MODULUS OF ELASTICITY OF ROCKS:

AND

VELOCITIES OF SEISMIC WAVES:

WITH

A HINT TO THE FREQUENCY OF AFTER-SHOCKS.

BY

S. KUSAKABE, *Rigakushi.*

With Plates I.–XIV., Containing 75 Figures.

May. 1904.

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1. Introduction.

The present experiments, the expenses of which were defrayed by the Earthq. Inv. Committee out of the fund specially alloted for the investigation of the elastic properties of rocks, serves as a complement to the note, recently published by the author, on the modulus of rigidity of rocks.* Some of the specimens were one and the same with those used in the last investigation, and the others were prepared also in a similar manner. The principal object of the present investigation is not to determine any accurate value of the modulus of elasticity, but to determine whether the modulus is constant within tolerably wide limits or not, and if it is not constant, how it varies with the amount of stress or with time, and other factors which affect the change. The modulus is measured by the method of flexure; but the apparatus is not so simple like one which is generally employed. It would not be therefore superfluous to describe the details of the apparatus in the next section.

2. Arrangements and Flexure-apparatus.

In the measurement of flexure, the methods of cathetometre or of

* This Publication. No. 14. 1903. Tōkyō. The Jour. of the Col. of Sci., Imp. Univ. of Tōkyō, Japan. Vol. XIX, Art. 6. 1903.

micrometrescrew are generally put aside. The method of mirror and scale, Fig. 1, Pl. I, as modified by A. König,* is generally adopted though that of optical interference is more accurate. The apparatus as designed in the present experiment combines the advantages of König's arrangement with other necessary appliances. The principal features of the improvements are:—(1) to bend the specimen cyclically from one side to the other, with increasing and decreasing force passing through zero continuously; (2) to eliminate any external disturbance such as minute rotation of the specimen or slight displacements of the scale and telescope.

First arrangement:—A rough sketch of the first arrangement is shown in Fig. 2, Pl. I. The specimen is placed horizontally and bent righthandedly or lefthandedly, so that the plane of curvature is horizontal. There are necessarily four fulcrums and two scales instead of two and one respectively. As in König's method, the twice-reflected image of the first scale S_1 is seen in the field of the telescope T; and beside this an image of the second scale S_2, once reflected by the mirror M_2 is also to be seen.

Second arrangement:—In Fig. 3, Pl. I, the second arrangement is shown in its rough sketch. As in the first arrangement, the specimen is placed horizontally and, when it is bent, its plane of curvature is also horizontal. There is only one scale; but two more mirrors M_3 and M_4 are rigidly fixed to the support, while M_1 and M_2 are atached to the specimen. Four different images of one and the same scale S could be seen in the field of the telescope T, Fig. 4, Pl. I. They are all reflected twice by the following mirrors respectively:—

Right upper image reflected by the mirrors M_1 and M_2,
Right lower ,, ,, ,, ,, ,, M_1 and M_4,
Left upper ,, ,, ,, ,, ,, M_3 and M_2,
Left lower ,, ,, ,, ,, ,, M_3 and M_4.

* A. König. Ueber eine neue Methode zur Bestimmung des Elasticitäts moduls. Wied. Ann. 28, 1886.

Flexure Apparatus:—In Pl. I, Figs. 5 and 6 show front and side-view of the flexure apparatus while Fig. 7 shows it in the plan. The two mirrors M_1 and M_2 rotate as the specimen is bent, while the other mirrors M_3 and M_4 are fixed unless the apparatus itself is displaced. The fulcrums F_1, F_2, F_3 and F_4 in Fig. 7 are so adjusted that the edges of any two of them lie in a vertical plane. A small frame-work F, which is shown in Figs. 6 and 7 and more minutely in Fig. 8 Pl. II, serves to apply bending force to the specimen. The frame-work consists of two wedges, one fixed (W_1) and the other movable (W_2) inside a proper case. After having put a specimen between the two wedges, the movable wedge W_2 can be pushed firmly against the specimen by a fixed screw S. In the extremities of the strings S_1 and S_2, which run over small pulleys P_1, P_2 etc. to the neighbourhood of the observer, some weights are hang which give the bending force. The support of the fulcrums is made of soft iron, which is rigidly screwed on a wooden block. From Fig. 5, Pl. 1, it would be easily seen that, when equal weights are hang on both S_1 and S_2, no bending force acts on the specimen, and that it is the difference of weights hang on the two strings which is effective to bend the specimen. That is to say, if m_1 and m_2 are the two weights hang on the two strings S_1 and S_2 respectively, then their sum $m_1+m_2=M_0$ resists the action of the bending force, the last of which is due to their difference $m_2-m_1=M$. For future reference, M_0 and M will be called the Resisting mass and the Effective mass respectively.

When the effective mass is positive, i.e. as more weight is hang on the string S_2 than on S_1, the specimen is supported by the fulcrums F_3 and F_4, and it becomes comvex towards the righthand side. In the other case, it becomes convex towards the lefthand side, supported by the fulcrums F_1 and F_2. The fulcrums standing face to face, i.e. F_1 and F_3 or F_2 and F_4, are clamped not to push too tightly against the specimen, as there is a possibility of the bending of the specimen being hindered by friction.

A telescope, provided with a micrometer-screw, is rigidly clamped on a tripod. The scale, engraved on a ground-glass plate, 20 cm. long and 2 cm. wide is covered by a black board having a slit, 8 mm. wide, and is illuminated by a row of small gas flames.

3. Order of Observation.

Order of observation is generally as follows:—

1. To begin with, equall weights, each $\frac{1}{2}$ M_0, are hang on the strings S_1 and S_2.

2. A specimen is put between the fulcrums, passing through the framework F, the last of which is to be clamped on the middle part of the specimen. The planes passing through the edges of the fulcrums standing face to face should be normal to the length of the specimen.

3. The mirrors are so clamped in their proper positions that the images of the scale reflected by them stand side by side within the field of the telescope. This adjustment requires much experience.

4. The constants of the micrometer-screw for all images are determined. They are nearly equal to each other, but not strictly. One mm. of the scale division is equal to about 20 divisions of the micrometer-screw, which is again equivalent to rotation of 5.176×10^{-6} rad.$=1''.068$.

5. Zero-readings are taken for all images in a fixed order; i.e. (i) right upper image, (ii) right lower image, (iii) left upper image, (iv) left lower image.

6. The suspended weights consists of some fourty pieces of equal weights. A definite number of pieces, say $\frac{1}{2}$ m, was taken off from one string and added on the other. The bending force due to this is obviously mg, where g represent the value of gravity. The time-record corresponding to this transposition of weights is taken.

7. After a certain time, the readings are noted for all images in the same order as in the case of zero-reading.

8. Second transposition of weights; the time recorded; scale

readings noted; and so on till a definite amount of bending forces is reached.

9. The weight is then transposed in the opposite way so that the force diminishes gradually and ultimately becomes oppositely directed. In this way, a series of observations are made to complete the cyclic process several times.

10. From the amount of the deviations of the images, the amount of bending due to corresponding force are calculated, by a process to be explained in the next section.

4. Process of Calculation.

Process of calculation naturally divides itself into two according as the first or the second arrangement is adopted.

For the first arrangement we have :— In Fig. 9, Pl. II, let

SPE be the initial position of the specimen,

Sm_1 and Em_1' ,, ,, ,, ,, ,, ,, mirrors M_1 and M respectively,

ab and AB be the initial positions of the scales,

T be the position of the telescope.

Suppose, for the first case, the effective mass M to be positive so that the specimen takes the position $SP'E$ and the mirror Sm_1 and Em_1' takes the positions Sm_2 and Em_2', rotating through an angle $\pm\alpha$ respectively. And also suppose, then, the specimen to be rotated through an angle β so that the last positions of the mirrors are $S'm_3$ and $E'm_3'$ respectively. Note that the rotation of the specimen is generally not negligible, although the fulcrums as well as the apparatus itself are absolutely fixed. If the specimen be perfect square prism and the plane passing through the edges of the fulcrums F_1 and F_2 or F_3 and F_4 perpendicular to that passing through those of the fulclums F_1 and F_3 or F_2 and F_4, the specimen can not rotate. The above condition, however, could not be satisfied in the actual case, so that the specimen rotates whenever the effective mass changes its sign.

Let $\overline{MM'}=c,\quad \overline{AM'}=D,\quad \overline{Ma}=d,$ then it may be easily seen that

$$4\alpha=\text{arctg}\left\{\frac{1}{d}\left(\overline{ab}-\frac{c}{D}\overline{AB}\right)\right\}$$

or

$$4d\alpha=\overline{ab}-\frac{c}{D}\overline{AB}-\frac{1}{3}\frac{1}{d^2}\left\{\overline{ab}-\frac{c}{D}\overline{AB}\right\}^3+\ldots\ldots$$

Now, in so far as the present experiment goes, the first two terms in the righthand member are only effective: e.g. for the case of a piece of sandstone which seems to be one of the rocks having the smallest modulus of elasticity, we had

$M=3000$ grams, $\overline{ab}=3.139$ cm., $\overline{AB}=1.525$ cm., $c=12.8$ cm., $d=235.3$ cm., $D=248.5$ cm.

so that $\quad \overline{ab}-\frac{c}{D}\overline{AB}=3.060,\qquad \frac{1}{3}\frac{1}{d^2}\left\{\overline{ab}-\frac{c}{D}\overline{AB}\right\}^3=0.00017.$

Thus, the magnitude of the third term, and *a prioli* all the following terms, is within the errors of observation. Hence we have

$$\alpha=\frac{1}{4d}\left\{\overline{ab}-\frac{c}{D}\overline{AB}\right\}.$$

It may be easily proved that, in other cases where either the curvature or the rotation of the specimen or both of them change their signs, the same formula holds good, provided $\overline{ab}$ and $\overline{AB}$ are considered as the algebraic quantities having positive or negative signs.

The amount of rotation, i.e. β, is calculated from the formula

$$\alpha+\beta=\frac{1}{2}\ \text{arctg}\frac{\overline{AB}}{D}\doteqdot\frac{1}{2}\frac{\overline{AB}}{D}.$$

i.e. $$\beta=\frac{\overline{AB}}{2D}-\alpha.$$

In the example above cited, we have

$\alpha+\beta=3.07\times10^{-3};\quad \alpha=3.25\times10^{-3};\quad \beta=-0.18\times10^{-3}$ radians.

Here it will be noted that the value of β was generally small and constant relative to that of α, except in some particular cases which could be easily expected. When any small error occured in the evaluation of α, it caused an enormous discontinuity upon the value of β, so that the error could be easily discovered on tracing the curve of β.

For the second arrangement we have:—In Fig. 10, Pl. II, let the zero-readings be taken when the telescope is in T while the mirrors M_1 and M_3 in the position Mm_1 and the mirrors M_2 and M_4 in the position $M'm_2$. In reality, the reflections of light by the mirrors took place, as a matter of course, in the space of three dimensions; but, for the sake of simplicity, let us assume that the path of the ray of light lies wholly on the plane of the paper. Let $a\,b$ be the position of the scale, and suppose that a is a point which gives its images in the field of telescope after reflecting at S and S'. Suppose that, after a certain number of operations, the specimen is bent, it is rotated and also the telescope is displaced and rotated relative to the scale. Let their respective values be given by

α=angle through which the mirror M_2 is rotated as the specimen is bent,

$-\alpha$=angle through which the mirror M_1 is rotated as the specimen is bent,

β=angle through which the specimen is rotated,

δ=the component of the displacement of the telescope parallel to the scale.

Note that the component perpendicular to the scale is negligible relative to the distance between the scale and the telescope.

γ=The amount of rotation of the telescope.

Then, if θ and ω denote the angles between the mirrors M_2 and M_4, M_1 and M_3 respectively, we have.

TABLE. I.

Specimen convex to	righthand side		lefthand side	
Rotation of the specimen:	clockwise	counter clockwise	clockwise	counter clockwise
θ	$\alpha-\beta$	$\alpha+\beta$	$-\alpha+\beta$	$-\alpha-\beta$
ω	$-\alpha-\beta$	$-\alpha+\beta$	$\alpha+\beta$	$\alpha-\beta$

That is to say, provided α and β are taken as algebraic quantities having proper signs, we have

$$\theta=\alpha+\beta$$
$$\omega=\beta-\alpha.$$

Let T' be the last position of the telescope, and put

$R.U.$ = the deviations of the right upper image, which is reflected by the mirrors M_1 and M_2,

$R.L.$ = the deviations of the right lower image, which is reflected by the mirrors M_1 and M_4,

$L.U.$ = the deviations of the left upper image, which is reflected by the mirrors M_2 and M_4,

$L.L.$ = the deviations of the left lower image, which is reflected by the mirrors M_3 and M_4.

Then, from simple geometry, it may be easily proved that

$$L.L.=\delta+(D+c+d)\gamma,$$
$$L.U.=\delta+(D+c+d)\gamma-2d\omega,$$
$$R.L.=\delta+(D+c+d)\gamma+2(c+d)\theta,$$
$$R.U.=\delta+(D+c+d)\gamma-2d\omega+2(c+d)\theta.$$

where $\overline{ss'}=c,\quad \overline{sa}=d,\quad Ts'=D.$

If there is no disturbance, evidently we have

$$\beta=0,\quad \gamma=0,\quad \delta=0,$$

So that

$$L.L.=0.$$
$$L.U.=2d\alpha,$$
$$R.L.=2(c+d)\alpha,$$
$$R.U.=2(c+d)\alpha+2d\alpha=4\{d+\tfrac{c}{2}\}\alpha,$$

the last of which is a well known form.

In all cases, we must have a relation between the four values as follows:—

$$L.L.+R.U.=R.L.+L.U.$$

The difference of the two sums indicates an error of observation; whence it gives the means of selecting correct ones from numerous observations. For instance, in the case of a piece of sandstone we have:—

TABLE. II.

$M.$	$R.U.$	$R.L.$	$L.U.$	$L.L.$	$R.U.+L.L.$	$R.L.+L.U.$	Error.
grs. 900	c 1.129	c 0.529	c 0.593	c −0.004	c 1.125	c 1.122	c 0.003
1200	1.670	0.789	0.877	−0.006	1.664	1.666	−0.002
1500	2.204	1.039	1.159	−0.007	2.197	2.198	−0.001

To calculate the amount of bending, we have four equations containing four unknown quantities. There is, however, one functional relation between the four equations. At the same time, the unknown quantities also may be reduced into three, as δ and γ appear always in one and the same combination.

Put
$$x=L.L.$$
$$x+y=L.U.$$
$$x+z=R.L.$$
Then
$$x+y+z=R.U.$$

Taking any three of the four equations, we may solve them. It is preferable, however, to use all four equations, since none of them is strictly correct. Applying the method of least squares, we have the normal equations

$$4x+2y+2z=L.L.+L.U.+R.L.+R.U.$$
$$2x+2y+z\ =L.U.+R.U.$$
$$2x+\ y\ +2z=R.L.+R.U.$$

The solution is given by

$$x=\frac{1}{4}\left\{3.L.L.+L.U.+R.L.-R.U.\right\},$$

$$y=\frac{1}{2}\left\{(L.U.+R.U.)-(L.L.+R.L.)\right\},$$

$$z=\frac{1}{2}\left\{(R.L.+R.U.)-(L.L.+L.U.)\right\},$$

where
$$x=\delta+(c+d+D)\gamma,$$
$$y=-2d\omega,$$
$$z=2(c+d)\theta,$$

and $$\theta=\alpha+\beta$$
$$\omega=\beta-\alpha.$$

Eliminating x, y, z, θ and ω from the above equations, we have

$$\alpha=\frac{1}{8}\frac{c+2d}{d(c+d)}\left\{(R.U.-L.L.)+(L.U.-R.L.)\frac{c}{c+2d}\right\}$$
$$\beta=\frac{1}{8}\frac{c+2d}{d(c+d)}\left\{(R.L.-L.U.)+(L.L.-R.U.)\frac{c}{c+2d}\right\}$$

In the case lately cited, we have

TABLE. III.

$c=12.4$ cm., $d=241.5$ cm., $\frac{c}{c+2d}=2.503\times10^{-2}$, $\frac{1}{8}\frac{c+2d}{d(c+d)}=1.0099\times10^{-3}$

M	$R.U.-L.L.$	$R.L.-L.U.$	α	β
900 grams.	1.133 cm.	−0.064 cm.	11.46×10^{-4} rad.	-0.93×10^{-4} rad.
1200	1.676	−0.088	16.95	−1.31
1500	2.211	−0.120	22.36	−1.76

It is to be noticed that in the above calculation, tangent and arc of an angle are taken to be equal to each other. The greatest angle to be dealt with, indeed, is $\gamma+2\theta-2\omega$ which is of an order of 10^{-2} radian: whence the difference between the tangent and the arc is of an order 10^{-6}; that is to say, it is of an order of 10^{-4} of their own amounts, which is within the error of observation.

The relation between the modulus of elasticity and the amount of bending is given by the well known formula

$$E=\frac{3}{4}\frac{Mgl^2}{ab^3\alpha}$$

where a and b are the breadth and thickness of the specimen, while l is the distance between the corresponding fulcrums.

5. Preliminary Experiments.

The last investigation* with regard to the modulas of rigidity proved a great deviation from Hooke's law even in the case of the

* Loc. cited.

smallest strain. Preliminary experiments showed it to be alike also in the case of bending. Fig. 11 in Pl. II, shows the result of observation on a piece of micaschist, No. 42_2. The ordinate represents the amount of diviation of the image in the first arrangement, while the abscissae represents the corresponding effective mass in grams. It is clear at a glance that there is a great amount of residual bending when the force becomes nil, and also that the residual vanishes very rapidly when an oppositely directed force is applied. Such is always the case in magnetic hysteresis, especially at heigh temperatures,* and no one would doubt the result.

Examinning more accurately, however, it was proved that the latter fact was false. The deviation above stated was not due to the bending of the specimen, but also to its rotation. Evaluating these quantities separately in the manner above given, we have the results given in Fig. 12 and Fig. 13 in the same plate. Fig. 12 can be taken as the hysteresis curve in the relation between the bending and the force, while Fig. 13 gives the amount of rotation of the specimen during a cycle. When a series of observation, is completed within one or two hours, rotation of the specimen is the only correction required, so that the first arrangement is sufficient to be relied upon. As the experiment, however, is to be continued during several hours or even for days, it is safer to use the second arrangement.

