Mutual Relations between Torsion and Magnetization in Iron and Nickel Wires.

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With Plate XVI.

The various effects of stress on the magnetization of ferromagnetic metals are of such a complex character that no simple relation seems to exist among them. The strains caused by magnetizing the ferromagnetics are of no less complex a nature, so that the co-ordination of these two classes of complicated phenomena is, up to the present, still a matter of doubt. Various isolated facts, such as the analogies between the change of magnetization by longitudinal pull and that of length by magnetization, the relation between the twist caused by the interaction of longitudinal and circular magnetizations and the circular or longitudinal magnetization produced by twisting a longitudinally or circularly magnetized wire respectively, were long considered as affording a clue to the explanation of these phenomena. So far as we are aware, no attempt has yet been made to place all of these different phenomena on a common footing. Some time ago[1], we hinted at the probable connections which exist between the twist caused by passing an electric current through a longitudinally magnetized wire and the change of volume and of length in ferromagnetic metals produced by magnetization. The said relation can also be extended to the explanation of other phenomena; namely, the transient current produced by twisting a magnetized wire and the longitudinal magnetization caused by twisting a circularly magnetized wire. It is our object in the present paper to show that these different phenomena can be linked together in a common bond.

§ 1. Twist produced by the interaction of circular and longitudinal magnetizations.

The subject was first studied by G. Wiedemann[2] who established remarkable reciprocal relations with the longitudinal magnetization produced by twisting a circularly magnetized wire. Dr. Knott[3] found that the direction of twist in iron is opposite to that in nickel; Bidwell[4] afterwards discovered that the twist in iron is reversed in high fields and takes place in the same direction as in nickel. Unfortunately some of the experiments were undertaken with wires which were longer than that of the coil, so that the magnetization was far from being uniform. It will suffice for qualitative tests, but we can not hope for any

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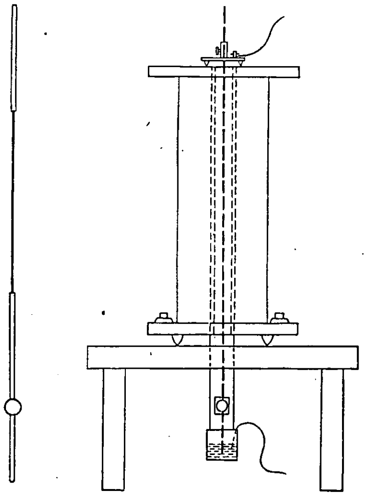
[1] Nagaoka and Honda, this Journ. **13**, p. 57, 1900; Phil. mag. **49**, p. 341, 1900.

[2] G. Wiedemann, Pogg. Ann. **103**, p. 571, 1858; **106**, p. 161, 1859; *Elektricität*, **3.**

[3] Knott, Trans. Roy. Soc. Edinb., **32** (1), p. 193, 1882/83; **35** (2), p. 377, 1889; **36** (2), p. 485, 1891.

definite quantitative results. The position of maximum twist in nickel shows a large difference in the present from the corresponding experiment by Dr. Knott.

The twist produced by longitudinal magnetization of a circularly magnetized wire was measured in the following way. To the extremities of an iron or nickel wire 21 cm. long were brazed stout brass wires, and a light plane mirror was attached to the lower one. The end of the lower brass wire was dipped in a mercury pool, while the upper brass wire was clamped to a small tripod, which rested on the top of a magnetizing coil provided with hole, slot, and plane arrangement. One end of the accumulator was connected with the tripod, while the other was led to a mercury pool. The wire hung vertically in the axial line of the coil, which was 30 cm. long and gave a field of 37.97 C.G.S. units at the centre by passing a current of one ampere. The vertical component of the terrestrial magnetic field was compensated by placing another coil in the interior of the magnetizing coil. The lower part of the wire to be tested was protected against air current by enclosing it in a wide brass tube with a small window, just where the reflecting mirror was attached. The twist was measured by scale and telescope method, by which the deflection of 0.3″ per. cm. was easily read. The current was measured by Kelvin graded amperemeters, whose constants were



from time to time checked by means of an ampere balance. The experiment was conducted in the following manner:—

1. The circularly magnetizing current was kept constant, and the amount of twist measured by varying the longitudinally magnetizing current.

