On the Mutual Influence between Longitudinal and Circular Magnetizations in Iron and Nickel.

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With Plates XXIV & XXV.

The change of magnetization of iron and nickel wires due to electric current passing through them attracted the attention of several physicists. Marianini[1] first observed the decrease in the moment of a permanent magnet by discharging a Leyden jar through it. The effect of longitudinal current on the intensity of magnetization of an iron wire placed in constant magnetizing field was examined by G. Wiedemann.[2] He arrived at the same result as Marianini, but he found further that when the current is broken, the magnetization is considerably increased. Villari[3] undertook similar experiments in connection with the effect which bears his name, and obtained analogous results. He concludes that longitudinal current increases the magnetization of soft iron in weak magnetizing fields and decreases it in the strong, and that on breaking the current, there is always increase of

1) Ann. de chem. et phys. **16**, 436, 1846.
2) Pogg. Ann. **117**, 213, 1862.
3) Pogg. Ann. **126**, 103, 1865.

magnetization. The experiment was repeated by Werner Siemens.[1] He wound two coils on an iron tube, one about the wall of the tube and the other perpendicular to it; the result of thus magnetizing the tube both longitudinally and transversely at the same time, as examined by means of a magnetometer, led to a considerable diminution of longitudinal magnetization. This result was afterwards confirmed by the experiments of W. H. Schultze,[2] C. G. Knott,[3] and F. H. Pitcher.[4]

The effect of longitudinal field on circular magnetization was also examined by Villari[5] with an iron tube. By passing a magnetizing current, the intensity of circular magnetization increased, and increased again when the current was removed.

The present investigation is, to a great part, the critical study of the results already obtained by these physicists.

I. Arrangement.

In the present experiment, the intensity of longitudinal magnetization and its change were measured by means of a magnetometer, while the circular magnetization was examined by the ballistic method.

The magnetometer consisted of a small bell magnet suspended in a thick copper case by a quartz fibre and provided with a plane mirror. It was placed due magnetic west of a magnetizing coil, and its deflection was read by means of a telescope and scale which was 1.90 m. apart from the magnetometer.

The magnetizing coil was 29 cm. long and wound in two

1) Berl. Monatsber. **23**, Juni 1881; Wied. Ann **14**, 685, 1881.
2) Wied. Ann. **24**, 643, 1895.
3) Phil. Mag. **30**, 244, 1890; Proc. Roy. Soc. Edinb. **18**, 124, 1891.
4) Phil. Mag. **47**, 421, 1899.
5) Mem. di Bologna [5] **2**, 443, 1892; Beibl. **17**, 670.

layers with copper wire of 1 mm. diameter, and gave the field of 29.5 C.G.S. units at the middle of the coil due to the current of one ampère. The internal diameter of the coil was 2.7 cm. and the external 3.4 cm. It was placed in horizontal position with its axis perpendicular to the magnetic meridian.

The strength of the magnetizing current was measured by Thomson graded galvanometer which was carefully compared with deciampère balance both before and after each experiment.

The ballistic galvanometer was of Wiedemann form, with copper damping. Its resistance was about 52 ohms, and its period of vibration was about 6 seconds. It was 4 m. distant from the magnetizing coil. The constant of the galvanometer was determined by means of a standard coil which was always placed in the secondary circuit. The last mentioned coil was wound in a single layer with fine copper wire on a glass tube, whose diameter was 2.724 cm. and had 500 turns. It was placed in another coil whose field at the centre due to the current of one ampère was 36.0 C.G.S. units, and the constant of the galvanometer was determined by the ballistic throw at the make and break of the current in the primary coil.

The ferromagnetics used in the present investigation had the following dimensions :—

1. Wire of Swedish iron :

   length=21.00 cm.; diameter=1.34 mm.; demagnetizing factor=0.0019.

2. Tube of Swedish iron :

   length=20.70 cm.; external diam.=0.416 ; internal diam. =0.335 cm.; demagnetizing factor=0,0063.

3. Tube of soft iron :

   length=16.79 cm.; external diam.=0.966 cm.; internal diam.=0.842 cm.; demagnetizing factor=0.0308.

4. Nickel tube:

length=17.02 cm.; external diam.=1.328 cm.; internal diam.=1.252 cm.; demagnetizing factor=0.0261.

During the experiment, it was almost impossible to avoid the change of temperature due to circularly magnetizing current. It was thus necessary at the first step to determine the effect of temperature on the intensity of magnetization of iron or nickel wire between the temperatures 10° C and 100° C.

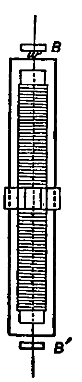
An iron wire was heated by passing steam through the space between two co-axial tubes of brass; it was placed in the axial line of the tubes, and the magnetizing coil was wound on the external tube. The effect of heating on the magnetization of iron is very small, the greatest change amounting only to 0.10 C.G.S. units per degree rise of temperature in the field of 21.2 C.G.S. units (I=1060 C.G.S. units). In fields less than 11 C.G.S. units, it is to increase and above that field to decrease the intensity of magnetization. With nickel wire, the character of the change of magnetization is similar with that of iron, but the amount of the change is comparatively large. The greatest increase of magnetization amounted to 0.35 C.G.S. units per degree rise of temperature in the field of 7.6 C.G.S. units (I=121 C.G.S. units). Thus the change of magnetization due to a small rise of temperature may be neglected for iron as well as for nickel.

The effect of torsional strain of iron or nickel caused by the magnetizing field and the longitudinal current on the intensity of magnetization was also carefully examined and found to be negligibly small.

To study the action of longitudinal current passing through the magnetized wire by the ballistic method, a secondary coil was wound on a glass tube of external diameter 0.748 cm. with fine copper wire, and placed in the magnetizing coil, in the axial line of which the wire was placed. The wire was soldered to a brass wire (2 mm. in diameter and 10 cm. in length) at each end, through which longitudinal current entered or left the wire. The ballistic throw produced by passing a given longitudinal current gave the change in magnetization.

The smaller iron tube was not circularly magnetized by means of a current passing directly through the tube, but was placed in the magnetizing field due to a current through an insulated copper wire of 1.8 mm. diameter in the axial line of the tube. To avoid heating, cold water was kept constantly flowing through the interspace between the wire and the inner wall of the tube.

To measure the effect of longitudinal current on magnetization by the magnetometric method, we must eliminate the direct action of the current on the magnetometer. For this purpose, another copper wire of the same diameter was bent so as to form a rectangle 6 cm. by 36. This wire was fixed in the horizontal plane passing through the centre of the magnetizing coil, with its longer sides parallel to the central line and symmetrical with respect to the centre of the coil as shown in the annexed figure. The central copper wire laid on V-shaped grooves of brass plates B and B′ at the ends of the coil, and its terminals were dipped in small mercury pools, bored in the middle of these grooves. On the side of the magnetometer, a short copper wire W (1 cm. in length) was placed parallel to and



under the central copper wire to connect the middle point of the shorter side of the rectangular copper wire with the brass plate B. The corresponding point on the opposite side of the copper wire and the brass plate B′ on the same side were connected with the poles of an accumulator. The current thus entered the brass piece B′, then flowed through the straight copper wire, and passed to two branches of the wire.