Influence of the Resisting Mass :—To get rid of the influence of friction, etc, the resisting mass is kept constant during the whole experiment. Now it may be doubted that, as the resisting mass increases with the total mass to be moved, it might have some influence uppon the hysteresis curve. To examine this point, a series of experiments was made on a piece of sandstone, No. 3_4. The resisting mass was 1000, 1300, 1600, 1900 grams in each successive experiment respectively, while all other conditions remained the same through the four experiments. During the experiment, $1^h50^m - 5^h45^m$ P.M., 19^{th}

* D.K. Morris. On the Magnetic properties and Electrical Resistance of Iron as dependent upon Temperature. Phil. Mag. Vol. 44. 1897.

March 1903, the temperature of the room changed from 12°·8 to 12°·2, whose influence upon the elasticity may be neglected as we shall see soon after. To get rid of any influence of its initial state, the result was adapted after two complete cycles in each experiment. In Pl. III., the curves in Fig. 14 show the relations between the amount of bending and the corresponding effective mass for successive experiments. The curves in Fig. 15 show the variation of the modulus of elasticity due to the change of bending force during one complete cycle on the above experiments.

It may be necessary to write a remark on the meaning of the term "Modulus of Elasticity." As there is a great amount of hysteresis in the relation of stress to strain, the ordinary conception of the modulus of elasticity is necessarily vague and uncertain. The actual resistance to the deformation in any state whatever, be it already bent or twisted, elastic or plastic at that state, will be taken as the measure of elasticity at that state. Hence, in the above example, the modulus is measured, step by step, by the increase of bending per 200 grams increase of the effective mass. In each curve of Fig. 15, the right and left branches correspond to the cases where the specimen was bent convex towards righthand or lefthand side respectively, while the lower branches correspond to the increasing stress and the upper to the decreasing one. Comparing the curves in these figures, we may safely conclude that the presence of the resisting mass has no sensible influence, or, if any, it is negligibly small.

6. Yielding and Recovery from the Yielding.

The phenomenon of yielding, though it is not so enormous as in the case of torsion, is still sufficiently great to be dealt with. A piece of sandstone, No. 3_4, was loaded with $M_0=3300$, $M=3000$ grams at 4^h27^m P.M. 9^{th}. Feb. 1903.

At the instant of loading we had $\alpha=27.95\times10^{-4}$ for the amount of bending which increased to $\alpha=33.86\times10^{-4}$ at 8^h6^m P.M. and to $\alpha=$

60.57×10^{-4} at 8^h33^m of the next morning. Further observation was continued during about two weeks till the yielding, though it was steadily increasing, was much obliterated by the influence of the temperature-change. The amount of yielding since 3^h30^m P.M. 10^{th} is given in the following table.*

TABLE. IV.

Specimen No. 3_4. Sandstone. Loaded at 4^h27^m P.M. 9^{th} Feb., 1903. $M=3000$ grs., $M_0=3300$ grs.

Total amount of bending $\alpha=\alpha_0+\alpha_1+\alpha_2$, where $\alpha_0=27.95\times10^{-4}$, $\alpha_1=32.62\times10^{-4}+x$. in radian.

Time 10^{th} P.M.	Temperature.	α_2	Time 11^{th} A.M.	Temperature.	α_2
3^h 30^m	12°.0	0.00×10^{-4} rad.	11^h 53^m	10°.2	10.25×10^{-4} rad.
			P.M.		
3 51	11.3	0.02	0 48	10.1	10.43
4 10	10.8	0.15	1 14	10.1	10.54
4 35	10.0	0.25	1 40	10.6	10.53
5 6	9.5	0.58	2 5	11.5	10.85
6 19	9.2	1.44	2 30	11.9	11.69
6 39	9.1	1.82	2 54	12.1	13.55
7 5	9.3	2.17	3 14	12.2	14.99
7 31	9.2	2.38	3 34	12.3	16.87
7 58	9.0	2.52	3 52	12.4	18.41
8 24	9.0	3.19	4 13	12.4	20.31
8 45	8.8	3.26	4 40	12.1	22.66
9 12	8.8	3.52	5 1	11.7	23.86
9 35	8.7	3.70	5 21	11.5	24.44
9 47	8.6	3.96	6 10	11.4	25.53
11^{th} A.M.					
8 25	6.4	8.79	6 28	11.5	25.83
8 47	6.7	9.07	6 52	11.5	26.04
9 10	7.0	9.31	7 16	11.4	26.25
9 30	7.3	9.29	7 37	11.3	26.50
9 52	7.9	9.52	7 56	11.0	26.70
10 14	8.4	9.77	8 24	11.0	26.85
10 42	8.7	9.91	8 44	10.8	26.91
11 0	8.9	9.94	9 6	10.7	26.96
11 28	9.6	10.11	etc. etc.	etc.	etc.

* On noon of this day, the telescope was slightly disturved so that the total sum of the yielding could not be known. After this event, I devised the second arrangement.

It will be seen that the amount of bending increased, in a course of two and half days, to, at least, more than three times of its initial amount. The above result as well as the further yielding of the same specimen are plotted on Pl. IV. Even after some tenth of thousands of minutes, steady increase of bending could be seen. It is, indeed, questionable whether there is any limit to the yielding or not. One instance where a plate of marble, resting horizontally on four posts at the corners, in a course of about half a century, was considerably bent by its own weight, is reported by T. J. J. See.* Here it may be remarked that, in Fig. 17, there is an abnormal increase of bending. Close examination showed that it was the effect of temperature-rise, as we shall see in the next section.

Recovery from the yielding:—It is of no small interest to examine whether the yielding above stated is elastic or permanent. From an investigation by F. D. Adams and J. T. Nicolson,† it is evident that even such a comparatively rigid rock as marble may become wholly plastic under suitable condition. In that case, an enormous change of shape occurred in a comparatively short time and it seemed to be permanent. For instance, the diameter of a cylinder increased by 1.388 times of its initial, bulging out by endpressure in only 18 minutes. The structure of the marble deformed in 64 days was essentially the same in character as that which was deformed to the same extent in 10 minutes. The folding of rocks and other kindered phenomena pertaining to the manifold change of shape in rocks are found in great abundance. It is never out of question whether Such phenomena had occurred in a short time under wholly plastic condition and now is in permanent set, or they are the results of yielding, progressing from time to time, wrought by the continuous action of stress, and always ready to recover from their over-strained state.

* The secular bending of a marble slab under its own weight. Nature. Nov. 20. 1902.

† An experimental investigation into the flow of marble. Phil. Trans. of the R. S. A. Vol. 195. 1901.

If the latter were the case, it would not be wholly unconcievable that an overstrained portion of the earth's crust recovers gradually after its stress have been removed by some geological disturbance. Then, this phenomenon of recovery may be, to be sure, one of the causes of after-shocks of an earthquake, since ultimate result of this phenomenon must be equal that which may be produced by an oppositely directed stress.

A piece of sandstone No. 3_4 remained loaded, $M=3000$, $M_0=3300$ grams, since 4^h27^m P.M. 9th Feb. 1903 till 7^h50^m P.M. 23rd of the same month, during 14d 3h 23m i.e. 20363 minutes. Then it was unloaded, $M=0$, $M_0=3300$ grams, and the amount of residual bending was observed from instant to instant. As in the case of torsion, it recovered gradually and incesantly. The result of the experiment is given in the following table.

TABLE. V.

Specimen No. 3_4. Sandstone.
Loaded at 4^h27^m P.M. 9th Feb. $M=3000$ grs. $M_0=3300$ grs.
Unloaded at 7^h 50^m P.M. 23rd Feb. $M=0$ „ $M_0=3300$ grs.
Amount of residual bending $=x-\alpha$; α = amount of recovery.

Time.	α	Time.	α
23rd P.M. 7^h 51^m	20.87×10^{-4} rad.	24th P.M. 1^h 38^m	27.50×10^{-4} rad.
52	21.59	2 37	28.06
55	21.87	3 15	28.30
57	22.02	4 33	29.06
8 0	22.30	5 34	29.28
4	22.51	6 43	31.09
6	22.37	7 14	31.27
9	22.59	8 18	31.47
17	22.69	9 41	31.84
24	22.95	25th A.M. 10 0	34.44
57	23.32	P.M. 6 42	37.63
9 35	24.52	27th A.M. 8 12	45.27
24th A.M. 8 3	25.91	P.M. 5 28	47.38

Thus, the amount of recovery became increased, in a course of about four days, by more than twice its initial value. The result is also shown in Figs. 18 and 19, Pl. IV.

Effect of recovery on cycles is to be seen from Fig. 20 in Pl. V. Another piece of sandstone, No. 4_3, was loaded with $M=3000$, $M_0=3300$ grams during 2647 minutes. Then it was released gradually from the load and treated in a cyclical manner as usual. The result is given in the following table :—

TABLE. VI.

Specimen No. 4_3. Sandstone.
Loaded with $M=3000$, $M_0=3300$ grams-weight,
from 6^h 35^m P.M. 20th May, 1903.
to 2^h 42^m P.M. 22nd of the same month:
then it is unloaded and loaded cyclically.

M. in grams weight.	First cycle. α in radians.	Second cycle. α in radians.	Third cycle. α in radians.
3000	24.21×10^{-4}	19.12×10^{-4}	17.90×10^{-4}
2700	23.80	18.59	17.51
2400	23.24	18.14	17.08
2100	22.47	17.51	16.50
1800	21.59	16.54	15.72
1500	20.36	15.66	14.71
1200	18.97	14.50	13.50
900	17.25	13.25	12.22
600	15.52	11.59	10.65
300	13.50	9.96	9.02
000	11.31	7.83	7.07
—300	9.02	5.51	4.92
—600	6.68	3.41	2.59
—900	4.29	1.30	0.64
—1200	1.84	—0.62	—1.13
—1500	—0.70	—2.68	—3.42
—1800	—3.12	—5.03	—5.49
—2100	—5.38	—7.60	—7.78
—2400	—9.33	—9.62	—9.69
—2700	—11.17	—11.62	—11.86
—3000	—13.47	—13.86	—13.81
—2700	—13.04	—13.28	—13.15
—2400	—12.46	—12.82	—12.60
—2100	—11.83	—12.20	—11.97
—1800	—11.10	—11.35	—11.30
—1500	—	—10.70	—10.60
—1200	—8.95	—9.64	—9.31
—900	—7.55	—8.01	—7.99
—600	—6.02	—6.50	—6.57
—300	—3.95	—4.33	—4.61
000	—2.02	—2.57	—2.56
300	0.03	—0.28	—0.70
600	2.11	1.40	1.32
900	4.18	3.54	3.42
1200	6.19	5.45	5.38
1500	8.26	7.51	7.33
1800	10.57	9.72	9.45
2100	12.54	11.81	11.47
2400	14.75	13.99	13.46
2700	16.93	15.82	15.35

The result is also shown in Fig. 20. Pl. V.

As a natural consequence of yielding and recovery, a piece of rock under over-strain is not indifferent of the direction of the second stress to be applied, even after the piece has long been in the strained condition. For instance, a piece of sandstone, No. 3_4, was subjected to a force due to $M=-1500$, $M_0=3300$ grams during 1641 minutes and then a second force was applied in the same direction as the first one. In the next place, the same piece, was acted by the same force during 1234 minutes and then a second force was applied in the opposite direction as the first one. The changes of flexure for equal change of force were very different in these two cases, as shown in the following table:—

TABLE. VII.

$M=M_1+M_2$. Subjected under $M_1=-1500$ grams during 1641 minutes.		Subjected under $M_1=-1500$ grams during 1234 minutes.		Started from neutral state: i.e. $M_1=0$.	
M_2	a	M_2	a	M_2	a
—300	-2.00×10^{-4}	300	0.59×10^{-4}	300	2.58×10^{-4}
—600	—4.29	600	2.25	600	6.43
—900	—7.48	900	4.39	900	11.46
—1200	—11.17	1200	6.25	1200	16.95
—1500	—15.06	1500	11.20	1500	22.36
etc.	etc.	etc.	etc.	etc.	etc.

The result is also shown in Fig. 21, Pl. V.

7. Effect of Temperature.

The curve of yielding given in Fig. 17. Pl. IV, as it was noted in the last section, is very irregular, increasing with abnormal rapidity in some moments. Comparing this with the curve of temperature-change during the same time, we may percieve at a glance that the disturbance of yielding corresponds to the variation of temperature, In Pl. VI, Figs. 22–25, the temperature and the amount of bending are plotted against the laps of time respectively, In Fig. 22, there is an abrupt increase of bending between 46th and 49th hours. In the corresponding part, we notice the rise of temperature. There is, of course, more or less time-lag on the part of yielding, since a piece of rock requires more time to get on the same temperature as the surrounding air than a mercury thermometer.

Here it must be remarked that the rise of temperature is never an ultimate cause of the increase of bending. It is neither more nor less than an inducer. The amount of increase of bending induced by a rise of temperature depends wholly on the capability to yield at that instant. When the capability is large, an enormous increase of bending takes place induced by a little rise of temperature. Looking on the figures, for instance, we see that the rise of temperature was more rapid during 64th—70th hours, while the amount of yielding during the same time was comparatively small.

In so far as the temperature-rise is a mere inducer, the resulting flexure must be irreversible. The flexure, however, decreased a little during 71th hour, when the temperature descended with great rapidity. The above shows that one part of the flexure is reversible with regard to the change of temperature. Whence we must conclude that the effect of temperature is double: firstly, it acts as an inducer,—i.e. it facilitates the flexure-change caused by other agent; seconlly, it is one of the ultimate causes of the flexure-change. For the first part, there can be no numerical relation between the variations of both flexure and temperature. For the second part, however, a functional relation must exist connecting the two variations. These two effects always appears hand in hand. When the capability to yield is large, the second part must be obliterated by the first one. After a long time, as the capability to yield tend to vanish, the second part becomes the principal one. This was really the case with the present experiments, as we shall see in the annexed figures in Pl. VI.

In Figs. 23–25, the corresponding curves are similar to each other, with an exception to be explained in the following lines. Left parts of the two curves in Fig. 23 and right parts of those in Fig. 25 are not parallel; on the contrary, they converge towards the right. Examining them more minutely, we percieve that this abnormity takes place, when and only when the temperature is lower than a definite degree Ca. 9°.C. When the temperature is higher than this, the two curves proceed parallel to each other; in other case they are symmetrical to each other. That is to say, the amount of bending increases with the rise of temperature in the first case, but it decreases in the second case, and vice versa.

The above fact is also contained implicitely in the curve of bending in Fig. 22. From 40^{th} to 48^{th} hour, the temperature rose with great rapidity, but the abrupt increase of bending took place only after 46^{th} hour, which indirectly shows that, during 40^{th}–46^{th} hour, as the temperature was below the neutral point, the rise of temperature, on the contrary, diminished the yielding.

The relation between temperature and bending is given more clearly in Fig. 26 of the same plate. The curve has a neutral point in the neighbourhood of Ca. 9°C. That is to say, the elasticity of sandstone is maximum at that temperature in so far the change due to temperature-variation is concerned. In the case of the rigidity-modulus, we had a result strictly analogous to this effect.

To determine any general relation between elasticity and temperature requires further investigation by a special arrangement. Here it is sufficient to remark only that though the change of flexure due to the variation of temperature is unexpectedly great, yet it is almost negligible as compared with the total amount of flexure. Indeed, the increase of bending per degree of temperature-rise in the case of sandstone is of an order 10^{-3} of its total amount.

8. Hysteresis Curve.

Looking on the curve in Fig. 12, Pl. II, we see that there is a tendency on the part of the rock to persist in any strained state which it may have acquired, especially when the variation of the stress changes its sign. The curve is closed and it is also of simple and regular form, though its path during the increase of stress differs entirely from that during the decrease. A more close examination will prove that the on-curve, when it is turned through two right angles, becomes coincident with the off-curve. All rocks, in so far as the author has investigated on, have this property in common, though they differs in the curvature of the curves and other minute details. Figs. 27–50, Pls. VII–IX, show the hysteresis curves of several rocks of different kinds.

From what was stated above, it is evident that all these specimens had symmetry on both sides, with the exception of one or two schistose rocks. When a specimen was in a strained state, or when it had a crack or the like, the curve, however, lost its symmetry so that the curve did not close, or else the on-curve could not be brought in coincidence with off-curve.

There is one important fact which deserves to be here mentioned. Although the hysteresis curve is of a definite form and traces one and the same curve when a specimen is bent and unbent many times cyclically, the direction of the elongation of curve does not remain fixed when the amplitude of the cycle (i.e. the greatest amount of stress applied to the specimen during the cycle) is varied. As a general rule, the hysteresis curve becomes more and more vertical when the amplitude of the cycle is further and further increased. Figs. 51–53 fully explains this fact. When the centre of the cycle corresponds to the strained state, Fig. 53, this variation of the direction seemed to be rather rapid.

The amount of hysteresis, which is to be measured by the area enclosed by the curve or by some function of it, is not necessarily greater for the rock having smaller modulus of elasticity. For example, red schalsteine No. 14_1 has smaller modulus of elasticity (2.39–3.09×10^{11}) than marble No. 6_3 (3.24–3.51×10^{11}), but the former has smaller hysteresis than the latter, as it will be seen from Fig. 31, Pl. VII. and Fig. 41, Pl. VIII. Generally speaking, however, the amount of hysteresis is least for archaean rocks and increases rapidly for new rocks.

9. Variation of the Modulus of Elasticity During the Cycle.

As it was noted in the former sections, the modulus of elasticity is never constant for a given piece of rock. In the case of sandstone, given in Fig. 15, Pl. III, it is evident that the variation of the modulus obeys one and the same law for both on- and off-curve, in so far as the centre of cycle is at the neutral state of the specimen. That is to say, the curves in Fig. 15 are symmetrical with respect to the axis of ordinate.

Further examples showing how the modulus of elasticity varies during one complete cycle are given in Figs. 54–65, Pls. XI and XII., for several different kinds of rocks. Each kind of rock seems to have its special character. If Hooke's law were to hold good, four branches of each curve would all shrink to a single horizontal stright line. In the case where no hysteresis exists, both the upper and the lower branches would coinside with each other to make a line not necessarily straight.

For all cases of rocks here experimented upon, the upper branch is concave towards the positive axis of the ordinate. As to its character, however, the variety is very abundant; circular, hyperbolic, parabolic, oval, and other curves of higher order of complexity. The left lower branch, indeed, is a continuation of the right upper one, and vice versa. But, in saying circular or parabolic, etc., we assume left upper branch as a continuation of the right upper one, and vice versa. The curvature of the lower branches is turned sometimes upwards and other times downwards. That is to say, one piece of rock, e.g. Fig. 61, chloriteschist No. 26_1, becomes more and more stiff as it is bent further and further, while other piece, e.g. Fig. 62, marble No. 6_3, becomes more and more flexible.

