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Change of the Modulus of Elasticity in Ferromagnetic Substances by Magnetization.

By

K. Honda, *Rigakushi*,
S. Shimizu, *Rigakushi*,
and
S. Kusakabe, *Rigakushi*.

With 1 Plate.

1. It has been generally admitted that magnetization has very little effect upon the elasticity of ferromagnetic substances. Wertheim[1] measured with a micrometer the elongation of an iron wire due to tension in the magnetized and unmagnetized states and obtained exactly the same results. Guillemin[2] placed an iron bar horizontally, fixing it at one end, while from the other, which was left free, he hung a small weight. The magnetization of the bar by a co-axial coil produced a slight raising of the weight. Since there is an attraction between the bar and the coil, when magnetized, the above effect may not be totally due to the increase of the elasticity of the bar; but to ascribe the effect wholly to the attraction, as G.

1) Wertheim, Ann. de Chim. et de Phys. (3) **12**, 610, 1842.
2) Guillemin, Comp. Rend. **22**, 264 and 432, 1846.

Wiedemann[1] did does not seem to be justifiable. Wartmann[2] used Chladui's figure to investigate the change of elasticity of magnetized iron and steel plates, and also examined the sound accompanying longitudinal and transversal vibrations of magnetized iron wires. No influence of magnetization was observed. Trèves[3] put in vibration two tuning forks having the same period of vibration. When one of them was put in a coil and magnetized by a strong electric current, its vibration was accelerated producing the beat; but when the current was broken, the beats were no longer audible and the two notes were in unison. This shows an increace of elasticity by magnetization. H. Tomlinson[4] found that the elongation of an iron wire by loading is independent of magnetization. Bock[5] found the effect to be less than ½%, if there was any. By passing an electric current through a stretched pianoforte wire, M.G. Noyes[6] noticed an increase of elasticity, which was less than 1%. But in his later experiment[7], he did not accept the conclusion, to which he was led by his former experiment. We can, however, have from his tables an increase of elasticity; but he attributed it to the effect of temperature. Maurain[8] also found a small increase of frequency in a tuning fork placed in a very strong magnetic field. In the investigation of the effect of tension upon the magnetic elongation of a pianoforte wire, B. Brackett[9] observed that the effect of tension was to diminish the magnetic elongation, and he ascribed it to an increase of elasticity. J. S.

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1) Wiedemann's Electricität III. 813.
2) Wartmann, Ann. de Chim. et de Phys. **24**, 360, 1848.
3) Trèves, Comp. Rend. **67**, 321, 1868; Archives des soc nat. N. S. **33**, 74, 1868.
4) Tomlinson, Proc. Roy Soc. **40**, 447, 1886.
5) Bock, Wied. Ann. **54**, 442, 1895; Phil. Mag. (5) **39**, 548, 1895.
6) Noyes, Phy. Rev. (4) **2**, 277, 1895.
7) Noyes, Phy. Rev. (6) **3**, 432, 1896.
8) Maurain, Comp. Rend. **121**, 248, 1895.
9) Brackett, Phy. Rev. (5) **5**, 257, 1897.

Stevens and H. G. Dorsey[1] used the method of pressure and applied the interference fringes to measure the amount of depression. The effect of magnetization upon a loaded iron or steel bar was found to be very small ; it showed a minute increase of elasticity, amounting only to $\frac{1}{300}\%$ for the strongest current used. The effect also increased with magnetizing current. In the next year, Stevens measured the magnetic elongation of steel wires under different tensions, and ascribed the change of elongation to that of elasticity by magnetization, as Brackett did. The result was an increase nearly proportional to the magnetizing force. Lately K. Tangl[2] has published his results on the same subject. He made use of the principle that the moment of a bifilar suspension increases with tension applied to its lower end. By magnetizing the wire under constant tension, he measured the amount of the magnetic elongation. The tension was, then, so varied that the wire returned to its initial length. The ratio of the tension so varied to the magnetic elongation was taken as proportional to the increase in the modulus of elasticity in that field. Besides iron, he also examined nickel wires which showed a small increase of elasticity. In fields ranging from 200 to 480 C. G. S. units, the maximum increase amounted to about 1.02% for iron as well as for nickel. He also investigated the effect of tension, but the result does not seem to be satisfactory.

All of these experiments show that the magnetization increases slightly the modulus of elasticity of iron and nickel, and that the change increases with the magnetizing force, but its law is not clearly brought out.

2. Different methods, by which previous experimenters determined the said effect may be grouped under three heads. The first

1) Stevens and Dorsey, Phy. Rev. (2) **9**, 116, 1899; Phy. Rev. (2) **11**, 95, 1900; Phy. Zeitschr. **2**, 682, 1900.

