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Vanishing theorems for the sheaf of microfunctions with holomorphic parameters

—Flabbiness of the sheaf of 2-microfunctions—

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Introduction

This note deals with a global vanishing theorem for the sheaf $\mathcal{CO}$ of microfunctions with holomorphic parameters and some related results.

Noro-Tose [7] obtained a similar vanishing theorem for the domains of type {a real open set}×{a Stein domain}. Our approach is not based on theirs, but on Schapira-Zampieri's ([11]); a reduction to the flabbiness of microfunctions. We also prove in §3 the cohomological triviality of $\mathcal{A}^2$ (see §1.3 for its definition), which will be used to prove the flabbiness of $\mathcal{C}^2$ in §4.

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§1. Preliminary

1.1. Sato's Microlocalization

Let X be a C^∞ manifold and Y be a closed submanifold of X. $D^+(X)$ denotes the derived category of bounded complexes of sheaves of modules on X. Refer to [2] for the notion of derived categories and derived functors.

For $F \in Ob(D^+(X))$, Sato's microlocalization of F along Y is defined by

$$\mu_Y(F) = R\Gamma_{S_Y^*X}(\pi_{Y/X}^{-1}F)^a. \tag{1.1}$$

Here $\pi_{Y/X}$ is the projection from ${}^Y\tilde{X}^* = (X-Y) \cup S_Y^*X$ onto X, $a: S_Y^*X \to S_Y^*X$ is the antipodal map of the fibers, and $G^a = a^{-1}G$ for any $G \in Ob(D^+(S_Y^*X))$. We remark that ${}^Y\tilde{X}^*$ is the comonoidal transformation of X with center Y (see [8]).

1.2. Microfunctions with holomorphic parameters

Let $M=R_t^p\times R_x^q$ be with its complexification $X=C_w^p\times C_z^q$. We set

$$N=X\cap\{\mathrm{Im}\, w=0\}\xrightarrow{\sim} R_t^p\times C_z^q,\qquad \tilde{\Sigma}=S_N^*X\xrightarrow{\sim} iS^*R_t^p\times C_z^q. \tag{1.2}$$

We take a coordinate $(t, x;\ i\tau dt+i\xi dx)$ of S_M^*X with $(\tau, \xi)\neq 0$. Then $\tilde{\Sigma}$ is called a partial complexification of an involutory submanifold

$$\Sigma=\{(t, x;\ i\tau, i\xi)\in S_M^*X;\ \xi=0\} \tag{1.3}$$

in S_M^*X. $\tilde{\Sigma}$ is endowed with the sheaf $\mathcal{C}^p\mathcal{O}^q$ (or $=\mathcal{C}_{\tilde{\Sigma}}$) of microfunctions with holomorphic parameters z (see [8], [4]). Explicitly, $\mathcal{C}^p\mathcal{O}^q$ is defined as

$$\mathcal{C}^p\mathcal{O}^q=\mu_N(\mathcal{O}_X)[p]. \tag{1.4}$$

Here we remark that $\mathcal{C}^p\mathcal{O}^q$ is concentrated in degree 0.

1.3. 2nd-microlocalizations

We follow the notation above. M. Kashiwara introduced the sheaf of 2nd-microfunctions $\mathcal{C}_\Sigma^2$ long time ago in Nice, which is explicitly defined as

$$\mathcal{C}_\Sigma^2=\mu_\Sigma(\mathcal{C}_{\tilde{\Sigma}})[q]. \tag{1.5}$$

Here this complex is concentrated in degree 0. We also define the sheaves of 2nd-hyperfunctions and 2nd-real analytic functions respectively by

$$\mathcal{B}_\Sigma^2=\mathcal{H}_\Sigma^q(\mathcal{C}_{\tilde{\Sigma}})\quad\text{and}\quad \mathcal{A}_\Sigma^2=\mathcal{C}_{\tilde{\Sigma}}|_\Sigma. \tag{1.6}$$