From the above fact, it seems that the procedure of bending had opposite effect on the elasticity of the specimens in the two cases, chloriteschist and marble. This fact, however, may be easily understood from the following explanation. When a piece of rock is bent by an increasing force further and further, there must be a limit as to the amount of strain beyond which it can not go on, or else it breaks down: i,e. The piece firstly becomes more flexible and then, after having reached to a minimum elasticity, it begins to become more and more stiff until it breaks down at last. Hence whether the curve is convex or concave up or down depends wholly upon the ratio of the actual stress to the breaking one. This fact is typically shown by the case of graphiteschist, Fig. 59, Pl. XI.

Though it is not easy to determine any law according to which the modulus of elasticity varies with the phase of the cycle, we may find, as a first approximation, an empirical expression for each specimen. For instance, in the case of sandstone whose experimental result is figuratively given in Fig. 15, Pl. III, we have two parabolic equations expressing the upper and the lower branches respectively. If y and x represent $E \times 10^{-11}$ and the phase respectively, we have

for the upper branch, $y_1 = 0.243 + 0.92\,x^2$ with a probable error $= \pm 0.013$,

and for the lower branch, $y_2 = 0.243 + 0.043\,x^2$ with a probable error $= \pm 0.004$,

As a matter of fact, the constant term of y_1 is equal to that of y_2; it represents the modulus of elasticity at the state where no external force is acting. These two parabolic expressions are traced in Fig. 66, in Pl. XIII. Assuming this relation between the modulus of elasticity and the phase, we may easily calculate the amount of flexure for the specimen corresponding to any bending force not greater than that due to 1000 grams of weight. The result of calculation is given in Fig. 67, Pl. XIII, which coincide with the observed value within a probable error $\pm 7.5 \times 10^{-6}$ while the amount of bending due to $M = 1000$ grams is 1334×10^{-6}.

In the case of a piece of chloriteschist, No. 26_1, Fig. 61, Pl. XII the upper branch can be represented by an hyperbola

$$\frac{y^2}{(5.75)^2}-\frac{x^2}{(0.35)^2}=1,$$

or $$y=\sqrt{\{33.13+122.52x^2\}} \quad , \quad \text{probable error}=\pm 0.13.$$

For one kind of granite, No. 9_1, Fig. 54, Pl. XI, on the other hand, the relation is given by an ellipse

$$\frac{(y-10.09)^2}{(6.21)^2}+\frac{x^2}{(0.93)^2}=1 \quad ,$$

or $$y=10.09-\sqrt{\{38.56-44.4x^2\}} \quad , \quad \text{probable error}=\pm 0.12.$$

This expression, of course, is applicable only within a certain limit i.e. phase <0.93.

In so far as the author experimented upon, the modulus of elasticity of all rocks can be expressed as a simple and definite function of the phase in a cyclical process, provided the specimen is not too near its breaking state.

In the following table, the constant term of the expression for every specimon is given as the modulus of elasticity of several rocks. It corresponds therefore to the value of the modulus of elasticity at the instant when the bending force became zero during the specimen, whose section is about one centimetre square and the distance between the fulcrums is 10 cm., was bent cyclically on both sides by a force varying between those due to $M=3000$ grams weight. The value at any other state under different conditions should be, with all probability, greater than those given in this table.

TABLE. VIII.

No.	Rock.	Locality.	Kind.	Density.	Mod. of elasticity.	Mean E.	Velocity of Long. Wave.
			ARCHÆAN ROCKS.		$\times 10^{11}$(c.g.s.)		$\frac{\text{Kilom.}}{\text{Second.}}$
31_1.	Quartzschist.	Chichibu.	Metamorphic.	2.67	10.48—7.07	8.78	5.73
46_1.	Quartzschist.	Gumma.	„	2.62	8.41—8.40	8.41	5.67
8_1.	Serpentine.	Chichibu.	Eruptive (altered).	2.72	7.73—7.21	7.47	5.24
40_1.	Micaschist.	Ibaraki.	Metamorphic.	2.54	6.49—5.92	6.21	4.94
18_1.	Chloriteschist.	Chichibu.	„	2.88	8.63—5.39	7.01	4.93
7_2.	Peridotite.	Kuji.	Eruptive (altered).	2.61	6.73—5.83	6.28	4.91
26_1.	Chloriteschist.	Chichibu.	Metamorphic.	2.82	7.03—6.29	6.66	4.86
22_1.	Gabbro.	„	Eruptive.	2.71	6.21—5.57	5.89	4.66
24_1.	Graphiteschist.	„	Metamorphic.	2.59	5.12—4.93	5.03	4.41
23_1.	Graphiteschist.	„	„	2.56	3.69—3.37	3.53	3.71
42_2.	Micaschist.	Ibaraki.	„	2.63	1.29—1.16	1.23	2.16
			PALÆOZOIC ROCKS.				
34_1.	Adinoleslate.	Gumma.	Sedimentary.	2.64	10.99—10.23	10.61	6.34
12_2.	Clayslate.	Aumi.	„	2.71	10.71—9.08	9.90	6.04
9_1.	Granite.	Mikage.	Eruptive.	2.54	4.31—3.66	3.99	3.96
21_1.	Limestone.	Chichibu.	Sedimentary (Metamorphosed).	2.64	4.14—3.65	3.90	3.84
6_3.	Marble.	Kuji.	„	2.68	3.51—3.24	3.38	3.55
14_1.	Red Schalstein.	Aumi.	Sedimentary.	2.43	3.09—2.39	2.74	3.36
32_1.	Pyroxenite.	Gumma.	„	2.90	2.96—2.91	2.94	3.18
10_2.	Granite.	Kagawa.	Eruptive.	2.57	2.30—2.10	2.20	2.93
29_1.	Limestone.	Gumma.	Metamorphic.	2.66	2.06—1.92	1.99	2.74
			TERTIARY ROCKS.				
35_1.	Sandstone.	Chichibu.	Sedimentary.	2.47	3.55—3.51	3.53	3.78
50.	Two Pyroxene Andesite.	Awomori.	?	2.70	4.04—2.38	3.21	3.44
2_3.	Tuff.	Izu.	Sedimentary.	1.90	1.39—1.36	1.38	2.69
5_3.	Rhyolite.	Yechizen.	Eruptive.	2.40	0.90—0.77	0.84	1.87
4_3.	Sandstone.	Kii.	Sedimentary.	2.25	0.68—0.57	0.63	1.67
3_4.	Sandstone.	Chōshi.	„	2.21	0.34—0.20	0.27	1.11
			DILUVIUM ROCKS.				
17_2.	Andesite.	Gumma.	Eruptive.	2.63	4.36—4.31	4.34	4.06
16.	Andesite.	Gumma.	„	2.32	0.68—0.63	0.66	1.69

10. Mean Value of the Modulus of Elasticity During One Cycle.

In the last section we learned that the modulus of elasticity varies, during one cycle according to a definite law. When a piece of rock is bent by a force and unbent by virtue of its elastic property, it is not, evidently, the modulus of elasticity at any perticular state, which determines the vibratory motion of the rock. Modulus of elasticity at all different phases of the vibratory motion equally play their parts in causing the motion. Hence, to know the apparent modulus of elasticity during one complete vibration, we must take mean value of the modulus of elasticity at all different phases.

Let $\mathfrak{E}$ be the mean value, then we have

$$\mathfrak{E}=\int_0^1 y\,dx$$

where y is the modulus of elasticity expressed as a function of the phase x.

If the relation between x and y is parabolic, we have

$$\mathfrak{E}=\int(a+bx^2)\,dx=a+\tfrac{1}{3}b.$$

For the case where it is hyperbolic, we have

$$\mathfrak{E}=\int\frac{b}{a}\sqrt{a^2+x^2}\,dx=\frac{b}{2a}\left\{x\sqrt{a^2+x^2}+a^2\log\left(x+\sqrt{a^2+x^2}\right)\right\}.$$

We may assume, with all probability, that the function is a power series

$$y=a_0+a_1x+a_2x^2+\ldots\ldots,$$

Then
$$\mathfrak{E}=\int\sum a_nx^n dx=\sum\frac{a_n}{n+1}.$$

Eor a piece of sandstone, when the maximum bending force during the cycle was equal to that due to $M=3000$ grams, we have

for the upper branch, $y_1=0.152\ +2.35x^2$

for the lower branch, $y_2=0.152\ +0.082x^3$.

Hence, in this case, we have

$$\mathfrak{E}\times 10^{-11}=\frac{1}{2}\int_0^1 y_1\,dx+\frac{1}{2}\int_0^1 y_2dx=0.152+\frac{1}{6}\left\{2.35+0.082\right\}$$

i.e. $\mathfrak{E}=0.152\times 3.67\times 10^{11}$.

Thus, the mean vabue $\mathfrak{E}$ taken for one complete cycle is 3.67 times greater than the value E taken at the state of no bending force.

11. Diminution of the Modulus of Elasticity Due to Increase of the Amplitude of Cycle.

As it was lately noted, the hysteresis curve becomes more and more vertical when the amplitude of cycle is further and further increased. This fact expresses that the mean elasticity gradually weakens as the amplitude of cycle increases. The variation of the modulus of elasticity in each cycle given in Fig. 52, Pl. X. is shown in Fig. 68, Pl. XII. At a glance, we see that the greater the amplitude, the lower the curve. In Fig. 69, Pl. XIII, the same result is shown in a somewhat different manner. The modulus is expressal as a function of the phase in the cycle. It will be seen that the modulus of elasticity corresponding to one and the same phase of the cycle is generally greater when the amplitude is smaller.

In this case, also, the curves can all be represented by a series of parabolic expressions. The constant terms of them are, of course, not equal to each other: i.e. they are

TABLE IX.

Amp. (i.e. $\mathfrak{M}$ in grams.)	300	600	1200	1800	2400	3000
$E.$ ($c.g.s \times 10^{11}$)	0.65	0.46	0.33	0.27	0.21	0.15

This relation between the amplitude of cycle and the modulus of elasticity is also shown in Fig. 70, Pl. XII. Thus, it is very important to notice how the modulus of elasticity diminishes when the amplitude of the cycle increases.

12. Increase of the Modulus of Elasticity due to the Strained State of the Specimen.

In the case of the modulus of rigidity, it was noted that the modulus is comparatively greater in a strained than in the neutral state. This is also the case for the modulus of elasticity. Here it will suffice to give one example. Firstly, a piece of sandstone No. 4_3 was applied with a force increasing by $\Delta M = 300$ grams step by step. Secondly, the same specimen, after having been under a constant force $M = -3000$ grams during 4027 minutes, was applied, also, with a force increasing by $\Delta M = 300$ grams step by step. The modulus in each step is given in the annexed table, and also it is graphically shown in Fig. 71, Pl. XIII.

TABLE X.

Second force: *M.* in grams.		300	600	900	1200	1500	1800	2100	2400	2700	3000
E.	From neutral state: $F_0 = 0$.	1.104	1.168	0.810	0.727	0.664	0.578	0.601	0.565	0.517	0.540
$\left\{ \begin{matrix} c.g.s. \\ \times 10^{11} \end{matrix} \right\}$	From strained state: $F_0 = 3000$	2.962	2.169	1.577	1.380	0.987	0.874	0.706	0.702	0.601	0.555

Tis example will fully erpress how the modulus of elasticity in a strained state is greater than that in its neutral state.

13. The Velocities of Propagation of Seismic Waves.

In the author's publications above cited it was experimentally as well as theoretically explained that, in the case of distorsional waves, the velocity of propagation is a function of the amplitude of the wave as there exist more or less yielding in the rocks through which the waves propagate, and also that, in view of this inference, we do not see the necessity to assume the path of the tremors to be different from that of the principal shocks. The present experiment relating to other modulus of elasticity, i.e. Young's modulus, gives, it seems to me, more strong foundation to support the above view.

In so far as the above-mentioned experiments can be confidentially relied upon, the following inference may safely be drawn from them, provided we agree that the velocity of wave varies as the square root of the elastic constant of the medium through which the wave propagates.

In § 10, it was noted that the elastic constant varies during one cycle of bending and that all values at different phases of the cycle equally play their parts in causing the motion, i.e. the apparent value of the elastic constant during one complete vibration must be the mean value of all the values at different phases. Now the elastic constant which determines the velocity of wave is, as a matter of course, not its value at any particular phase, but its mean value taken for one complete cycle.

Assuming $y = a_0 + a_1 x + a_2 x^2 + \ldots\ldots$

we have $\mathfrak{E} = \int_0^1 \sum a_n x^n \, dx = \sum \frac{a_n}{n+1}$

In usual case, what is given as the value of the elastic constant is

$$E = a_0 + a_1 f(x_1)$$

where x_1 is a certain small quantity particular to the mode of experiment.

In Table VIII, the value of a_0 is given as the value of the modulus of elasticity; accordingly the value of velocity there given is calculated by the formula

$$v = \sqrt{\frac{a_0}{\rho}}.$$

It will be noted that these values, as a matter of course, are generally smaller than those given by Professor H. Nagaoka in his valuable report:* $V = \sqrt{\frac{E}{\rho}}$

In the actual case, what determines the velocity, is neither v nor V, but, with all probability, it should be given by

$$\mathfrak{V} = \sqrt{\frac{\mathfrak{E}}{\rho}}$$

Since the mean elasticity $\mathfrak{E}$ is remarkably greater than the values a_0 or E, the actual velocity of propagation $\mathfrak{V}$ must be somewhat greater than those v or V given in the tables. For instance, in the case of

* H. Nagaoka. Elastic Constant of Rocks and the Velocity of the Seismic Waves. This Publication No. 4. 1900.

a piece of sandstone, the result of experiment showed that the mean value $\mathfrak{E}$ is 3.67 times greater than the constant term a_0. Whence, we must have

$$\frac{\mathfrak{V}}{v}=\sqrt{3.67}=1.92,$$

that is to say, the actual velocity would be probably twice the value given in the table. In general, the actual velocity would be $\sqrt{\left\{\sum \frac{a_n}{n+1}/a_0\right\}}$ times greater than that given in the table.

14. Diminution of the Velocity due to Increase of Amplitude.

The velocity must necessarily diminish with increase of the amplitude of the wave, since the elastic constant diminishes in that case as fully explained in the eleventh section. From the instance given in that page we have a good example to show how the velocity changes with the amplitude.

TABLE XI.

Ratio of Amplitudes.	1	2	4	6	8	10
Ratio of Velcoities.	2.08	1.75	1.48	1.34	1.18	1.00

This relation between the amplitude and the velocity is more clearly shown in Fig. 72, Pl. V. The velocity increases twice or more while the amplitude diminishes from $\mathfrak{M}=3000$ to $\mathfrak{M}=300$ grams.

Examining the course of the curve, we may safely conclude that any further diminution of the amplitude will give very rapid increase of the velocity.

15. Increase of the Velocity due to the Strained State of Stratum.

As the elastic constant is comparatively greater in the strained than in the neutral state, the velocity must, as a matter of course, be correspondingly greater in the former state than in the latter. In the example shown in Table X, the ratio of the values of elastic constant for the first step of flexure is 2.962 : 1.104 so that the ratio of the velocities will be 1.72 : 1.05 or 1.64. In other words, the velocity in that strained state must be 1.64 times greater than that in the neutral state. This is, of course, nothing but an example; general relation between them, however, would not much deviate in its quality.

16. Diminution of the Velocity due to Increase of Temperature.

Though the variation of the elastic constant due to temperature-rise is comparatively small, it can never be neglected in so far as the velocity of seismic waves is concerned, since the underground temperature rapidly rises with the increase of depth. According to Professor A. Tanakadate,* the underground temperature increases at a rate greater than 25°C per kilometre of depth.

The change of the elastic constant, on the other hand, is Ca. 0.5 % per degree of temperature-change for the case of sandstone. That is to say, the elastic constant diminishes by 12.5 % per kilometre of depth, provided all other conditions remain the same. Then the velocity would proportionally diminish as the depth gradually increases, so that the influence of the underground temperature on the velocity of propagation would be too great to be neglected.

Here a remark may be added on the existence of a stratum of maximum velocity of propagation, proposed by Professor H. Nagaoka. That the modulus of elasticity increases in a greater rate from cainozoic to archæan rocks than the density does, is surely an established fact. When the effect of temperature is taken into acount, however, the depth of a stratum where such old rocks aboundant can

* A. Tanakadate. The Pub. of the I.E.C. in Japanese Language. No. 45. 1903.

never be put out of consideration, since the increase of depth tends to increase the density but to diminish the elastic constants for one and the same kind of rock, if the elastic constant of rocks with earth's crust behave in the same manner as sandstone as regard temperature. According to geologists,* the thickness of each of quatery, tertiary and mesozoic rocks is some three or four kilometres, while that of palaeozoic attains twenty five or more kilometres in some regions. The summation of the thickness of the strata of different ages, of course, includes only the part which was revealed to the earth's surface by some geological disturbances. Whence it follows that to attain the main stratum of palaeozoic rocks we must go deep in, at least, some ten kilometres, and for a stratum of archaean rocks, at least, thirty kilometres. The underground temperature, however, at such a great depth must be tolerably high, and the consequence of it would probably in a great diminution of the elastic constants, so that the depth of the stratum of maximum velocity, if it exist at any rate, would be greatly shortened. The author would not, however, insist in this opinion till more accurate observations on the change of the elastic constants due to temperature-rise, which are in course of preparation, will fully elucidate the relation between the elastic constants and the temperature.

17. Note on the Existence of Paths of Maximum Velocity and Seismic Shadow.

Notwithstanding the doubt about existence of a *stratum* of maximum velocity, there may exist, with all probability, several *paths* of maximum velocity of propagation within the earth's crust. As geology teaches us, within the earth's crust there are scattered everywhere veins and dikes of different kinds of old rocks, uplifted by geological disturbances, some of which run over many hundreds or thousands of kilometres.

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* Encyclopædia Britannica: Geology.

In this case, it is true, the velocity along such a vein or the like is greater than that through any of the surrounding strata, so that the seismic waves mainly propagate through that region.

As a consequence of the above result, if an observing station be situated on such a vein, not only the number of earthquakes abserved at the station is greater than that observed at any of the stations in the neighbourhood, but the direction of motion would not necessarily indicate the position of the seismic centre. It is of daily experience that observers in some districts always feel any earthquakeshock to come from one particular direction even when the seismic centre is in entirely different direction, provided the centre is not too near the observers.

