curve at last cuts the zero line. At a sufficiently high temperature, the change of expansion is always negative.

The change of the coefficient of expansion in annealed cobalt (Fig. 82) is rather abnormal, and its maximum amount is considerably large, being about 2% of the coefficient itself. The change of the mean coefficient of expansion between the ordinary and liquid air temperatures steadily decreases with the field, though its amount is small. As the temperature rises, the amount of the diminution increases rapidly and then decreases. Here the course of the curve shows a minimum decrease of the change. As the temperature is further increased, a considerable amount of increase is observed, which steadily increases with field. It attains a maximum and again decreases, till it changes its sign.

The values of $\delta\alpha$ thus obtained may possibly differ from the values obtained by heating the specimens in a constant magnetic field. They are the changes of α by magnetization, when the temperature is first raised and then the field applied. However, any experiment for the determination of the change of α, in which the thermal expansion is directly observed in magnetic field, must be welcomed, as affording a counterpart for the analogous comparison.

Again, if we put, in the last equation, the tension T per unit area for H, we get

$$\left(\frac{\partial l}{\partial t}\right)_T - \left(\frac{\partial l}{\partial t}\right)_0 = \frac{\partial (l_T - l_0)}{\partial t},$$

or $$a_T - a_0 = \delta a = \frac{\partial (l_T - l_0)}{l_0 \partial t};$$

but since

$$\frac{l_T - l_0}{l_0} = \frac{T}{E},$$

we have finally

$$\frac{\delta a}{T} = \frac{\partial \frac{1}{E}}{\partial t} = -\frac{1}{E^2}\frac{\partial E}{\partial t}.$$

Thus the effect of tension upon the coefficient of thermal expansion can be found from the temperature coefficient of the modulus of elasticity. The relation was first obtained by Dahlander,* and verified by him to agree well with the experiment. In problems which do not relate to magnetism, the hysteresis effect is generally very small, so that the agreement might have been expected.

*) Dahlander, Pogg. Ann. **145**, p. 147, 1872.

Fig. 1. Pure Nickel.

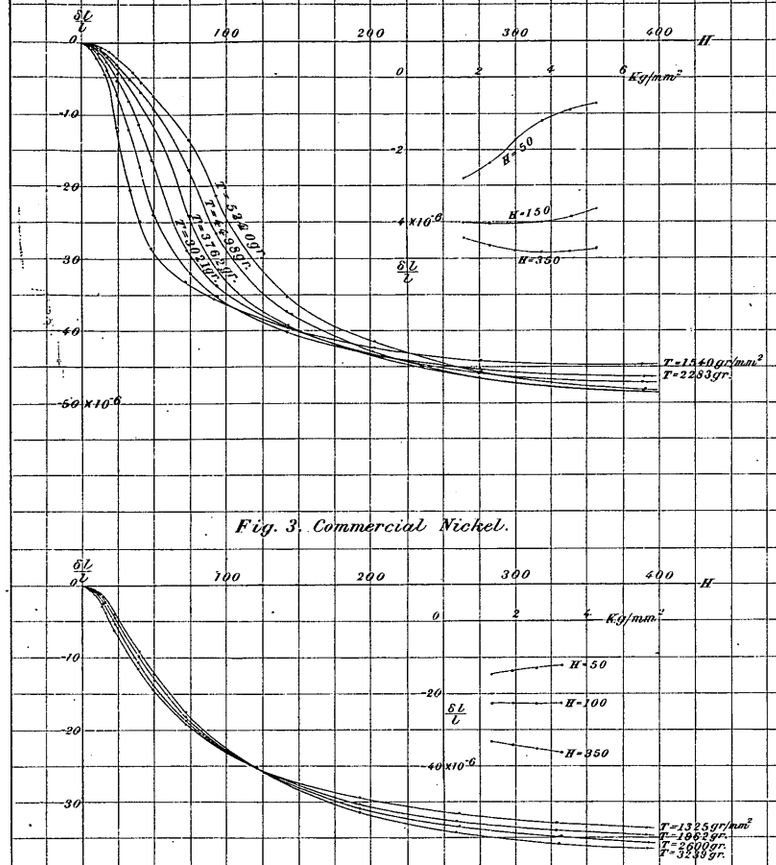


Fig. 2. Pure Nickel.

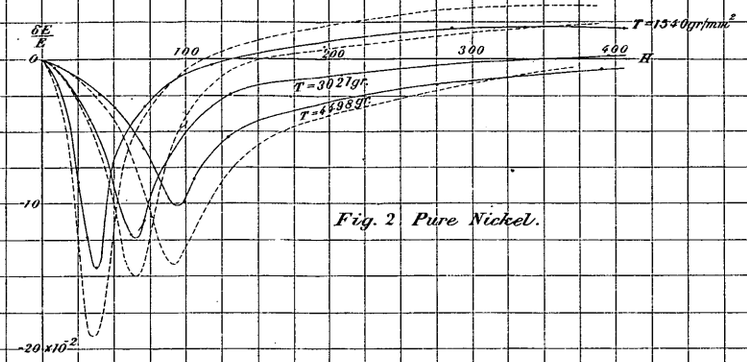


Fig. 3. Commercial Nickel.

Fig. 4. Commercial Nickel.

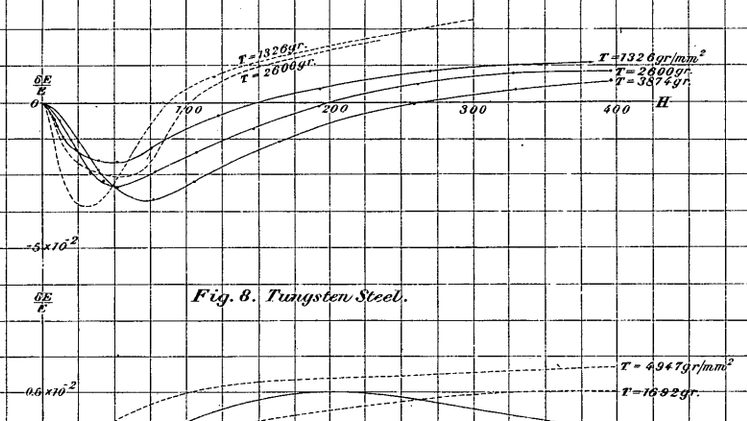


Fig. 5. Swedish Iron.

