VI. On the Preliminary Tremors and the Velocities of Transit of the Earthquake Motion.

32. *The durations of the preliminary tremors.*—Let us first inquire about the relation that may exist between the durations of the 1st and 2nd preliminary tremors. From Tables II, III, IV, VI and VII, it will be seen that in general these two durations are not much different from one another ; the duration of the 1st preliminary tremor varying in the cases of the distant earthquakes (Group I) between 1 m 11 s and 17 m 35 s, and that of the 2nd preliminary tremor between 1 m 2 s and 16 m 40 s. Confining our attention to the earthquakes of Group I (Table II), there are 28 cases in which the durations of the two preliminary tremors were definitely measured, as tabulated below.

TABLE XII.—Durations of the 1st and 2nd Preliminary Tremors.
(Distant earthquakes, Group I.)

No. of eqke.	Duration of the 1st P. T.	Duration of the 2nd P. T.
2	7m 21s	7m 14s
22	8 43	4 41
29	8 26	10 26
33	5 37	4 18
44	6 0	3 7
58	4 15	2 0
69	9 54	9 10
127	9 17	9 38
143	17 35	17 21
147	8 24	7 43
149	3 40	2 48
163	2 48	4 59
165	4 29	2 8
177	6 7	3 35
186	8 20	16 40
188	9 33	9 49
190	1 11	1 2
193	7 36	9 38
196	7 38	6 53
197	7 43	6 30
198	2 5	1 36
201	10 19	12 0
207	5 46	3 39
217	4 23	4 12
218	4 36	4 30
220	6 20	4 23
222	3 10	3 23
235	4 51	4 21

Taking means of these 28 case. we find :—

the mean duration of the 1st P. T.$=y_1'=6$ m 39 s.

„ „ „ „ „ 2nd P. T.$=y_2'=6$ m 21 s.

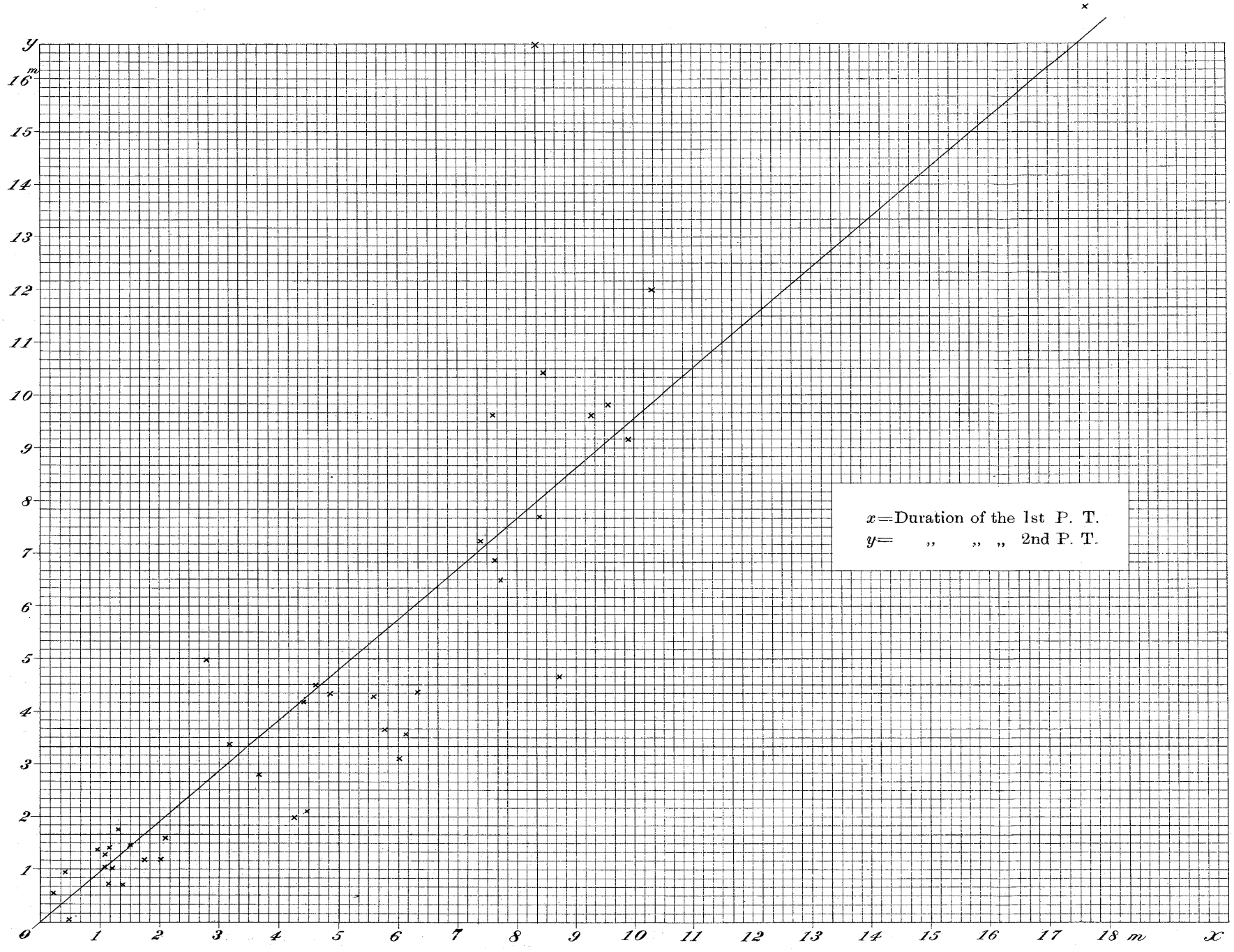
Consequently $\frac{y_1'}{y_2'}=1{,}05$ (2)