As other consequence, there may exist *seismic shadow;* or, in other words, *seismic waves may be shielded by a vein or dike of old rocks.* As the elastic constants is neither negligibly small nor infinitely great for any rock, the shielding, of course, is not perfect. Strong earthquakes, however, may be reduced to weak ones and weak shocks to insensible tremors. After all, earthquakes originating in one region may always be well observed in the station while those originating in the other region may fail to be observed in the station.

In Professor F. Ōmori's paper* we find the most interesting example to support the above consideration. Of the earthquakes which happened between Sept. 1887 and July 1889 in Central Japan, those whose origins were situated within certain boundaries were always not felt in Tōkyō, though the weaker earthquakes of more distant origins were easily felt.

18. A Hint to the Frequency of After-shocks.

Even at the present day, after all that has been written on earthquakes, but little is really known as to the frequency of after-shocks. An empirical formula,

* F. Ōmori. This Publication. No. 11. 1902.

$$F=\frac{k}{h+t}$$

in which k and h are constants, was proposed by Professor F. Ōmori * about ten years ago. How his hyperbolic formula gives satisfactory results is sufficiently shown by him in the valuable paper "On the after-shocks of Earthquakes," in which the formula is applied to the three recent great earthquakes in Japan; namely, those of Kumamoto in 1889, of Mino-Owari in 1891, and of Kagoshima in 1893. Lately, other formula, in a form of logarithmic function was obtained by O. Enya,† founded on three assumptions. A result of labourious calculation is given by him to show that the logarithmic formula is better than the hyperbolic.

To get any reliable formula for the frequency of after-shocks, the first step for it is to know what is the cause of after-shocks. Probably several distinct causes should be recognized, for it is hardly to be supposed that all subterranean disturbances, differing as they do so widely in intensity and in duration, should be referable to one common mechanism. Each succeeding geological disturbance, the fracturing, dislocation, caving-in of ill-supported regions, and also establishment of lines of freedom for the exhibition of volcanic activity which would accompany these changes, would promote the aftershocks.

According to the intrinsic meaning of the name "After-shock," however, the nearest cause must be due to residual disturbance in the geotechtonic condition after the primitive-shock has recovered. An earthquake not recieving any participation of this residual is no after-shock, but an independent earthquake. Standing on this principle, with an assumption that the frequency of after-shocks in a given instant is proportional to the rate with which the earthcrust recovers from the residual disturbance, one more general formula for the frequency may be easily obtained. The above assumption is, to say in other words, that if the earthcrust were perfectly elastic, the fre-

* F. Ōmori. The Jour. of the coll. of Science Vol. VII. Tōkyō.

† O. Enya. The Pub. of the E.I.C. in Japaneses Language. No. 35. 1901.

quency of earthquakes would be proportional to the rate with which the disturbance of the earthcrust would be produced.

From what has been explained in the sixth section we know, as a matter of fact, that a piece of rock yields under a constant action of force, and also that the residual strain surviving the force diminishes from instant to instant, i.e. the rock recovers from its overstrain. This last phenomenon must be the essential cause of the after-shocks. Thus, the first step is to find any formula expressing the rate of recovery, or the rate with which the residual varies with time. One form of such a formula, however, was lately deduced from the logarithmic law of yielding, as it was given in the author's paper above cited. The formula is

$$\rho = k \log \frac{\Gamma(2p+1)\ [\Gamma(p+t+1)]^2}{[\Gamma(p+1)]^2\ \Gamma(2p+t+1)\ \Gamma(t+1)} \quad \ldots\ldots\ldots\ (\mathrm{I})$$

where ρ is the total amount of recovery at the instant t, both ρ and t being reckoned from the instant when the external force is wholly withdrawn, while k and p are constants, of which the former specifys the rock and the latter specifies the time-lapse required by the force to attain its maximum.

Let F be the frequency, then the above assumption is

$$F \propto \frac{\Delta\rho}{\Delta t}$$

i.e. if c is a constant, we have

$$F = c\frac{\Delta\rho}{\Delta t}$$

$$= ck \log \frac{[p+t+1]^2}{(2p+t+1)(t+1)}$$

$$= ck \log \left\{1 + \frac{p^2}{(2p+1)+2(p+1)t+t^2}\right\}. \quad \ldots\ldots\ (\mathrm{II})$$

Thus we have a logarithmic form for the frequency of after-shocks. One curve of frequency for some particular values of c, k and p is drawn in Fig. 73, Pl. XIV. A little consideration on the nature of the constant p will make it reasonable to neglect the term $\frac{t^2}{p^2}$ so long as t is not very large. Then we have, to the first approximation,

$$F=k' \log \left\{1+\frac{1}{A+Bt}\right\} \dots\dots\dots\dots\dots \text{(III)}$$

which is the same with that of Enya. Again, expanding the logarithmic function and taking its first term only, we have Professor Ōmori's formula

$$F=\frac{k''}{k+t}. \dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(IV)}$$

Though the resulting formula for the frequency are tolerably well found in so far as they were tested by Professor Ōmori and Mr. Enya, the original formula (I) for the recovery is not wholly out of question. The assumptions under which the formula (I) is deduced are very far from being the case in an earthquake. The force acting on rock is assumed to increase intermittently, and, to be worse, it is assumed to be withdrawn not suddenly but slowly and intermittently. The following may be more close to the actual case.

Whatever may be the real cause of an earthquake, it is reasonable to consider the force as increasing constantly with time, i.e.

$$df=k\,dt$$

where k is a constant, and attaining a sufficient amount F to cause an earthquake at time T, so that we have

$$F=k\,T.$$

Suppose the logarithmic law of yielding, which was empirically established in the last series of experiments, to be granted, so that

$$d\eta=K\,df \log(t+\tau),$$

where η is the amount of yielding and K a constant specifying the kind of rock while τ is a constant refering to the choice of origin of time t. Then we have

$$\eta=K\,k\int_0^T \log(t+\tau)dt$$

$$=k\,K\,(T+\tau)\log\frac{T+\tau}{e}+k\,K.$$

If the total force F is suddenly withdrawn at the instant $t=T$, it may be easily proved that the residual strain at any instant $t=T+t'$ is given by

$$\sigma=kK\{T+t'+\tau\}\log\left\{\frac{T+t'+\tau}{t'+\tau}\right\}.$$

Now, as the frequency is assumed to be proportional to the rate of recovery, we have

$$F=-c\frac{d\sigma}{dt}$$

$$=\frac{c.k.K.T}{t+\tau}-c.k.K\log\left\{1+\frac{T}{t+\tau}\right\},$$

where τ,c,k,K and T are all constants, and t is written for t' whose origin may be any instant, provided proper value is given to the constant τ.

Here the frequency F may be considered to be composed of two terms F_1 which is hyperbolic and F_2 which is logarithmic, so that h being a constant

$$F_1=\frac{h}{t+\tau},$$

$$F_2=\frac{h}{T}\log\left\{1+\frac{T}{t+\tau}\right\}.$$

As the constant T is, with all probability, very great compared to other constants c,k and K, the main term is the first one, so that the curve of frequency F is a little different from a hyperbola.

When h is given, the curve of F_1 takes a definite form, but the curve of F_2 is wholly indefinite in so far as T, i.e. the time required by the force to attain its greatest amount, is not given. That is to say, if the time during which the causal agent of the earthquake existed is long, the curve of frequency approaches the hyperbola represented by F_1, but it deviates more and more from the latter curve as the duration T becomes shorter and shorter. Since F_2 generally increases as T decreases, the number of after-shocks of an earthquake of rapid generation is necessarily smaller than that of other earthquake of gentle generation, in so far as the other conditions required to cause the earthquakes remain constant. Numerical example given below will show it more clearly.

TABLE. XII.

$h=1000$; $\tau=1$.		F_2		F	
t	F_1	$T=100$	$T=1000$	$T=100$	$T=1000$
1	500	39	6	461	494
2	333	35	6	298	327
3	250	33	6	217	244
4	200	30	5	170	195
5	167	29	5	138	162
6	143	27	5	116	138
7	125	26	5	99	120
8	111	25	5	86	106
9	100	24	5	76	95
10	91	23	5	68	86
11	83	22	4	61	79
12	77	22	4	55	73
13	71	21	4	50	67
14	67	20	4	47	63
15	63	20	4	43	59
16	59	19	4	40	55
17	56	19	4	37	52
18	53	18	4	35	49
19	50	18	4	32	46
20	48	18	4	30	44
21	45	17	4	28	41
22	43	17	4	26	39
23	42	16	4	26	38
24	40	16	4	24	36
29	33	15	4	18	29
32	30	14	3	16	27
39	25	12	3	13	22

The curves of F_1 and F are drawn in Fig. 74, Pl. XIV. In this example, suppose the unit of time to be one month, then the number of after-shocks during the first, second, third, etc. one month would be either 461, 298, 217, etc. or 494, 327, 244, etc. respectively, according as the time required by the force to attain its greatest amount was a hundred or a thousand months.

19. Curves of Iso-frequency of After-shocks.

The discussion in the last section refers to the frequency of after-shocks in the very centre of disturbance, i.e. the seismic focus of the primitive earthquake. It must be borne in mind, however, that such a centre, so far from being anything like a mathematical point, is in nature a subterranean region, which in many cases, is undoubtedly of very large dimensions. Whatever may be the real origin of the after-shocks, it is reasonable to regard them as proceeding from the seismic focus. If the earth were a homogeneous solid, perfectly isotropic, the curves of iso-frequency would take the form of a series of closed curves around the seismic focus. As a matter of fact, however, the earthcrust is made up of rocks varying greatly in physical properties, each having its own density and elasticity. To make the variation more discontinuous, rocks of all geological ages are mingled together, as it were, by a series of geological disturbances, and they are scattered about through the earth crust.

As to the relation between the frequency y at any place, distant R from the seismic focus, and the mean radius r of the curve of iso-frequency, which is assumed to be a circle, corresponding to that place, Professor F. Ōmori proposed an empirical formula

$$y = a\, b^{-r}$$

where both a and b are constants. Here a hint is given to show how the existence of hysteresis plays a great part in the frequency of after-shocks, i.e. $r \sim R$ increases very rapidly as there exist different amounts of hysteresis in the stratum of rocks forming the path of the seismic wave.

From the figures in Pls. VII and VIII, it will be easily seen that, as a general rule, the amount of hysteresis decreases with an increases of the modulus of elasticity. Though nothing can be said about any numerical relation between hysteresis and elasticity, but in the rocks so far examined, certain relation between these two physical constants seems to exist. Thus it would not be a wild conjecture to say that the amount of hysteresis gradually diminishes from cainozoic, e.g. sandstone and rhyolite, to archaean rocks, e.g. peridotite and quartzschist, in a definite, though not yet known, ratio.

It is a matter of course that a seismic wave propagating through a medium of greater hysteresis fades more rapidly than that propagating through other medium of less hysteresis. The consequence is that, provided the frequency of after-shocks at the seismic focus is given, the frequency at any place having a given distance from the seismic focus increases with the geological age of rocks forming the path of wave between the focus and the place. That is to say, the distance between two successive lines of iso-frequency is greatest where archaean rocks lie and becomes less and less through palaeozoic and mesozoic rocks to reach its least value at the region where cainozoic rocks extend.

Now examining the curves of iso-frequency in the great Japanese earthquakes carefully drawn by Professor F. Ōmori in his paper above cited as well as many others, it is clearly seen that the irregularitis of the curves conforms with that of the geological distribution of rocks in those regions. For example, in the case of Mino-Owari earthquake, in which the total number of after-shocks during one year, 1892, amounted to more than eight hundreds, four curves of iso-frequency are shown, with corresponding geological distribution of rocks, in Fig. 75, Pl. XIV. The iso-frequency curve of $F=500$ lies wholly within quatery rocks and is in an elongated form extending neary north and south between Gifu and Nagoya. The central region of the after-shocks may be in a similar form. The succeeding curves of iso-frequency, however, so far from being similar to the first, are in quadrantal forms. In the western part, indeed, where the curves lie within quaterly rocks, they are all parallel to each other; but in other three directions they shrink in or swell out with all possible irregularities. These irregularities, however, turn to a regularity when the geological distribution of rocks in the corresponding regions is taken into account. To express in a simple terseness, *it swells out where palaeozoic or better archaean* *rocks predominate and shrinks in where cainozoic rocks extend.* This simple law is sufficiently satisfied up to very minute portions, as the figure proves it most clearly.

An inference to be drawn from the above is that the conductivity of seismic wave, if we are allowed to employ such a term from some analogies in heat and electricity, is least for cainozoic rocks and, increasing step by step from mesozoic to palaeozoic, it becomes many times greater for archaean rocks. As a corollary, since the geological map indicates only surface distribution of rocks, we may conclude that the seismic wave mainly transmits through the earth's surface, or quite probably, seismic action is mainly due to surface waves discussed by Lord Rayliegh, and recently propounded by Lamb for isotropic medium. Any further discussion, however, as to the seismic wave-conductivity of several rocks requires more precise quantitative investigation of the amount of hysteresis for these rocks.

20. Conclusions.

In so far as the present experiments extend, we have the following conclusions.

1. In most rocks, Hooke's law does not hold even for stress equivalent to a few percentages of their breaking forces.

2. Any rock yields slowly but progressively under a constant force, though the rate of yielding diminishes with the increase of duration. It recovers from the yielding when it is released from the force; but it seems to require an infinite time to return to its initial state by itself.

3. If the modulus of elasticity is proportional to the ratio of the change of force to the change of flexure produced by that force-variation, it is wholly indeterminate, in so far as its previous history as well as its present condition is not completely known.

4. When a piece of rock is bent further and further, the modulus of elasticity firstly diminishes and then, passing through its minimum

value, increases again to a certain amount till the piece is broken. The modulus of elasticity in a virgin piece seems, as a matter of course, to be much greater than that usually obtained by assuming Hooke's law.

5. When a piece of rock is bent right and left cyclically, the modulus of elasticity varies not only from point to point in a cycle, but also from cycle to cycle, provided the amplitude of the cycle does not remain constant.

6. The mean modulus of elasticity taken for one complete cycle is much greater than the modulus of elasticity estimated in the usual manner.

7. The modulus of elasticity diminishes when the amplitude of the cycle increases, and the variation is more rapid for smaller amplitude than for larger one. This last fact is in contradiction to the assumption that for small amplitude the modulus of elasticity may be considered as a constant.

8. The modulus of elasticity of a piece of rock increases when the piece is already in a strained state.

9. The modulus of elasticity generally increases from cainozoic to archæan rocks. The amount of hysteresis, on the contrary decreases from the new rocks to the old ones.

The inferences to be drawn from the above results, in so far as seismology is concerned, are as follows ;—

1. The velocity of propagation of seismic wave is least for quatery rocks and, increasing with age, it becomes greatest for archæan rocks.

2. The velocity, as it is a function of the mean modulus of elasticity taken for one complete vibration, must be much greater than that estimated in the usual manner.

4. The velocity must diminish when the amplitude of seismic wave increases : the rate of diminution is greater when the amplitude is smaller than in the other case.

4. Along some veins or dikes of old rocks running through the

earth's crust, the velocity of propagation is maximum. Consequently, the intensity as well as the direction of seismic motion may be very different for two neighbouring stations. Further consequence results on seismic shadow; i.e. seismic wave may be shielded by a vein of old rocks.

5. Seismic wave-conductivity of rocks is least for cainozoic rocks and, increasing from mesozoic to palæozoic rocks, it becomes many times greater for an archæan rocks. Consequently, the curves of iso-frequency of after-shocks swells out or shrinks in very rapidly according as the region consists of archæan or cainozoic rocks.

6. Actual coinsidence of the curves of iso-frequency with the geological distribution of rocks in the region, which is the case, e.g., for Mino-Owari earthquake where the number of after-shocks during one year, 1892, amounts to 867 at its central region, may be explained on the supposition that the seismic waves are mainly transmitted along the earth's surface. It is quite probable that seismic action is mainly due to surface waves discussed by Lord Rayliegh, and recently propounded by Lamb for isotropic medium.

7. As to the frequency of after-shocks at a central region, the frequency F at any time t is given by

$$F=\frac{c.k.K.T}{t+\tau}-c.k.K\log\left\{1+\frac{T}{t+\tau}\right\}$$

where τ,c,k,K and T are constants of different kinds: i.e.

τ refers to the choice of the origin of time t,

c determines the amount of strain required to produce one after-shock,

k is the force produced in unit time, by accumuration of which the primitive earthquake took place,

K is a constant specifying the rate of yielding of the rock in the central region,

T is time required by the accumurating force to attain its greatest amount.

Consequently, other conditions being equal, the frequency of after-shocks is greater for an earthquake which is slowly generated than for rapidly generated one. The curve of frequency is nearly hyperbolic: its deviation from hyperbola is greater in the latter case than in the former.

REMARKS:—Preliminary experiments with sandstone show that the modulus of elasticity is much affected by the variation of temperature: i.e. ca. 0.5 % per degree. It does not, however, necessarily diminish with an increase of temperature when the temperature is low: i.e. it is maximum at ca. 9°c.
If the above fact be true for most rocks, the velocity of propagation would diminish with the increase of temperature of the medium. Consequently, any cause of the temperature-rise would also be a cause of the velocity-diminution. For instance, as the underground temperature rises very rapidly with the increase of depth, the velocity would diminish proportionally, provided all other conditions remain the same.

In conclusion, I wish to express my great indebtedness to Mr. Fukuchi for valuable informations conserning the geological characters of the specimens. My best thanks are also due to Professor H. Nagaoka, under whose most kind guidance I carried out this experiment.

EXPLANATION OF PLATES.

Pl. I.

Fig. 1. Sketch of the modification of König's arrangement.

,, 2. Sketch of the first arrangement.

,, 3. Sketch of the second arrangement.

,, 4. View of the four images visible in the field of telescope.

,, 5. Front view of the flexure apparatus.

,, 6. Side ,, ,, ,, ,, ,,

,, 7. Plan of the same.

Pl. II.

Fig. 8. A little iron framework by which the bending force is applied to the specimen. W_1 and W_2 are two wedges, of which the former is fixed while the latter is movable. The specimen is tightly clamped between them by the screw S. The bending force is due to the difference of weights suspended at the extremities of the strings S_1 and S_2.

Fig. 9-10. Sketches of the first and second arrangements respectively. These show the geometrical relation between the bending of the specimen and the deviations of the images within the telescope-field.

Fig. 11. This shows the result of preliminary observation on a piece of micaschist. The ordinate represents the amount of deviation of the image in first arrangement, while the abscissa represents the corresponding effective mass in grams weight.

,, 12. This shows the relation between the amount of bending of the above specimen and the corresponding force applied to the specimens.