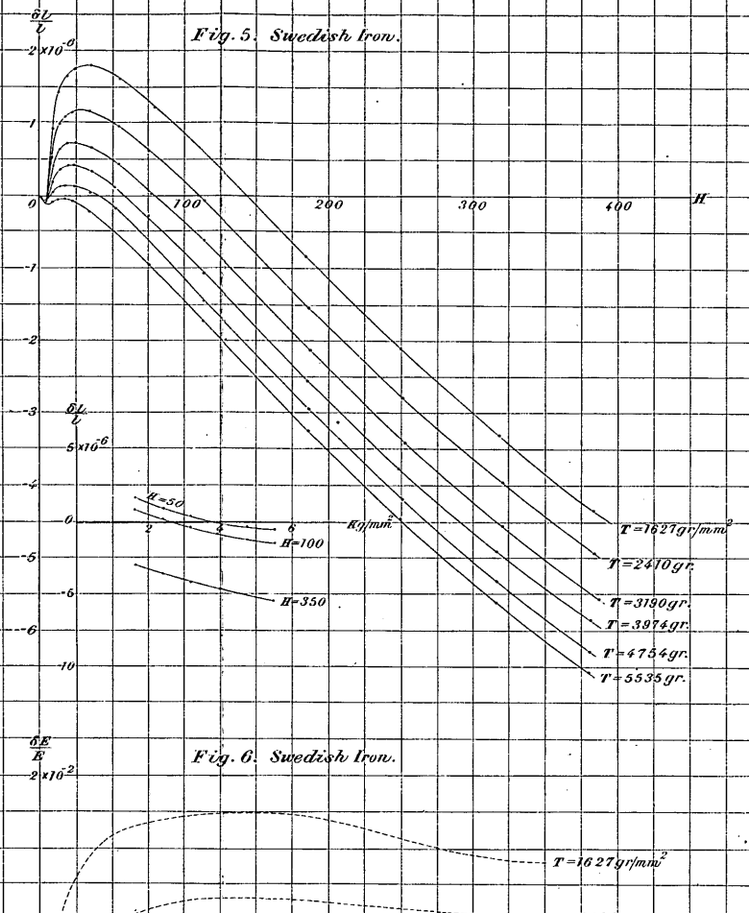


Fig. 6. Swedish Iron.

Fig. 7. Tungsten Steel.

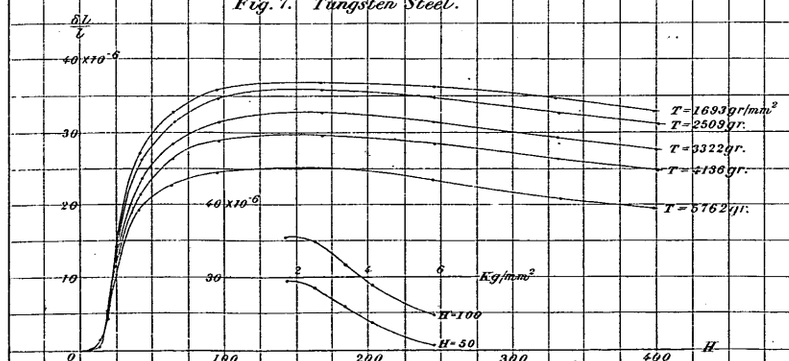


Fig. 8. Tungsten Steel.

Fig. 9. 28.74% Ni.

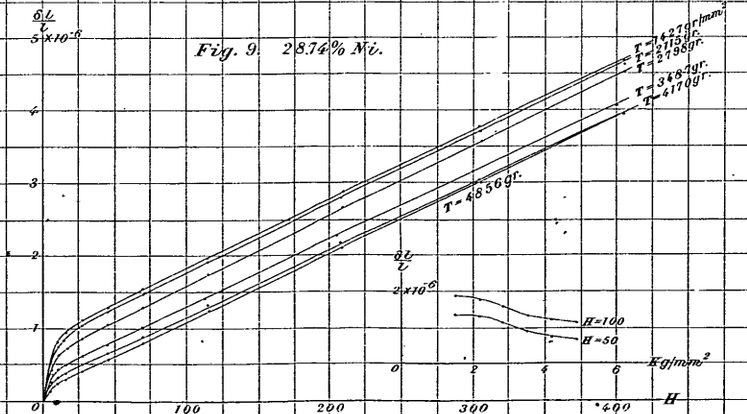


Fig. 10. 28.74% Ni.

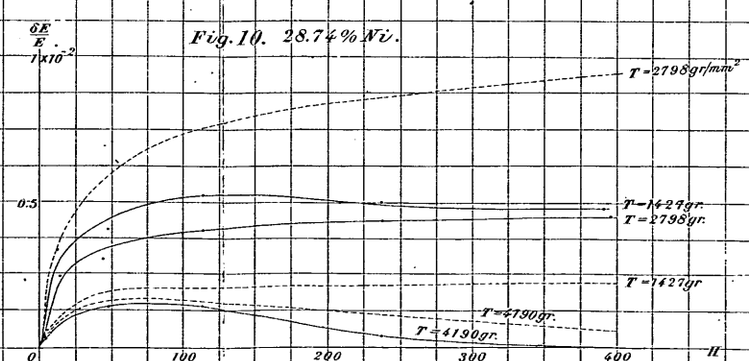


Fig. 11. 50.72% Ni.

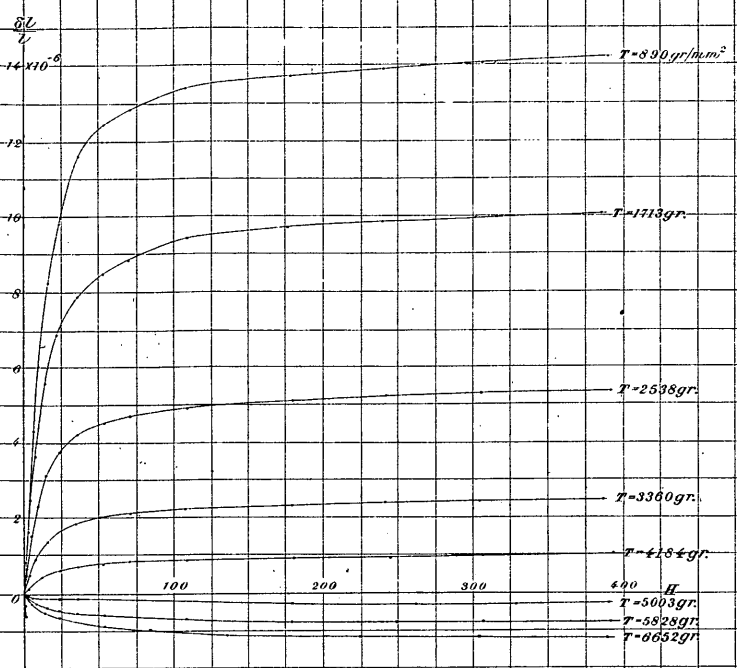


Fig. 15. 70.32% Ni.