2) Tangl, Ann. der Phys. **6**, 34, 1901.

method makes use of the acoustic phenomena and can not be used for accurate determinations.

The second is the method of elongation. Since, the elongation due to the change of elasticity is only a small fraction of the total elongation, this method is only suitable for the accurate measurement of the effect in question, when a differential method is applicable. Unfortunately this is not the case in the present instance ; for if we first stretch the wire by a tension and then magnetize it, there is always magnetic elongation which is far greater than that due to the change of elasticity.

We may, however, modify the measurement in the following way, as Bidwell and others have done. The wire is first brought under tension, and then the magnetic elongation in different fields is determined. This process is repeated with several loadings. From these sets of observation, we may decide the question.—How is the elasticity of a ferromagnetic wire affected by magnetization ?

Let E′ and E be the modulii of elasticity within and without the magnetizing field, and e′ and e the magnetic elongations per unit of length with and without the tension, respectively. We first load the wire with the tension T per square centimeters, and then magnetize it; the total elongation in 1cm will be

$$\frac{T}{E}+e'\left(1+\frac{T}{E}\right).$$

Next, changing the order of operations, we first magnetize the wire and then stretch it ; then elongation will be

$$e+(1+e)\frac{T}{E}.$$

If the elongation is independent of the order of operations, we get, neglecting small quantities,

$$\frac{E'-E}{E'E}=\frac{(e'-e)}{T};$$

putting $E'-E=\delta E$, we have

$$\frac{\delta E}{E}=\frac{(e-e')E}{T-(e-e')E}.$$

If we wish to compare two curves corresponding to the tensions T and $T+\Delta T$, the above equation becomes

$$\frac{\delta E}{E}=\frac{\Delta e.E}{\Delta T-\Delta e.E},$$

where Δe is the difference of magnetic elongations corresponding to the tensions T and $T+\Delta T$.

In the preceding paper, two of us studied the effect of tension on magnetic elongation for iron, Wolfram steel, nickel, and nickel steel From these results, we calculated the change of elasticity, as shown in the following tables : —

Soft iron

$\frac{T}{\Delta T}$	$\frac{167gr.}{660}$	$\frac{2145gr.}{659}$	$\frac{4125gr.}{1320}$
H	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$
30	2.24×10^{-2}	0.83×10^{-2}	0.59×10^{-2}
80	2.23	0.98	0.71
300	3.22	1.43	0.67

Wolfram steel

$\frac{T}{\Delta T}$	$\frac{4430gr.}{3540}$	$\frac{7965gr.}{7070}$	$\frac{15030}{10600}$
H	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$
100	0.10×10^{-2}	0.21×10^{-2}	0.14×10^{-2}
300	0.18	0.28	0.24
500	0.28	0.31	2.27

Nickel

$\frac{T}{\Delta T}$	$\frac{863gr.}{688}$	$\frac{2239gr.}{658}$	$\frac{4304gr.}{1376}$
H	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$
10	-3.85×10^{-2}	-1.45×10^{-2}	-0.44×10^{-2}
20	−5.93	−6.33	−1.36
60	−1.46	−7.98	−8.77
120	+3.14	−2.54	−7.53
260	+6.90	+1.97	−1.93

Nickel steel (45% Ni)

$\frac{T}{\Delta T}$	$\frac{156gr.}{307}$	$\frac{770gr.}{601}$	$\frac{3842gr.}{1230}$
H	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$	$\frac{\delta E}{E}$
50	14.10×10^{-2}	4.58×10^{-2}	2.00×10^{-2}
100	14.90	5.60	2.50
150	15,00	6.80	2.40
300	15.00	7.13	2.57
500	15.00	6.45	2.53

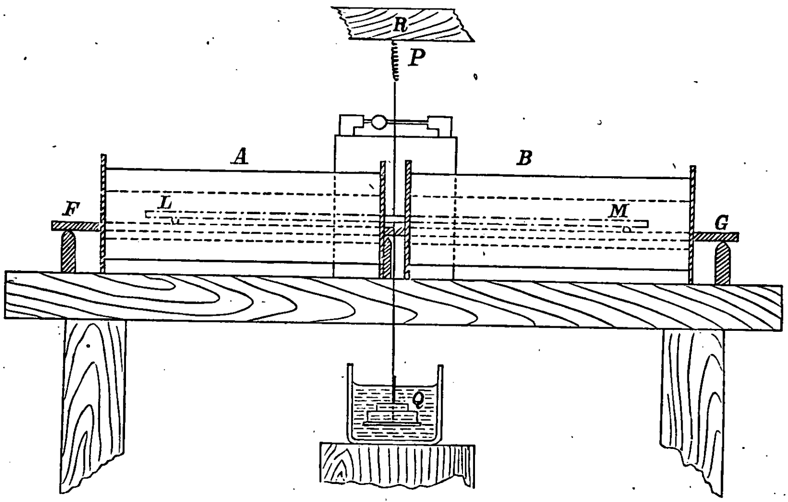
Here H denotes the external field and T the tension per square millimeters. The calculation of $\frac{\delta E}{E}$ from Steven's results for a pianoforte wire gives 1.9×10^{-2} in a field of 40 C.G.S. units, which approximately agrees with the results for soft iron.

