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On the $\mathcal{D}$ modules described by the inverse image of the smooth map

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0. Introduction.

Let X and Y be two complex manifolds and $f: X \to Y$ a smooth holomorphic map (i.e. $df: TX \to TY$ is surjective). Then we define a canonical injection $\rho: X \times_Y T^*Y \hookrightarrow T^*X$ and a canonical projection $\varpi: X \times_Y T^*Y \to T^*Y$. Denote by $\mathcal{D}_X^\infty$ the sheaf of differential operators of infinite order on X.

Suppose $\mathcal{M}$ is an admissible $\mathcal{D}_X^\infty$ module whose characteristic variety is contained in $\rho\varpi^{-1}(T^*Y)$. Then employing differential operators of infinite order, we can show $\mathcal{M}$ is isomorphic to inverse image of some admissible $\mathcal{D}_Y^\infty$ module (cf. [S-K-K]).

In this article, we give a similar result using only differential operators of finite order under suitable conditions.

1. Preliminary.

Let X be a complex manifold and $\pi: T^*X \to X$ the cotangent bundle of X. We denote by $\mathcal{D}_X$ the sheaf of differential operators of finite order with holomorphic coefficients. The ring $\mathcal{D}_X$ is endowed with the filtration by order, and the associated graded ring $\mathrm{Gr}(\mathcal{D}_X)$ is identified with a subsheaf of $\pi_*(\mathcal{O}_{T^*X})$.

Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$ module. Then there exists locally a good filtration of $\mathcal{M}$, and the characteristic variety of $\mathcal{M}$ is defined by

$$\mathrm{Char}(\mathcal{M}) := \mathrm{Supp}\Big(\mathcal{O}_{T^*X} \underset{\pi^{-1}\mathrm{Gr}(\mathcal{D}_X)}{\otimes} \pi^{-1}\mathrm{Gr}(\mathcal{M})\Big).$$

In the definition of inverse image of $\mathcal{D}_X$ modules with respect to a holomorphic map $f: X \to Y$, the sheaf $\mathcal{D}_{X\to Y} := \mathcal{O}_X \underset{f^{-1}\mathcal{O}_Y}{\otimes} f^{-1}\mathcal{D}_Y$ plays an important role. Remark that $\mathcal{D}_{X\to Y}$ is a sheaf of $(\mathcal{D}_X, f^{-1}\mathcal{D}_Y)$ bi-module (for details, refer to [S], [S-K-K]).

2. The variety $\mathrm{Char}^1_f(\mathcal{M})$ (cf. [S]).

Let $f: X \to Y$ be a smooth morphism of complex manifolds. Now we recall the definition of the relative differential operators and the relative 1-micro-characteristic variety on the relative cotangent bundle.

The relative cotangent bundle $\pi: T^*(X/Y) \to X$ is defined by the exact sequence of fiber spaces:

$$0 \longrightarrow X \underset{Y}{\times} T^*Y \longrightarrow T^*X \longrightarrow T^*(X/Y) \longrightarrow 0,$$

and the ring $\mathcal{D}_{X/Y}$ of relative differential operators is defined by

$$\mathcal{D}_{X/Y} = \{P \in \mathcal{D}_X : [P, f^{-1}\mathcal{O}_Y] = 0\}.$$

Then $\mathcal{D}_{X/Y}$ is a coherent subring of $\mathcal{D}_X$ and endowed with the filtration induced from $\mathcal{D}_X$ (i.e. $F^k(\mathcal{D}_{X/Y}) = \mathcal{D}_{X/Y} \cap F^k(\mathcal{D}_X)$). The associated graded ring $\mathrm{Gr}(\mathcal{D}_{X/Y})$ is identified with the subsheaf of $\pi_* \mathcal{O}_{T^*(X/Y)}$.

Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$ module and let $\mathcal{M}_0$ be a coherent $\mathcal{D}_{X/Y}$ module which generates $\mathcal{M}$ over $\mathcal{D}_X$. (Such a $\mathcal{M}_0$ locally exists and has a good filtration.)