§ 2. Vanishing Theorems for $\mathcal{C}\mathcal{O}$

2.1. Statement of Theorems

Let L be a p-dimensional oriented real analytic manifold. Then, for $M=L\times R_x^q$, we can generalize the definitions in 1.2 of $X, N, \tilde{\Sigma}$ and $\mathcal{C}_{\tilde{\Sigma}}(=\mathcal{C}^p\mathcal{O}^q)$ in a natural way; that is,

$$\tilde{\Sigma}\simeq iS^*L\times C_z^q\xrightarrow{\ \gamma\ } iS^*L, \tag{2.1}$$

where γ is the canonical projection. An open set D in C_z^q is said to be of *product type* iff D has the form

$$D=D_1\times\cdots\times D_q$$

with some open subsets $D_1, \cdots, D_q$ of $\boldsymbol{C}$.

THEOREM 2.1. *Let U, V be open subsets of $\tilde{\Sigma}$ satisfying $U \supset V$. Suppose that, for every $t^* \in iS^*L$,*

$$(2.2)_q \qquad \gamma^{-1}(t^*) \cap (U-V) \neq \varnothing \Longrightarrow \gamma^{-1}(t^*) \cap (U-V) = \gamma^{-1}(t^*) \cap U = \text{an open set of product type in } \boldsymbol{C}_z^q.$$

Then we have

$$(2.3)_q \qquad H^k_{U-V}(U, \mathcal{C}^p\mathcal{O}^q) = 0 \qquad \text{for every } k \geqq 1.$$

As a corollary of $(2.3)_q$, we have in a weak sense the partial flabbiness of $\mathcal{C}^p\mathcal{O}^q$ with respect to microfunction parameters; that is,

THEOREM 2.2. *Let U, V be open subsets of $\tilde{\Sigma}$ satisfying $U \supset V$. Suppose the condition $(2.2)_q$. Then the restriction map*

$$(2.4)_q \qquad \Gamma(U, \mathcal{C}^p\mathcal{O}^q) \longrightarrow \Gamma(V, \mathcal{C}^p\mathcal{O}^q)$$

is surjective.

REMARK 2.3. i) As the simplest example of (U, V) in these theorems, we can take $U = W_1 \times D_1 \times \cdots \times D_q$, $V = W_2 \times D_1 \times \cdots \times D_q$ with some open sets W_1, W_2 in iS^*L and with some open sets $D_1, \cdots, D_q$ in $\boldsymbol{C}$. ii) Schapira-Zampieri proved a theorem equivalent to Theorem 2.1 for $q=1$ (that is, a vanishing theorem for "$\mathcal{C}_{N|X}$"), and as a corollary they obtained the flabbiness of "$\mathcal{C}_{\Omega|X}|_{T^*_M X}$", which is a sheaf similar to $\mathcal{C}^2$ for $q=1$ (see [11]).

2.2. Proof of Theorem 2.1

We prove Theorem 2.1 by induction on q. In case $q=0$, $(2.3)_0$ is meaningful and true because of the flabbiness of the sheaf of microfunctions. So we can assume that $q \geqq 1$ and that $(2.3)_{q-1}$ holds for every L and U, V satisfying the conditions.

Consider the Riemann sphere $\boldsymbol{P}^1 = \boldsymbol{C} \cup \{\infty\}$ with its underlying real manifold as $\boldsymbol{S}^2 = \boldsymbol{R}^2 \cup \{\infty\}$. Put $L' = L \times \boldsymbol{S}^2$, $\tilde{\Sigma}' = iS^*L' \times \boldsymbol{C}^{q-1}_{z'}$ with $z' = (z_2, \cdots, z_q)$, and consider the following imbedding:

$$(2.5) \qquad \tilde{\Sigma} = iS^*L \times \boldsymbol{C}^q_z \ni (t;\ i\tau dt;\ z_1, z') \longrightarrow (t, \operatorname{Re} z_1, \operatorname{Im} z_1;\ i\tau dt;\ z') \in iS^*L' \times \boldsymbol{C}^{q-1}_{z'} = \tilde{\Sigma}'.$$

Hereafter, we identify $\tilde{\Sigma}$ with a locally closed subset of $\tilde{\Sigma}'$. Then we have an exact sequence

