JOURNAL OF THE COLLEGE OF SCIENCE, IMPERIAL UNIVERSITY,
TOKYO, JAPAN.

VOL. XX., ARTICLE 9.

Modulus of Elasticity of Rocks*

AND

SOME INFERENCES RELATING TO SEISMOLOGY.

By

S. Kusakabe, *Rigakushi.*

With two plates.

CONTENTS.

INTRODUCTION.

The present experiments, detailed descriptions of which are to be found in "The Publications of the Earthquake Investigation Committee in Foreign Languages" No. 17, Tokyo, 1904, serve as a complement to the note, recently published by the

---

* A short abstract is to be found in the "Proceedings of the Tokyo Physico-Mathematical Society" Vol. II., No. 11. May, 1904.

author, on the modulus of rigidity of rocks.* Some of the specimens were identical with those used in the last investigation, and the others were prepared in a similar manner. The principal object of the present investigation is not to determine any accurate value of the modulus of elasticity, but to determine whether the modulus is constant within tolerably wide limits or not, and if it is not constant, how it varies with the amount of stress or with time and other factors which affect the change. The modulus is measured by the method of flexure; but the apparatus is more complex than the one which is generally employed. It may therefore not be superfluous to give the following detailed description of it.

ARRANGEMENTS AND FLEXURE-APPARATUS.

In the measurement of flexure, the use of the cathetometre or of the micrometrescrew are generally dispensed with. The method with mirror and scale, as modified by A. König,† is generally adopted, though that by optical interference is more accurate. The apparatus as designed in the present experiment combines the advantages of König's arrangement with other necessary appliances. The principal improvements are:—(1) the bending of the specimen cyclically from one side to the other, with increasing and decreasing force passing through zero continuously; (2) the elimination of external disturbances such as any minute rotation of the specimen or slight displacements of the scale and telescope.

* Ibid. Vol. I., No. 14, Oct. 1902. Pub. of the E. I. C. in F. L. No. 14, Tokyo, 1903. The Journal of the College of Science, Imperial University of Tokyo. Vol. XIX., Art. 6. 1903.

† A. König. Ueber eine neue Methode zur Bestimmung des Elasticitätsmoduls. Wied. Ann. 28, 1886.

A rough sketch of the arrangement is shown in Fig. 1. The specimen is placed horizontally and, when it is bent, its plane of curvature is also horizontal. There are necessarily one scale, four fulcrums and four mirrors, of which two mirrors M_1 and M_2 are attached to the specimen as in KÖNIG's method, while the others M_3 and M_4 are rigidly fixed to the support. Four different images of one and the same scale S are to be seen in the field of the telescope T, Fig. 2. They are all reflected twice by the following mirrors respectively:—

Right upper image	reflected	by	the	mirrors	M_1 and M_2,
right lower	„	„	„	„	M_1 and M_4,
left upper	„	„	„	„	M_3 and M_2,
left lower	„	„	„	„	M_3 and M_4.

The apparatus is shewn in Figs. 3, 4 and 5, in its front- and side-views as well as in its plan. The two mirrors M_1 and M_2 rotate as the specimen is bent, while the other mirrors M_3 and M_4 are fixed unless the apparatus itself is displaced. The fulcrums F_1, F_2, F_3 and F_4 are so adjusted that the edges of any two of them lie in a vertical plane. A small framework F, which is shown in Fig. 6, serves to apply bending force to the specimen. The frame-work consists of two wedges, one fixed (W_1) and the other movable (W_2) inside a proper case. After placing a proper specimen between the two wedges, the movable wedge W_2 may be pushed firmly against the specimen by the fixed screw S. At the extremities of the strings S_1 and S_2, which run over small pulleys P_1, P_2 etc. towards the observer, some weights are hung which supply the bending force. The support of the fulcrums is made of soft iron, which is rigidly screwed on a wooden block.

From what has been just described, it may be easily seen that, when equal weights are hung on both S_1 and S_2, no bending force is exerted on the specimen, and that it is only the difference of the weights attached to the two strings which is effective in bending it. That is to say, if m_1 and m_2 are the two weights attached to the strings S_1 and S_2 respectively, then their sum $m_1+m_2=M_0$ regists the action of the bending force, the last of which is due to their difference $m_2-m_1=M$. For future reference, M_0 and M will be called the *resisting mass* and the *effective mass* respectively.

When the effective mass is positive, the specimen is supported by the fulcrums F_3 and F_4, and it becomes convex towards the righthand side. In the other case, it becomes convex towards the lefthand side, supported by the fulcrums F_1 and F_2. The fulcrums standing face to face, i.e. F_1 and F_3 or F_2 and F_4, are clamped so as not to push too tightly against the specimen, as there is a possibility that the bending of the specimen will be hindered by friction.

