Experimental facts.

Twisting Apparatus. {See figures in Pl.II.}

The framework of the apparatus was one and the same with that used by Professor H. Nagaoka in his experiments above cited. One more pulley P_2 was added on the other side of P, symmetrical to P_1; the specimen to be tested being firmly clamped to P. Two flexible strings s_1 and s_2, attached to the lowest part of the pulley P, going upwards along its rim, one in each direction, were slung around other pulleys P_1 and P_2, whence they hung vertically downwards. Two another flexible strings s_1' and s_2', attached to the heighest part of the middle pulley P, passed over it along its rim, one in each direction, and hung down vertically. The relative positions of these three pulleys were so adjusted that the lines of passage from the middle pulley to the side ones were in vertical direction. The lower extremities of the strings were connected by thin brass pieces C_1 and C_2, two and two, as shown in Fig 3, Pl. II. One of these two pieces, *i. e.* C_2, was a strip of rectangular cross section whose dimensions were nearly 2×10 in millimetres: while the other C_1 was of the shape of a rectangular box without bottom, in order that the string s_2' may freely run vertically down through it. The weight of C_2 was equal o that of C_1. By proper adjustment of the length of the two pieces, the four strings above mentioned remained all vertical. From the middle points of these two pieces, another flexible strings s_1'' and s_2'' hung vertically, which, after passing over the pairs of small pulleys P_1', P_1'' and P_2', P_2'' respectively, were attached to pans carrying the weights. By a proper arrangement, the lines of passage remained vertical from C_1 and C_2 to P_1' and P_2' and horizontal from P_1' and P_2' to P_1'' and P_2''. I say simply a proper arrangement, since it would lead too far if I attempt to describe the details of the actual arrangement. Thus it will be easily seen that by mounting proper weights on the pans Q_1 and Q_2, any couple whatever, positive or negative, could be produced without any increase of load to be supported by the knife-edge.

Two mirrors, by the difference of whose deflections the amount of torsion was to be measured, were attached to two frameworks, one in each, which were clamped to the prism, one near each end. It will suffice to give a rough sketch of the frame. (See Fig. 5). The arm carrying the mirror was of such a length that the two mirrors stood side by side, one mm. or a little more distant from each other. Three superfluous limbs served to counterbalance the mirror. The deflections were measured by a vertical scale divided on ground glass and illuminated from behind by difused sunlight; it stood 270.5 cm. apart from the prism, and a telescope mounted on a tripod stand;—not two as in Professor H. Nagaoka's experiment. That is to say, two images of the scale, reflected by two different mirrors, were brought within the field of one and the same telescope. The readings of scale division were noted at a single glance. (See Fig. 4.)

Method of Observation.

The method of observation was as follows:—

1. To begin with, the prism of rock was firmly clamped in horizontal position and perpendicular to the plane of the twisting pulley. Other auxilially pulleys lay in one vertical plane with the last one.
2. The direction of the two mirrors was so adjusted that the two images of the vertical scale stood side by side within the field of one and the same telescope.
3. Equal weights, each $\frac{1}{2}$ M_0, were put on the pans, which, as a matter of course, gave no twisting couple to the prism, since they acted in opposite directions. The reason why such superfluous weights were mounted will be seen in the later section.
4. Zero-readings were taken up to $\frac{1}{20}$ mm.; firstly for the right image which was reflected from the mirror attached near

to the fixed end of the prism; then, as fast as I could, for the left image which corresponded to the free end. The interval between these two operations was about ten seconds.

5. The mounted weights consisted of some fourty pieces of equal weights. A definite number of pieces, say $\frac{1}{2}m$, was taken off from the one of the pans and put on the other. The resulting couple is obviously mgr, if r and g represent the radius of the pulley P and the value of gravity respectively. The time-record, corresponding to this transposition of weights, was taken.
6. After a certain definite time, the readings were taken for both images, as in the case of zero-readings.
7. Again, second transposition of weights; the time recorded, scale-reading taken; and so on till a definite amount of couple was reached.
8. Then the transposition of weights in the opposite way, so that the couple diminished gradually and ultimately became oppositely directed. In this way, a series of observations was made to complete the cycle several times. The maximum amount of the couple applied during one cycle will be called the amplitude of the cycle.
9. From the difference of deviations of the two images, the angle of twist due to corresponding couple was calculated. One scale-division $=1.845\times10^{-4}$ radian $=38.''1$; couple due to one gram of suspended weight $=6.712\times10^{3}$ *c. g. s.* unit.

After all these operations were over, it will be easily seen that the result of observation was never affected by any external disturbance such as the yielding of the scale-support or minute displacement of the telescope, or flexure of the floor on which the twisting apparatus rested, etc. One instance is given in Pl. III. It is plotted from the result of observation made on a piece of clay slate, No. 12_1, a kind of sedimentary rock of palæozoic age. The curve A represents the deflection of the mirror M_2 which was attached near the fixed end of the prism, while the other curve B represents the difference of deflections of the two mirrors M_1 and M_2. As will be expected, the irregularity of the course of A is enormous; while the course of B is quite free from it.

Determination of the dissipative function expressing the loss of twisting couple due to the imperfectness of the arrangement.

The actual arrangement is inevitably imperfect. The friction of the pulleys is evidently the chief source of error; but the imperfect flexibility and the extensibility of the strings can not also be neglected. Suspending more weights, the twisting couple increases. But this increase of couple can never be strictly equal to the product of the arm and the increase of weights. One part of it must be spent to rotate the pulleys, to bend in one part and to straighten in the other, to stretch out along all the length of the string.

Let the dissipative function stated above be represented by $f\{\mu, mg\}$. Then the effective couple due to the weight mounted on the pan Q_1, whose mass is m_1, is

$$+\{m_1 g - \Sigma f(\mu, m_1 g)\}r$$

where r is the radius of the twisting pulley. Similarly the weight m_2 mounted on the pan Q_2 gives an effective couple

$$-\{m_2 g - f(\mu, m_2 g)\}r.$$

These two couples are oppositely directed; *i. e.* the former in counter-clockwise direction while the latter is clockwise. The dissipative function $f\{\mu, mg\}$ may generally, as in the case of friction, be assumed to be simply proportional to the force mg, at least in so far as that function is not the main object of the investigation. Then the two couples are

$$+m_1 g\{1-\sigma_1\}r$$
$$-m_2 g\{1-\sigma_2\}r.$$

When these two weights are hung at the same time, the resultant couple is

$$m_1g\{1\mp\sigma_1\}r-m_2g\{1\pm\sigma_2\}r=\{(m_1-m_2)\mp(\sigma_1m_1+\sigma_2m_2)\}gr,$$

where the upper or the lower of the double sign must be taken according as (m_1-m_2) is increasing or decreasing. Here it must be observed that the sign of σ depends not upon the sign of (m_1-m_2) but upon the sign of the rate of its variation, so that σ_2 in the first case and σ_1 in the last case changes its sign. The reason why this is so will be easily understood if a little consideration be taken of the nature of the opposing forces which always act to resist the motion. Thus, when (m_1-m_2) increases, the pulley P tends to make counter-clockwise rotation and σ_1 as well as σ_2 resist it. In the opposite case, where (m_1-m_2) decreases, the pulley tends to rotate in clockwise direction and again the rotation is resisted by σ_1 as well as by σ_2.

Now the two branches for the transmission of couple being as nearly similar to each other as possible, one and the same factor of proportionality σ may serve in both branches. Then the couple is

$$C=\{(m_1-m_2)\mp\sigma(m_1+m_2)\}rg,$$

where the upper or the lower of the double sign must be selected according as (m_1-m_2) is increasing or decreasing.