,, 13. This shows the amount of rotation of the specimen as a whole during one complete cycle of bending in the above experiment.

Pl. III.

Fig. 14. These are the hysteresis curves for bending of a sandstone. The resisting masses are different for the four curves.

„ **15.** These curves show the variation of the modulus of elasticity due to the change of bending force during one complete cycle in the above experiments. In each curve, the right and left branches correspond to the cases where the specimen was bent convex towards righthand or lefthand side respectively, while the lower branches correspond to the increasing stress and the upper to the decreasing one. Comparing these four curves, we may safely conclude that the presence of resisting mass has no sensible influence, or, if any, it is negligibly small.

Pl. IV.

Figs. 16 17. Yielding of sandstone under constant force, $M=3000$, $M_0=3300$ grams. That is to say, the amount of bending of the specimen is plotted against the time during which the specimen was subjected to the constant force.

„ **18-19.** Recovery from the yielding of the above specimen, which was loaded with $M=3000$, $M_0=3300$ grams during 20363 minutes.

Pl. V.

Fig. 20. Effect of recovery from the yielding on cycles. A piece of sandstone, loaded with $M=3000$, $M_0=3300$ grams during 2647 minutes, was treated in cyclical process as usual.

„ **21.** Curve A represents the relation between the amount of bending and the corresponding amount of force acting upon the specimen, which was initially in a natural state. In curves B and C, the specimen was initially in a strained state; i.e. in curve *B*, the specimen was subjected to $M_1 = -1500$ grams during 1234 minutes, while in curve *C* it was subjected to the same force during 1641 minutes. Comparing these three curves with each other we see that, in the neighbourhood of starting point, of the three curves, *A* is more steep than *C* which is more steep than *B*.

„ **72,** The ordinate represents the velocity of propagation of seismic waves through a stratum of sandstone, while the abscissa represents the amplitude or the maximum stress given to the rock during one complete vibration.

Pl. VI.

Figs. 22-25. In each figure, the upper curve shows the variation of temperature with time, while the lower that of bending, under constant force.

Fig. 26. The amount of bending is plotted against the corresponding temperature.

Pls. VII-IX.

Fig. 27-50. Examples of flexural hysteresis in several rocks of different kinds.

Pl. X.

Fig. 51-53. The curves in these figures show how the hysteresis curve varies when its amplitude is varied.

Pls. XI. and XII.

Figs. 54-65. In each figure, the four branched curve represents the variation of the modulus of elasticity during one complete cycle of bending. Refer to the explanation of Fig. 15.

Pl. XII.

Fig. 68. This shows how the four branched curve changes its form with the amplitude of cycle.

„ **70.** Relation between the modulus of elasticity at the state of no stress with the maximum amount of stress applied to the specimen during the cycle.

Pl. XIII.

Fig. 66. Variation of the modulus of elasticity during one complete cycle, calculated by parabolic formula for sandstone.

„ **67.** Relation between the amount of bending and the force during one complete cycle, calculated by the above parabolic formula. This is to be compared with the curves in Fig. 14, Pl. III.

„ **69.** The ordinate represents the modulus of elasticity and the abscissa the ratio of the actual force to the maximum force applied to the specimen during the cycle. The maximun force is 300 grams weight for the upper most curve, and increasing step by step, it is 3000 grams weight for the lowest one.

„ **71.** Both curves A and B represent the variation of the modulus of elasticity with the change of the amount of force. Curve A starts from the natural state of the specimen, while curve B starts from a state where the specimen was subjected under a force $M = -3000$ grams during 2652 minutes.

Pl. XIV.

Fig. 73. Curve of frequency of after-shocks of an earthquake calculated by logarithmic formula.

„ **74.** Curves of frequency of after-shocks. Curve a; calculated by hyperbolic formula. Curves b and c; calculated by the formula deduced from the principle of residual strain.

„ **75.** Geological map of Mino, Owari and the neighbouring districts, with the iso-frequency curves of after-shocks, during 1892, of the Mino-Owari earthquake in 28 Oct. 1891. The iso-frequency curves are copied from Professor Ōmori's paper published in "The Jour. of the Coll. of Sci., Imp. Univ. of Tōkyō, Japan, Vol. VII."

Fig. 1.

König's arrangement.

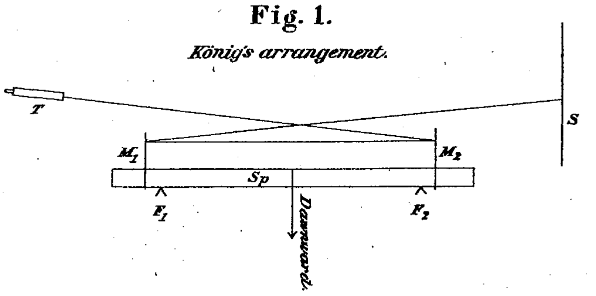


Fig. 2.

First arrangement.

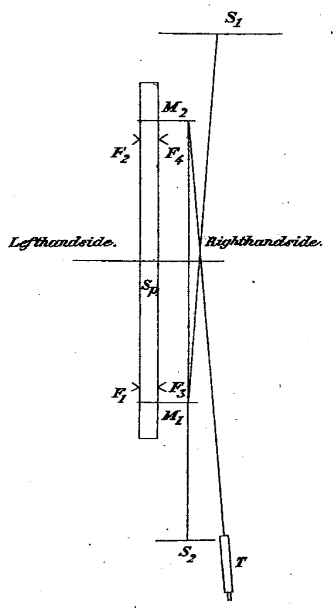


Fig. 3.

Second arrangement.

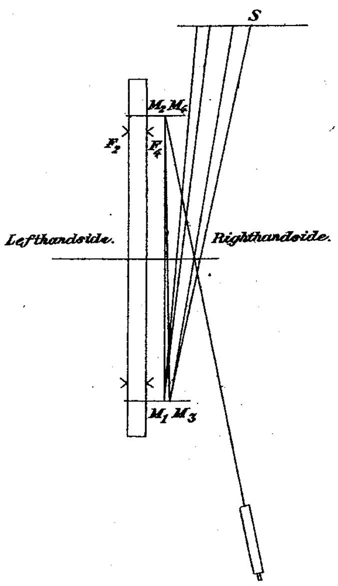


Fig. 4.

The field of telescope.

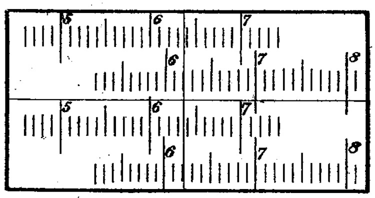


Fig. 5.

Front view.

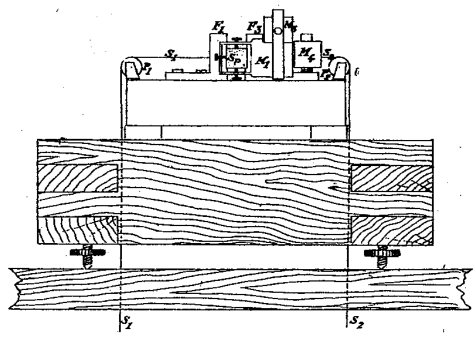


Fig. 6.

Side view.

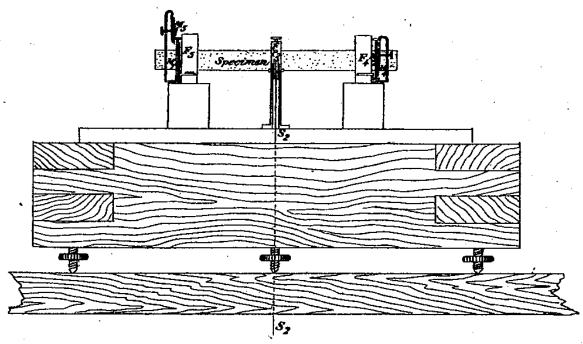


Fig. 7.

Plan of the apparatus.

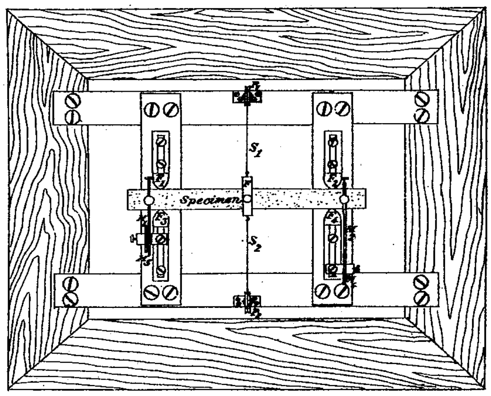


Fig. 9.

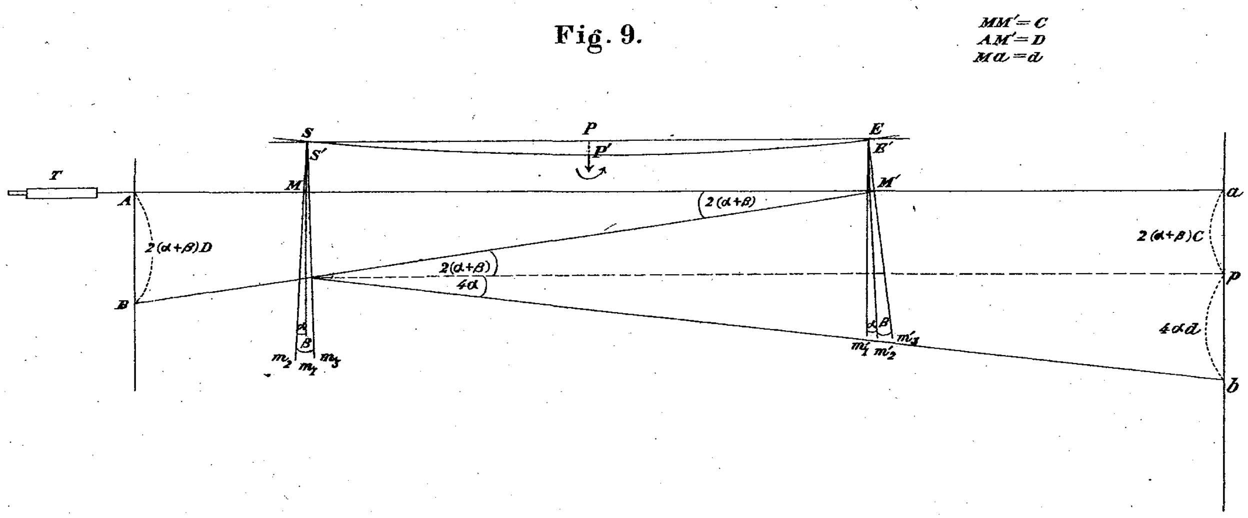


Fig. 10.

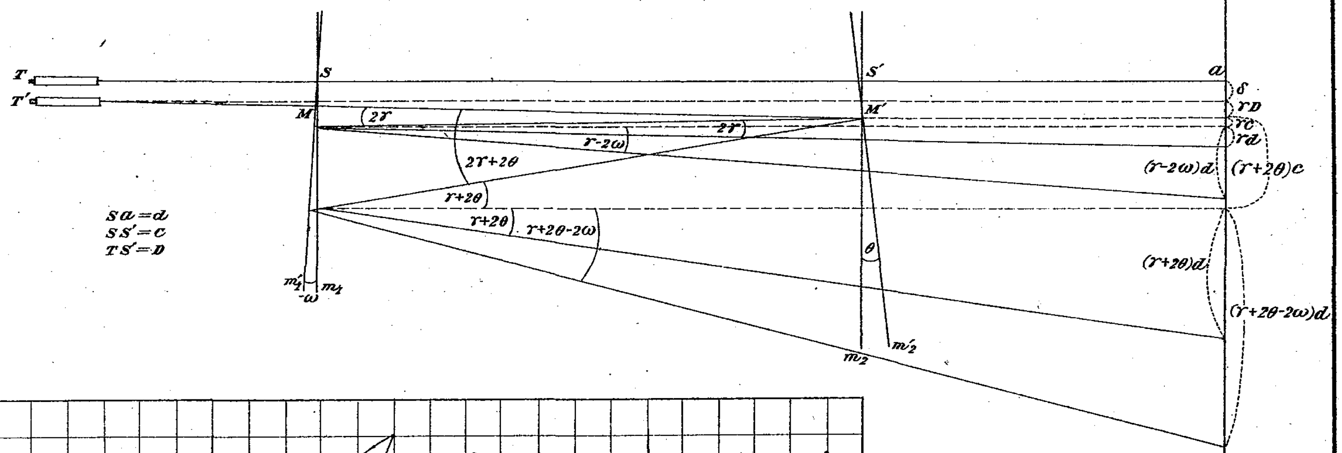
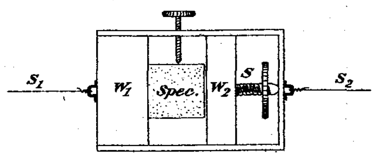
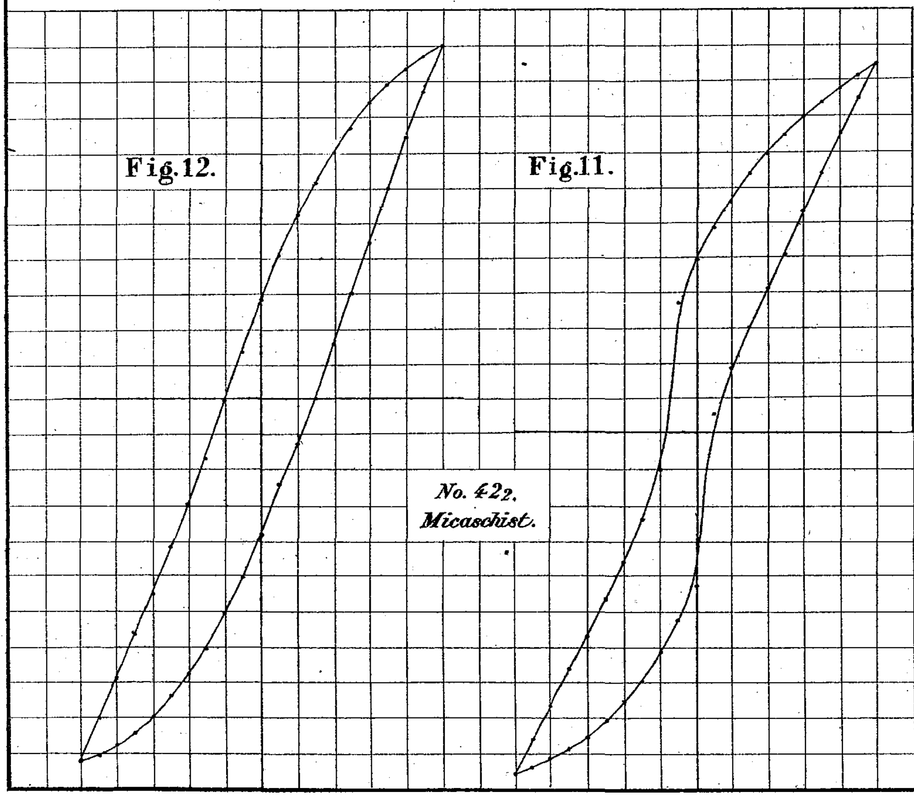
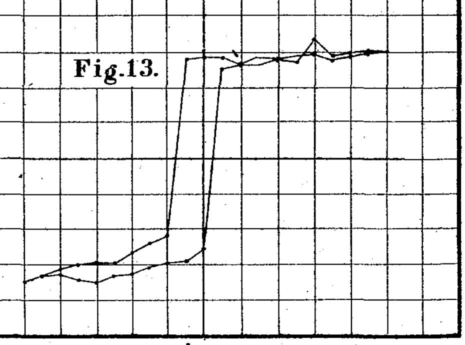


Fig. 8.







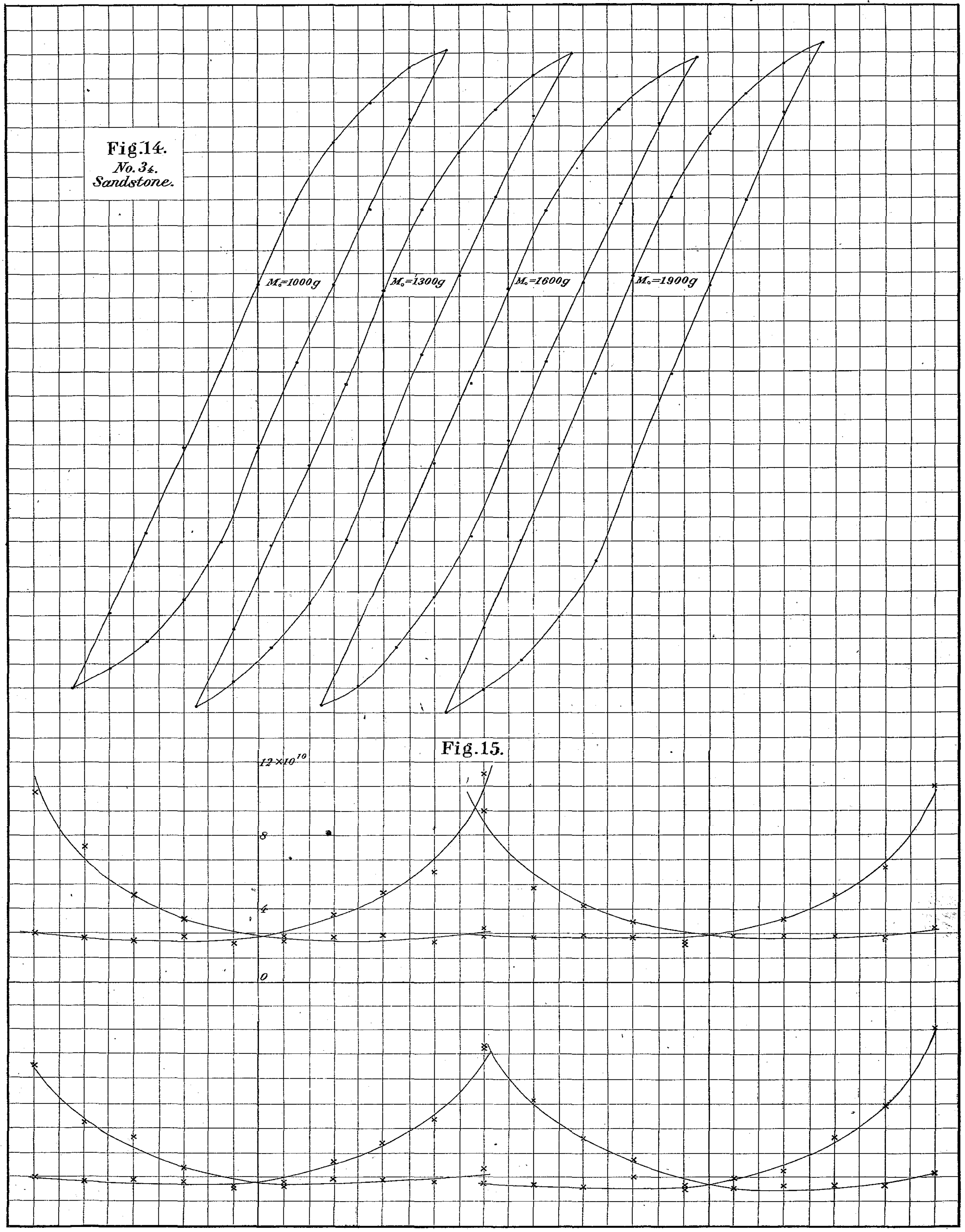


Fig. 14.
No. 34.
Sandstone.
M₀=1000g
M₀=1300g
M₀=1600g
M₀=1900g
Fig. 15.
12×10¹⁰
8
4
0