2. The longitudinally magnetizing current was kept constant, and the amount of twist measured by varying the circularly magnetizing current.

Before each experiment, care was taken to demagnetize the wire completely either longitudinally or circularly by passing an alternate current of gradually diminishing intensity.

Twist by varying the longitudinal field (Fig. 1).—The direction of twist in iron, so long as the longitudinal magnetizing field is not strong, is such that if the current is passed down the wire from the fixed to the free end and the wire is magnetized with north pole upwards, the free end, as seen from above, twists in the direction of the hands of a watch. By keeping the circular field constant, the amount of twist increases at first, till it reaches a maximum in a field of about 20 units; it then goes on diminishing till it ultimately changes the direction and continues to twist in the opposite direction with increasing field. The field at which the twist is reversed increases with the circularly magnetizing field. In nickel, the direction of twist is opposite to that in iron, but the general feature is similar to iron, the only difference being that even in strong longitudinal fields, the twist is not reversed. For wires of the equal thickness, the amount of twist in nickel is greater than that in iron—the maximum twist in iron wire of 1 mm. diam. by passing 6 amperes through it amounts to about 28″ per cm., while with nickel wire of 0.83 mm. diam. under similar conditions, the maximum twist amounts to about 200.″

Twist by varying the circular field (Fig. 2).—Here we notice a slight dissimilarity between iron and nickel. In iron, the twist increases with the strength of the circular field, if the longitudinal field remains constant. Such is also the case with nickel in moderate and strong fields. In low longitudinal fields, however, the twist does not continue to increase with the circular, but we notice a maximum as will be clear in the figure. There is great experimental difficulty in increasing the circular field, inasmuch as the heating of the wire becomes very great and thus materially deteriorates the result.

The hysteresis accompanying the cyclical change of the circular magnetization deserves special notice (see Fig. 3). If the longitudinal field be such that with the increase of the circularly magnetizing force, the twist reaches a maximum, the curve of twist goes below the former course on weakening the circular magnetization. The twist, however, goes on slowly increasing, till it crosses the *on*-curve and then reaches a maximum, whence it gradually diminishes and ultimately vanishes in negative field. The course after passing this point is exactly the reverse of that already described. The character of twist is exactly the same for iron as for nickel, when we take the opposite character of twist into account. The nature of the hysteresis is nearly the same when the longitudinal magnetizing field is made to vary, while the circular field remains constant.

The results thus far obtained are in accordance with the experiments of Wiedemann and Knott; we have only to notice the discrepancy as regards the position of maximum twist in nickel. In Dr. Knott's experiment, the said point occurs in tolerably high field, while in the present experiment, it occurs nearly in the same field as in iron. It may partly be due to the difference in the method of measuring the twist and partly to the non-uniformity of the field, as was often the case in most of the older experiments.

The observed angles of twist in iron and nickel are exhibited in the following tables, where C denotes the longitudinal current per sq. mm. in amperes, H the field strength in C.G.S. units and τ the angle of twist per cm. expressed in seconds.

Circular Field being Constant.

Iron wire: diam.=0.98 mm.

C=1.06		C=3.38		C=6.95	
H	τ	H	τ	H	τ
5.6	18.6″	5.6	21.9″	5.6	16.4″
12.4	21.7	11.3	35.6	11.3	30.0
26.0	13.1	22.6	32.8	22.6	36.4
41.3	7.9	36.2	24.7	49.2	27.8
90.5	2.5	79.2	13.7	78.0	19.7
112.0	1.6	97.3	10.7	96.2	16.4
215.0	− 0.5	191.0	6.0	132.0	8.2
492.0	− 1.9	442.0	2.7	455.0	2.5

Nickel wire: diam.=0.83 mm.