A telescope, provided with a micrometer-screw, is rigidly clamped on a tripod. The scale, engraved on a ground-glass plate, 20 cm. long and 2 cm. wide is covered with a black board having a slit, 8 mm. wide, and is illuminated by a row of small gas jets.

PROCESS OF OBSERVATION AND CALCULATION.

The observation is generally as follows :—

1. To begin with, equal weights, each $\frac{1}{2} M_0$, are hung on the strings S_1 and S_2.

2. A specimen is put between the fulcrums, passing through the frame-work F, the last of which is to be clamped on the

middle part of the specimen. The planes passing through the edges of the fulcrums standing face to face should be normal to the length of the specimen.

3. The four mirrors are so clamped in their proper positions that the images of the scale reflected by them stand side by side within the telescope-field. To adjust them properly requires much practice.

4. The constants of the micrometer-screw for all images are determined. They are nearly equal to each other but not strictly so. One mm. of the scale division is equal to about 23 divisions of the micrometer-screw, which is again equivalent to a rotation of $5{\cdot}176 \times 10^{-6}$ rad. i.e. $1''{\cdot}068$.

5. Zero-readings are taken for all images in a fixed order; i.e. (i) right upper image, (ii) right lower image, (iii) left upper image, (iv) left lower image.

6. The suspended weights consist of some forty pieces of equal weight. A definite number of pieces, say $\frac{1}{2}$ *m.*, are taken off from one string and added on the other. The bending force due to this is obviously mg, where g represent the value of gravity. The time-record corresponding to this transposition of weights is taken.

7. After a certain time, the readings are noted for all images in the same order as in the case of the zero-reading.

8. Second transposition of weights; the time recorded; scale-readings noted: and so on till a definite amount of bending force is reached.

9. The weight is then transposed in the opposite way so that the force diminishes gradually and ultimately becomes oppositely directed. In this way, a series of observations is made to complete the cyclic process several times.

10. From the amount of the deviations of the images, the amount of bending due to each corresponding force is calculated, by the following method.

In Fig. 7, let the zero-reading be taken when the telescope is in T while the mirrors M_1 and M_3 are in the position Mm_1 and the mirrors M_2 and M_4 in the position $M'm_2$. In reality, the reflections of light by the mirrors take place, as a matter of course, in the space of three dimensions; but, for the sake of simplicity, let us assume that the path of the ray of light lies wholly on the plane of the paper. Let *ab* be the position of the scale, and suppose that *a* is a point which gives its images in the field of the telescope after reflecting at s and s'. Suppose that, after a certain number of operations, the specimen is bent, it is rotated and also the telescope is displaced and rotated relatively to the scale. Let their respective values be given by

α=angle through which the mirror M_2 is rotated as the specimen is bent,

$-\alpha$=anlge through which the mirror M_1 is rotated as the specimen is bent,

β=angle through which the specimen is rotated,

δ=the component of the displacement of the telescope parallel to the scale.
Note that the other component is negligible relatively to the distance between the scale and the telescope.

γ=the amount of rotation of the telescope.

Then, if θ and ω denote the angles between the mirrors M_2 and M_4, M_1 and M_3 respectively, we have

TABLE I.

Specimen convex to:	Righthand side.		Lefthand side.	
Rotation of the specimen.	Clockwise.	Counter-clockwise.	Clockwise.	Counter-clockwise.
θ	$\alpha-\beta$	$\alpha+\beta$	$-\alpha+\beta$	$-\alpha-\beta$
ω	$-\alpha-\beta$	$-\alpha+\beta$	$\alpha+\beta$	$\alpha-\beta$

That is to say, provided α and β are taken as algebraic quantities having proper signs, we have simply

$$\theta=\alpha+\beta$$
$$\omega=\beta-\alpha$$

Let T′ be the last position of the telescope, and put

R.U. = The deviation of the right upper image,
R.L. = ,, ,, ,, ,, right lower image,
L.U. = ,, ,, ,, ,, left upper image,
L.L. = ,, ,, ,, ,, left lower image.