DEFINITION 2.1. The *relative* 1-*micro-characteristic variety* of $\mathcal{M}$ over Y is defined by

$$\mathrm{Char}^1_f(\mathcal{M}) := \mathrm{Supp}\Big(\mathcal{O}_{T^*(X/Y)} \underset{\pi^{-1}\mathrm{Gr}(\mathcal{D}_{X/Y})}{\otimes} \pi^{-1}\mathrm{Gr}(\mathcal{M}_0)\Big).$$

This definition does not depend on the choice of $\mathcal{M}_0$ and not on the good filtration of $\mathcal{M}_0$. Remark that $\mathrm{Char}^1_f(\mathcal{M})$ is a closed analytic subset of $T^*(X/Y)$.

If $\mathcal{M}$ is generated in particular by one generator (i.e. $\mathcal{M} \simeq \mathcal{D}_X/\mathcal{J}$, $\mathcal{J}$ is a left ideal of the ring $\mathcal{D}_X$), we have

$$\mathrm{Char}^1_f(\mathcal{M}) = \{\theta \in T^*(X/Y) : \delta(P)(\theta) = 0,\ \forall P \in \mathcal{J} \cap \mathcal{D}_{X/Y}\}. \tag{2.1}$$

Here we denote by δ the symbol map from $\mathcal{D}_X$ to $\mathrm{Gr}(\mathcal{D}_X)$.

We need the following lemma in the proof of the main theorem. Let $f: X \to Y$ and $g: Y \to Z$ be smooth morphisms of complex manifolds. Then we define three canonical morphisms:

$$T^*(X/Z) \overset{i}{\hookleftarrow} X \underset{Y}{\times} T^*(Y/Z) \overset{\varpi}{\longrightarrow} T^*(Y/Z).$$

$$T^*(X/Z) \overset{p_r}{\longrightarrow} T^*(X/Y).$$

In the above situation, we have

LEMMA 2.2. (1) *Let $\mathcal{N}$ be a coherent $\mathcal{D}_Y$ module. Then*

$$\mathrm{Char}^1_{\{g\circ f\}}\Big(\mathcal{D}_{X\to Y}\underset{f^{-1}\mathcal{D}_Y}{\otimes} f^{-1}\mathcal{N}\Big)=i\circ\varpi^{-1}\,\mathrm{Char}^1_g(\mathcal{N}).$$

(2) *Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$ module. Then we have the inclusion*

$$\mathrm{Char}^1_f(\mathcal{M})\supset p_r\,\mathrm{Char}^1_{\{g\circ f\}}(\mathcal{M}).$$

Conversely if $\mathrm{Char}^1_{\{g\circ f\}}(\mathcal{M})\subset X$, *then we have the inclusion* $\mathrm{Char}^1_f(\mathcal{M})\subset X$ (*here we identify X with the zero-section of $T^*(X/Z)$ or $T^*(X/Y)$*).

PROOF. First we reduce the problem to the case that $\mathcal{M}$ or $\mathcal{N}$ is generated by one generator. By (2.1) we can easily prove (1) and the first assertion of (2). For the second assertion of (2), since $\mathrm{Char}^1_{\{g\circ f\}}(\mathcal{M})\subset X$, there exists a $\mathcal{D}_{X/Z}$ module $\mathcal{M}_0$ which is a coherent $\mathcal{O}_X$ module and generates $\mathcal{M}$ over $\mathcal{D}_X$. Then $\mathcal{M}_0$ is in particular a $\mathcal{D}_{X/Y}$ module. This implies $\mathrm{Char}^1_f(\mathcal{M})\subset X$. ■

Next we recall the pure dimensionality of $\mathcal{D}_X$ modules.

DEFINITION 2.3 (cf. [S-K-K]). A coherent $\mathcal{D}_X$ module $\mathcal{M}$ is *purely i-dimensional* if and only if

$$\mathcal{E}xt^j_{\mathcal{D}_X}(\mathcal{M},\mathcal{D}_X)=0\quad(j\neq i).$$

3. Statement of the main theorem.

Let $f:X\to Y$ be a smooth morphism of complex manifolds. We define the following two categories.

$\underline{D}(X):=\{$Category of coherent $\mathcal{D}_X$ modules on $X\}$.
$\underline{D}(X/Y):=\{\mathcal{M}\in\underline{D}(X):\ \mathrm{Char}^1_f(\mathcal{M})\subset X$ (zero-section of $T^*(X/Y)$)$\}$.

