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The theory of Radon transformations and 2-microlocalization (I)

—Vanishing theorem for the sheaf of microfunctions with holomorphic parameters—

Dedicated to Professor Seizô Itô on his 60th birthday

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Plan

Introduction

The theory of 2nd microlocalization was initiated by M. Kashiwara in 1975 in Nice. He constructed the sheaf of 2-microfunctions from the sheaf $\mathcal{CO}$ of microfunctions with holomorphic parameters.

Let $M=\boldsymbol{R}^{n-d}_t\times\boldsymbol{R}^d_x$ and its complexification $X=\boldsymbol{C}^{n-d}_w\times\boldsymbol{C}^d_z$. We set $N=\boldsymbol{R}^{n-d}_t\times\boldsymbol{C}^d_z$ in X and set $\tilde{\Lambda}=S^*_NX$ $(\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d_z)$ and $\Lambda=S^*_MX\cap\tilde{\Lambda}$ $(\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{R}^d_x)$. $\tilde{\Lambda}$ is endowed with the sheaf $\mathcal{CO}$ of microfunctions with holomorphic parameter z. M. Kashiwara constructed the sheaf $\mathcal{C}^2_\Lambda$ on $S^*_\Lambda\tilde{\Lambda}$, by which we can study the properties of microfunctions defined on Λ precisely. More explicitly, there exists the sheaf $\mathcal{B}^2_\Lambda$ of 2-hyperfunctions on Λ which satisfies the exact sequences

$$0\longrightarrow\mathcal{CO}|_\Lambda\longrightarrow\mathcal{B}^2_\Lambda\longrightarrow\pi_*\mathcal{C}^2_\Lambda\longrightarrow0$$

and

$$0 \longrightarrow \mathcal{C}_M|_\Lambda \longrightarrow \mathcal{B}^2_\Lambda.$$

Here $\pi: S^*_\Lambda\tilde{\Lambda} \longrightarrow \Lambda$. See Kashiwara-Laurent [7] for $\mathcal{C}^2_\Lambda$ and Y. Laurent [9] for 2-microdifferential operators which act on $\mathcal{C}^2_\Lambda$. See also N. Tose [14], [15], [16] and [17] for their applications to the propagation of singularities for some classes of microdifferential equations.

In this paper, we reconstruct the sheaf $\mathcal{C}^2_\Lambda$ in the framework of the cohomological Radon transformations developed by K. Kataoka [3]. As mentioned above, the sheaf of 2-microfunctions $\mathcal{C}^2_\Lambda$ is defined purely cohomologically from the sheaf $\mathcal{CO}$. By virtue of the global vanishing theorem for $\mathcal{CO}$, we can express 2-microfunctions as boundary values of microfunctions with holomorphic parameters by expressing the cohomology groups as Čech cohomology. But the choice of coverings itself is not canonical. If we take the covering depending on the fibers of $S^*_\Lambda\tilde{\Lambda} \to \Lambda$ and replace the sums in Čech cohomology group to the integration along fibers of $S^*_\Lambda\tilde{\Lambda} \to \Lambda$, we gain the notion of cohomological Radon transformation. Roughly we express 2-microfunctions and 2-hyperfunctions as a microfunction valued $(d-1)$ form $\sum_{|J|=d-1} f_J(t, x, x^*)dx^{*J}$ depending holomorphically on x and x^*. Here x^* denotes the fiber coordinate of $S^*_\Lambda\tilde{\Lambda} \to \Lambda$. Then the global vanishing theorem for $\mathcal{CO}$ (Theorem 1.1) will play an essential role. See also the introduction of K. Kataoka [3] where an intuitive explanation of Radon transformation can be found.

Now we give the plan of this paper. In 1° we prepare some notation about the sheaf of microfunctions with holomorphic parameters, the abstract form of the theorem of Edge of the Wedge, 2-microfunctions and the nuclear properties of the sheaf of holomorphic functions with smooth parameters. Again we recall the crucial fact used to apply the theory of Kataoka [3] is given in Theorem 1.1, which will be proved in 2°. In 3° we define the sheaf of microfunctions with both holomorphic parameters and smooth parameters and study its cohomological properties, which will be used to construct the cohomological Radon transformation in 4°. In 5° we give a morphism $\mathcal{C}_M|_\Lambda \to \mathcal{B}^2_\Lambda$ using the Radon transformation. In 6° we relate the Radon transformation of 2-microfunctions to the Čech cohomology group with value in $\mathcal{CO}$. In 7° we consider the curvilinear wave expansion of microfunctions with holomorphic parameters and give a theorem of Edge of the Wedge of Martineau type for $\mathcal{B}^2_\Lambda$.

Applications of this note will appear in the subsequent paper [18], where several operations for 2-microfunctions and 2-hyperfunctions will be defined with the aid of the Radon transformation. Moreover we will give theorems about the interdependence between the support of a microfunction and its 2-singular spectrum.

A few words on the process how this joint paper has come to being. Actually, a substantial part of the result of this paper is contained in a Thesis for Master's Degree of the first author submitted to the University of Tokyo in March of 1985 (M. Noro [11]). Then the first author took his current job which is not tightly connected with this field of mathematical research and had some difficulty in completing the manuscripts for publication. Under these circumstances, the second author, who believes in the importance of this work and who already contributed to the progress of the study through continual discussions with the first author at the Komatsu Seminar for a long period, has joined in order to complete the present English version with some elaboration.

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1° Perliminaries

1.1° Sato's microlocalization

Let X be a C^∞ manifold and Y be a closed submanifold of X. $D^+(X)$ denotes the derived category of bounded below complexes of sheaves of modules on X. For $\mathcal{F} \in \mathrm{Ob}(D^+(X))$, the Sato's microlocalization of $\mathcal{F}$ along Y is defined by

$$(1.1) \qquad \mu_Y(F) = R\Gamma_{S_Y^*X}(\pi^{-1}\mathcal{F})^a$$

where $\pi_{Y|X} : \mathrm{Mon}_Y^*(X) = (X \setminus Y) \cup S_Y^*X \to X$ is the comonoidal transformation of X along Y and $a : S_Y^*X \to S_Y^*X$ is the antipodal map of fibers. (For $\mathcal{G} \in \mathrm{Ob}(D^+(S_Y^*X))$ $\mathcal{G}^a$ stands for the inverse image of $\mathcal{G}$ by a.) Here we remark that $\mathrm{Mon}_Y^*(X)$ is expressed as $\widetilde{{}^YX^*}$ in Sato-Kawai-Kashiwara [13].

We also define the monoidal transformation of X along Y as

$$(1.2) \qquad \tau_{Y|X}: \mathrm{Mon}_Y(X) = (X \setminus Y) \cup S_YX \longrightarrow X,$$

which is written as $\widetilde{{}^YX}$ in Sato-Kawai-Kashiwara [13].

1.2° Microfunctions with holomorphic parameters

Let

$$M=\boldsymbol{R}_x^d\times\boldsymbol{R}_t^{n-d} \tag{1.3}$$

and X be its complexification $\boldsymbol{C}_z^d\times\boldsymbol{C}_w^{n-d}$. We set

$$N=X\cap\{\operatorname{Im} w=0\}\simeq\boldsymbol{C}_z^d\times\boldsymbol{R}_t^{n-d} \tag{1.4}$$

and

$$\tilde{\Lambda}=S_N^*X. \tag{1.5}$$

We take a coordinate of S_M^*X as $(x,t;\sqrt{-1}\,(\xi\cdot dx+\tau\cdot dt)\infty)$. Then $\tilde{\Lambda}$ is called a partial complexification of an involutive submanifold of S_M^*X:

$$\Lambda=\{(x.\,t;\sqrt{-1}(\xi,\tau))\in S_M^*X;\,\xi=0\}. \tag{1.6}$$

$\tilde{\Lambda}$ is endowed with the sheaf $\mathcal{CO}$ or $\mathcal{C}_{\tilde{\Lambda}}$ of microfunctions with holomorphic parameter z, which is constructed in Theorem 2.2.5 of Chapter 3 of Sato-Kawai-Kashiwara [13]. See also Kashiwara-Laurent [7]. Explicitly, $\mathcal{CO}$ is defined as

$$\mathcal{CO}=\mu_N(\mathcal{O}_X)[n-d]. \tag{1.7}$$

Here we remark that $\mathcal{CO}$ is concentrated in degree 0. We put $N_0=\boldsymbol{R}_t^{n-d}$ and $X_0=\boldsymbol{C}_w^{n-d}$ and have the identifications:

$$\operatorname{Mon}_N^*(X)\simeq\operatorname{Mon}_{N_0}^*(X_0)\times\boldsymbol{C}^d \tag{1.8}$$

and

$$S_N^*X\simeq S_{N_0}^*X_0\times\boldsymbol{C}^d\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d. \tag{1.9}$$

The following global vanishing theorem for $\mathcal{CO}$ is essential in the following construction of the Radon transformation of 2-microfunctions and will be proved in 2°.

THEOREM 1.1. *For an open convex subset U of $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$ and a Stein open set D of $\boldsymbol{C}^d$, we have*

$$H^j(U\times D,\mathcal{CO})=0\quad(j\geqq 1). \tag{1.10}$$

1.3° Abstract form of the theorem of Edge of the Wedge

We quote the abstract form of Edge of the Wedge which is proved by Kashiwara-Laurent [7]. We remark that the prototype of it can be found in M. Kashiwara [4].

