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Kinetic Measurements of the Modulus of Elasticity

FOR

158 SPECIMENS OF ROCKS:

AND

A NOTE ON THE RELATION BETWEEN THE STATIC AND THE KINETIC VALUES OF THE SAME.*

By

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With one plate.

CONTENTS.

* The essence of this report was read in the meetings of the Tōkyō Physico-Mathematical Society for Sep. 1904 and April 1905; and was published in Nos. 15 and 22, Vol. II, Proc. of the Society.

I. INTRODUCTION.

As to the elastic constants of several rocks which compose the so-called outer-crust of our planet, we have already a valuable paper published by Professor H. NAGAOKA,* in which the elastic constants for one hundred specimens are given, with some notes relating to seismology. The author also has reported some experimental researches relating to the hysteresis and the variation of the constants under different conditions.† A little consideration, however, will make it evident that the values of elastic constants determined by a statical method may be far from what ought to be used in the discussion of seismic waves.

The importance of measuring them again by a kinetic method may thus be readily recognized. Not only are the rapid alterations of state concerned in the propagation of a wave attended with a thermal effect, which goes to change the elastic constants beyond their value statically obtained; but also in the case of rocks, the phenomenon of yielding may have great influence in making them deviate further from one another.

In the author's papers above cited it has been shown that the modulus of elasticity varies, during one cycle of strain, according to a definite law. When a piece of rock is bent by a force and unbent by virtue of its own elasticity, it is not, evidently, the modulus of elasticity at any particular state, which determines the vibratory motion of the rock. Moduli of elasticity at all the different phases of the vibratory motion equally take part in

* H. NAGAOKA. The Pub. of the E.I.C. in F.L. No. 4, 1900, and Phil. Mag. 1900.

† S. KUSAKABE. This Journal. Vol. XIX., Art. 6. and Vol. XX., Art. 9. The Pub. of the E.I.C. in F.L. Nos. 14 and 17.

causing the motion. Hence for the apparent modulus of elasticity during one complete vibration, we must take the mean value of the moduli at all different phases.

For a piece of sandstone, e.g., when the maximum bending force during the cycle was equal to that due to $M=3000$ grams-weight, we had a mean value 3.67 times greater than that taken at the state of no bending. In other words, the mean kinetic modulus for this specimen, provided the vibration be assumed to take place at as slow a rate as in this case, is nearly three and a half times greater than the static modulus.

Possibly sound-experiments furnish the best means of ascertaining the kinetic modulus of elasticity. The results deduced from such infinitely small strains as occur in sound vibration is no doubt of great significance as regards the elastic property of rocks. The fact that it seems strange to speak of the vibration of such loose rocks as sandstone, shows at once that the measurement of the kinetic modulus of elasticity in them is much more defficult than in metallic substances.

The method to be adopted here is a new application of Melde's experiment*, combined with the principle of resonance. The number of specimens examined amounts to one hundred and fifty eight, collected from various lacalities in the main islands of Japan, containing 23 Archæan, 65 Palæozoic, 12 Mesozoic and 58 Cainozoic rocks. This whole series of experiments is but a beginning in this field of inquiry and may be thrown aside as a wreck in the path of progress; yet the author hopes it may prove a help to later investigators.

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* MELDE. Phil. Mag. Vol. 47, 1874. Lowery. Ibidem. Vol. 48, 1874.

II. METHOD OF MEASUREMENT AND APPARATUS.

The essence of Melde's experiment is that, on one hand, a massive fork excited by a bow or sustained in permanent operation by an electro-magnet, produces its vibrations in approximate independence of the reactions of any light body, which may be connected with it; and, on the other hand, the period of the forced vibration of the light body is determined solely by the period of the force which is supposed to act on the system from without. The principle of resonance is merely that the kinetic energy or the amplitude of any forced vibration is the greatest possible, when the period of the external force is that in which the system would vibrate freely under the influence of its own elasticity.

The present method is simply as follows :—

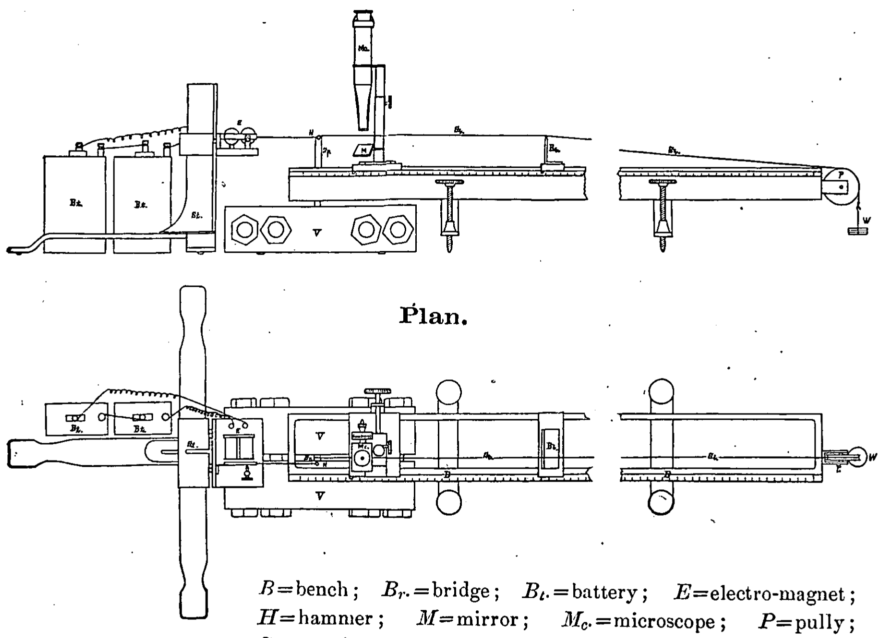
a. A specimen of rock, one end of which is tightly clamped by a massive vice, is maintained in free vibration.

b. A fine wire of known linear density w is connected with the free end of the specimen and stretched with a known tension.

c. The length l of the above wire, in which the wire vibrates with maximum amplitude is measured.

The annexed figures show the plan and elevation of the whole arrangement, which is laid directly on stone floor of the laboratory.

Elevation.



B=bench; B_r.=bridge; B_t.=battery; E=electro-magnet; H=hammer; M=mirror; M_c.=microscope; P=pully; S_p.=specimen; S_r.=string *i. e.* copper wire; S_t.=stand; V=vice; W=weight.

A specimen *Sp.* clamped in the vice V is tapped by a hammer, H, which is supported by a stand *St.* and maintained in constant vibration electro-magnetically. A fine copper wire *Sr.* of ca. 0.05 mm. in radius is connected by means of bee's wax to the upper end of the specimen and stretched over a bridge *Br.* by a tension due to the suspended weight, W. The copper wire, whose breaking tension is ca. 140 grams, is strained by a tension of 100 grams during some tens of minutes. Both the bridge and the microscope *Mc.* may slide along an iron bench B of ca. 118 cm. in length, in which a scale is graduated for each mm.