Equation (2) gives the relation of the durations of the two preliminary tremors of distant earthquakes. Turning now our attention to the earthquakes of comparatively near origin, namely those of Groups II, III, V and VI,* we find 13 cases in which the durations of the 1st and 2nd preliminary tremors were distinctly measured, as follows :—

TABLE XIII.—Durations of the 1st and 2nd Preliminary Tremors.
(Earthquakes of near origins.)

Group.	No.	Duration of the 1st P. T.	Duration of the 2nd P. T.
II.	160	1m 4s	1m 4s
	232	2 0	1 12
III.	16	0 56	1 24
	70	1 21	0 42
	90	0 28	0 4
	105	1 4	1 17
	148	1 18	1 46
	151	1 43	1 12
	167	0 12	0 33
	230	1 30	1 28
	239	1 9	1 25
V.	100	24	0 57
VI.	112	1 7	0 44

* In the earthquakes of the remaining Groups IV, VII, VIII and IX, the 1st and 2nd preliminary tremors were not generally separately distinguishable.


y
m
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
m
x
x=Duration of the 1st P. T.
y= „ „ „ 2nd P. T.

Taking means of the above 13 cases, we find :—

the mean duration of the 1st P. T.$=y_1'=1$ m 6 s. .

„ „ „ „ „ 2nd P. T.$=y_2'=1$ m 4 s.

Consequently $\frac{y_1'}{y_2'}=1{,}04$ (3)

Thus the ratio of the durations of the 1st and 2nd preliminary tremors is found to be the same for the distant earthquakes as well as for those of near origins. I shall therefore put generally

$$\frac{y_1}{y_2}=1{,}05 \text{ ; or } y_2=0{,}95\times y_1 \qquad (4)$$

in which y_1 denotes the duration of the 1st preliminary tremor and y_2 that of the 2nd preliminary tremor, of any given earthquake. Equation (4) shows that the durations of the two preliminary tremors are approximately equal to one another. (See § 34).

The accompanying figure illustrates the relation of the two preliminary tremors for the 41 earthquakes contained in the above two Tables ; the mean curve being the straight line *op* bisecting the angle between the axes of x and y.