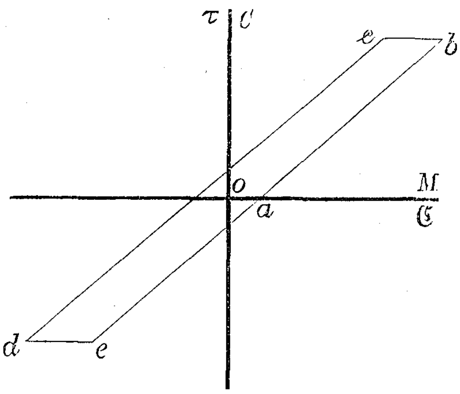
Let $m_1+m_2=M_0$ be a constant and $m_1-m_2=M$ be a variable such that M lies between $+M_0$ and $-M_0$, then for the effective couple we have

$$C=\{M-\sigma M_0\}rg.$$

Here-after M and M_0 will be called *Effective mass* and *Resisting mass* respectively. In this expression, σ has a constant positive value $+\sigma_0$ or a constant negative value $-\sigma_0$ according as M is increasing or decressing, except in the case where M increases or decreases for the first time so that the sign of σ is to be changed. In this critical case, the value of σ changes continuously from $\mp\sigma_0$ to $\pm\sigma_0$ passing through all the values lying between the two. During this transition, the amount of the effective couple remains constant while on the other hand the effective mass is varying.

Expressed in mathematical language, the variation of the twisting couple C is simply proportional to the variation of the effective mass M and independent of the friction, etc. arising from the imperfectness of the arrangement, with one exception that when the variation of M changes its sign the effective couple C remains constant until the former reaches a certain definite value (very small.)

Or graphically:—The relation between M and C may be expressed by a narrow parallelogram whose shorter sides are parallel to the axis of M and the angle contained by the adjacent sides is independent of the dissipative function such as friction.



Now it will be easily seen that the relation between the effective mass M and the amount of twist producal by the couple due to the effective mass is strictly analogous to the above one, provided Hooke's law holds true. Or in other words, if the couple $\mathfrak{C}$ is calculated from the effective mass M without any correction due to dissipation in several parts of the arrangement, the relation between the amount of twist τ and the apparent couple $\mathfrak{C}$ may be represented by the above diagram just stated, provided one of the units is properly chosen.

Here it must be observed that such a curve as above can never be said to represent the hysteresis of τ in the relation of $\mathfrak{C}$. It may be true that the energy proportional to the area is transformed into heat; but it is never within the twisted specimen where the heat is generated.

The following result of observation shows the order of magnitude of the dissipative function due to imperfectness of the arrangement. Two specimens were prefered to all the others, because their limit of elasticity seemed to be comparatively wide so that Hooke's law was approximately obeyed within the region tested. Starting from their neutral state, the specimens were twisted to and fro cyclically up to the amplitudes given below:

1. $\theta = \pm 4 \times 6.712 \times 10^5$ *c. g. s.* unit.
2. $\theta = \pm 10$,,
3. $\theta = \pm 15$,,
4. $\theta = \pm 24$,,
5. $\theta = \pm 30$,, .

The following table, which is a part of the actual record of the experiment, will fully explain the mode of observation.

TABLE I. (Pyroxenite)

Time.	Effective mass	Left reading	Deviation of M_2	Right reading	Deviation of M_1	Difference	Remarks.
h m		c	c	c	c	c	
A.M. 9 9.0	0×100	10.100		10.350			Specimen No. 32_1.
10.0	1	10.145	0.045	10.370	0.020	0.025	Date. 7 Sept. 1902.
11.0	2	10.205	0.105	10.390	0.040	0.065	Temp. 27°0 *C.*
12.0	3	10.275	0.175	10.405	0.055	0.120	$M_0 = 3300$ grams.
13.0	4	10.330	0.230	10.425	0.075	0.155	By mistake, 500 gr. for a short time.
							Effective mass of
14.0	3	10.330	0.230	10.425	0.075	0.155	one gram gives a
15.0	2	10.285	0.185	10.410	0.060	0.125	couple
16.0	1	10.215	0.115	10.395	0.015	0.070	$= 6.712 \times 10^3$ in *c.*
17.0	0	10.155	0.055	10.380	0.030	0.025	*g. s.* unit.
18.0	−1	10.095	−0.005	10.355	0.005	−0.010	
							$\overset{c}{1} = 381$ seconds of angle of twist.
19.0	−2	10.015	−0.085	10.325	−0.025	−0.060	$= 1.845 \times 10^{-3}$
20.0	−3	9.955	−0.145	10.310	−0.040	−0.105	radian.
21.0	−4	9.890	−0.210	10.290	−0.060	−0.150	
22.0	−3	9.910	−0.190	10.295	−0.055	−0.135	
23.0	−2	9.945	−0.155	10.305	−0.045	−0.110	
24.0	−1	10.000	−0.100	10.320	−0.030	−0.070	
25.0	0	10.050	−0.050	10.335	−0.015	−0.035	
26.0	1	10.105	0.005	10.365	0.015	−0.010	
27.0	2	10.180	0.080	10.385	0.035	0.045	
28.0	3	10.240	0.140	10.405	0.055	0.085	Rain.
29.0	4	10.310	0.210	10.425	0.075	0.135	
30.0	3	10.300	0.200	10.420	0.070	0.130	
31.0	2	10.230	0.130	10.410	0.060	0.100	
32.0	1	10.205	0.105	10.395	0.045	0.060	
33.0	0	10.140	0.040	10.380	0.030	0.010	

Time.	Effective mass	Left reading	Deviation of M_2	Right reading	Deviation of M_1	Difference	Remarks.
h m		c	c	c	c	c	
A.M. 9.34.0	-1×100	10.085	-0.015	10.355	0.005	-0.020	
35.0	-2	10.025	-0.075	10.335	-0.015	-0.060	
36.0	-3	9.950	-0.150	10.310	-0.040	-0.110	
37.0	-4	9.885	-0.215	10.2[illegible]0	-0.030	-0.155	
38.0	-3	9.900	-0.200	10.300	-0.050	-0.150	
39.0	-2	9.925	-0.175	10.310	-0.040	-0.135	
40.0	-1	9.985	-0.115	10.320	-0.030	-0.085	
41.0	0	10.045	-0.055	10.340	-0.010	-0.045	
42.0	1	10.115	0.015	10.370	0.020	-0.005	
43.0	2	10.175	0.075	10.385	0.035	0.040	
44.0	3	10.225	0.125	10.400	0.050	0.075	
45.0	4	10.310	0.210	10.425	0.075	0.135	
46.0	6	10.465	0.365	10.480	0.130	0.235	
47.0	8	10.610	0.510	10.520	0.170	0.340	
48.0	10	10.770	0.670	10.580	0.230	0.440	
49.0	8	10.700	0.600	10.555	0.205	0.395	
50.0	6	10.595	0.495	10.515	0.165	0.330	
51.0	4	10.465	0.365	10.480	0.130	0.235	Cloudy.
52.0	2	10.315	0.215	10.430	0.080	0.135	
53.0	0	10.180	0.080	10.395	0.045	0.035	
54.0	-2	10.040	-0.060	10.340	-0.010	-0.050	
55.0	-4	9.890	-0.210	10.300	-0.050	-0.160	
56.0	-6	9.745	-0.355	10.250	-0.100	-0.255	
57.0	-8	9.595	-0.505	10.200	-0.150	-0.355	
58.0	-10	9.430	-0.670	10.150	-0.200	-0.470	
59.0	-8	9.490	-0.610	10.170	-0.180	-0.430	$L = \overset{c}{9.15}$
10. 0.0	-6	9.595	-0.505	10.200	-0.150	-0.355	
1.0	-4	9.720	-0.380	10.235	-0.115	-0.265	
etc.	etc.	etc.	etc.	etc.	etc.	etc.	