Then, from simple geometry, it may be easily proved that

$$\text{L.L.}=\delta+(\text{D}+\text{c}+\text{d})\gamma,$$
$$\text{L.U.}=\delta+(\text{D}+\text{c}+\text{d})\gamma-2\text{d}\omega,$$
$$\text{R.L.}=\delta+(\text{D}+\text{c}+\text{d})\gamma+2(\text{c}+\text{d})\theta,$$
$$\text{R.U.}=\delta+(\text{D}+\text{c}+\text{d})\gamma-2\text{d}\omega+2(\text{c}+\text{d})\theta,$$

where $\text{ss}'=\text{c}\quad,\quad \text{sa}=\text{d}\quad,\quad \text{TS}'=\text{D}.$

If there is no external disturbance, evidently we have

$$\beta=0\quad,\quad\gamma=0\quad,\quad\delta=0,$$

so that

$$\text{L.L.}=0$$
$$\text{L.U.}=2\text{d}\alpha,$$
$$\text{R.L.}=2(\text{c}+\text{d})\alpha,$$
$$\text{R.U.}=2(\text{c}+\text{d})\alpha+2\text{d}\alpha,$$
$$=4\left\{\text{d}+\frac{\text{c}}{2}\right\}\alpha,$$

the last of which is a well known form.

In all cases, we have between the four values the following functional relation.

$$L.L.+R.U.=R.L.+L.U.$$

Thus, the difference of the two sums indicates an error of observation: whence it gives the means of rejecting from numerous observations those which are incorrect. For instance, in the case of a piece of sandstone we had:

TABLE II.

M.	R.U.	R.L.	L.U.	L.L.	R.U.+L.L.	R.L.+L.U.	Error.
900^g	1·129^c	0·529^c	0·593^c	−0·004^c	1·125^c	1·122^c	0·003^c
1200	1·670	0·789	0·877	−0·006	1·664	1·666	−0·002
1500	2·204	1·039	1·159	−0·007	2·197	2·198	−0·001

To calculate the amount of bending, we have four equations containing four unknown quantities. There is, however, one functional relation between the four equations. At the same time, the unknown quantities also may be reduced to three, as δ and γ appear always in one and the same combination.

Put $$x=L.L.$$

$$x+y=L.U.$$

$$x+z=R.L.$$

then $$x+y+z=R.U.$$

Taking any three of the four equations, we may solve them. It is preferable, however, to use all equations, since none of them is strictly correct. Applying the method of least squares we have

$$x=\tfrac{1}{4}[3L.L.+L.U.+R.L.-R.U.],$$

$$y=\tfrac{1}{2}[(\mathrm{L.U.}+\mathrm{R.U.})-(\mathrm{L.L.}+\mathrm{R.L.})],$$
$$z=\tfrac{1}{2}[(\mathrm{R.L.}+\mathrm{R.U.})-(\mathrm{L.L.}+\mathrm{L.U.})],$$

where

$$x=\delta+(c+d+D)\gamma,$$
$$y=-2d\omega,$$
$$z=2(c+d)\theta,$$

and

$$\theta=\alpha+\beta$$
$$\omega=\beta-\alpha.$$

Eliminating x, y, z, θ and ω from the above equations, we have

$$\alpha=\tfrac{1}{8}\frac{c+2d}{d(c+d)}\left\{(\mathrm{R.U.}-\mathrm{L.L.})+(\mathrm{L.U.}-\mathrm{R.L.})\frac{c}{c+2d}\right\},$$
$$\beta=\tfrac{1}{8}\frac{c+2d}{d(c+d)}\left\{(\mathrm{R.L.}-\mathrm{L.U.})+(\mathrm{L.L.}-\mathrm{R.U.})\frac{c}{c+2d}\right\}.$$

In the example above cited, we have

TABLE III.

$c=12{\cdot}4$ cm.	$d=241{\cdot}5$ cm.	$\frac{c}{c+2d}=2{\cdot}503\times10^{-2}$,	$\tfrac{1}{8}\frac{c+2d}{d(c+d)}=1{\cdot}0099\times10^{-3}$.	
M.	R.U. – L.L.	R.L. – L.U.	α	β
900^{g}	$1{\cdot}133^{\mathrm{c}}$	$-0{\cdot}064^{\mathrm{c}}$	$11{\cdot}46\times10^{-4}$ rad.	$-0\ 93\times10^{-4}$ rad.
1200	1·676	–0·088	16·95	–1·31
1500	2·211	–0·120	22·36	–1·76

It is to be noticed that, in the above calculation, the tangent and arc of an angle are taken to be equal to each other. The greatest angle to be dealt with is of the order of 10^{-2} radian: whence the difference between the tangent and the arc is of the order 10^{-6}, that is to say, it is of the order of 10^{-4} of their own amounts, which is within the error of observation.

The relation between the modulus of elasticity and the amount of bending is given by the well known formula.

$$E = \frac{3}{4} \frac{Mgl^2}{ab^3\alpha}$$

where a and b are the breadth and thickness of the specimen, while l is the distance between the corresponding fulcrums.