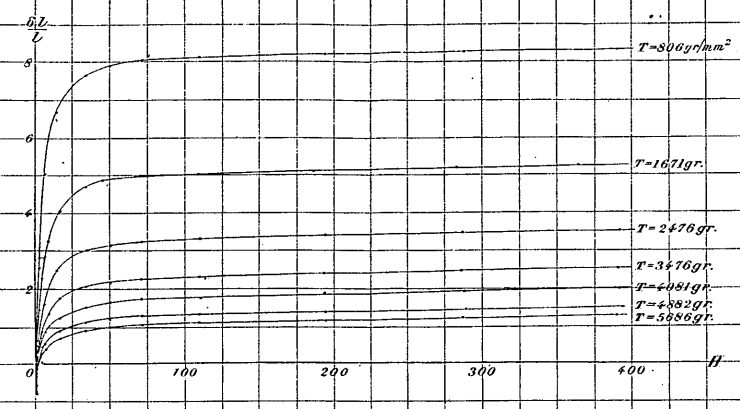


Fig. 17. 70.32% Ni.

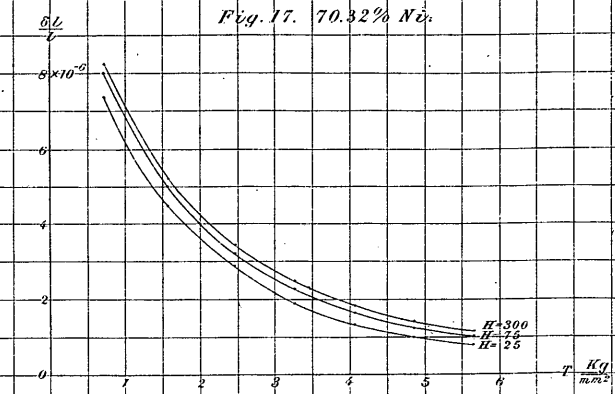


Fig. 12.

50.72% Ni.

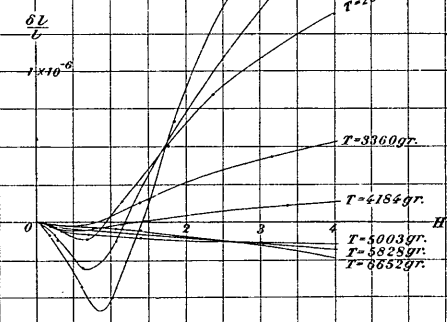


Fig. 19.

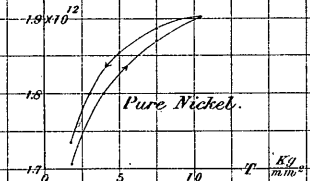


Fig. 13. 50.72% Ni.

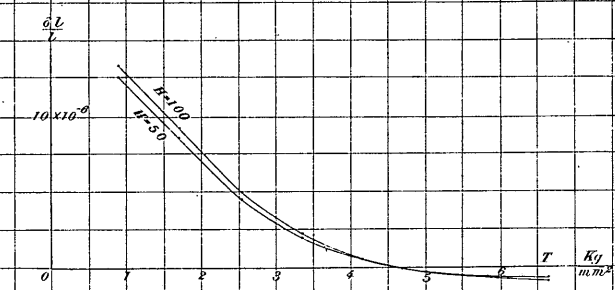


Fig. 14. 50.72% Ni.

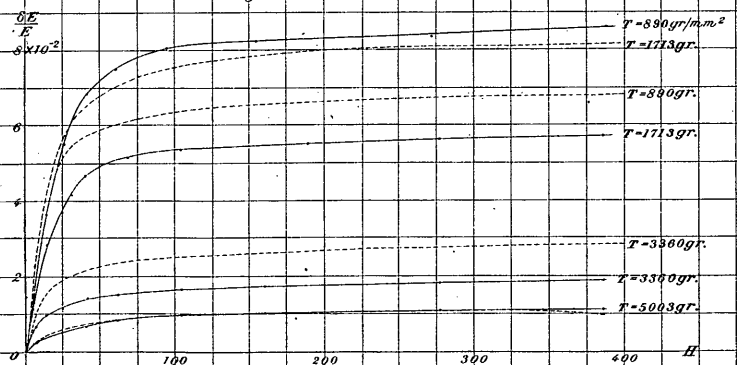


Fig. 16.

70.32% Ni.

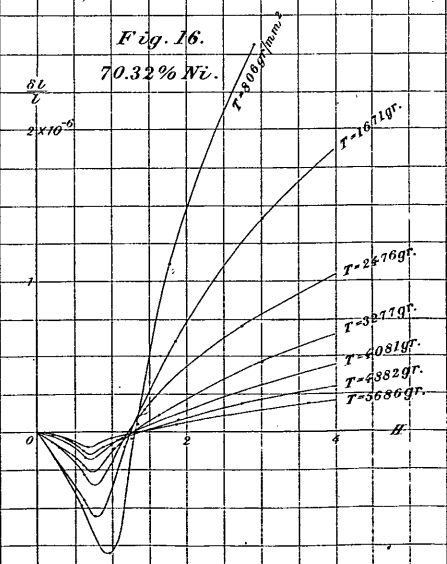


Fig. 20.

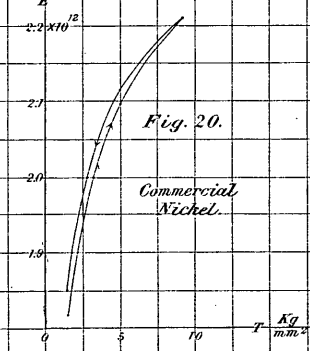


Fig. 22.

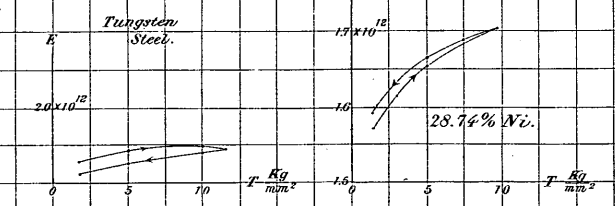


Fig. 23.

Fig. 18. 70.32% Ni.

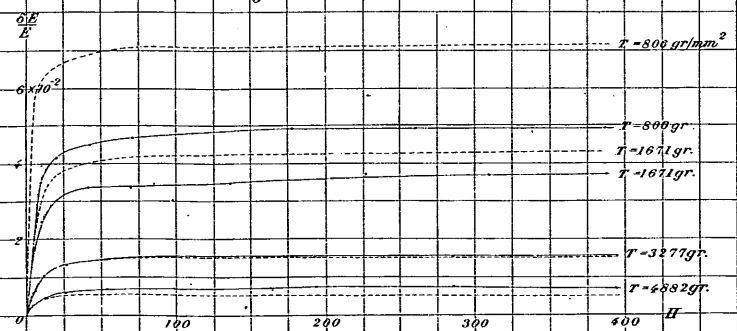


Fig. 21.

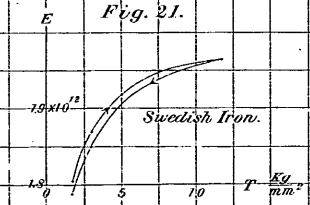


Fig. 25.