By this arrangement, the direct action of the current was nearly eliminated; the accuracy of the balance in the branches of the lead wire was tested before each experiment. The current flowing symmetrically with respect to the central line along each side of the rectangular wire produces nearly equal but opposite field on the iron tube, and is 3 cm. apart from it; hence the disturbance of the circularly magnetizing field would be insensible. This arrangement was found inconvenient for experimenting with thick tubes.

With larger iron and nickel tubes, I used another method of eliminating the heating by current. Each tube was wound uniformly with an insulated copper wire around the wall along its generating lines as to produce a nearly uniform circular field by a current passing through the wire, as was already done by Siemens and Schultze. The coil was protected by paraffin wax. It was then fixed in the central line of the magnetizing coil, and cold water kept constantly flowing in the tube.

The magnetization and its change were mostly measured by the magnetometric method. To determine them by means of the ballistic method, a secondary coil was wound on each tube. Placing the tube inside the magnetizing coil, and passing a current of cold water constantly round it, the change of longitudinal magnetization due to a circularly magnetizing field was measured, and compared with that obtained by the magnetometric method. The results obtained by these two different methods nearly coincide, showing that there is no trace of the effect of heating as determined by the magnetometric method.

To investigate the effect of the longitudinal field on the circular magnetization of iron or nickel tube, I also wound very carefully a secondary coil over the circularly magnetizing coil and parallet to the tube, and measured the change by the ballistic method.

The total number of turns of the primary circularly magnetizing coil was 40 for the iron tube and 60 for the nickel tube, and the mean field due to the current of one ampère was 17.7 C.G.S. units for iron and 18.6 C.G.S. units for nickel. Owing to the thickness of these tubes, the greatest deviation of the field from the mean was 14 % for iron and 4.5 % for nickel. The number of turns of the secondary coil for the measurement of longitudinal magnetization was 303 for iron and 1,000 for nickel, and that for circular magnetization was 100 for iron and 302 for nickel.

II. Result of Experiments.

1. Iron Wire.

The wire was first well annealed and put into the secondary coil, and the coil was then fixed in the central line of the magnetizing coil. After the wire was carefully demagnetized, it was placed in constant magnetizing field. The longitudinal current was then sent through the wire, and the corresponding throw of the ballistic galvanometer was read. The wire was again demagnetized and the same process repeated several times by changing the strength of the longitudinal current. Since the deflection is almost instantaneous, the effect of heating would be negligibly small.

Three sets of observation for iron wire are given in the following table and in Fig. I. In this and the following figures, curves are drawn through every point obtained by experiments.

TABLE I.

IRON WIRE.

H=0.9		H=1.7		H=10.4	
Long. cur. per sq. cm.	δ I	Long. cur. per sq. cm.	δ I	Long. cur. per sq. cm.	δ I
1.3	4	1.2	10	1.2	1
2.9	19	2.9	43	2.9	4
6.6	68	6.6	116	6.6	14
11.2	97	10.2	115	11.2	10
17.0	104	17.1	77	16.9	-17
30.2	95	30.5	4	30.5	-104
37.8	87	38.4	-30	38.5	-173
47.0	82	47.6	-61	44.4	-247

Here H denotes the effective magnetizing field, and δ I the change of the intensity of magnetization. Currents are given in ampères.

The hysteresis attending cyclic changes of longitudinal current is shown in Fig. 2. The magnetizing field was 6.7 C.G.S. units and the corresponding intensity of magnetization was 684 C.G.S. units. When the cyclic state arrived, the initial intensity of magnetization increased to 827 C.G.S. units. The strongest current applied was only 35 ampères per square centimeter so that the effect of heating would be small.

2. Smaller Iron Tube.

In the first place, putting the auxiliary copper wire in the central line of the magnetizing coil, the direct action of the longitudinal current on the magnetometer was eliminated, and then the compensation of the magnetizing coil brought to perfection. Replacing the auxiliary wire by the iron tube which was carefully annealed, it was magnetized longitudinally, and then gradually increasing the axial current, the corresponding deflections of the magnetometer were noted. The strongest current used in this experiment was 40 ampères.

The following table contains 3 sets of the observed results; 6 curves for different magnetizing fields are given in Fig. 3.

TABLE II.

H=4.5		H=6.6		H=19.0	
Long. cur. in amp.	δ I.	Long. cur. in amp.	δ I.	Long. cur. in amp.	δ I.
1.3	24.0	0.8	11.6	0.8	0.7
2.9	72.6	1.9	28.6	1.9	6.1
3.9	88.6	4.2	51.1	3.3	6.5
5.4	85.6	6.9	35.1	4.2	3.1
8.3	72.6	10.2	10.6	8.1	-30.3
11.7	61.4	11.5	8.2	11.3	-70.2
15.8	52.5	15.7	-11.9	15.3	-106.2
39.5	22.5	39.3	-57.9	39.4	-199.4

Next changing the order of magnetization, the tube was first circularly magnetized by a constant axial current, and then magnetized longitudinally, beginning with zero field up to about 60

C.G.S. units. The results are given in the following table and in Fig. 4.

TABLE III.

Long. cur.=0 amp.		Long. cur.=15.3 amp.		Long. cur.=40.9 amp.	
H	I	H	I	H	I
1.4	45	1.8	52	1.6	38
2.6	118	3.7	198	3.8	179
3.6	221	8.4	531	6.1	345
5.5	460	18.6	773	13.0	582
10.2	745	27.7	886	24.7	746
19.3	921	35.5	952	34.0	816
36.4	1044	44.4	1005	42.8	871
45.2	1085	57.9	1067	56.8	946

It is to be noticed that the intensity of magnetization in the longitudinal direction is, in this case, always less than that in the former case for the same longitudinal and transverse fields, as will be seen from Figs. 3 and 4.

Next keeping the longitudinal field constant, the axial current was changed cyclically between +12.5 and −12.7 ampères, and the hysteresis curve as shown in Fig. 5 was obtained. The magnetizing field was 11.6 C.G.S. units and the corresponding intensity of magnetization was 776 C.G.S. units. It increased to 845 C.G.S. units, when the cyclic state arrived.

The hysteresis curve attending the cyclic changes of longitudinal field, while the circular field is kept constant, is the same as the ordinary hysteresis curve, except that the area is considerably reduced, in accordance with what is well known.

3. Larger Iron Tube.

I first studied the behaviour of the tube by the magnetometric method. The magnetizing coil was well compensated, and the tube fixed in the central line of the coil; then closing both ends of the coil with cork, cold water was kept constantly flowing in the interspace.

The effect of transverse field on the intensity of magnetization in longitudinal direction is given in the following table and Figs. 6 and 7.

TABLE IV.

H=6.9			H=9.2			H=53.4		
h	δ I	δ I/I	h	δ I	δ I/I	h	δ I	δ I/I
1.0	44	0.092	1.2	18	0.023	2.0	2	0.002
2.0	52	0.107	2.0	26	0.032	3.0	3	0.002
3.0	64	0.132	3.0	36	0.045	6.7	–4	–0.004
6.6	54	0.112	6.7	8	0.010	13.3	–35	–0.028
13.1	19	0.040	13.3	–70	–0.087	28.4	–127	–0.104
27.8	–44	–0.091	27.4	–184	–0.228	40.0	–206	–0.168
39.0	–80	–0.166	39.0	–244	–0.302	53.8	–294	–0.240
54.3	–124	–0.256	49.2	–287	–0.355	74.9	–407	–0.332
83.5	–190	–0.392	73.9	–373	–0.462	88.9	–470	–0.384
104.0	–227	–0.470	95.6	–428	–0.530	103.8	–520	–0.425

Here H and I have the same meanings as before, and h denotes the transverse magnetizing field.