The distinction of forced and free vibrations is very important, and must be clearly explained. If a vibration is the response of the system to a force imposed upon it from without and is maintained by the continued operation of that force, it is obviously a forced vibration. It must, however, be remembered that any free vibration which we shall have in a laboratory experiment takes its origin necessarily from a force acting upon it from without. At first, there is a forced vibration not less important than its rival, but when the force is removed, though there is no discontinuity in velocity or displacement, yet the period of the force is at once exchanged for that natural to the system and the forced vibration is converted into a free vibration.

In the present case, the frequency of the hammer-blow, which is easily adjustable within wide limits by varying its moment of inertia, is about ten per second; while that of the vibration of the specimen lies between some three hundreds and a thousand per second. In other words, between two consecutive tappings, the number of vibrations of the specimen amounts to some thirty or one hundred, so that the mode of vibration is necessarily of a free nature.

III. TEST OF THE APPARATUS WITH A TUNING-FORK AND A PRISM OF SOFT IRON.

So far as the principle is concerned, the above statements are sufficient and nothing more is required. In laboratory work, however, there are several experimental difficulties to be overcome; especially, maintenance of the vibration and the fulfilment of the condition of a clamped end. The fixedness of the clamped end is absolutely important, since the frequency varies inversely as square of the length of the vibrating portion.

To test the apparatus, the experiment was made with a tuning-fork of known frequency, giving the following result.

Tuning-fork. Sol_2. 384 VS.					June 8, 1904.		
$W=40{\cdot}595$ grs.		$w=4{\cdot}77\times10^{-4}$ grs./cm.			$g=979{\cdot}8$ in the laboratory.		
Measured to	1st Obs.	2nd Obs.	3rd Obs.	4th Obs.	Mean.	l'	calculated.
I node.	24·0	24·1	24·1	24.0	24·0	24·00	24·0
II node.	47·7	48·4	48·0	47·6	47·9	23·95	47·8
III node.	71·6	71·6	71·4	71·4	71·5	23·83	71·6
IV node.	95·1	95·8	95·2	95·6	95·4	23·85	95·4

Here it is necessary to remark that, although the vibration of the string is due to the periodic force imparted from the specimen, the point of application of the force never corresponds to a loop, but, on the contrary, there is a node in that vicinity. Assuming the impressed force given by the vibrating rock, which is connected with the string at a point whose distance from the bridge is $x=b$, to vary as $F \cos pt$, the amplitude of the motion between the bridge and the rock is approximately given by

$$\gamma\left\{\frac{\sin^2\frac{px}{a}+\frac{f^2x^2}{4a^2}\cos^2\frac{px}{a}}{\sin^2\frac{pb}{a}+\frac{f^2b^2}{4a^2}\cos^2\frac{pb}{a}}\right\}^{\frac{1}{2}}$$

where γ is the amplitude of the rock measured at the point where the string is connected, and f is the coefficient of friction of the string. From this expression it is evident that the amplitude of the forced vibration attains its maximum value when the point of application of the impressed force is a node.

In the above example, it is reasonable to assume that a nodal point lies somewhere at a point near the origin of the scale-measurement. Let this distance be denoted by a, then the above values of l' correspond to $l+\frac{a}{N}$ where N is the number of loops contained in the observed segment of the string. Thus a and l being two unknown quantities, they may be easily calculated by the method of least squares. The result is :—

$$a = 0^{\cdot c}24$$
$$l = 23^{\cdot c}79.$$

The result of the second experiment with another string and smaller tension is

$$W = 20{\cdot}908$$
$$w = 5{\cdot}27 \times 10^{-4}$$
$$l = 16{\cdot}32.$$

The number of free vibrations of gravest mode for a perfectly flexible string is given by

$$n_1 = \frac{1}{2l}\sqrt{\frac{W.g}{w}}$$

where W is the suspended weight to which the tension is due. But if we consider the string as not being infinitely thin, its stiffness must be taken into account. In the case where the extremity of the wire is constrained to be a node by stretching it over a bridge but no couple acts to fix its direction, the correction for a circular wire is given by

$$n_2 = \frac{\pi w^{\frac{3}{2}} E'}{16 l^3 \rho'^2 \sqrt{W.g}.}$$

where ρ' and E' are the specific density and the modulus of elasticity of the wire respectively.

Thus, the number of vibrations n of the specimen, which is identical with that of the connected wire, is given by

$$n = n_1 + n_2 = n_1 + \frac{\pi w E'}{32 \rho'^2}\,\frac{1}{n_1 l^4}$$

Another term of correction due to the effect of the wire being connected at the free end may be easily ascertained as we see that the effect of the small load dM is the same as a lengthening of the specimen, whose weight is M, in the ratio

$$M : M + dM.$$

Now, calculating the number of vibrations for the tuning-fork from the data above given, we have

$n = 191{\cdot}9$ from the first experiment,

$n = 191{\cdot}6$,, ,, second ,,

The difference between each of these and the registered value 192 is within the error of observation.

As to the verification for the fulfilment of the condition of a clamped end, the case of a tuning-fork with two prongs is wholly out of place, so that it may not be superfluous to cite here the following observations with a prism of soft iron. Any continuation of the specimen beyond the clamped section would be without effect, as it acquires no motion; but as the first clamp is relaxed, the pitch rapidly falls, in consequence of the increase of the length. Hence, in tapping the specimen care must be taken to give no impact to the clamped section, i.e. the specimen should be tapped at a position corresponding to the centre of percussion with respect to the clamped section.

The result of the first observation is as follows :—(c.g.s. units).

$$\rho = 7{\cdot}779$$
$$b = 0{\cdot}763$$
$$L = 13{\cdot}4$$
$$W = 30{\cdot}718$$
$$w = 5{\cdot}19 \times 10^{-4}$$
$$E' = 1{\cdot}221 \times 10^{12}$$
$$\rho' = 8{\cdot}667$$

$$g = 979{\cdot}8$$
$$l = 10{\cdot}87$$

whence
$$n_1 = 350{\cdot}3$$
$$n_2 = 0{\cdot}2$$
$$n = 350{\cdot}5$$

The relation between the number of vibrations n and the modulus of elasticity E of the specimen is given by the well known formula

$$n = \frac{k}{2\pi L^2} m^2 \sqrt{\frac{E}{\rho}}$$

where k is the radius of gyration and ρ is the specific density of the specimen whose length is L; while m is a constant satisfying the equation

$$\cos m \cosh m + 1 = 0.$$

The smallest root, 1·875, corresponds to the gravest mode of vibration. The first over-tone is ca. 2·6 octaves higher than the gravest tone, so that in an actual case the succeeding roots of the equation have no importance.