EXPERIMENTAL RESULTS.

The last investigation with regard to the modulus of rigidity proved a great deviation from Hooke's law even in the case of the least strain. Preliminary experiments showed it to be the same also in the case of bending. Looking at the curve in Fig. 8, we see that there is a tendency on the part of the rock to persist in any strained state which it may have acquired, especially when the variation of the stress changes its sign. The curve is closed and it is also of simple and regular form, though its path during the increase of stress differs entirely from that during the decrease. All rocks, so far as the author has investigated, have this property in common, though they differ in the character of the curves and in other minute details.

It may be suspected that, as the resisting mass increases with the total mass to be moved, this may have some influence upon the hysteresis curve. The comparison of the results of four successive experiments, in which the resisting mass was 1000, 1300, 1600 and 1900 grams respectively, while all other conditions remained the same throughout, showed that the influence of the resisting mass might be safely neglected.

There is one important fact which deserves to be mentioned here. Although the hysteresis curve is of a definite form and traces one and the same curve when a specimen is bent and unbent many times cyclically, the direction of the elongation of the curve does not remain fixed when the amplitude of the cycle —i.e. the greatest amount of stress applied to the specimen during the cycle—is varied. As a general rule, the hysteresis curve becomes more and more vertical when the amplitude of the cycle is further and further increased. One instance is given in Fig. 9.

The amount of hysteresis, which is to be measured by the area enclosed by the curve or by some function of it, is least for Archæan rocks and increases rapidly for new rocks.

Although the modulus of elasticity is never constant during a cyclical strain, its variation obeys one and the same law for both the on- and the off-curve, in so far as the centre of the cycle coinsides with the neutral state of the specimen. It may be necessary to make a remark on the meaning of the term "Modulus of Elasticity." As there is a great amount of hysteresis in the relation of stress to strain, the ordinary conception of the modulus of elasticity is ultimately vague and uncertain. The actual resistance to the deformation in any state whatever, be it already bent or twisted, elastic or plastic in that state, will be taken as the measure of elasticity in that state, so that in the present experiment, the modulus is measured, step by step, by the increase of bending per 200 grams increase of the effective mass.

The curve expressing the relation between the modulus of elasticity and the amount of strain is symmetrical with respect to the axis of ordinate. One instance for Limestone No. 29, is given in Fig. 10. Each kind of rock seems to have its own special character. If Hooke's law were to hold good, four branches of curve, of which the right and left branches correspond to the cases where the specimen is bent convex righthand or lefthand side respectively, while the lower branches correspond to the increasing stress and the upper to the decreasing one, would all shrink to a single horizontal straight line. In the case where no hysteresis exists, both the upper and the lower branches would coincide with each other to make a line not necessarily straight.

For all cases of rocks here experimented upon, the upper branch is concave towards the positive axis of the ordinate. As to its character, however, the variety is very abundant: circular, hyperbolic, oval and other curves of higher order of complexity. The curvature of the lower branches is turned sometimes upwards and at other times downwards. Although it is not easy to determine any law according to which the modulus varies with the phase of the cycle, we may find, as a first approximation, an empirical expression for each specimen. For instance, in the case of sandstone, we have.

for the upper branch, $y_1 = 0{\cdot}243 + 0{\cdot}92x^2$

for the lower branch, $y_2 = 0{\cdot}243 + 0{\cdot}043x^2$

where y and x represent $E \times 10^{-11}$ and the phase respectively. As a matter of fact, the constant term of y_1 is equal to that of y_2, representing the modulus of elasticity at the state where no external force is acting.

In the following table, the constant term of the expression for every specimen is given as the modulus of elasticity of several rocks. It corresponds therefore to the value of the modulus of elasticity in the state when the bending force became zero, during which the specimen, whose section was about one centimeter square and the distance between the fulcrums was 10 centimeters, was bent cyclically on both sides by a force varying between those due to $M=\pm 3000$ grams-weight. The value in any other state under different conditions must necessarily be different from those given in the table.

TABLE IV.

