PUBLICATIONS OF THE EARTHQUAKE INVESTIGATION COMMITTEE. NO. 22. SECTION B, ART I.

Strains produced by Surface Loading over a Circular Area with Applications to Seismology.

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With Plates I–III.

§ 1. The problem dealing with the strains of an isotropic elastic solid, bounded on one side by an infinite plane, was treated by Boussinesq and Cerruti. The former has published a series of researches on the allied subject in his treatise "Applications du Potentiel," in which full information of the problem in its varied aspects is to be obtained. A separate chapter is devoted to problems dealing with the strains produced by a vertical pressure applied over a circular area on the bounding horizontal plane, but unfortunately only a few particular cases of strains are worked out. Those who are interested in seismology may find it serviceable to have an approximate solution of the problem concerning the strain of the earth's crust, when there is surface loading, which may be likened to atmospheric pressure or to weight of rainfall exerted over a circular portion of the surface, in order to deduce the strains arising therefrom and thus obtain in some measure indications of the surface deformations due to the said natural agencies. With this object in view, interesting applications to seismology have already been made by Chree;* he discussed the influence of surface loading over a rectangular area and calculated the deviations

* Chree, Phil. Mag, 43, 173, 1897.

of the vertical and the tilting arising from it. A further extension of the problem to surface loading over a circular area is desirable, because the region of low or high pressure, as it is actually observed on the earth's surface, more resembles a circle than a rectangle. Moreover the loading is never uniform, so that the investigation of the problem or heterogeneous loading is not without interest. If the solution for an imperfect elastic solid, and for solids distributed in different strata were possible, the approximation to the actual problem as regards seismology would be of great value. The mathematical difficutly is however almost insurmountable, so that we are at present compelled to relinquish the attempt and satisfy ourselves with the solution for isotropic bodies.

Seismologists are familiar with the tremors associated with the change in the conditions of the atmosphere, sometimes in places quite outside the domain of low or high pressure. The question naturally arises how far from the place of surface loading the strain is appreciable, and how it depends on the extent of the stressed surface. Unfortunately the statical effect is generally difficult to observe, but the existence of the tremors itself indicates that the strain caused by the loading gives rise to elastic disturbance, calling forth free vibrations, which are presumably determined by the contour and geological structure of the region surrounding the place of observation. The surface wave thus called forth by distant surface loading is rather difficult of treatment, but we may obtain a crude idea from the calculation of the statical effect how far the influence of the stress is to be traced.

§ 2. Taking xy-coordinate plane as horizontal and z-axis vertically downwards, the components of elastic displacements u, v, w at point xyz in an isotropic elastic medium extending in positive direction of z axis, are given by

$$
(1)\begin{cases}
u=-\dfrac{1}{4\pi\mu}\dfrac{\partial^2}{\partial x\partial z}\displaystyle\int rdP-\dfrac{1}{4\pi(\lambda+\mu)}\dfrac{\partial}{\partial x}\int\log(z+r)\,dP\\
v=-\dfrac{1}{4\pi\mu}\dfrac{\partial^2}{\partial y\partial z}\displaystyle\int rdP-\dfrac{1}{4\pi(\lambda+\mu)}\dfrac{\partial}{\partial y}\int\log(z+r)\,dP\\
w=-\dfrac{1}{4\pi\mu}\dfrac{\partial^2}{\partial z^2}\displaystyle\int rdP+\dfrac{2\lambda+3\mu}{4\pi\mu(\lambda+\mu)}\int\dfrac{dP}{r}
\end{cases}
$$

where λ, μ are Lamé's constants, and the integration with respect to pressure P at point x', y', o, extends over the stressed area. The radius vector r is given by

$$r^2=(x-x')^2+(y-y')^2+x^2$$

Since

$$\frac{\partial^2 r}{\partial z^2}=\frac{1}{r}-\frac{z^2}{r^3},$$

the vertical displacement on the horizontal plane is given by

$$w_0=\frac{\lambda+2\mu}{4\pi\mu(\lambda+\mu)}\int\frac{dP}{r} \tag{2}$$

and the horizontal displacement for loading over a circle amounts to

$$U=-\frac{1}{4\pi(\lambda+\mu)}\int\frac{1}{r}\frac{\partial r}{\partial a}dP \tag{3}$$

where $a^2=x^2+y^2$, the centre of the circle being taken for the coordnate origin.

§ 3. For the calculation of the strain due to stress over a cirular area, we shall have to use the following notations.

R : radius of the circle.

a : distance of the centre of the circle from the point Q under consideration (x, y, o).

ρ : distance of surface point Q' (x', y' o) from the centre of the circle.

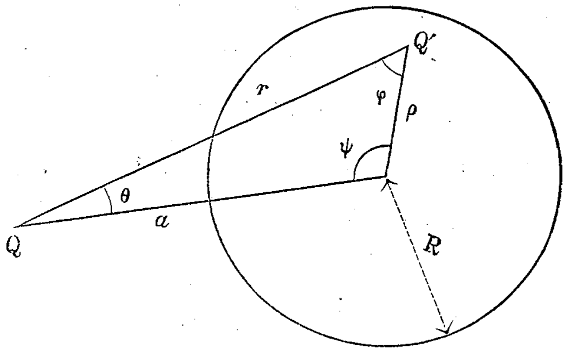
r : radius vector from the point on the surface at which the strain is sought.

θ : angle between r and a.

φ : angle subtended by a at the point Q' (x', y', o).

ψ: angle subtended by r at the centre.