From these tables, we see that in iron and nickel steel, the magnetization considerably increases the modulus of elasticity, the amount of the change for a given load increasing with the magnetizing field. The increase also varies with tension; it decreases as the tension is increased. Wolfram steel shows a small increase of elasticity; under a constant field, the increase reaches a maximum as the tension is increased. In nickel, the elasticity decreases in the weak fields and increases in the strong. The change is also a function of the tension; in weak fields, the diminution reaches a maximum and then gradually decreases as the tension is increased. In strong fields, the increase becomes less and less and at last changes its sign with the increase of tension. The field in which the change of elasticity vanishes becomes greater as the tension is increased.

The third is the method of flexure. The advantage of this method lies in the fact that the differential effect can be measured, by suspending a weight at the middle of a ferromagnetic bar and then measuring the change of depression caused by magnetizing it. Such a bar elongates or contracts by magnetizition, while its thickness diminishes or increases; but as we shall soon indicate, the lateral elongation or contraction will be very small compared with the change of depression due to that of elasticity.

Hence, of these three methods, that of flexure is the most suitable for studying the change of elasticity. We therefore used this method to investigate the said effect and also to test the results of the elongation method just referred to.

3. Our method of measurement was similar to that of Stevens and Dorsey, as shown in the annexed figure.



A and B were two magnetizing coils of the same dimensions, which rested horizontally in a co-axial line. FG was a stout brass rod of rectangular section extending between two fulcrums ; it was also supported at the midde point by another fulcrum. The coils can, therefore, be moved independently of the bar. LM was a rod to be tested placed in the axial line of the coils. It was supported at L and M by two fulcrums ; one of them was an ordinary wedge fixed to the brass rod, while the other consisted of a cylinder, which could rotate about its own axis. Q was the weight suspended from the middle of the bar. At the center of the bar, a fine copper wire, the diameter of which was about 0.08mm, was soldered and stretched vertically upwards by means of a weak spring P. This wire was wound once round a rotating cylinder, to which a small mirror was attached, and then stretched upwards, as used in Hertz's dynamometer.*

* Hertz, Instrumentenkunde, **3**, 17, 1883 ; Gesammelte Werke Ba. 1.

The rotation of the cylinder was observed by means of a vertical scale and a telescope.

The dimensions of each part of our arrangement were as follows :—

Length of each coil	= 39.90 cm,
Its internal diameter	= 5.80 cm,
$4\pi n$	= 393.5,
Distance between the coils in air gap	= 2.5 cm for iron and steel, = 2.0 cm for nickel and cobalt,
Distance between two fulcrums L and M	= 59.91 cm for iron and steel, = 21.59 cm for nickel and cobalt,
Diameter of the rotating cylinder	= 0.172 cm,
Scale distance	= 261.3 cm.

The sensibility of our apparatus was such that the displacement of one division of the image of the vertical scale in the field of the telescope corresponded to a change of depression of 1.72×10^{-5}cm in the middle of our ferromagnetic rod. It was neccessary to protect the mirror and the thin copper wire from air currents in order to prevent minute vibrations of the mirror.

The measurements were conducted in the following order. The bar to be tested was placed in the axial line of the coils and then loaded by a weight. The tension of the fine copper wire was then suitably adjusted by means of a screw fixed to the support R, and the mirror was directed towards the telescope. This adjustment was effected as in the experiment described in the preceding paper. To begin with, a current through the coils was made or broken and the working of the arrangement tested. The bar was then demagnetized and the initial reading taken. A current was then passed through the coils and the corresponding deflection noted. These processes were repeated with successively increasing currents.

Since the resistance of the coils did not exceed 30 ohms, no heating of the core due to current was observed during the time, in which the deflection was taken ; hence we dispensed with water-jacketing arrangement.

