Combined Effect of Longitudinal and Circular Magnetizations on the Dimensions of Iron, Steel and Nickel Tubes.

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With Plates VIII. and IX.

1. The change of length in the direction of magnetization has been made a subject of investigation by several experimentalists, but few of them have measured the change in the direction perpendicular to that of magnetization. Joule[1] first observed the diminution of length of an iron gas-piping by passing a current through an insulated wire inserted into it, and bent over the sides, so as to form a circular magnetizing coil of $1\frac{1}{2}$ convolutions. His experiment was modified by Bidwell[2] who measured the change of dimensions in an iron ring. He found that the ring becomes thicker in a strong field and thinner in a weak one. From the measurement of the internal as well as the external change of volume for iron, steel, nickel, and cobalt tubes, Knott[3]

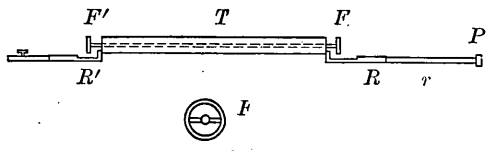
1) Joule, Scientific papers I, 263.
2) Bidwell, Proc. Roy. Soc. **56**, 94, 1895.
3) Knott, Trans. Roy. Soc. **39**, 457, 1898.

calculated the change of lateral dimension in these tubes. The result for iron coincided qualitatively with that of Bidwell.

The first experiment on the change of length of an iron wire by the combined action of longitudinal and circular magnetizations was made by Beatson[1] who observed the diminution of length at the moment when an electric current was passed through a magnetized wire. A similar result was afterward obtained by Righi.[2] The same experiment was also repeated by Bidwell,[3] who observed a large increase in the change of length by longitudinal magnetization of an iron wire carrying a current.

2. Through the kindness of Prof. Nagaoka, his apparatus[4] for the measurement of the minute change of length was placed at my disposal. The apparatus consists of a small optical lever with an arrangement for temperature compensation on the same principle as the gridiron pendulum. The rod, by which the change of length is made sensible to the lever, was slightly modified.

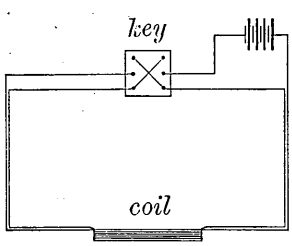
In the annexed figure, T is the tube to be tested, F and F' are two circular brass rings protruding from the tube at a distance of 1 cm. from the ends, and soldered to a brass rod passing through the axis of the tube. The magnetizing coil was wound round the tube parallel to its length extending from F to F' to envelope it completely, and so arranged that the tube could slide in the coil with little friction. F in the lower part of the figure shows



1) Beatson, Archives des. Sc. phys. et nat. **2**, 113, 1846.
2) Righi, Mem. di Bologna **4**, 1, 1879; Beibl. **4**, 802.
3) Bidwell, Proc. Roy. Soc. **51**, 495, 1892; Beibl. **17**, 582.
4) Nagaoka, Phil. Mag. **27**, 131, 1894; Wied. Ann. **53**, 487, 1894.

the front view of these rings. R and R' were two rods in contact with the ends of the tube. The ends of these rods were bent upwards and so filed down, that they could easily slide between two parallel wires of the coil, which were specially fixed at a distance of 1 mm. from each other. The rod r served to communicate the motion to the prism P. The other parts of the apparatus remained unchanged. The apparatus was put into a magnetizing coil, 30 cm. long and wound in 12 layers with copper wire of 2 mm. diameter. The field at the centre of the coil due to a current of one ampere was 37.97 C.G.S. units. The current through the outer coil produced the change of length by longitudinal magnetization and that through the inner coil gave rise to the change of length by circular magnetization.

To study the effect of temperature on the change of length, the circular magnetizing coil was wound, not by a single wire, but by double wires; thus connecting the four ends of these wires to a reversing key as shown in the figure, the circular field can be made or annulled by turning the key one way or the other. The total number of turns of the circular magnetizing coil was 44 for the nickel tube, 40 for the wolfram steel tube and 36 for the soft iron tube. The magnetizing currents were measured by Thomson graded galvanometers which were compared with a deciampere balance before each experiment.



3. The samples used in the present experiment had the following dimensions:

material	length. (cm.)	external diam. (cm.)	internal diam. (cm.)	demagnetizing factor.
nickel	17.02	1.328	1.252	0.0261
wolfram steel	20.30	1.124	1.048	0.0162
soft iron	16.97	0.966	0.842	0.0308

The tubes of nickel and soft iron are the same as that used in the study of the mutual influence between longitudinal and circular magnetizations. It was found by analysis that the nickel was nearly chemically pure, the trace of impurity being inmeasurably small.

Results of Experiments.

1. Nickel Tube.

4. The tube was carefully annealed, before the circular magnetizing coil was wound round it. The change of length due to longitudinal field alone was then measured in the usual way. The results were compared with that obtained after the circular magnetizing coil was wound round the tube. The comparison showed that there was in general small difference between these two, and that the change of length in the former case was always greater than that in the latter, the difference amounting to nearly 2 or 3 %. This is evidently due to the resistance to contraction experienced by the tube, although it can easily slide along the coil. Whether the apparatus executed its function correctly or not was tested before each experiment by making a longitudinal field and comparing the deflection so obtained with that in the free state; otherwise serious mistakes would sometimes have arisen.