Now, calculating the modulus of elasticity for the specimen of soft iron from the above data, we have

$$E = 2{\cdot}029 \times 10^{12} \text{ c.g.s. unit.}$$

In the above experiment, the direction of the motion of the specimen was perpendicular to the elongation of the string. It is well known that, if the direction of motion is parallel to the string, the period of the vibration of the string becomes double that of the specimen. That is to say, the frequency n of the specimen is equal to twice the frequency (n_1+n_2) of the string. The second observation with another string and smaller tension was made under the last mode of vibration, giving the following result :—

$$W = 11{\cdot}065$$
$$w = 5{\cdot}67 \times 10^{-4}$$
$$l = 12{\cdot}47$$

whence

$$n_1 = 175{\cdot}3$$
$$n_2 = 0{\cdot}2$$
$$n = 351{\cdot}0$$
$$E = 2{\cdot}034 \times 10^{12} \text{ c.g.s. unit.}$$

IV. PRELIMINARY EXPERIMENTS AND THE PROCESS OF CALCULATION.

Although the above testings gave satisfactory result, preliminary experiments with sandstone and tuff were so very ambiguous as to wholly confound the observer. At first sight, there seems to be no definite length with which the string may vibrate with maximum amplitude. In the case of sandstone, e.g., lengths corresponding to maximum amplitude were as follows:—

(1) 9·28	(1) 9·93	(1)10·64	(1)11·42	(1)12·34	(1)13·40	(1)14·63	(1)16·23
(2)17·39	(2)18·41	(2)19·69	(2)21·04	(2)22·61	(2)24·49	(3)26·06	(2)26·70
(3)27·58	(1)29·43*	(3)31·53	(2)32·53	(3)33·89	(4)34·62	(1)36·76*	etc. etc.

The small numbers in brackets are the number of loops contained in the segment. Those marked with * correspond to a peculiar mode of vibration.

It may be argued *prima facie* that the elasticity of such loose materials as rocks which compose the earthcrust is not unique and therefore, as the facts prove, that the velocity of the propagation of seismic waves is diverted between wide limits. If we assume that all these lengths equally correspond to the proper vibrations of the specimen, then the velocity of longitudinal waves should be proportionally many-valued. In the case of sandstone No. 3_4, e.g. we have,

	$l =$	13·48 (c)	15·33	16·36	17·20	19·25	21·68	24·67	28·72	34·82	42·89
whence	$V \propto$	12·9	11·3	10·6	10·1	9·0	8·0	7.0	6·0	5·0	4·0

In Professor F. Ōmori's papers relating to seismometory we frequently find what correspond to the above, calculated as the velocities of seismic waves in their successive phases.

Repeated experiments, however, showed that this confusion was an effect of the tapping by the hammer, so that varying the period of the impressed force we might obtain another series of maximum values. Although the vibration is really of a free nature, it is rendered intermittent by the periodic interposition of an obstacle, so that a very different result is arrived at. In this case, a vibration of a frequency n varies in its amplitude with a frequency m, which last is the frequency of the hammer. The amplitude increases very suddenly and it is always positive so that the motion may be assumed, though by a very rough approximation, to be represented by the expression,

$$Y = A_0 \cos 2\pi\, nt + \sum_{\upsilon=1}^{\upsilon=\alpha} A_\upsilon \left[\cos 2\pi\,(n+2\upsilon\, m) + \cos 2\pi\,(n-2\upsilon\, m)\right].$$

It is obvious that, in such a case as the above, the amplitude takes its maximum value when the length of the strings corresponds to any one of the numerous component vibrations. The relative magnitudes of the several maximum amplitudes differ very much from one another and in such a way that the greatest maximum corresponds to the vibration in the natural period of the specimen, and the smaller the amplitude the more it is affected by the impressed force.

When m is not too small to be compared with n, each maximum may be distinctly observed; but their consecutives more and more approach each other as the ratio m/n becomes smaller and smaller. In the case where the ratio is one-fiftieth or smaller than that, the series of maxima become approximately continuous and practically constitute one maximum with a small gradient, as in the case of soft iron.

Again, varying the period of the impressed force from m to m', all the lengths corresponding to maximum amplitudes, except that which corresponds to the natural period of the specimen, are changed. When these two facts are taken into account, it is not a difficult matter to determine the natural period of vibration for any specimen at hand.

Let l_α and l_β be the lengths of string, vibrating with one loop, corresponding to the number of vibrations $(n+2\alpha m)$ and $(n+2\beta m)$ respectively, then for a certain length l, which is a common multiple of l_α and l_β, the string vibrates in a peculiar manner apparently with one loop, nodes of one mode of vibration being over-lapped by loops of another mode and *vice versa*.

For example, in the case of sandstone, the result of one experiment with a brass hammer showed that the string might vibrate with a maximum amplitude corresponding to any one of the number of vibrations

$$410{\cdot}0 \pm 25{\cdot}7\,\upsilon$$

where υ is an integer, so that we have

$$n = 410{\cdot}0$$
$$2m = 25{\cdot}7.$$

Another experiment with a lead hammer, in which the moment of inertia was increased, gave a different result, as follows:—

$$n = 409{\cdot}3$$
$$2m' = 15{\cdot}3.$$

These two values for the frequency in free vibration are equal to each other within the error of observation.

In the above observations, the tension of the string remained constant and equal to

$$W = 30{\cdot}718 \text{ grams.}$$

To test whether the error of observation is affected by the variation of the constant tension or not, two other observations were made on the same specimen, giving the result;

$$W = 30{\cdot}718, \qquad n = 409{\cdot}7$$
$$W = 20{\cdot}908, \qquad n = 411{\cdot}4$$
$$W = 11{\cdot}065, \qquad n = 409{\cdot}1$$

Now, taking the mean of the above values, the data required to calculate the modulus of elasticity of the sandstone are

$$L = 9{\cdot}9$$
$$b = 1{\cdot}16$$
$$\rho = 2{\cdot}25$$
$$n = 410{\cdot}1$$

whence $E_K = 10{\cdot}36 \times 10^{10}$ c.g.s. unit.

It may be here noted that the value determined by the static method is nearly half of the above, i.e.

$$E_S = 5{\cdot}7 \times 10^{10} \text{ c.g.s. unit.}$$

So far as the principle is concerned, the present method is very simple, and it may be easily understood at a glance. The mode of observation and the process of calculation, however, are so tedious and complicated that the author considers it in no case superfluous to recapitulate them.

With a known tension $W = W_1$ and a linear density $w = w_1$ of the string and a certain frequency of the hammer $m = m_1$, we proceed as follows:—

Node. / Obs.	1st	2nd	—	rth	—	qth
1st	l_{11}	l_{12}	—	l_{1r}	—	l_{1q}
2nd	l_{21}	l_{22}	—	l_{2r}	—	l_{2q}
3rd	l_{31}	l_{32}	—	l_{3r}	—	l_{3q}
Mean.	l'_1	l'_2	—	l'_r	—	l'_q
l'_r/r	l_1	l_2	—	l_r	—	l_q

Now, a and l_a being two unknown quantities, we have the relation

$$l_a+\frac{a}{r}=l_r \qquad \text{where} \quad r=1,\, 2,\ldots\ldots q.$$

By the method of least squares, l_a may be easily found and from it the frequency of the string n_a may be calculated by the formula given in the third chapter.

$$n_1 \quad n_2 \quad n_3 \quad — \quad n_\upsilon \quad — \quad n_{t-1} \quad n_t,$$

which are connected by the relation

$$n+2m_1\upsilon=n_\upsilon \qquad \text{where} \quad \upsilon=1,2,\ldots\ldots t.$$

From these values, the frequency of the free vibration of the specimen may be calculated by the method of least squares, n and m_1 being two unknown quantities. Thus we have the first value for the frequency, i.e.

$$n=n'.$$

Now, varying the tension, linear density of string and period of the hammer, i.e.

$$W=W_2,\ w=w_2,\ m=m_2$$

we proceed as in the former case and obtain a second value

$$n=n''.$$

A third series of observations, with $W=W_3$, $w=w_3$ and $m=m_3$, gives a third value

$$n=n'''.$$

These three values of the frequency, i.e. n', n'' and n''' are nearly equal to one another provided there be no blunder or mistake in the observation or in the reduction of the result. Taking the mean of these three values as the frequency of the free vibration of the specimen, with a small correction due to the stiffness of the string, the modulus of elasticity is calculated by the formula given in the third chapter.