The lateral contraction or elongation due to magnetization was at most of the order 2×10^{-6} cm for iron and 7×10^{-6} cm for nickel. For, the maximum elongation or contraction per centimeter in the field strength used in the present experiment was about 4×10^{-6} and 27×10^{-9} cm for iron and nickel respectively. Hence, assuming the change of volume to be negligibly small compared with that of length, the maximum lateral contraction or elongation was approximately 2×10^{-6} and 7×10^{-6} cm for the three metals respectively. But in our experiments, the displacement of 1mm of the vertical scale in the field of the telescope corresponds to a change of depression of 1.72×10^{-5} cm. Thus the lateral change of dimensions due to magnetization is within the limit of experimental errors. The disturbance of the results due to magnetic elongation or contraction in the longitudinal direction was eliminated by means of the rotating cylinder, which served as one of fulcrums.

The bar bent slightly downwards if loaded ; hence when it was magnetized, it would strive to make itself straight. This may cause an apparent increase of elasticity ; but it was confirmed by a direct experiment that the effect was negligibly small. For, the reading obtained by inclining the two coils with respect to the bar to a degree greatr than the actual case was almost the same as in the case when the coils rested in a coaxial line.

Since the bar was considerably shorter than the whole length of the coils, it lay nearly in a uniform field, except at the middle. The effect of the air gap between the coil was also studied, varying its width by 1 or 2 cm ; however, such a change had no sensible effect on our results.

4. The dimensions of our specimens and their modulii of elasticity are given in the following table :—

Metal	Soft iron	Steel	Wolfram steel	Nickel	Cobalt
Length.	64.00 cm	64.00 cm	64.00 cm	24.20 cm	27.30 cm
Breadth.	0.903	0.920	0.948	0.510	radius
Thickness.	0.901	0.913	0.953	0.511	=0.680
Elasticity.	2.02×10^{12}	2.01×10^{12}	2.05×10^{12}	1.96×10^{12}	1.79×10^{12}

The present arrangement was not suitable for the absolute measurement of the modulus of elasticity, since the yielding effect of several parts of the arrangement disturbs the result. Hence the modulus of elasticity was determined by the ordinary method of flexure with two mirrors.

The intensity of magnetization of these specimens was determined by the magnetometric method. The results are graphically shown in Fig. 1. Ordinate represents the intensity of magnetization and abscissa the effective field ($H=H'-N\,J$).

The magnetic change of length was found to have an intimate relation to the change of elasticity ; it was therefore measured for each specimen. To each end of the bar, a brass rod of the same thickness and 15 cm long was soldered. The bar was there vertically suspended co-axial with the magnetizing coil by means of a screw adjustment. From its lower end, a weight of 1 or $\frac{1}{2}$ kilograms was hung by a copper wire. The rotating cylinder with a mirror was brought in contact with the wire under suitable pressure, to prevent sliding. The weight was dipped in water so as to avoid its vibratory disturbance. The magnetizing coil was so long that the bar lays nearly in a uniform field. The measurement of the magnetic elongation gave the following results :—

Soft iron.		Steel.		Wolfram steel.		Nickel.		Cobalt.	
H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$	H	$\frac{\delta l}{l}$
9.6	0.45×10^{-6}	19.3	0.08×10^{-6}	16.8	0.13×10^{-6}	16.8	-0.60×10^{-6}	17.0	-0.00×10^{-6}
13.4	0.80	29.4	0.13	24.7	1.18	24.3	−2.66	——	——
20.1	1.65	52.0	0.83	38.6	3.16	34.4	−4.52	38.6	−0.18
30.6	2.63	90.6	0.95	58.3	4.26	54.1	−7.97	70.0	−0.47
67.1	3.48	137.1	0.75	83.9	4.99	76.4	−11.96	113.3	−0.77
113.3	3.06	179.5	0.28	138.4	5.51	124.2	−17.60	176.8	−1.30
204.1	1.40	260.1	−0.58	217.8	5.66	282.5	−24.58	262.6	−2.18
349.9	−1.48	398.2	−1.88	393.7	5.24	405.6	−26.18	399.5	−3.41
510.2	−3.73	512.8	−2.93	500.4	4.99	516.5	−26.84	512.8	−4.12

Here H denotes the effective field and $\frac{\delta l}{l}$ the elongation or contraction per centimeter due to magnetization. These results are also drawn in Fig. 2. The curves for soft iron, steel, and Wolfram steel are quite ordinary ; that for nickel, which is not annealed, is less steep than for ordinary annealed nickel. Professor Nagaoka and one of us have already pointed out that the magnetic character of cobalt is much affected by annealing. The curve for cobalt, which was well annealed, shows this abnormity. We shall soon observe that the elasticity of a substance undergoing large magnetic change of length is also much influenced by magnetization.