Figures in Pls. IV and V show the general feature of the relation between the torsion and couple. It will be seen, at a glance, that

1. The curve may be devided into several branches at the positions where the variation of couple changes its sign; and then each branch except one which starts from the origin is approximately straight, provided comparatively small part immediately after the turning point is left out of consideration.
2. If a straight line is drawn, representing each branch, as nearly coinciding with it as possible, none passes through the origin.

3. Each representative of the branch for which the variation of couple is positive lies below the origin; *i. e.* it passes through the fourth quadrant.
4. Each reprsentative of the branch for which the variation of couple is negative passes through the second quadrant.
5. Of several representatives, that which represents the branch whose amplitude of twist is greater lies more distant from the origin.
6. That which lies more distant from the origin makes greater angle with the axis of couple.

Of these facts, the first expresses that Hooke's law is approximately true, provided the change of couple always takes place in one direction. The second, taken together with the first, shows that there must be certain disturbing cause especially prevailing in the neighbourhood of the starting point, and at the turning points of the course. The second, third and fourth taken together show that there is dissipation for each cycle of twisting and untwisting, whether due to the imperfect elasticity of the twisted rocks or due to the imperfectness of the twisting apparatus. Observe that the latter is a constant in the present arrangement. The fifth teaches us that the dissipation above stated increases more and more, when the twist becomes greater and greater. Whence evidently one part of the dissipation is due to the imperfect elasticity of the specimen. The sixth means that the specimen becomes less and less elastic when it is twisted further and further.

To show these facts quantitatively, I give the following numerical values:—

Assuming Hooke's law, each branch, at least in the neighbourhood of twist$=0$ or couple$=0$ *i. e.* parts lying in the second and fourth quadrants, may be represented by

$$\alpha+\beta\ \mathfrak{C}=\tau$$

where, $\mathfrak{C}$ and τ being the couple and the twist respectively, α is proportional to apparent residual surviving the twisting couple $\mathfrak{C}$, while β is inversely proportional to the modulus of rigidity of the specimen at that state.

As a matter of course, α and β are constants specifying the stage of the treatment; or in other words, they are constants having different values in different branches of the curve, *i. e.* functions of θ. For each value of $\mathfrak{C}$, the corresponding value of τ being given in Table I and the like, I have calculated the values of α and β by the method of least squares, with the following results:—

TABLE II.

$\alpha+\beta\mathfrak{C}=\tau$.	Serpentine. No. 8_2. (Chichibu).			Pyroxenite. No. 32_1. (Gumma).			Resisting mass =3300 grams.
Θ	α	$-\gamma=\frac{\alpha}{\beta}$	β	α	$-\gamma=\frac{\alpha}{\beta}$	β	
±0	∓0.0111	±0.0434	0.255	∓0.0074	±0.0390	0.248	Θ is expressed in so many hundred grams. One hundred grams give rise to couple equal to 6.71×10^5 *c. g. s.* unit.
±4	∓0.0293	±0.0691	0.424	∓0.0322	±0.0752	0.428	
±10	∓0.0529	±0.1091	0.485	∓0.0538	±0.1082	0.497	
±15	∓0.0639	±0.1254	0.508	∓0.0873	±0.1732	0.504	
24	∓0.0987	±0.1924	0.513	∓0.1229	±0.2324	0.529	
±30	∓0.1212	±0.2300	0.537				

Remarks. Here the constants are not expressed in absolute units, but α and β satisfy the equation:—Difference of the deflections of the two mirrors $=\alpha+\beta\times$ effective mass. Of the double sign, the upper or the lower must be chosen such that the required twist corresponds to increasing or decreasing state of the effective mass. In the first column, $\Theta=\pm4$ means that the specimen was twisted to and fro cyclically, the twisting couple varying between those due to the effective mass $M=\pm400$ grams.

The values of both α and β, as was noted a little before, increase with θ. It is more natural to write the relation in the form

$$\beta\{M-\gamma\}=\delta$$

where γ is positive or negative according as δ is observed by increasing or decreasing M. Then γ is equal to the effective mass which is required to bring the specimen into the state of no torsion. The corresponding values of γ are also given in the above table. Here it must be observed that γ consists of two terms, σM_0 and $f\{\theta\}$ say, where the first is due to the imperfectness of the arrangement so that it is constant in the present case, while the last is due to the energy dissipated within the twisted specimen, so that it is a function of the amplitude of the cycle. Thus

$$\gamma=\sigma M_0+f\{\theta\}.$$

The value of γ is plotted against the corresponding value of θ in Fig. 9. Pl. VI. It may be seen at a glance that each series of points lies approximately on a straight line. That is to say, the variation of γ is simply proportional to the variation of θ, so that the above equation becomes

$$\gamma=\sigma M_0+h\theta,$$

where h is a constant.

From the corresponding values of θ and γ given in Table II, I calculated the values of σ and h, having the following ones for the results:—

No. $8_{.}$.	$\sigma_1=0.0118$	;	$h_1=0.0817$.
No. 32_1.	$\sigma_2=0.0132$	;	$h_2=0.0619$.

Observe that though both σ and h are constants, the nature of constancy is not the same: *i. e.* σ has one and the same value for all experiments, while h has generally different values for different specimens of rocks. The above values of σ_1 and σ_2 coincide tolerably well with each other within the errors of observation. Taking $\sigma_0=0.0125$ for their mean value, the effective couple due to the effective mass M under the resisting mass M_0 is given by (with one exception)

$$C=\{M-\sigma M_0\}gr,$$

where $\sigma=+0.0125$ or $\sigma=-0.0125$ according as M is increasing or decreasing. In one exceptional case, which occurs when the sign of σ is to be changed, σ takes such a definite value lying between $+0.0125$ and -0.0125 that the effective couple C remains constant, although M is varying; *i. e.* C remains constant until the effective mass M is changed from its last extreme value $\mathfrak{M}$ to the value $\mathfrak{M}-0.025M_0$ or $\mathfrak{M}+0.025M_0$, according as $\mathfrak{M}$ was the greatest or the least value of M.

Elastic Yielding.

Though the specimens above cited obeyed Hooke's law sufficiently well, this is not always the case. In certain specimens, the elastic yielding is considerably great. Indeed, it is doubtful if there is any limit to the yielding of certain rocks, at least, when the twisting couple is sufficiently great. The following experiment will show the enormous elastic yielding of some rocks.