V. EXPERIMENTAL RESULTS.

For the complete discussion of the elastic nature of rocks, as many different elastic constants as the number of symmetry planes, which can be drawn in the rock, must be determined. As we have, however, no simple means of examining these symmetry planes, a single modulus of elasticity was determined relating to two mutually perpendicular directions, on the supposition that the material was isotropic.

The above enormously complicated method was applied to one hundred and fifty eight specimens of different kinds of several ages, and cost the author immense labor during one complete year. For all this labor and trouble expended to obtain but a poor result, he is consoled with Boyle's thought that, "men are oftentimes obliged to suffer as much wet and cold and dive as deep to fetch up sponges as to fetch up pearls."

The table at the end of the text contains the results arranged in the order of geological age; for the same geological age, those with the larger modulus come before those with the smaller. The velocity for longitudinal waves, calculated by the formula

$$V=\sqrt{\frac{E}{\rho}}$$

is also given in the table. The actual velocities of longitudinal waves in various rocks may differ from those given; the table, however, will probably furnish a rough estimate which may be of some use in seismometry.

Expressing the elastic constant of the rocks, classified according to the age of formation, by means of "the height from a fixed base line," Fig. I, we find a distinct gradation as we pass from the rocks of the Archæan age to those of the Cainozoic. Some of recent age may, of course, have a greater modulus of elasticity than those belonging to the older periods. As a whole, however, Archæan rocks come in first of all, while Cainozoic rocks come in last. The greatest and the least of the Archæan group are greater than those of the Palæozoic group respectively, and so on in turn for other periods. In the mean, the greater part of Archæan rocks have a greater modulus of elasticity than the greater part of Palæozoic rocks and so on.

The modulus of elasticity for a given rock may vary within wide limits as the density and other physical properties differ for each specimen. In the case of granite, e.g., No. 59_1 has the greatest value $E=5{\cdot}93\times10^{11}$ while No. 71* the least value $E=1{\cdot}25\times10^{11}$. c.g.s. unit. For engineers, it would be well to remember that in the case of any rock with unusually great density it does not necessarily follow that the modulus of elasticity is correspondingly great. For example, a piece of granite No. 63_1 has a density twelve percent greater than that of the other piece of granite No. 9_1, while the latter has its modulus of elasticity, on the contrary, forty-eight percent greater than the former. The possession of a greater modulus of elasticity and yet less density is what makes a material the more valuable.

As a general rule, however, rocks of recent formation have a smaller modulus of elasticity and, at the same time, less density than those of older periods. The modulus of elasticity of old rocks increases very rapidly, more rapidly, indeed, than is proportional, as the density increases slowly.

The velocity of propagation for longitudinal waves as shown in the table, also increases with the age of formation of the rock in question. It may here be noted that, in so far as the present experiments go, the curve expressing the relation between the density and the velocity is somewhat concave towards the positive part of the axis along which the velocity is measured. That is to say, the increase of velocity is more rapid than the corresponding increase of density as the age of the formation becomes older.

VI. EFFECT OF MOISTURE.

In the present case, it being impossible to give specimens any desired amount of moisture, it is only intended to test whether the modulus of elasticity is or is not largely affected by the amount of moisture which might be present within the specimen.

To begin with, a specimen of sandstone was clamped in the usual way, and the wire was stretched with a known tension. When the specimen was in the ordinary dry state, it was found that the wire vibrated most violently when the half wave-length was

$$l=10{\cdot}39 \text{ cm.}$$

Then the specimen was wrapped in a wet cloth in its clamped state and fed with a constant supply of water dropped upon it for forty-two hours to moisten it throughout. It was then found that the corresponding half wave length had increased to about double, i.e.

$$l=20{\cdot}25 \text{ cm.}$$

which indicated that the modulus of elasticity had decreased to about one-fourth of its original value! Not without some doubt as to the result the author waited for one complete day till the specimen had apparently become dry, when it was found that it had nearly returned to its original state of elasticity; i.e.

$$l=10{\cdot}65 \text{ cm.}$$

On heating the specimen by hot air to drive out all the moisture, and then rapidly cooling it to the ordinary temperature, its elasticity increased slightly; i.e.

$$l=9{\cdot}89 \text{ cm.}$$

After three hours, it having returned to its ordinary state with regard to temperature and moisture, the elasticity became weakened to

$$l=10{\cdot}62 \text{ cm.}$$

The result of such cyclical observations shows clearly that the enormous diminution of the modulus of elasticity is actually caused by the effect of moisture. The difference between the initial $l=10{\cdot}39$ and the final $l=10{\cdot}62$ may be due to some variations of surrounding conditions,—probably a little relaxation of the clamp. As the first clamp is relaxed it results in an increase of the effective length of the specimen. The last difference corresponds to an increase of one mm. (or a little more) of the effective length.

The following result of the experiment may serve to give a rough notion as to the effect of moisture.

Rock.	Kind and age.	Density.		Elasticity × 11⁻¹⁰	
		Dry.	Wet.	Dry.	Wet.
Sandstone.	Sedimentary, Mesozoic.	2·230	2·351	10·15	3·17
Mica schist.	Metamorphic, Palæozoic.	2·647	2·669	17·54	9·21
Serpentine.	Eruptive, Archæan.	2·708	2·711	74·55	62.86

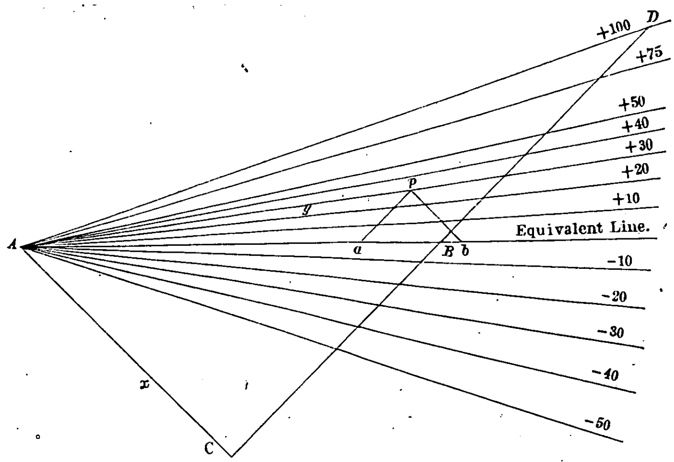
A glance at the above table will show what an important effect on the elastic nature of the earth crust is produced by the drops of water, which permeate beneath the earth's surface.