5. The experiments on the change of length by circular magnetization, namely, on the change of dimension in a direction perpendicular to the magnetic field, were conducted in the following manner: The tube was first demagnetized and a circular magnetizing current was made only for a moment and the corresponding deflection read. The change of length due to magnetization followed almost instantaneously, but the change due to the heating of the coil became sensible somewhat later; hence these two effects were unmistakably distinguishable so long as the magnetizing current was not strong. On this account the highest field did not exceed 100 C.G.S. units.

The effect of the longitudinal field on the change of length by circular magnetization was also measured. A constant longitudinal field was first made and the corresponding deflection observed; then currents of different strength were momentarily passed through the circular magnetizing coil, and the additional deflection was read. These results are given in the following table and also in Fig. 1:

TABLE I.

H=0		H=6.9		H=22.1		H=182.9	
h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$
7.6	4.4	8.2	2.7	8.2	0.5	8.5	1.6
16.1	19.0	17.5	40.8	17.5	18.0	13.6	2.2
26.3	46.2	31.1	87.1	31.2	76.2	21.3	4.4
40.6	75.1	40.9	110.4	40.9	124.0	31.5	9.2
50.5	92.5	49.9	130.6	50.2	157.8	49.0	27.2
64.5	108.8	57.5	141.4	64.2	190.4	64.2	59.9
72.1	117.0	72.1	163.2	70.9	201.3	69.5	81.6
83.2	119.7	81.7	168.6	78.8	217.7	87.6	130.6

Here H and h denote effective longitudinal and circular fields respectively, both in C.G.S. units. $\frac{\delta l}{l}$ represents additional change of length by circular field. All these changes were measured at a constant temperature of about 25° C.

Fig. 1 shows that the change of length by circular magnetization increases at first showly and then rapidly. With the further increase of the circular field, the rate of increase becomes gradually less. This result agrees in quality with Knott's calculation. The circular magnetization combined with a constant longitudinal one is always to increase the length which is first shortened by the longitudinal magnetization. In weak circular fields, the curve of the change of length with a constant longitudinal field lies below the curve with no such field; but in strong fields, the first curve lies above the second. The point of intersection of these two curves is displaced into a higher field with the increase of the longitudinal.

6. We shall next pass on to the change of length by longitudinal magnetization with a constant circular field. The tube was first demagnetized by reversals, and then the deflections for longitudinal magnetizing currents of different strength were measured. During the experiment, the temperature at the centre of the magnetizing coil was 18.8° C. The tube was then carefully demagnetized both as regards the longitudinal and circular magnetizations. Then a constant current was passed through the circularly magnetizing coil so that the field strength became null. Owing to the heating of the coil, the tube rapidly expanded at first, but usually after an hour or two, it reached a stationary state; when that state was reached, the measurement of the change of length by longitudinal field alone was commenced, which gave the length change at a higher temperature. After

the observation was finished, the key was reversed, so that the circular magnetizing current was then called into play. During this process, no gradual displacement was observed, showing that the temperature of the tube remained unchanged during the reversal, but at the same time an instantaneous deflection was noticed, which showed the change of length by circular magnetization. By reading the displaced position of the line in the micrometer ocular, the deflection corresponding to the longitudinal magnetization was noted. The tube was then demagnetized as regards the longitudinal magnetization, the circular magnetization remaining constant. The same process was repeated for stronger fields, till a set of observations was completed.

7. How the rise of temperature affects the change of length by magnetization will be seen from Fig. 2. The change of length at ordinary temperature is somewhat less than that which Prof. Nagaoka and myself[1] have obtained for an ovoid made of the same specimen. The difference may perhaps be explained by that of annealing and of the geometrical shape of these samples. The temperature was measured by inserting a mercury thermometer inside the tube. Its effect is thus tolerably large; the rise of temperature is attended with an increase of the change of length in weak fields, and is accompanied with a decrease in strong fields. From the same figure, we obtain the relation of temperature to the change of length at a constant field as shown in Fig. 3. It is well known that the magnetization of nickel increases with temperature in low fields and decreases in strong ones; but under the temperature of 100° C, the change of magnetization is too small to account for the change of length.

1) Nagaoka and Honda, Preceding paper.

So far as I am aware, Barrett[1] is the only physicist who has investigated the effect of temperature on the change of length; his experiment resulted in the decrease of about one-fourth of the change of length by a rise of temperature by about 50° C. Perhaps his field was too strong to cause an increase.

8. The results of the change of length by longitudinal magnetization with a constant circular field are given in the following table and in Fig. 4. The change of length was reduced to the temperature of 18.8° C by using the results above obtained.