The accompanying figure will show these quantities at a glance.



§ 4. Uniform Pressure over a Circular Area.

We have to consider the present problem in two aspects; (1) when the point Q under consideration is outside the stressed circle, (2) when the the point is inside the circle.

Let the pressure per unit area be denoted by p_0, then $dp=p\,\rho\,d\rho\,d\psi$, and since

$$r^2=a^2+\rho^2-2a\rho\cos\psi,$$

$$(4)\quad U=-\frac{1}{4\pi(\lambda+\mu)}\int\frac{1}{r}\frac{\partial r}{\partial a}dP=-\frac{p_0}{2\pi(\lambda+\mu)}\int_0^R\int_0^\pi\frac{(a-\rho\cos\psi)\rho d\rho d\psi}{a^2+\rho^2-2a\rho\cos\psi}$$

The integral

$$\int^\pi\frac{(a-\rho\cos\psi)d\psi}{a^2+\rho^2-2a\rho\cos\psi}=\frac{\pi}{2a}-\frac{\rho^2-a^2}{2a}\int_0^\pi\frac{d\psi}{a^2+\rho^2-2a\rho\cos\psi}$$

$$=\frac{\pi}{a}\quad\text{for}\quad a>\rho$$

$$=0\quad ,,\quad a<\rho$$

Thus when the point Q lies inside the circle, we have to divide the integral into two parts

$$\int_0^R=\int_0^a+\int_a^R$$

of which the last part vanishes.

Consequently we obtain

$$(5)\qquad \begin{cases} U=-\dfrac{p_0 R^2}{4(\lambda+\mu)a} & \text{for an } \textit{external} \text{ point} \\ U=-\dfrac{p_0\, a}{4(\lambda+\mu)} & \text{,, ,, } \textit{internal} \text{ point.} \end{cases}$$

For positive pressure, the displacement tends towards the centre of the circle, and for negative pressure from it. For an external point, the horizontal displacement is directly proportional to the compressed area, and inversely proportional to the distance from the centre; for an internal point, it is simply proportional to the distance, so that the horizontal component is maximum at the periphery of the circle of pressure. Fig. 1. shows the graph of the displacement.

The vertical displacement is given by (2) in the form

$$(6)\qquad w_0=\frac{\lambda+2\mu}{4\pi\mu\,(\lambda+\mu)}\,p\iint dr\,d\theta$$

But $\rho^2=a^2+r^2-2ar\cos\theta$ and $\sin\theta=\dfrac{\rho}{a}\sin\varphi$, consequently

$$dr=\frac{\rho\,d\rho}{\sqrt{\rho^2-a^2\sin^2\theta}}$$

$$d\theta=\frac{\rho}{a}\,\frac{\cos\varphi}{\sqrt{1-\dfrac{\rho^2}{a^2}\sin^2\varphi}}\,d\varphi$$

Thus

$$(7)\qquad \frac{2\pi\mu\,(\lambda+\mu)}{\lambda+2\mu}\,w_0=\frac{p_0}{a}\int_0^R\int_0^\pi\frac{\rho\,d\rho\,d\varphi}{\sqrt{1-\dfrac{\rho^2}{a^2}\sin^2\varphi}} \quad \text{for an } \textit{external} \text{ point.}$$

At an internal point, we have to divide the circle into two concentric circles, such that the point Q lies on the periphery of the internal circle. The displacement w_0 consists of two parts w_0' and w_0'', of which

$$(7_a)\quad \begin{cases} \dfrac{2\pi\mu(\lambda+\mu)}{\lambda+2\mu} w_0' = \dfrac{p_0}{a}\displaystyle\int_0^a\int_0^\pi \dfrac{\rho\, d\rho\, d\varphi}{\sqrt{1-\frac{\rho^2}{a^2}\sin^2\varphi}} \\ \dfrac{2\pi\mu(\lambda+\mu)}{\lambda+2\mu} w_0'' = p_0\displaystyle\int_a^R\int_0^\pi \dfrac{d\rho\, d\varphi}{\sqrt{1-\frac{a^2}{\rho^2}\sin^2\varphi}} \end{cases}$$

and $w_0 = w_0' + w_0''$ for an *internal* point

To evaluate (7), let us put $\dfrac{\rho}{a} = \kappa$, then

$$\int_0^R\int_0^{\frac{\pi}{2}} \frac{\rho\, d\rho\, d\varphi}{\sqrt{1-\frac{\rho^2}{a}\sin^2\varphi}} = \int_0^R K\rho\, d\rho = a^2\int_0^{\frac{R}{a}} K\kappa\, d\kappa$$

where K stands for a complete elliptic integral of the first kind. Using the wellknown formula

$$K\kappa\, d\kappa = d\left(\kappa\,\kappa'^2\frac{\partial K}{\partial\kappa}\right)$$

$$\int_0^{\frac{R}{a}} K\kappa\, d\kappa = -k'^2K(k) + E(k)$$

where E in an elliptic integral of the second kind and $k = \dfrac{R}{a}$, $k' = \sqrt{1-\dfrac{R^2}{a^2}}$.

