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Change of the Modulus of Rigidity in Ferromagnetic Substances by Magnetization.

By

K. Honda, *Rigakushi*, S. Shimizu, *Rigakushi*,

and

S. Kusakabe, *Rigakushi*.

1. We have already seen that the change of elasticity by magnetization is not so small as has generally been admitted. The present experiment deals with the change of rigidity by magnetization. The investigation is especially important, inasmuch as the change of rigidity by magnetization is reciprocally related to that of magnetization by torsion.

In the course of his experiments on the mutual relations between torsion and magnetism, G. Wiedemann[1] observed that the torsion of an iron wire was diminished by magnetization. This shows an increase of rigidity. C. Barus[2] hung two identical iron wires in the same vertical line, separated by a rigid piece of brass, which carried the index mirror; to the lower end of the wire, a weight was attached. The wire was twisted and then either the upper or the lower end of the system fastened. If both ends were twisted equally

1) Wiedemann's Electricität III, 796.
2) Barus, Amer. Jour; **34**, 175, 1887; Phy. Rev. XIII, 257, 1901.

in opposite directions, the position of the mirror remained unchanged. A magnetizing coil was placed co-axially with the upper wire. If the rigidity of the wire were changed by magnetization, the mirror would rotate in either direction. Barus found that for soft iron and steel, the change of rigidity was 0.24% and 0.08% respectively. In his later experiment, he observed a change of rigidity amounting to 1% for soft iron. With a similar arrangement, H. D. Day[1] investigated the same subject in iron. He found that the change of rigidity increased with the field, and that it became generally less as the initial twist was increased. The maximum value obtained was only 0.8%.

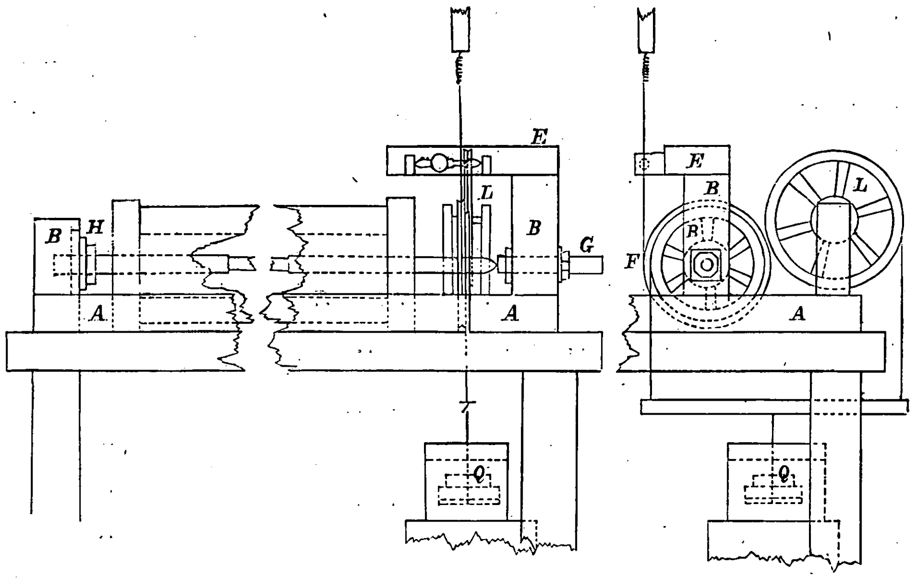
In the experiments of Barus and Day, the tensile stress, which was found to produce a notable effect, would complicate the change to be sought for. Moreover, the lower wire was not perfectly free from magnetization, and the mirror would not give the perfect differential effect.

The experiment of J. S. Stevens[2] for iron and steel rods gave an increase of rigidity by magnetization. The change amounted to 2.3% for soft iron and 0.43% for steel in a field of 138 C.G.S. units. It also increased with the magnetizing force. In his experiment, the length of the magnetizing coil was much less than that of the rod, so that the magnetization was far from being uniform.