It is to be noticed that though the transverse magnetizing coil was wound round the wall of the tube very carefully along its generating lines, small longitudinal magnetization was still to be traced due to a current through the coil, when no magnetizing field in that direction was acting, so that this was corrected for each experiment. It is probably due to the magnetic æolotropy of the tube.

Although the longitudinal magnetizing field was kept constant, the effective part of the field must vary, because the change of longitudinal magnetization is necessarily accompanied by that of the demagnetizing force. The influence of the change on the intensity of longitudinal magnetization is considerable in weak fields, but not so prominent in strong fields. In weak fields, the effective part decreases by the application of transverse field, so that the increase of magnetization in weak fields is considerably reduced. In stronger fields, it is somewhat increased. Since the demagnetizing factor is very small for the iron wire and the smaller tube, the effect is not so remarkable as in the case of the larger tube. The small increase of magnetization, in weak fields, for the latter is mainly due to this cause. H in the tables I, II, IV denotes the initial effective field.

The same change was also measured by the ballistic method to assure the absence of the effect of heating when it was examined by the magnetometric method. Starting every time from a magnetically neutral state, I measured the change of magnetization by the ballistic galvanometer, and obtained the presupposed result. An example is given in the following table for the sake of comparison.

TABLE V.

H=3.1 (ballistic).		H=3.4 (magnetometric).	
h	δ I	h	δ I
1.2	13	1.0	13
3.2	47	2.0	32
6.9	56	3.0	53
14.3	42	6.7	51
22.2	33	13.3	40
30.4	25	28.2	22
44.2	13	39.5	11
56.6	7	54.8	–4
60.8	–3	76.1	–24

The magnetizing fields are not the same for these two cases, so that we can not find an exact coincidence between the corresponding values of δ I, but we observe at the same time that the coincidence must become closer, if H were the same in these two cases. From this and other results, I find that the effect of heating of the transverse magnetizing coil is not sensible in the case of iron.

Using the magnetometric method, I next examined the change of longitudinal magnetization by altering the order of magnetizations. The tube was first placed in a constant circular magnetizing field, and then magnetized longitudinally from zero field up to 18 C.G.S. units. The results are given in the following table and in Fig. 8.

TABLE VI.

h=0		h=2.2		h=37.8	
H	I	H	I	H	I
1.1	27	1.0	23	1.0	22
2.4	85	1.6	55	2.6	76
3.1	169	2.1	93	4.7	171
3.8	257	3.0	173	6.9	261
4.4	346	3.6	260	—	—
5.1	426	4.7	420	—	—
6.8	644	7.0	662	—	—
10.0	842	9.6	815	10.7	371
13.3	963	11.7	883	14.6	467
17.7	1035	18.5	983	24.7	631

Here again, from Figs. 6 and 8 we observe that the intensity of magnetization is, in this case, always less than that in the former case for the same transverse and longitudinal fields, so that the increase in weak longitudinal fields soon vanishes when the transverse field is increased.

Figs. 1, 3 and 6 show that in all longitudinal magnetizing fields less than 55 C.G.S. units, the magnetization increases slowly at first, and then rapidly, till it reaches a maximum. Then the change of magnetization begins to decrease very slowly in weak longitudinal fields, but rapidly in the stronger, and finally becomes less than its initial value. The decrease of magnetization in strong transverse or circular fields tends to approach a certain limiting value as the field is increased.

The transverse field at which the change of magnetization attains a maximum value decreases slowly as the longitudinal field is increased; and the point at which the change of magnetization vanishes recedes rapidly towards the origin as the longitudinal field is increased.

For weak longitudinal fields, the change of magnetization is linearly related to the strength of circular field, if it is not very weak. In strong transverse fields, the decrease of magnetization reaches a maximum and then again diminishes as the longitudinal field is increased.

If the axial current be broken gradually or suddenly, the intensity of magnetization always increases to a value greater than the initial. In weak fields, the increase is very great, amounting sometimes to double its initial value.

Fig. 7 shows that the value of $\delta I/I$ is comparatively large in weak fields, but decreases rapidly as the longitudinal field is increased. Hence the rate of increase of magnetization increases as the longitudinal field decreases, till it reaches a certain value, beyond which the rate again diminishes.

Figs. 4 and 8 show the curves of longitudinal magnetization when different circular fields are in action. Here the magnetization in low fields is increased when the transverse field is weak. In strong fields, it decreases for all transverse fields.

The hysteresis curves shown in Figs. 2 and 5 are similar in appearance with that of torsion,[1] but as will be seen hereafter, it arises from an entirely different cause. The symmetry of the

1) Phil. Trans. 1878; Reprint of Papers, Vol. II. P. 380.

curves with respect to the axis of δI shows that the change of longitudinal magnetization is independent of the direction of the transverse field, as first observed by Wiedemann.[1]

The experiments of several previous investigators on this subject are somewhat discrepant. As was already mentioned, G. Wiedemann noticed only decrease of magnetization; Villari observed an increase of magnetization by passing a current through an iron bar placed in a constant longitudinal field of moderate strength, and a decrease in a strong. He[2] also found a decrease of magnetization in strong circular field, and the discordance between the two was ascribed to the effect of heating by the current. On the other hand, Knott, Siemens, Schultze and Pitcher did not observe any increase of magnetization.

All these results are however not altogether discordant, and can be easily reconciled from the results of the present experiment.

I shall next describe the experiment on the effect of longitudinal field on circular magnetization.

The magnetizing coil was so placed that it gave no effect on the galvanometer. Putting the tube in the magnetizing coil and demagnetizing it by reversals, the effect of longitudinal field on the circular magnetization, when there is no circularly magnetizing field, was first examined by the ballistic method. The effect was very small and can be easily corrected. Keeping the circular magnetizing field constant, the tube was longitudinally magnetized with the following result.

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1) loc. cit.
2) loc. cit.

TABLE VII.

h=2.5		h=6.2		h=18.3		h=55.1	
H	δI	H	δI	H	δI	H	δI
0.0	0	0.0	0	0.0	0	0.0	0
2.1	110	2.1	23	2.2	3	2.2	3
3.2	214	3.2	34	3.2	1	3.2	–3
5.2	296	5.1	0	5.1	–27	5.1	–24
8.5	272	8.9	–103	8.5	–86	8.5	–52
15.7	176	15.9	–240	15.4	–172	15.4	–87
30.9	90	32.3	–390	32.0	–316	31.7	–153
38.2	79	40.3	–421	39.8	–661	39.1	–175
45.5	76	47.5	–440	46.8	–407	47.1	–195
64.0	58	65.4	–486	65.9	–483	65.9	–266
74.2	52	103.5	–536	76.7	–523	102.4	–379

To calculate H, we must subtract from the external field the demagnetizing force corresponding to the intensity of magnetization when the circular field is acting; but in the above table, I only took account of the demagnetizing force corresponding to no circular field, so that there will be some difference in the value of H.