Thus

$$(8)\quad w_0 = \frac{\lambda+2\mu}{\pi\mu(\lambda+\mu)} p_0 a\{E(k) - k'^2K(k)\} \text{ for an external point.}$$

On the periphery $\dfrac{R}{a} = 1$, and $E(1) = 1$; thus,

$$w_0 = \frac{\lambda+2\mu}{\pi\mu(\lambda+\mu)} p_0 a.$$

At an internal point

$$\frac{2\pi\mu(\lambda+\mu)}{\lambda+2\mu} w'_0 = 2p_0 a$$

and

$$\frac{2\pi\mu(\lambda+\mu)}{\lambda+2\mu}w''_0=p_0\int_0^{\pi}\int_a^{R}\frac{d}{d\rho}(\sqrt{\rho^2-a^2\sin\varphi})d\rho$$

$$=2p_0(RE(k)-a) \qquad \text{where} \quad k=\frac{a}{R}.$$

Thus

$$(8_a) \qquad w_0=\frac{\lambda+2\mu}{\pi\mu(\lambda+\mu)}p_0RE(k) \quad \text{for} \quad R>a.$$

The two expressions (8) and (8_a) coincide when $R=a$. The vertical displacement is greatest at the centre of the circle of pressure, and gradually decreases towards the periphery, but after passing out of the stressed area, the rate of diminution becomes extremely slow and ultimately vanishes in an asymptotic manner. Fig. 2 shows the amount of depression diagrammatically. These figures will bette- illustrate the general features of the horizontal and vertical displacer ments at a glance that a mere inspection of the mathematical formulas will show.

It may not be superfluous to give expressions for w_0 in terms of Weierstrass's notations. An easy calculation shows that

$$w_0=\frac{\lambda+2\mu}{\pi\mu(\lambda+\mu)}p_0\, a\frac{(\eta_1+e_2\omega_1)}{\sqrt{e_1-e_3}} \quad \text{where} \quad k=\frac{R}{a}$$

$$a\geqq R$$

$$w_0=\frac{\lambda+2\mu}{\pi\mu(\lambda+\mu)}p_0R\left(\frac{\eta_1+e_1\omega_1}{\sqrt{e_1-e_3}}\right) \quad \text{where} \quad k=\frac{a}{R}$$

$$R\geqq a$$

For the purposes of numerical calculation, it is convenient to have these expressions tranfsormed into ϑ-functions, in order that we may employ q-series with advantage.

Evidently

$$w_0=-\frac{2(\lambda+2\mu)}{\pi^2\mu(\lambda+\mu)}p_0a\vartheta_3(o)\vartheta''_3(o) \quad \text{for} \quad a\geqq R$$

and

$$w_0 = -\frac{2(\lambda+2\mu)}{\pi^2\mu(\lambda+\mu)} p_0 R \frac{\vartheta''_2(o)}{\vartheta_2(o)\vartheta_3^2(o)} \quad \text{for} \quad a \leqq R.$$

both of which are easily expressed in terms of q.

Fig. 1.

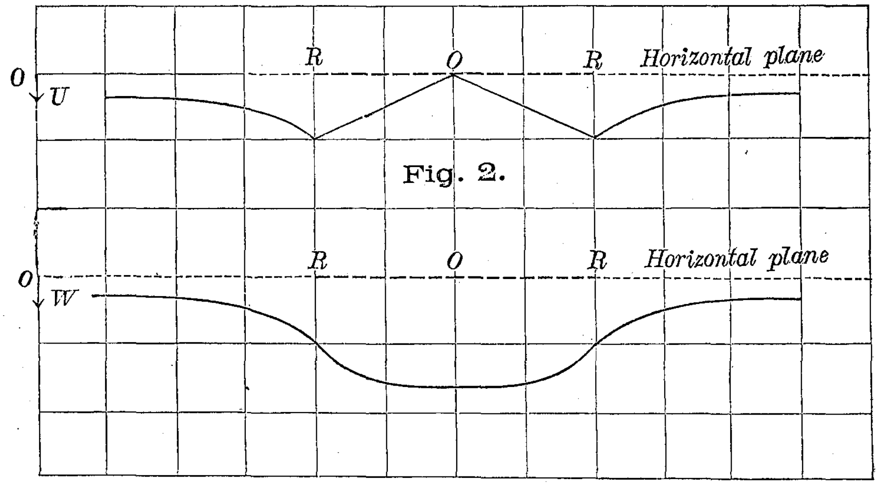


Fig. 2.

§ 5. Pressure over a Circular Area given by

$$p = P_0 - \rho^{2n}$$

In the present case we have to calculate

$$\int_0^R \int_0^\pi \frac{(P_0 - \rho^{2n})\rho d\rho d\varphi}{\sqrt{a^2 - \rho^2 \sin^2 \varphi}} \tag{9}$$

for finding the vertical displacement at an external point.

For the sake of brevity, let us put

$$J_n = \int_0^R \frac{\rho^{2n+1} d\rho}{\sqrt{a^2 - \rho^2 \sin^2 \phi}}$$

$a^2 = \alpha, \quad \sin^2 \phi = \beta, \quad \rho^2 = x;$

then

$$J_n = \frac{1}{2} \int_0^{R^2} \frac{x^n dx}{\sqrt{\alpha + \beta x}}$$

Putting $z^2 = \alpha + \beta x$, we obtain

$$J_n = \int^{a\Delta} \frac{(z^2-\alpha)^n}{\beta^{n+1}} dz \quad \text{where} \quad \Delta = \sqrt{1 - \frac{R^2}{a^2}\sin^2\varphi}$$

which is easily integrable.

Let us consider two particular cases.