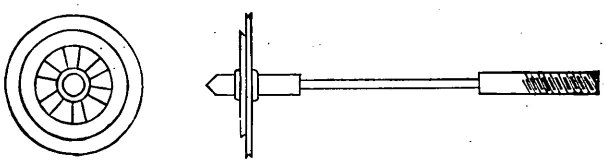
2. Our method of twisting the ferromagnetic rod was the same as that of Professor Nagaoka[3] used for studying the elastic constants of rocks; but the sensibility of the apparatus was 106.0 times greater for the same scale-distance.

The front and side views of the apparatus are given in the annexed figures.

1) Day, Electrician, **39**, 480, 1897.
2) J. S. Stevens, Phy. Rev. (3) **10**, 161, 1900.
3) Phil. Mag. **50**, 53, 1900.



AA was a stout wooden frame rectangular in shape. BB were the parts projecting from the frame ; to the one, a brass rod, to which a ferromagnetic rod was soldered, was clamped by means of a screw nut H, and to the other, a screw G, which carried an agate cup in one of its extremities, was clamped. F was a double pulley whose axis was a thick brass cylinder ; a point made of nonmagnetic nickel steel was firmly fixed to one of its extremities, while the ferromagnetic rod was soldered to the other, as shown in the following figure. The



inner circumference of the pulley served to twist the rod, and the other to multiply the sensibility of the apparatus. C was a magnetizing coil and E a block of wood, to which a rotating cylinder was affixed as in the case of the former experiment. A fine copper wire, well annealed by passing through it an electric current, was attached to a point on the outer circumference of the pulley and went vertically upward around it. The wire after passing round the cylinder was stretched by a weak spring in the usual way. The deflection of the mirror attached to the rotating cylinder was observed by means of a scale and telescope. The details of the apparatus will be easily understood from the above figures.

The dimensions of each part of our arrangement were as follows :—

Length of the coil....................	=	30.0	cm,
Its internal diameter..................	=	3.0	,,
$4\pi n$	=	379.7	,,
Outer radius of the pulley }	=	8.93	,,
Inner ,, ,, ,, ,, }	=	7.15	,,
Diameter of the rotating }	=	0.160	,,
cylinders }	=	0.280	,,
Diameter of the copper wire.........	=	0.008	,,
Scale-distance	=	230.8	,,

If a ferromagnetic rod is twisted through a small angle φ, the thin vertical wire is pulled down through a small distance $R\varphi$, R being the outer radius of the pulley. This causes an elongation of the weak spring attached to the copper wire, and consequently the rotating cylinder, whose radius is r, is turned through an angle $R\varphi/r$ Hence the angle of torsion is magnified in the ratio R : r ; in the actual calculation, we must take into account the thickness of the thin wire. The ratio was in our case 106 : 1 ; with this arrangement, we were able to measure a change of angle amounting to only $1.92'' \times 10^{-3}$ per cm of the ferromagnetic rod.

3. The measurement was conducted in the following order. The specimen to be tested was fixed in the axial line of the magnetizing coil so as to lie nearly in a uniform field. If the steel pivot on one end of the bar carrying the specimen was left free, and a magnetizing current passed through the coil, a deflection of the mirror was observed, though there was no twisting couple. The deflection

is evidently not due to the twisting of the rod, but to its bending by magnetization. The case corresponds to the experiment of Guillemin described in the preceding paper. The nature of the deflection and its amount coincided with the change of elasticity by magnetization, which we have already studied.

The steel pivot was then slightly brought in contact with the agate cup; if this contact was made in a suitable degree, the deflection due to magnetization, when acted on by no twisting couple, could be made negligibly small. In case the deflection could not be sufficiently reduced, it was always corrected for. The contact being so adjusted, a couple was applied by suspending a weight. The tension of the copper wire was next adjusted and the working of the apparatus tested by adding successively weights of 1, 10, 50 grams to the pan. If the deflections of the mirror were proportional, the adjustment was considered to be correct.