From the above table, we see that the general nature of the change of magnetization is the same as that already described. In the present case, the demagnetizing force in the direction of circular magnetizing field does not exist, and therefore h remains constant during the application of longitudinal field; hence we must have greater increase of magnetization inweak longitudinal fields than in the former experiment. The large increase of magnetization in the weak field h=2.5 C.G.S. units must partly be due to this cause.

It was still a question whether the magnetizations in the longitudinal and transverse directions were equal, as the isotropy of the material was not well ascertained, and moreover the magnetized body was a toroid as regards circular magnetization, while with respect to longitudinal magnetization, it was a hollow cylinder. To settle this point, the magnetizations in these two directions were measured with the following result:

TABLE VIII.

H	I (long.)	H	I (circul.)
1.0	23	0.7	12
1.8	59	2.0	43
2.6	141	2.7	334
3.6	367	6.7	767
5.2	594	12.8	903
7.6	828	19.8	962
9.9	919	26.9	1004
20.3	1083	37.3	1056
31.9	1154	59.3	1131
70.7	1281	76.1	1175
88.3	1317	88.7	1200
114.3	1360	108.7	1243

The curve of circular magnetization is steeper than that of the longitudinal as seen from Fig. 9; hence the increase of circular magnetization in weak fields must be greater than that of longitudinal magnetization for the same field strengths for the reason hereafter given. The large increase of magnetization in the weak field noticed in the preceding experiment is to be explained partly by the difference in the behaviour of the longitudinal and circular magnetization. The difference in the decreases of magnetizations in longitudinal and transverse directions in strong fields is also due to the cause just mentioned as well as to the absence of demagnetizing force in the transverse direction.

Villari only observed increase of magnetization in the transverse direction by applying axial magnetizing fields, because his fields were too weak. Thus the effect of longitudinal field on circular magnetization is the same as that of circular field on longitudinal magnetization.

4. Nickel Tube.

The change of magnetization in longitudinal direction due to transverse field was examined by the magnetometric method as was done with the iron tube. The results are given in the following table and in Fig. 10.

TABLE IX.

H=6.6			H=9.2			H=23.2		
h	δI	$\delta I/I$	h	δI	$\delta I/I$	h	δI	$\delta I/I$
6.8	15.6	0.083	6.9	10.8	0.036	3.1	1.6	0.004
13.9	30.5	0.161	13.8	23.7	0.078	6.9	2.3	0.006
21.1	43.2	0.229	28.6	23.5	0.078	13.8	3.2	0.008
28.6	52.2	0.276	39.1	7.6	0.025	28.6	−9.7	−0.024
39.0	53.1	0.281	47.6	−14.2	−0.047	39.2	−26.9	−0.066
58.7	13.1	0.069	58.7	−45.3	−0.150	58.5	−70.4	−0.173
73.0	−27.6	−0.146	74.8	−97.2	−0.321	81.9	−132.3	−0.325
88.6	−63.9	−0.338	90.7	−141.6	−0.467	90.5	−155.0	−0.380

The same effect was also measured by the ballistic method. The results obtained by these two different methods agreed fairly well with each other, showing that the effect of heating of the coil, if any, must be very small even for nickel, as the following table will show :

TABLE X.

H=15.6 (ballistic)		H=14.9 (magnetometric)	
h	δI	h	δI
3.4	0.8	6.9	4.6
6.5	1.2	13.8	5.3
16.1	2.5	21.3	3.5
26.8	-2.8	28.6	-3.7
38.7	-16.0	39.0	-22.7
61.8	-67.3	47.7	-43.9
73.5	-102.2	58.7	-72.6
85.5	-140.0	75.1	-122.1
93.8	-166.7	91.2	-167.7

By the magnetometric method, I also measured the magnetizations with constant circular fields, the result of which is graphically shown in Fig. 11.

The hysteresis attending the cyclic changes of transverse field is shown in Fig. 12. The dotted line represents successive steps, till the change becomes at last cyclic.

Figs. 10, 11 and 12 show that the general feature of the change of magnetization is the same as that for iron. Although there is some difference in quantitative details, the qualitative results are generally common to iron and nickel.

The remarkable difference between iron and nickel lies in the field strength at which the increase of magnetization reaches a maximum. Such a field in nickel is, in the present experiment, greater than 35 C.G.S units in weak longitudinal fields, while in iron it is about 4 C.G.S. units.

Next I shall describe the result of experiments on the effect of longitudinal field upon the transverse intensity of magnetization measured by the ballistic method. Proceeding as in the case of iron, I obtained the following numbers:

TABLE XI.

h=6.7		h=20.9		h=41.4	
H	δ I	H	δ I	H	δ I
0	0	0	0	0	0
3.4	3.3	3.4	11.3	3.4	5.0
6.4	11.2	6.3	19.2	6.3	0.4
15.9	10.0	15.9	0.8	15.9	–40.6
31.1	3.8	31.9	–18.9	31.9	–59.3
49.3	–2.1	50.6	–36.0	50.6	–113.9
59.3	–9.2	88.9	–58.9	88.6	–159.0
77.7	–11.9	117.0	–67.2	102.1	–171.5
97.8	–14.2	131.0	–75.2	131.0	–190.5
143.3	–16.4	147.5	–79.9	146.6	–198.8

Here, in the calculation of H, the demagnetizing force corresponding to the magnetization with no circular field was only taken into account as in the case of iron.

The general feature of the change of magnetization is the same as in the former experiment. In this case, however, the increase in weak fields is generally small. As was the case with iron, the large increase of magnetization must have taken place in the above experiment, because the demagnetizing force in the direction of circularly magnetizing field does not exist. That this was not the case is to be ascribed to the magnetic æolotropy of the nickel tube. The tube was made of a plate 6.5 mm. thick. It was well annealed both before and after the hole was bored; but the æolotropy was, notwithstanding, still to be traced. This and the geometrical shape of the tube are the causes of the difference in longitudinal and circular magnetizations as shown in the following table:

TABLE XII.

H	I (long.)	H	I (circul.)
0.9	33.7	2.9	12.6
2.1	58.0	6.5	28.0
4.2	99.6	13.2	61.5
6.2	221.4	20.5	103.7
10.4	359.3	26.5	145.9
16.1	396.0	36.6	204.9
22.5	416.0	57.1	308.5
42.5	446.5	72.7	365.7
55.9	456.8	83.1	393.3
98.2	473.7	96.9	419.3
144.4	482.3		

These numbers plotted against H are given in Fig. 13. The longitudinal magnetization in the above table was measured by the magnetometric method, that determined by the ballistic method being almost coincident. It does not seem probable that the æolotropy can be entirely effaced simply by the process of annealing. The result here obtained is confirmed by the experiment of E. Rhoads[1] on the effect of fibrous structure introduced by rolling thin iron plate, with regard to the intensity of magnetization and the change of length by magnetization. From Fig. 13, we see that the curve of circular magnetization resembles that of steel; hence it is to be expected, that the change of circular magnetization by longitudinal field is comparatively small. In general, the change of magnetization is less with harder (magnetically) iron or nickel.

It was also observed in the iron and nickel tubes that the intensity of circular magnetization when we first magnetize it circularly and then longitudinally is always greater than when the order of magnetizations is reversed.

III. Theory.

From the molecular theory of Weber, W. Siemens[2] concluded that the magnetization in a given direction should be different, according as the field perpendicular to that direction exists or not, and that in strong fields near saturation the magnetization must diminish.