To study more closely the effect of moisture, and especially its effect as combined with high temperatures, a special arrangement is now in preparation, the author hopes to be able to give some observations on that special subject in the near future.

VII. RELATION BETWEEN THE KINETIC AND STATIC VALUES.

Of the one hundred and fifty eight specimens given in the table, a greater part have their elastic constants determined according to the statical method by either Professor H. NAGAOKA or by the author himself. Now it is of no small interest to compare the kinetic and the static values for one and the same specimen of rocks.

A short remark may here be added with respect to a new system of co-ordinates specially fitted for percentage representation.



Take a rightangled-isoscelestriangle ABC with its vertex C below the horizontal base AB. Divide each of the three sides into one hundred equal segments, and also in the produced part BD mark off any number of segments equal to one of the last. Join the vertex A with each of the points of section on the side BC and its produced part.

Let α and β be the corresponding values of any physical quantity in two different conditions $\mathfrak{A}$ and $\mathfrak{B}$. From A measure a length Ax equal to α, the length of one segment being taken as the unit of measurement, on the side AC or its produced part. Then, from the end point x measure again a length xy equal to β in the direction parallel to the other side CB. If the last point y falls on the n^{th} of the previously drawn lines numbered from the base AB, then the physical quantity is said to have increased or decreased n percent while the condition varied from $\mathfrak{A}$ to $\mathfrak{B}$, according as the line is above or below the base.

These premised, we have a new system of co-ordinates, in which the radius vector and the percentage are two independent variables. It resembles a system of polar co-ordinates, but one variable 'percentage' is not proportional to an angle measured from any fixed line. If a point p (ρ_1, n_1) is given, then describe a rightangled-isosceles triangle apb, with the given point as the vertex and its base ab coinciding with AB. The two segments Aa and Ab represent the corresponding values in the two different conditions referred to, provided one of the last segments is taken as the unit of measurement.

In Fig. 2, the modulus of elasticity is represented in this co-ordinate system, its static and kinetic values corresponding to α and β respectively. It will be seen, at a glance, that when the radius vector is relatively small almost every point lies on the positive side, while a majority of those in which the radius vector is somewhat greater lie on the negative side. Generally speaking, so far as is shown by the present experiment, the percentage is enormously large for a small radius vector, but diminishes as the radius vector increases and ultimately it becomes even negative.

That the percentage diminishes as the radius vector increases is a matter of course, in so far as the phenomenon of yielding is the principal cause which makes the two values differ from each other. A negative value, however, can never be expected unless some other cause or causes exist beside the phenomenon of yielding.

Examining more closely, we see that the percentage rapidly diminishes and becomes even negative especially for those rocks which have a foliated structure as in the case of schists and slates. In schistose rocks, the percentage is generally negative as they have large moduli of elasticity and, at the same time, a distinctly foliated structure. Any eruptive rock, of whatever age it may be, has a positive percentage. Of sedimentary rocks, those in which the moduli of elasticity are comparatively small have also a positive percentage, but some of them having large moduli may have negative percentages. The following table will show the facts more clearly.

Rock.	Total no. of specimens.	Mean value of the percentage.	Mean value of the modulus of elasticity.
Sandstone.	6	+38	$1{\cdot}57 \times 10^{11}$
Tuff.	7	+23	1·92
Andesite.	13	+20	2·12
Granite.	5	+11	3·14
Slate.	17	− 6	4·69
Peridotite-serpentine.	6	+ 4	6·29
Schists.	10	−10	6·39

Here it must be noted that I do not mean to say that the percentage would be always negative for any old rock having a distinctly foliated structure. That the percentage diminishes as the radius vector increases would in all probability be true; its becoming negative, however, may be due to some other causes. The two series of experiments have been made under very different surrounding conditions. The effect of moisture, for instance, is never negligible as some observations were made during the wet summer season; while the others, during the dry cold of winter.

Any complete discussion of the relation between the static and the kinetic values must be postponed, at least, until the effects of temperature and moisture on the elastic constants have been clearly investigated. The numerical values hitherto given for the moduli of elasticity of several rocks must be taken only as rough estimates.

The only closing remark that can be properly added is that the result of these experiments as a whole has only brought the author, who at first sight thought he had found a quarry of research that would soon be exhausted, to the threshold of a labyrinth, where many paths invites him to proceed further.

In conclusion, I wish to express my great indebtedness to Dr. N. FUKUCHI of the Geological Laboratory, for valuable information concerning the geological characters of the specimens. My best thanks are due to Professor H. NAGAOKA, under whose kind guidance I have carried out the experiments.

June 1905.

Physical Laboratory, College of Science,
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Modulus of Elasticity for 158 Specimens of Rocks.

Specimen No.	Rock.	Locality.	Kind.	Density.	Modulus of Elasticity. E_1	E_2	E	Velocity of longit. wave.
		ARCHÆAN ROCKS.			c.g.s. × 10^{11}			Km./Sec.
50.*	Chlorite schist.	Chichibu.	Metamorphic.	2·96	9·79	12·39	11·09	6·12
9.*	"	"	"	2·98	10·69	10·88	10·78	6·02
7_2.	Peridotite (Serpentinized).	Kuji, Ibaraki.	Eruptive (Altered).	2·61	7·72	9·74	8·73	5·78
8_1.	Serpentine.	Chichibu.	"	2·72	9·05	6·98	8·02	5·43
46_1.	Quartz schist.	Kashiwagi, Gumma.	Metamorphic.	2·64	5·99	8·91	7·45	5·31
31_1.	Quartz schist.	Onishi, Gumma.	Metamorphic.	2·67	7·66	7·19	7·43	5·28
18_1.	Chlorite schist.	Nogami, Chichibu.	"	2·88	6·59	7·19	6·89	4·89
16.*	Peridotite serpentine.	Ibaraki.	Eruptive.	2.83	6·55	7·07	6·81	4·91
22_1.	Gabbro.	Nogami, Chichibu.	"	2·71	5·95	7·26	6·61	4·94
26_2.	Chlorite schist.	Kunikami, Chichibu.	Metamorphic.	2·87	5·21	6·35	5·78	4·49
91_1.	Chlorite schist.	Chichibu.	Metamorphic.	2·88	5·78	5·37	5·58	4·41
45_1.	"	Nogami, Chichibu.	"	2·77	5·46	5·54	5·50	4·46
26_1.	"	Kunikami, Chichibu.	"	2·82	5·55	5·35†	5·45	4·39
28.	Graphite schist.	Onishi, Gumma.	"	2·59	5·39	5·07	5·23	4·50
41_a.*	Peridotite serpentine.	Kuji, Ibaraki.	Eruptive.	2·78	5·13	4·96	5·04	4·27
24_1.	Graphite schist.	Kunikami, Chichibu.	Metamorphic.	2·59	4·88	5·15	5·02	4·41
49_1.	Chlorite schist.	Nogami, Chichibu.	"	2·77	4·84	5·00	4·92	4·22
41_b.*	Peridotite serpentine.	Kuji, Ibaraki.	Eruptive.	2·79	4·52	4·86	4·69	4·10
60_1.	Gneiss.	Shinshiro, Mikawa.	Metamorphic.	2·59	4·02	4·98	4·50	4·17
17.*	Peridotite serpentine.	Kuji, Ibaraki.	Eruptive.	2·57	4·21	4·67	4·44	4·16
47_1.	Quartz schist.	Onishi, Gumma.	Metamorphic.	2·63	4·82	3·34	4·08	3·94
23_1.	Graphite schist.	Kunikami, Chichibu.	"	2·56	4·05	4·02	4·04	3·98
30.	Chlorite schist (Decomposed).	Onishi, Gumma.	"	2·64	3·96	4·11	4·03	3·91
		PLAÆOZOIC ROCKS.						
67_1.	Schalsteine.	Asakuma-dake, Ise.	Sedimentary.	2·98	9·35	9·90	9·62	5·68
12_2.	Clayslate.	Miyanomaye, Aumi.	"	2·71	9·26	8·84	9·05	5·78
69_1.	Pyroxenite.	Isosuzu-gawa, Ise.	Metamorphic.	3·05	8·85	8·87	8·86	5·40
78_b.*	Schalsteine.	China.	Sedimentary.	2·77	8·01	8·60	8·30	5·48
78_a.*	"	"	"	2·77	8·34	7·87	8·11	5·41