In these experiments, to begin with, equal weights were put on the two pans so that the specimen was free from couple, and the readings corresponding to that state were taken. At a known instant' a definite number of weights was transfered from the one to the other pan. Then the readings were taken from instant to instant. One part of the experimental notes is given in the following table:—

TABLE III. *(Sandstone).*

Time	Duration	Left reading	Deviation	Right reading	Deviation	Difference	Remarks
A.M. h m		c d		c d			
9.30.00	m	2.8−4.9	c	3.7+0.3			Specimen No. 3_2.
30.25	0.25	6.7−1.8	3.915		c		Date. 30. Nov. 1901.
30.90	0.90			4.5+2.0	0.808	c	Temp. 15°.5 C.
31.67	1.67	6.9−5.0	4.100			3.283	$M_0=700$ grams.
32.50	2.50			4.5+7.0	0.831		$M\ =500$ grams.
							c is so many centi-
33.09	3.09	6.9+12.0	4.182			3.350	metres read from
35.59	5.59			4.5+10.2	0.816		the image of the
36.80	6.80	7.1−8.2	4.284			3.428	scale.
37.81	7.81			4.6−7.7	0.863		d is the number of
38.70	8.70	7.1−2.5	4.312			3.447	division of micro-
							metre screw su-
39.80	9.80			4.6−6.2	0.870		ffixed to the obser-
41.00	11.00	7.1+4.4	4.345			3.472	ving telescople.
42.34	12.34			4.6−3.4	0.883		
43.60	13.60	7.2−3.8	4.405			3.523	Left: $\overset{c}{1}=\overset{d}{207}$.
44.70	14.70			4.6−3.8	0.881		
"							Right: $\overset{c}{1}=\overset{d}{215}$.
47.00	17.00	7.2+4.8	4.447			3.559	
48.00	18.00			4.6−2.4	0.887		$\overset{c}{1}=1.845\times10^{-3}$ in
49.50	19.50	7.3−6.6	4.492			3.600	radian.
50.70	20.70			4.6−1.3	0.892		$=6'\ 21''$.
52.00	22.00	7.3−3.5	4.507			3.612	
52.70	22.70			4.6−0.3	0.897		
53.70	23.70	7.3+3.2	4.539			3.642	
54.37	24.37			4.6+0.1	0.899		
56.70	26.70	7.3+9.4	4.569			3.668	
57.25	27.25			4.6+0.5	0.901		
58.80	28.80	7.4−6.1	4.594			3.691	Temp.=16°.0C.
59.95	29.95			4.6+1.0	0.905		
10. 2.95	32.95	7.4−4.0	4.604			3.697	
3.75	33.75			4.6+2.6	0.911		
5.97	35.97	7.4−3.6	4.606			3.693	
etc.	etc.	etc.	etc.	etc.	etc.	etc.	

In this experiment, the resisting mass was 700 grams while the effective mass was 500 grams. Superfluous mass of 200 grams! Would it not, as may be suspected, have any influence upon the result? Possibly, more or less influence. To examine this point, the next experiment, was made, in which the effective mass was the same, while the resisting mass was increased to 1500 grams. The result of these two experiments is graphically shown in Fig. 10, Pl. VII. The upper of the two corresponds to the first case and the lower to the last. At first sight, they seem to be almost coincident. Examining closely, however, we find one difference that the curvature of the former is decidedly greater than that of the latter in that part, which corresponds to the first few minutes. That is to say, during the first few minutes, the elastic yielding becomes greater if the resisting mass is less, or the elastic yielding, in this case, approaches its limit, if any, more quickly than in the other case. If this is generally true, the following remark will not be superfluous.

The force and its arm being expressed by F and r respectively, the amount of couple is measured by Fr. In laboratory experiment, the force F generally is due to a weight of mass M, so that we have $F=Mg$. When F is given, we may consider M and g to vary in any manner: *c. g.*

(*a*) g takes enormously great value g_1 next to infinity so that $M_1=\text{const.}/g_1$ is nearly zero.

(*b*) g takes infinitely small value g_2 so that $M_2=\text{const.}/g_2$ is next to infinity.

In these two cases, is the result the same? At the very instant when the couple began to act, they may have equal activity; but to twist the specimen actually through any finite angle θ, the mass M_1 or M_2 must be displaced through a distance equal to $D=\theta r$. Even in the case where the mass is free to move, the time required to move through that distance is

$$t_1'=\sqrt{\frac{2D}{g_1}}=\sqrt{\frac{2DM_1}{\text{const.}}} \quad \text{or} \quad t_2'=\sqrt{\frac{2D}{g_2}}=\sqrt{\frac{2DM_2}{\text{const.}}} .$$

Now in actual case, the force F has two functions: *i. e.* to overcome the elastic force and to displace the heavy mass. Thus evidently, to twist the specimen through any finite angle θ, it requires finite time given by

$$t_1>\sqrt{\frac{2DM_1}{\text{const.}}} \quad \text{or} \quad t_2>\sqrt{\frac{2DM_2}{\text{const.}}} .$$

In the first case where M_1 is infinitely small, t_1 may be infinitismal; but in the second case, since M_2 is very great, t_2 must be correspondingly great. In common torsion problem, couple is considered abstractly. It corresponds to the limiting case $M=0$; or the solution is only true for the result after infinite time. If the specimen were magnetized transversally at the free end and the twisting couple be due to a circular magnetic field, there is no retarding cause but the moment of inertia of the mass to be twisted so that it approaches to the ideal case. In actual case, where M is never zero, apparent elastic yielding due to this fact must be expected even for perfectly elastic substance. I have, however, to remark that this time of retardation, is negligibly small for rough experiment.

Secondly I examined the influence of the magnitude of the constant couple upon the elastic yielding. The specimen was No. 4, sandstone (Chōshi), a kind of sedimentary rocks of tertiary age. The effective mass was 300, 500, 700, 900, 1100, 1300 and 1500 grams in each successive experiment respectively, while the resisting mass 2000 grams remained unchanged throughout the experiments: one gram of mass gives 6.712×10^3 *c.g.s.* unit of couple. The result of the experiments is shown in Fig. 11, Pl. VIII.

As will be expected, all curves are of similar forms. In this case also, there is one remarkable difference, that of any two curves, that whose couple is smaller approaches to horizontality more quickly than the other. From this fact, it is obvious that the elastic yielding becomes greater when the couple is further increased. Any quantitative relation between the couple and the elastic yielding can never be established from a few experiments.

Here it must be noted that absolute amount of twist is never known since the zero-reading in each experiment corresponds not necessarily to the virgin state of the specimen. Indeed, the value of σ being assumed to be ± 0.0125, so-called zero-reading is nothing but the reading which corresponds, as the case may be, to certain twisting couple lying between $+1.68 \times 10^5$ and -1.68×10^5 *c.g.s.* unit. In addition to this the effect of hysteresis must be taken into account. In short, all curves in Fig. 11. are not referred to one and the same origin of coordinates.

Rate of elastic yielding, however, is independent of the so-calleding, so that it may be directly compared in different curves. The figures show that the elastic yielding is very great at first and then decreases slowly. From the values of the twist and corresponding time, I calculated the ratio

$$\frac{\triangle\delta}{\triangle t}=\frac{\delta(t_2)-\delta(t_1)}{t_2-t_1}$$

where $\delta_{(t)}$ represents the value of δ at the instant t.

As a matter of course, the result of calculation for any one experiment is too discontinuous to be traced as a single curve. But plotting the result of all the eight observations together, we may at once percieve one character common to all. Fig. 12, Pl. IX. shows how the ratio $\frac{\triangle\delta}{\triangle t}$ relates to the whole duration t. The general feature of the curve, if such a one is drawn passing through a mean position, much resembles a rectangular hyperbola. To test whether this is true or not, I took their logarithms instead of $\frac{\triangle\delta}{\triangle t}$ and t themselvs. The result is shown in Fig. 13 on the same plate. At a glance, it is evident that the most probable as well as simple curve, is a stright line. One straight line, whose equation is

$$\log\frac{\triangle\delta}{\triangle t}+\log t=\log\frac{1}{10}$$

is drawn in the figure, which seems nearly to pass through the mean position. Then the corresponding equation of the hyperbola is

$$t.\frac{\triangle\delta}{\triangle t}=\frac{1}{10}$$

which is also traced in the corresponding figure.