G. Wiedemann[3] also deduced the same conclusion from the hypothesis of molecular magnets, and added that in weak fields magnetization increases.

Using the equations established by Poisson and Kirchhoff, W. H. Schultze[4] concluded that the magnetization in a given direction must increase by applying a magnetizing force perpendicular to it,

1) Phy. Rev. 7, 1898, 65.
2) loc. cit.
3) Wiedemann's Electricität, 3, 725.
4) loc. cit.

when the resultant field is less than that corresponding to the maximum susceptibility of iron, and that it must decrease, when the former field is greater than the latter.

Proceeding from a similar idea as Schultze, I believe that all the phenomena observed in the present experiment can be explained in the following manner.

Suppose the ferromagnetics to be isotropic, and to have no residual magnetism; then the intensity of magnetization at any point can be expressed in the form

$$I=f(H),$$

where H is the resultant field. Taking H for the abscissæ and I for the ordinates, the curve $I=f(H)$ represents that of magnetization.

Now let there be magnetizing forces l and t acting in perpendicular directions, then if $H=\sqrt{l^2+t^2}$, we have

$$I=f(\sqrt{l^2+t^2}),$$

the component of which along l is

$$i=I\cos(H,l)=f(\sqrt{l^2+t^2})\frac{l}{\sqrt{l^2+t^2}};$$

hence if k be the susceptibility, we have

$$i=kl.$$

If we suppose that l is kept constant and t is changed,

$$\frac{\partial i}{\partial t}=l\frac{\partial k}{\partial t}=l\frac{\partial k}{\partial H}\frac{\partial H}{\partial t}=\frac{lt}{H}\cdot\frac{\partial k}{\partial H},$$

from which we see that the whole phenomenon depends upon the change of k considered as function of H.

Now we know that k increases at first, till it reaches a maximum, and then gradually decreases; hence so long as the resultant field H is less than the field corresponding to the maximum susceptibility, the value of $\frac{\partial i}{\partial t}$ is always positive, so that there must be increase of magnetization in the direction of l by increasing t. When H is greater than that field, $\frac{\partial i}{\partial t}$ is negative, and we have decrease of magnetization. When H becomes just equal to that field, $\frac{\partial i}{\partial t}$ is zero, and the value of i reaches a maximum. This field is determinate for a particular substance, so that the value of t corresponding to the maximum value of i must diminish when l is increased, such that l^2+t^2 is constant.

In soft iron or nickel, $\frac{\partial k}{\partial H}$ in weak fields is greater than that for harder one; hence increase of magnetization in weak fields for the former metal is greater than that for the latter, as already remarked.

The convexity or concavity of the curve of i plotted against t will be seen from the sign of the expression

$$\frac{\partial^2 i}{\partial t^2} = l\left(\frac{t}{H}\right)^2 \frac{\partial^2 k}{\partial H^2} + \left(\frac{l}{H}\right)^3 \frac{\partial k}{\partial H}.$$

From the experimental values of $\frac{\partial k}{\partial H}$ and $\frac{\partial^2 k}{\partial H^2}$, we see that when l is small and less than the critical field, $\frac{\partial^2 i}{\partial t^2}$ is at first positive, but it soon becomes negative, and when t is still further increased, it again changes its sign. When l is greater than the critical field, $\frac{\partial^2 i}{\partial t^2}$ is negative at first, and passing an inflexion point, it changes sign.

Thus when l is less than the critical field, the curve is at first convex towards the axis of t; but it soon becomes concave towards the same axis. When t is further increased, it again becomes convex. When l is greater than the critical field, the curve is at first concave and then becomes convex towards the axis of t in strong fields.

In the above considerations, we have supposed our ferromagnetics to be magnetically isotropic and to have no residual magnetism; but these suppositions are not admissible for iron or nickel tube. Moreover we have not considered the variation of the effective part of the longitudinal field caused by the change of demagnetizing force with increasing transverse field. Hence we can not, strictly speaking, compare the above results with those obtained by experiments. But qualitatively all these points just mentioned are verified in the present experiment, as shown by Figs. 1, 3, 6 & 10.

In order to get a clearer insight of the phenomenon, I compared the result calculated from the curve of longitudinal or circular magnetization by the above considerations with that obtained by experiments. The comparison was made only in strong longitudinal fields, in which the suppositions introduced in the theoretical consideration were comparatively small. In general, the change of magnetization in weak transverse fields fairly agrees with theory, but in the strong, the deviation becomes more pronounced. By taking the theoretical results calculated from the circular magnetization instead of the longitudinal, the coincidence between theory and experiment becomes closer, especially in strong fields. These results are given in the following table:

TABLE XIII.

SMALLER IRON TUBE.

Cur. in amp.	H=19.0		H=40.7	
	δI (exp.)	δI_1 (theor.)	δI (exp.)	δI_1 (theor.)
2	7	–9	4	–2
4	4	–21	3	–6
8	–30	–61	–10	–18
15	–103	–168	–63	–56
25	–162	–310	–128	–130
40	–201	–435	–181	–254

LARGER IRON TUBE.

h	H=27.8			H=53.4		
	δI(exp.)	δI_1(theor.)	δI_2(theor.)	δI(exp.)	δI_1(theor.)	δI_2(theor.)
10	–32	–55	–47	–17	–19	–17
30	–219	–322	–280	–138	–143	–128
50	–353	–522	–454	–271	–307	–271
70	–454	–651	–572	–383	–445	–399
90	–526	–736	–647	–475	–561	–495

The theoretical values of the change of magnetization in the above table were simply obtained graphically from the curves of magnetization: δI_1 from the longitudinal and δI_2 from the circular magnetization. With the smaller iron tube, the field due to axial current c at a point distant r from the axis was calculated on the supposition that it is equal to $\frac{2c}{r}$. This assumption is evidently a rough approximation.

Since the æolotropy of the nickel tube is considerable, the values deduced from the circular and the longitudinal magnetizations are widely discordant with each other and also with the experimental results. But in strong longitudinal fields, the experimental and the theoretical results obtained from the curve of circular magnetization agree fairly well.

One of the principal causes of the discrepancy between theory and experiment lies in the fact that in deducing the theoretical conclusion, I have not taken account of the æolotropy. As I have observed, the intensity of magnetization in a longitudinal direction, when longitudinal and transverse fields, l and t, act simultaneously, is expressed by kl, where k is the susceptibility corresponding to the resultant field $\sqrt{l^2+t^2}$. Hence a small change in the curvature of the curve of magnetization has a great influence upon the intensity of longitudinal magnetization, and thus it is easily seen that the supposition of isotropy leads to a discordan result.

In nickel, the field at which the change of longitudinal magnetization by transverse field reaches the maximum was far greater than in iron. This arises also from the magnetic susceptibility of the nickel tube. By applying a gradually increasing transverse field to a weak longitudinal field, the direction of the resultant field rapidly inclines towards that of the transverse field. Hence under weak longitudinal fields, the magnetization by the resultant field resembles that of the circular field rather than that of the longitudinal. Since the susceptibility of the nickel tube by circular magnetization increases in comparatively slower manner than in iron, the maximum increase of the longitudinal magnetization must recede into higher circular field. Thus the occurrence of the maximum increase of longitudinal magnetization in high fields is not characteristic for nickel; with a magnetically softer nickel, it would recede towards a weak field as in the case of iron.