Let T be a topological space. We assume that there exists a functor $\mathcal{F}$

(1.11) $\{X : X \text{ is a complex manifold}\} \xrightarrow{\mathcal{F}} \{\text{sheaves of vector spaces on } X\times T\}$

$$X \longmapsto \mathcal{F}_X.$$

Moreover for a holomorphic map between complex manifolds $\phi : X\to X'$, we have the operation of substitution

$$\phi^* : (\phi\times \mathrm{id}_T)^{-1}\mathcal{F}_{X'} \longrightarrow \mathcal{F}_X \tag{1.12}$$

which satisfies the following conditions.

(H1) Let U and $V (\subset U)$ be two open subsets of a complex manifold X such that U is connected and that V is nonempty. Let W be an open set of T. Then we have

$$\Gamma_{(U\setminus V)\times W}(U\times W, \mathcal{F}_X)=0.$$

(H2) $f : X\to \boldsymbol{C}$ is a holomorphic map with $df\neq 0$ on X. Let $Y=f^{-1}(0)$ and $i : Y\to X$ be the canonical injection. Then we have an exact sequence

$$0\longrightarrow \mathcal{F}_X \longrightarrow \mathcal{F}_X \xrightarrow{i^*} \mathcal{F}_Y \longrightarrow 0.$$

(H3) Let X and Y be two complex manifolds such that Y is compact. We put $f : X\times Y\times T\to X\times T$. Then we have for any $q\geqq 0$

$$R^q f_* \mathcal{F}_{X\times Y} = \mathcal{F}_X \underset{\boldsymbol{C}}{\otimes} H^q(Y, \mathcal{O}_Y).$$

In the above situation, we have

THEOREM 1.3. *Let G be a closed subset in $\boldsymbol{C}^n$ and x be a point of G. We assume there exists no $\boldsymbol{C}$-linear affine subvariety L in $\boldsymbol{C}^n$ with $\dim L=n-q-1$ such that $L\ni x$ and $L\cap G$ is a neighborhood of x in L. Then we have for any $t\in T$*

$$H^k_{G\times T}(\mathcal{F}_{\boldsymbol{C}^n})_{(x,t)}=0 \qquad (k<q). \tag{1.13}$$

1.4° 2-microfunctions

We follow the notation in 1.2°.

M. Kashiwara introduced the sheaf of 2-microfunctions in [5]. Explicitly, the sheaf of 2-microfunctions $\mathcal{C}^2_\Lambda$ is defined as

$$\mathcal{C}^2_\Lambda = \mu_\Lambda(\mathcal{C}_{\tilde{\Lambda}})\,[d]. \tag{1.14}$$

Here $\mathcal{C}^2_\Lambda$ is concentrated in degree 0.

We also define the sheaf of 2-hyperfunctions as

$$\mathcal{B}_{\Lambda}^{2}=\mathcal{H}_{\Lambda}^{d}(\mathcal{C}_{\tilde{\Lambda}}). \tag{1.15}$$

We set

$$\mathcal{A}_{\Lambda}^{2}=\mathcal{C}_{\tilde{\Lambda}}|_{\Lambda}. \tag{1.16}$$

There exists the canonical spectral map

$$Sp_{\Lambda}^{2}:\pi^{-1}\mathcal{B}_{\Lambda}^{2}\longrightarrow\mathcal{C}_{\Lambda}^{2} \tag{1.17}$$

where $\pi: S_{\tilde{\Lambda}}^{*}\tilde{\Lambda}\to\Lambda$. For a 2-hyperfunction u, we set

$$SS_{\Lambda}^{2}(u)=\mathrm{supp}\,(Sp_{\Lambda}^{2}(u)), \tag{1.18}$$

which is called the 2-singular spectrum of u.

For details about 2-microfunctions, see Kashiwara-Laurent [7].

1.5° A theorem about the nuclearity

Let $X=\boldsymbol{C}_{w}^{n_1}\times\boldsymbol{C}_{z}^{n_2}\times\boldsymbol{R}_{u}^{n_3}$. There exists the sheaf $\mathcal{O}\mathcal{O}\mathcal{L}$ of smooth functions with holomorphic parameters z and w on X. Then we have

PROPOSITION 1.4. *Let W be an open subset of $\boldsymbol{C}^{n_1}$ satisfying*

$$\dim_{C} H^{k}(W,\mathcal{O}_{X})<\infty. \tag{1.19}$$

We take an open subset D in $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$ such that

$$H^{j}(D,\mathcal{O}\mathcal{L})=0 \qquad (j\geqq 1), \tag{1.20}$$

where $\mathcal{O}\mathcal{L}$ is the sheaf of smooth functions on $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$ with holomorphic parameters w. Then we have

$$H^{k}(W\times D,\mathcal{O}\mathcal{O}\mathcal{L})\simeq H^{k}(W,\mathcal{O})\underset{C}{\otimes}\Gamma(D,\mathcal{O}\mathcal{L}). \tag{1.21}$$

We can prove the proposition above in the same way as Lemma 2.4 of the following section 2.

2° Vanishing theorem for the sheaf of microfunctions with holomorphic parameters

We follow the notation prepared in 1.2°.

2.1° Proof of the Theorem 1.1

Because S_N^*X is purely d codimensional with respect to $\pi^{-1}\mathcal{O}_X$ (π : $\mathrm{Mon}_N^*(X)\to X$), we have

$$H^j(U\times D, \mathcal{C}_{\tilde{\Lambda}}) = H^{j+d}_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X). \tag{2.1}$$

Here $\tilde{U}=\Omega\cup U^a$ is an open set of $\mathrm{Mon}_{N_0}^*(X_0)$ with Ω open in $\boldsymbol{C}^d\setminus\boldsymbol{R}^d$. ($U^a$ is the inverse image of U by $a: S_N^*X\to S_N^*X$.) Consider the long exact sequences

$$\to H^j_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X)\to H^j(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X)\to H^j(\Omega\times D, \mathcal{O}_X)\xrightarrow{+1}. \tag{2.2}$$

By Lemma 2.4, if $\dim_C H^k(\Omega, \mathcal{C}_{X_0})<\infty$, there exists an isomorphism

$$H^k(\Omega\times D, \mathcal{O}_X)\simeq H^k(\Omega, \mathcal{O})\otimes\Gamma(D, \mathcal{O}). \tag{2.3}$$

On the other hand, we have

$$H^k(\Omega, \mathcal{O})=0 \qquad (k\geqq d) \tag{2.4}$$

by a theorem of B. Malgrange. Thus

$$H^k(\Omega\times D, \mathcal{O}_X)=0 \qquad (k\geqq d) \tag{2.5}$$

follows.

Since we have an isomorphism

$$H^j_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X)\xrightarrow{\sim} H^j(\tilde{U}\times D, \mathcal{O}_X) \tag{2.6}$$

by (2.2) and (2.5), it is sufficient to prove

$$H^j(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X)=0 \qquad (j\geqq d). \tag{2.7}$$

Take a flabby resolution of $\mathcal{O}_X$ as

$$0\to\mathcal{O}_X\to\mathcal{L}^{\cdot}. \tag{2.8}$$

Then

$$0\to\pi^{-1}\mathcal{O}_X\to\pi^{-1}\mathcal{L}^{\cdot} \tag{2.9}$$

is an exact sequence. Moreover we can prove

LEMMA 2.1.

$$H^j(\tilde{U}\times D, \pi^{-1}\mathcal{L}^k)=0 \qquad (j\geqq 1,\ k\geqq 0). \tag{2.10}$$

PROOF. Consider the long exact sequence

$$\rightarrow H^j_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{L}^k)\rightarrow H^j(\tilde{U}\times D, \pi^{-1}\mathcal{L}^k)\rightarrow H^j(\Omega\times D, \mathcal{L}^k)\xrightarrow{+1}. \tag{2.11}$$

Since $\mathcal{L}^k$ is flabby, we have

$$H^j(\tilde{U}\times D, \mathcal{L}^k)=0 \qquad (j\geqq 1). \tag{2.12}$$

On the other hand, by Proposition 1.2.4 of Chapter 1 of [13], we have

$$H^j_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{L}^k)\simeq \varinjlim_{Z} H^j_Z(X, \mathcal{L}^k). \tag{2.13}$$

Here Z runs the family of locally closed subsets of X satisfying the condition of Proposition 1.2.4 of Chapter 1 of [13]. Thus

$$H^j_{U^a\times D}(\tilde{U}\times D, \pi^{-1}\mathcal{L}^k)=0 \qquad (j\geqq 1) \tag{2.14}$$

follows and we have proved Lemma 2.1. (q.e.d. for Lemma 2.1.)

Using Lemma 2.1 above, there is an isomorphism

$$H^j(\tilde{U}\times D, \pi^{-1}\mathcal{O}_X)\simeq H^j(\Gamma(\tilde{U}\times D, \pi^{-1}\mathcal{L}^{\cdot})). \tag{2.15}$$

Hereafter we calculate the right side of (2.15).