To begin with, the ferromagnetic rod was demagnetized by reversals, and then a current passed, taking the deflection as soon as possible. These processes were repeated with successively increasing currents. In order to prevent minute oscillations of the mirror, the thin copper wire and the mirror should be protected from the air currents.

The resistance of the magnetizing coil was only $0.6\,\Omega$, so that the heating of the core due to current was negligibly small up to the strongest current used in the present experiment, and the creeping of the image of the scale was not at all observed. But we were careful to read the deflection as quickly as possible.

Since the couples corresponding to 1, 10, 50 grams produced torsions proportional to their respective weights, the friction at the pivot did not seem to disturb our results. That the observed deflection was really due to a rotation, but not to the depression or elevation of the pulley, was verified in the following way. The fine copper wire was fixed to the axial line of the pulley and the depression or elevation of the axis itself due to magnetization of the ferromagnetic rod was observed by means of a rotating cylinder carrying a mirror. A minute deflection amounting to only a fraction of a millimeter was noticed, whereas the deflection was several centimeters when the copper wire was fixed to a point on the circumference of the pulley. Hence the actual depression or elevation of the axis, if any, was negligibly small.

The samples to be tested were the same as those used in the former experiment, except in the case of the nickel rod.

In the present experiment, the length of each rod was reduced to 22 cm, and the diameter of the cobalt bar also to 1.082 cm. The nickel rod, used in the former experiment was turned into a square rod from a plate, and the mechanical process, which the specimen underwent, hardened it in magnetic quality. Moreover the nickel was not sufficiently thick for the torsion experiment, so that another nickel bar, the diameter of which was 1.117 cm, was substituted for it. The new specimen was turned into a cylindrical form from a thick bar, and was magnetically much softer.

Our apparatus was not suited for the absolute measurement of the modulus of rigidity and therefore its determination was, in the usual manner, carried on with Professor Nagaoka's apparatus above referred to. The results were:

Metal	Soft iron	Steel	Wolfram steel	Nickel	Cobalt
Rigidity	7.92×10^{11}	7.89×10^{11}	8.57×10^{11}	7.41×10^{11}	6.04×10^{11}

4. *Soft iron.* The rigidity of soft iron is always increased by magnetization, as will be seen from the following table and Fig. 1.

SOFT IRON.

$N=7.89\times10^6$ $\theta=22''.11$		$N=14.36$ $\theta=40''.21$		$N=20.87\times10^6$ $\theta=58''.45$		$N=27.48\times10^6$ $\theta=76''.96$		$N=33.96\times10^6$ $\theta=95''.13$	
H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$
7.5	4	7.3	14	7.3	15	7.3	23	7.3	25
11.0	11	11.1	25	11.1	38	11.1	50	11.1	50
17.2	34	17.0	58	17.2	84	17.0	105	17.0	119
29.6	61	28.6	123	29.4	186	28.6	218	28.6	260
57.4	117	55.5	224	56.8	329	54.3	406	55.8	481
99.2	159	92.4	299	98.4	446	92.4	550	96.4	661
189.9	193	187.8	379	171.0	530	169.9	676	169.9	783
295.4	214	284.7	423	295.4	603	295.4	776	295.4	908
542.9	230	442.9	456	542.9	649	438.3	843	443.0	994
587.5	247	587.5	477	585.3	680	580.9	887	583.1	1054
802.9	257	802.9	500	802.9	711	800.5	923	805.1	1095

Here N denotes the moment of force expressed in C.G.S. units, θ'' the angle of torsion per cm corresponding to the moment, as calculated from the rigidity, and $\delta\theta''$ the observed change of torsion due to magnetization, given in seconds of arc. $\delta\theta$ is taken positive when the change of twist indicates an increase of rigidity and taken negative when it indicates a decrease. H is the effective force ($=H_0-IN$).

Thus, the untwisting of the rod always increases with magnetization, its amount increasing in the same way that the intensity of magnetization is related to magnetizing force. As the moment of force increases, the amount of untwisting increases proportionally, so that the change of rigidity is fairly independent of the twisting couple for all magnetizing fields. The form of the curves is similar to that of the curves of depression in the former experiment, except in very weak fields. In the present case, the initial minute depression of the curves was not observed.