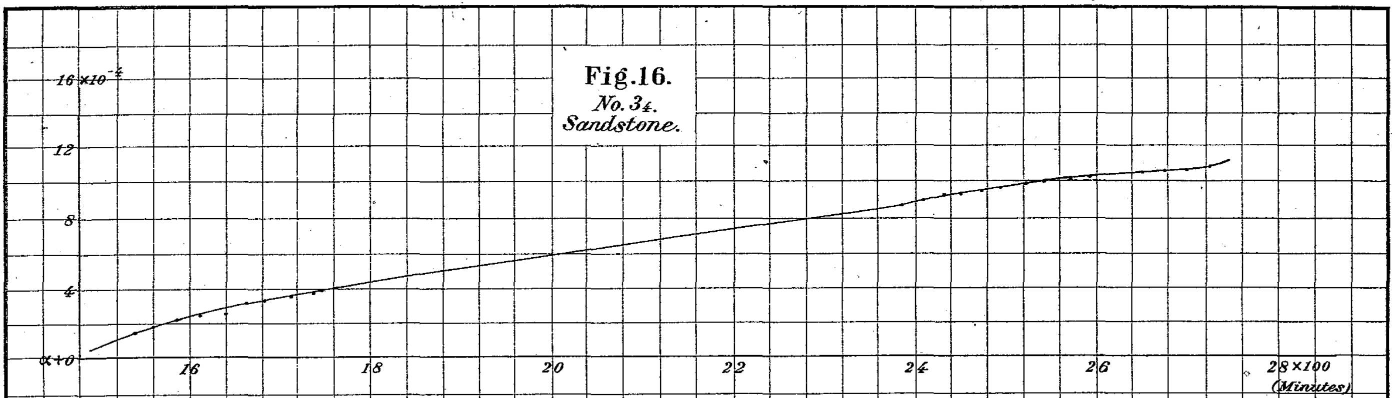


Fig. 16.
No. 34.
Sandstone.

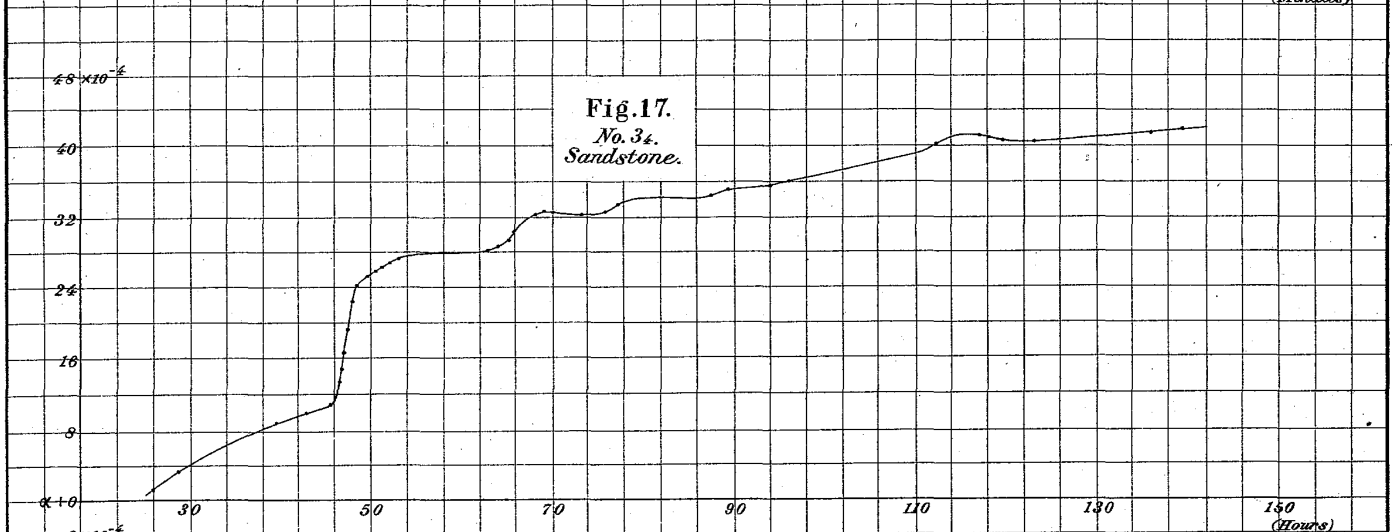


Fig. 17.
No. 34.
Sandstone.

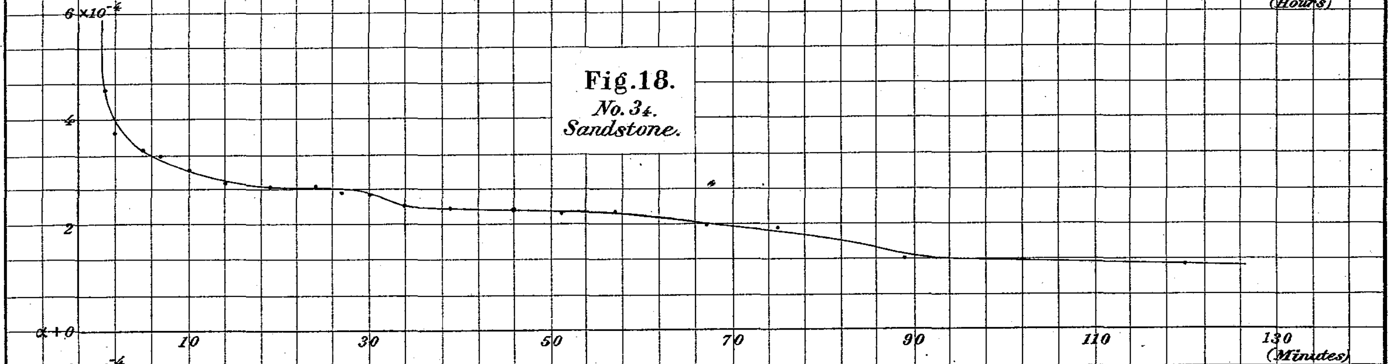


Fig. 18.
No. 34.
Sandstone.

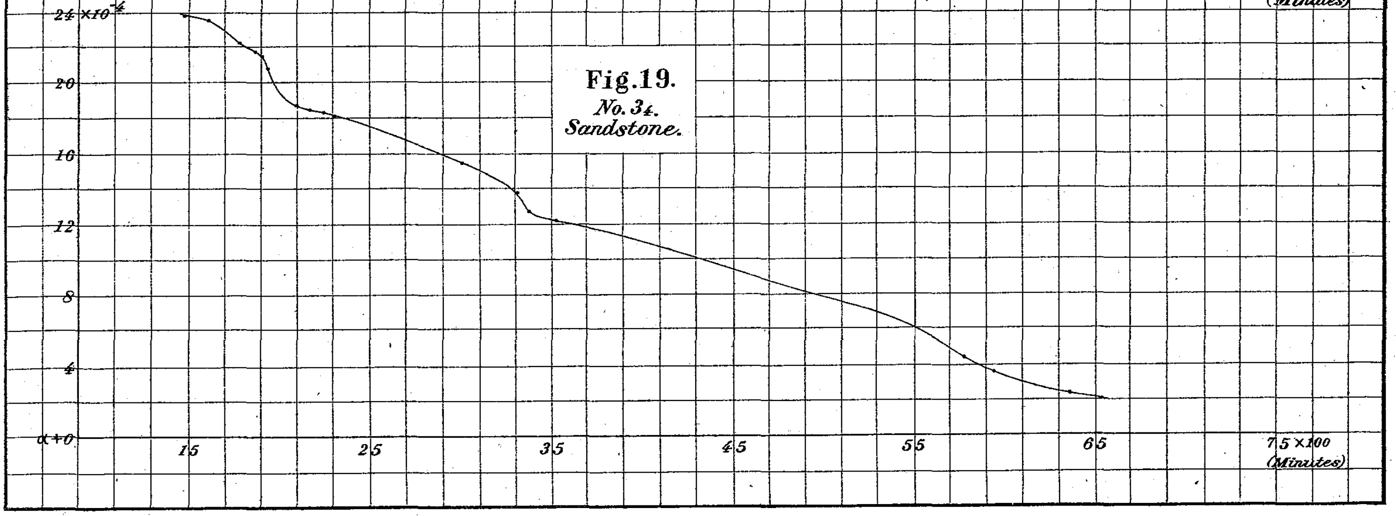


Fig. 19.
No. 34.
Sandstone.

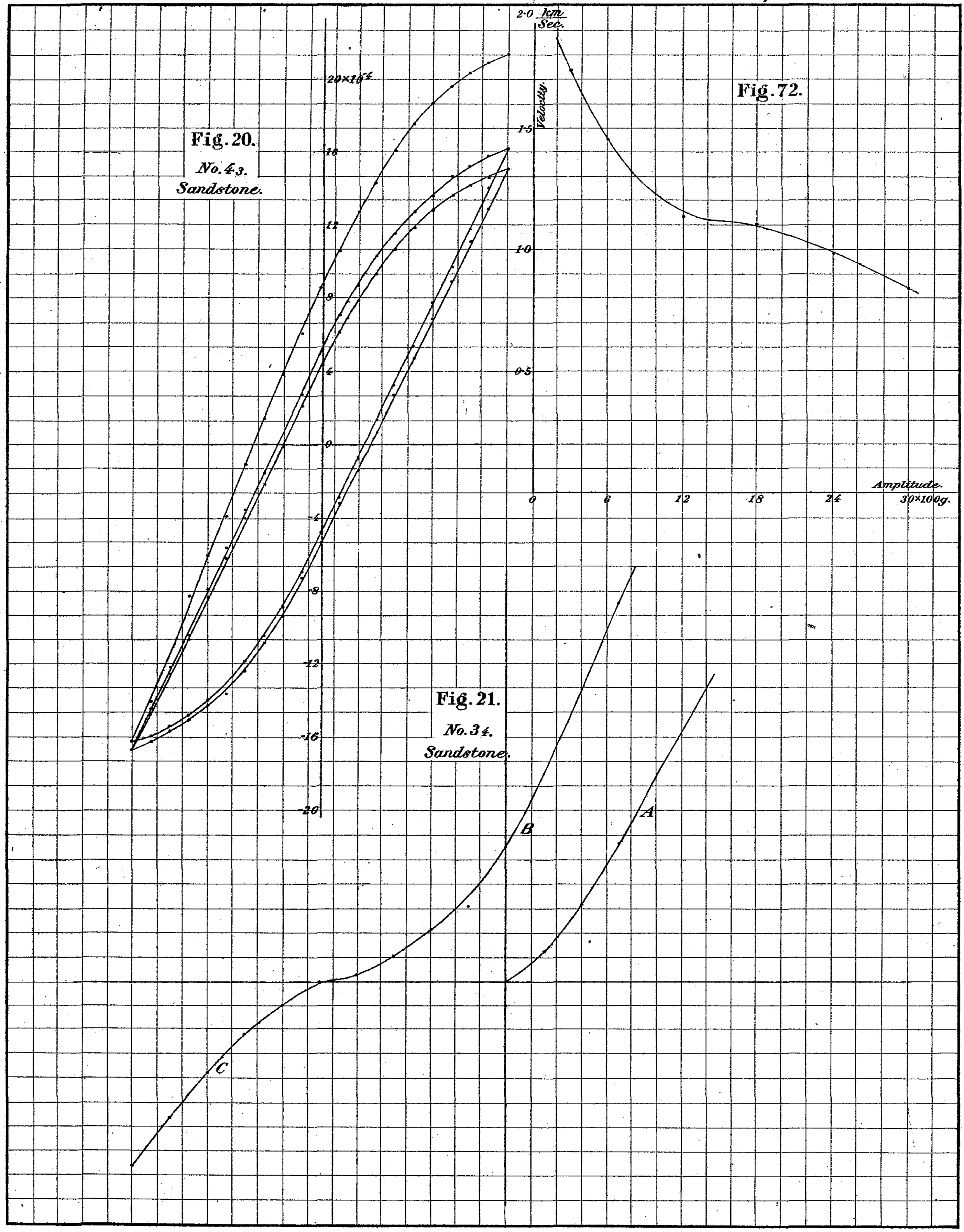
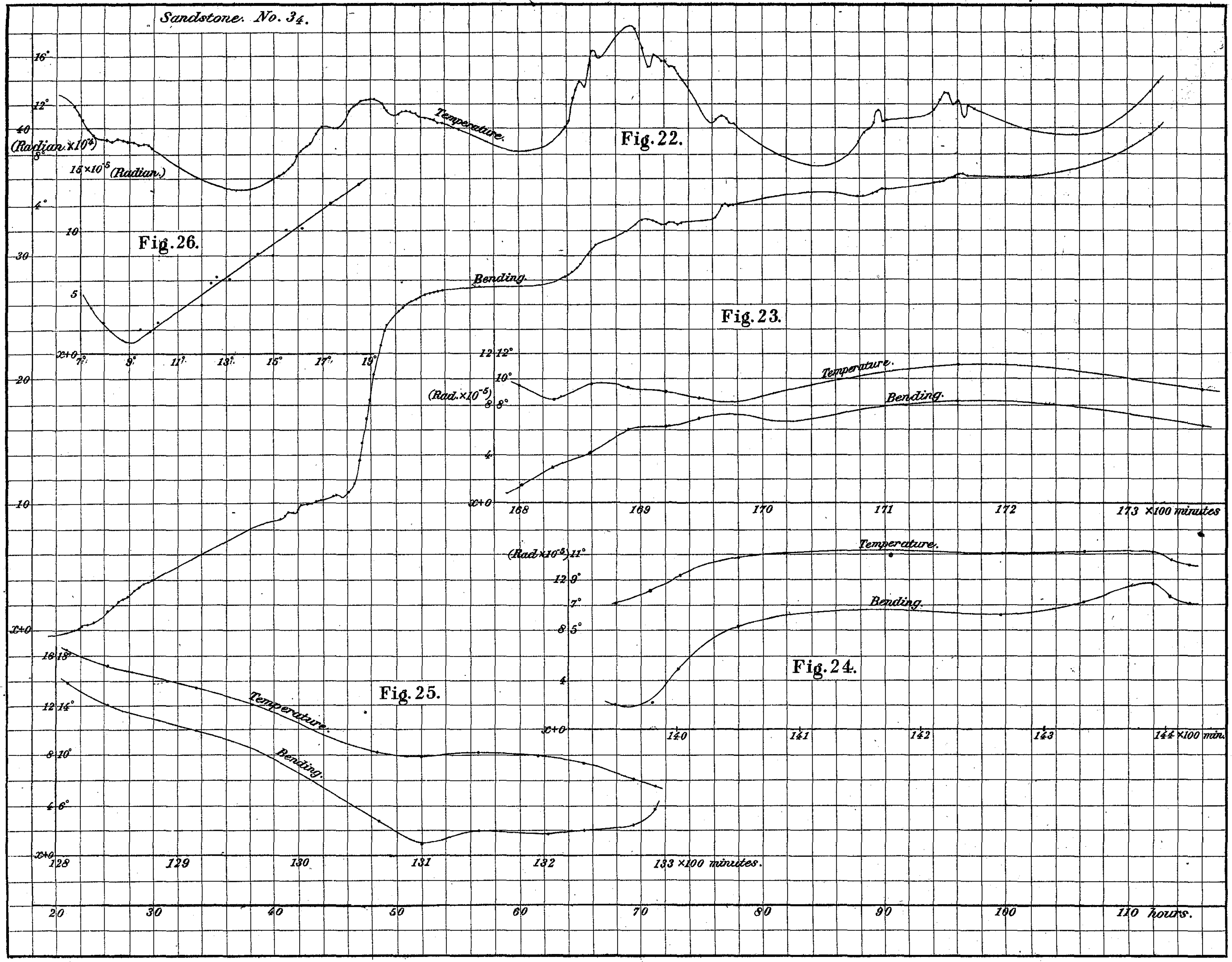


Fig. 20.
No. 43.
Sandstone.
20×10⁻⁴
16
12
8
4
0
-4
-8
-12
-16
-20
Fig. 72.
2·0 km/Sec.
Velocity.
1·5
1·0
0·5
Amplitude.
0
6
12
18
24
30×100g.
Fig. 21.
No. 34.
Sandstone.
A
B
C


Sandstone. No. 34.
Fig. 22.
Temperature.
Bending.
(Radian × 10⁴)
15 × 10⁻⁵ (Radian.)
Fig. 26.
Fig. 23.
(Rad. × 10⁻⁵)
Temperature.
Bending.
168
169
170
171
172
173 × 100 minutes
Fig. 24.
(Rad × 10⁻⁵)
Temperature.
Bending.
140
141
142
143
144 × 100 min.
Fig. 25.
Temperature.
Bending.
128
129
130
131
132
133 × 100 minutes.
20
30
40
50
60
70
80
90
100
110 hours.