(2.6) $$0 \longrightarrow \mathcal{C}_{\tilde{\Sigma}} \longrightarrow \mathcal{C}_{\tilde{\Sigma}'} \xrightarrow{\partial/\partial \bar{z}_1} \mathcal{C}_{\tilde{\Sigma}'} \longrightarrow 0$$

on $\{(t, x, y;\ i\tau, i\xi, i\eta;\ z') \in \tilde{\Sigma}';\ (x, y) \neq \infty\}$, where $\mathcal{C}_{\tilde{\Sigma}'} = \mathcal{C}^{p+2}\mathcal{O}^{q-1}$ and $\mathcal{C}_{\tilde{\Sigma}}$ is defined as 0 in $\tilde{\Sigma}' - iS^*L \times \boldsymbol{S}^2 \times \boldsymbol{C}_{z'}^{q-1}$. Let U, V be as in the theorem. Then (2.6) implies a long exact sequence:

(2.7) $$\begin{array}{l} 0 \longrightarrow H^0_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}}) \longrightarrow H^0_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) \longrightarrow H^0_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) \longrightarrow \\ \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \\ \longrightarrow H^k_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}}) \longrightarrow H^k_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) \longrightarrow H^k_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) \longrightarrow, \end{array}$$

where U' and V' are open sets in $\tilde{\Sigma}'$ defined as follows:

(2.8) $$U' = U \cup (\tilde{\Sigma}' - iS^*L \times \boldsymbol{S}^2 \times \boldsymbol{C}_{z'}^{q-1}), \qquad V' = V \cup (\tilde{\Sigma}' - iS^*L \times \boldsymbol{S}^2 \times \boldsymbol{C}_{z'}^{q-1}).$$

Remark that $U' - V' = U - V$, and that the pair (U', V') satisfies $(2.2)_{q-1}$ for the projection

(2.9) $$\gamma' \colon iS^*L' \times \boldsymbol{C}_{z'}^{q-1} \longrightarrow iS^*L'.$$

Hence we know by the induction hypothesis that $H^k_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) = 0$ for every $k \geqq 1$. Thus we conclude $(2.3)_q$ for every $k \geqq 2$. Further, $(2.3)_q$ for $k = 1$ is equivalent to the surjectivity of

(2.10) $$\Gamma_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}) \xrightarrow{\partial/\partial \bar{z}_1} \Gamma_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}).$$

Take any section $u(t, x, y, z')$ of $\Gamma_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'})$, and put

(2.11) $$g(t, x, y, z') = (1 + x^2 + y^2)^2 \cdot u(t, x, y, z') \in \Gamma_{U'-V'}(U', \mathcal{C}_{\tilde{\Sigma}'}).$$

In order to extend g as a section of $\mathcal{C}_{\tilde{\Sigma}'}$ to $\boldsymbol{S}^2$ with respect to (x, y), we remark that $U' - V'$ is written as

(2.12) $$U' - V' = U - V = \gamma^{-1}(\gamma(U - V)) \cap U = \bigcup_{t^* \in \gamma(U-V)} \{t^*\} \times D_1(t^*) \times \cdots \times D_q(t^*).$$

Here $D_1(t^*), \cdots, D_q(t^*)$ are some non-void open sets in $\boldsymbol{C}$ depending on $t^* \in \gamma(U - V)$. Since $D_1(t^*) \times \cdots \times D_q(t^*)$ is the section of an open set U, each $D_j(t^*)$ depends lower-semicontinuously on t^*; that is, for every $t_0^* \in \gamma(U - V)$ and every compact $K \subset D_j(t_0^*)$, there exists a neighborhood G of t_0^* in iS^*L such that

(2.13) $$K \subset D_j(t^*) \qquad \text{for any} \quad t^* \in \gamma(U - V) \cap G.$$