These categories are abelian. Important is the fact that $\mathcal{M}\in\underline{D}(X/Y)$ implies $\mathrm{Char}(\mathcal{M})\subset\rho\varpi^{-1}(T^*Y)$ (cf. [S]). Moreover we define two functors $\underline{f}_*$ and $\underline{f}^*$.

$$(3.1)\qquad \begin{aligned}\underline{f}_*:\underline{D}(X/Y)&\longrightarrow\underline{D}(Y)\\ \mathcal{M}&\longmapsto f_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y},\mathcal{M}).\end{aligned}$$

$$
\begin{aligned}
\underline{f}^{*}: \underline{D}(Y) &\longrightarrow \underline{D}(X/Y) \\
\mathcal{N} &\longmapsto \mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} f^{-1}\mathcal{N}.
\end{aligned} \tag{3.2}
$$

Now we show under a suitable condition the above two categories are equivalent.

THEOREM 3.1. *Let $f: X\to Y$ be a smooth surjective holomorphic map of complex manifolds. Assume each fiber of f is connected and simply connected. Then the category $\underline{D}(Y)$ and the category $\underline{D}(X/Y)$ are equivalent and the equivalence is given by the functor* (3.1) *and* (3.2).

PROOF. The proof is similar to that of [S-K-K; Th. 5.3.1]. We need several lemma.

LEMMA 3.2. *Let $f: X\to Y$ be a smooth surjective map with connected fibers.*

(1) *If $\mathcal{F}$ is a sheaf on Y, then we have the isomorphism:*

$$\mathcal{F}\simeq f_{*}f^{-1}\mathcal{F}.$$

(2) *Let $\mathcal{G}$ be a sheaf on X which is locally constant on each fiber of f. Moreover we assume each fiber of f is simply connected, then:*

$$\mathcal{G}\simeq f^{-1}f_{*}\mathcal{G}.$$

PROOF. (1) is a special case of [K-K-K; Prop. 2.1.3]. (2) It is sufficient to prove

$$\Gamma(f^{-1}f(U), \mathcal{G}) \longrightarrow \Gamma(U, \mathcal{G}) \tag{3.3}$$

is isomorphic for any open set U in X with each fiber of $f|_U$: $U\to f(U)$ connected and simply connected. Since $\mathcal{G}$ is locally constant on the fibers of f and the fibers are connected, (3.3) is injective.

To show the surjectivity of (3.3), we first construct the morphism of stalks which are on the same fiber of f. Let $y\in Y$ and $x_0, x_1\in f^{-1}(y)\subset X$. Fix the path $\gamma\subset f^{-1}(y)$ from x_0 to x_1. Then we define the morphism

$$I_\gamma: \mathcal{G}_{x_0} \longrightarrow \mathcal{G}_{x_1},$$

by the continuation along the path γ (this is possible because $\mathcal{G}$ is locally constant on the γ). Since the fiber of f is simply connected, I_γ depends only on x_0 and x_1, and is independent of the choice of the path $\gamma\subset f^{-1}(y)$. Thus we denote I_{x_1,x_0} instead of I_γ. If $x_0, x_1, x_2\in f^{-1}(y)$, we get:

(3.4) $$I_{x_2,x_1}\circ I_{x_1,x_0}=I_{x_2,x_0}.$$

Let $s\in\Gamma(U,\mathcal{G})$. Then we associate the map $\tilde{s}$ from $f^{-1}f(U)$ to the sheaf space $\mathcal{G}$ as follows:

$$\begin{aligned}\tilde{s}:f^{-1}f(U) &\longrightarrow f^{-1}f(U)\underset{X}{\times}\mathcal{G}\\ x &\longmapsto (x,\ I_{x,\tilde{x}}s_{\tilde{x}}).\end{aligned}$$

Here $\tilde{x}$ is an arbitrary point in $f^{-1}f(x)\cap U$. Then $\tilde{s}$ is well defined by (3.4). Now it is easy to see $\tilde{s}$ is continuous (i.e. $\tilde{s}\in\Gamma(f^{-1}f(U),\mathcal{G})$) and $\tilde{s}|_U=s$. ■

LEMMA 3.3 ([S]). *Let $\mathcal{M}\in\underline{D}(X/Y)$. Then $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y},\mathcal{M})$ is locally constant on the fiber of f.*

PROOF. By Theorem 4.3.4 [S],

$$\mathrm{SS}(\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y},\mathcal{M})|_{f^{-1}(y)})\subset\mathrm{Char}^1_f(\mathcal{M})\underset{X}{\times}f^{-1}(y)\quad(\text{for }\forall y\in Y).$$