C=2.45		C=4.33		C=6.05	
H	τ	H	τ	H	τ
4.9	− 94.1″	4.9	− 97.6″	4.9	− 88.6″
11.2	−137.0	11.2	−164.4	11.2	−159.8
24.3	−125.5	24.3	−179.4	24.3	−196.4
38.8	−102.1	38.8	−156.6	38.5	−182.6
53.6	− 85.1	53.4	−134.1	53.2	−159.2
84.0	− 62.9	83.8	−101.8	84.0	−119.7
102.2	− 54.5	102.2	− 88.0	103.4	−101.8
225.0	− 31.7	222.0	− 47.9	226.0	− 59.3
415.0	− 19.6	389.0	− 29.8	414.0	− 40.2

Longitudinal Field being Constant.

Iron Wire: diam. = 1.05 mm.

H=2.67		H=5.92		H=19.38	
C	τ	C	τ	C	τ
0.35	0.3″	0.58	0.3″	0.40	0.0″
0.80	1.1	1.03	0.6	0.89	0.0
1.06	2.0	1.73	2.0	1.36	0.0
1.80	3.7	2.20	3.0	1.92	0.3
2.22	5.7	2.96	5.1	2.66	0.4
3.04	8.8	3.55	6.2	3.48	0.6
4.35	14.2	4.78	9.1	4.88	1.4
6.79	21.5	6.73	13.6	6.25	2.5
7.52	23.2	7.34	14.7	7.16	2.8

Nickel wire: diam. = 0.83 mm.

H=5.7		H=39.7		H=96.3	
C	τ	C	τ	C	τ
1.03	22.1″	0.95	22.6″	0.81	8.5″
1.89	48.1	2.06	59.3	1.76	19.9
2.82	70.2	3.34	101.4	2.82	32.3
4.50	102.0	4.10	123.6	4.29	50.5
7.25	121.2	5.90	153.6	6.22	73.2
9.40	116.4	7.95	172.8	8.65	99.6
11.60	107.4	11.40	178.8	11.50	127.5
13.38	99.0	13.10	176.4	13.22	141.6

§ 2. Circular magnetization produced by twisting a longitudinally magnetized wire.

By twisting a longitudinally magnetized ferromagnetic wire, circular magnetization is developed. If, therefore, two ends of the wire are connected by a conducting wire, a transient current due to the circular magnetization appears in the circuit at the moment when the twist is applied. Some years ago, one[1] of us investigated the transient current for iron and nickel wires. It was then found that the current due to twisting was opposite in direction in these two metals and that it reached a maximum in moderate fields. As the magnetizing current was not very strong, no conclusive measurements were made as regards the nature of the transient current in strong fields. In order to make this point clear and see if any intimate relation with the Wiedemann effect could be traced, fresh experiments were undertaken by the same method as before. We have to notice that the ferromagnetic wire was so placed in the axial line of the magnetizing coil that it lay in nearly uniform field.

Some of the measurements of the transient current for iron and nickel wires are given in the following table and in Fig. 4.

Iron wire : diam. = 1.33 mm. length = 20.90 cm.				Nickel wire : diam. = 1.09 mm. length = 20.80 cm.			
$\theta = 15°$		$\theta = 50°$		$\theta = 15°$		$\theta = 50°$	
H	$Q \times 10^8$	H	$Q \times 10^8$	H	$Q \times 10^8$	H	$Q \times 10^8$
3.2	25.4	3.2	30.7	1.3	− 5.8	1.1	− 2.5
5.3	29.0	4.8	33.6	3.7	− 7.1	2.7	− 6.2
15.6	24.9	15.8	42.0	16.0	−17.7	5.3	−18.5
32.6	16.3	30.5	38.4	33.7	−21.2	10.2	−27.1
54.5	10.9	48.6	30.5	53.4	−21.5	25.6	−38.0
87.4	6.4	86.0	18.5	81.2	−20.1	44.9	−41.7
120.8	2.5	139.4	8.1	115.4	−19.2	67.6	−40.7
165.6	1.5	183.8	3.7	157.0	−16.4	107.4	−40.0
242.0	− 0.1	327.6	− 3.2	213.7	−12.6	160.2	−35.7
495.4	− 1.7	447.4	− 5.0	319.6	−10.3	241.5	−29.1
650.2	− 2.9	711.0	− 5.1	530.3	− 6.5	316.4	−26.7
908.0	− 3.3	959.0	− 5.2	703.0	− 5.4	699.8	−13.9
1298.0	− 3.1	1521.0	− 4.7	1214.0	− 3.9	1150.0	−10.6
1790.0	− 2.5	1872.0	− 4.4	1815.0	− 2.0	1853.0	− 8.9

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[1] Nagaoka, Journ. Sci. Coll., Tōkyō, **4**, p. 323, 1891.