We consider the problem in the general situation as follows. Let

$$M=\boldsymbol{R}^m\simeq\{0\}\times\boldsymbol{R}^m\hookrightarrow\boldsymbol{R}^l_x\times\boldsymbol{R}^m_t=N \tag{2.16}$$

and F be a sheaf on N. We take the comonoidal transformation of N along M:

$$\pi:\mathrm{Mon}^*_M(N)=\{(\boldsymbol{R}^l\setminus\{0\})\cup S^{l-1}_\xi\}\times\boldsymbol{R}^m\rightarrow N=\boldsymbol{R}^l\times\boldsymbol{R}^m \tag{2.17}$$

and take an open proper convex subset U of $S^*_MN=S^{l-1}\times\boldsymbol{R}^m$. We regard S^{l-1}_ξ as a unit sphere in $\boldsymbol{R}^{l-1}\setminus\{0\}$. Then we set in $M\setminus N=(\boldsymbol{R}^l\setminus\{0\})\times\boldsymbol{R}^m$

$$\Omega=\{(x,t)\in M\setminus N;\ \text{there exists a point } (\xi,t)\in U \text{ such that } \langle x,\xi\rangle\geqq 0\}. \tag{2.18}$$

Here we remark that $\tilde{U}=\Omega\cup U^a$ is an open subset in $\mathrm{Mon}^*_M(N)$. Because U is proper convex, we can take a sequence $\{K_j\}_{j\in N}$ of compact proper convex subset of U satisfying

$$K_j\subset K_{j+1} \text{ and } U=\bigcup_{j\in N}K_j. \tag{2.19}$$

We set in Ω

(2.20) $\Omega_j=\{(x,t)\in\Omega\,;\,|x|\leqq j$ and there exists a point $(\xi,t)\in K_j$ satisfying $\langle x,\xi\rangle\geqq 0\}$.

We define

$$\tilde{K}_j=\Omega_j\cup K_j. \tag{2.21}$$

Then $\tilde{K}_j$ is a neighborhood of $\operatorname{int}K_j$ in $\mathrm{Mon}_M^*(N)$ and satisfies

$$\tilde{K}_j\subset\operatorname{int}(\tilde{K}_{j+1})\ \text{and}\ \tilde{U}=\bigcup_j\tilde{K}_j. \tag{2.22}$$

Thus we have

$$\Gamma(\tilde{U},\pi^{-1}\mathcal{F})=\varprojlim_j\Gamma(\tilde{K}_j,\pi^{-1}\mathcal{F}). \tag{2.23}$$

Here we give

LEMMA 2.2. *We have an isomorphism*

$$\Gamma(\tilde{K}_j,\pi^{-1}\mathcal{F})\xleftarrow{\sim}\Gamma(\pi(\tilde{K}_j),\mathcal{F}). \tag{2.24}$$

PROOF. We can show $\tilde{K}_j$ is compact in $\mathrm{Mon}_M^*(N)$. Thus

$$p=\pi|_{\tilde{K}_j}:\tilde{K}_j\longrightarrow\pi(\tilde{K}_j)$$

is a closed map with connected fibers. We show that the canonical morphism $\mathcal{F}\to p_*p^{-1}\mathcal{F}$ is isomorphic. Take a point $q\in\pi(\tilde{K}_j)$. We have the following isomorphism.

$$(p_*p^{-1}\mathcal{F})_q=\varinjlim_{W\ni q}\Gamma(p^{-1}(q),p^{-1}\mathcal{F})\xrightarrow{\sim}\varinjlim_{V\supset p^{-1}(q)}\Gamma(V,p^{-1}\mathcal{F}). \tag{2.25}$$

Here the second isomorphism is due to the closedness of p. Moreover we have the morphisms

$$\varinjlim_{V\supset p^{-1}(q)}\Gamma(V,p^{-1}\mathcal{F})\longrightarrow\Gamma(p^{-1}(q),p^{-1}\mathcal{F})\xrightarrow{\sim}\mathcal{F}_q. \tag{2.26}$$

Here the first morphism is injective. The second one is isomorphic since p has connected fibers. After all we have a commutative diagram

$$\begin{tikzcd} & (p_*p^{-1}\mathcal{F})_q \arrow[dr, "\beta"] & \\ \mathcal{F}_q \arrow[ur, "\alpha"] \arrow[rr, "\mathrm{id}"'] & & \mathcal{F}_q. \end{tikzcd} \tag{2.27}$$

Because β is injective, α is isomorphic. We have proved that the canonical morphism

$$\mathcal{F} \longrightarrow p_* p^{-1}\mathcal{F} \tag{2.28}$$

is isomorphic. Thus

$$\Gamma(\pi(\tilde{K}_j), \mathcal{F}) \xrightarrow{\sim} \Gamma(p^{-1}(\pi(\tilde{K}_j)), p^{-1}\mathcal{F}) \xrightarrow{\sim} \Gamma(\tilde{K}_j, \pi^{-1}\mathcal{F}). \tag{2.29}$$

(q.e.d. for Lemma 2.2)

We get back to the original situations.

We take Ω and $\tilde{K}_j = \Omega_j \cup K_j$ in $\mathrm{Mon}^*_{N_0}(X_0)$ as defined above in the general situations. Moreover, since D is Stein in $\boldsymbol{C}^d$, we can take a sequence of compact analytic polyhedra in $\boldsymbol{C}^d$ satisfying

$$D_j \subset D_{j+1} \text{ and } D = \bigcup_j D_j. \tag{2.30}$$

Then we have by Lemma 2.2,

$$\Gamma(\tilde{U} \times D, \pi^{-1}\mathcal{L}^k) \xrightarrow{\sim} \varprojlim \Gamma(\pi(\tilde{K}_j) \times D_j, \mathcal{L}^k). \tag{2.31}$$

Thus we get an isomorphism

$$H^k(\tilde{U} \times D, \pi^{-1}\mathcal{O}) \simeq H^k(\varprojlim \Gamma(\pi(\tilde{K}_j) \times D_j, \mathcal{L}^\cdot)). \tag{2.32}$$

Now we quote a classical Lemma about Mittag-Leffler's argument.

LEMMA 2.3 (cf. M. Kashiwara [20]). *Let*

$$\longrightarrow \mathcal{F}^\cdot_j \longrightarrow \mathcal{F}^\cdot_{j+1} \longrightarrow \mathcal{F}^\cdot_{j+1} \longrightarrow \tag{2.33}$$

be a projective system of complexes of modules. We assume for any i $\{\mathcal{F}^i_\cdot\}$ satisfies the condition

(ML) $\{\mathrm{Im}\,(\mathcal{F}^i_{j+\nu} \to \mathcal{F}^i_j)\}_\nu$ *is stationary for any j.*

Then
1) *the canonical morphism*

$$\phi_k : H^k(\varprojlim_j \mathcal{F}^\cdot_j) \longrightarrow \varprojlim_j H^k(\mathcal{F}^\cdot_j) \tag{2.34}$$

is surjective.
2) *Moreover if $\{H^k(\mathcal{F}^\cdot_j)\}_j$ satisfies the condition* (ML), *then ϕ_{k+1} is isomorphic.*

Let $\mathcal{F}^k_j = \Gamma(\pi(\tilde{K}_j) \times D_j, \mathcal{L}^k)$. Since $\mathcal{L}^k$ is flabby, $\{\mathcal{F}^k_\cdot\}$ satisfies the condition (ML). Here we remark

$$H^k(\mathcal{F}_j^\cdot)=H^k(\pi(\tilde{K}_j)\times D_j, \mathcal{O}_X). \tag{2.35}$$

Thus if we show

$$H^k(\pi(\tilde{K}_j)\times D_j, \mathcal{O}_X)=0 \qquad (k\geqq d), \tag{2.36}$$

we can prove by Lemma 2.3

$$H^{k+1}(\tilde{\Omega}\times D, \pi^{-1}\mathcal{O}_X)=0 \qquad (k\geqq d). \tag{2.37}$$

Since $\pi(\tilde{K}_j)$ and D_j are compact, there is an isomorphism

$$H^k(\pi(\tilde{K}_j)\times D_j, \mathcal{O}_X)=\varinjlim_{W_1\times W_2\supset\pi(\tilde{K}_j)\times D_j} H^k(W_1\times W_2, \mathcal{O}_X) \tag{2.38}$$

where the inductive limit in (2.38) is taken for any open subset W_1 in C^{n-d} and any open subset W_2 in C^d satisfying the condition in (2.38). Because D_j is a compact analytic polyhedron, there exists a fundamental systems of neighborhoods of D_j which consists of Stein open sets. Thus we can show that the right side of (2.38) vanishes in case $k\geqq d$ by using Lemma 2.4. After all we have got the desired vanishing of cohomology groups. (q.e.d. for Theorem 1.1)

2.2° Lemma about the nuclear property of the sheaf $\mathcal{O}_X$

We prove a lemma concerning the nuclear property of the sheaf $\mathcal{O}_X$ of germs of holomorphic functions.