TABLE II.

h=0		h=10.7		h=16.8		h=22.9	
H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$
5.3	− 1.5	6.9	− 10.9	8.1	− 17.6	7.2	− 13.0
8.5	− 25.6	14.6	− 61.7	17.8	− 73.5	14.2	− 49.4
17.0	− 71.7	27.8	−118.1	27.4	−118.1	22.3	− 93.9
29.4	−107.6	42.7	−161.6	46.7	−174.9	37.6	−149.2
41.2	−134.7	62.8	−198.6	71.9	−223.4	65.1	−198.7
61.0	−163.9	103.3	−238.4	——	——	94.0	−232.7
101.3	−202.4	131.6	−255.2	131.9	−271.0	129.7	−269.3
184.9	−241.8	176.8	−279.2	177.7	−291.5	175.2	−293.4
274.9	−261.3	254.0	−301.7	255.4	−313.7	255.5	−317.0
354.9	−274.1	361.0	−312.9	363.4	−329.7	359.3	−333.2
468.6	−279.2	505.0	−324.0	516.1	−339.4	516.1	−347.4
709.2	−289.5	720.2	−331.6	779.0	−344.5	725.2	−351.4

1) Barrett, Phil. Mag. [4] **47**, 51, 1874; Nature **26**, 515 586, 1882; Beibl. **7**, 201.

The comparison of Figs. 1 and 4 shows that the general character of the change of length is the same in these two cases, except that the sign of the change is opposite. Hence similar remarks as in the former hold good in the present case.

In the experiment with nickel and cobalt wires traversed by an electric current, Bidwell found that the effect was inmeasurably small. The discrepancy in nickel perhaps arises from the effect of temperature, which he did not take into account; the difference in the method adopted in the present experiment for obtaining a circular field and in that of Bidwell does not seem to play an important part in accounting for the said discrepancy. According to the present experiment, the rise of temperature occasioned a comparatively large diminution of the length change in strong fields. Hence it can not be denied that in Bidwell's experiment, the effect of circular magnetization was just as great as that of temperature. The same remark will perhaps apply to his experiment with cobalt; but having no cobalt tube at my disposal, the experimental verification must be postponed till some future date. However, a theoretical deduction in favor of the view above stated will be given in the last part of the paper.

It would not be out of place to remark that a klinging note of the nickel tube was heard at the make and break of circular magnetizing current, a well known phenomenon. Even with such a weak current as we obtain from a single Daniell's cell was sufficient to produce a distinctly audible sound.

9. It will sometimes happen that it is convenient to have a simple expression for the change of length. For nickel, the change of length is very well given by an empirical formula of the form

$$\frac{\delta l}{l} = -\frac{\alpha H^n}{1+\beta H^n},$$

where α, β and n are constants and H is assumed to be positive. The determination of these constants from the experimental curve gave the following results:

$$\alpha = 5.18,\ \beta = 0.0164 \text{ and } n = 1.017.$$

In the calculation, only the fields H=20, 80, 320 were chosen to simplify the calculation. Using these values of the constants, the change of length due to fields of different strength was calculated and compared with the experimental value as shown in the following table:

TABLE III.

H	$\frac{\delta l}{l}$ (cal.)	$\frac{\delta l}{l}$ (exp.)
10	-46×10^{-7}	-40×10^{-7}
20	-81	-81
30	-108	-109
50	-148	-148
80	-185	-185
120	-215	-214
150	-230	-229
200	-247	-246
250	-258	-258
320	-269	-269
400	-278	-278
500	-285	-284
600	-292	-289
700	-293	-291

Thus except in weak fields, the coincidence between these two is very close; the difference does not amount to 1 %. This formula applies, not only for the change of length, but also for every curve which has only one inflexion point and becomes asymptotic when one of the co-ordinates increases indefinitely, such as the curve of magnetization.

2. Wolfram Steel Tube.

10. The method of procedure with the steel tube was exactly the same as in the corresponding case of the nickel tube. The result of the change of length by circular magnetization, e.g., the dilatation in a direction perpendicular to the field, as well as the effect of longitudinal field on the change of length by circular magnetization are given in the following table and graphically shown in Fig. 5. These observations were taken at a constant temperature of about 17° C.

TABLE IV.

H=0		H=15.1		H=31.8		H=81.3	
h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$	h	$\frac{\delta l}{l} \times 10^7$
14.0	− 0.0	13.2	− 0.4	13.6	− 0.4	12.9	− 0.4
20.8	− 8.6	31.7	−12.9	30.9	− 2.1	31.2	− 0.9
35.2	−20.2	41.9	−32.2	41.9	−14.6	41.6	− 5.2
51.8	−22.8	51.7	−40.4	51.1	−25.8	51.1	−12.9
65.1	−26.6	63.7	−45.1	63.7	−38.7	63.7	−25.8
78.7	−27.9	74.7	−47.3	74.7	−48.1	75.6	−30.9
99.5	−27.9	88.6	−49.4	91.4	−58.0	88.6	−40.0

Here we observe that circular magnetization produces contraction which increases very slowly at first, but afterwards quite rapidly, till it reaches a nearly constant value. The existence of the field of maximum contraction is still a question. The result is somewhat discordant as compared with that of Bidwell with an iron ring, in which case the diminution vanishes in a field of about 86 C. G. S. units. Since the behaviour of wolfram steel as regards the change of dimensions by magnetization is very different from that of soft iron, the cause of the discrepancy is probably to be sought for in the difference of the specimens.