Here θ denotes the angle of torsion and Q the time-integral of the transient current expressed in C.G.S. units. The resistance of the whole circuit was 4.5 ohms. The nickel wire here used was made of the same specimen as the nickel prism used in our former experiments.

As is well known, the direction of the transient current, and therefore that of the circular magnetization, is opposite in iron and nickel. The current for constant amount of twist increases with the strength of the longitudinal field ; it, however, soon reaches a maximum, whence it gradually diminishes. In nickel, the transient current attains asymptotic values in strong fields without changing its direction, while in iron, it is reversed in a field of about 200 C.G.S. units, when the twist is small. The increase after the reversal is not pronounced, but becomes finally asymptotic.

§ 3. Longitudinal magnetization produced by twisting a circularly magnetized wire.

The longitudinal magnetization produced by twisting a circularly magnetized wire presents the same character as the transient current above described. The experiment is very difficult on account of the heating of the wire. To avoid the rise of temperature, the iron or nickel wire to be tested was covered with *urushi* (Japan lac) which has the special property of being a very good insulator while, at the same time, the melting temperature is comparatively high. The wire thus insulated was stretched in the axial line of a secondary coil whose diameter was 1.5 cm. and whose total number of turns was 540, and the current of cold water was kept flowing about it to keep the temperature of the wire uniform. Thus maintaining the electric current in the wire constant, it was twisted and the induced current in the secondary circuit due to the longitudinal magnetization thereby developed was measured by the ballistic method.

Some of the results of observations are given in the following table and graphically shown in Fig. 5.

Iron wire: diam.= 0.888 mm. length=20.74 cm.				Nickel wire: diam.= 0.965 mm. length=20.94 cm.			
$\theta=15°$		$\theta=50°$		$\theta=15°$		$\theta=50°$	
C	$Q\times10^9$	C	$Q\times10^9$	C	$Q\times10^9$	C	$Q\times10^9$
0.21	0.7	0.21	16.1	0.21	−45	0.14	−133
0.85	3.7	0.69	45.3	0.66	−111	0.23	−294
1.53	20.5	1.49	89.1	1.56	−205	0.90	−397
2.36	31.4	2.19	111.0	2.40	−239	2.05	−448
3.93	36.5	3.27	125.6	3.35	−256	2.87	−450
4.72	33.6	4.65	124.2	4.36	−265	4.42	−452
6.55	29.2	5.91	115.4	5.86	−272	7.37	−448
7.89	21.9	8.29	109.6	7.95	−269	10.34	−431
12.82	13.9	12.48	86.2	10.89	−256	15.33	−407
19.01	10.9	17.08	65.7	14.07	−243	20.85	−397
24.29	5.8	24.37	51.8	20.18	−219	23.04	−378
28.64	5.8	29.14	43.2	26.46	−206	26.13	−362

C denotes the total current through the wire expressed in amperes; θ and Q have the same meanings as before.

As will be seen from the figure, the quantity of induced electricity in the secondary circuit, and therefore the longitudinal magnetization developed, by twisting a circularly magnetized iron wire attains a maximum, when the mean circular field is about 10 units. It then decreases, but in spite of the constant stream of water, the heating due to electric current prevented the experiment from being pushed to the point where the direction of the current is reversed. However, to judge from the course of the curve, the tendency is such that there is a reversal. In nickel, the direction of the induced current is opposite to that in iron, and the total quantity of the current attains a maximum, whence it continually diminishes, but not to such an extent that the current ultimately changes its direction.

These experiments show that the twist produced by the combined action of the longitudinal and circular magnetizations, the circular magnetization produced by twisting a longitudinally magnetized wire, and the longitudinal magnetization caused by twisting a circularly magnetized wire are characterized by having various peculiarities, which are common to all of them. This can not be a mere chance coincidence; we shall have to ascribe these allied phenomena to the same common cause.