5. In observing the displacement of the image of the vertical scale in the field of the observing telescope by passing a current through the coils, we were struck with the large effect contrary to

the results of previous experimenters. The largest deflections for soft iron and wolfram steel amounted to adout 9 cm with a scale at a distance of 2.6 meters for a field of 500 C.G.S. units.

The change of depression corresponding to different loadings in soft iron is given in the following table and graphically shown in Fig. 3.

T=110gr		T=610gr.		T=1630gr.		T=2650gr.	
H	δs	H	δs	H	δs	H	δs
7.6	$0{,}26\times10^{-4}$	13.1	1.03×10^{-4}	7.6	-0.17×10^{-4}	10.6	-0.26×10^{-4}
11.7	0.69	18.8	1.89	15.8	1.20	20.1	4.90
15.4	1.35	26.1	4.13	17.9	2.58	26.9	8.77
20.9	2.41	29.0	4.82	24.5	6.19	33.9	11.52
23.6	2.75	36.2	5.50	33.5	8.52	52.9	13.50
33.5	3.61	59.0	6.36	41.4	9.12	69.4	14.27
52.9	4.13	133.4	6.36	55.8	9.63	129.0	14.45
103.7	4.30	231.4	6.36	194.5	9.63	211.7	14.62
233.9	4.13	290.8	6.36	206.8	10.15	289·6	14.79
365.2	4.30	382.5	6.36	437.0	11.04	397.5	15.31

In the above table, the change of depression δs is taken positive when it indicates an increase of elasticity and taken negative, when it indicates a decrease. H is the effective field, and T the suspended weight. It is to be observed that owing to the weight of the bar itself, the depression is caused by magnetization, when there is no suspended weight.

The general course of the curves in Fig. 3 resembles that of magnetization. In weak fields, however, we notice a minute decrease of elasticity, when the load exceeds about 1.5 kilograms. Iron contracts laterally when magnetized by weak currents, and this contraction may produce such an apparent decrease of elasticity ; but the calculation shows that it is more than can be accounted for by the lateral contraction. When the field increases beyond this region, the change of depression increases rapidly and soon reaches its asymptotic value, after which the increase takes place quite slowly. When the weight is added under a given field, the change of depression is increased. The rate of increase is large with a small weight, and decreases in amount as the weight is increased, approaching an asymptotic value.

From the change of depression, we may calculate the ratio of the change to the modulus itself. The depression due to the suspended weight as well as to its own weight in an unmagnetized bar is given by the approximate formula* $s=\frac{l^3g}{4Eab^3}(T+\frac{5}{8}W)$,
where l, a, b are the length, breadth and thickness of the bar, T and W are the suspended weight and the weight of the bar itself respectively, l and W refer to the part of the bar lying between two fulcrums. The observed change of depression divided by this is the ratio in question, that is, $\frac{\delta E}{E}$. Some of our results of calculation are given in the following table :—

H \ $T\times\frac{5}{8}W$	329 gr.	829 gr.	1349 gr.	1849 gr.	2869gr.
20	1.64×10^{-2}	0.77×10^{-2}	0.50×10^{-2}	0.47×10^{-2}	0.44×10^{-2}
30	2.79	1.47	1.09	1.08	0.88
50	3.15	1.84	1.35	1.28	1.16
100	3.36	1.92	1.48	1.37	1.28
250	3.40	1.93	1.51	1.40	1.32
400	3.40	1.93	1.51	1.40	1.32

*) Clebsch's Elasticität 375 ; Winkelmann's Physik I, 266.

The above results for soft iron approximately agree with those given by the method of elongation both qualitatively and quantitatively. But, the specimens in these two cases are not the same, and moreover $\frac{\delta E}{E}$ is a function of a stress; hence we can not, in a strict sense, conform these two results. Our results, when compared with those of previous experimenters, are markedly large, especially with small loadings.

Wolfram steel. The change of elasticity by magnetization for wolfram steel is quite similar to that of soft iron. The curves of depression is less asymptotic to the axis of the magnetizing force. The initial small decrease of elasticity is more marked than in the case of soft iron and occurs even with the smallest load. The following table and Fig. 4 show the character of the change of depression.