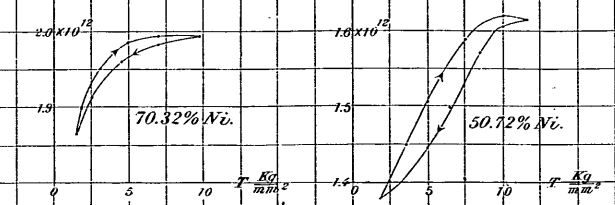


Fig. 24.

Fig. 26. Soft Iron.

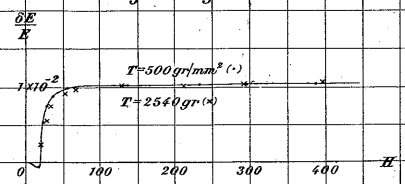


Fig. 27. Tungsten Steel.

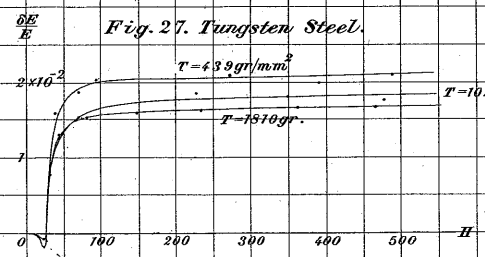


Fig. 28. Nickel.

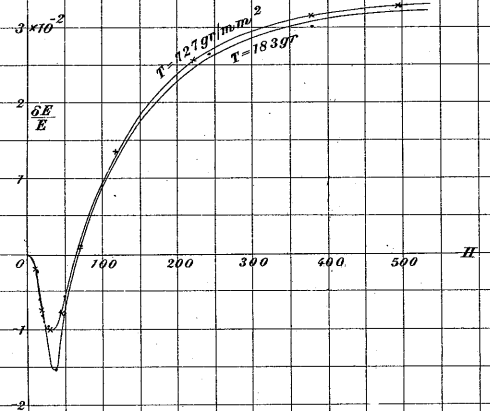


Fig. 29. Nickel.

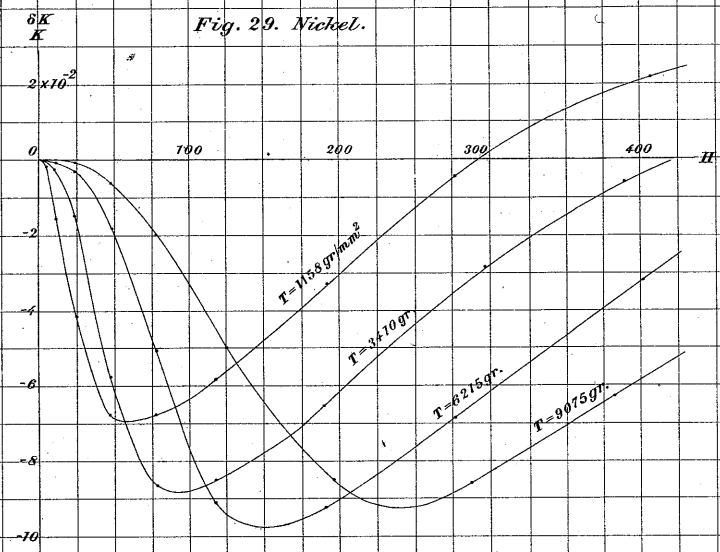


Fig. 30. Swedish Iron.

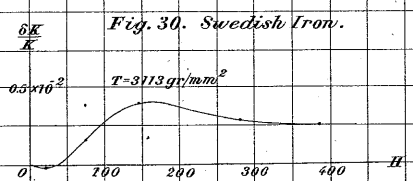


Fig. 31. Tungsten Steel.

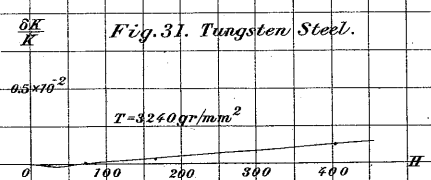


Fig. 33. 50.72% Ni.

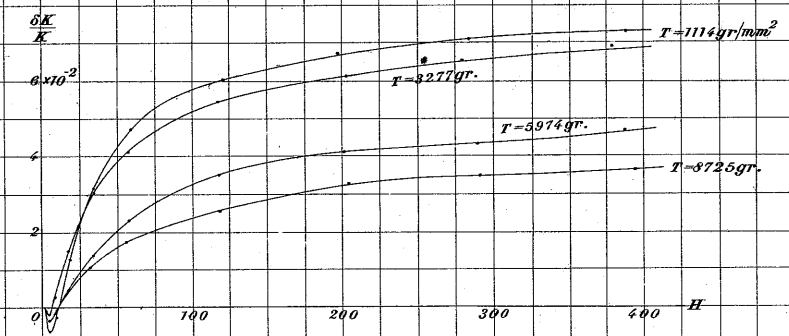


Fig. 34. 70.32% Ni.

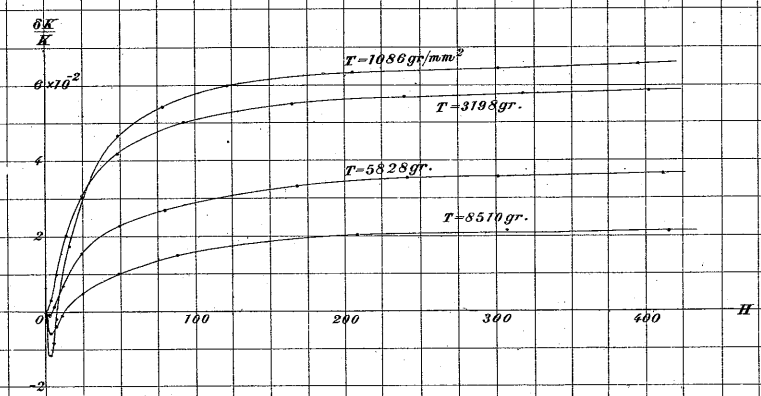


Fig. 35. Pure Nickel.

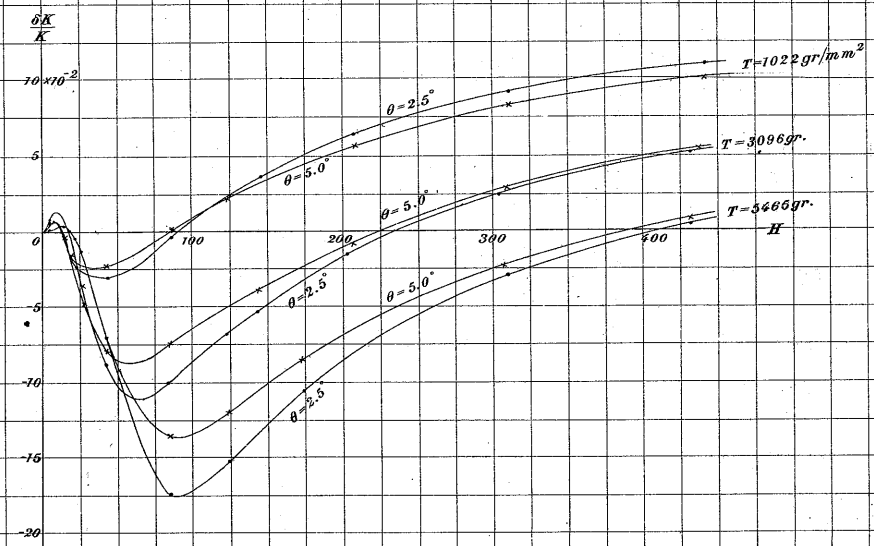


Fig. 32. 28.74% Ni.