Assuming that the relation between $\frac{\triangle\delta}{\triangle t}$ and t is given by this equation we may write

$$d\delta=\frac{1}{10}\,\frac{dt}{t}$$

which when integrated becomes

$$\delta=\frac{1}{10}\log t+\text{const.}$$

Let the value of $\tau=\frac{\delta}{2DL}$ at the time $t_0=1$ be represented by τ_0, then we have, D and L being the scale-distance and the reduced length of the specimen respectively,

$$\eta=\tau-\tau_0=\frac{1}{20DL}\log t,$$

as the first approximation to the value of the twist due to elastic yielding, provided the amount of yielding is measured after a lapse of unit of time.

Here it must be remarked that since this empirical formula represents only a general feature common to all the curves corresponding to different twisting couples, it can not be expected that this formula is applicable to calculate the amount of twist corresponding to any particular twisting couple. In Pl. VIII, Fig. 14, the curves expressed by the equation

$$\delta-\delta_0=k\log t^{\frac{1}{10}}$$

are traced, which may be considered as the standard curves of the elastic yielding.

Again, taking the amount of couple into account, the equation of elastic yielding must be of the form

$$\delta-\delta_0=f\{C\}.\log t^{\frac{1}{10}} \qquad \text{or} \qquad \frac{\delta-\delta_0}{\log t^{\frac{1}{10}}}=f\{C\},$$

where $f\{C\}$ represents a function of the twisting couple C.

From the results of eight observations above mentioned, I calculated the value of $f\{C\}$ for a series of different values of C. Then, by a similar process as above, I intended to determine the function f. One of the probable relation, which I arrived at,

is
$$\log\frac{\triangle\delta}{\triangle C}=0.50\log C+0.61$$

which gives after integration

$$\delta-\delta_0=2.7\ C^{\frac{3}{2}}\log t^{\frac{1}{10}}$$

or
$$\tau-\tau_0=1.7\times10^{-8}.\ C^{\frac{3}{2}}\log t,$$

where τ_0 is the value of τ at the instant from which the elastic yielding is to be measured, both expressed in radian, and C is expressed in terms of the effective mass in grams.

I never insist, however, to propose this empirical formula as a general expression, since it is deduced from a few experiments made on a single specimen. Moreover, the data, especially the time-record, can never be said to be sufficiently accurate for such manifold reductions. I rather consider the elastic yielding to be simply proportional to the couple C instead of its two-third power.

The above is nothing but one example which shows how the elastic yielding proceeds with time. To obtain empirically any plausible expression for the elastic yielding, the time-variation of the twist must be more accurately measured.

Limit of elastic yielding, if any, is desirable to be evaluated. With certain specimen, it is questionable whether there is any limit to the elastic yielding or not. The following experiments will show how the elastic yielding goes on further even after a sufficiently long time. The specimen to be tested remained for a long time subjected under a constant couple, and then the time variation of its twist was observed, as is given below.

TABLE IV. (*Sandstone*).

Time	Duration	Left reading	Deviation	Right reading	Deviation	Difference	Remarks.
11th h m	m	c d	c	c d	c	c	
A.M. 9. 6	1108	18.3 − 3.9		11.9 − 5.2			Temp. = 13°.7
36	1138	+ 7.1	0.052	+ 0.0	0.025	0.027	
44	1146	+ 10.9	0.070	+ 0.5	0.027	0.043	Loaded at P.M. $2^{h}.38^{m}$
57	1159	18.4 − 5.2	0.094	+ 1.1	0.030	0.064	10th Disc. 1901.
10. 1	1163	+ 1.5	0.126	+ 3.5	0.041	0.085	Temp. = 15°.7
							M_0 = 2000 grams.
12	1174	+ 1.5	0.126	+ 2.6	0.037	0.089	M = 1500 grams.
24	1186	+ 4.5	0.140	+ 4.6	0.047	0.093	Temp. = 16°.3
34	1196	+ 6.5	0.149	+ 5.6	0.051	0.098	
51	1213	+ 8.9	0.161	+ 5.7	0.052	0.109	A little shock at $10^{h}\ 1^{m}$.
11. 19	1241	+ 13.0	0.180	+ 5.0	0.049	0.131	Specimen No. 4_1,
39	1261	+ 14.2	0.186	+ 2.6	0.037	0.149	Temp. = 15°.2
49	1271	+ 15.7	0.193	+ 1.9	0.034	0.159	
P.M. 0. 4	1286	+ 12.5	0.178	+ 0.6	0.028	0.150	= 15 .8
3. 16	1478	+ 10.1	0.167	− 10.7	− 0.026	0.193	= 16°.4
41	1503	+ 10.0	0.166	− 11.8	− 0.031	0.197	$1^{c} = 215^{d}$.
4. 8	1530	+ 4.6	0.140	− 14.9	− 0.046	0.186	Temp. = 16°.0
28	1550	+ 7.7	0.155	− 14.6	− 0.045	0.200	15°.5
43	1565	+ 9.5	0.164	− 13.4	− 0.039	0.203	15°.4
5. 5	1587	+ 7.0	0.152	− 16.5	− 0.054	0.206	14°.7

In the above table, the last column but one expresses the amount of the twist of the prism. The increase of the twist is unquestionably steady, though it is very slow. In Pl. X, Figs. 15—18, these, with the result of other experiments, are graphically shown. Even after a lapse of many thousand minutes, steady increase of twist may be clearly noticed.

Effect of Temperature.

The present arrangement being unsuited for heating or cooling the specimen, I rather intended to know only in what order the result is influenced by neglecting the correction due to temperature-variation in the laboratory. When the elastic yielding prevails, it is scarcely possible to obtain any credible result for the effect of temperature, since the temperature-variation can not be made comparably rapid.

To get rid of the effect of the elastic yielding, I put the specimen under a constant couple, $M=1500$ (*i.e.* 1.007×10^7 *c.g.s.* unit of couple) and $M_0=2000$ grams, during three days and nights so that the creeping due to the elastic yielding became almost negligibly small. Then I observed the variation of twist due to temperature-change. One of the results is given in the following table.

TABLE V. (*Sandstone*).

Time	Duration	Left reading	Deviation	Right reading	Deviation	Difference	Temperature
13th Disc. 1901.							
h m	m	c d	c	c d	c	c	
A.M. 9. 27	4009	18.7+ 6.8		11.4− 2.6			9.5°C.
40	4022	+13.2	0.030	− 0.5	0.010	0.020	10.0
51	4033	+16.5	0.046	+ 0.2	0.013	0.033	10.7
10. 9	4051	18.8+ 2.2	0.078	+ 0.1	0.013	0.065	11.0
26	4038	+ 9.2	0.111	+ 0.9	0.017	0.094	12.2
48	4090	+14.9	0.139	+ 4.3	0.033	0.106	13.8
11. 25	4127	+21.5	0.170	+ 5.3	0.038	0.132	14.0
11. 51	4153	+22.2	0.173	+ 5.3	0.038	0.135	15.0
P.M. 1. 19	4241	+ 6.5	0.099	+ 0.0	0.012	0.087	12.8
2. 27	4309	− 1.5	0.030	− 5.5	−0.014	0.074	11.9
4. 9	4411	18.7+10.3	0.017	− 8.6	−0.029	0.046	11.6
5. 13	4475	+ 9.9	0.015	− 4.7	−0.010	0.025	10.8
5. 55	4517	+11.2	0.021	− 3.7	−0.005	0.026	10.3
6. 19	4541	+12.3	0.023	+ 0.7	0.016	0.010	10.0
7. 11	4593	+17.1	0.049	+ 5.8	0.040	0.009	9.7
8. 0	4642	+20.6	0.063	+11.4	0.067	−0.001	8.7
8. 40	4682	18.8+ 7.5	0.103	+16.6	0.091	0.012	8.4
9. 22	4724	+15.5	0.141	11.5+ 0.3	0.114	0.027	8.0
9. 50	4752	+19.4	0.160	+ 1.3	0.119	0.041	7.5