T=110 gr.		T=549 gr.		T=1130 gr.		T=1918 gr.	
H	δs	H	δs	H	δs	H	δs
15.8	-0.05×10^{-4}	14.0	-0.03×10^{-4}	15.1	-0.05×10^{-4}	18.2	-0.34×10^{-4}
20.5	−0.26	19.4	−0.22	18.2	−0.21	20.5	−0.69
23.9	0.34	23.1	−0.03	22.4	−0.24	23.0	−0.21
30.2	0.83	25.4	0.34	25.6	0.36	38.4	8.43
40.7	1.46	36.6	3.44	30.2	3.41	56.5	10.23
62.6	1.93	63.1	4.64	41.1	5.76	75.7	11.09
91.9	2.34	91.9	5.07	68.5	7.09	144.3	11.74
211.9	2.86	269.9	6.02	224.2	8.95	230.2	12.38
384.0	3.44	386.5	6.28	348.1	9.20	360.5	13.07
484.9	3.97	486.0	6.79	472.4	9.56	460.0	13.41

The values of $\frac{\delta E}{E}$ are given in the following table:—

$T+\frac{2}{5}W$ / H	358 gr.	797 gr.	1378 gr.	2206 gr.
70	1.79×10^{-2}	1.88×10^{-2}	1.67×10^{-2}	1.56×10^{-2}
100	2.05	2.02	1.79	1.64
200	2.55	2.27	2.00	1.80
300	2.93	2.42	2.10	1.89
400	3.26	2.60	2.18	1.92
500	3.54	2.68	2.23	1.97

From the last table, we see that the increase of elasticity under a constant field generally becomes less as the load is increased, except in weak fields, in which we notice a maximum as in the case of the elongation method. The ratio of $\frac{\delta E}{E}$ for wolfram steel is several times greater than that by the method of elongation. The principal cause of the discrepancy may probably be due to the fact that the wolfram steel used in the present experiment was magnetically much softer than the specimen used in the former experiment, for the latter was hardened by stretching. It is a well established fact that a hardened iron or steel wire suffers comparatively small magnetic elongation and that the effect of tension on the elongation is also very small. Hence it is to be expected that the result of the method of elongation comes out to be much smaller than that given by the present experiment.

6. *Steel.* Steel shows a comparatively small increase of elasticity; the results of observation are given in the following table and graphically drawn in Fig.5.

T=1005gr.		T=1918gr.		T=2830gr.	
H	δs	H	δs	H	δs
11.8	0.00×10^{-4}	13.4	0.09×10^{-4}	10.3	0.17×10^{-4}
18.8	0.07	27.6	0.43	19.3	0.26
32.2	0.26	43.7	0.95	33.0	1.38
50.5	0.77	85.2	1.89	47.1	2.06
94.7	1.03	170.7	2.58	85.6	2.75
215.6	1.20	199.6	2.49	205.7	3.61
298.5	1.38	292.2	3.05	275.0	3.96
391.6	1.39	386.6	3.27	398.8	4.56
491.1	1.46	487.4	3.44	498.6	4.99

Thus the general character of the change of depression is similar to that of soft iron ; but the initial decrease is not observed. The course of the curve is much steeper than in soft iron and wolfram steel, and less asymptotic to the axis of the magnetizing force.

The values of $\frac{\delta E}{E}$ are given in the following table :—

H \ $T+\frac{5}{8}W$	1251gr.	2184gr.	3096gr.
50	0.14×10^{-2}	0.15×10^{-2}	0.17×10^{-2}
100	0.22	0.25	0.25
200	0.27	0.33	0.31
300	0.28	0.37	0.35
400	0.30	0.40	0.39

Thus the increase of elasticity under a constant field reaches a maximum with a load of between 1270 and 2200 grams. On both sides of the load, it gradually decreases as the load is increased or decreased. These results approximately agree with those of wolfram steel obtained by the method of elongation.

Cobalt. The cobalt bar was too thick, and the maximum deflection in the field of the telescope was only 1.5 mm, so that we can not claim for cobalt the same accuracy as in the case of the other specimens. But we notice a distinct increase, as shown in the following table and in Fig.6.

T=1005 gr.			T=2830 gr.		
H	δs	$\frac{\delta E}{E}$	H	δs	$\frac{\delta E}{E}$
21.6	0.00×10^{-4}	0.00×10^{-2}	35.1	0.01×10^{-4}	0.02×10^{-2}
53.0	0.01	0.06	70.3	0.02	0.04
180.5	0.07	0.47	195.6	0.09	0.21
313.4	0.09	0.58	312.1	0.16	0.37
455.1	0.11	0.76	455.1	0.16	0.37

For cobalt, the depression due to the suspended weight is calculated by the formula, neglecting the weight of the bar itself;

$$s = \frac{1}{12\pi} \frac{Tl^3}{ER^4},$$

where l and R are the length and the radius of the bar respectively.

Thus the character of the change of elasticity in cobalt is much the same as that in steel.