$n = 1$:—

$$J_1 = \frac{a^3}{\sin^4\varphi}\left(\frac{\Delta^3}{3} - \Delta + \frac{2}{3}\right)$$

$n = 2$:—

$$J_2 = \frac{a^5}{\sin^6\varphi}\left(\frac{\Delta^5}{5} - \frac{2\Delta^3}{4} + \Delta + \frac{8}{15}\right)$$

Putting $\int_0^\varphi \frac{d\varphi}{\Delta} = u$, we have

$$\Delta = dnu, \quad \sin\varphi = snu,$$

whence for

$$I_n = \int_0^{\frac{\pi}{2}} J_n d\varphi$$

we get

$$I_1 = a_3 \int_0^K \frac{1}{sn^4u}\left(\frac{dn^4u}{3} - dn^2u\right)du + \frac{2a^3}{3}\int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sin^4\varphi}$$

$$I_2 = a^5 \int_0^K \frac{1}{sn^6u}\left(\frac{dn^6u}{5} - \frac{2dn^4u}{3} + dn^2u\right)du + \frac{8a^5}{15}\int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sin^6\varphi}$$

The integrands can be easily reduced to the following forms.

$$\frac{1}{sn^4u}\left(\frac{dn^4u}{3} - dn^2u\right) = \frac{k^4}{3} + \frac{1}{3}\frac{k^2}{sn^2u} - \frac{2}{3sn^4u}$$

$$\frac{1}{sn^6u}\left(\frac{dn^6u}{5} - \frac{2}{3}dn^4u + dn^2u\right) = k^6 - \frac{16}{15}\frac{k^4}{sn^2u} - \frac{4}{15}\frac{k^2}{sn^4u} + \frac{8}{15\,sn^6u}$$

Using the formula of recursion

$$sn^{2n-3}u\ cn\,u\,dn\,u=(2n-3)\int sn^{2n-4}u\,du-(2n-2)(1+k^2)\int sn^{2n-2}u\,du$$
$$+(2n-1)k^2\int sn^{2n}u\,du$$

we easily find

$$\int\frac{du}{sn^2u}=-\frac{cn\,u\,dn\,u}{sn\,u}+k^2\int sn^2u\,du$$

$$\int\frac{du}{sn^4u}=-\frac{cn\,u\,dn\,u}{3\,sn^3u}+\frac{2}{3}(1+k^2)\left(k^2\int sn^2u\,du-\frac{cn\,u\,dn\,u}{sn\,u}\right)-\frac{k^2u}{3}$$

$$\int\frac{du}{sn^6u}=-\frac{cn\,u\,dn\,u}{5\,sn^5u}+\frac{4}{3}(1+k^2)\int\frac{du}{sn^4u}-\frac{3}{5}k^2\int\frac{du}{sn^2u}$$

Remembering that at the limits

$u=0$	and	$u=K$
$dn(0)=1$		$dn(K)=k'$
$sn(0)=0$		$sn(K)=1$
$cn(0)=1$		$cn(K)=0$
$Z(0)=0$		$Z(K)=0$

we get

$$\int_0^K\left(\frac{k^4}{3}+\frac{k^2}{3sn^2u}-\frac{2}{3sn^4u}\right)du+\frac{2}{3}\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sin^4\varphi}=\frac{4(k^2-1)}{9}K+\frac{4+k^2}{9}E.$$

After all the necessary reductions, we find that at points external to the circle, the vertical displacement of the horizontal plane amounts to

$$(10)\qquad w_0=\frac{2(\lambda+2\mu)}{9\pi\mu(\lambda+\mu)}\frac{p_0a^3}{R^2}\left\{2(1-2k'^2)E-k'^2(1-3k'^2)K\right\}$$

when the pressure distribution is given by

$$p=p_0(R^2-\rho^2)$$

In the interior of the circle, we have to calculate

$$\frac{2\pi\mu(\lambda+\mu)}{(\lambda+2\mu)p_0}w_0''=\int_a^R\int_0^{\frac{\pi}{2}}\frac{(R^2-\rho^2)\rho d\rho d\varphi}{\sqrt{\rho^2-a^2\sin^2\varphi}}$$

$$=\frac{2R^2}{3}\int_0^K dn^4u\,du - Ra + \frac{R^2ak^2}{3} + \frac{2a^3}{9}$$

$$=\frac{2R^3(2(1+k'^2)E-k'^2K)}{9} - Ra^2 + \frac{5a^3}{9}$$

$$\text{where}\quad k^2=\frac{a^2}{R^2},\quad k'^2=\frac{R^2-a^2}{R^2}$$

Add to this

$$\int_0^a\int_0^{\frac{\pi}{2}}\frac{(R^2-\rho^2)\rho d\rho d\varphi}{\sqrt{a^2-\rho^2\sin^2\varphi}}=Ra^2-\frac{5a^3}{9}.$$

Consequently,

(10$_a$) $$w_0=\frac{(\lambda+2\mu)p}{9\pi\mu(\lambda+\mu)}R\{2(1+k'^2)E-k'^2K^3\}\quad\text{where } k=\frac{a}{R}$$

and $R \geqq a$.