LEMMA 2.4. *Let $X=C_z^p\times C_w^q$. Let W be an open subset in C^p and D be an open subset of C^q. We assume*

$$H^j(D, \mathcal{O}_{C^q})=0 \quad (j\geqq 1) \tag{2.40}$$

and

$$\dim_C H^k(W, \mathcal{O}_{C^p})<\infty. \tag{2.41}$$

Then we have an isomorphism

$$H^k(W\times D, \mathcal{O}_X)\simeq H^k(W, \mathcal{O}_{C^p})\otimes\Gamma(D, \mathcal{O}_{C^q}). \tag{2.42}$$

PROOF. First we remark that for any open subset D of C^q, $\Gamma(D, \mathcal{O})$ is a Fréchet nuclear space. Thus by H. Komatsu [22] (see also A. Douady [19]), $\cdot\hat{\otimes}\,\Gamma(D, \mathcal{O})$ is an exact functor for topological short exact sequences of Fréchet spaces. Thus we have for any open subset U [resp. V] of C^p [resp. C^q],

$$\mathcal{L}(U)\hat{\otimes}\mathcal{O}(V)\simeq\mathcal{L}\mathcal{O}(U\times V) \tag{2.43}$$

and

$$\mathcal{O}(U)\hat{\otimes}\mathcal{O}(V)\simeq\mathcal{O}(U\times V). \tag{2.44}$$

Here $\mathcal{L}$ denotes the sheaf of smooth functions and $\mathcal{L}\mathcal{O}$ takes for that of smooth functions with holomorphic parameters $w\in \boldsymbol{C}^q$.

We can take the partial Dolbeault resolution with respect to z variables on X as

$$0\longrightarrow\mathcal{O}_X\longrightarrow\mathcal{L}^{(0,0)}\mathcal{O}\xrightarrow{\bar{\partial}_z}\mathcal{L}^{(0,1)}\mathcal{O}\xrightarrow{\bar{\partial}_z}\cdots\xrightarrow{\bar{\partial}_z}\mathcal{L}^{(0,p)}\mathcal{O}\longrightarrow 0. \tag{2.45}$$

On the other hand, since $H^j(D,\mathcal{O}_{\boldsymbol{C}^q})=0$ $(j\geqq 1)$, we can show

$$H^j(W\times D,\mathcal{L}^{(0,i)})=0 \qquad (j\geqq 1, i\geqq 0) \tag{2.46}$$

by using Andreotti-Grauert [1]. Thus

$$\begin{aligned} H^j(W\times D,\mathcal{O}_X)&\simeq H^j(\Gamma(W\times D,\mathcal{L}^{(0,\cdot)}\mathcal{O}))\\ &\simeq H^j(\Gamma(W,\mathcal{L}^{(0,\cdot)})\hat{\otimes}\Gamma(D,\mathcal{O})) \end{aligned} \tag{2.47}$$

follows.

Let

$$0\longrightarrow\mathcal{O}_{\boldsymbol{C}^p}\longrightarrow\mathcal{L}^{(0,\cdot)} \tag{2.48}$$

be the Dolbeault resolution on $\boldsymbol{C}^p$. We set

$$B^i=\mathrm{Im}\,(\Gamma(W,\mathcal{L}^{(0,i-1)})\longrightarrow\Gamma(W,\mathcal{L}^{(0,i)})) \tag{2.49}$$

and

$$Z^i=\ker\,(\Gamma(W,\mathcal{L}^{(0,i)})\longrightarrow\Gamma(W,\mathcal{L}^{(0,i+1)})). \tag{2.50}$$

Because $\dim_{\boldsymbol{C}}H^k(W,\mathcal{O}_{\boldsymbol{C}^p})<\infty$, B^k is a closed subspace in $\Gamma(W,\mathcal{L}^{(0,k)})$ by Theorem IV. 3.4.9 of H. Komatsu [21]. Thus

$$0\to Z^{k-1}\hat{\otimes}\Gamma(D,\mathcal{O})\to\Gamma(W\times D,\mathcal{L}^{(0,k-1)})\to B^k\hat{\otimes}\Gamma(D,\mathcal{O})\to 0 \tag{2.51}$$

and

$$0\to B^k\hat{\otimes}\Gamma(D,\mathcal{O})\to Z^k\hat{\otimes}\Gamma(D,\mathcal{O})\to H^k(W,\mathcal{O})\otimes\Gamma(D,\mathcal{O})\to 0 \tag{2.52}$$

are exact.

On the other hand, since the functor $\cdot\hat{\otimes}\Gamma(D,\mathcal{O})$ is a left exact functor, we have for any i

$$(2.53)_i \qquad Z^i \hat{\otimes} \Gamma(D, \mathcal{O}) \simeq \ker(\Gamma(W\times D, \mathcal{L}^{(0,i-1)}\mathcal{O}) \to \Gamma(W\times D, \mathcal{L}^{(0,i)}\mathcal{O})).$$

By (2.50) and $(2.53)_{k-1}$, we get

$$(2.54) \qquad B^k \hat{\otimes} \Gamma(D, \mathcal{O}) \simeq \mathrm{Im}\,(\Gamma(W\times D, \mathcal{L}^{(0,k-1)}) \to \Gamma(W\times D, \mathcal{L}^{(0,k)})).$$

After all,

$$(2.55) \qquad H^k(\Gamma(W\times D, \mathcal{L}^{(0,\cdot)}) \simeq H^k(W, \mathcal{O}_{C^p}) \hat{\otimes} \Gamma(D, \mathcal{O}^1_{C^q})$$

follows from $(2.53)_k$, (2.54) and (2.52).

3° Sheaf of microfunctions with smooth and holomorphic parameters

We define the sheaf of microfunctions with both holomorphic parameters and smooth parameters and give some vanishing theorems concerning it.

Let $X = \boldsymbol{C}^{n_1}_w \times \boldsymbol{C}^{n_2}_z \times \boldsymbol{R}^{n_3}_u$ $(w = t + \sqrt{-1}\,s)$ and $N = \{\mathrm{Im}\, w = 0\} \cap X \simeq \boldsymbol{R}^{n_1}_t \times \boldsymbol{C}^{n_2}_z \times \boldsymbol{R}^{n_3}_u$. We put $N_0 = \boldsymbol{R}^{n_1}_t$ and $X_0 = \boldsymbol{C}^{n_1}$. Then we have

$$(3.1) \qquad \mathrm{Mon}^*_N(X) \simeq \mathrm{Mon}^*_{N_0}(X_0) \times \boldsymbol{C}^{n_2} \times \boldsymbol{R}^{n_3}.$$

For a complex manifold W, there exists the sheaf $\mathcal{O}_W\mathcal{O}\mathcal{L}$ of smooth functions on $W \times \boldsymbol{C}^{n_2}_z \times \boldsymbol{R}^{n_3}_u$ which depend holomorphically on $W \times \boldsymbol{C}^{n_2}_z$. The shenf $\mathcal{O}_W\mathcal{O}\mathcal{L}$ is also denoted by $\mathcal{F}_W$ in this section. $\mathcal{F}_{C^{n_1}}$, which is the sheaf on X, is also expressed as $\mathcal{O}\mathcal{O}\mathcal{L}$.

Here we give

PROPOSITION 3.1.

$$(3.2) \qquad \mathcal{H}^k(\mu_N(\mathcal{O}\mathcal{O}\mathcal{L})) = 0 \qquad (k \neq n_1).$$

PROOF. $\mathcal{F}_W$'s trivially satisfy the condition (H1) and (H2) in the subsection 2.2° when we put $T = \boldsymbol{C}^{n_2} \times \boldsymbol{R}^{n_3}$.

When Z is a compact complex manifold, we have

$$(3.3) \qquad \dim_C H^k(Z, \mathcal{O}_Z) < \infty \qquad (\text{for } k \geqq 0)$$

by a theorem of Cartan. Using Proposition 1.4, we can show that $\mathcal{F}_W$'s satisfy the condition (H3) in 1.2°. Thus we can apply Theorem 1.3 for $\mathcal{F}_W$'s.

We identify

(3.4) $$S_N^*X \simeq \sqrt{-1}\, S^*\boldsymbol{R}^{n_1} \times \boldsymbol{C}^{n_2} \times \boldsymbol{R}^{n_3}$$

and take a coordinate of S_N^*X as $(t, \sqrt{-1}\,\tau dt\infty ; z, u)$ with $\tau \in \boldsymbol{R}^{k_1} \setminus \{0\}$.

Take a point $\rho \in S_N^*X$. We may assume from the begining $\rho = (0, \sqrt{-1}\,\tau_0 dt\infty ; 0, 0)$ with $\tau_0 = (1, 0, \cdots, 0)$.

We take n_1 points of $\boldsymbol{R}^{n_1} \setminus \{0\} : \tau_1, \cdots, \tau_{n_1}$ such that the convex hull of $\{\tau_0, \tau_1, \cdots, \tau_{n_1}\}$ is a neighborhood of the origin in $\boldsymbol{R}^{n_1}$. We put $\tau = (\tau_1, \cdots, \tau_{n_1})$ and

(3.5) $$G_\tau = \{(w, z, u) \in X ; \langle \operatorname{Im} w, \tau_l \rangle \geqq 0 \quad (l = 1, \cdots, n_1)\}.$$

Then we have

(3.6) $$\mathscr{H}^k(\mu_N(\mathcal{O}\mathcal{O}\mathcal{L}))_\rho \simeq \varinjlim_\tau \mathscr{H}^k_{G_\tau}(\mathcal{O}\mathcal{O}\mathcal{L})_{\pi(\rho)}$$

where the inductive limit is taken for τ satisfying the condition above.