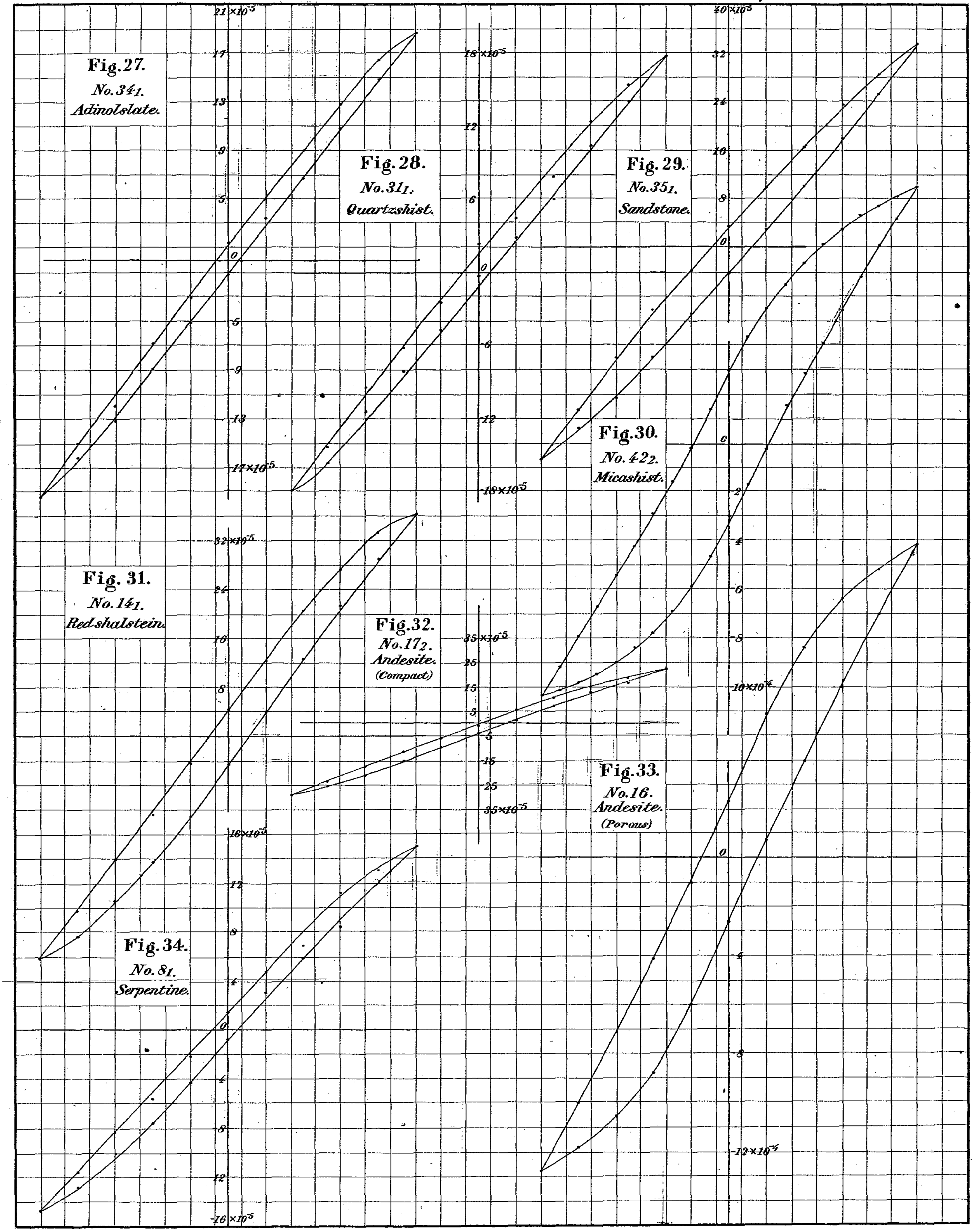


Fig. 27.
No. 34₁.
Adinolslate.
Fig. 28.
No. 31₁.
Quartzshist.
Fig. 29.
No. 35₁.
Sandstone.
Fig. 30.
No. 42₂.
Micashist.
Fig. 31.
No. 14₁.
Red shalstein.
Fig. 32.
No. 17₂.
Andesite.
(Compact)
Fig. 33.
No. 16.
Andesite.
(Porous)
Fig. 34.
No. 8₁.
Serpentine.
21×10⁻⁵
17×10⁻⁵
18×10⁻⁵
-18×10⁻⁵
40×10⁻⁵
32×10⁻⁵
35×10⁻⁵
-35×10⁻⁵
16×10⁻⁵
-16×10⁻⁵
10×10⁻⁴
-12×10⁻⁴

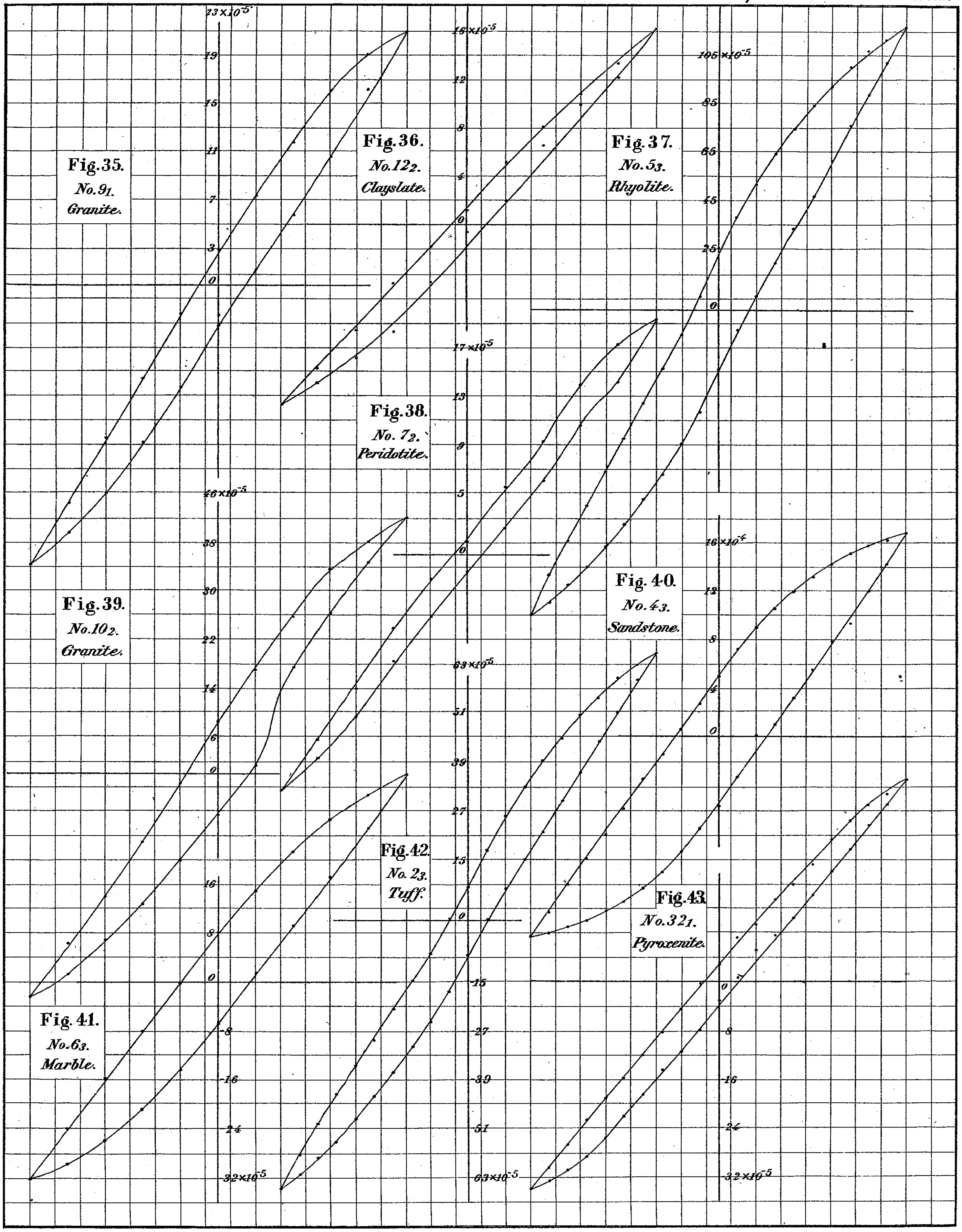


Fig. 35.
No. 91.
Granite.
Fig. 36.
No. 122.
Clayslate.
Fig. 37.
No. 53.
Rhyolite.
Fig. 38.
No. 72.
Peridotite.
Fig. 39.
No. 102.
Granite.
Fig. 40.
No. 43.
Sandstone.
Fig. 41.
No. 63.
Marble.
Fig. 42.
No. 23.
Tuff.
Fig. 43.
No. 321.
Pyroxenite.

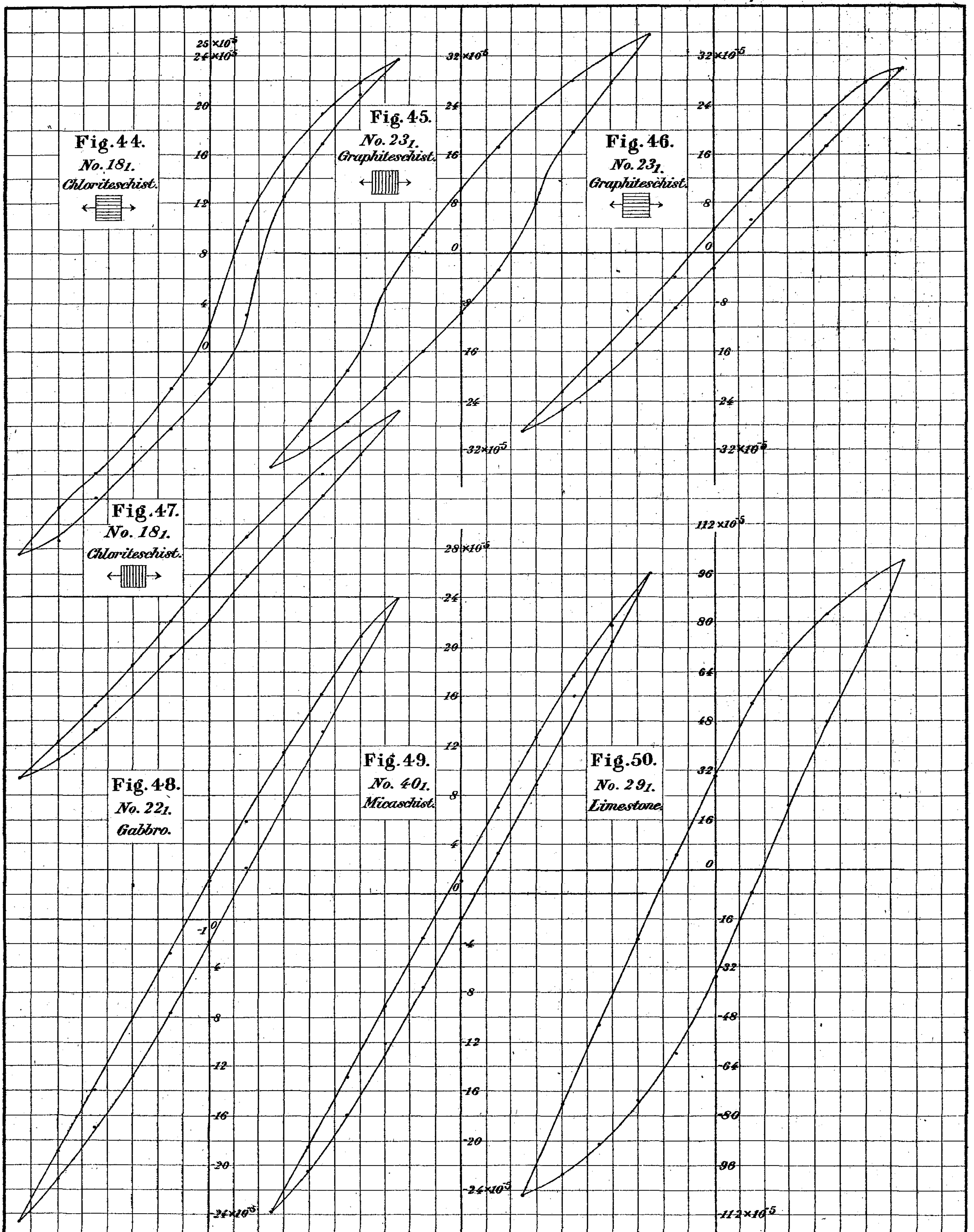


Fig. 44.
No. 18₁.
Chloriteschist.
Fig. 45.
No. 23₁.
Graphiteschist.
Fig. 46.
No. 23₁.
Graphiteschist.
Fig. 47.
No. 18₁.
Chloriteschist.
Fig. 48.
No. 22₁.
Gabbro.
Fig. 49.
No. 40₁.
Micaschist.
Fig. 50.
No. 29₁.
Limestone.

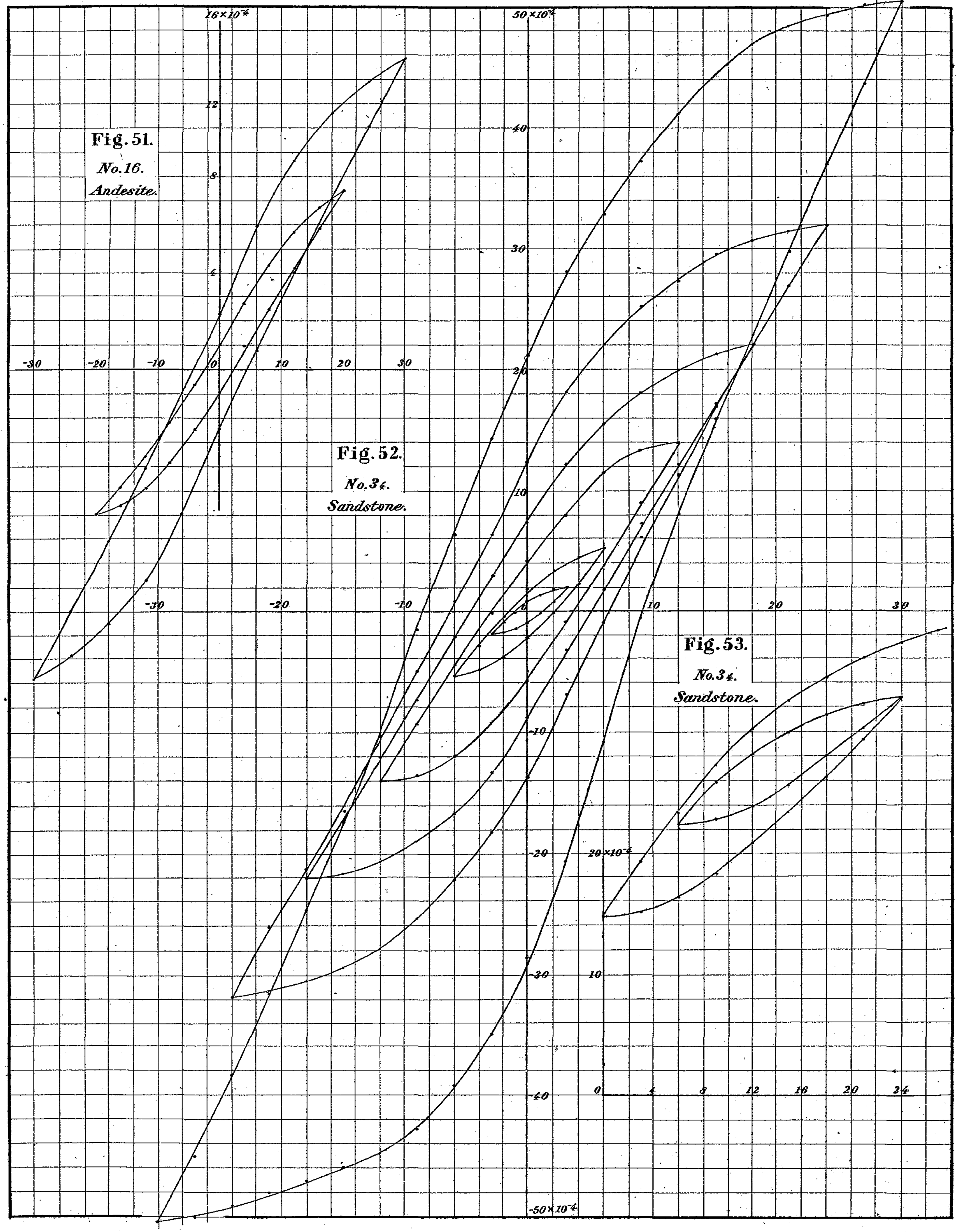


Fig. 51.
No. 16.
Andesite.
Fig. 52.
No. 34.
Sandstone.
Fig. 53.
No. 34.
Sandstone.
16×10⁻⁴
50×10⁻⁴
-50×10⁻⁴
20×10⁻⁴

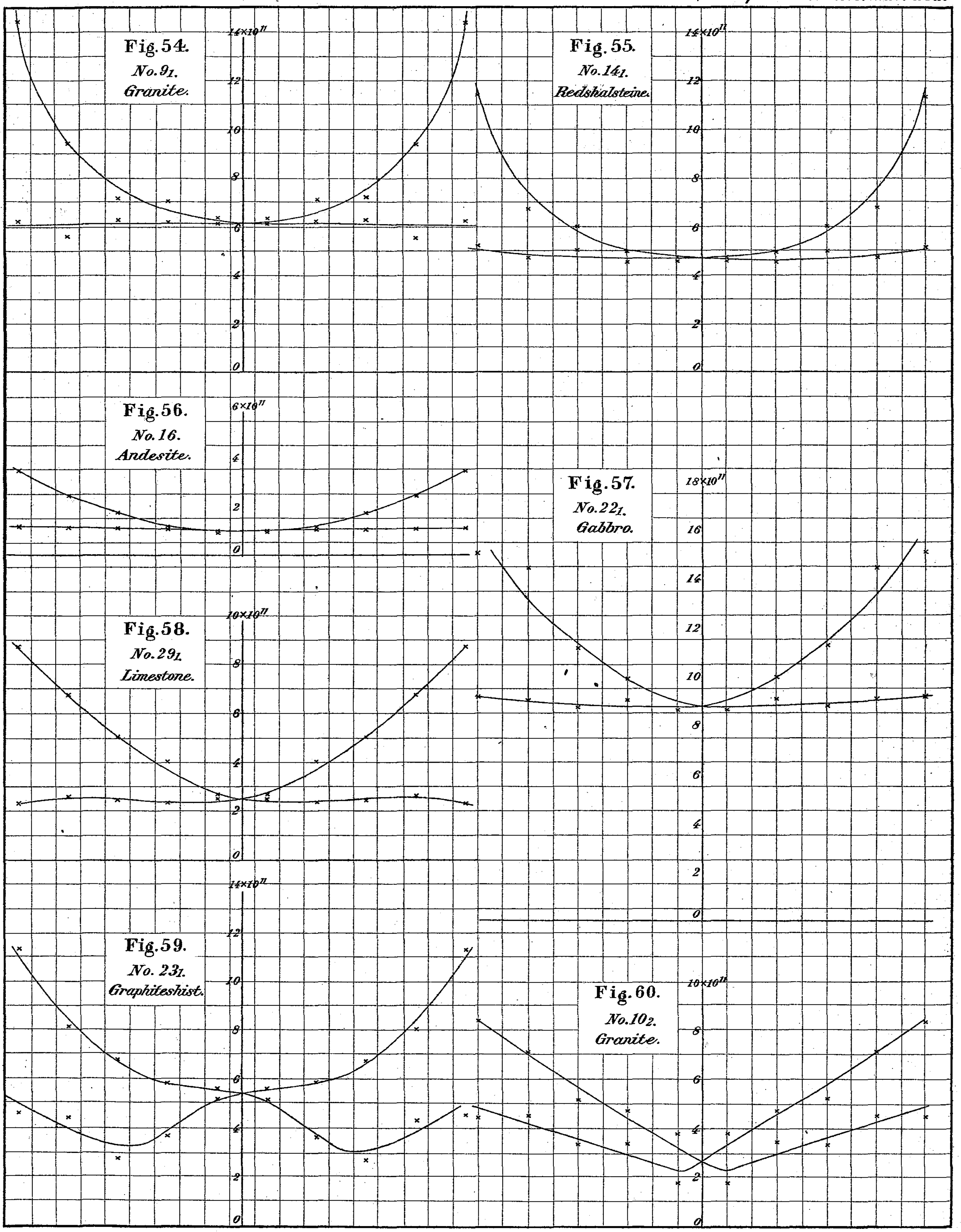


Fig. 54.
No. 9_1.
Granite.
14×10¹¹
12
10
8
6
4
2
0
Fig. 55.
No. 14_1.
Redshalsteine.
14×10¹¹
12
10
8
6
4
2
0
Fig. 56.
No. 16.
Andesite.
6×10¹¹
4
2
0
Fig. 57.
No. 22_1.
Gabbro.
18×10¹¹
16
14
12
10
8
6
4
2
0
Fig. 58.
No. 29_1.
Limestone.
10×10¹¹
8
6
4
2
0
Fig. 59.
No. 23_1.
Graphiteshist.
14×10¹¹
12
10
8
6
4
2
0
Fig. 60.
No. 10_2.
Granite.
10×10¹¹
8
6
4
2
0