Here we denote by $\mathrm{SS}(\mathcal{F})$ the micro-support of a sheaf $\mathcal{F}$ (Refer to [K-S] for the definition.). Hence $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y},\mathcal{M})$ is locally constant on the fiber of f (Prop. 4.1.2 [K-S]). ■

LEMMA 3.4. *Assume the same condition in the theorem. Let $\mathcal{N}\in\underline{D}(Y)$. Then we have the isomorphism:*

$$\underline{f}_*\underline{f}^*\mathcal{N}\simeq\mathcal{N}.$$

PROOF. First we remark

$$\begin{aligned}\underline{f}_*\underline{f}^*\mathcal{N}&\simeq f_*\mathcal{H}om_{\mathcal{D}_X}\Big(\mathcal{D}_{X\to Y},\ \mathcal{D}_{X\to Y}\underset{f^{-1}\mathcal{D}_Y}{\otimes}f^{-1}\mathcal{N}\Big)\\ &\simeq f_*f^{-1}\mathcal{N}\simeq\mathcal{N}.\end{aligned}$$

Here we have used Lemma 3.2 (1) and the isomorphism $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y},\mathcal{D}_{X\to Y})\simeq f^{-1}\mathcal{D}_Y$ as a right $f^{-1}\mathcal{D}_Y$-module. ■

By Lemma 3.4, it remains to prove for $\mathcal{M}\in\underline{D}(X/Y)$

(i) $\underline{f}_*\mathcal{M}$ is a coherent $\mathcal{D}_Y$ module,

(ii) the isomorphism $\mathcal{M}\simeq\underline{f}^*\underline{f}_*\mathcal{M}$.

First we study the special case: $X=\boldsymbol{C}^{n+l}=(x_1,\cdots,x_n,t_1,\cdots,t_l)$, $Y=\boldsymbol{C}^n=(x_1,\cdots,x_n)$, $f(x,t)=x$, and $x_0\in X$. Then by Lemma 2.2, we may assume $l=1$ by induction on the fiber dimension of f.

Since $\mathrm{Char}^1_f(\mathcal{M})$ is contained in the zero-section of $T^*(X/Y)$, we can find locally a coherent $\mathcal{O}_X$ module which generates $\mathcal{M}$ over $\mathcal{D}_X$ and is stable under D_t actions. Then we find locally an exact sequence

$$\phi : (\mathcal{D}_{X\to Y})^{m_1} \longrightarrow \mathcal{M} \longrightarrow 0$$

and set $\mathcal{L}=\mathrm{Ker}\,\phi$. Then we have:

$$0 \longrightarrow \mathcal{L} \longrightarrow (\mathcal{D}_{X\to Y})^{m_1} \longrightarrow \mathcal{M} \longrightarrow 0. \tag{3.5}$$

Since $\mathcal{L} \in \underline{D}(X/Y)$, we repeat the same procedure to $\mathcal{L}$ and obtain in a neighborhood U of x_0 the resolution of $\mathcal{M}$

$$(\mathcal{D}_{X\to Y})^{m_2} \longrightarrow (\mathcal{D}_{X\to Y})^{m_1} \longrightarrow \mathcal{M} \longrightarrow 0. \tag{3.6}$$

Since the fiber dimension is equal to 1, $\mathcal{E}xt^2_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{L})=0$. Apply the functor $R\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \bullet)$ to the sequence (3.5) and consider the long exact sequence

$$\longrightarrow \mathcal{E}xt^1_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{D}_{X\to Y}) \longrightarrow \mathcal{E}xt^1_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M}) \longrightarrow \mathcal{E}xt^2_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{L}) \longrightarrow .$$

Then we conclude $\mathcal{E}xt^1_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M})=0$ for all $\mathcal{M} \in \underline{D}(X/Y)$. This implies the functor $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \bullet)$ is exact on the category $\underline{D}(X/Y)$.