(Continued.)

Specimen No.	Rock.	Locality.	Kind.	Density.	Modulus of Elasticity.			Velocity of longit. wave.
					E_1	E_2	E	
65_1.	Schalsteine.	Kamuro, Kii.	Sedimentary.	2·84	8·32	7·80	8·06	5·33
71_1.	Limestone.	Akasaka, Gifu.	"	2·71	7·82	7·54	7·68	5·33
32_1.	Pyroxenite.	Hominoyama, Gumma.	"	2·90	7·81	7·52	7·67	5·15
2_b.*	Clayslate.	?	"	2·69	7·61	7·47	7·54	5·30
34_1.	Adinolslate.	Yonowo, Gumma.	"	2·64	7·54	7·45	7·50	5·33
13.*	Limestone.	Kanagawa.	Sedimentary.	2·65	7·43	7·52	7·48	5·31
2_a.*	Clayslate.	?	"	2·67	7·77	7·13	7·45	5·28
29.*	Limestone.	Mikawa.	"	2·68	6·68	7·11	6·89	5·07
79.*	Schalsteine.	Rikuchū.	"	2·65	6·75	6·70	6·72	5·04
74_1.	Limestone.	Akasaka, Gifu.	"	2·69	6·66	6·65	6·65	4·97
55.*	Limestone.	Musashi.	Sedimentary.	2·63	6·55	6·62	6·58	5·01
71_2.	"	Akasaka, Gifu.	"	2·69	6·67	6·30	6·48	4·91
11_b.*	Marble.	Ibaraki.	"	2·63	6·26	6·14†	6·20	4·86
73.*	Sandyslate.	Rikuchū.	"	2·64	6·13	6·05	6·09	4·81
80.*	Schalsteine.	"	"	2·82	5·77	6·23	6·00	4·62
72_1.	Limestone.	Akasaka, Gifu.	Sedimentary.	2·69	5·87	6·13	6·00	4·72
59_1.	Granite.	Okazaki, Mikawa.	Eruptive.	2·63	5·92	5·94	5·93	4·75
74.*	Clayslate.	Nikkō.	Sedimentary.	2·15	5·69	5·72	5·70	5·15
1_a.*	Weathered clayslate.	Umebatake.	"	2·31	5·13	5·93	5·53	4·89
87_1.	Ophicalcite.	Arakawa, Chichibu.	Eruptive.	2·65	4·91	5·30	5·11	4·39
21_1.	Limestone.	Nogami, Chichibu.	Sedimentary.	2·64	5·31	4·76	5·04	4·37
1_b.*	Weathered clayslate.	Umebatake.	"	2·30	4·97	4·58	4·77	4·55
9_1.	Granite.	Mikage, Settsu.	Eruptive.	2·54	4·88	4·61	4·75	4·32
76_1.	Slate.	Atago, Yamashiro.	Sedimentary.	2·24	4·61	4·83	4·72	4·59
40_1.	Micaschist.	Fudō-tōge, Ibaraki.	Matemorphic.	2·54	4.25	4·81	4.53	4.22
69*.	Granite.	Kagawa.	Eruptive.	2·57	4·53	4·42	4·47	4·17
81_1.	"	Tamba.	"	2·62	4·46	4·46	4·46	4·13
64_1.	"	Nishiura, Mikawa.	"	2·61	4·53	4·30	4·42	4·11
66_2.	"	Shirakawa, Kyōto.	"	2·62	4·28	4·01	4·14	3·98
79_1.	Slate.	Narutaki, Kyōto.	Sedimentary.	2·45	3·90	4·26	4·08	4·09
75_1.	Limestone.	Akasaka, Gifu.	Sedimentary.	2·72	3·55	4·59†	4·07	3·87
66_1.	Granite.	Shirakawa, Kyōto.	Eruptive.	2·62	4·03	—	—	3·93
6_3.	Marble.	Maiyama, Ibaraki.	Sedimentary (Metamorphosed.)	2·68	3·48	3·89	3·69	3·71
3_b.*	Clayslate.	Tamba.	"	2·39	3·63	—	—	3·90
60_b.*	Weathered clayslate.	Yamashiro.	"	2·31	3·65	3·49	3·57	3·94

(*Continued.*)

Specimen No.	Rock.	Locality.	Kind.	Density.	Modulus of Elasticity.			Velocity of longit. wave.
					E_1	E_2	E	
44_1.	Granite.	Hyōgo, Settsu.	Eruptive.	2·59	3·49	3·33	3·41	3·63
63_2.	"	Nishiura, Mikawa.	"	2·84	3·35	3·37	3·36	3·44
12_a.*	Marble.	Ibaraki.	Sedimentary.	2·65	3·35†	—	—	3·56
10_2.	Granite.	Kitaki, Kagawa.	Eruptive.	2·57	3·24	—	—	3·55
63_1.	"	Nishiura, Mikawa.	"	2·84	3·40	3·02†	3·21	3·36
60_a.*	Weathered clayslate.	Yamashiro.	Sedimentary.	2·32	3·66	2·72†	3·19	3·72
3_a.*	Clayslate.	Tamba.	"	2·37	3·17	—	—	3·65
78_2.	Slate.	Narutaki, Kyōto.	"	2·42	3·13	3·16	3·14	3·59
12_b.*	Marble.	Ibaraki.	"	2·65	3·68†	2·60†	3·14	3·44
78_1.	Slate.	Narutaki, Kyōto.	"	2·44	3·08	—	—	3·55
80_1.	Slate.	Atago, Kyōto.	Sedimentary.	2·36	2·99	3·00	3·00	3·56
27_1.	Micaschist.	Motoizumi, Chichibu.	Metamorphic.	2·63	2·71	3·15	2·93	3·35
29_1.	Limestone.	Onishi, Gumma.	"	2·66	2·40	2·85	2·63	3·15
64_a.*	Clayslate.	Tochigi.	Sedimentary.	2·46	2·63	2·46	2·54	3·21
82_1.	Contact slate.	Tamba.	"	2·33	2·75	2·23	2·49	3·27
68.*	Granite.	Kitaki, Kagawa.	Eruptive.	2·55	2·19	1·67†	1·98	2·79
52.*	"	Ibaraki.	"	2·50	1·80†	—	—	2·68
7_a.*	Weathered clayslate.	"	Sedimentary.	2·50	1·53	1·58	1·56	2·50
65_a.*	"	"	"	2·49	1·54	1·53	1·53	2·48
7_b.*	"	"	"	2·50	1·51	1·44	1·47	2·43
65_b.*	Weathered clayslate.	Ibaraki.	Sedimentary.	2·50	1·38	1·37	1·37	2·34
56.*	Granite.	"	Eruptive.	2·53	1·36	1·20	1·28	2·25
71.*	"	"	"	2·59	1·28	1·23	1·25	2·20
42_1.	Mica schist.	Tsukioka, Ibaraki.	Metamorphic.	2·64	1·08	1·29	1·19	2·12
61.	Pegmatite.	Hōjō, Ibaraki.	Eruptive.	2·57	1·16	1·10	1·13	2·10