This, with the result of other experiments, is plotted in Pl. XI. In each of the figures 19, 20 and 21, the lower curve expresses the relation between the twist and time, while the upper, that between the temperature and time. Similarity of each pair of the curves teaches us that the change of twist is simply proportional to the temperature-variation. To make this point more clear, I plotted the increase of twist against the corresponding rise of temperature, which is Fig. 22. The curves, as a whole, express the simple proportionality between the change of twist and temperature-variation. We find, however, one remarkable fact that the temperature-variation of the twist has a minimum value in the neighbourhood of $9^0\,C$. If this is free from errors of observation, the rigidity assumes its maximum value at that temperature. It is questionable whether this is general property of all rocks or merely special to that specimen alone. It may be also the variation of twist due to other causes than the temperature-change, *e.g.* change of humidity. To decide this experimentally is so interesting a problem, that I intend to solve the question in a next series of experiments.

Here it can not be omitted to remark that though the temperature-variation of the twist is tolerably great, as it may be seen from the figures above given, it is almost negligible compared with the total amount of twist. Indeed the increase of twist per degree of temperature-rise is less than 3×10^{-3} of its total amount.

Modulus of Rigidity in Virgin Piece.

As it was already spoken of, the modulus of rigidity is never a constant but a function of the couple itself. Moreover, it depends also upon the previous history through which the specimen was brought to that state. From what is stated in page 13 and shown quantitatively in subsequent pages, it is obvious that the modulus of rigidity may be assumed, to the first approximation, as a function of the amplitude alone, *i.e.* the amount of maximum couple in opposite direction, under which it was subjected just before. Thus, in Table II, β represents the inverse of the modulus of rigidity (not in absolute unit) which increases with θ. All the values of β there given correspond equally to non-strained state of the specimen. That they differ from each other is due to the specimens having been formerly subjected to different amounts of twisting couple.

By the modulus of rigidity in virgin state, I mean the modulus of rigidity corresponding to non-strained state in the limiting case where the amplitude, or the maximum stress to which it was subjected, was infinitely small. From the data given in Table II, we may calculate this value as follows:—

From Table II, plotting each value of α against the corresponding value of θ, I obtained Fig. 23 in Pl. VI. Within the errors of observation, the most probable relation, if any, to connect them, must be of a form

$$\alpha = a + b\theta.$$

By the method of least squares, I obtained the equations

for No. 8_2: $\alpha=0.0111+0.0349\ \theta$,

for No. 32_1: $\alpha=0.0097+0.0508\ \theta$.

Here, a, *i.e.* the limiting value of α for $\theta=0$, must be the amount of twist corresponding to the resisting couple due to the friction in the arrangement. This latter, however, is given in Table II as the value of γ corresponding to zero-amplitude. Thus, as γ and and a being the twisting couple and twist produced by it respectively, the modulus of rigidity in the neighbourhood of virgin state is given by

$$\frac{1}{\beta_0}=\frac{\gamma_0}{\alpha_0}=\frac{\sigma M_0}{a}.$$

As the result of actual calculation, for the two specimens, I obtained the values

$$\beta_0=0.255 \quad \text{and} \quad \beta_0=0.249.$$

Expressing these as well as the corresponding ones in Table II in *c.g.s.* unit, the modulus of rigidity in non-strained state is given in the following table.

TABLE VI. (*Modulus of rigidity*).

Max. couple (Arbit. unit) / Specimen.	1	30	75	115	185	231
Serpentine(Chichibu)	52.2×10^{10}	31.4×10^{10}	27.4×10^{10}	26.2×10^{10}	25.9×10^{10}	24.8×10^{10}
Pyroxenite (Gumma)	49.0×10^{10}	28.4×10^{10}	24.5×10^{10}	24.1×10^{10}	23.0×10^{10}	—

The relation between the modulus of rigidity in non-strained state and the maximum amount of couple to which the specimen was subjected in the cycle is graphically shown in Fig. 24, Pl. VI.

Modulus of rigidity is, thus, doubly indefinite. Since the modulus is also a function of the couple to which the specimen is at that instant subjected, it is evident that we shall have another series of curves corresponding to the modulus of rigidity in the strained state to several amounts. Thus, the modulus of rigidity is doubly indefinite even in the case when all external conditions such as temperature, humidity, etc., are actually given.

1. When the maximum amount of strain, to which the specimen was just previously subjected, is given, the modulus of rigidity is a function of the actual strain at that instant.

2. When the actual strain at any instant is given, the modulus of rigidity is a function of the maximum amount of strain to which the specimen was just previously subjected.

Of these two, the first expresses simply that Hooke's law is not strictly true, while the latter shows the existence of hysteresis in the relation of the twist to the twisting couple. Here it must be remarked that as any deviation from Hooke's law never necessitates the existence of hysteresis, obedience to that law never proves non-existence of hysteresis. For example, elongation and contraction may be proportional to tension and pressure respectively, but their factors of proportionality are not necessarily one and the same.

Hooke's law and Hysteresis. In addition to this when the Hooke's law is only approximately obeyed, the factor of proportionality depends entirely upon its previous history. A single example will be sufficient to show how the influence of previous history is great even when Hooke's law is approximately obeyed. For this purpose I prefered a piece of sandstone to all the others, since in it this property was strikingly visible. As it is shown in Figs. 25 and 26, Pl. XII, the result of observation was tolerably reliable within the errors of observation. When any particular branch, which may be either on- or off-curve, is considered independently of the others, there is little objection to assume Hooke's law. Moreover, the factor of proportionality for the on-curve is nearly equal to that for the corresponding off-curve in the two series of observations. The factor of proportionality, however, never remains the same for the two series, as it is indicated by the difference of inclinations of the curves. There is no reason to reject either of them. To assure the reliability of observation, I give here one part of the actual record in the following table.

TABLE VII. (*Sandstone*)

Time	Effective mass	Left reading	Deviation	Right reading	Deviation	Difference	Remarks.
h m	grams	c	c	c	c	c	
P.M. 2. 56.0	$0.\times10^2$	7.1	−4.4	9.6	−0.7	−3.7	Specimen No. 4_2.
57.7	2.	9.6	−1.9	10.0	−0.3	−1.6	Date. 13th May 1902.
59.6	4.	12.2	0.7	10.5	0.2	0.5	Temp.=14.°7 C.
3. 1.5	6.	14.8	3.3	10.9	0.6	2.7	M_0=2000 grams.
3.1	8.	17.4	5.9	11.4	1.1	4.8	Calm, fair.
							$\Theta=\pm1600$ grams.
14.2	−2.	13.7	2.2	10.7	0.4	1.8	Change of effective
15.7	−4.	11.2	−0.3	10.3	0.0	−0.3	mass always 200
17.6	−6.	8.6	−2,9	9.8	−0.5	−2.4	grams at a time.
18.9	−8.	5.9	−5.6	9.4	−0.9	−4.7	Initial zero-readings:
20.7	−10.	3.2	−8.3	8.9	−1.4	−6.9	c
33.0	0.	7.0	−4.5	9.6	−0.7	−3.8	Left=11.5
34.7	2.	9.5	−2.0	10.0	−0.3	−1.7	Right=10.3
36.2	4.	12.2	0.7	10.5	0.2	0.5	The record begins
37.9	6.	14.8	3.3	10.9	0.6	2.7	after one cycle.
39.5	8.	17.6	6.1	11.4	1.1	5.0	1 gr = 6.712 × 10^3
							c.g.s. unit of
53.7	−2.	13.8	2.3	10.7	0.4	1.9	couple.
55.1	−4.	11.1	−0.4	10.2	−0.1	−0.3	
56.6	−6.	8.3	−3.2	9.8	−0.5	−2.7	
58.1	−8.	5.6	−5.9	9.3	−1.0	−4.9	
59.8	−10.	2.8	−8.7	8.8	−1.5	−7.2	

Calculating the modulus of rigidity from this and other similar data, I obtained the following values:—

$$\mu_1=0.72\times10^{10} \quad ; \quad \mu_2=1.25\times10^{10}.$$

Nothing but the difference of previous history can be accounted for the cause of such a wide discrepancy. Thus, any conformity with Hooke's law within small portion is no assurance for non-existence of hysteresis.