No.	Rock.	Locality.	Kind.	Density.	Mod. of elasticity.	Mean E.	Velocity of Long. Wave.
			ARCHÆAN ROCKS.		$\times 10^{11}$(c.g.s.)		$\frac{\text{Km.}}{\text{Sec.}}$
31_1.	Quartzschist.	Chichibu.	Metamorphic.	2·67	10·48—7·07	8·78	5·73
46_1.	„	Gumma.	„	2·62	8·41—8·40	8·41	5·67
8_1.	Serpentine.	Chichibu.	Eruptive (altered).	2·72	7·73—7·21	7·47	5·24
40_1.	Micaschist.	Ibaraki.	Metamorphic.	2·54	6·49—5·92	6·21	4·94
18_1.	Chloriteschist.	Chichibu.	„	2·88	8·63—5·39	7·01	4·93
7_2.	Peridotite.	Kuji.	Eruptive (altered).	2·61	6·73—5·83	6·28	4·91
26_1.	Chloriteschist.	Chichibu.	Metamorphic.	2·82	7·03—6·29	6·66	4·86
22_1.	Gabbro.	„	Eruptive.	2·71	6·21—5.57	5·89	4·66
24_1.	Graphiteschist.	„	Metamorphic.	2·59	5·12—4·93	5·03	4·41
23_1.	„	„	„	2·56	3·69—3·37	3·53	3·71
42_2.	Micaschist.	Ibaraki.	„	2·63	1.29—1·16	1·23	2·16
			PALÆOZOIC ROCKS.				
34_1.	Adinoleslate.	Gumma.	Sedimentary.	2·64	10·99—10·23	10·61	6·34
12_2.	Clayslate.	Aumi.	„	2.71	10·71—9·08	9·90	6·04
9_1.	Granite.	Mikage.	Eruptive.	2·54	4·31—3·66	3·99	3·96
21_1.	Limestone.	Chichibu.	Sedimentary (Metamorphosed).	2·64	4·14—3·65	3·90	3·84
6_3.	Marble.	Kuji.	„	2·68	3·51—3.24	3·38	3·55
14_1.	Red Schalstein.	Aumi.	Sedimentary.	2·43	3·09—2.39	2·74	3·36
32_1.	Pyroxenite.	Gumma.	„	2.90	2·96—2·91	2·94	3·18
10_2.	Granite.	Kagawa.	Eruptive.	2·57	2·30—2·10	2·20	2·93
29_1.	Limestone.	Gumma.	Metamorphic.	2·66	2·06—1.92	1·99	2·74

No.	Rock.	Locality.	Kind.	Density.	Mod. of elasticity.	Mean E.	Velocity of Long. Wave.
			TERTIARY ROCKS.		$\times 10^{11}$(c.g.s.)		$\frac{\text{Km.}}{\text{Sec.}}$
35_1.	Sandstone.	Chichibu.	Sedimentary.	2·47	3·55—3·51	3·53	3·78
50.	Two Pyroxene Andesite.	Mutsu.	Eruptive.	2·70	4·04 2·38	3·21	3·44
2_3.	Tuff.	Izu.	Sedimentary.	1·90	1·39—1·36	1·38	2·69
5_3.	Rhyolite.	Yechizen.	Eruptive.	2·40	0·90—0·77	0·84	1·87
4_3.	Sandstone.	Kii.	Sedimentary.	2·25	0·68—0·57	0·63	1·67
3_4.	„	Chōshi.	„	2·21	0·34—0·20	0·27	1·11
			DILUVIUM ROCKS.				
17_2.	Andesite.	Gumma.	Eruptive.	2·63	4·36—4·31	4·34	4.06
16.	„	„	„	2·32	0·68—0·63	0·66	1·69

As the hysteresis curve becomes more and more vertical when the amplitude of the cycle increases further and further, the mean elasticity necessarily weakens when the amplitude of the strain increases. For instance, in a case of sandstone where the curves could all be represented by a series of parabolic expressions, the constant term of them was :—

TABLE V.

Amp. (in gramsweight.)	300	600	1200	1800	2400	3000
E. (c. g. s. $\times 10^{11}$).	0·65	0·46	0·33	0·27	0·21	0·15

Thus it is important to notice how the modulus of elasticity diminishes when the amplitude of the strain increases.

Here it will suffice to remark that, as in the case of the modulus of rigidity, the modulus of elasticity also is comparatively greater in a strained than in the neutral state.

The phenomenon of yielding, though it is not so enormous as in the case of torsion, is still sufficiently great to be dealt with. For a piece of sandstone, e.g., which was loaded with $M_0 = 3300$ and $M = 3000$ gramsweight, the amount of bending increased, in the course of two and a half days, to, at least, more than three times its initial value. It is, indeed, questionable whether there is any limit to the yielding.

Also the amount of residual surviving the bending force does not remain constant, but recovers gradually and uninterruptedly. The amount of recovery, in the case of the above specimen just referred to, increased, in the course of about four days, by more than twice its initial value.