MESOZOIC ROCKS.

Specimen No.	Rock.	Locality.	Kind.	Density.	E_1	E_2	E	Velocity of longit. wave.
53.*	Clayslate.	Rikuzen.	Sedimentary.	2·70	7·80	7·39	7·59	5·30
77.*	Schalsteine.	Akamagaseki.	"	2·78	6·46	6·52	6·49	4·85
72.*	Clayslate.	Rikuzen.	"	2·71	6·54	6·38	6·46	4·89
76.*	"	?	"	2·71	6·42	6·38	6·40	4·86
62_a.*	"	Tsushima.	"	2·68	4·39	4·25	4·32	4·02
68_1.	Granite.	Yoshima, Kagawa.	Eruptive.	2·61	3·42	3·40	3·41	3·62
5.*	Sandstone.	Kii.	Sedimentary.	2·22	1·32	1·50	1·41	2·52
85_1.	"	Kyūshū.	"	2·34	1·29	1·30	1·30	2·36
6_b.*	"	Kii.	"	2·22	1·23	1·00	1·11	2·23
83_1.	"	Kyūshū.	"	2·09	1·03	1·07	1·05	2·24

(*Continued.*)

Specimen No.	Rock.	Locality.	Kind.	Density.	Modulus of Elasticity.			Velocity of longit. wave.
					E_1	E_2	E	
84_1.	Sandstone.	Kyūshū.	Sedimentary.	2·24	1·02	1·05	1·03	2·15
6_a.*	„	Kii.	„	2·24	1·02	0·95†	0·98	2·09

CAINOZOIC ROCKS.

Specimen No.	Rock.	Locality.	Kind.	Density.	E_1	E_2	E	Velocity of longit. wave.
51_1.	Andesite.	Shinano.	Eruptive.	2·61	4·37	4·35	4·36	4·09
17_2.	„	Haruna, Gumma.	„	2·63	4·17	3·94	4·06	3·94
1_1.	„	Nebukawa, Sagami.	„	2·59	3·96	—	—	3·91
54.*	„	„	„	2·56	4·12	3·53	3·83	3·87
35_1.	Sandstone.	Mitagawa, Chichibu.	Sedimentary.	2·47	3·65	3·41	3·53	3·78
1_2.	Andesite.	Nebukawa, Sagami.	Eruptive.	2·59	3·45	3·59	3·52	3·69
15*.	„	Komatsu, Sagami.	„	2·20	3·32	3·10	3·21	3·82
52_1.	Rhyolite.	Hōraiji, Mikawa.	„	2·20	3·20	3·08	3·14	3·80
57_1.	Porphylite.	Arumi, Mikawa.	„	2·29	3·31	2·94	3·12	3·70
51.*	Rhyolite.	Honbun, Izu.	„	2·32	3·12	3·10	3·11	3·66
50.	Andesite.	Nakatsugaru, Mutsu.	Eruptive.	2·69	2·79	2·90	2·85	3·26
14.*	Tuff.	Nanasawa, Sagami.	Sedimentary.	2·22	2·91	2·66	2·79	3·55
28.*	Andesite.	Izu.	Eruptive.	2·17	2·49	2·87	2·68	3·52
30.*	Tuff.	Kiga, Izu.	Sedimentary.	2·17	2·51	2·64	2·57	3·45
8_a.*	Rhyolite tuff.	Iyo.	„	2·35	2·48	2·36	2·42	3·21
39.*	Andesite.	Yokone, Izu.	Eruptive.	2·40	2·39	2·35	2·37	3·14
10.*	Tuff.	Yoshida, Izu.	Sedimentary.	2·28	2·29	2·39	2·34	3·21
70.*	Andesite.	Komatsu, Sagami.	Eruptive.	2·46	2·40	2·23	2·31	3·07
58_1.	Sandstone.	Yebi, Mikawa.	Sedimentary.	2·21	2·33	2·27	2·30	3·23
8_b.*	Rhyolite tuff.	Iyo.	„	2·32	2·22	2·38	2·30	3·15
19_b.*	Tufaceus sandstone.	Tomioka, Gumma.	Sedimentary.	2·32	2·15	2·31	2·23	3·10
56_1.	„	Yebi, Mikawa.	„	2·12	2·17	2·12	2·15	3·19
63_a.*	Rhyolite tuff.	Nakura, Mikawa.	„	2·15	2·15	2·11	2·13	3·14
19_a.*	Tufaceus sandstone.	Tomioka, Gumma.	„	2·31	2·08	2·01	2·04	2·97
59_b.*	Rhyolite.	Gumma.	Eruptive.	2·45	1·94	1·93†	1·94	3·82
53_1.	Rhyolite tuff (Weathered).	Hōraiji, Mikawa.	Sedimentary.	2·16	1·78	1·87	1·83	2·91
4_a.*	Tuff.	Sawada, Izu.	„	1·84	1·78	1·84†	1·81	3·14
59_a.*	Rhyolite.	Gumma.	Eruptive.	2·47	1·88	1·64	1·76	2·67
67_a.*	Andesite tuff.	Midera, Yechizen.	Sedimentary.	2·44	1·74	1·73	1·73	2·66
57.*	Tuff.	Nawazi, Izu.	„	2·04	1·67	1·70†	1·69	2·88

(*Continued.*)