Another principal cause of the discrepancy is the effect of residual magnetism. When the longitudinal field is kept constant and the transverse field is gradually increased, the residual magnetism due to the gradual change of direction of the resultant field must have some effect on the successive resultant intensity of magnetization. This effect we shall now proceede to consider.

It was found by experiments that magnetization of iron or nickel in a given direction was very little affected by the existence of residual magnetism in the same direction at the beginning of magnetization, provided the magnetizing field is not less than that which caused the residual magnetism. Let us now consider any two consecutive stages of applying transverse fields, and let I_1 and I_2 be the corresponding intensities of magnetization. We shall then investigate the contribution of I_1 as a residual magnetism on the formation of I_2. We resolve the residual magnetism due to I_1 into two components, one along and the other perpendicular to the intensity of magnetization I_2' which gives I_2 when acted on by the residual magnetism. Of these two, the parallel component very little affects the intensity of magnetization, as we have seen from experiments. The only effective part is the component perpendicular to I_2', the action of which tends to change its direction. Hence we may conclude that in whatever manner the residual magnetization may act, it is only to prevent the shifting of the direction of magnetization towards that of the resultant field. The intensity of magnetization in the longitudinal or transverse direction of our tubes must therefore depend on the order of application of the two perpendicular fields. This was verified experimentally for each tube.

The magnetic æolotropy also causes the direction of the resultant field to differ from that of the corresponding magnetization; but as will be seen from the present experiment, the effect is very small.

In strong longitudinal fields, the demagnetizing force diminishes with increasing transverse field, and therefore the effective part of the field increases, so that the intensity of longitudinal magnetization becomes greater than if the longitudinal field were constant. This influence can also be traced in the preceding table.

The transverse field also appears to produce molecular disturbance, and tends to increase the magnetization. It is generally very small in comparison with other effects.

Thus we have three principal causes of the discrepancy between theory and experiment, i.e., the magnetic æolotropy, the residual magnetism and the change of the demagnetizing force.

G. Wiedemann[1] explained the large increase of magnetization, when the transverse field is removed, by the instability of molecular magnets caused by releasing the transverse field. The increase is also explained by residual magnetism; if we subject the ferromagnetics to a gradually increasing transverse field, while the longitudinal field is kept constant, it will be strongly magnetized, and therefore when the transverse field is gradually or suddenly removed, there must remain a large amount of residual magnetism and hence we observe a large increase of magnetization. That the increase becomes greater as the longitudinal field diminishes, amounting sometimes to double its initial value,

1) loc. cit.

gives an additional evidence to the above explanation. Moreover that a sudden break of the transverse field gives less increase of magnetization than a gradual diminution, and that the increase becomes greater and greater as the transverse field is increased, however slow the breaking process may take place, seem to favor the explanation.

Summary.

The following is a summary of the results of experiments.

RESULTS IN IRON.

1) In constant longitudinal fields, the magnetization, so long as the field is weak, of iron wire or tube increases at first, till it reaches a maximum, and then decreases to a value less than the initial, by applying a gradually increasing transverse field.

2) In fields greater than 55 C.G.S. units, the magnetization in the longitudinal direction decreases from the outset.

3) The effect of longitudinal field on the intensity of transverse magnetization is exactly the same as the effect of transverse field on the intensity of longitudinal magnetization.

4) By first applying the magnetizing field either in the direction of the tube or perpendicular to it, and then another perpendicular to *it*, the magnetization in the direction of the first application of the field is always greater than when the order of magnetizations is reversed.

5) The change of longitudinal or transverse magnetization by transverse or longitudinal field is independent of the direction of the field.

6) The hysteresis curve attending the cyclic change of the transverse field, while the longitudinal field is kept constant, is

similar to that of torsion. The hysteresis by the cyclic change of the longitudinal field, while the transverse field is kept constant, is the same as the ordinary hysteresis, except that the area of the loop is considerably reduced.

Results in Nickel.

7) With regard to the effect of tranverse field on longitudinal magnetization and that of longitudinal field on transverse magnetization, nickel behaves qualitatively like iron; but the effect is generally small. The hysteresis curves have also similar forms as in the case of the former metal.

In conclusion, I wish to express my thanks to Prof. H. Nagaoka, under whose direction this investigation was carried out, and also to Prof. A. Tanakadaté for useful suggestions, and to Messrs. D. Sudo and S. Sano for kind advice and assistance in carrying out the present experiments.

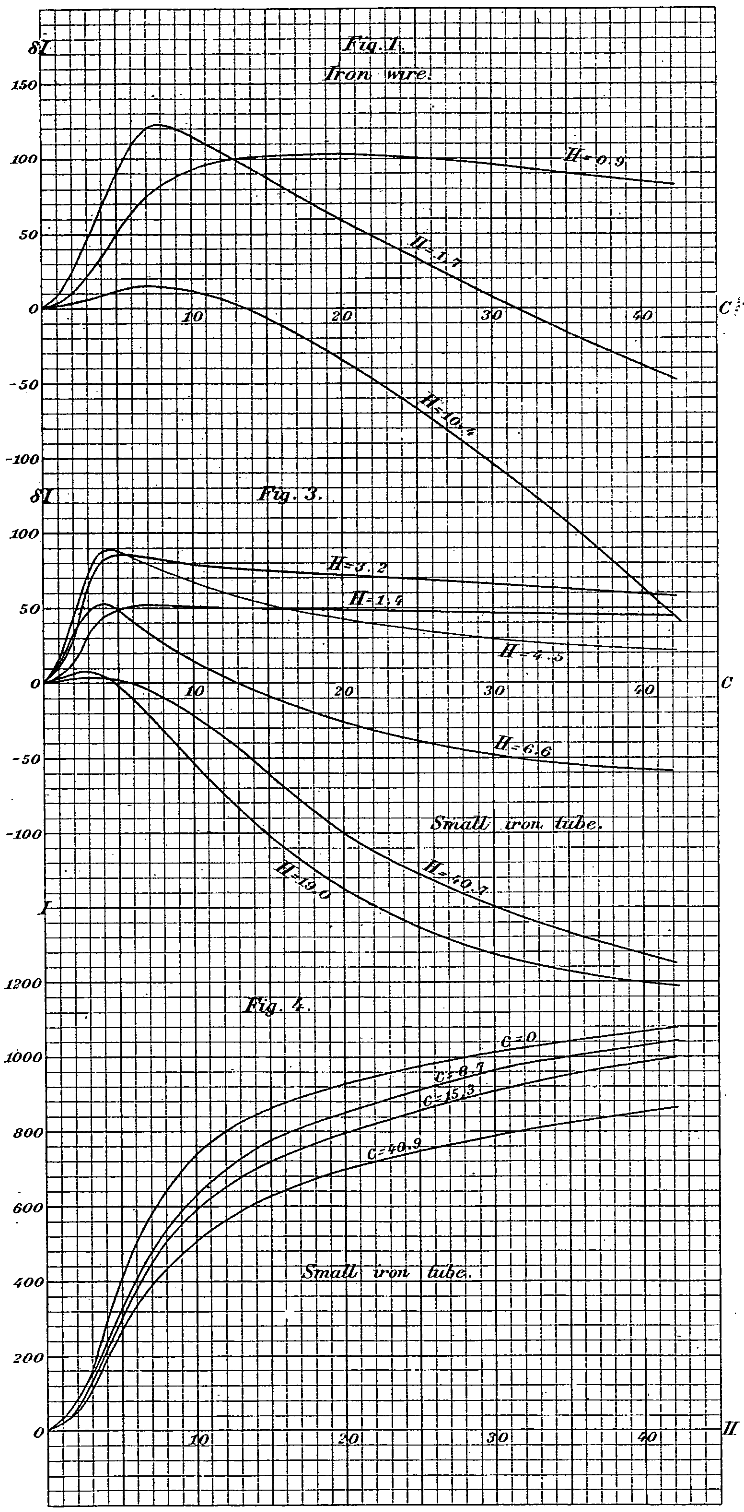
Fig. 1.
Iron wire.
δI
150
100
50
0
-50
-100
10
20
30
40
C
H=0.9
H=1.7
H=10.4
Fig. 3.
δI
100
50
0
-50
-100
10
20
30
40
C
H=3.2
H=1.4
H=4.5
H=6.6
H=19.0
H=40.7
Small iron tube.
Fig. 4.
I
1200
1000
800
600
400
200
0
10
20
30
40
H
C=0
C=6.7
C=15.3
C=40.9
Small iron tube.