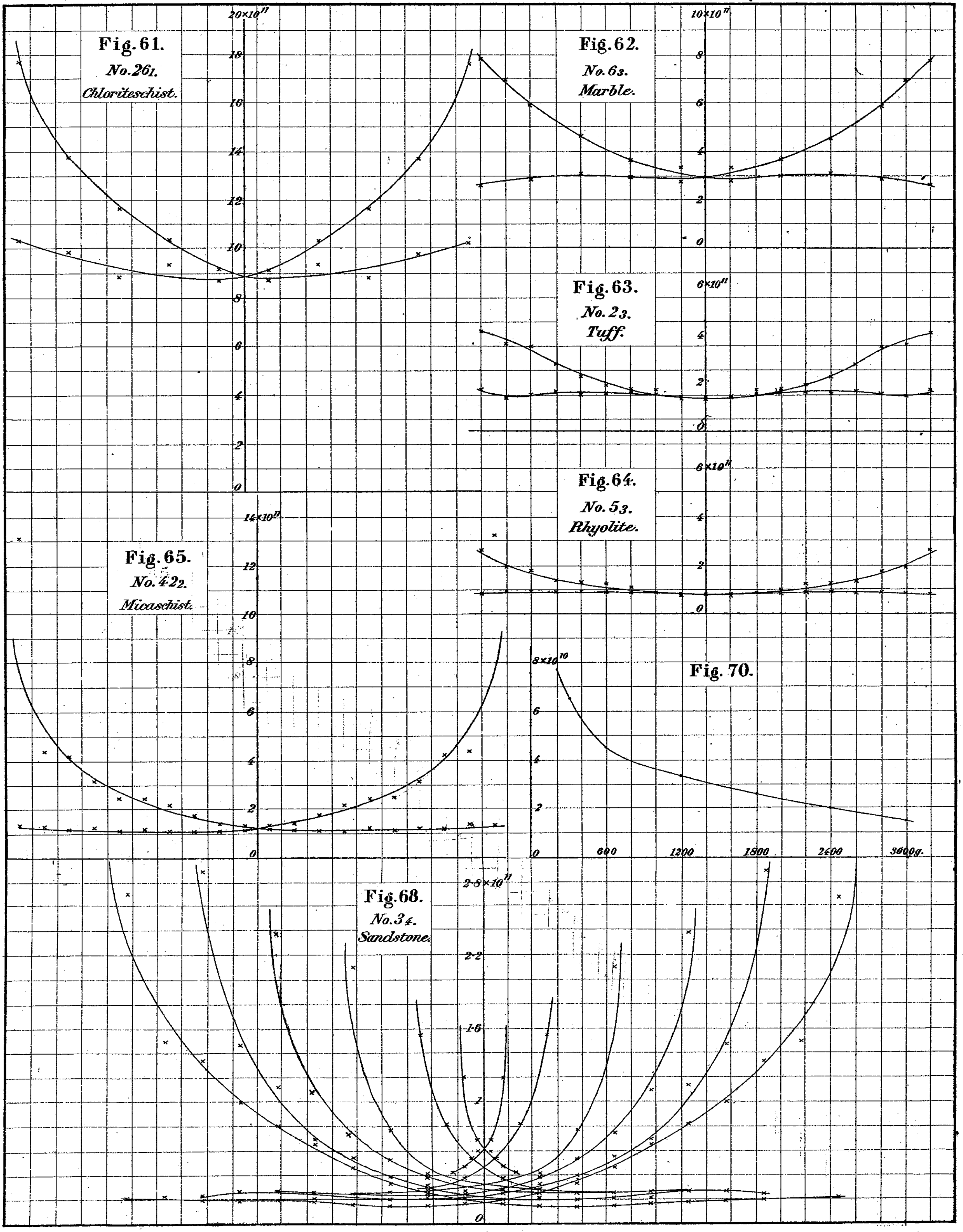


Fig. 61.
No. 261.
Chloriteschist.
20×10^11
18
16
14
12
10
8
6
4
2
0
Fig. 62.
No. 63.
Marble.
10×10^11
8
6
4
2
0
Fig. 63.
No. 23.
Tuff.
6×10^11
4
2
0
Fig. 64.
No. 53.
Rhyolite.
6×10^11
4
2
0
Fig. 65.
No. 422.
Micaschist.
14×10^11
12
10
8
6
4
2
0
Fig. 70.
8×10^10
6
4
2
0
600
1200
1800
2400
3000g.
Fig. 68.
No. 34.
Sandstone.
2·8×10^11
2·2
1·6
1
0

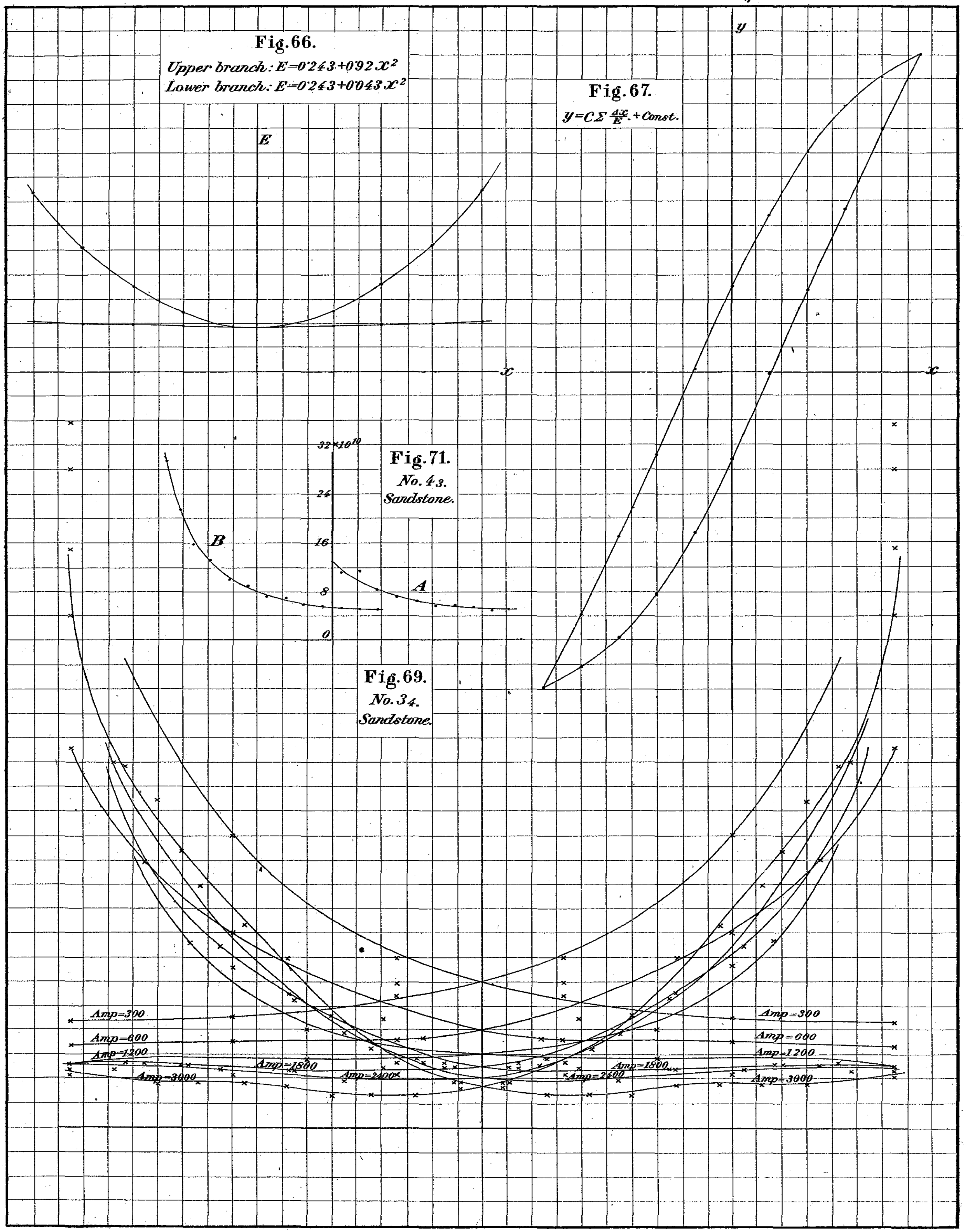
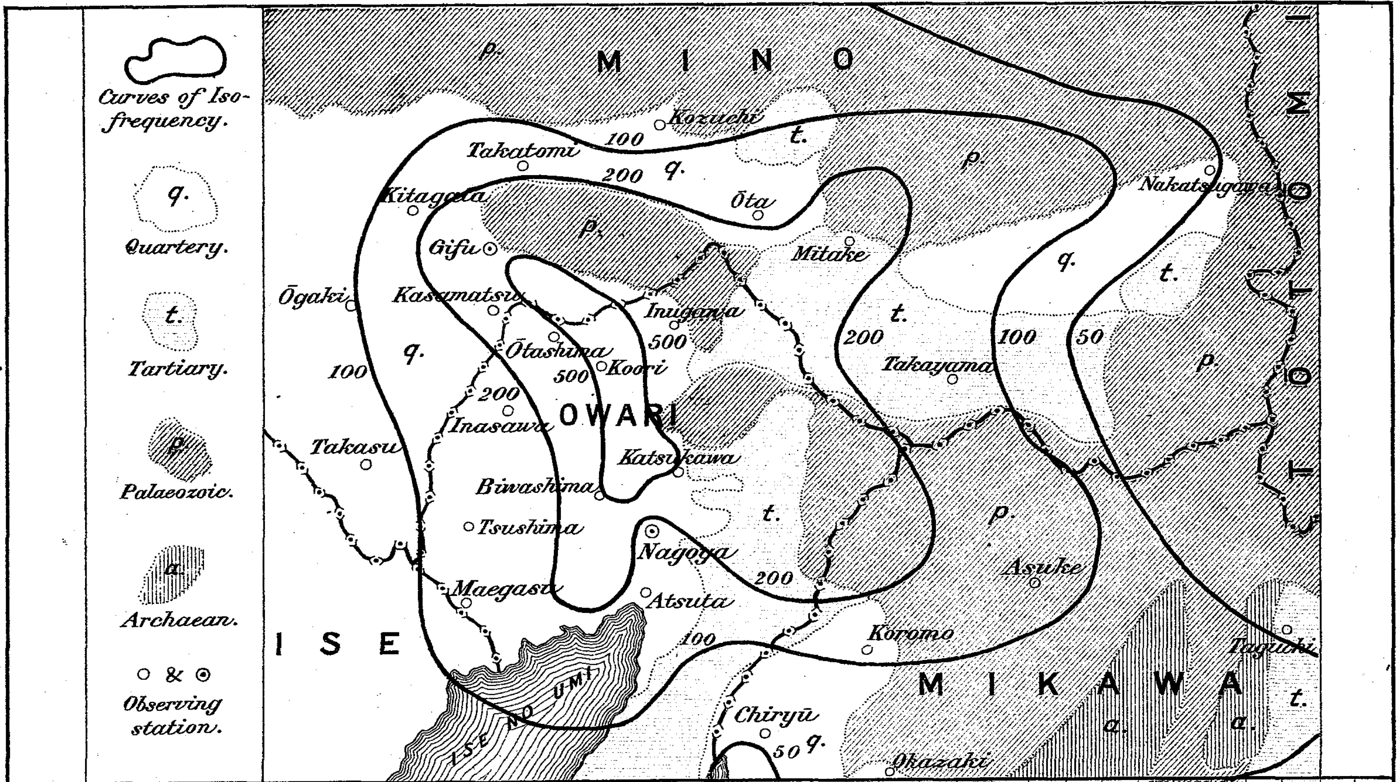


Fig. 66.
Upper branch: E=0·243+0·92 x²
Lower branch: E=0·243+0·043 x²
E
x
Fig. 67.
y=CΣ Δx/E + Const.
y
x
Fig. 71.
No. 43.
Sandstone.
32×10¹⁰
24
16
8
0
A
B
Fig. 69.
No. 34.
Sandstone.
Amp=300
Amp=600
Amp=1200
Amp=1800
Amp=2400
Amp=3000

Fig. 75.



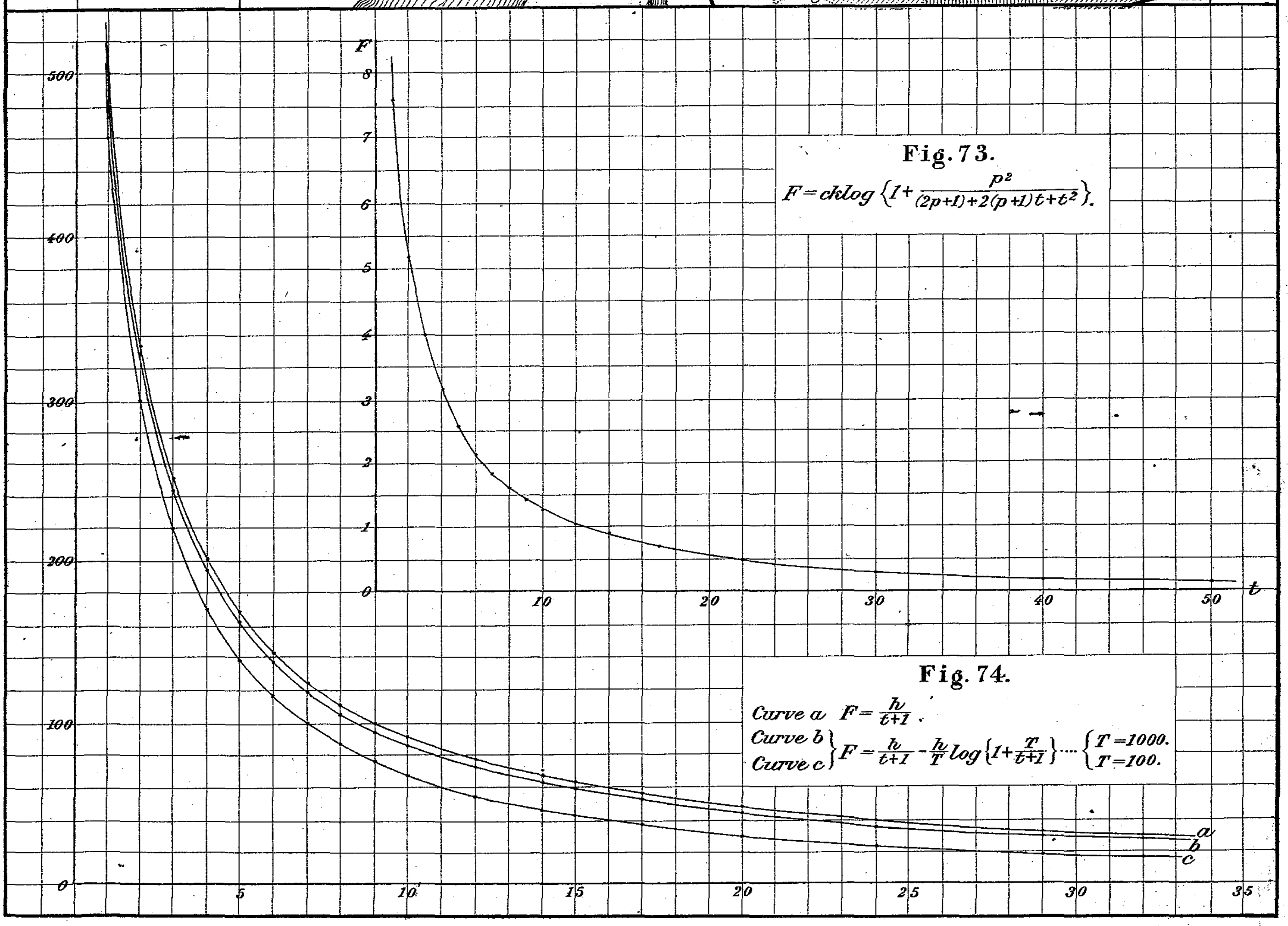


Fig. 73.

$$F = ck\log\left\{1 + \frac{p^2}{(2p+1)+2(p+1)t+t^2}\right\}.$$

Fig. 74.

Curve a $F = \frac{h}{t+1}$.

Curve b, Curve c $\left.\right\} F = \frac{h}{t+1} - \frac{h}{T}\log\left\{1+\frac{T}{t+1}\right\} \cdots \left\{\begin{array}{l} T=1000. \\ T=100. \end{array}\right.$