Hysteresis.

The curves in Pls. IV. and V., which have been drawn to show the effect of cyclic twisting process in pyroxenite and serpentine, have the important feature in common that there is a tendency on the part of the rock to persist in any strained state which it may have acquired. This tendency is especially conspicuous when the variation of the twisting couple changes its sign. Thus, when the twisting couple has been raised to its maximum value, we find, on releasing the specimen gradually from the couple, that the rate, at which the specimen becomes untwisted with withdrawal of the twisting couple, is always, especially at the beginning of the alteration, less than that at which it was twisted with the application of the couple.

As one result of this persistency, there remains residual twist when the twisting couple is entirely withdrawn. The results go further thus, if after withdrawing the twisting couple, we begin to reapply it, we find that the specimen is no more indifferent to the direction of the couple to be applied. It is apparently more rigid in one direction than in the other. In addition to this, combined with the effect of elastic yielding, the torsion-gradient may become even negative; apparently far beyond infinite rigidity according to the definition of the term already given. This fact is graphically shown in Pl. XIII, Fig. 27. Starting from the origin O, corresponding to its neutral state, the specimen was twisted to and fro repeatedly, and it was brought to the state P in the direction QP. Here the specimen being free from couple, the twist represented by OP is residual. When the re-applied couple was of same sign with that which was previously applied, the curve thus obtained was PS, while by the application of oppositely directed couple I obtained the curve PR. These two branches starting from one and the same state are wholly different in their nature.

Though the rate at which the specimen becomes twisted by reapplying the couple of the same sign is much less than that on the first application, it improves gradually and when the couple has been completely restored, we find that the specimen was twisted nearly as much as in the first case. Consequently, when the specimen was twisted to and fro cyclically within any definite limits of couple, the curve of twist becomes a nearly closed one of simple and regular form; but its path (on-curve) during the increase of couple differs entirely from that (off-curve) during the decrease. The two branches form a loop. These facts will be better understood from the curves which I have drawn in Figs. 28–39, Pls. XIV. and XV. All curves present similar feature. Though the on-curve differs entirely from the off-curve, they are so related to each other that if one of them is turned through two right angles, it becomes coincident with the other.

Figures in Pls. XVI. and XVII. give further illustrations of the effects of torsional hysteresis in causing a loop to be formed on the curves of twist when the twisting couple experiences several cyclic changes of different amplitudes. It shows, in addition to the large loop produced by reversal of the twisting couple, many smaller ones produced within more narrow limits and again two smallest ones which were obtained by withdrawing the couple wholly and then reapplying it in the same direction as before.

Many other experiments have shown that similar loops are formed when there are two or more different cyclic changes of the twisting couple. They are, however, not necessarily similarly situated; in general, the greater the amplitude of the cycle, the more they tend to become vertical. This is directly related to the fact that the modulus of rigidity becomes less and less when the couple bocomes greater and greater.

Elastic Recovery.

As the elastic yielding under constant stress is inevitable, it is a matter of course that the residual twist after over-strain must diminish with elapse of time. How this recovery takes place will be seen from the following experiment. The specimen No. 4_1 was subjected to constant twisting couple, $M_0=2000$ and $M=1500$ in grams, (*i.e.* 1.007×10^7 *c.g.s.* unit) during a week, precisely 167 hours, and then it was suddenly released from the couple. The amount of residual was observed from instant to instant and the corresponding time was also noted. The result of observation is shown in Figs. 42 and 43, Pl. VII. This curve of elastic recovery much resembles, in its broad aspect, to that of the elastic yielding.

Effect of recovery on cycles was also studied in the next experiment. The specimen No. 3_3 was subjected to constant twisting couple$=5.37\times10^6$, *i.e.* $M_0=2000$ and $M=800$ in grams, from 6^{h} 0^{m} *P.M.* 2^{nd}, July, to 8^{h} 31^{m} *A.M.* 15^{th} of the same month, *i.e.* during 306 h 31 m. Then starting from this state, the specimen was twisted to and fro cyclically. The result is plotted in Fig. 44, Pl. XVIII. Gradual recovery will be manifestly traced along the whole cycle. As the result of recovery from over-strain, any complete cycle is not closed. To make the influence of recovery from overstrain clearer, it would be better to compare this with the curve in Pl. XVI. In these two curves, Fig. 40 and Fig. 44, the cyclic change of the twisting couple were conducted through entirely one and the same process. The only difference is, that in the first the centre of symmetry of the cyclic process was the neutral state of the specimen while in the second, it was in the over-strained state.

One more experiment was made on the same specimen, whose result is shown in Fig. 45, Pl. XIX. Here, the form of the curve, on the whole, has a centre of symmetry coinciding, as in Fig. 40, Plate XVI, with that of the cyclic process (provided a slight mistake in the cyclic process is taken into account.) Now comparing the two curves, Fig. 44 and Fig. 45, we find, at a glance, so conspicuous difference that they may be scarcely said to be of one and the same type.

In both of these experiments, the centre of symmetry of the cyclic process was in equally strained state corresponding to equal amount of twisting couple$=5.37\times10^6$, *i.e.* $M_0=2000$ and $M=800$ in grams. The only difference is that in the first case the specimen was constrained under the twisting couple during very long duration, *i.e.* more than 306 hours. No possible explanation for this noncoincidence of the two curves can be found but that in the first, the effect of elastic yielding having been very great, the specimen had more tendency than in the other to recover from the strained state when the twisting couple was withdrawn.

Lastly, the following experiment may be cited to show how the position of the centre of symmetry of the cyclic process affects the loop. Starting from the strained state, as in the last experiment, the same specimen was twisted to and fro cyclically, the centre of symmetry of each cycle being so chosen that it corresponds to a definite state as regards the amount of the twisting couple, and thus it was different from cycle to cycle. The result is graphically shown in Fig. 46, Plate XX. The greatest cycle lies between -200 and $+800$; the next greater between 0 and $+500$; the third between either -200 and $+200$, or $+200$ and $+600$, or $+600$ and $+1000$; the least between 0 and $+300$. It will clearly be seen that all of the loops produced by these cycles are of one and the same shape. Indeed, they are nearly similar, as they are so widely different in their dimensions. It is, however, also evident that when the amplitude of the cycle is given with regard to the amount of the twisting couple, the loop, whose centre of symmetry is nearer to the neutral state, is more vertical than the other. This fact shows that the specimen becomes *more and more rigid*, when it is more and more strained.