Thus at the centre of the circle, the depression amounts to

$$w_0=\frac{\lambda+2\mu}{6\mu(\lambda+\mu)}R$$

and at the boundary

$$w_0=\frac{2(\lambda+2\mu)}{9\pi\mu(\lambda+\mu)}R$$

As regards the horizontal displacement, we have for the law

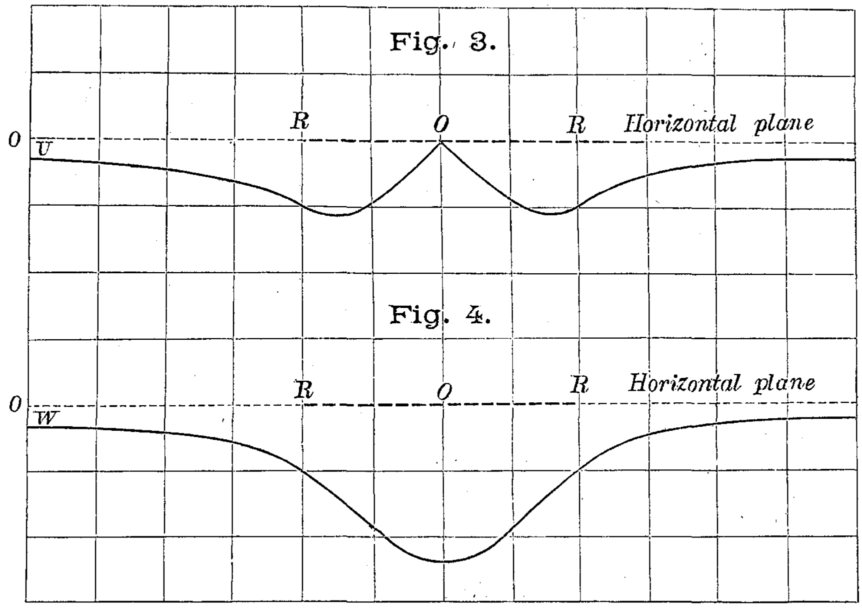
$$p=p_0(R^2-\rho^2)$$

(11) $$U=-\frac{p_0 R_2}{8(\lambda+\mu)a}$$ for an *external* point

and

(11$_a$) $$U=-\frac{p_0a(2R^2-a^2)}{8(\lambda+\mu)R^2}$$ „ „ *internal* point.

The formulas (11) (11$_a$) show that the horizontal displacement for the pressure distribution $p=p_0(R^2-\rho^2)$ is somewhat similar to that already found for uniform pressure. The difference arises in the neighbourhood of the periphery $\rho=R$, where the maximum displacement is to be found. Fig. 3 gives the diagram of the horizontal displacement. As to the vertical displacement, and the complicated functions by which it is expressed is rather difficult to judge, but constructing a diagram (Fig. 4), we easily find that the depression is quite large at the centre, and the tilting is generally more uniform than for constant distribution of pressure. At distant points, the asymptotic approach to the horizontal plane is evident from the figures.



§ 6. Finally, let us find the depression in the vertical line through the centre. Evidently the sinking in depth z is given by

$$w_z = \frac{\lambda + 2\mu}{4\pi\mu(\lambda+\mu)} \int \frac{dP}{r} + \frac{z^2}{4\pi\mu} \int \frac{dP}{r^3}$$

where $r^2 = \rho^2 + z^2$, x and y being put equal to zero. For uniform pressure p_0.

$$w_z = \frac{p_0(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)} \int_0^R \int_0^{2\pi} \frac{\rho\, d\rho\, d\theta}{\sqrt{z^2+\rho^2}} + \frac{p_0 z^2}{4\pi\mu} \int_0^R \int_0^{2\pi} \frac{\rho\, d\rho\, d\theta}{(z^2+\rho^2)^{\frac{3}{2}}}$$

Putting $\frac{R}{z} = tg\, \alpha$, we get

$$w_z = \frac{p_0 R}{2\mu}\, tg\, \frac{\alpha}{2} \left(2 \sin^2 \frac{\alpha}{2} + \frac{\mu}{\lambda + \mu} \right) \tag{12}$$

when the pressure distribution is given by $p=p_0(R^2-\rho^2)$

$$w_z=\frac{(\lambda+2\mu)p_0}{4\pi\mu(\lambda+\mu)}\int_0^R\int_0^{2\pi}\frac{(R^2-\rho^2)\rho\, d\rho\, d\theta}{R^2\sqrt{z^2+\rho^2}}+\frac{z^2}{4\pi\mu}\int_0^R\int_0^{2\pi}\frac{(R^2-\rho^2)\rho\, d\rho\, d\theta}{R^2(z^2+\rho^2)^{\frac{3}{2}}}$$

which by easy integration becomes

$$(13)\qquad w_z=-\frac{p_0R}{4\eta}\frac{\sin\frac{\alpha}{2}}{\cos^3\frac{\alpha}{2}}\left\{\cos 2\alpha+\frac{(\lambda+2\mu)(1-3\cos^2\frac{\alpha}{2})}{3(\lambda+\mu)}\right\}.$$

These two formulas (12) and (13) show that the vertical depression in the central line of the stressed circle diminishes gradually, so that we may expect the effect to be felt at a considerable depth, provided the radius of the region of pressure is large. In the application to technical problems, R may be a small quantity, but in geophysical questions as regards the region of low pressure or of heavy rainfall, the result above arrived at will be of special interest. Fig.'s 5 and 6 show how the depression diminishes with the depth.

$R=$ Fig. 5.