Therefore, considering $\gamma(U - V) = \gamma(U) - \gamma(V)$ (see $(2.2)_q$), we know that the following sets U'', V'' are open in $\tilde{\Sigma}'$;

$$U''=\bigcup_{t^*\in\gamma(U-V)}\{t^*\}\times S^2\times D_2(t^*)\times\cdots\times D_q(t^*)\cup\gamma(V)\times S^2\times C_{z'}^{q-1}\cup(\tilde{\Sigma}'-iS^*L\times S^2\times C_{z'}^{q-1}), \tag{2.14}$$

$$V''=\gamma(V)\times S^2\times C_{z'}^{q-1}\cup(\tilde{\Sigma}'-iS^*L\times S^2\times C_{z'}^{q-1}). \tag{2.15}$$

Hence we have an exact sequence:

$$\Gamma_{U''-V''}(U'',\mathcal{C}_{\tilde{\Sigma}'})\longrightarrow\Gamma_{U'\setminus V''}(U',\mathcal{C}_{\tilde{\Sigma}'})\longrightarrow H^1_{U''-(U'\cup V'')}(U'',\mathcal{C}_{\tilde{\Sigma}'}). \tag{2.16}$$

Since $U''-(U'\cup V'')=\bigcup_{t^*\in\gamma(U-V)}(\{t^*\}\times(S^2-D_1(t^*)))\times D_2(t^*)\times\cdots\times D_q(t^*)$, the pair $(U'',U'\cup V'')$ satisfies condition $(2.2)_{q-1}$ for γ'. Thus by the induction hypothesis we have $H^1_{U''-(U'\cup V'')}(U'',\mathcal{C}_{\tilde{\Sigma}'})=0$. As a result, the element $g(t,x,y,z')$ of $\Gamma_{U'-V'}(U',\mathcal{C}_{\tilde{\Sigma}'})=\Gamma_{U'\setminus V''}(U',\mathcal{C}_{\tilde{\Sigma}'})$ has an extension $\tilde{g}(t,x,y,z')\in\Gamma_{U''-V''}(U'',\mathcal{C}_{\tilde{\Sigma}'})$. By using $\tilde{g}$, we define $h(t,x,y,z')$ as

$$h=\frac{1}{\pi}\int_{R^2\cup\{\infty\}}\left(\frac{Y(\tilde{y}-y)}{x+iy-(\tilde{x}+i\tilde{y})-i0}+\frac{Y(y-\tilde{y})}{x+iy-(\tilde{x}+i\tilde{y})+i0}\right)\times\tilde{g}(t,\tilde{x},\tilde{y},z')\frac{d\tilde{x}d\tilde{y}}{(1+\tilde{x}^2+\tilde{y}^2)^2}, \tag{2.17}$$

where $Y(x)$ is the Heaviside function. In fact, the integrand is well-defined at $(x,y)=\infty$, and so h is well-defined as a section of $\Gamma_{U'-V'}(U',\mathcal{C}_{\tilde{\Sigma}'})$. Further it is easy to see that $\frac{\partial}{\partial\bar{z}_1}h(t,x_1,y_1,z')=u(t,x_1,y_1,z')$ in $\Gamma_{U'-V'}(U',\mathcal{C}_{\tilde{\Sigma}'})$. Thus the map (2.10) is surjective. Hence the proof is completed.

§ 3. Vanishing Theorem for $\mathcal{A}^2$

Let L be a p-dimensional oriented real analytic manifold. Then, for $M=L\times R_x^q$, we can generalize the definitions in 1.2 of $X,N,\Sigma,\tilde{\Sigma},\mathcal{C}_{\tilde{\Sigma}}$ and $\mathcal{A}^2_{\Sigma}$ in a natural way; that is,

$$\Sigma=\{(t,x;\ i\tau dt+i\xi dx)\in iS^*M;\ \xi=0\}\xrightarrow{\sim}iS^*L\times R_x^q. \tag{3.1}$$

THEOREM 3.1. *Let U be an open subset of Σ. Then we have*

$$H^k(U,\mathcal{A}^2_{\Sigma})=0\qquad\textit{for every}\quad k\geqq1. \tag{3.2}$$

PROOF. Put $M_0=L\times R^{q+1}_{x_0,x}$ with its complexification $X_0=\tilde{L}\times C^{q+1}_{z_0,z}$, where $\tilde{L}$ is a complexification of L. Further we define $N_0,\tilde{\Sigma}_0,\Sigma_0$ in the same way as $N, \tilde{\Sigma}, \Sigma$:

