On the Modulus of Rigidity of Rocks.*

BY

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Introduction.

The present experiments were undertaken to extend Professor H. Nagaoka's investigation on elastic constants of rocks.† The experiments are wholly related to torsion, as my first object was to show how Hooke's law does not hold even for very small strain and how great hysteresis there exists in the relation of torsion to couple.

The specimens to be examined were prepared in the same manner as those experimented by Professor Nagaoka; i. e. in the shape of a rectangular parallelopiped whose dimensions are about $15 \times 1 \times 1$ centimeters.

The arrangement for the experiment also was, on the whole, one and the same as that already used by him, but with some improvements, necessary to make the twisting couple cyclical, as will be described below.

Preliminary experiments with the apparatus, as he noted in the publication above cited, showed that the deviation from Hooke's law as well as elastic after-effect are strikingly prominent even for very small torsion, so that it is scarcely possible to make the measurement

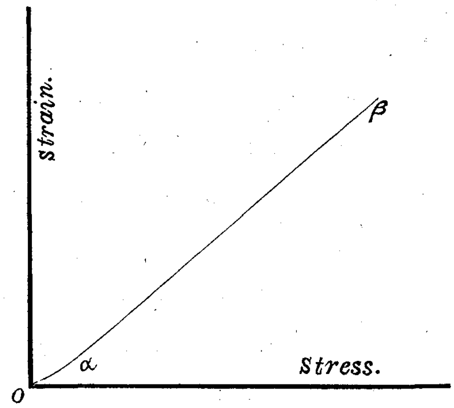
* The expenses of experiments were defrayed by the Earthquake Investigation Committee out of the fund specially allotted for the investigation of the elastic properties of rocks.

† H. Nagaoka. Elastic constants of Rocks and the velocity of the Seismic Waves. Pub. of the E. I. committe in Foreign Languages. No. 4. 1900; Phil Mag. Vol. 2, July. 1900.

within the limit of elasticity, as the instrument is not sensitive enough.

The part obeying Hooke's law, if any, lies, as a matter of course, in the neighbourhood of infinitely small strain. So I intended firstly to find the ratio of stress to strain in the neighbourhood of zero-strain, which may be said to be the modulus of rigidity properly taken within the limit of elasticity.

In ordinary method, be it the determination of Young's modulus by flexure or of the modulus of rigidity by torsion, etc., no credible result of measurement can be obtained in the neighbourhood of zero-strain, by virtue of the friction called into play in the arrangement, and other annoying causes. The consequence is that even in the case of substance obeying Hooke's law quite accurately, there exists a short part $o\alpha$, in the neighbourhood of the starting point, which apparently does not obey Hooke's law, although the limit of elasticity extends to β far beyond the point α. This is, of course, due to the imperfectness of the instrument, inevitable in every practical arrangement. Thus it is common to reject the first part $o\alpha$, and to calculate the required modulus from measurement made on the part $\alpha\beta$. But in the case of some substances whose limit of elasticity is very narrow, the point β comes short of α, which shows that the limit of elasticity is too small to be experimented by the arrangement at hand. Moreover, even in the case where there exists apparently a nearly straight part, it can never be safely said that the specimen is examined within the limit of elasticity, as it will be proved in the later sections.



With most of rocks, the above was the case. One instance will suffice to show how the matter stands. Fig. 1. in Pl. I. shows the general feature of the relation of torsion to couple in a piece of sandstone, a kind of sedimentary rocks of tertiary age. The twisting couple is measured on the abscissa while the corresponding amount of twist is taken as ordinate, the units adopted being arbitrary. If the meaning of the term "*Modulus of rigidity*" be extended to express simply *the resistance to distorsion in a given state,* no matter whether the substance be in the neutral or in the strained state or be it then elastic or plastic, the trigonometrical tangent of the angle contained between the stress-axis and the tangent to the curve at any point on it is proportional to the modulus of rigidity of the substance at that state.

Looking at the form of the curve, it will be seen that the tangent line to the curve is nearly horizontal at the origin and gradually tends to become vertical. The horizontality of the tangent means infinite rigidity, or the specimen is perfectly rigid at the state of zero-strain. When the tangent becomes strictly vertical, the rigidity must be zero, so that the substance tends to a fluid state. This last statement way be true, but the former, *i. e.* infinite rigidity at origin is not plausible. This behavior is chiefly to be attributed to friction called into play in the arrangement.

Now, gradually releasing the specimen from the couple and again starting *ab initio* the tangent becomes horizontal and even passes over to the negative side, *i. e.* the directions of the twisting couple and of the twist produced by it are opposite to each other. Thus twisting in one direction makes the twist take place in the opposite direction. This is indeed a striking paradox, but observe that this is due to the lagging of the strain after the stress; or, as the phenomenon is usually called, there is *Hysteresis* in the relation of the twist produced to the twisting couple.

From the above, it will be easily seen that the modulus of rigidity is a function of the stress as well as of the history through which the specimen was brought to the present condition. In such a case as above, it is very interesting to find the value of the modulus of rigidity in the virgin state. Since the disturbing causes must predominate within the region near zero-strain, it is evident that we arrive at no trustworthy result if the above arrangement is employed.

The essence of my improvements of the arrangement was, firstly, to get rid of the influence of the friction in the different parts of the instrument, and secondly, to twist the specimen at first in one direction and then in the opposite direction cyclically, with increasing and decreasing couples passing through zero continuously.

Examined by the arrangement thus improved, it was found that the modulus of rigidity in virgin state is much greater than it is commonly believed. It seems to me, that if the modulus of elasticity is liable to a similar variation, there is no need to assume that the path of the tremors in earthquake is different from that of the principal shocks. Wide difference of velocities for several parts of seismic waves may be simply accounted for by the fact that the velocity of elastic wave diminishes when its amplitude increases, in so far as there is elastic-yielding in the rocks through which the wave propagates.