That the effect of longitudinal field on the change of length by circular magnetization is of the same nature as in the case of nickel, except that the sign of the change is opposite, is also apparent from the same figure. As we have remarked, Beatson and Righi observed the same phenomenon.

11. The middle curve in Fig. 6 represents the change of length by longitudinal magnetization at the temperature of the room. The lower curve was obtained at 80.2° C, and the upper curve at the same temperature by reversing the key so as to produce circular field. From the figure, we see that the behaviour of wolfram steel as regards the change of length is widely different from that of other sorts of iron. It is remarkable that the length of the tube, after reaching the maximum elongation, diminishes very slowly as the field is increased, a fact already noticed by the experiment[1] referred to. In that case, the maximum elongation was somewhat less than in the present experiment. The discordance between the two is probably due to the difference of annealing and also of the shape of the specimens.

1) Nagaoka and Honda, loc. cit.

The effect of temperature is to decrease the change of length; the diminution increases with the field, till it reaches a maximum, and then decreases very slowly. Barrett[1] did not find the effect in the case of iron and cobalt. The upper curve shows that the influence of circular magnetization on the change of length is large for steel.

12. The effect of circular field on the change of length by longitudinal magnetization is shown in the following table and in Fig. 7. The results are reduced to the temperature of 17.2° C.

TABLE V.

h=0		h=10.8		h=17.7		h=25.8	
H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$
12.0	0.0	19.3	3.7	14.5	0.2	11.3	0.0
17.7	8.3	30.5	18.3	26.1	11.9	21.0	2.1
29.3	16.2	37.6	24.5	31.9	18.0	31.2	17.2
49.4	27.3	53.1	35.8	50.8	37.2	57.7	41.8
93.0	38.8	84.6	46.3	75.7	50.5	83.7	56.8
125.1	42.9	135.3	52.4	105.5	54.5	120.3	67.6
170.0	44.9	182.5	54.2	168.4	60.1	165.8	72.5
348.5	43.3	233.3	55.0	244.5	63.7	246.5	75.2
441.0	42.9	325.5	55.6	350.0	63.9	351.5	73.3
545.0	42.5	505.0	54.2	461.5	63.6	459.5	72.3
728.0	41.3	708.8	52.4	615.5	62.8	671.6	71.0

Thus the longitudinal magnetization combined with a constant circular one is always to increase the length which is first

1) loc. cit.

shortened by the circular magnetization. In weak longitudinal fields, the curve of the change of length with a constant circular field lies slightly below the curve with no circular field; but in strong fields, the first curve lies markedly above the second. The point of intersection of these two curves shifts into a high field as the circular field is increased. The field of the maximum elongation seems to increase with the circular field.

3. Soft Iron Tube.

13. The experiments of the change of length by circular magnetization and of the effect of longitudinal field on the change of length led to the following results, which are graphically shown in Fig. 8. The observations were taken at the temperature of 18° C.

TABLE VI.

H=0		H=5.7		H=25.8		H=67.6	
h	$\frac{\delta l}{l}\times 10^7$	h	$\frac{\delta l}{l}\times 10^7$	h	$\frac{\delta l}{l}\times 10^7$	h	$\frac{\delta l}{l}\times 10^7$
5.3	− 7.8	5.3	− 4.2	5.3	− 0.0	5.3	− 0.5
14.0	−13.0	13.8	−11.9	14.0	− 5.2	13.8	− 1.0
21.4	−15.6	20.7	−16.6	21.4	−10.4	21.0	− 4.2
37.5	−15.6	35.7	−20.8	37.3	−20.8	37.3	− 9.9
53.2	−14.5	51.8	−22.3	53.3	−26.0	53.2	−14.0
69.4	−12.5	66.6	−22.3	69.2	−28.0	67.8	−18.2
81.6	− 9.3	81.3	−20.8	81.6	−26.0	80.5	−20.8
98.8	− 7.8	97.7	−19.7	98.0	−23.4	98.1	−20.8

By circular magnetization, the length of the tube diminishes rapidly at first, till it reaches a minimum, then it gradually

recovers. The field at which the tube returns to its former length is not yet reached so far as the present experiment extends. The result agrees qualitatively with that of Bidwell and the calculation of Knott.

The general form of the curve does not change by the application of a constant longitudinal field, but the field of maximum contraction shifts into high field as the longitudinal field increases. The amount of the maximum contraction increases with the longitudinal field, till it reaches a maximum, and then it gradually decreases. In weak circular fields, the change of length diminishes with the increase of the longitudinal.

14. As in the case of wolfram steel, three curves in dotted lines are given in Fig. 9, two of which correspond to the change of length at the temperatures of 18.7°C and 76.1° respectively. When the key in the circuit of the circularly magnetizing coil was reversed so as to produce a field, the change of length corresponding to the third curve was obtained.