The yielding of specimens under a constant force having become comparatively small after a few days, the temperature-variation of the flexure may be clearly observed. The relation between temperature and bending for a piece of sandstone is given in Fig. 11. The curve, as a whole, expresses the simple proportionality between the two elements. We find, however, the amount of flexure has a minimum value in the neighbourhood of about 9°C. In the case of the rigidity-modulus, we had a result strictly analogous of this effect. It may be, however, the effect of moisture. To determine any general relation between elasticity and temperature requires further investigation by a special arrangement.

SOME INFERENCES RELATING TO SEISMOLOGY.

In the author's publications above cited, it was experimentally as well as theoretically explained that, in the case of distorsional waves, the velocity of propagation is a function of the amplitude of the wave, as there exists more or less yielding in the rocks through which the waves propagate, and also that, in view of this inference, we do not see the necessity of assuming the path of the tremors to be different from that of the principal shocks. The present experiment relating to other modulus give, it seems to me, still stronger foundations to support the above view. We must not however forget that, it is unfortunately the common rule rather than the exception that a theory, however perfect it may be, does not explain all the facts connected with it and also that almost every phenomenon has more than a single cause, and this is particularly true in the case of earthquakes.

As the elastic constant varies during one cycle of bending and all values at different phases of the cycle equally play their parts in causing the vibratory motion, the apparent value of the elastic constant during one complete vibration must be the mean value of all the values at different phases. Now the mean elasticity for one complete cycle being distinctly greater than what is commonly adopted, the actual velocity of propagation for seismic waves must be correspondingly greater than those given in the above table, which are calculated by taking the square root of the elasticity-density ratio. In the case, e.g., of a piece of sandstone, the result of the experiment shows that the mean value is 3·67 times greater than the constant term. Whence we may infer that the actual velocity, in this case, would be probably twice the value given in the table.

Again, the velocity must necessarily diminish with an increase of the amplitude of the wave, since the elasticity diminishes in that case as explained above. From the example given there, we may deduce the following to show how the velocity changes with the amplitude.

TABLE VI.

Ratio of Amplitudes.	1	2	4	6	8	10
Ratio of Velocities.	2·08	1·75	1·48	1·34	1·18	1.00

Though the variation of the elastic constant due to temperature-rise is comparatively small, it can never be neglected in so far as the velocity of seismic waves is concerned, since the underground temperature rapidly rises with the increase of depth. Although the elastic constants increase from Cainozoic to Archæan rocks in a greater ratio than the density does, to attain the main stratum of Palæozoic rocks we must go deep down some ten kilometres, at least, and for a stratum of Archæan rocks, at least, thirty kilometres, where the underground temperature must be tolerably high. Any conclusive deductions should, however, be postponed until the more accurate observations on the change of the elastic constants due to temperature-rise, which are in course of preparation, shall have fully elucidated the relation between the elastic constants and the temperature.

As a matter of fact, there are scattered everywhere within the earth's crust veins and dikes of different kinds of old rocks, uplifted by geological disturbances, some of which run over many hundreds or thousands of kilometres. The velocity along such a vein or dike must necessarily be greater than that through any of the surrounding strata, so that the seismic waves mainly propagate through that region. As a consequence of the above result, if an observing station be situated near such a vein, not only will the number of earthquakes observed at the station be greater than those observed at any place in the vicinity, but the direction of the motion will not necessarily indicate the position of the seismic centre. It is a matter of daily experience with us who live in an earthquake country located in the 'girdle of fire of the Pacific, that observers in some districts feel all shocks as if coming from one particular direction even when the seismic centre lies in an entirely different direction.

As another consequence, there may exist seismic shadows; or, in other words, seismic waves may be partially shielded by a vein or dike of old rocks. Earthquakes originating in one region may always be well observed in the station while those originating in another region may fail to be observed in the station. In Prof. F. ŌMORI's paper* we find a most interesting example to support the above consideration. Of the earthquakes which happened between Sept. 1889 and July 1886 in Central Japan, those whose origins were situated within certain boundaries were never felt in Tokyo, though the weaker ones of more distant origins were clearly felt there.

The frequency of earthquakes as related to the geological distribution of rocks will be fully discussed in a following paper under a special title.

In conclusion, I wish to express my great indebtedness to Mr. FUKUCHI for valuable information concerning the geological characters of the specimens. My best thanks are due to Professor H. NAGAOKA, under whose kind guidance I have carried out this experiment.

---

* F. ŌMORI. The Pub. of the E. I. C. in F. L. No. 11. 1902.

Fig. 1.