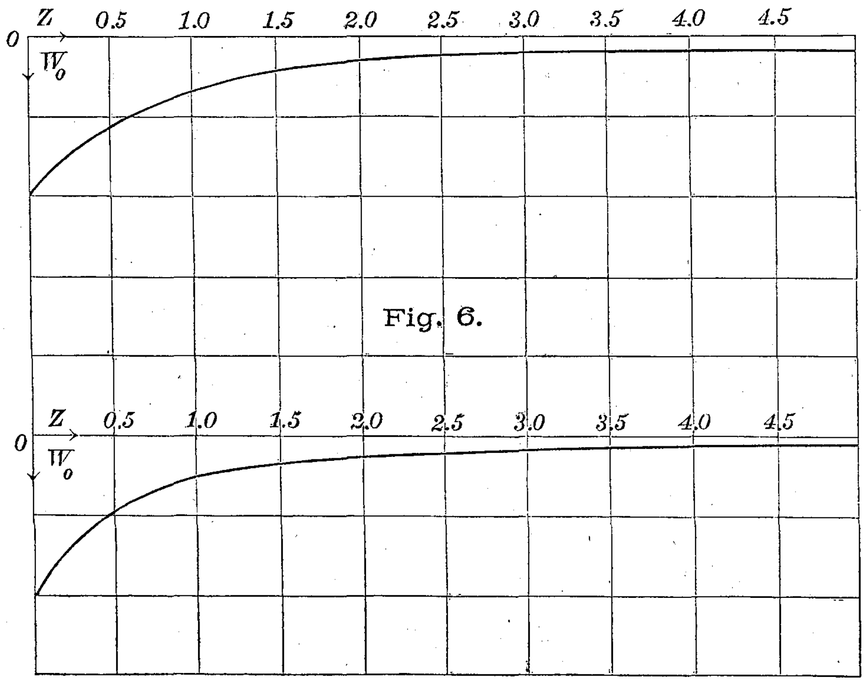


Fig. 6.

§ 7. For the distributions of stress above considered, I have constructed the contour lines of equal depressions as shown in Fig.'s 7 and 8 Pl. I. From these diagrams, it is evident that the gradient of depression is greater near the periphery of the area of pressure. It is less marked for the case in which the pressure reaches a maximum value at the centre than for a uniform distribution.

When the depression due to pressures over two circular areas is to be found, we have to apply the principle of the superposition of small displacements. The effect can be most easily effected by using the same method of procedure as finding the equipotential lines by adding systems of such lines due to distinct sources. The depressions due to two positive or negative pressures over circular areas are represented in Fig. 9 and 10 *a*, *b*, *c*, Pl. II. These diagrams resemble the equipotential lines due to two attracting or repelling sources. When nagative pressure is exerted over one portion and positive pressure over the other, the lines of equal depression take the form given in Fig. 11 *a*, *b*, Pl. III.

§ 8. For those interested in seismology, it would not be out of place to state one or two instances of the amount of depression due to surface loading. Supposing the elastic medium to consist of andesite, we have to use the following elastic constants.*

$$\mu = 6\times 10^{10} \text{ (C. G. S.)}$$

$$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu} = \text{Young's modulus} = 8.80\times 10^{10} \text{ (C. G. S.)}$$

When the pressure is uniform and $p_0 = 1$ cm. weight of mercury per cm^2, and $R = 50$ km., the depression at the periphery

$$w = 1.15 \text{ cm.}$$

and at the centre

$$w_0 = 1.80 \text{ cm.}$$

so that the mean tilting amounts to about $0''.02$.

For $p = p_0(R^2 - \rho^2)$, and $p_0R^2 = 1$ cm. weight of mercury

* H. Nagaoka. Pub. Earthq. Inv. Comm., No. 4, p. 60.