The change of length by longitudinal magnetization at ordinary temperature is somewhat less than those obtained by previous experimenters. The difference is probably to be ascribed to the well annealed state[1] of the tube; also, the resistance to the elongation experienced by the tube due to the friction of the circular magnetizing coil was found to affect the result slightly. The general feature of the change of length is so well known that farther remarks are unnecessary. It is only to be noticed that here the field of the maximum elongation is greater by 20 C. G. S. units than that of the minimum contraction due to circular magnetization.

The rise of temperature is to diminish the change of length

1) Bidwell, Phil. Mag. **55**, 228, 1894.

in weak fields and to increase it in strong ones. The field at which the temperature produces no effect is about 52 C. G. S. units. In the case of wolfram steel, this field, if it exists, seems to be pushed to an intensely strong field. We also observe that the effect of circular field on the change of length by longitudinal magnetization is tolerably large, as observed by Bidwell.

15. The results of the experiments on the change of length by longitudinal magnetization with a constant circular field are summed up in the following table and graphically shown in Fig. 10, these results being reduced to 18.7° C.

TABLE VII.

h=0		h=5.7		h=9.2		h=26.2	
H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$	H	$\frac{\delta l}{l} \times 10^7$
5.3	1.1	6.9	4.7	5.3	1.1	5.3	1.7
10.3	12.8	—	—	11.2	17.2	10.3	9.6
21.5	17.1	17.9	22.4	22.9	29.6	20.5	22 7
41.3	19.2	37.8	28.8	39.4	37.4	40.6	35.6
70.3	19.2	61.8	32.1	64.0	42.3	61.2	40.0
97.9	17.1	111.1	31.0	97.9	42.2	90.9	41.1
144.3	11.1	142.6	27.8	145.0	38.8	143.5	38.2
223.0	3.6	218.0	21.4	217.6	31.0	217.2	34.5
318.5	− 4.9	320.3	12.4	320.0	20.2	312.5	25.3
481.4	−20.3	490.0	− 4.3	493.8	4.2	475.0	8.6
697.0	−32.5	704.0	−13.5	684.2	− 5.7	647.0	− 2.6

Thus the nature of the change of length is the same as in the reciprocal case already mentioned, except that the sign of the change is opposite. As shown in the figure, in strong fields, the curve corresponding to the change of length with a constant circular field lies always above that with no circular field.

In weak fields, the curves nearly coincide with each other. The field of maximum elongation slightly increases with the circular, and the amount of the elongation, after reaching a maximum, begins to decrease with further increase of the circular field. Though Bidwell did not observe this point, the present experiment agrees quite well with his result.

16. So far the experiments made on the tubes of nickel, steel and iron show that the effect of circular field on the change of length by longitudinal magnetization is of the same nature as the effect of longitudinal field on the change of length by circular magnetization.

From the results of the change of length by longitudinal and circular magnetizations, the change of volume by magnetization can easily be calculated, provided we assume the material to be isotropic, as was already done by Bidwell. If u and v represent these two dilatations respectively, the volume change σ is given by the formula $\sigma = u + 2v$.

Assuming the isotropy of our specimens, we find the calculation leads to the following results:

TABLE VIII.

H	Nickel	Wolfram steel	Soft iron
	$\frac{\delta v}{v}$	$\frac{\delta v}{v}$	$\frac{\delta v}{v}$
10	-18.5×10^{-7}	0.0×10^{-7}	-9.6×10^{-7}
20	-21.0	-7.2	-13.1
30	0.0	-18.8	-12.8
40	18.0	-22.2	-12.2
60	46.5	-21.8	-7.7
80	54.0	-19.2	-1.9
100		-16.4	1.5

We thus obtain incredibly large values for the change of volume. In nickel and soft iron, there is at first decrease of volume, and then follows an increase; in wolfram steel, the diminution of volume reaches a maximum and then gradually decreases. The above result for soft iron agrees fairly well with that of Bidwell[1] for unannealed iron ring. But in the experiment with ovoids[2] made of the same specimens, there was always small increase of volume for nickel, steel and soft iron. The amount of the change at the field of 100 C. G. S. units was 0.7×10^{-7}, 3.1×10^{-7} and 2.8×10^{-7} for these metals respectively. Hence the question now arises whether the change of volume is so influenced by the shape of these metals. To settle this point, fresh experiments on the change of volume were undertaken with a dilatometer. The answer was in the negative, the result being in rough agreement with that for the ovoids. The initial decrease of volume was never observed, but the volume always increased with the increase of the magnetizing field. The discrepancy between the calculated and the experimental result is perhaps due to the æolotropy of the materials. For, if it were not isotropic, the lateral dilatation by longitudinal magnetization would not coincide with the change of length by circular magnetization. It will also be explained by the æolotropy of the specimens that in weak fields, Bidwell's calculation resulted in the large diminution of volume of iron rings in contradiction to the experimentally established fact.

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1) loc. cit.
2) Nagaoka and Honda, loc. cit.

Concluding Remarks.