(3.3) $N_0 = L \times C^{q+1}_{z_0, z}$, $\tilde{\Sigma}_0 = S^*_{N_0} X_0 \simeq iS^* L \times C^{q+1}_{z_0, z}$,
$\Sigma_0 = \{(t, x_0, x;\ i\tau dt + i\xi_0 dx_0 + i\xi dx) \in iS^* M_0;\ \xi_0 = 0, \xi = 0\} \simeq iS^* L \times R^{q+1}_{x_0, x}$.

Let $P = \partial^2/\partial x_0^2 + \cdots + \partial^2/\partial x_q^2$ be the Laplace operator, and $\mathcal{C}^P_{M_0}$ be the solution-subsheaf for $Pu=0$ in $\mathcal{C}_{M_0}$. Then we have the exact sequence

$$0 \longrightarrow \mathcal{C}^P_{M_0}|_{\Sigma_0} \longrightarrow \mathcal{C}_{M_0}|_{\Sigma_0} \xrightarrow{P} \mathcal{C}_{M_0}|_{\Sigma_0} \longrightarrow 0 \tag{3.4}$$

by the solvability of partially elliptic operators due to Bony and Schapira [1] (see also [5]). Moreover they proved the isomorphism

$$\mathcal{C}^P_{\tilde{\Sigma}_0}|_{\Sigma_0} \xrightarrow{\sim} \mathcal{C}^P_{M_0}|_{\Sigma_0} \tag{3.5}$$

for general partially elliptic operators P. We identify

(3.6) $M = M_0 \cap \{x_0 = 0\}$, $X = X_0 \cap \{z_0 = 0\}$, $\tilde{\Sigma} = \tilde{\Sigma}_0 \cap \{z_0 = 0\}$ and $\Sigma = \Sigma_0 \cap \{x_0 = 0\}$.

By the theorem of Cauchy-Kowalevsky type with value in $\mathcal{CO}$ due to Schapira [10] (essentially proved in [1]), we have

$$\mathcal{C}^P_{\tilde{\Sigma}_0}|_{\tilde{\Sigma}} \xrightarrow{\sim} (\mathcal{C}_{\tilde{\Sigma}})^2. \tag{3.7}$$

After all, we have the following exact sequence on Σ:

$$0 \longrightarrow (\mathcal{A}^2_{\Sigma})^2 \longrightarrow \mathcal{C}_{M_0}|_{\Sigma} \xrightarrow{P} \mathcal{C}_{M_0}|_{\Sigma} \longrightarrow 0. \tag{3.8}$$

Since $\mathcal{C}_{M_0}|_{\Sigma}$ is flabby, (3.2) reduces to the surjectivity of

$$\Gamma(U, \mathcal{C}_{M_0}|_{\Sigma}) \xrightarrow{P} \Gamma(U, \mathcal{C}_{M_0}|_{\Sigma}). \tag{3.9}$$

In fact we can prove this by using the fundamental solution for P in R^{q+1} and the flabbiness of $\mathcal{C}_{M_0}$ (use the partial compactification $L \times (R^{q+1} \cup \{\infty\})$ in the same way as in the proof of Theorem 2.1).

REMARK 3.2. The method above was suggested by Prof. Schapira, who mentioned to the first author that the cohomological triviality of the sheaf of germs of real analytic functions on R^n due to Malgrange can be proved in the same way (see [9]).