In the experiments of this and the last paragraphs, we were assisted by Mr. S. Shimizu, a post-graduate in physics, to whom our best thanks are due.

§ 4. Theory.

As already remarked in our last paper on magnetostriction, Kirchhoff's theory can be extended to the study of the relation between torsion and magnetization, exactly, in the same manner as was done by Maxwell and Chrystal to explain the Wiedemann effect. There we found that the *mean circular magnetization* called into play by twisting a ferromagnetic wire of radius R through angle ω amounts to

$$-\frac{1}{3}\omega\, k''\, HR. \qquad (A)$$

in field H, and that the *mean longitudinal magnetization* caused by twisting a ferromagnetic wire carrying an electric current C amounts to

$$-\frac{1}{2}\omega\, k''\, C. \qquad (B)$$

The reciprocal relation between these two phenomena is thus apparent at a glance. We shall next show how the same phenomena are reciprocally connected with the torsion produced by the interaction of the longitudinal and circular magnetizations.

The stress components in a magnetic medium as given by Kirchhoff are as follows:

$$X_x = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\alpha^2 + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(\alpha^2 + \beta^2 + \gamma^2\right).$$

$$Y_y = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\beta^2 + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(\alpha^2 + \beta^2 + \gamma^2\right).$$

$$Z_z = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\gamma^2 + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(\alpha^2 + \beta^2 + \gamma^2\right).$$

$$Y_z = Z_y = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\beta\gamma.$$

$$Z_x = X_z = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\gamma\alpha.$$

$$X_y = Y_x = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)\alpha\beta.$$

Taking the axis of z in the axial line of the wire, and two other axes in the plane perpendicular to it, we see that the component magnetic forces in a longitudinally magnetized wire traversed by an electric current are

$$\alpha = -h \sin\theta, \qquad \beta = h\cos\theta, \qquad \gamma = H,$$

where h denotes circular field given by

$$h = \frac{2Cr}{R^2},$$

C being the current, r the distance of the point from the axis of the wire, R the radius, and θ the angle between r and the axis of x.

The stress components in ferromagnetic medium acted upon by the forces above specified are given by

$$\begin{aligned}
X_x &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)h^2 \sin^2\theta + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(H^2 + h^2\right), \\
Y_y &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)h^2 \cos^2\theta + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(H^2 + h^2\right), \\
Z_z &= -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)H^2 + \frac{1}{2}\left(\frac{1}{4\pi} + k - k'\right)\left(H^2 + h^2\right), \\
Y_z &= Z_y = -\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)h\, H\cos\theta, \\
Z_x &= X_z = \left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)h\, H\sin\theta, \\
X_y &= Y_x = \left(\frac{1}{4\pi} + k + \frac{k''}{2}\right)h^2 \sin\theta\cos\theta.
\end{aligned}$$

The moment about the axis of the wire is given by

$$\begin{aligned}
N &= \int\!\!\int \left(Z_y\, x - Z_x\, y\right) dxdy. \\
&= -\int\!\!\int \left(\frac{1}{4\pi} + k + \frac{k''}{2}\right) h\, H\, r\, dxdy. \\
&= -\frac{2}{R^2}\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right) CH \int_0^R \int_0^{2\pi} r^3\, drd\theta. \\
&= -\pi\left(\frac{1}{4\pi} + k + \frac{k''}{2}\right) CHR^2.
\end{aligned}$$

Since $\frac{1}{4\pi}$ and k are very small compared with k'', the torsional couple twisting the wire amounts nearly to

$$-\frac{\pi k''}{2}CHR^2 = -\frac{k''\,CH}{2} \times \textit{Cross section.} \qquad (C)$$

Since the amount of torsion of a cylindrical wire by a given couple is inversely proportional to the fourth power of its radius, it is evident that for given longitudinal current and field, the angle of twist is inversely proportional to the square of the radius. This inference was approximately verified in the present experiments.

In deducing the three formulæ (A), (B), (C), we can not, strictly speaking, put k'' outside the sign of integration, because the strain coefficient depends on the field strength, which is not uniform in a wire traversed by electric current. Hence in these formulæ, we shall have to use a mean value to obtain a close approximation.