17. From the experiment on the relation between magnetism and twist, Knott[1] concluded that the pure strain effects on a ferromagnetic wire caused by tension and longitudinal current through it are of an opposite character, and also, on the ground of Maxwell's explanation for Wiedemann's effect, that in an iron or nickel wire carrying an electric current, the change of length by magnetization must be greater than when there is no longitudinal current. Since the change of length for cobalt is scarcely affected by tension, the same must also be the case for longitudinal current. The consideration is partially verified by the experiment of Bidwell and also by the present one.

The same phenomenon may also be more concisely explained in the following manner. Suppose our samples to be isotropic and to have no residual effect. Let l and t be two magnetic forces acting longitudinally and circularly in two perpendicular directions. When these two forces act simultaneously, we have a resultant force H; this force occasions the change of dimensions in our ferromagnetics. The dilatation in the direction of the resultant force, as well as that in the direction perpendicular to it, can be expressed by $f(H)$ and $F(H)$ respectively, which are even functions of H. To obtain the dilatation in the longitudinal direction, we have simply to construct a strain ellipsoid at any point of the ferromagnetics and to find the change of length of the radius vector in this direction. The simple calculation gives

$$\frac{\delta L}{L}=f(H)\frac{l^2}{H^2}+F(H)\frac{t^2}{H^2}.$$

1) Knott, Trans. Roy. Soc. Edinb. **36**, pt. II., 485.

In the case of nickel, the change of volume is negligibly small compared with that of length; hence we may put with tolerable accuracy $f(H)+2F(H)=0$. With steel and soft iron, the change of volume is not very small compared with the change of length. But if t does not exceed 50 C. G. S. units, the effect of volume change on the change of length by combined action of l and t is negligibly small, for in these strong fields at which the change of volume is pronounced, the ratio t^2/H^2 in the above expression becomes very small. Hence even for these metals, we may neglect the change of volume, provided the circular field is not very large, and the expression for $\frac{\delta L}{L}$ becomes, in all cases,

$$\frac{\delta L}{L}=\frac{l^2-\frac{1}{2}t^2}{H^2}f(H).$$

Since the material is supposed to be isotropic, $f(H)$ is the same as the ordinary change of length by longitudinal magnetization. Thus the change of length by longitudinal magnetization with a constant circular field can be calculated from the change of length by longitudinal magnetization alone. The same expression can also be used for the calculation of the change of length due to circular magnetization with a constant longitudinal field.

In order to compare the above result with that of the experiment, it is obviously necessary to subtract from $\frac{\delta L}{L}$ the expression $F(t)$ for the change of length by longitudinal magnetization with a constant circular field t, and $f(l)$ for the reciprocal case.

Assuming for the expression $f(H)$ a suitable empirical formula for iron, steel or nickel, a simple analytical discussion of

the expression $\frac{\delta L}{L}$, or numerical calculation of it for different values of l and t from the experimental curve of the ordinary change of length leads to the conclusion that for iron, steel and nickel, all the points, which we have remarked in connection with the curves shown in Figs. 1, 4, 5, 7, 8 and 10, are involved without exception in the expression of $\frac{\delta L}{L}$.

It may be observed that the behaviour of cobalt with regard to the change of length is just the reverse of that of iron, and therefore every result which we have obtained for iron is also applicable to the case of cobalt, provided the sign of the length change be properly reversed. Thus in strong fields, the length of a cobalt tube should, by the combined effect of longitudinal and circular magnetizations, become shorter than when acted upon by the former alone. In weak fields, the result should be just the opposite. The field of maximum contraction should increase with the circular, and the amount of the contraction, after reaching a maximum, gradually decrease. The circular field at which the maximum contraction occurs should be far greater than that for iron.

19. The comparison above made is qualitative; how the calculated and th eexperimental numbers agree with each other is seen from the following table:

TABLE IX.

Nickel Tube, t=10.7			
l	L' (cal.)	L' (exp.)	difference
10	-20×10^{-7}	-25×10^{-7}	5×10^{-7}
20	−74	−85	11
30	−112	−125	13
50	−162	−175	13
80	−204	−216	12
120	−237	−250	13
200	−270	−285	15
300	−291	−305	14
500	−309	−323	14
700	−318	−331	13

Here L' denotes $\frac{\delta L}{L}-F(t)$, and its value was calculated from the experimental curve for the ordinary change of length. A glance in the above table shows a fair agreement between the calculated and the experimental values. The difference between these numbers is not of a serious nature, if we remember that one scale division of the micrometer ocular corresponds to the change of 5.12×10^{-7} for nickel, and that the correction for temperature amounts to 11×10^{-7} in the most significant case.

The discrepancy is probably due to the residual effect and also to the æolotropy of the tube. If the tube, after it is magnetized both longitudinally and circularly, is demagnetized by reversals with regard to the longitudinal magnetization, the circular field remaining constant, as was actually the case in the present experiment, the elongation due to the circular field alone is usually increased by one or two scale divisions,

a phenomenon which is perhaps to be attributed to the residual effect noticed in my former paper[1]. The constancy of the difference in the above table furnishes additional evidence in support of this view. The æolotropy of the tube as regards the change of length evidently influences the experimental values. Moreover the change of the intensity of longitudinal magnetization due to the mutual interaction of longitudinal and circular fields is not taken into account in the calculation of the effective field. These causes, I believe, are sufficient to account for the said discrepancy.