From the angle of torsion and its change, we calculated the ratio of the change (δK) to the rigidity (K) itself. The ratio is fairly independent of the twisting couple for all fields; in the following table, mean values for the different couples are given.

H	20	60	100	200	400	600	800
$\frac{\delta K}{K}$	0.0019	0.0058	0.0076	0.0096	0.0110	0.0118	0.0122

These values are also plotted against the magnetizing force in Fig. 2; the course of the curve resembles that of magnetization, having one inflexion point and approaching to an asymptotic value as the field is increased.

The untwisting by magnetization forms a reciprocal relation to the well known fact that the magnetization of iron decreases by twisting.

The above results for soft iron agree in quality with those of previous experimenters, and the amount of the change nearly coincides with some of Barus' results. In the experiment of Day, the change of rigidity was a little smaller than in the present case, and was greatly influenced by the amount of the twisting couple in contradiction to our results. Stevens' experiment gave a much greater increase of rigidity.

Wolfram steel. As we have already found the change of elasticity in wolfram steel due to magnetization is nearly the same as that of soft iron both in quality and in quantity. This remark also applies to the present case, so that what we have said about the change of rigidity in soft iron equally applies to the case of wolfram steel.

This will be seen from the following tables and Figs. 3 and 2:

$N=7.89\times10^{6}$, $\theta=16.''58$		$N=14.36\times10^{6}$, $\theta=30.''17$		$N=20.87\times10^{6}$, $\theta=43.''85$		$N=27.48\times10^{6}$, $\theta=57.''74$		$N=33.96\times10^{6}$, $\theta=71.''37$	
H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$
15.3	6	15.3	21	15.4	31	15.3	27	15.4	33
19.2	27	19.1	62	19.1	85	19.1	88	19.2	121
23.6	60	22.7	108	23.6	163	23.5	188	23.6	223
35.1	90	33.1	167	33.1	240	34.9	267	35.2	348
55.0	115	54.6	206	54.6	289	54.5	360	54.8	441
92.1	138	91.1	246	91.1	354	90.3	446	92.1	546
169.5	156	169.5	294	169.5	414	169.5	529	169.5	658
290.1	175	291.0	321	289.5	456	289.1	596	289.1	731
437.6	183	436.9	342	436.9	490	434.1	631	429.8	783
579.1	194	579.1	356	579.1	510	576.9	664	574.6	818
785.6	200	787.8	375	785.6	533	783.6	702	783.4	864

H	20	60	100	200	400	600	800
$\frac{\delta K}{K}$	0.0015	0.0073	0.0085	0.0098	0.0110	0.0116	0.0122

5. *Steel.* We have seen that in steel, the change of elasticity by magnetization is much smaller than in soft iron. So, in the case of rigidity, we also observe a comparatively small increase. The following table contains the results of our observations :—

$N=7.89\times10^{6}$, $\theta=20.''82$		$N=14.36\times10^{6}$, $\theta=37.''88$		$N=20.87\times10^{6}$, $\theta=55.''05$		$N=27.48\times10^{6}$, $\theta=72.''50$		$N=33.96\times10^{6}$, $\theta=89.''60$	
H	$\delta\theta'''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$	H	$\delta\theta''\times10^{3}$
14.6	1	14.4	5	14.4	10	14.4	10	14.6	17
28.6	10	28.6	27	28.6	38	29.0	60	30.4	77
70.9	25	70.9	58	71.5	83	70.7	117	65.2	148
189.7	40	190.6	87	189.7	133	187.9	181	187.9	223
307.6	46	307·6	100	304.5	160	304.5	214	305.5	256
448.5	52	448.5	112	450.7	179	444.1	229	448.4	283
590.5	60	588.4	121	590.5	188	588.4	246	588.4	302
794.2	63	794.2	129	792.0	206	792.0	262	794.3	321

These numbers are graphically shown in Fig. 4. We see that the form of the curves is much less steep than in those for soft iron or wolfram steel.