The mutual relations between twist and magnetization are embodied in the three formulæ above given. There we notice that the strain coefficient k'' determines the nature of the three different phenomena studied in the above experiments. The fact that the coefficient k'' is principally determined by the elongation in the ferromagnetic metal accounts for the close analogy between the said phenomena and the elongations due to magnetization. As the above result imports, the analogy is not exact, inasmuch as the elongation is also affected by terms depending on k', which depends mostly on the change of volume.

In order to test the consequences of the theory as regards the twist produced by the joint action of circular and longitudinal magnetizations, we have calculated the twist by assuming the values of k'', calculated from the changes of volume and of length in iron and nickel ovoids. Graphically represented (Fig. 6.), the fields of maximum twist by calculation coincide nearly with that given by experiments, and the reversal of twist in iron takes place in low fields as actually found by observation. The quantitative differences are, however, tolerably large in iron, but in nickel the amount of twist is nearly coincident with the experimental values. Calculating, in the same manner, the quantity of the transient current produced by twisting longitudinally magnetized wires, we find a close coincidence between the experimental and theoretical values in nickel, but the difference is tolerably large in iron. In using the strain coefficients, we must always bear in mind that these values are widely different according to the nature of the specimen; especially with wires, we are not sure of its being magnetically isotropic. The apparent discrepancy would probably be lessened, if we could measure the twist as well as the strain coefficients on the same specimen. The remarkable qualitative coincidence as regards the existence of maximum twist and its reversal in iron are convincing proofs that the theory, so far as we know at present, admits of connecting various experimental facts in a common bond.

As regards the mutual relations among the three different phenomena above enumerated, it will suffice to state that several of them have already been noticed by G. Wiedemann in his researches on the relation between torsion and magnetism. He especially studied the relation between permanent torsion and the effect of magnetizing the twisted wire. The principal object of his researches was to expose the different aspects of the phenomena involved in the relation between torsion and magnetization in order to bring to light his ingenious theory of rotatory molecules.

Elegant as it at first sight appears to be, Wiedemann's theory abounds with hypotheses which we are not always warranted in making.

In his work on the applications of dynamics to physics and chemistry, J. J. Thomson has propounded a new method of investigating the mutual relations between the effects of various physical agencies. He showed that the existence of a certain phenomenon involves as a natural consequence that of another reciprocating with it. As an application of his method, he showed that if the wire be twisted by the interaction of longitudinal and circular magnetizations, a transient current will be produced simply by twisting a longitudinally magnetized wire and a longitudinal magnetization will be developed by twisting a circularly magnetized wire.

The peculiar feature of Kirchhoff's theory lies in the simple and natural way of elucidating the relations between the various kinds of strain caused by magnetization and the effects of stress on magnetization. Just as we can study the various elastic behaviour of isotropic bodies by knowing the bulk- and stretch-moduli, we have to deal, in Kirchhoff's theory, with the strain coefficients k' and k'' which play the *rôles* of different moduli in the theory of elasticity.

The reciprocal relations between the strain caused by magnetization, and the effects of stress on magnetization, as found by actual experiments, will be found to be of paramount importance in arriving at a correct theory of magnetostriction. The strain accompanying the magnetization of ferromagnetic metal will be determined, when we know the effects of stress on magnetization and vice versa. As regards the relations between twist and magnetization, we may conveniently place them under the following parallel statements:

Strains produced by magnetization.	**Effects of Stress on magnetization.**
(*a*)—(Experiment and theory). A longitudinally magnetized wire is twisted by circular magnetization.	(*a'*)—(Experiment and theory) Twisting a longitudinally magnetized wire gives rise to circular magnetization.
(*b*)—(Experiment and theory). A circularly magnetized wire is twisted by longitudinal magnetization.	(*b'*)—(Experiment and theory). Twisting a circularly magnetized wire gives rise to longitudinal magnetization.
(*c*)—(Experiment and theory). Up to moderate fields, the twist produced by the longitudinal and circular magnetizations of an iron wire is opposite to that in nickel.	(*c'*)—(Experiment and theory). Up to moderate fields, the transient current, or the longitudinal magnetization produced by twisting a longitudinally or circularly magnetized wire respectively, is opposite to that in nickel.
(*d*)—(Experiment and theory). The twist due to longitudinal magnetization of a circularly magnetized iron or nickel wire reaches a maximum in low fields.	(*d'*)—(Experiment and theory). The transient current produced by twisting a longitudinally magnetized iron or nickel wire reaches a maximum in low fields.
(*e*)—(Experiment and theory). In strong fields, the twist due to longitudinal magnetization of a circularly magnetized iron wire is reversed and takes place in the same direction as in nickel.	(*e'*)—(Experiment and theory). In strong fields, the direction of the transient current produced by twisting a longitudinally magnetized iron wire is reversed and is in the same direction as in nickel.