In former section, I remarked that, when the centre of symmetry of the cycle is given, the greater the amplitude of the loop, the more it tends to become vertical, which shows that the specimen becomes *more and more plastic* when it is twisted further and further. These two facts are contrary to each other; at first sight, it seems to be altogether inconsistent. The existence of torsional hysteresis, however, is sufficient to be accounted for, as it will be easily understood after a little consideration.

Multiplicity of the modulus of rigidity.

No definite twist is associated with any given couple. As it was repeatedly stated, the amount of twist due to any given twisting couple is entirely indefinite, in so far as its previous history is not wholly known. To specify the torsion, we must know, not only the twisting couple to which it is actually subjected, but also what changes it underwent in reaching that value and how long it was subjected to several twisting couples. By properly adjusting the application and removal of the twisting couple we can get the specimen twisted through any angle, which lies between comparatively wide limits, associated with a given twisting couple. In Fig. 27, Fig. 40, Fig. 41 and Figs. 44–46, we have sufficient examples to illustrate this fact. In these experiments, the couple was changed every one minute. One more experiment is added to show the effect of duration. In this experiment, the amplitude remained constant but the couple changed quite irregularly. The result is plotted in Fig. 47, Pl. XXI.

There is also no definite gradient in passing through a given state. Not only may the specimen be brought to any twisted state, within a certain limit, by a given couple, but it may have more than one torsional gradient in passing through that state. Hence, if the modulus of rigidity is measured by the limiting value of the ratio of the change of twisting couple to the change of twist, there is no definite value for it, in so far as the passage through which the specimen was brought to that state is not wholly known. Apparently inconsistent facts described in the last section are to be ascribed to the multiplicity of the gradients.

Neutral state and Non-twisted free state. One of the most interesting example of this effect of hysteresis is that even when a specimen is free from any twisting couple and finds itself in non-twisted state, it is not necessarily neutral. In virgin state, it is, of course, neutral and has single gradient for both positive and negative twisting couples. After having been twisted to and fro, the specimen may be treated in such a way that when the couple is wholly withdrawn there remains no residual twist: *i.e.* the specimen is again free from any couple and devoid of torsion. In every respect, there is no difference between this and a virgin piece. Tested with twisting couple, however, the specimen is no more neutral. Though there is no external evidence that the piece is anything but neutral, it is much more easy to be twisted in the direction opposite to that, towards which it has been already twisted, than to be twisted in the same direction. Or in other words, it retains latent traces of the twist from which it was lately released, and it is far from being the same as a virgin piece.

Again, as the effect of elastic recovery, it may also happen that seemingly a neutral but not really virgin piece, (a piece which is free from any couple and apparently without any torsion), becomes twisted with a lapse of time, although it is not subjected to any force at that moment. Thus, a piece which has been, at least, once twisted can not be really neutral as a virgin piece.

Neutralization of a twisted piece: There is, however, one method by which any twisted piece can again be brought to a practically neutral state. It consists in the following process:—Twist the piece to and fro, applying positive and negative couples alternately, in such a manner that the amplitude gradually diminishing one after another ultimately tends to become negligibly small. Then, as a matter of course, the piece attains its initial non-twisted state, as the experiment shows whose result is shown in Fig. 48, Pl. XXII. Strictly speaking, this neutral state can be attained only with an infinite number of reversals and after an infinite time. It resembles the process of demagnetization.

Lastly, to give general idea of the comparative magnitudes of the modulus of rigidity for several rocks, I add Fig. 50, Pl. XXIV. in which the results of observations made on several rocks are plotted in one and the same scale, together with that of soft iron.

The horizontal line, *i.e.* axis of couple, corresponds to a perfectly rigid substance. Soft iron is more rigid than all of rocks here examined. Next to it comes peridotite, an eruptive rock of Archæan age, followed by others gradually diminishing in their rigidity. The last of the series is mica-schist, a kind of metamorphic rock. The vertical line, *i.e.* axis of twist, corresponds, of course, to a perfect fluid. (Correction due to a little difference between the dimensions of the specimens is not taken into account.)

Numerical values, also, of the modulus of rigidity obtained in the present experiment are given in the following table. They serve, however, only to give general idea of the magnitude of the modulus of rigidity, since they are liable to great variation according to the history of the specimen.

TABLE VIII. *(List of the specimens).*

Specimen No.	Name	Locality	Kind	Mod. of Rigidity	Density	Velocity of Trans. Wave.
		ARCHÆAN.		$\times 10^{10}$		Kilom./second.
18	Chlorite-schist	Arakawa, Chichibu.	Metamorphic	20.0—24.1	2.82	2.70—2.89
8	Serpentine	Yokose, Chichibu.	Eruptive (Altered)	24.8—52.2	2.71	3.03—4.40
31	Quartz-schist	Onishi, Gumma.	Metamorphic	24.5—28.9	2.64	3.06—3.29
42	Mica-schist	Tsukioka, Ibaraki.	Metamorphic	1.77	2.64	0.82
7	Peridotite (Serpentinized)	Machiya, Kuji.	Eruptive (Altered)	22.3	2.61	2.93
		PALÆOZOIC.				
22	Pyroxenite	Hominoyama, Gumma.	Sedimentary (Metamorphized)	23.0—49.0	2.90	2.82—4.33
12	Clay-slate	Miyanomai, Aumi.	Sedimentary	2.93	2.74	1.03
21	Limestone	Arakawa, Chichibu.	Sedimentary	7.73	2.64	1.71
6	Marble	Maiyama, Kuji.	Sedimentary (Metamorphized)	8.63—9.15	2.64	1.82—1.85
13	Clay-slate	Ōyamadera, Aumi.	Sedimentary	10.82	2.58	2.05
10	Granite	Konokijima, Kagawa.	Eruptive	5.71	2.57	1.49
9	Granite	Mikage, Hyōgo.	Eruptive	16.9	2.54	2.58
14	Red Schalstein	Sugiyama, Aumi.	Sedimentary	7.79	2.43	1.79

Specimen No.	Name	Locality	Kind	Mod. of Rigidity	Density	Velocity of Trans. Wave.
		TERTIARY.		$\times 10^{10}$		$\frac{\text{Kilom.}}{\text{second.}}$
5	Rhyolite	Mitera, Yechizen.	Eruptive	2.74—3.12	2 36	1.07—1.16
4	Sandstone	Shinjō, Ki-i.	Sedimentary	0,72—1.25	2.21	0.57—0.94
3	Sandstone	Chōshi, Chiba.	Sedimentary	0.41—2.64	2.20	0.43—1.09
2	Tuff	Kawatsu, Izu.	Sedimentary	5.73—6.18	1,91	1.74—1.80
		DILUVIUM.				
17	Andesite	Haruna, Gumma.	Eruptive	8.09	2.63	1.75
1	Andesite	Nebukawa, Sagami.	Eruptive	8.02	2.59	1.76
16	Andesite (Porous)	Haruna, Gumma.	Eruptive	2.17	2.32	0.97

Remarks. In the fifth column, two values of the modulus of rigidity are given, which are extreme values obtained in the present experiment under different conditions. For instance, for Specimen No. 3, the value 0.41 corresponds to the modulus of rigidity taken in the region lying between the states given by twist$=0$ and couple$=0$ when the specimen, whose normal section was 1.17^2 in cm., was twisted cyclically by the couple which varied between $+5.37\times10^6$ and -5.37×10^6 *c.g.s.* unit: while the other value 2.64 corresponds to the case where the same specimen, being actually subjected under the couple 5.37×10^6, was further twisted cyclically by a second couple which varied between $+1.34\times10^6$ and -1.34×10^6 *c.g.s.* unit.