Here again, the change of rigidity is independent of the applied couple; the values of $\frac{\delta K}{K}$ for different fields are given in the following table and in Fig. 2:—

H	60	200	400	600	800
$\frac{\delta K}{K}$	0.0013	0.0022	0.0029	0.0032	0.0035

The results of the previous experimenters fairly agree with those of the present case. The reciprocal relation between torsion and magnetism also holds for steel.

Cobalt. As in the case of steel, the effect of magnetization upon the rigidity of a cobalt bar is very small. The rigidity always increases by magnetization, as shown in the following tables and Figs. 5 and 2:—

$N=14.36\times10^6$ $\theta=36.''46$		$N=27.48\times10^6$ $\theta=69.''78$		$N=40.48\times10^6$ $\theta=102.''79$	
H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$
46.4	3	46.4	7	46.4	11
96.3	17	106.3	38	106.0	60
217.9	46	209.2	90	210.2	142
328.9	69	330.9	133	330.9	190
470.7	81	472.8	163	474.9	233
609.2	96	611.5	189	611.5	269
808.2	106	820.4	214	820.4	317

H	100	200	400	600	800
$\frac{\delta K}{K}$	0.0005	0.0012	0.0021	0.0028	0.0031

Thus the course of the curves is less steep in cobalt than in steel ; the inflexion point is not so marked in the former metal as in the latter. The change of rigidity is also independent of the applied couple.

So far as we are aware, the effect of torsion on the magnetization of cobalt has not yet been studied, on account of the difficulty of getting the specimen in the form of a wire. But if the reciprocal relation holds in the case of cobalt, the above results show that the effect of torsion on the magnetization of cobalt is the same as in iron. We have seen from the experiment of Professor Nagaoka and one of us that the character of cast cobalt as regards magnetostriction is remarkably different from that of annealed cobalt. The cobalt in the present experiment was well annealed, so that the above inference is to be restricted to an annealed cobalt.

6. *Nickel.* The change of rigidity of the nickel bar was so large that it was necessary to reduce the sensibility of the apparatus by using a rotating cylinder of greater diameter. As in the case of elasticity, we again observe in the metal the singular phenomenon that the change of torsion by magnetization alters its sign as the magnetizing force is increased. The following table contains the results of the observation, which are also drawn in Fig. 6.

$N=4.65\times10^6$ $\theta=8.''47$		$N=7.89\times10^6$ $\theta=14.''37$		$N=11.09\times10^6$ $\theta=20.''19$		$N=14.36\times10^6$ $\theta=26.''14$		$N=17.56\times10^6$ $=31.''96$	
H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$	H	$\delta\theta''\times10^3$
11.5	— 3	11.4	— 15	11.7	— 30	11.4	— 59	11.4	— 69
14.8	— 33	14.6	— 69	14.6	—102	14.6	—157	14.6	—177
24.5	—108	24.5	—190	24.5	—272	24.5	—351	24.5	—433
39.1	—144	39.0	—242	39.0	—347	38.7	—433	38.0	—534
62.8	—102	62.7	—167	61.1	—256	61.8	—328	61.5	—406
96.1	— 7	96.1	— 10	95.3	— 33	94.2	— 39	94.2	— 49
140.5	102	139.5	177	137.8	252	138.9	315	138.4	374
229.3	262	223.9	452	226.1	645	226.6	822	244.8	1088
354.1	419	352.3	721	349.1	996	350.1	1274	349.1	1543
504.5	528	475.3	888	500.2	1248	504.7	1612	502.3	1946
649.6	596	649.1	1019	645.2	1415	649.7	1808	645.2	2214
867.1	655	867.1	1120	862.6	1566	864.8	1998	864.8	2447

Thus in weak fields, the deflection shows a farther twisting of the nickel, that is, a decrease of rigidity. This decrease reaches a maximum as the field becomes stronger ; it then begins to decrease, and in a field of about 100 C.G.S. units, the rigidity returns to its original value. When the field is farther increased, the rigidity rapidly increases, and after passing an inflexion point, its rate of increase becomes gradually less. Thus the character of the change is quite analogous to that of the change of elasticity.