7. *Nickel.* As regards the change of elasticity by magnetization, nickel shows an abnormal behaviour, as already pointed out in the case of the method of elongation. The following table and Fig.7. are the results of observations :—

T=93 gr.		T=276 gr.		T=549 gr.		T=820 gr.	
H	δs	H	δs	H	δs	H	δs
16.4	-0.15×10^{-4}	10.5	-0.09×10^{-3}	8.4	-0.09×10^{-4}	10.0	-0.28×10^{-4}
24.1	−0.38	16.9	−0,43	17.5	−0.98	16.7	−1.17
32.9	−0.45	24.8	−0.67	29.8	−1,43	21.9	−1.62
41.8	−0.55	34.5	−1.03	39.2	−1.26	26.4	−1.75
67.8	−0.83	48.1	−0.86	48.9	−1.08	39.6	−1.62
118.1	−0.72	68.2	−0.79	85.1	−0.52	68.2	−0.72
220.0	−0.46	171,7	−0.09	203.9	1.43	117.3	1.17
302.4	−0.17	240.0	0.55	302.4	2.34	217.5	3.18
386.0	0.05	376.0	1.07	399.7	2.79	377.2	4.47
490.8	0.28	496.0	1.46	490.8	3.06	492.1	4.88

The values of $\frac{\delta E}{E}$ are as follows :—

H \ T+$\frac{5}{8}$W	105 gr.	287 gr.	561 gr.	832 gr.
30	-1.80×10^{-2}	-1.60×10^{-2}	-1.30×10^{-2}	-1.08×10^{-2}
70	−4.08	−1.40	−0.70	−0.38
100	−3.84	−0.92	−0.21	+0.30
200	−2.50	+0.46	+1.20	+1.80
300	−0.88	+1.47	+2.07	+2.52
400	+0.42	+2.17	+2.56	+2.83
500	+1.34	+2.67	+2.84	+3.03

Thus, the modulus of elasticity considerably decreases in the weak fields and increases in the strong. The field of no change decreases as the load is increased. The change of depression also increases with the load. In weak fields, the rate of decrease diminishes as the load is increased ; in strong fields, however, the contrary is the case.

Comparing the above results with those given by the method of elongation, we notice that in the present case, the field of no change becomes less, whereas in the former, it becomes greater, as the load is increased, and that the amount of the change is much less in the present case than in the former. Our nickel rod was turned into a square rod from a plate, and the mechanical process, which the specimen underwent, hardened its magnetic quality. The nickel wire used in the preceding experiment was almost chemically pure, and magnetically softer. Hence the discrepancy with regard to the amount of the change may be explained by the difference of the specimens ; but the discrepancy with regard to the field of no change can scarcely be explained by the same fact.

Tangl's results are much smaller than ours, and moreover he did not observe the decrease of elasticity in weak fields, because his initial field was too strong to give such a decrease.

Since nickel steel of suitable dimensions for determining the modulus of elasticity by flexure was not at our disposal, we could not test the result of the method of elongation. But from the above results, we may conclude that the change of elasticity by flexure does not generally coincide with that of elasticity by elongation. As we have observed, the elasticity is no longer independent of the stress applied to the bar ; hence it is possible, and perhaps rather natural, to conclude that in magnetic fields, the elasticity as given by flexure is different from that given by elongation.

In conclusion, we have to express our best thanks to Prof. H. Nagaoka and also to Prof. A. Tanakadate for many valuable suggestions.