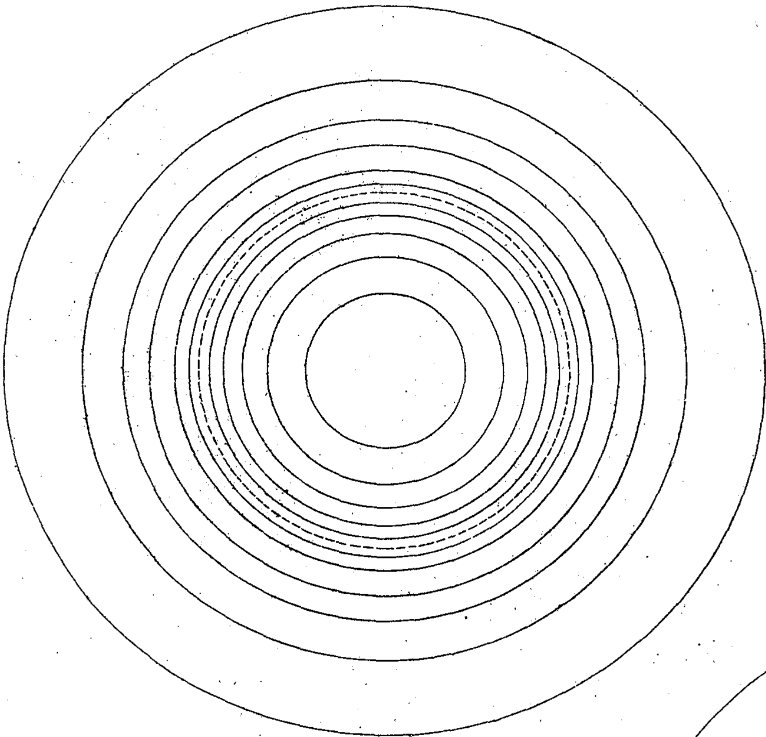
$w_0 = 0.52$ cm. at the periphery

$w_0 = 1.23$ cm. „ „ centre

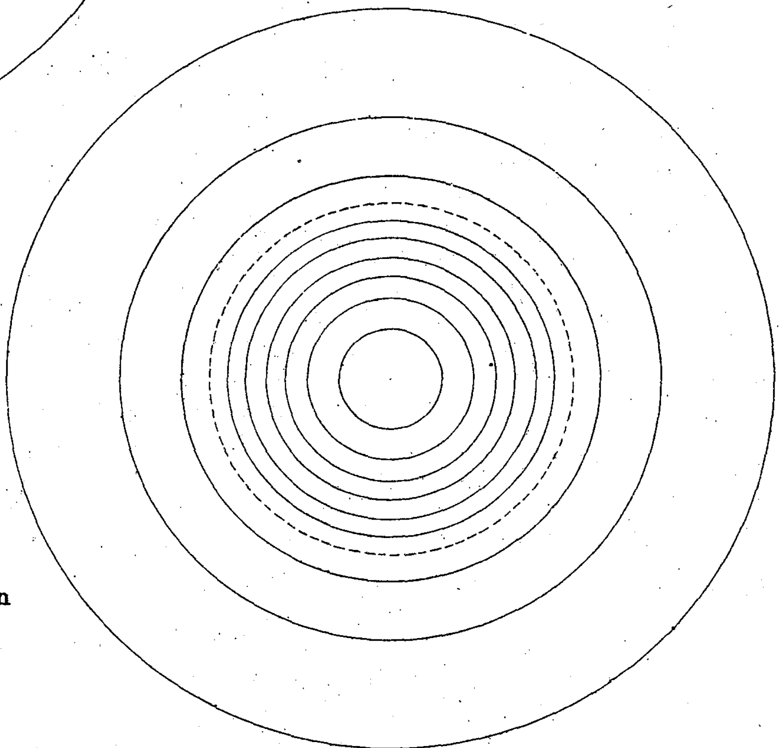
and the mean tilting is about 0.″02 as before. The elastic constants of the surface soil are evidently several times weaker than the andesite, so that the barometric change or rainfall might produce effects which would be within the limit of observation. With elastic solid of high plasticity, and with continuous application of pressure, there will be gradual increase of the strain, so that the effect may sometimes accumulate and at last attain an appreciable amount.

What will interest the seismologist in the present problem is the extent to which the strain due to vertical pressure is appreciable. If the area of pressure be confined to a small portion of the surface, the horizontal and the vertical components will practically vanish in its immediate neighbourhood, but the strained area increases almost in the same proportion as the stressed region, so that in the actual problem of the barometric change or of heavy rainfall, the effect would also be felt in places apparently remote from the place subjected to differences of surface pressure. The evanescence of the strain takes place almost asymptotically as already shown in the diagram, so that it will slightly disturb the elastic equilibrium and call forth vibrations or tremors. The surface wave which is most likely to be excited on the horizontal boundary and which is easily accessible to observation forms the next subject of discussion. Another problem, which will be interesting for seismology and volcanology is the strain on the horizontal surface, produced by internal pressure, a subject which I hope to be able to discuss in the near future.

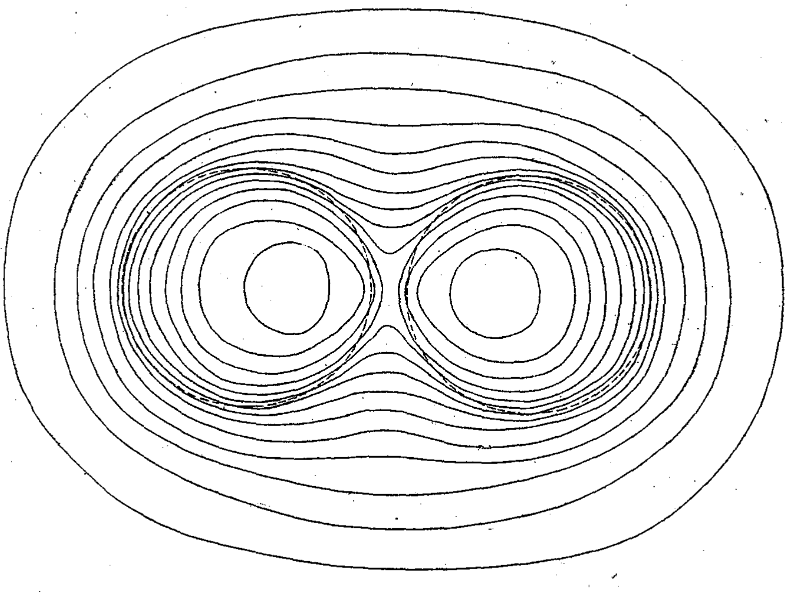
Depression due to $p = p_o$. (Fig. 7.)



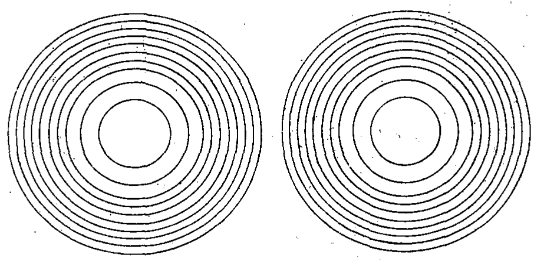
Depression due to $p = p_0(R^2 - \rho^2)$ (Fig. 8.)



Depression due to symmetrical distribution of pressure $p = p_o$. (Fig. 9.)