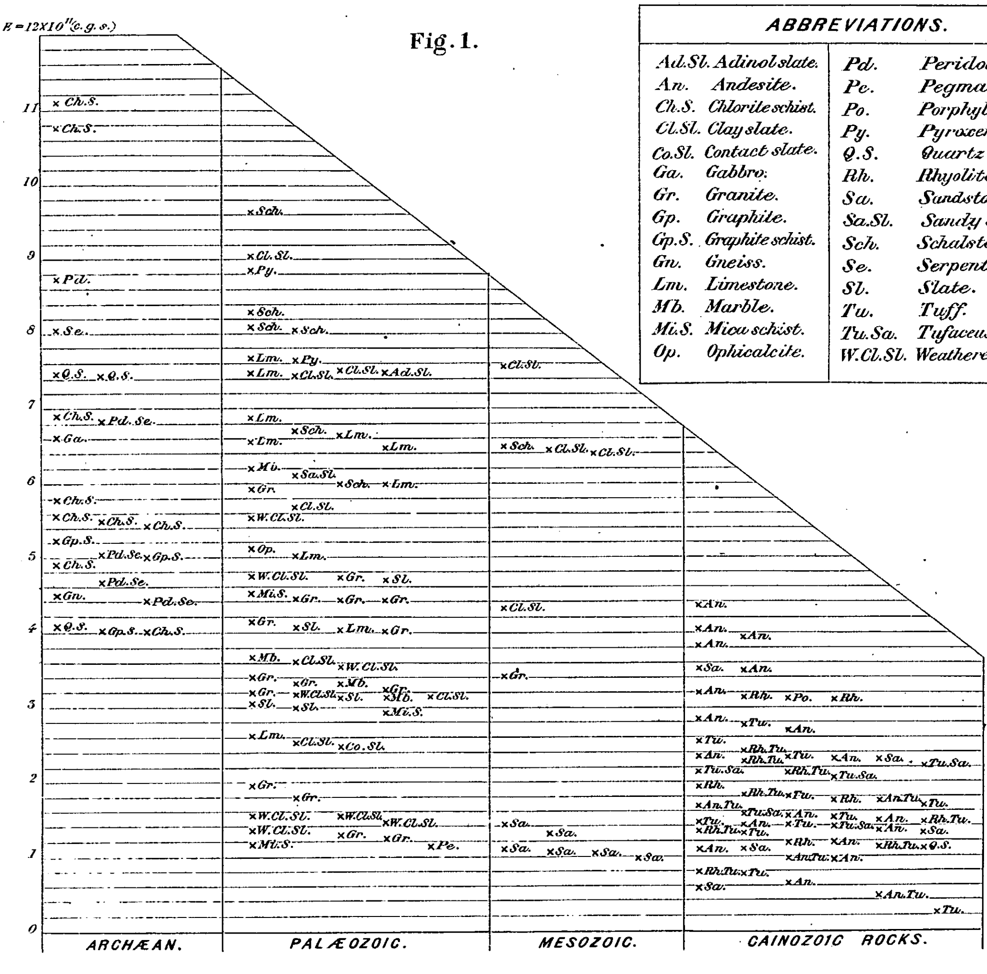
Specimen No.	Rock.	Locality.	Kind.	Density.	Modulus of Elasticity.			Velocity of longit. wave.
					E_1	E_2	E	
67_b.*	Andesite tuff.	Midera, Yechizen.	Sedimentary.	2·40	1·63	1·72	1·68	2·65
54_1.	Tufaceus sandstone.	Nebukawa, Sagami.	„	2·07	1·55	1·63	1·59	2·77
46_b.*	Andesite.	Manazuru, Sagami.	Eruptive.	2·55	1·56	1·55	1·55	2·47
2_1.	Tuff.	Kawatsu, Izu.	Sedimentary.	1·91	—	1·50	—	2·80
40.*	Andesite.	Sagami.	Eruptive.	2·30	1·54	1·45	1·49	2·54
61_b.*	Rhyolite tuff.	Iwashiro.	Sedimentary.	2·20	1·46	1·48	1·47	2·59
36.*	Tuff.	Izu.	„	1·85	1·65	1·27	1·46	2·81
49_b.*	Andesite.	„	Eruptive.	2·21	1·46	1·39	1·42	2·54
4_b.*	Tuff.	Kawatsu, Izu.	Sedimentary.	1·82	1·45	1·39	1·42	2·79
55_1.	Tufaceus sandstone.	Hōraiji, Mikawa.	„	2·09	1·44	1·35	1·39	2·58
47.*	Andesite.	Yenoura, Shiruga.	Eruptive.	2·43	1·36	1·35	1·35	2·36
61_a.*	Rhyolite tuff.	Iwashiro.	Sedimentary.	2·23	1·37	1·30	1·34	2·45
22.*	Tuff.	Teishi, Izu.	„	2·01	1·30	1·35	1·32	2·56
58.*	Sandstone.	Chōshi, Chiba.	„	2·35	1·35	1·26	1·30	2·35
5_3.	Rhyolite.	Midera, Yechizen.	Eruptive.	2·40	1·09	1·28	1·18	2·22
16.	Andesite.	Haruna, Gumma.	Eruptive.	2·32	1·21	1·14	1·18	2·26
32.*	Rhyolite tuff.	Mitaka, Izu.	Sedimentary.	1·89	1·21	1·02	1·11	2·42
37.*	Quartz sandstone.	Hizen.	„	2·23	1·02	1·21	1·11	2·23
38.*	Andesite.	Izu.	Eruptive.	1·94	1·05	1·17	1·11	2·39
4_3.	Sandstone.	Shinjō, Kii.	Sedimentary.	2·25	1·04	1·13	1·08	2·19
23.*	Andesite tuff.	Yema, Izu.	Sedimentary.	1·83	1·00	0·98	0·99	2·33
34.*	Andesite.	Manazuru, Sagami.	Eruptive.	2·02	0·96	0·95	0·95	2·17
66.*	Rhyolite tuff.	Amakusa, Hizen.	Sedimentary.	2·26	0·96†	0·62†	0·79	1·87
31.*	Tuff.	Teishi, Izu.	„	1·92	0·78	0·77	0·78	2·02
48.*	Andesite	Izu.	Eruptive.	2·10	0·63	—	—	1·73
3_2.	Sandstone.	Chōshi, Chiba.	Sedimentary.	2·20	0·56	0·62	0·59	1·63
21.*	Andesite tuff.	Izu.	„	1·50	0·42	0·50	0·46	1·76
35.*	Tuff.	„	„	1·29	0·26†	—	—	1·43

N.B. The specimens marked with * are those whose elastic constants were statically determined by Professor H. NAGAOKA and published in "The Pub. of the E.I.C. in F.L." No. 4. 1900. and Phil. Mag. 1900.

The specimens marked with † are those which were found to have been crushed or broken at the clamped section so that the value given in the table may be erroneous.

MODULUS OF ELASTICITY.

Fig. 1.



ABBREVIATIONS.

Ad.Sl.	Adinol slate.	Pd.	Peridotite.
An.	Andesite.	Pe.	Pegmatite.
Ch.S.	Chlorite schist.	Po.	Porphylite.
Cl.Sl.	Clay slate.	Py.	Pyroxenite.
Co.Sl.	Contact slate.	Q.S.	Quartz schist.
Ga.	Gabbro.	Rh.	Rhyolite.
Gr.	Granite.	Sa.	Sandstone.
Gp.	Graphite.	Sa.Sl.	Sandy slate.
Gp.S.	Graphite schist.	Sch.	Schalsteine.
Gn.	Gneiss.	Se.	Serpentine.
Lm.	Limestone.	Sl.	Slate.
Mb.	Marble.	Tu.	Tuff.
Mi.S.	Mica schist.	Tu.Sa.	Tufaceus sandstone.
Op.	Ophicalcite.	W.Cl.Sl.	Weathered Cl. Sl.

Fig. 2.