Applying Theorem 1.3, we have

(3.7) $$\mathscr{H}^k_{G_\tau}(\mathcal{O}\mathcal{O}\mathcal{L})_{\pi(\rho)} = 0 \qquad (k < n_1)$$

thus

(3.8) $$\mathscr{H}^k(\mu_N(\mathcal{O}\mathcal{O}\mathcal{L}))_\rho = 0 \qquad (k < n_1).$$

Let for a positive number δ

(3.9) $$U_\delta = \{(w, z, u) \in X ; |w| < \delta, |z| < \delta, |u| < \delta\}$$

and

(3.10) $$U_{\delta, l} = \{(w, z, u) \in U_\delta ; \langle \operatorname{Im} w, \tau_l \rangle < 0\}.$$

We remark that

(3.11) $$\mathscr{H}^k_{G_\tau}(\mathcal{O}\mathcal{O}\mathcal{L})_{\pi(\rho)} \simeq \varinjlim_\delta H^k_{G_\tau}(U_\delta, \mathcal{O}\mathcal{O}\mathcal{L})$$

and

(3.12) $$U_\delta \setminus G_\tau = \bigcup_{l=1}^{n_1} U_{\delta, l}.$$

Because $\{U_\delta, U_{\delta,1}, \cdots, U_{\delta, n_1}\}$ is a Leray covering for $\mathcal{O}\mathcal{O}\mathcal{L}$.

(3.13) $$H^k_{G_\tau}(U_\delta, \mathcal{O}\mathcal{O}\mathcal{L}) = 0 \qquad (k > n_1)$$

holds. Thus we have

$$\mathcal{H}^k_{G_\tau}(\mathcal{OOL})_{\pi(\rho)}=0 \qquad (k>n_1). \tag{3.14}$$

(q.e.d.)

DEFINITION 3.2. We set

$$\mathcal{COL}=\mathcal{H}^{n_1}(\mu_N(\mathcal{OOL})). \tag{3.15}$$

DEFINITION 3.3 (Andreotti-Grauert [1], Kataoka [3]). Let D be a connected open subset of $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$. D is called a regular family of Stein domains if the following conditions (3.16) and (3.17) are satisfied.

(3.16) Let $\pi:\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}\to\boldsymbol{R}^{n_3}$ be the natural projection. For any $x\in\pi(D)$, $\pi^{-1}(x)$ is Stein.

(3.17) For any $x\in\pi(D)$, there exists an open subset W_x in $\boldsymbol{C}^{n_2}$ and an open neighborhood U_x of x in $\boldsymbol{R}^{n_3}$ such that both $(W_x\times U_x, \pi^{-1}(U_x)\cap D)$ and $(\pi^{-1}(x)\cap D, W_x)$ are Runge pairs.

By Andreotti-Grauert [1], if D is a regular family of Stein domains in $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$, we have

$$H^j(D,\mathcal{OL})=0 \qquad (j>0). \tag{3.18}$$

Here $\mathcal{OL}$ is the sheaf of smooth functions on $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$ which are holomorphic with respect to the variables of $\boldsymbol{C}^{n_2}$.

Moreover, there exists a sequence of compact analytic polyhedra $\{Q_i\}_i$ in $\boldsymbol{C}^{n_2}\times\boldsymbol{R}^{n_3}$ satisfying

$$Q_i\subset Q_{i+1} \text{ and } D=\bigcup_i Q_i \tag{3.19}$$

and

(3.20) Q_i has a fundamental system of neighborhoods $\{W_\lambda\}$ such that $H^j(W_\lambda,\mathcal{OL})=0$ $(j>0)$ for any λ.

Using Proposition 1.4 and the remarks above, we can prove the following theorem in the same way as Theorem 1.1.

THEOREM 3.4. *Let U be a proper convex subset of $\sqrt{-1}S^*\boldsymbol{R}^{n_1}$ and D be a regular family of Stein domain. Then we have*

$$H^j(U\times D,\mathcal{COL})=0 \qquad (j>0). \tag{3.21}$$

We settle another notation. Let

$$X=\boldsymbol{C}^{n_1}\times\boldsymbol{R}^{n_2},\ N=\boldsymbol{R}^{n_1}\times\boldsymbol{C}^{n_2},\ \hat{X}=X\times\boldsymbol{R}^{n_3} \text{ and } \hat{N}=N\times\boldsymbol{R}^{n_3}. \tag{3.22}$$

We set

$$p:\hat{X}\longrightarrow X \text{ and } p:\mathrm{Mon}^*_{\hat{N}}(\hat{X})\longrightarrow\mathrm{Mon}^*_N(X). \tag{3.23}$$

$S^*_N X$ is endowed with the sheaf

$$\mathcal{CL}=\mathcal{H}^{n_1}(\mu_N(\mathcal{OL})) \tag{3.24}$$

of microfunctions with smooth parameters where $\mathcal{OL}$ is the sheaf of smooth functions on X holomorphic with respect to the variables of C^{n_1}. Here we give a vanishing theorem for $p^{-1}\mathcal{CL}$.

THEOREM 3.5. *We take an open subset D of $R^{n_2}\times R^{n_3}$ such that there exists a sequence of open subsets in D: $\{W_j\}$ satisfying*

$$\bigcup_j W_j=D \text{ and } W_j\Subset W_{j+1}\Subset D \tag{3.25}$$

and

(3.26) *for any $x\in R^{n_2}$, $p^{-1}(x)\cap W_j$ and $p^{-1}(x)\cap\overline{W}_j$ and $p^{-1}(x)\cap D$ are contractible $(p:R^{n_2}\times R^{n_3}\to R^{n_2})$. Then for any open proper convex subset U in $\sqrt{-1}S^*R^{n_1}$, we have*

$$H^k(U\times D, p^{-1}\mathcal{CL})=0 \qquad (k>0). \tag{3.27}$$

PROOF. First of all, we have

$$p^{-1}\mathcal{CL}=p^{-1}\mu_N(\mathcal{OL})\,[n_1]\simeq\mu_{\hat{N}}(p^{-1}\mathcal{OL})\,[n_1] \tag{3.28}$$

by Lemma 2.2.3 of Chapter 1 of Sato-Kawai-Kashiwara [13]. Thus

$$H^k(U\times D, p^{-1}\mathcal{CL})\simeq H^k_{U^a\times D}(\tilde{U}\times D, \pi^{-1}p^{-1}\mathcal{OL}) \tag{3.29}$$

follows with $\tilde{U}=U^a\cup\Omega$ (U^a denotes the image of U by the antipodal map $a:S^*_{\hat{N}}\hat{X}\to S^*_{\hat{N}}\hat{X}$). Here $\tilde{U}$ is an open subset of $\mathrm{Mon}^*_{R^{n_1}}(C^{n_1})$ and Ω is an open set in $C^{n_1}\setminus R^{n_1}$ and π is the comonoidal map $\pi:\mathrm{Mon}^*_{\hat{N}}(\hat{X})\to\hat{X}$.

Consider the long exact sequence

$$\begin{aligned}&\to H^k_{U^a\times D}(\tilde{U}\times D, \pi^{-1}p^{-1}\mathcal{OL})\to H^k(\tilde{U}\times D, \pi^{-1}p^{-1}\mathcal{OL})\\&\to H^k(\Omega\times D, p^{-1}\mathcal{OL})\to\end{aligned} \tag{3.30}$$

and a resolution of $p^{-1}\mathcal{OL}$

$$0\to p^{-1}\mathcal{OL}\to p^{-1}\mathcal{L}^{(0,\cdot)}\mathcal{L} \tag{3.31}$$

which is the inverse image of Dolbeault resolution of $\mathcal{OL}$ on X. By

Theorem 2.1.4 of K. Kataoka [3], we have

$$H^j(\Omega\times D, p^{-1}\mathcal{L}^{(0,k)}\mathcal{L})=0 \quad (j\geqq 1, k\geqq 0). \tag{3.32}$$

Thus

$$H^k(\Omega\times D, p^{-1}\mathcal{O}\mathcal{L})\simeq H^k(\Gamma(\Omega\times D, p^{-1}\mathcal{L}^{(0,\cdot)}\mathcal{L})) \tag{3.33}$$

follows. Moreover, since p is open and with connected fibers, the right side of (3.33) is isomorphic to $H^k(\Gamma(\Omega\times p(D), \mathcal{L}^{(0,\cdot)}\mathcal{L}))$. Thus we have

$$H^k(\Omega\times D, p^{-1}\mathcal{O}\mathcal{L})\simeq H^k(\Gamma(\Omega\times p(D), \mathcal{L}^{(0,\cdot)}\mathcal{L}))\simeq H^k(\Omega\times p(D), \mathcal{O}\mathcal{L}). \tag{3.34}$$

Here the last term in (3.34) vanishes in case $k\geqq n_1$. Hence we have

$$H^k_{\tilde{U}^a\times D}(\tilde{U}\times D, \pi^{-1}p^{-1}\mathcal{O}\mathcal{L})\simeq H^k(\tilde{U}\times D, \pi^{-1}p^{-1}\mathcal{O}\mathcal{L}) \qquad (k>n_1). \tag{3.35}$$

In the same way as the proof of Theorem 1.1, it is enough to prove

$$H^k(K\times \overline{W}_j, p^{-1}\mathcal{O}\mathcal{L})=0 \qquad (k>n_1) \tag{3.36}$$

for a compact subset K in C^{n_1}. But we have

$$H^k(K\times \overline{W}_j, p^{-1}\mathcal{O}\mathcal{L})\simeq H^k(K\times p(\overline{W}_j), \mathcal{O}\mathcal{L}), \tag{3.37}$$

because $p|_{K\times \overline{W}_j}$ is proper and with contractible fibers and $K\times \overline{W}_j$ is Hausdorff. Here right side of (3.37) vanishes in case $k>n_1$. (q.e.d.)