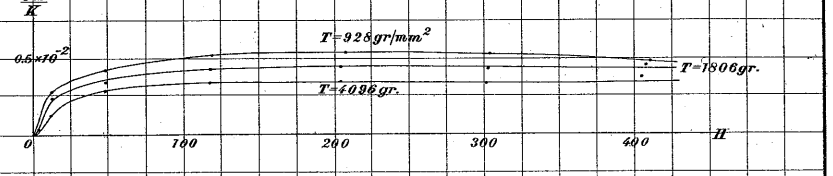


Fig. 36. Swedish Iron.

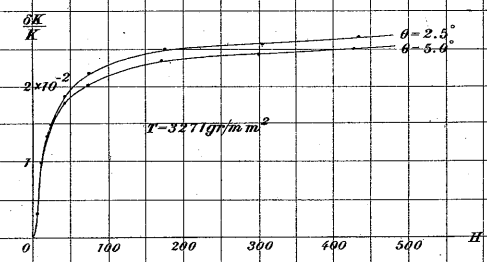


Fig. 37. Tungsten Steel.

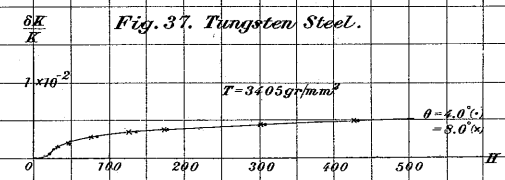


Fig. 38. 28.74% Ni.

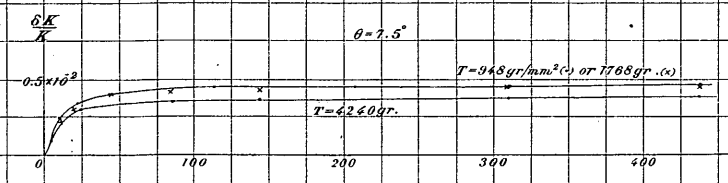


Fig. 39. 50.72% Ni.

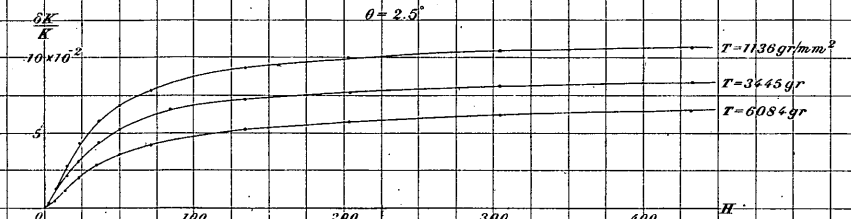


Fig. 40. 70.32% Ni.

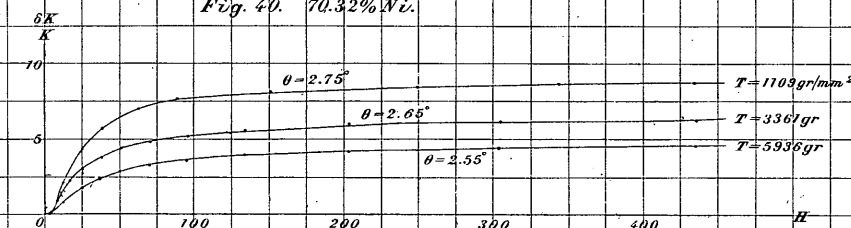


Fig. 41. Swedish Iron.

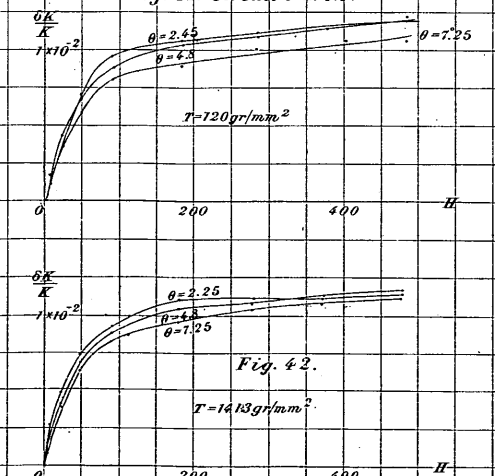


Fig. 42.

Fig. 43. Swedish Iron.

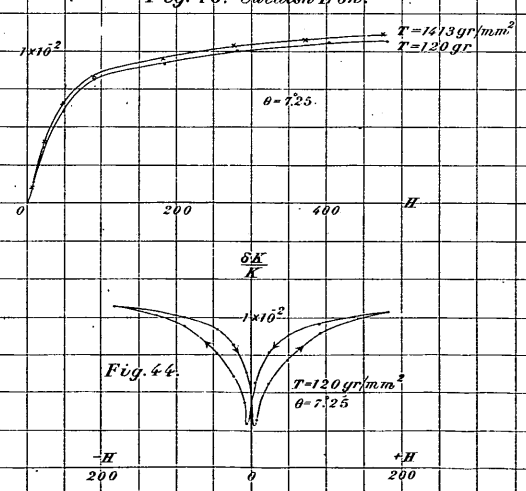


Fig. 44.

Fig. 45. 23.6% Nickel Steel.

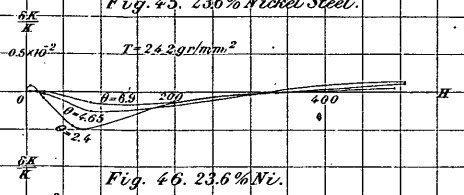


Fig. 46. 23.6% Ni.

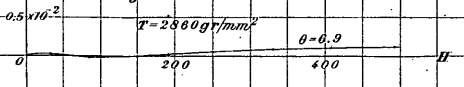


Fig. 47. 24.04% Ni.

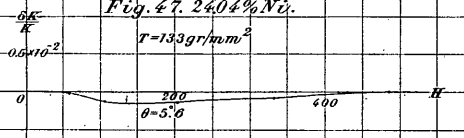


Fig. 48. 24.04% Ni.

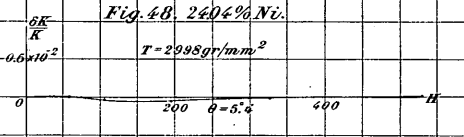


Fig. 49. 26.64% Ni.

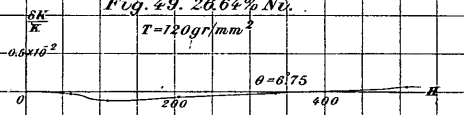


Fig. 50. 26.64% Ni.