In a given field, the amount of torsion or detorsion due to magnetization is proportional to the applied couple, so that the change of rigidity is independent of the couple for all magnetizing fields. It is also a proof of the fact that the curves corresponding to different couples pass through a point on the axis of the field. The ratio of the change to the modulus itself for different fields are given in the following table and in Fig. 2 :—

H	20	40	80	100	200	400	800
$\frac{\delta K}{K}$	—0.0096	—0.0168	—0.0067	0.0012	0.0263	0.0532	0.0748

Thus in nickel, the ratio is rather large compared with other ferromagnetics. In the former experiment, the change of elasticity of the same metal, even in a hardened state, was rather large. If we should study the change of elasticity with the present sample, proportionally large changes would be observed. That this inference is probably correct, may be seen from the results of the method of elongation ; in this case, a well annealed nickel wire was examined, and a large change of elasticity amounting to about 6 % was obtained.

According to Professor Nagaoka[1] and Zehnder[2], the magnetization of nickel increases by twisting in weak fields ; in strong fields, however, it diminishes by twisting. These results are reciprocally related to ours.

The change of torsion thus far described for iron, steel, cobalt and nickel is independent of the direction of the magnetizing force.

From the above result, we may conclude that in ferromagnetic substances, which undergo a large change of elasticity, there is also a proportionally large change of rigidity, and that the natures of their changes are parallel to each other.

7. In comparing the change of rigidity by magnetization with that of elasticity, we observe the one marked difference that the change of rigidity is independent of the applied stress, while that of the elasticity is largely influenced by it, especially in small stress. Hence it may be suspected that the observed change of elasticity may

1) Nagaoka, Jour. Coll. Sci., Tokyo **2**, 304, 1888 ; **3**, 189, 1889.
2) Zehnder, Wied. Ann. **41**, 210, 1890.

contain terms, which can not properly be considered as the change of elasticity. If this be the case, the change of elasticity by magnetization is only apparent.

In conclusion, it may be noted that the reciprocal relations between torsion and magnetism, as found by actual experiments, will be found to be of paramount importance in the theory of magnetostriction. We may conveniently place the results of our experiments in the following statements parallel with those of previous investigators :—

MAGNETIZATION TO TWIST.	TWIST TO MAGNETIZATION.
(*a*) The magnetization of iron decreases by twisting for all magnetizing fields.	(*a′*) The torsion of iron decreases in all magnetizing fields.
(*b*) The magnetization of nickel increases by twisting in weak fields.	(*b′*) The torsion of nickel increases in weak fields.
(*c*) The magnetization of nickel decreases by twisting in strong fields.	(*c*) The torsion of nickel decreases in strong fields.

A similar reciprocal relation would probably exist in the case of cobalt. The actual verification of the relation will be undertaken in the near future.

We have to express our cordial thanks to Professors H. Nagaoka and A. Tanakadate for valuable suggestions in the carrying out of the present experiment.