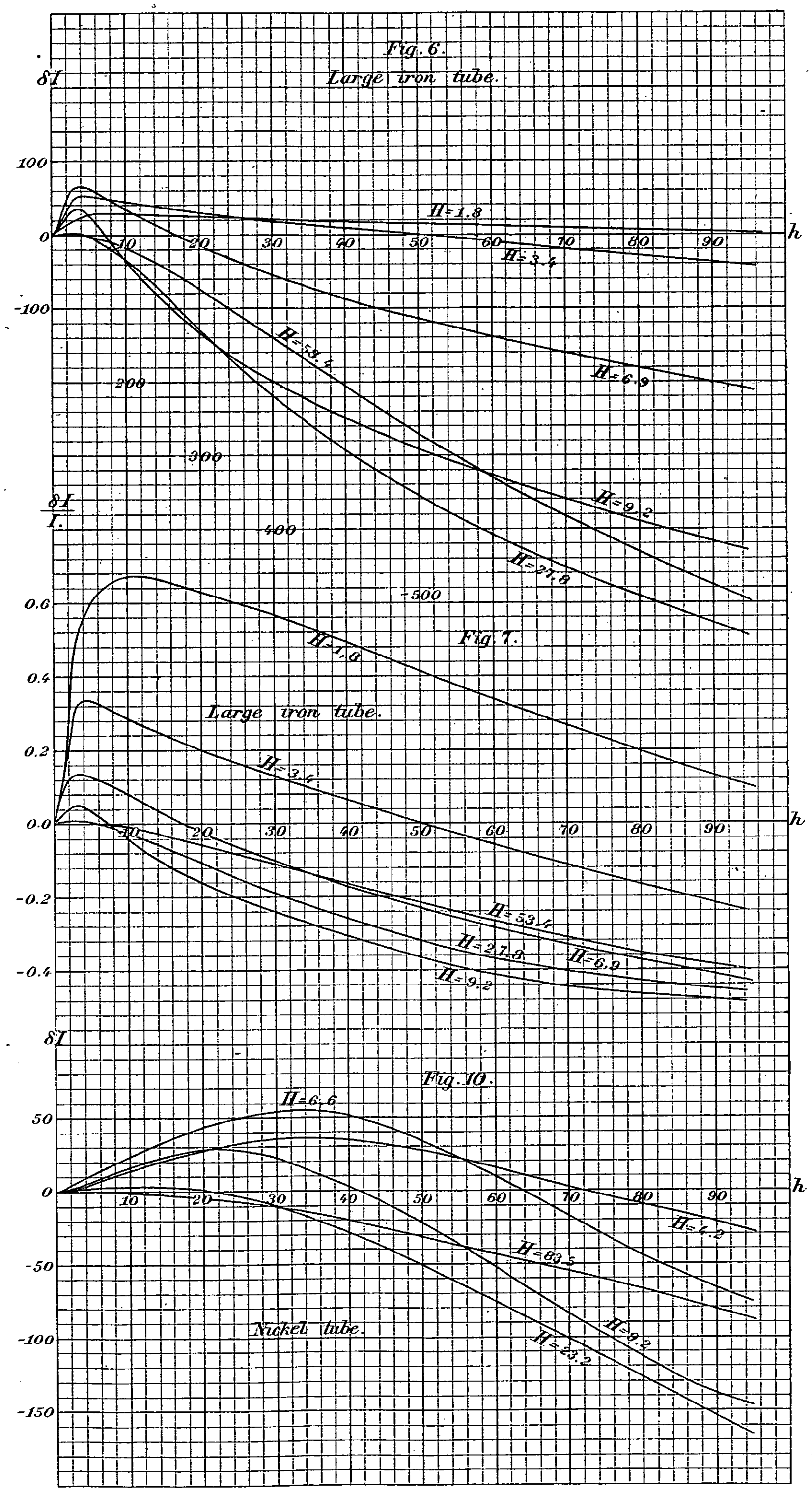
Fig. 6.
Large iron tube.
δI
100
0
-100
-200
-300
-400
-500
10
20
30
40
50
60
70
80
90
h
H=1.8
H=3.4
H=53.4
H=6.9
H=9.2
H=27.8
Fig. 7.
Large iron tube.
δI/I.
0.6
0.4
0.2
0.0
-0.2
-0.4
10
20
30
40
50
60
70
80
90
h
H=1.8
H=3.4
H=53.4
H=27.8
H=6.9
H=9.2
Fig. 10.
Nickel tube.
δI
50
0
-50
-100
-150
10
20
30
40
50
60
70
80
90
h
H=6.6
H=4.2
H=63.5
H=9.2
H=23.2