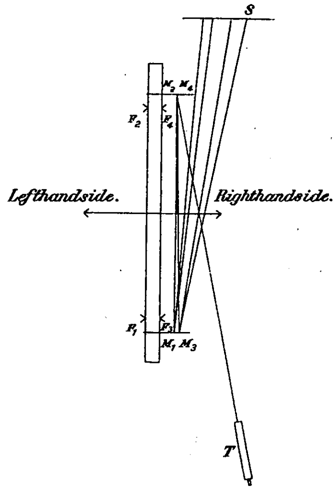


Fig. 3.
Front view.

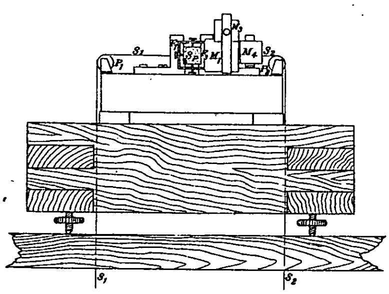


Fig. 4.
Side view.

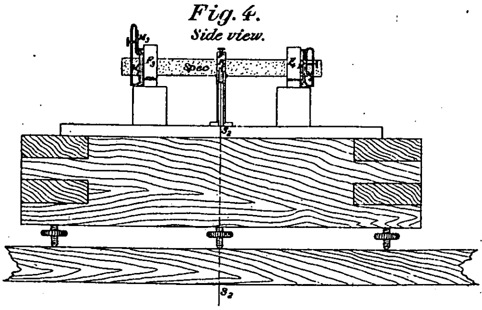


Fig. 2.
The field of telescope.

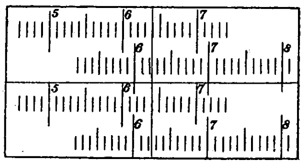


Fig. 5.
Plan of the flexure apparatus.

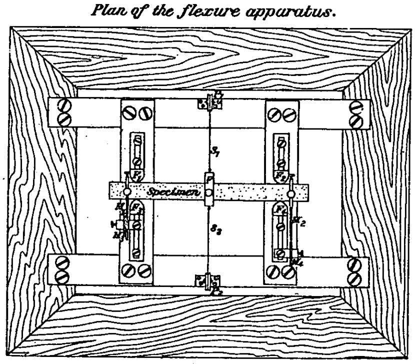
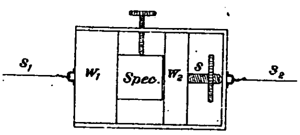


Fig. 6.



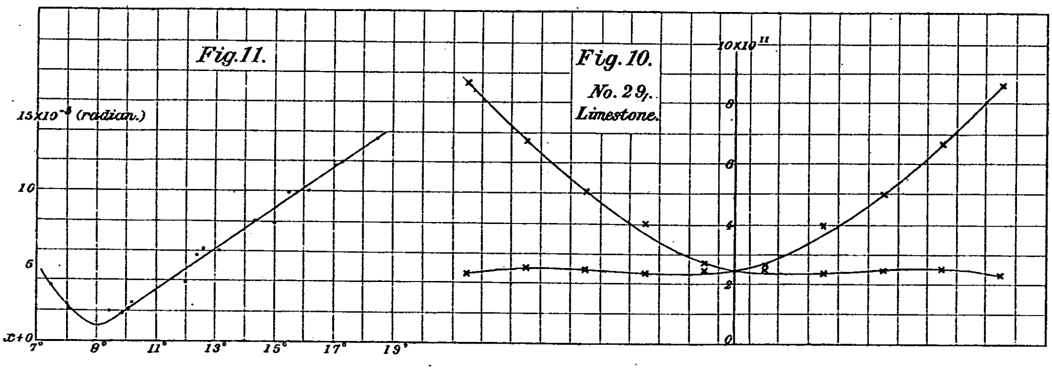
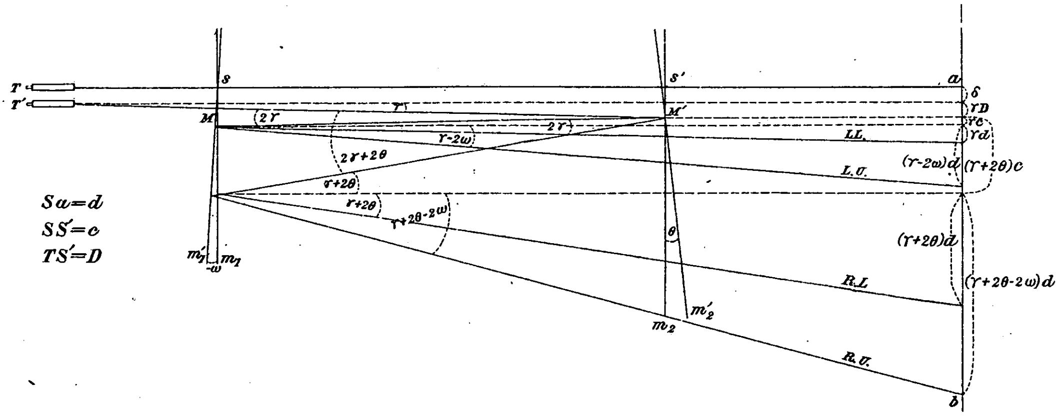


Fig. 7.