4° Cohomological Radon transformation for 2-microfunctions

4.0° We construct the cohomological Radon transformation for $\mathcal{B}^2_\Lambda$ and C^2_Λ in the same way as K. Kataoka [3].

Let $X=C^{n-d}_w\times C^d_z$ $(w=t+\sqrt{-1}\,s,\ z=x+\sqrt{-1}\,y)$ and

$$N=\{\mathrm{Im}\, w=0\}\cap X\simeq R^{n-d}_t\times C^d_z. \tag{4.1}$$

4.1° Take an r-dimensional complex manifold Y. We denote the sheaf of holomorphic relative l-forms with respect to Y on $X\times Y$ by $\mathcal{O}_X\mathcal{O}^{(l)}_Y$. Then we have an exact sequence on $X_1=X\times Y$

$$0\to p^{-1}\mathcal{O}_X\to\mathcal{O}_X\mathcal{O}^{(0)}_Y\to\mathcal{O}_X\mathcal{O}^{(1)}_Y\to\cdots\to\mathcal{O}_X\mathcal{O}^{(r)}_Y\to 0 \tag{4.2}$$

with $p: X_1=X\times Y\to X$. We put $N_1=N\times Y$ and microlocalize the exact sequence (4.2) along N_1. Then we can obtain an exact sequence (by Lemma 2.2.3 of Chapter 1 of Sato et al. [13])

$$0\to p^{-1}\mathcal{CO}\to\mathcal{COO}^{(0)}\to\mathcal{COO}^{(1)}\to\cdots\to\mathcal{COO}^{(r)}\to 0 \tag{4.3}$$

with $p: S^*_{N_1}X_1\simeq S^*_N X\times Y\to S^*_N X$. Here

$$\mathcal{CO}=\mathcal{H}^{n-d}(\mu_N(\mathcal{O}_X)) \tag{4.4}$$

and

$$\mathcal{COO}^{(l)}=\mathcal{H}^{n-d}(\mu_{N_1}(\mathcal{O}_X\mathcal{O}_Y^{(l)})). \tag{4.5}$$

On the other hand, we consider the relative Dolbeault resolution of $\mathcal{O}_X\mathcal{O}_Y^{(l)}$ with respect to $\boldsymbol{C}^d_z$

$$0\to\mathcal{O}_X\mathcal{O}_Y\to\mathcal{O}_{C^{n-d}}\mathcal{L}^{(0,\cdot)}\mathcal{O}_Y. \tag{4.6}$$

When we microlocalize (4.6) along N_1, we have an exact sequence

$$0\to\mathcal{COO}\to\mathcal{CL}^{(0,\cdot)}\mathcal{O}. \tag{4.7}$$

4.2° Take a smooth manifold Y of dimension d instead. $\mathcal{O}_{C^{n-d}}\mathcal{L}_{C^d}\mathcal{L}^{(l)}$ denotes the sheaf of smooth relative l-forms with respect to Y on $X_1=X\times Y$ depending holomorphically on $\boldsymbol{C}^{n-d}$. $\mathcal{OL}$ stands for the sheaf of smooth functions on X which are holomorphic in w.

Consider the relative de Rham's resolution on X_1:

$$0\to p^{-1}\mathcal{OL}\to\mathcal{O}_{C^{n-d}}\mathcal{L}_{C^d}\mathcal{L}_Y^{(\cdot)}. \tag{4.8}$$

Here $p: X\times Y\to X$. We microlocalize (4.8) along N_1 and obtain an exact sequence on $S^*_{N_1}X_1$:

$$0\to p^{-1}\mathcal{CL}\to\mathcal{CLL}^{(0)}\to\mathcal{CLL}^{(1)}\to\cdots\to\mathcal{CLL}^{(r)}\to 0 \tag{4.9}$$

with $p: S^*_{N_1}X_1\simeq S^*_N X\times Y\to S^*_N X$. Here

$$\mathcal{CL}=\mathcal{H}^{n-d}(\mu_N(\mathcal{OL})) \tag{4.10}$$

and

$$\mathcal{CLL}^{(l)}=\mathcal{H}^{n-d}(\mu_{N_1}(\mathcal{O}_{C^{n-d}}\mathcal{L}_{C^d}\mathcal{L}_Y^{(l)})). \tag{4.11}$$

On the other hand, $\mathcal{O}_X\mathcal{L}_Y^{(l)}$ is the sheaf of smooth relative l-forms with respect to Y on X_1 holomorphic with respect to (w, z). We can obtain the following exact sequence on $S^*_{N_1}X_1$ in the same way as (4.9).

$$0\to p^{-1}\mathcal{CO}\to\mathcal{COL}^{(0)}\to\mathcal{COL}^{(1)}\to\cdots\to\mathcal{COL}^{(r)}\to 0 \tag{4.12}$$

where

$$\mathcal{COL}^{(l)}=\mathcal{H}^{n-d}(\mu_{N_1}(\mathcal{O}_X\mathcal{L}_Y^{(l)})). \tag{4.13}$$

4.3° We follow the notation of 4.1° and take a complex manifold Y. Consider the diagram

$$\begin{array}{ccc} N_1=\boldsymbol{R}_t^{n-d}\times \boldsymbol{C}_z^d\times Y & \longrightarrow & \boldsymbol{C}_w^{n-d}\times \boldsymbol{C}_z^d\times Y=X_1 \\ \big\downarrow p & & \big\downarrow p \\ N=\boldsymbol{R}_t^{n-d}\times \boldsymbol{C}_z^d & \longrightarrow & \boldsymbol{C}_w^{n-d}\times \boldsymbol{C}_z^d=X \end{array} \tag{4.14}$$

and the commutative diagram on $S_{N_1}^*X_1$

$$\begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \to & p^{-1}\mathcal{CO} & \longrightarrow & \mathcal{COO}^{(0)} & \xrightarrow{d_Y} & \mathcal{COO}^{(1)} & \xrightarrow{d_Y} & \mathcal{COO}^{(2)} & \to \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \to & p^{-1}\mathcal{CL}^{(0,0)} & \to & \mathcal{CL}^{(0,0)}\mathcal{O}_Y^{(0)} & \to & \mathcal{CL}^{(0,0)}\mathcal{O}_Y^{(1)} & \to & \mathcal{CL}^{(0,0)}\mathcal{O}_Y^{(2)} & \to \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 0 & \to & p^{-1}\mathcal{CL}^{(0,1)} & \to & \mathcal{CL}^{(0,1)}\mathcal{O}_Y^{(0)} & \to & \mathcal{CL}^{(0,1)}\mathcal{O}_Y^{(1)} & \to & \mathcal{CL}^{(0,1)}\mathcal{O}_Y^{(2)} & \to . \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow \end{array} \tag{4.15}$$

Here $\mathcal{CL}^{(0,l)}$ is obtained by the microlocalization along N of the sheaf $\mathcal{O}_{C^{n-d}}\mathcal{L}^{(0,l)}$ of smooth relative $(0,l)$-forms on X with respect to $\boldsymbol{C}^d$ depending holomorphically in w. Explicitly

$$\mathcal{CL}^{(0,l)}=\mathcal{H}^{n-d}(\mu_N(\mathcal{O}_{C^{n-d}}\mathcal{L}^{(0,l)})). \tag{4.16}$$

$\mathcal{CL}^{(0,l)}\mathcal{O}_Y$ is derived from the sheaf $\mathcal{O}_{C^{n-d}}\mathcal{L}^{(0,l)}\mathcal{O}_Y$ of smooth relative $(0,l)$-forms with respect to $\boldsymbol{C}_z^d$ which are holomorphic in $\boldsymbol{C}^{n-d}$ and Y:

$$\mathcal{CL}^{(0,l)}\mathcal{O}_Y=\mathcal{H}^{n-d}(\mu_{N_1}(\mathcal{O}_{C^{n-d}}\mathcal{L}^{(0,l)}\mathcal{O}_Y)). \tag{4.17}$$