33. *The relation between the duration of the Ist preliminary tremor and the distance of the earthquake origin.*—In Vol. XI of the Jour. Coll. Sc. Imp. Univ. Tokyo, I have discussed the relation between the duration of the *preliminary tremor* and the distance of the earthquake origin, for the cases in which the latter quantity was under 1000 km. and above 100 km. I propose now to investigate similar relations with respect to distant earthquakes. The reason why I take here the 1st preliminary tremor, instead of the 2nd preliminary tremor or the total preliminary tremor, is that the duration of the 1st preliminary tremor can generally be determined much more accurately than either of the two others.

As already pointed out by several seismologists, the durations of the *preliminary tremor* of an earthquake increases with the distance of the origin. I give in the next table the durations of the 1st preliminary tremor ($=y_1$) and the distance between the earthquake origin and the place of observation measured along a great circle of the earth ($=x$), for the Tokyo observations of the great earthquakes of Alasca, Smyrna and of Java, which all took place in Sept., 1899, as well as for some large Japanese earthquakes observed in Europe.

TABLE XIV.—RELATION OF THE DURATION OF THE 1ST PRELIMINARY TREMOR AND THE DISTANCE OF THE EARTHQUAKE ORIGIN.

No.	Earthquake.	Date.	Place of Observation.	y_1 = duration of the 1st P. T.	x = distance (along the great circle) between the origin and place of observation.
a	Alascan eqkes.	Sept. 4th and 11th, 1899.	Tokyo.	7 m 39 s	6100 km.
b	Smyrna eqke.	Sept. 20th, 1899.	„	5 46	4800
c	Java „	„ 30th, „	„	10 19	9200
d	Japan „	Feb. 20th, 1897.	Potsdam.	9 55	8990
e	Japan eqkes.	———	Italy.	10 36	9580

In the above table, *a* denotes the three Alascan earthquakes of Sept. 4th and 11th, 1899 (Nos. 193, 196 and 197, of Group I); the y_1 is the mean duration of their 1st preliminary tremors and the x the distance deduced by assuming their origins to be situated all off Cape St. Elias, at about lat. 60° N and long. 140° W. *b* is the Smyrna earthquake (No. 201, Group I) and *c* the Java earthquake (No. 207, Group I); their distances y_1 have been calculated by assuming their origins to be situated respectively near Aidin, in Asia Minor, lat. 37° 50′ N and long. 29° E, and off the southern coast of the Island of Cerang at about lat. 6° S and long. 129° E. *d* is the strong Japanese

earthquake of Feb. 20th, 1897, (origin lat. 38° 30′ N, long. 143° 30′ E), which caused some damage in the vicinity of the city of Sendai in the north-eastern part of Main Island (Honshiu) ; this earthquake was observed in Potsdam by the present writer by means of a new Paschwitz horizontal pendulum apparatus, with a quick rate of the photographic paper of about 300 mm. per hour. e denotes the nine destructive or strong Japanese earthquakes given in § 37, observed in Italy ; y is the mean duration of the 1st preliminary tremors and x is the mean of all the distances between the earthquake origins and the places of observations, namely, Padova, Ischia, Rocca di papa, Rome, Verona, Pavia, Catania and Siena.

A glance at the table given above shows that y is nearly proportional to x. Let us therefore assume the following linear equation

$$ky+h=x, \qquad (5)$$

where k and h are constants. Determining the values of these two quantities from the data given in the same table by means of the method of Least Squares, we obtain

$$17{,}1y_1-1360=x, \qquad (6)$$

where y_1 is expressed in seconds and x in km. This equation, which is to be used strictly only for the cases of y_1 lying between about 5 m and 11 m, is fairly satisfactory, as will be seen from the 3rd and 4th columns of the following Table.

TABLE XV.—Estimation of the Distance of the Earthquake Origin from the Duration of the 1st Preliminary Tremor.