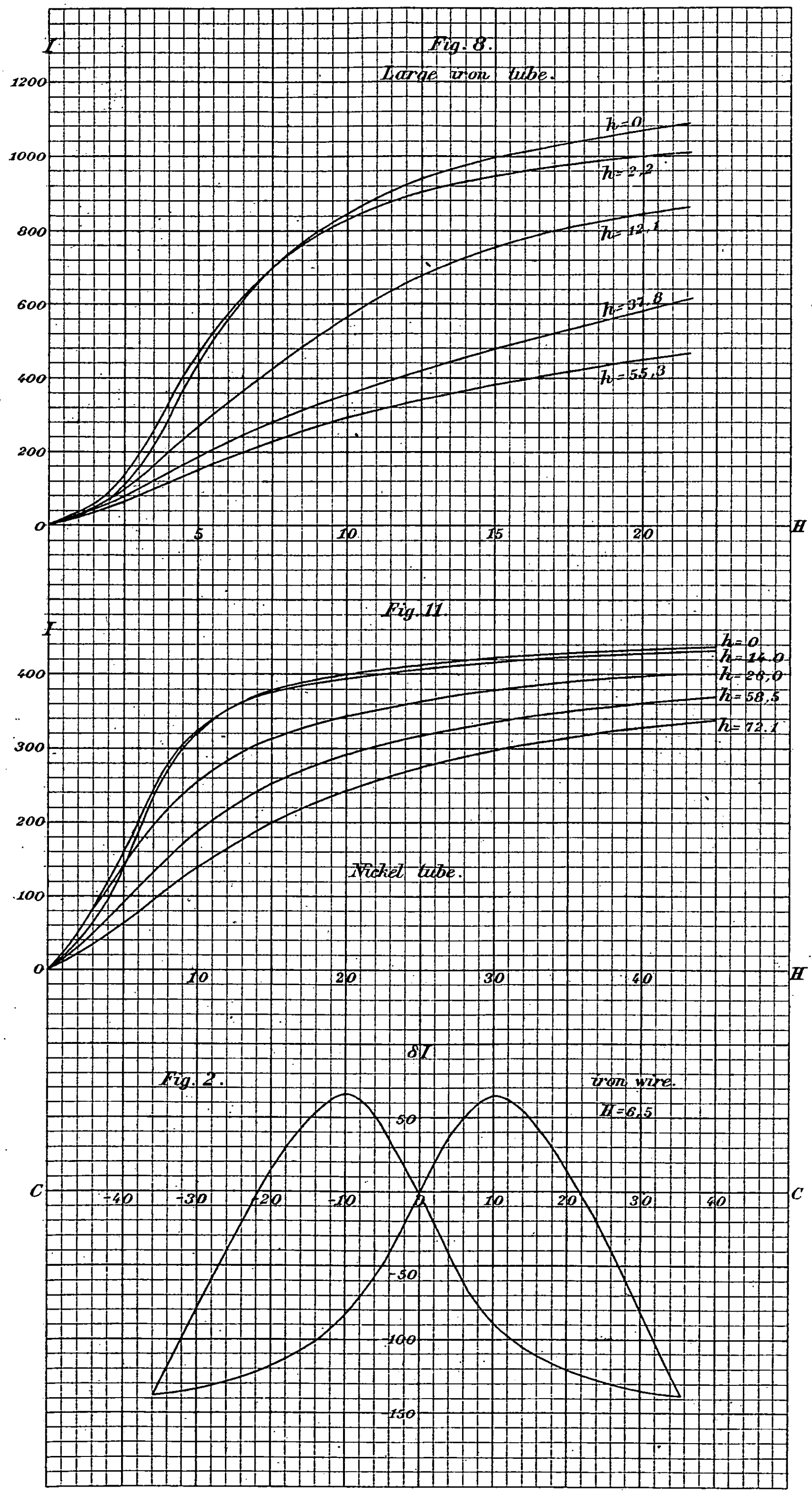
Fig. 8.
Large iron tube.
I
1200
1000
800
600
400
200
0
5
10
15
20
H
h=0
h=2.2
h=12.1
h=37.8
h=55.3
Fig. 11
I
400
300
200
100
0
10
20
30
40
H
h=0
h=14.0
h=26.0
h=58.5
h=72.1
Nickel tube.
Fig. 2.
δI
iron wire.
H=6.5
50
C
-40
-30
-20
-10
0
10
20
30
40
C
-50
-100
-150

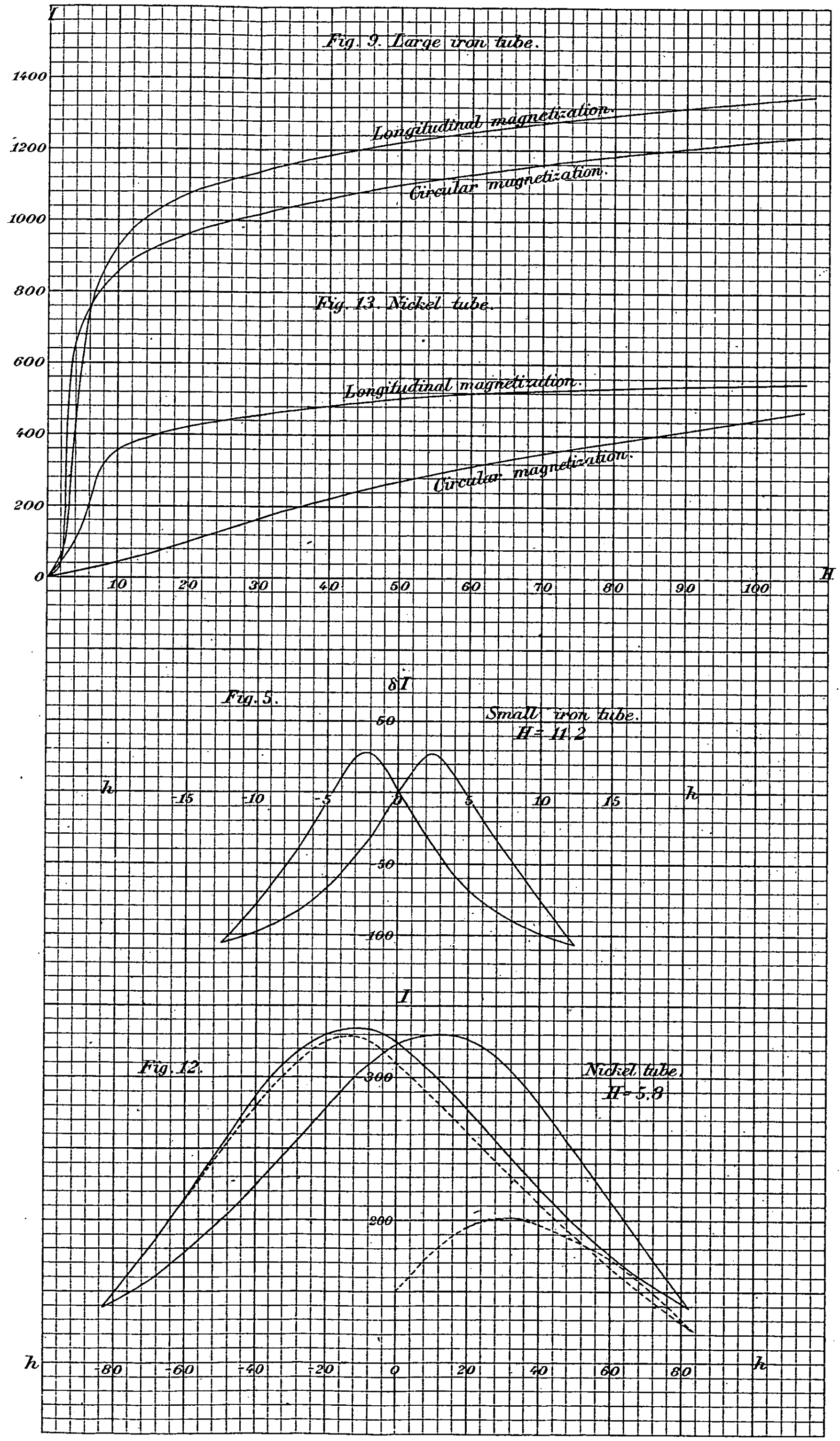
Fig. 9. Large iron tube.
I
1400
1200
1000
800
600
400
200
0
Longitudinal magnetization.
Circular magnetization.
Fig. 13. Nickel tube.
Longitudinal magnetization.
Circular magnetization.
10
20
30
40
50
60
70
80
90
100
H
Fig. 5.
δI
Small iron tube.
H=11.2
50
h
-15
-10
-5
0
5
10
15
h
-50
-100
I
Fig. 12.
Nickel tube.
H=5.8
300
200
h
-80
-60
-40
-20
0
20
40
60
80
h