In steel and soft iron, there are comparatively large differences between the calculated and the experimental numbers, as will be seen from the following table:

TABLE X.

l	Wolfram steel, t=17.7		Soft iron, t=26.2	
	L' (cal.)	L' (exp.)	L' (cal.)	L' (exp.)
10	2×10^{-7}	1×10^{-7}	9×10^{-7}	8×10^{-7}
30	22	18	23	30
50	31	37	29	38
80	39	51	31	41
120	46	57	29	41
200	48	62	21	36
300	48	64	12	26
500	47	63	− 5	7
700	45	62	−17	− 5

For iron and steel, the sensibility of the apparatus was about 2×10^{-7} and the correction for temperature amounted to 5×10^{-7}

1) K. Honda, Jour. Sc. Coll. XI., 311, 1899.

in the most significant case. I believe that the principal causes of discrepancy above enumerated are sufficient to account for the difference between the calculated and the experimental numbers.

19. Thus qualitatively the above result and the experiment are in complete agreement with each other, although there are some discrepancies in quantitative details; there are, however, probable causes to account for the discrepancies. According to Knott, the change of length in cobalt by longitudinal magnetization is very little affected by the presence of a circular field, but the above consideration leads to a result which contradicts his anticipation. Hence a single experiment on this point for cobalt will decidedly establish the correctness of the one explanation against that of the other.

In conclusion, I wish to express my best thanks to Prof. H. Nagaoka, and also to Prof. A. Tanakadate for useful advice and kind guidance.

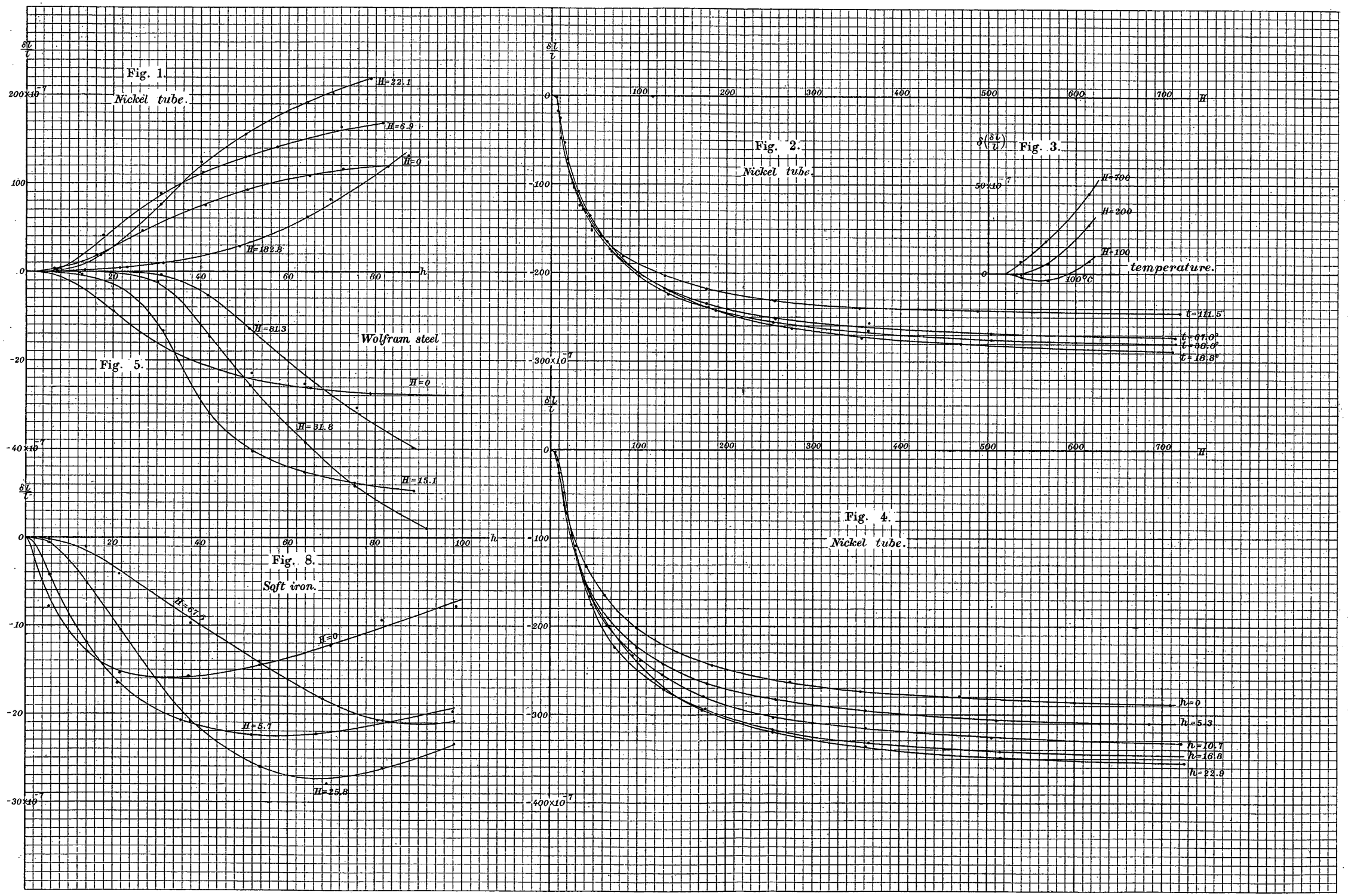
Fig. 1.
Nickel tube.
H=22.1
H=6.9
H=0
H=182.8
Fig. 2.
Nickel tube.
Fig. 3.
H=700
H=200
H=100
temperature.
t=111.5°
t=67.0°
t=38.0°
t=18.8°
Fig. 5.
Wolfram steel
H=81.3
H=0
H=31.6
H=15.1
Fig. 8.
Soft iron.
H=67.3
H=0
H=5.7
H=25.8
Fig. 4.
Nickel tube.
h=0
h=5.3
h=10.7
h=16.8
h=22.9