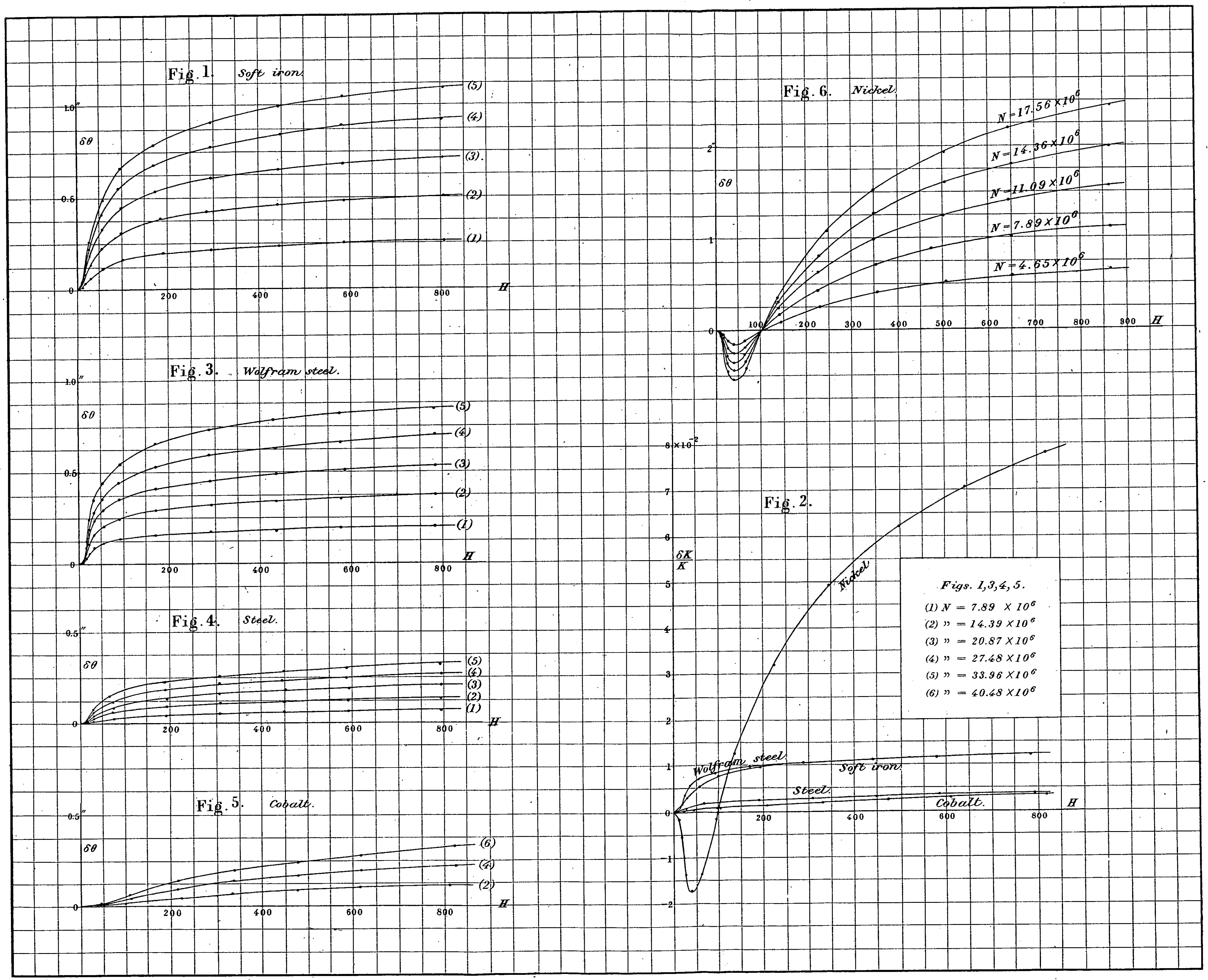


Fig. 1. Soft iron.
δθ
1.0″
0.5
0
200
400
600
800
H
(1)
(2)
(3)
(4)
(5)
Fig. 3. Wolfram steel.
Fig. 4. Steel.
Fig. 5. Cobalt.
(6)
Fig. 6. Nickel.
N = 17.56 × 10^6
N = 14.36 × 10^6
N = 11.09 × 10^6
N = 7.89 × 10^6
N = 4.65 × 10^6
Fig. 2.
δK/K
8 × 10^-2
Nickel
Wolfram steel
Soft iron
Steel
Cobalt
Figs. 1, 3, 4, 5.
(1) N = 7.89 × 10^6
(2) " = 14.39 × 10^6
(3) " = 20.87 × 10^6
(4) " = 27.48 × 10^6
(5) " = 33.96 × 10^6
(6) " = 40.48 × 10^6