Take a Stein open subset D in $\boldsymbol{C}^{n-d}\times Y$ satisfying the assumptions of Theorem 3.4 and Theorem 3.5 and an open subset U in $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$. Then in the commutative diagram

$$\begin{array}{ccccccccc} & & & & 0 & & 0 & & \\ & & & & \downarrow & & \downarrow & & \\ & & 0 & \longrightarrow & \Gamma(U\times D, \mathcal{COO}^{(0)}) & \longrightarrow & \Gamma(U\times D, \mathcal{COO}^{(1)}) & \longrightarrow \\ & & \downarrow & & \downarrow & & \downarrow & \\ 0 & \to & \Gamma(U\times D, p^{-1}\mathcal{CL}^{(0,0)}) & \to & \Gamma(U\times D, \mathcal{CL}^{(0,0)}\mathcal{O}^{(0)}) & \to & \Gamma(U\times D, \mathcal{CL}^{(0,0)}\mathcal{O}^{(1)}) & \to \\ & & \downarrow & & \downarrow & & \downarrow & \\ 0 & \to & \Gamma(U\times D, p^{-1}\mathcal{CL}^{(0,1)}) & \to & \Gamma(U\times D, \mathcal{CL}^{(0,1)}\mathcal{O}^{(0)}) & \to & \Gamma(U\times D, \mathcal{CL}^{(0,1)}\mathcal{O}^{(1)}) & \to \\ & & \downarrow & & \downarrow & & \downarrow & \end{array} \tag{4.18}$$

all columns and all rows are exact except the first row and the first column. Then by a lemma of Weil, we have the isomorphism

$$H^k(\Gamma(U\times D, \mathcal{C}\mathcal{O}\mathcal{O}^{(\cdot\cdot)}))\simeq H^k(\Gamma(U\times D, p^{-1}\mathcal{C}\mathcal{L}^{(0,\cdot\cdot)})). \tag{4.19}$$

Here the right side of (4.19) is isomorphic to $H^k(\Gamma(U\times p(D), \mathcal{C}\mathcal{L}^{(0,\cdot\cdot)}))$ ($p: C^d\times Y\to C^d$) because p is open and with connected fibers. Moreover, since

$$H^k(U\times p(D), \mathcal{C}\mathcal{L}^{(0,l)})=0 \qquad (k\geqq 1, l\geqq 0), \tag{4.20}$$

the isomorphism

$$H^k(\Gamma(U\times D, \mathcal{C}\mathcal{O}\mathcal{O}^{(\cdot\cdot)}))\simeq H^k(U\times p(D), \mathcal{C}\mathcal{O}) \tag{4.21}$$

follows.

4.4° We follow the notation prepared in 4.2°. That is,

$$\begin{array}{ccc} N_1=R_t^{n-d}\times C_z^d\times Y & \longrightarrow & C_w^{n-d}\times C_z^d\times Y=X_1 \\ \downarrow p & & \downarrow p \\ N=\ R_t^{n-d}\times C_z^d & \longrightarrow & C^{n-d}\times C_z^d=X. \end{array} \tag{4.22}$$

Take an open subset D in $C^d\times Y$ satisfying the assumption of Theorem 3.4 and Theorem 3.5. When we consider the resolution (4.11) i.e.

$$0\to p^{-1}\mathcal{C}\mathcal{O}\to\mathcal{C}\mathcal{O}\mathcal{L}^{(\cdot\cdot)}, \tag{4.23}$$

we can deduce the isomorphism

$$H^k(\Gamma(U\times D, \mathcal{C}\mathcal{O}\mathcal{L}^{(\cdot\cdot)}))\simeq H^k(U\times p(D), \mathcal{C}\mathcal{O}) \tag{4.24}$$

with $p: C^d\times Y\to C^d$.

4.5° Radon transformation for $\mathcal{C}_\Lambda^2$ with smooth parameters

We follow the notation prepared in 4.0°. We also set

$$M=\{(w, z)\in X; \operatorname{Im} z=0, \operatorname{Im} w=0\}\simeq R_t^d\times R_x^{n-d}. \tag{4.25}$$

We take a coordinate of S_M^*X ($\simeq\sqrt{-1}\,S^*M$) as $(t, x; \sqrt{-1}(\tau dt+\xi dx)\infty)$ and set

$$\Lambda=\{(x, t; \sqrt{-1}\,(\tau, \xi))\in S_M^*X; \xi=0\}\simeq\sqrt{-1}\,S^*R^{n-d}\times R_x^d \tag{4.26}$$

and

$$\tilde{\Lambda}=S_N^*X\simeq\sqrt{-1}\,S^*R^{n-d}\times C_z^d. \tag{4.27}$$

We consider the diagram

$$
(4.28)\qquad
\begin{tikzcd}
\sqrt{-1}\,S^*\mathbf{R}^{n-d}\times \mathbf{C}^d_z\times S^{d-1}_\xi \arrow[r, "p"] \arrow[d, "\tau"] & \sqrt{-1}\,S^*\mathbf{R}^{n-d}\times \mathbf{C}^d_z=\tilde{\Lambda} \\
\sqrt{-1}\,S^*\mathbf{R}^{n-d}\times \mathbf{R}^d_x\times S^{d-1}_\xi \arrow[r] & \sqrt{-1}\,S^*\mathbf{R}^{n-d}\times \mathbf{R}^d_x=\Lambda. \arrow[u, hook]
\end{tikzcd}
$$

We represent a point of $\sqrt{-1}\,S^*\mathbf{R}^{n-d}$ by ρ^* for short.

For a C^2 function $g:[0,\infty)\to[0,\infty)$ satisfying $g(0)=g'(0)=0$ and $g'(x), g''(x)\geqq 0$, we define

$$(4.29)\qquad D_{g,\varepsilon}=\{(\rho^*, z, \xi)\in\tilde{\Lambda}\times S^{d-1}\,;\,|y|<\varepsilon,\ y\xi-g(\{y^2-(y\xi)^2\}^{1/2})>0\}$$

with $y=\operatorname{Im} z$. Let

$$(4.30)\qquad \mathcal{CS}^k_{g,\varepsilon}=(\tau|D_{g,\varepsilon})_*\mathcal{CO}\mathcal{L}^{(k)}$$

and

$$(4.31)\qquad \mathcal{CS}^k_g=\varinjlim_{\varepsilon}\mathcal{CS}^k_{g,\varepsilon}.$$

When we identify

$$(4.32)\qquad S^*_\Lambda\tilde{\Lambda}\simeq\sqrt{-1}\,S^*\mathbf{R}^{n-d}\times\mathbf{R}^d_x\times S^{d-1}_\xi$$

and regard $\mathcal{CS}^k_g$ as a sheaf on $S^*_\Lambda\tilde{\Lambda}$, we can give

THEOREM 4.1. *We have the exact sequences*

$$(4.33)\qquad 0\to\pi^{-1}\mathcal{A}^2_\Lambda\to\mathcal{CS}^0_g\to\mathcal{CS}^1_g\to\cdots\to\mathcal{CS}^{d-1}_g\xrightarrow{\sigma}\mathcal{C}^2_\Lambda\to 0$$

on $S^*_\Lambda\tilde{\Lambda}$ *and*

$$(4.34)\qquad 0\to\mathcal{A}^2_\Lambda\to p_*\mathcal{CS}^0_g\to p_*\mathcal{CS}^1_g\to\cdots\to p_*\mathcal{CS}^{d-1}_g\to\mathcal{B}^2_\Lambda\to 0$$

on Λ. *Here* $\pi:S^*_\Lambda\tilde{\Lambda}\to\Lambda$.

We can prove the theorem above in the same way as K. Kataoka [3] using (4.24) and the theorem of edge of wedge for $\mathcal{CO}$.

REMARK 4.2. Take an open proper convex subset U in $\sqrt{-1}\,S^*\mathbf{R}^{n-d}$ and an open convex subset V in $\mathbf{R}^d_x$. Then $\{U\times(V+\sqrt{-1}\,\mathbf{R}^d_y)\times S^{d-1}\}\cap D_{g,\varepsilon}$ satisfies the assumptions of Theorem 3.4 and Theorem 3.5. Thus we have

$$(4.35)\qquad H^{d-1}(\Gamma(U\times V\times S^{d-1}, \mathcal{CS}^{\boldsymbol{\cdot}}_{g,\varepsilon}))\simeq H^{d-1}(U\times W, \mathcal{CO}).$$

Here $W=(V+\sqrt{-1}\{|y|<\varepsilon\}\setminus \boldsymbol{R}^d$. Thus we have

$$H^{d-1}(U\times W, \mathcal{CO})\simeq H^d_{U\times V}(U\times V^c, \mathcal{CO})\simeq \Gamma(U\times V, \mathcal{B}^2_\Lambda) \tag{4.36}$$

for $d\geqq 2$, where V^c is a Stein neighborhood of V in $\boldsymbol{C}^d$. Hence any 2-hyperfunction is represented by a global section of $\mathcal{COL}^{(d-1)}$. This fact can be shown directly in case $d=1$.