No.	y_1	x (actual.)	x, calculated by equation (6)	x, calculated by equation (8)
a	7^m 39^s	6100 km.	6500 km.	6750 km.
b	5 46	4800	4560	5100
c	10 19	9200	9240	9090
d	9 55	8990	8840	8750
e	10 36	9580	9540	9360

As an illustration, let us apply equation (6) to the Sumatra earthquake of January 6th, 1900, whose origin was, according to Prof. E. Rudolph, at about lat. 3° 10′ S. and long. 102° 44′ E. The duration of the 1st preliminary tremor in Tokyo was 7m 24s, which gives, according to equation (6), 6240 km. for the distance between the earthquake origin and Tokyo, being not much different from the actual distance of 5,800 km.

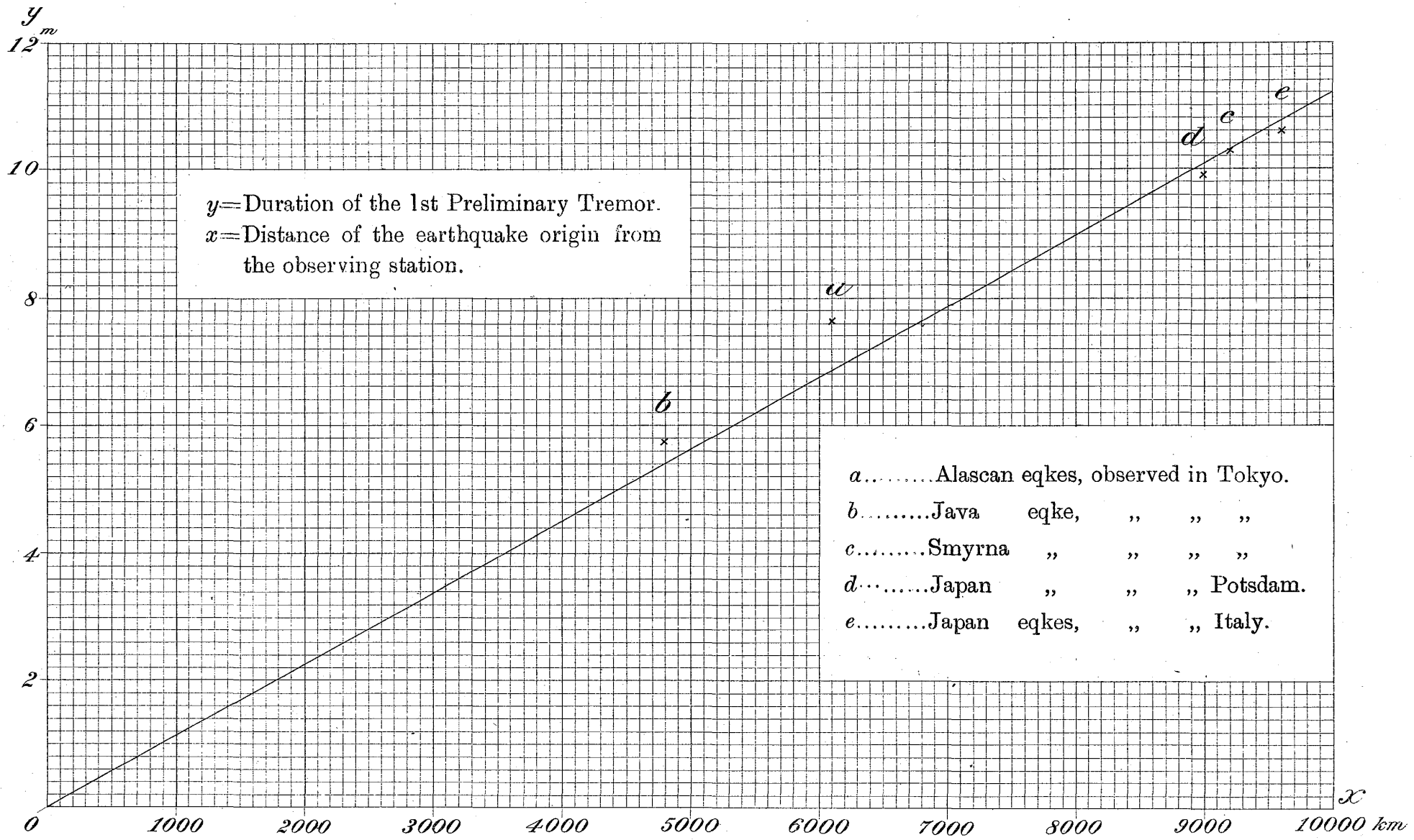
Instead of equation (5), we may, without much error, assume the following form of the relation between x and y_1 :