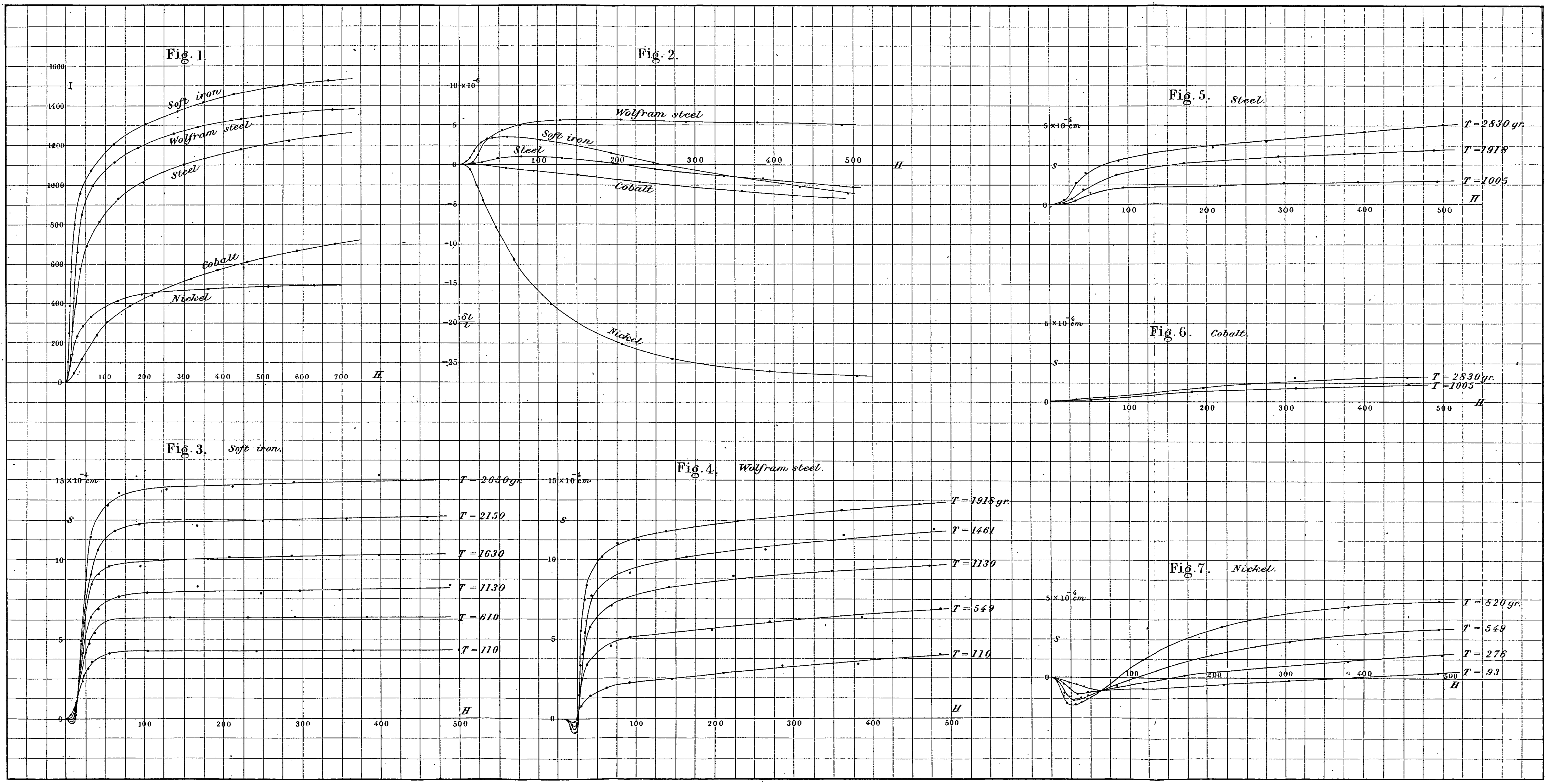


Fig. 1.
I
Soft iron
Wolfram steel
Steel
Cobalt
Nickel
H
Fig. 2.
Wolfram steel
Soft iron
Steel
Cobalt
Nickel
H
Fig. 3. Soft iron.
T = 2650 gr.
T = 2150
T = 1630
T = 1130
T = 610
T = 110
H
Fig. 4. Wolfram steel.
T = 1918 gr.
T = 1461
T = 1130
T = 549
T = 110
H
Fig. 5. Steel.
T = 2830 gr.
T = 1918
T = 1005
H
Fig. 6. Cobalt.
T = 2830 gr.
T = 1005
H
Fig. 7. Nickel.
T = 820 gr.
T = 549
T = 276
T = 93
H

ERRATA.

P. 2, line 2, read 'Chladni's' for 'Chladui's'.
P. 2, line 17, read 'trace' for 'have'.
P. 3, line 1, read 'flexure' for 'pressure'.
P. 3, line 17, omit 'increase in the'.
P. 4, line 17, read 'with' for 'within'.
P. 4, line 24, read 'the', for 'then'.
P. 5, tables, in $\frac{T}{\Delta T}$, '—' is not the sign of division.
P. 9, line 12, read 'these' for 'three'.
P. 10, line 17, read 'then' for 'there'.
P. 11, line 16, insert after '5'. '*Change of elasticity in soft iron.*'
P. 12, line 2, read 'about' for 'adout'.
P. 13, line 12, insert after 'depression' 'and the modulus of elasticity'.
P. 13, table, read '$T+\frac{5}{8}W$' for '$T\times\frac{5}{8}W$'.
P. 14, line 5, read 'compare' for 'conform'.
P. 15, table, read '$T+\frac{5}{8}W$' for '$T+\frac{8}{5}W$'.
P. 16, line 3, read 'less steep' for 'much steeper'.
P. 18, table, read '—4' for '—3'.