Thus applying the functor $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \bullet)$ to (3.6), we get the exact sequence on U

$$(f^{-1}\mathcal{D}_Y)^{m_2} \longrightarrow (f^{-1}\mathcal{D}_Y)^{m_1} \longrightarrow \mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M}) \longrightarrow 0. \tag{3.7}$$

Applying the exact functor $\mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} (\bullet)$ to the above sequence (3.7) and comparing with the sequence (3.6), we deduce

$$\mathcal{M}|_U=\Big(\mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} \mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M})\Big)\Big|_U.$$

Shrinking U, we may assume U is an open convex set. By Lemma 3.2 and Lemma 3.3,

$$\begin{aligned}
\mathcal{M}|_U &= \mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} (\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M})|_U) \\
&= \mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} (f|_U)^{-1}(f|_U)_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M}) \\
&= \Big(\mathcal{D}_{X\to Y} \underset{f^{-1}\mathcal{D}_Y}{\otimes} f^{-1}f_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M})\Big)\Big|_U \\
&= (\underline{f}^*\underline{f}_*\mathcal{M})|_U.
\end{aligned}$$

To show $\underline{f}_*\mathcal{M}$ is coherent, we apply the functor $(f|_U)_*$ to (3.7) and get the exact sequence

$$\mathcal{D}_Y^{m_2} \longrightarrow \mathcal{D}_Y^{m_1} \longrightarrow (f|_U)_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M}) \longrightarrow 0.$$

Moreover since $(f|_U)_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M}) \simeq f_*\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_{X\to Y}, \mathcal{M})|_U$, $\underline{f}_*\mathcal{M}$ is a coherent $\mathcal{D}_Y$ module.

In general case, for any point $x_0 \in X$ there exists an open neighborhood U of x_0 where we reduce the problem to the special case. Then we get

$$\begin{aligned}\mathcal{M}|_U &= (\underline{f}|_U)^*(\underline{f}|_U)_*\mathcal{M} \\ &= (\underline{f}^*\underline{f}_*\mathcal{M})|_U,\end{aligned}$$

on account of Lemma 3.2 and Lemma 3.3. This completes the proof. ∎

REMARK. If the fiber of f is not simply connected, two categories are not equivalent. But locally $\mathcal{M} \in \underline{D}(X/Y)$ is isomorphic to the inverse image of a coherent $\mathcal{D}_Y$ module.

COROLLARY. *Let X and Y be two complex manifolds with* $\dim(Y) \geq 1$ *and $f: X \to Y$ a smooth map. Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$ module.*

We assume the following conditions (1), (2), (3).

(1) *$\mathcal{M}$ is purely dimensional.*

(2) $\mathrm{Char}(\mathcal{M})$ *is equal to $\rho\varpi^{-1}(T^*Y)$.*

(3) $\mathrm{Char}_f^1(\mathcal{M})$ *is contained in zero-section of $T^*(X/Y)$.*

Then $\mathcal{M}$ is locally isomorphic to a direct summand of $\mathcal{D}_{X\to Y}$.

REMARK. If $\dim(Y) \geq 1$, we can not remove the condition (1) and (3) to ensure the same result. But if $\dim(Y) = 0$, the condition (2) is equivalent to $\mathrm{Char}(\mathcal{M}) \subset T_X^*X$ which implies the conditions (1) and (3). Thus it is sufficient to assume the condition (2) only. Moreover in this case, $\mathcal{D}_{X\to Y} \simeq \mathcal{O}_X$. Thereby we conclude that if $\mathrm{Char}(\mathcal{M}) \subset T_X^*X$, $\mathcal{M}$ is locally the de Rham system (i.e. $\mathcal{M} \simeq \bigoplus_{\text{finite}} \mathcal{O}_X$) (cf. [K]).

Example. Let $X = \boldsymbol{C}^2 = (z_1, z_2)$, $Y = \boldsymbol{C}^1 = (z_2)$, and $f: X \to Y$ be a projection $(z_1, z_2) \mapsto (z_2)$. Let $\mathcal{M}$ be the system of differential operators over X:

$$\mathcal{M} = \frac{\mathcal{D}_X}{\mathcal{D}_X(D_{z_1}^2 + D_{z_2})}.$$

Then it is known (cf. [S-K-K]),

$$\mathcal{M}^{\infty}\left(=\mathcal{D}_X^{\infty}\underset{\mathcal{D}_X}{\otimes}\mathcal{M}\right)\simeq\frac{\mathcal{D}_X^{\infty}}{\mathcal{D}_X^{\infty}(D_{z_1}^2)}.$$

But since $\mathrm{Char}_Y^1(\mathcal{M})=T^*(X/Y)$, $\mathcal{M}$ is not isomorphic to $\dfrac{\mathcal{D}_X}{\mathcal{D}_X(D_{z_1}^2)}$ by Theorem 3.1.

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