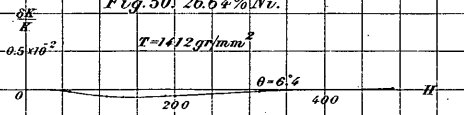


Fig. 51. 26.64% Ni.

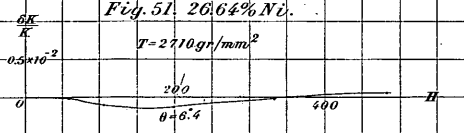


Fig. 52. 35% Ni.

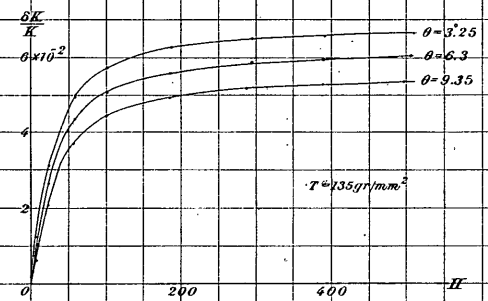


Fig. 53. 35% Ni.

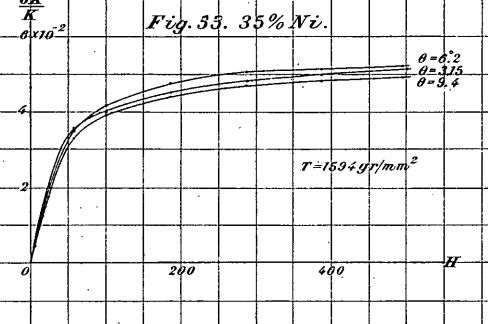


Fig. 54. 35% Ni.

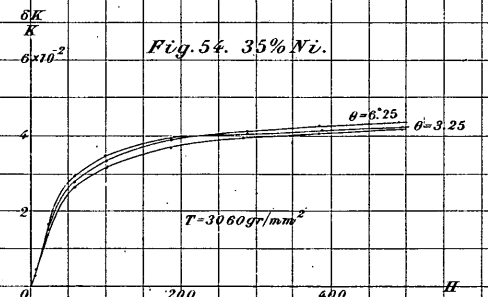


Fig. 55. 35% Ni.

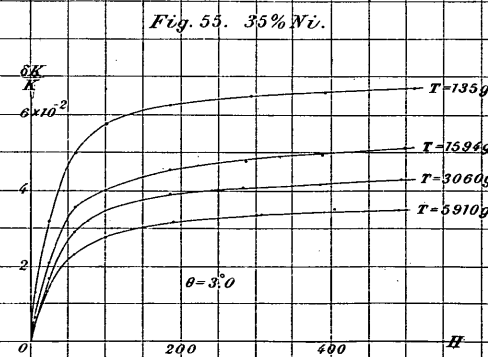
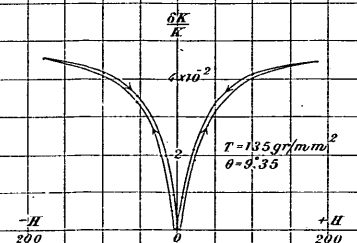


Fig. 56. 35% Ni.



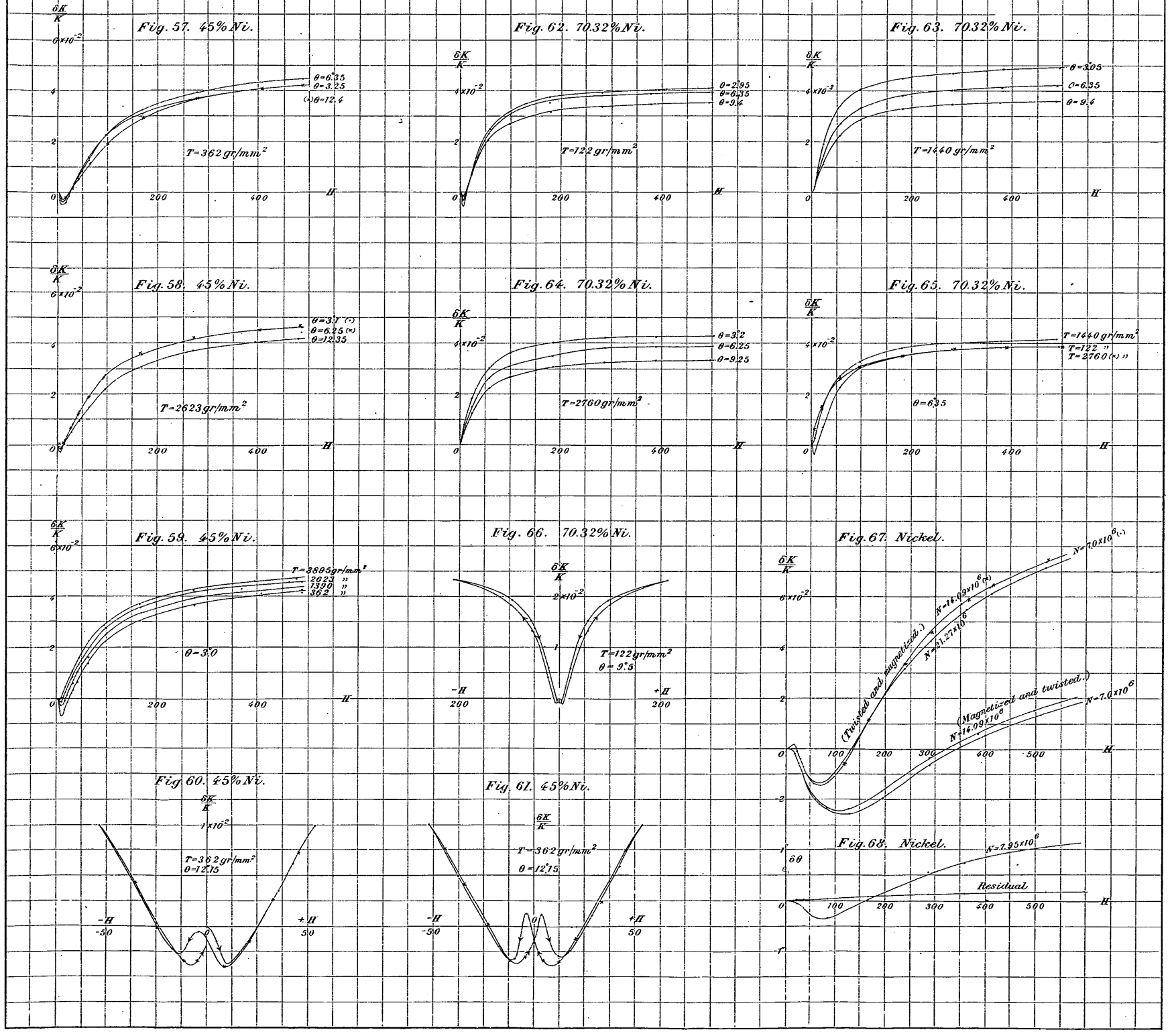
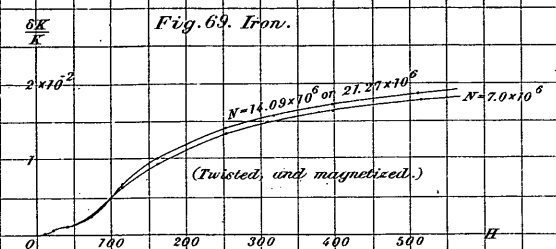