§ 4. Flabbiness of the sheaf of 2nd-microfunctions

By using the vanishing theorem for $\mathcal{A}^2_\Sigma$ prepared in § 3, we prove the flabbiness of the sheaf of 2nd-microfunctions. We inherit the notation $L, M=L\times \boldsymbol{R}^q_x, \cdots$ etc. from § 3. First we show

THEOREM 4.1. *$\mathcal{B}^2_\Sigma/\mathcal{A}^2_\Sigma$ is flabby.*

PROOF. $\mathcal{B}^2_\Sigma$ is a flabby sheaf because $\boldsymbol{R}\Gamma_\Sigma(\mathcal{C}_{\tilde{\Sigma}})=\mathcal{B}^2_\Sigma[-q]$ is purely q-codimensional and flabby-dimension of $\mathcal{C}_{\tilde{\Sigma}}\leqq q$ (recall the partial $\bar{\partial}$-resolution of $\mathcal{C}_{\tilde{\Sigma}}$ by microfunctions). Hence by the cohomological triviality of $\mathcal{A}^2_\Sigma$ due to Theorem 3.1, we can conclude the flabbiness of $\mathcal{B}^2_\Sigma/\mathcal{A}^2_\Sigma$.

THEOREM 4.2. *$\mathcal{C}^2_\Sigma$ is flabby.*

PROOF. At first we remark that $\mathcal{B}^2_\Sigma/\mathcal{A}^2_\Sigma$ is flabby for any $\Sigma=iS^*L\times K$ associated to an oriented real analytic manifold $M=L\times K$ of product-type because flabbiness is a local property. In particular, we can take the second factor K as $\boldsymbol{R}^q_x\times \boldsymbol{S}^{q-1}_\xi=\{(x,\xi)\in \boldsymbol{R}^q\times\boldsymbol{R}^q;\ \xi^2=1\}$. That is, we set $M', \Sigma', \tilde{\Sigma}'$ as follows:

$$M'=L\times \boldsymbol{R}^q_x\times \boldsymbol{S}^{q-1}_\xi,\quad \Sigma'=iS^*L\times \boldsymbol{R}^q_x\times \boldsymbol{S}^{q-1}_\xi,\quad \tilde{\Sigma}'=iS^*L\times \boldsymbol{C}^q_z\times\{\zeta\in \boldsymbol{C}^q;\ \zeta^2=1\}. \tag{4.1}$$

Then, following the method of proving the flabbiness of microfunctions ([8], [3]), we define sheaf homomorphisms Φ and Ψ:

$$\begin{array}{rccc} \Phi: & \mathcal{C}^2_\Sigma & \longrightarrow & \mathcal{B}^2_{\Sigma'}/\mathcal{A}^2_{\Sigma'}, \\ & \cup\!\!\!\!\raisebox{1pt}{\scriptsize$\shortmid$} & & \cup\!\!\!\!\raisebox{1pt}{\scriptsize$\shortmid$} \\ & f(t,x) & \longmapsto & \displaystyle\int f(t,y)K_{(q+1)/2}(x-y,\xi)dy \end{array} \tag{4.2}$$

$$\begin{array}{rccc} \Psi: & \mathcal{B}^2_{\Sigma'}/\mathcal{A}^2_{\Sigma'} & \longrightarrow & \mathcal{C}^2_\Sigma \\ & \cup\!\!\!\!\raisebox{1pt}{\scriptsize$\shortmid$} & & \cup\!\!\!\!\raisebox{1pt}{\scriptsize$\shortmid$} \\ & g(t,x,\xi) & \longmapsto & \displaystyle\sum_{j=1}^{q}\sum_{k=0}^{j-1} C_{jk}\iint g(t,y,\xi)L_{j,k}(x-y,\xi)dy\cdot\omega(\xi). \end{array} \tag{4.3}$$

Here

$$K_\lambda(x,\xi)=(x\xi+2i(x^2-(x\xi)^2)+i0)^{-\lambda},\quad L_{j,k}(x,\xi)=(x^2-(x\xi)^2)^k K_j(x,\xi), \tag{4.4}$$

$\{C_{jk}\}$ are some constants, and $\omega(\xi)$ is the volume-element on $\boldsymbol{S}^{q-1}$. In the same way as in Chapter 3 of [3], we can choose constants C_{jk}'s such that $\Psi\cdot\Phi=$identity: $\mathcal{C}^2_\Sigma\to\mathcal{C}^2_\Sigma$. Hence the flabbiness of $\mathcal{B}^2_\Sigma/\mathcal{A}^2_\Sigma$ implies the flabbiness of $\mathcal{C}^2_\Sigma$.

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