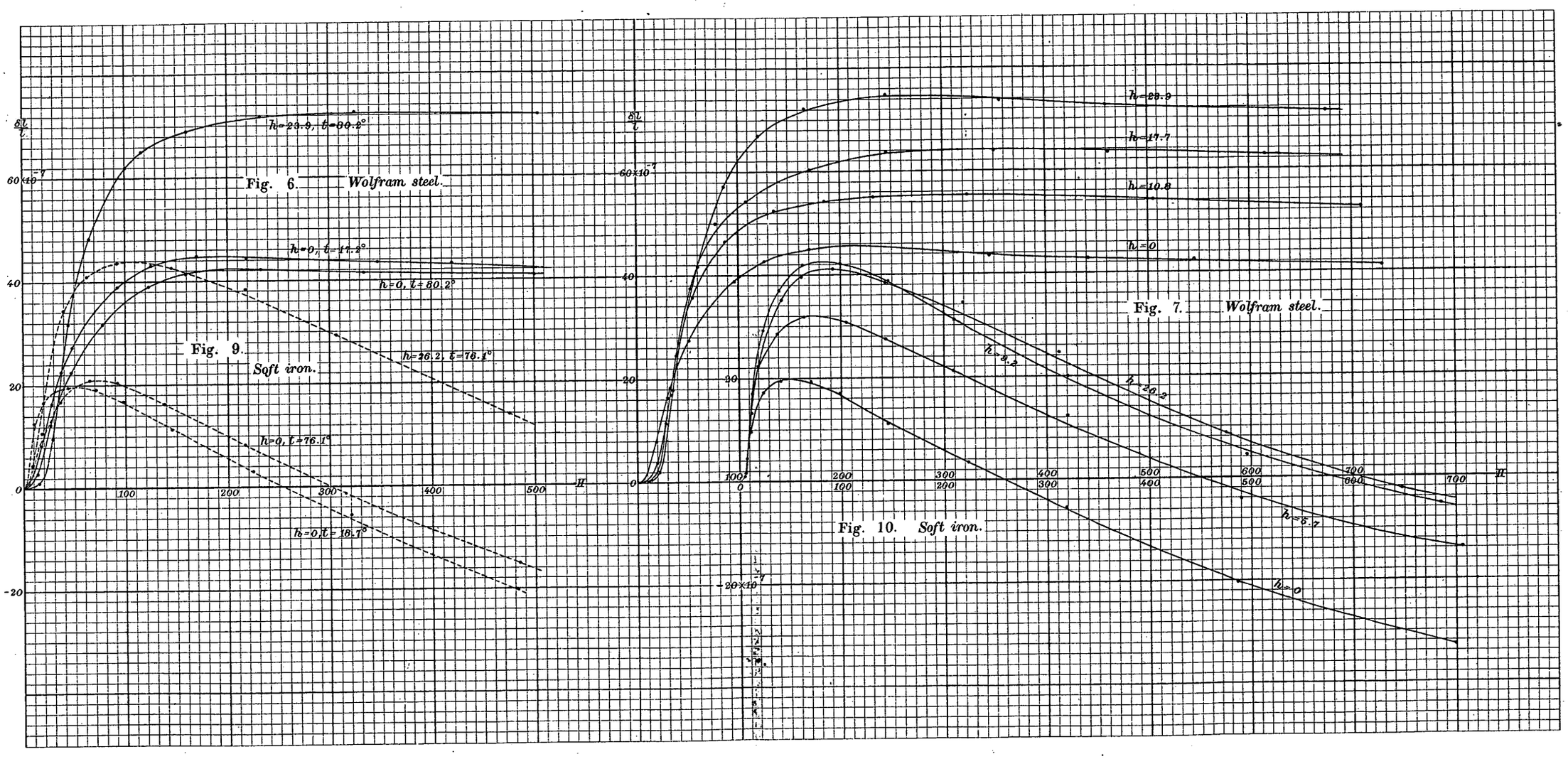

Fig. 6. *Wolfram steel.*

Fig. 9. *Soft iron.*

Fig. 7. *Wolfram steel.*

Fig. 10. *Soft iron.*