$$ky_1 = x \quad \ldots\ldots\ldots\ldots (7)$$

where k is a constant. From the data in Table XIV, k is found to be 14,7, or

$$14{,}7\, y_1 = x, \quad \ldots\ldots\ldots\ldots (8)$$

y_1 and x being expressed in the same units as in equation (6). Equation (8) gives also fairly good results, as shown in the last column of Table XV and has, besides being simpler, the advantage of being applicable even to earthquakes of near origin. As illustrations, I shall choose the Kiushiu earthquakes, Group VI. (See Table VII). Among the six earthquakes of this group, eqke No. 112 showed distinctly the 1st preliminary tremor, whose duration was 1m 7s. According to equation (8), this gives 980 km. for the distance between Tokyo and the earthquake origin, being not much different from the actual distance of 830 km.—With respect to the Kiushiu earthquakes, we may proceed more exactly as follows. The duration of the (total) preliminary tremor was definitely measured in the cases of the four earthquakes Nos. 14,66,112 and 236, the mean value being 1m 51s. Now according to equation (4) the durations of the 1st and 2nd preliminary tremors ought to be very nearly identical to one another. Hence we may infer from the above that the mean duration of the 1st


y
m
12
10
8
6
4
2
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000 km
x
y=Duration of the 1st Preliminary Tremor.
x=Distance of the earthquake origin from the observing station.
a
b
c
d
e
a.....Alascan eqkes, observed in Tokyo.
b.....Java eqke, ,, ,, ,,
c.....Smyrna ,, ,, ,, ,,
d.....Japan ,, ,, ,, Potsdam.
e.....Japan eqkes, ,, ,, Italy.

preliminary tremor for the Kiushiu earthquakes to be $\frac{1}{2}\times$ (1m 51s)= 56s, which gives according to equation (8) 820 km for the mean of the distances between the earthquake origins and Tokyo. Actually the distances in question for the four earthquakes were respectively 900,830,820 and 840 km; their mean, namely, 850 km, being very near to the value given by equation (8).

The relation between x and y_1, as contained in Table XIV, is graphically illustrated in the accompanying figure, the line *op* being drawn according to equation (8).

34. *Relation of the duration of the 1st preliminary tremor of distant earthquakes to that of the preliminary tremor of earthquakes of near origin.*— For earthquakes of near origin the relation between the duration of the *preliminary tremor* (=y) and the distance between the earthquake origin and the place of observation (=x), referred to at the commencement of § 33, is expressed by the following equation:—

$$7{,}51\,y=x-24{,}9 \qquad \ldots\ldots(9)$$

y being expressed in seconds and x in km.