Fig. 57. 45% Ni.
Fig. 62. 70.32% Ni.
Fig. 63. 70.32% Ni.
Fig. 58. 45% Ni.
Fig. 64. 70.32% Ni.
Fig. 65. 70.32% Ni.
Fig. 59. 45% Ni.
Fig. 66. 70.32% Ni.
Fig. 67. Nickel.
Fig. 60. 45% Ni.
Fig. 61. 45% Ni.
Fig. 68. Nickel.
δK/K
H
T=362 gr/mm²
T=122 gr/mm²
T=1440 gr/mm²
T=2623 gr/mm²
T=2760 gr/mm²
T=3895 gr/mm²
θ=3.0
θ=9.5
θ=12.15
(Twisted and magnetized.)
(Magnetized and twisted.)
Residual
δθ


δK/K
Fig. 69. Iron.
2×10⁻²
N=14.09×10⁶ or 21.27×10⁶
N=7.0×10⁶
(Twisted and magnetized.)
0
100
200
300
400
500
H

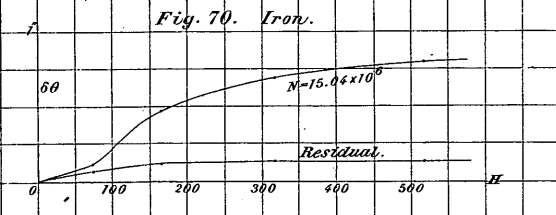
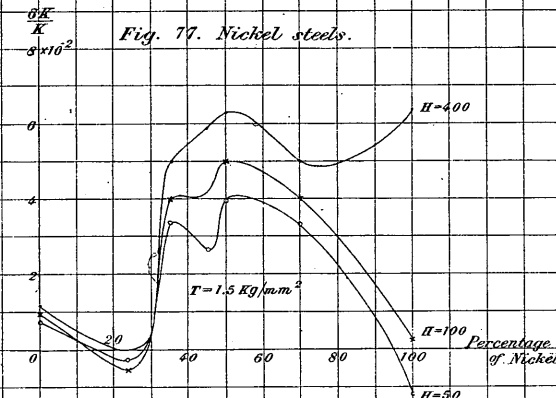


Fig. 70. Iron.
δθ
N=15.04×10⁶
Residual.
0
100
200
300
400
500
H


δK/K
Fig. 77. Nickel steels.
8×10⁻²
H=400
T=1.5 Kg/mm²
H=100
H=50
Percentage of Nickel.
0
20
40
60
80
100

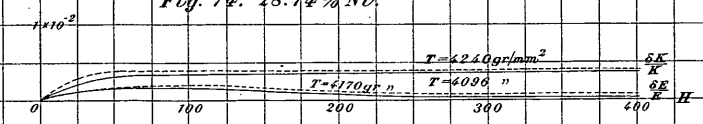


Fig. 74. 28.74% Ni.
1×10⁻²
T=4240gr/mm²
T=4170gr "
T=4096 "
δK/K
δE/E
0
100
200
300
400
H

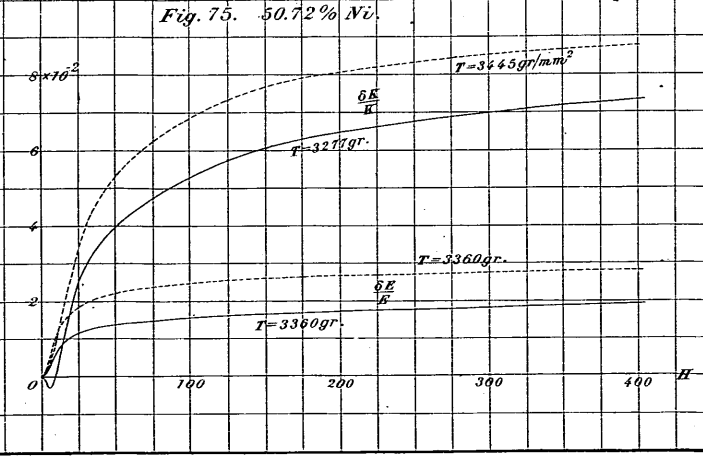


Fig. 75. 50.72% Ni.
8×10⁻²
T=3445gr/mm²
δK/K
T=3277gr.
T=3360gr.
δE/E
T=3360gr.
0
100
200
300
400
H

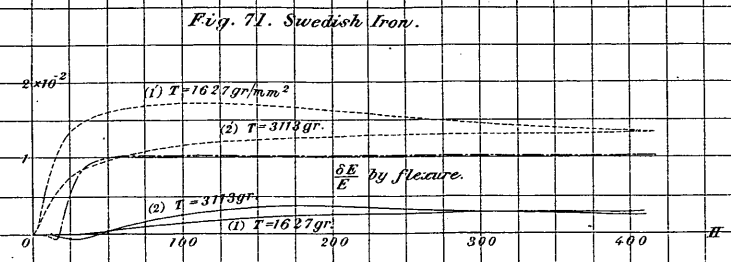


Fig. 71. Swedish Iron.
2×10⁻²
(1) T=1627gr/mm²
(2) T=3113gr.
δE/E by flexure.
(2) T=3113gr.
(1) T=1627gr.
0
100
200
300
400
H

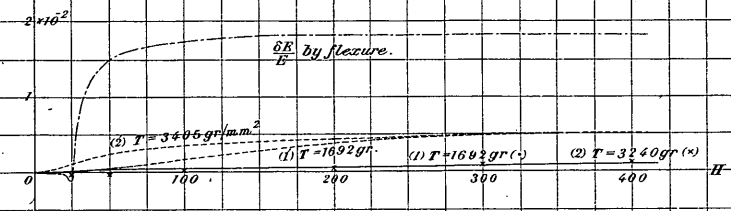


Fig. 72. Tungsten Steel.
2×10⁻²
δK/E by flexure.
(2) T=3405gr/mm²
(1) T=1692gr.
(1) T=1692gr (·)
(2) T=3240gr (×)
0
100
200
300
400
H