T
T′
S
S′
a
M
M′
δ
γD
γc
γd
2γ
γ
2γ
γ-2ω
2γ+2θ
γ+2θ
γ+2θ
γ+2θ-2ω
L.L.
L.U.
R.L.
R.U.
(γ-2ω)d
(γ+2θ)c
(γ+2θ)d
(γ+2θ-2ω)d
θ
m′₁
m₁
-ω
m₂
m′₂
b
Sa=d
SS′=c
TS′=D

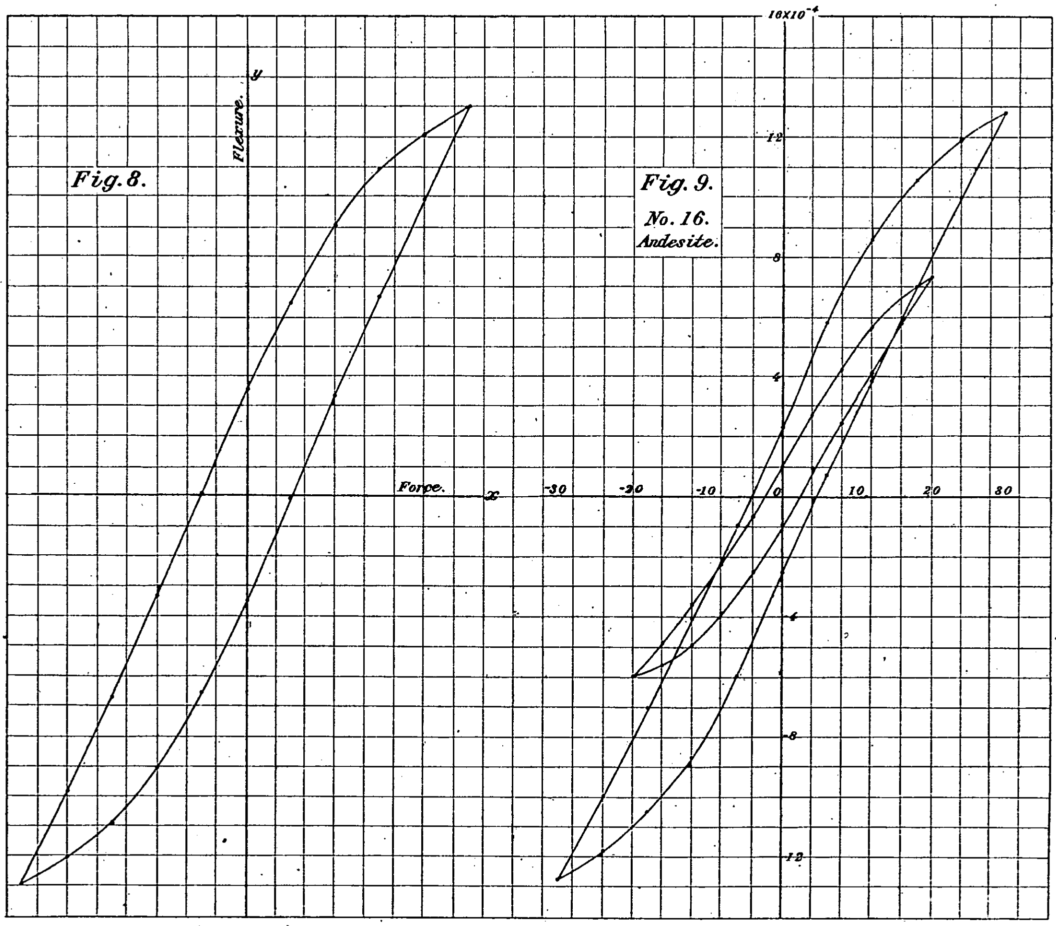


Fig. 8.
Flexure.
y
Force.
x
Fig. 9.
No. 16.
Andesite.
16x10⁻⁴
12
8
4
0
4
8
12
-30
-20
-10
10
20
30