Equation (9) was deduced from the observations in Tokyo of several Japanese earthquakes, whose x varied between 70 and 900 km. and ought consequently to be used only within the corresponding limits of y. It will, however, be seen that equation (9) is in reality approximately identical with equation (8). For, when the distance x is great the constant term in (9) may be neglected and we have

$$7{,}51\,y=x \qquad \ldots\ldots(9')$$

Here y denotes the duration of the entire *preliminary tremor*, which comprises the 1st and 2nd preliminary tremors. On the other hand, equation (8) may be written, by virtue of equation (4), approximately as follows:—

$$14{,}7 \times \frac{y}{2} = x,$$

or
$$7{,}35\ y = x, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8')$$

y being the duration of the total preliminary tremor. Equation (8′) is practically identical with equation (9′). Hence I conclude that the relation between the preliminary tremor and the distance of the earthquake origin is essentially the same both for earthquakes of distant origin and for those of near origin. Taking the mean of the coefficients of y in equations (8′) and (9′) and introducing again the constant term of 24,9, we obtain

$$\left.\begin{array}{l} 7{,}43\ y = x - 24{,}9 \\ 14{,}9\ y_1 = x - 24{,}9 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

or

The applicability of these equations is limited, till further investigations are made, by the following condition

$$100 \text{ km} < x < 10000 \text{ km},$$

or
$$10 \text{ s} < y < \text{about } 11 \text{ m}.$$

34. *The velocities of transit of earthquake motion proceeding from distant origin.*—In the description of the seismograms, the complete earthquake motion was divided into four main divisions of the 1st preliminary tremor, the 2nd preliminary tremor, the principal portion and the end portion; the principal portion being further divided, in most cases, into the three successive stages of the *initial phase*, the *slow-period phase* and the *quick-period phase*. Hence, setting aside the end portion, we may distinguish in distant earthquakes the following four essentially different epochs of motion :—

the 1st preliminary tremor;
the 2nd ,, ,,

the *initial phase* (sometimes continuous with the slow-period phase) of the principal portion ;
and the *quick-period phase* of the principal portion.

Let v_1 v_2 v_3 and v_5 denote the velocities of transit obtained by taking the moments of arrival of the beginning of the above four successive parts, and let us investigate what are the values of, and what may be the mutual relation between, these four velocities.

Denoting by y_1, y_2 and $y_{3,4}$ respectively the durations of the 1st preliminary tremor, the 2nd preliminary tremor, and of the joint initial and slow-period phases of the principal portion, we have, according to §§ 16 and 33,

$$y_2 = y_1 \times 0{,}95 \text{ ; and } y_{3,4} = y_1 \times 1{,}15 \text{ ;}$$

or $$y_1 = y_2 \times 1{,}05 = y_{3,4} \times 0{,}87 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Thus we see that the three durations are nearly equal to one another, and we may write

$$y_1 \doteqdot y_2 \doteqdot y_{3,4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

In the following discussion, however, I shall use equation (11).

If now t denotes the time interval between the occurrence of an earthquake and the arrival of the 1st preliminary tremor at a given station, and x the distance of the latter from the earthquake origin, we have the following relations :—

$$\begin{cases} v_1 = \dfrac{x}{t}, \\ v_2 = \dfrac{x}{t + y_1}, \\ v_3 = \dfrac{x}{t + y_1 + y_2}, \\ v_5 = \dfrac{x}{t + y_1 + y_2 + y_{3,4}} \end{cases} \qquad (13)$$

From these we obtain, in virtue of equation (10), where the constant term is rejected :—

$$\begin{cases} \dfrac{1}{v_1}=\dfrac{t}{x}, \\ \dfrac{1}{v_2}=\dfrac{t}{x}+\dfrac{y_1}{x}=\dfrac{1}{v_1}+\dfrac{1}{14,9}, \\ \dfrac{1}{v_3}=\dfrac{t}{x}+\dfrac{1}{x}(y_1+0,95\ y_1)=\dfrac{1}{v_1}+\dfrac{1,95}{14,9}, \\ \dfrac{1}{v_5}=\dfrac{t}{x}+\dfrac{1}{x}(y_1+0,95\ y_1+1,15\ y_1)=\dfrac{1}{v_1}+\dfrac{3,10}{14,9} \end{cases} \qquad (14)$$