In his paper on the principle of least action, Helmholtz[1] has placed the reciprocal relations of a dynamical system under three heads. Denoting the generalized co-ordinates, the veloci-

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[1] Helmholtz, Crelle's Journal **100**, p. 137, 1886; *Abh.*, **3**, p. 203, 1895.

ties, the accelerations and the forces by p's, q's, q''s, and P's, the relations are generally expressible by the equations

$$(1)\quad \frac{\partial P_a}{\partial p_b} = \frac{\partial P_b}{\partial p_a},$$

$$(2)\quad \frac{\partial P_a}{\partial q_b} = - \frac{\partial P_b}{\partial q_a},$$

$$(3)\quad \frac{\partial P_a}{\partial q'_b} = \frac{\partial P_b}{\partial q'_a}.$$

It will be easily seen that the relations above cited belong to case (2).

The greatest difficulty that we encounter in establishing the relations between the effects of stress on magnetization and the strain caused by magnetization lies in the great difference of strain coefficients according to the nature of the specimen. If all the experiments be performed in a proper manner on one and the same specimen of ferromagnetic metals, we may feel assured of being able to discern the true merits of the theory, or to detect its various defects, not only from qualitative points of view, but also in various quantitative details.

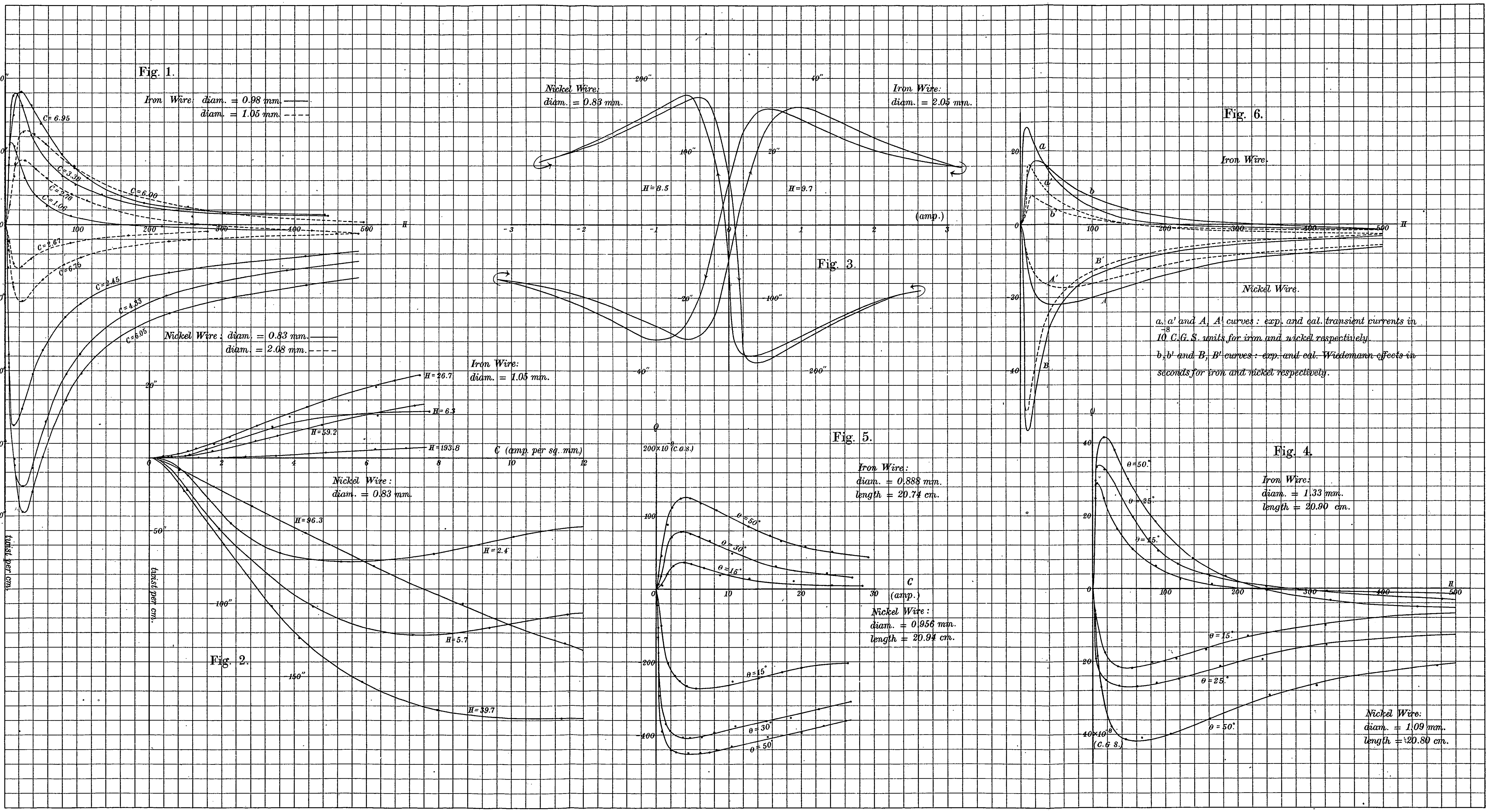
Fig. 1.
Iron Wire: diam. = 0.98 mm. ——
diam. = 1.05 mm. - - - -
C = 6.95
C = 3.38
C = 6.00
C = 2.76
C = 1.06
C = 2.67
C = 6.75
C = 2.45
C = 4.53
C = 6.05
Nickel Wire: diam. = 0.83 mm. ——
diam. = 2.08 mm. - - - -
40″
20″
0
-50″
-100″
-150″
-200″
100
200
300
400
500
H
twist per cm.
Fig. 2.
Iron Wire:
diam. = 1.05 mm.
H = 26.7
H = 6.3
H = 59.2
H = 193.8
C (amp. per sq. mm.)
2
4
6
8
10
12
Nickel Wire:
diam. = 0.83 mm.
H = 96.3
H = 2.4
H = 5.7
H = 39.7
20″
50″
100″
150″
twist per cm.
Fig. 3.
Nickel Wire:
diam. = 0.83 mm.
Iron Wire:
diam. = 2.05 mm.
200″
100″
40″
20″
H = 8.5
H = 9.7
(amp.)
-3
-2
-1
0
1
2
3
-20″
-100″
-40″
200″
Fig. 6.
Iron Wire.
a
a′
b
b′
A′
B′
A
B
Nickel Wire.
20
40
0
100
200
300
400
500
H
a, a′ and A, A′ curves: exp. and cal. transient currents in 10^{-8} C.G.S. units for iron and nickel respectively.
b, b′ and B, B′ curves: exp. and cal. Wiedemann effects in seconds for iron and nickel respectively.
Fig. 5.
Q
200×10⁻⁸ (C.G.S.)
Iron Wire:
diam. = 0.888 mm.
length = 20.74 cm.
θ = 50°
θ = 30°
θ = 15°
100
0
10
20
30
C
(amp.)
Nickel Wire:
diam. = 0.956 mm.
length = 20.94 cm.
200
400
θ = 15°
θ = 30°
θ = 50
Fig. 4.
Q
40
20
0
Iron Wire:
diam. = 1.33 mm.
length = 20.90 cm.
θ = 50°
θ = 35°
θ = 15°
100
200
300
400
500
H
θ = 15°
θ = 25°
θ = 50°
20
40×10⁻⁸
(C.G.S.)
Nickel Wire:
diam. = 1.09 mm.
length = 20.80 cm.