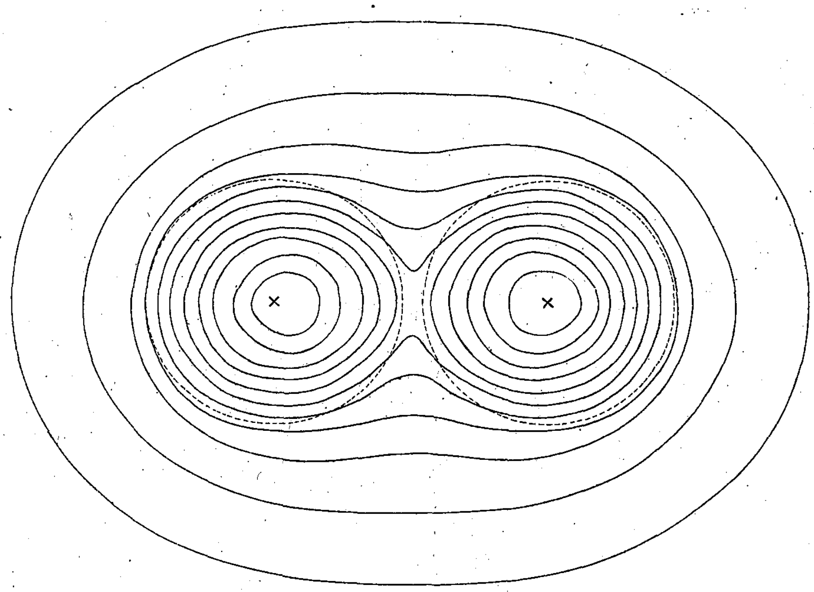


Pressure distribution. (Fig. 10_a)



Depression due to symmetrical distribution of pressure $= p_0(R^2 - \rho^2)$

(Fig. 10_b)

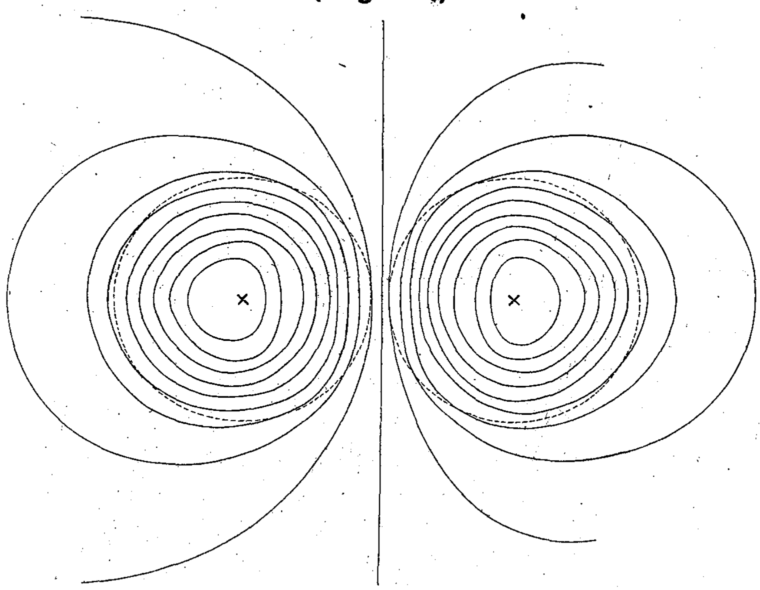


$p = p_0(R^2 - \rho^2)$

Elevation due to $p = -p_0(R^2 - \rho^2)$

Depression due to $p = p_0(R^2 - \rho^2)$

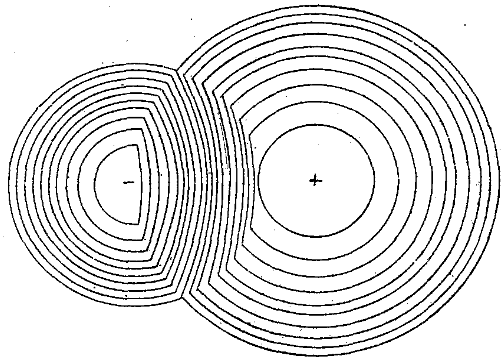
(Fig. 10_c)



$p = -p_0(R^2 - \rho^2)$

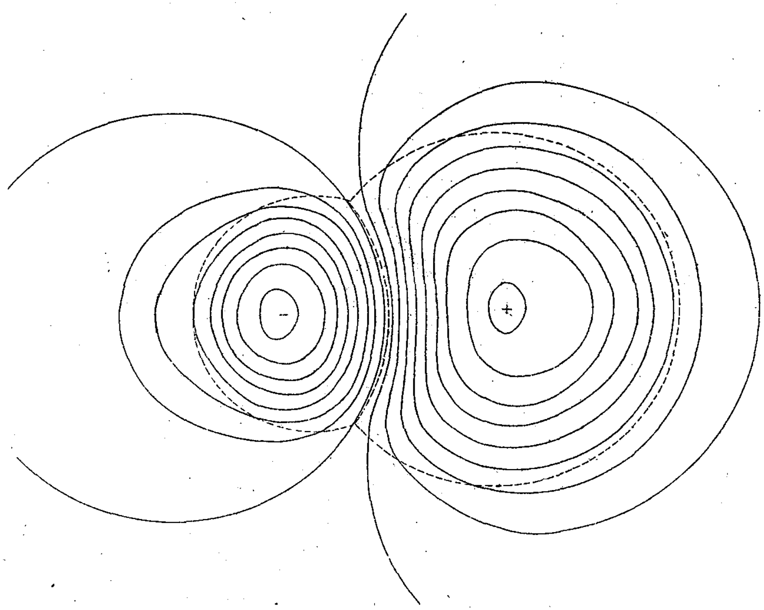
$p = p_0(R^2 - \rho^2)$

(Fig. 11_a)



Distribution of pressure

(Fig. 11_b)



Depression due to unsymmetrical distribution of pressure.