According to § 46 later on, the recent large Japanese earthquakes, observed in Italy and Germany, give 12,8 km. for the velocity v_1. Using this value, equations (14) give

$$\begin{cases} v_1=12,8 \text{ km. per sec.} \\ v_2=\ 6,9\ \text{,,}\quad \text{,,}\quad =v_1\times\dfrac{1}{1,9} \\ v_3=\ 4,8\ \text{,,}\quad \text{,,}\quad =v_1\times\dfrac{1}{2,7} \\ v_5=\ 3,5\ \text{,,}\quad \text{,,}\quad =v_1\times\dfrac{1}{3,7} \end{cases}$$

Thus roughly speaking v_2, v_3 and v_5 are respectively $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of v_1,

or $v_1 \doteqdot 2v_2 \doteqdot 3v_3 \doteqdot 4v_5$ .....(15)

In the cases of the Japanese earthquakes observed in Italy and Germany, the mean values of v_2 and v_5 are respectively 7,1 and 3,3 km. per sec.

It is here to be remarked that the value of v_5 is very near to the velocity of transit of 3,3 km. per sec. found from an extended seismic triangulation in Tokyo, started in 1894 by the late Prof. S. Sekiya and myself, and since 1895, very ably continued by Mr. A. Imamura; the velocity here quoted being the mean of observations of eight earthquakes.*

* The details of these experiments are given in Vols. XXI and XXXII of the (Japanese) Reports of the Earthquake Investigation Committee.

Equations (12) and (15) may possibly have certain physical meaning.

35. *The wave-length of the earthquake motion proceeding from distant origins.*—As stated in §§ 13—19, the principal vibrations in the 1st preliminary tremor have an average period of 8,03 s, while those in the initial phase of the principal portion have an average period of 31,3 s. The corresponding wave-lengths, λ_1 and λ_3, would, according to the results obtained in the preceding paragraph, be as follows :—

$$\lambda_1 = 8{,}03^{s} \times 12{,}9^{km} = 104 \text{ km},$$
$$\lambda_3 = 31{,}3 \times 4{,}8 = 150 \text{ „}$$

For the wave-length λ_5 of the quick-period phase of the principal portion, we have

$$\lambda_5 = 10{,}7^{s} \times 3{,}5^{km} = 38 \text{ km}.$$

The wave-lengths of the vibrations in the 2nd preliminary tremor may be similarly estimated.

The knowledge of the wave-lengths is important in considering the relation between the earthquake motion and the formation of the ground, as well as in the discussion of the nature of the slow-period earthquake undulations.

36. *On the paths of the earthquake waves through the earth's crust.* —As v_5 is very nearly equal to the velocity of propagation of the earthquake waves found by the seismic triangulation, it is probable that the waves in the quick-period phase of the principal portion are transmitted along the surface of the earth's crust. As regards other sets of the earthquake waves, those having the transit velocities of v_2 and v_3 may be propagated at some small depth within the earth's crust.

The transit velocity of the vibrations of the 1st preliminary tremor, namely v_1, is very great and no known rock has an elastic

modulus* sufficiently large to propagate with such a high velocity, whether the vibrations be longitudinal or transverse. Hence we must conclude that the waves of the 1st preliminary tremor are transmitted along some path within the earth's crust. As, however, the duration of the 1st preliminary tremor at a given station is very nearly proportional to the superficial distance between the latter and the earthquake origin, and, further, as this relation is the same for earthquakes both of distant and of near origin, it seems likely that the waves of the 1st preliminary tremor are transmitted nearly parallel to the surface of the earth and at a certain (probably constant) depth below it; the law of the 1st preliminary tremor, or generally the preliminary tremor, being explained on the supposition that the waves of the 1st and 2nd preliminary tremors and of the principal portion are all generated simultaneously at the earthquake origin, but are gradually separated from one another as the disturbance spreads from the latter on account of the difference of the transit velocities. The constant depth here assumed would be a small fraction of the radius of the earth.

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* See Prof. H. Nagaoka's paper:—*The elastic constants of rocks and velocity of earthquake waves.* No. IV of the *Publications.*