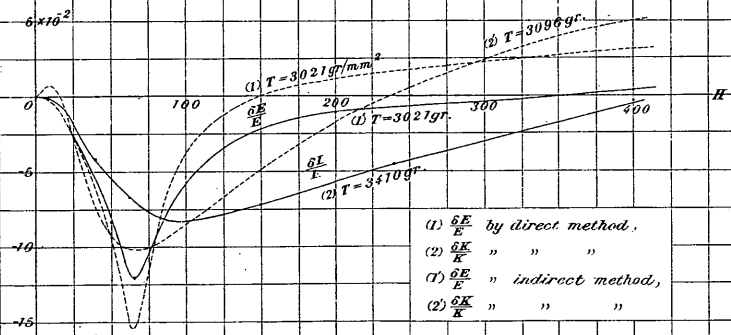


Fig. 73. Nickel.
6×10⁻²
(2) T=3096gr.
(1) T=3021gr/mm²
(1) T=3027gr.
δE/E
δL/L
(2) T=3410gr.
(1) δE/E by direct method,
(2) δK/K " " "
(1') δE/E " indirect method,
(2') δK/K " " "
0
-5
-10
-15
100
200
300
400
H

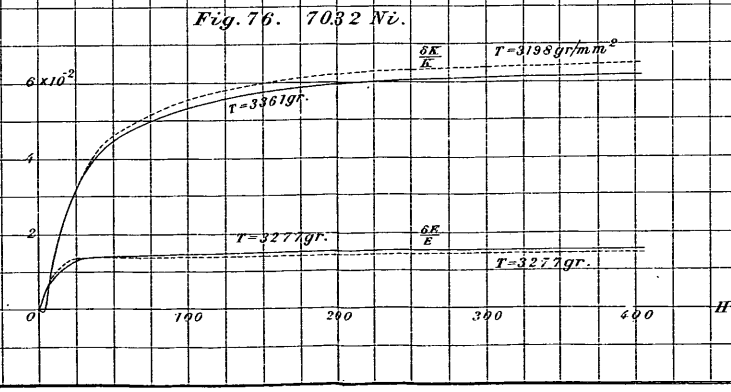


Fig. 76. 7032 Ni.
6×10⁻²
δK/K
T=3198gr/mm²
T=3361gr.
T=3277gr.
δE/E
T=3277gr.
0
100
200
300
400
H

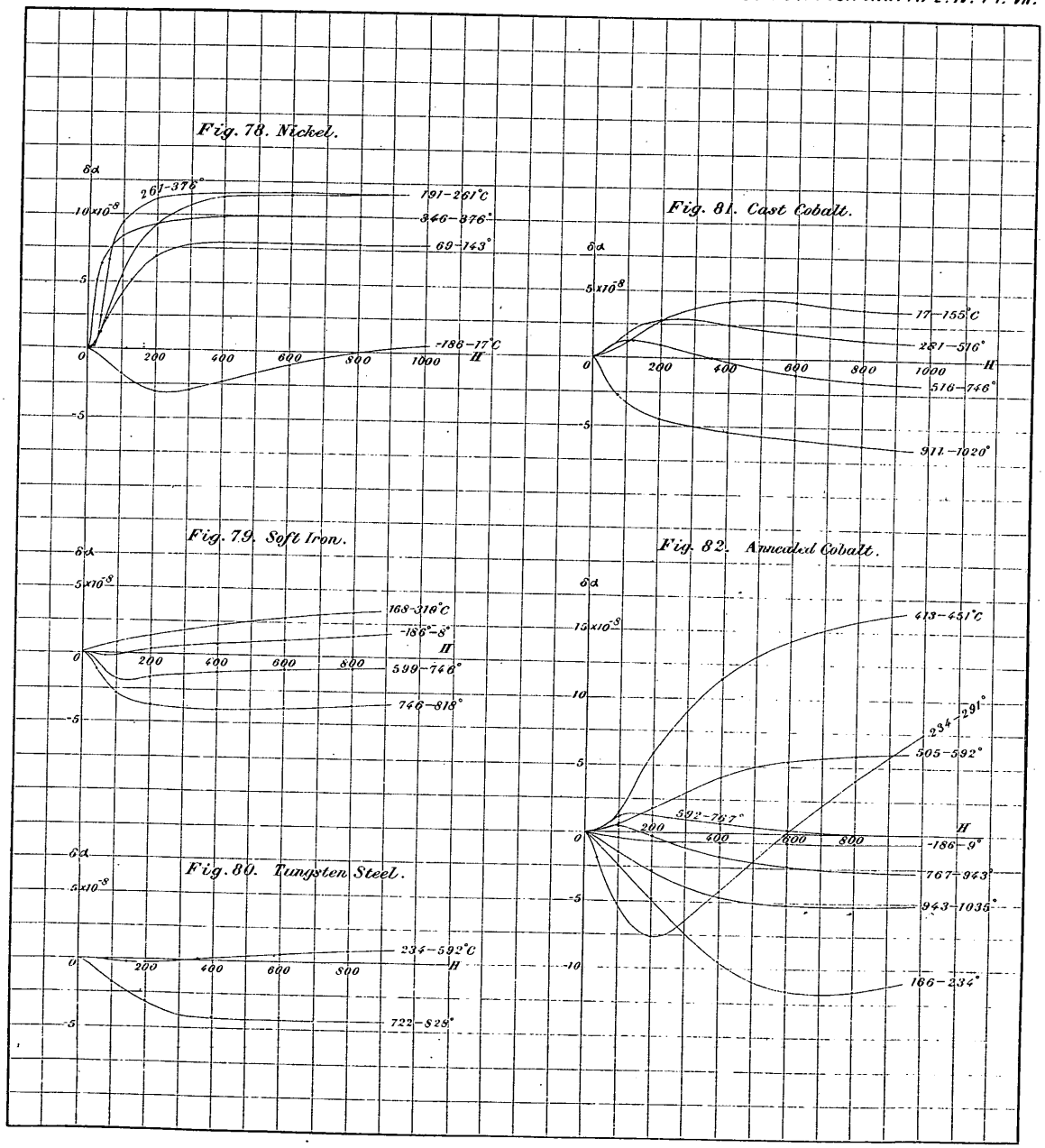


Fig. 78. Nickel.
δd
10×10⁻⁸
5
0
-5
200
400
600
800
1000
H
267-376°
791-261°C
346-376°
69-143°
-186-17°C
Fig. 79. Soft Iron.
δd
5×10⁻⁸
0
-5
200
400
600
800
H
168-319°C
-186°-8°
599-746°
746-818°
Fig. 80. Tungsten Steel.
δd
5×10⁻⁸
0
-5
200
400
600
800
H
234-592°C
722-828°
Fig. 81. Cast Cobalt.
δd
5×10⁻⁸
0
-5
200
400
600
800
1000
H
17-155°C
281-516°
516-746°
911-1020°
Fig. 82. Annealed Cobalt.
δd
15×10⁻⁸
10
5
0
-5
-10
200
400
600
800
H
413-451°C
234-291°
505-592°
592-767°
-186-9°
767-943°
943-1035°
166-234°

CORRIGENDA.

Page 1, *for* plate I–X *read* plate I–VII.