4.6° Radon transformation for C^2_Λ with real analytic parameters (plane wave decomposition)

We consider the situation in (4.26) and (4.27). We set

$$N_\varepsilon=\{\zeta\in \boldsymbol{C}^d_\zeta\,;\, \zeta^2=-1, |\operatorname{Re}\zeta|<\varepsilon\}\ (\zeta=\xi+\sqrt{-1}\eta) \tag{4.37}$$

and

$$\begin{array}{ccc} \sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times \boldsymbol{C}^d_z\times N_\varepsilon & \xrightarrow{p} & \sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times \boldsymbol{C}^d_z=\tilde{\Lambda} \\ \Big\downarrow \tau & & \Big\Uparrow \\ \sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times \boldsymbol{R}^d_x\times \sqrt{-1}\,S^{d-1} & \longrightarrow & \sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times \boldsymbol{R}^d_x=\Lambda \end{array} \tag{4.38}$$

where τ is given as $(\rho^*, z, \zeta)\to(\rho^*, \operatorname{Re} z, \sqrt{-1}\operatorname{Im}\zeta/|\operatorname{Im}\zeta|)$. We put

$$D_\varepsilon=\{(\rho^*, z, \zeta)\in \sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times \boldsymbol{C}^d_z\times N_\varepsilon\,;\, |y|<\varepsilon, \operatorname{Re}(z\zeta)+(|\xi|/\varepsilon)<0\} \tag{4.39}$$

and

$$\mathcal{CJ}^j_\varepsilon=(\tau|_{D_\varepsilon})_*\mathcal{COO}^{(j)} \tag{4.40}$$

and

$$\mathcal{CJ}^j=\varinjlim_{\varepsilon}\mathcal{CJ}^j_\varepsilon. \tag{4.41}$$

We regard $\mathcal{CJ}^j$ as the sheaf on $S^*_\Lambda\tilde{\Lambda}$ and give

THEOREM 4.3. *We have the exact sequences*

$$0\to\pi^{-1}\mathcal{A}^2_\Lambda\to\mathcal{CJ}^0\to\mathcal{CJ}^1\to\cdots\to\mathcal{CJ}^{d-1}\to\mathcal{C}^0_\Lambda\to 0 \tag{4.42}$$

on $S^*_\Lambda\tilde{\Lambda}$ and

$$0\to\mathcal{A}^2_\Lambda\to p_*\mathcal{CJ}^0\to p_*\mathcal{CJ}^1\to\cdots\to p_*\mathcal{CJ}^{d-1}\to\mathcal{B}^2_\Lambda\to 0 \tag{4.43}$$

on Λ.

We can prove the theorem above in the analogous way as K. Kataoka [3].

5° Morphism $\mathcal{C}|_\Lambda \to \mathcal{B}_\Lambda^2$

5.1° Let $M=\boldsymbol{R}_t^{n-d}\times\boldsymbol{R}_x^d$ and $X=\boldsymbol{C}_w^{n-d}\times\boldsymbol{C}_z^d$. We take a coordinate of S_M^*X as $(t, x\,;\sqrt{-1}\,(\tau dt+\xi dx)\infty)$ and set

$$\Lambda=\{(t, x\,;\sqrt{-1}\,(\tau, \xi)\infty)\in S_M^*X\,;\,\xi=0\}\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{R}^d. \tag{5.1}$$

We put $N=\boldsymbol{R}_t^{n-d}\times\boldsymbol{C}_z^d$ in X and

$$\tilde{\Lambda}=S_N^*X\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}_z^d. \tag{5.2}$$

It is shown in Kashiwara-Laurent [7] that there exists a canonical morphism $\mathcal{C}_M|_\Lambda\to\mathcal{B}_\Lambda^2$. Moreover the morphism above is proved to be injective. In this section we construct another morphism $\mathcal{C}_M|_\Lambda\to\mathcal{B}_\Lambda^2$ through the Radon transformation for $\mathcal{C}_M$ and $\mathcal{B}_\Lambda^2$.

5.2° First we take the Radon transformation of $\mathcal{C}_M$ with smooth parameters following K. Kataoka [3]. We regard S_M^*X as $\boldsymbol{R}_{(t,x)}^n\times S_{(\tau,\xi)}^{n-1}$ and define the sheaf $\mathcal{S}^k$ as follows. We set in $\boldsymbol{C}^n\times S^{n-1}$

$$D_\varepsilon=\{(w, z\,;\tau, \xi)\,;\,|\mathrm{Im}\,w|<\varepsilon, |\mathrm{Im}\,z|<\varepsilon, \langle\mathrm{Im}\,w, \tau\rangle+\langle\mathrm{Im}\,z, \xi\rangle>0\} \tag{5.3}$$

and $\tau:\boldsymbol{C}^n\times S^{n-1}\to\boldsymbol{R}^n\times S^{n-1}$

$$(w, z\,;(\tau, \xi)\infty)\longmapsto(\mathrm{Re}\,w, \mathrm{Re}\,z, (\tau, \xi)\infty).$$

We define the sheaf $\mathcal{S}^k$ on S_M^*X as

$$\mathcal{S}^k=\varinjlim_{\varepsilon}(\tau|_{D_\varepsilon})_*\,\mathcal{OL}^{(k)} \tag{5.4}$$

where $\mathcal{OL}^{(k)}$ is the sheaf of smooth relative k-forms with respect to S^{n-1} on $\boldsymbol{C}^n\times S^{n-1}$ depending holomorphically on $\boldsymbol{C}^n$. Then we have an exact sequence

$$0\to\pi^{-1}\mathcal{A}_M\to\mathcal{S}^0\xrightarrow{d_{(\tau,\xi)}}\mathcal{S}^1\to\cdots\to\mathcal{S}^{n-1}\xrightarrow{\sigma}\mathcal{C}_M\to0 \tag{5.5}$$

on S_M^*X by K. Kataoka [3].

5.3° On the other hand, we have

$$0\to\mathcal{A}_\Lambda^2\to p_*\mathcal{CS}^0\xrightarrow{d_\xi}p_*\mathcal{CS}^1\to\cdots\to p_*\mathcal{CS}^{d-1}\to\mathcal{B}_\Lambda^2\to0 \tag{5.6}$$

by Theorem 4. 1.

5.4° Take a point $\rho_0=(0, 0\,;\sqrt{-1}\,\tau_0 dt\infty)\in\Lambda$ with $\tau_0=(1, 0, \cdots, 0)$. Then

$f\in\mathcal{C}_M|_{\rho_0}$ can be expressed as

$$f=\sigma(F(w,z;\tau,\xi)\,d\sigma(\tau,\xi)) \tag{5.7}$$

by $F\in\mathcal{OL}(\{|w|<\varepsilon,\ |z|<\varepsilon,\ |(\tau,\xi)-(\tau_0,0)|<\varepsilon,\ \langle\mathrm{Im}\,w,\tau\rangle+\langle\mathrm{Im}\,z,\xi\rangle>0\})$. Here $d\sigma(\tau,\xi)$ denotes the standard volume form on S^{n-1}.

In the same way, $g\in\mathcal{B}^2_\Lambda$ can be written as

$$g=\sigma(G(t,z,\xi)d\sigma(\xi)) \tag{5.8}$$

by $G\in\mathcal{COL}(\{(t,\sqrt{-1}\,\tau dt\infty;z,\xi)\in\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1};\ |t|<\varepsilon,\ |\tau-\tau_0|<\varepsilon,\ \langle\mathrm{Im}\,z,\xi\rangle>0,\ |z|<\varepsilon\})$. Here $\mathcal{COL}$ is the sheaf on $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1}$ defined in 4.2° and $d\sigma(\xi)$ is the standard volume form on S^{d-1}.

5.5° We consider the Radon transformation of $\mathcal{COL}$ with smooth parameters. We regard $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1}$ as $\Sigma=\boldsymbol{R}^{n-d}_t\times S^{n-d-1}_\tau\times\boldsymbol{C}^d_z\times S^{d-1}_\xi$ and put $\tilde{\Sigma}=\boldsymbol{C}^{n-d}_w\times S^{n-d-1}_\tau\times\boldsymbol{C}^d_z\times S^{d-1}_\xi$. We take an open subset in $\tilde{\Sigma}$ as

$$\hat{D}_\varepsilon=\{(w,\tau,z,\xi)\,;\ |\mathrm{Im}\,w|<\varepsilon,\ \langle\mathrm{Im}\,w,\tau\rangle>0\} \tag{5.9}$$

and set

$$\begin{aligned}\tau:\tilde{\Sigma}&\longrightarrow\Sigma\\ (w,\tau,z,\xi)&\longmapsto(\mathrm{Re}\,w,\tau,z,\xi).\end{aligned} \tag{5.10}$$

We denote the sheaf of smooth relative l-forms with respect to τ on $\tilde{\Sigma}$ holomorphic with respect to w and z as $\mathcal{OL}^{(l)}\mathcal{OL}$. We set

$$\mathcal{S}^{(l)}_\varepsilon\mathcal{OL}=(\tau|_{\hat{D}_\varepsilon})_*\mathcal{OL}^{(l)}\mathcal{OL} \tag{5.11}$$

and

$$\mathcal{S}^{(l)}\mathcal{OL}=\varinjlim_{\varepsilon}\mathcal{S}^{(l)}_\varepsilon\mathcal{OL}. \tag{5.12}$$

Then we have an exact sequence

$$\begin{aligned}0\to\pi^{-1}(\mathcal{OOL}|_{\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1}})\to\mathcal{S}^{(0)}\mathcal{OL}\to\mathcal{S}^{(1)}\mathcal{OL}\to\cdots\\ \cdots\to\mathcal{S}^{(n-d-1)}\mathcal{OL}\to\mathcal{COL}\to0\end{aligned} \tag{5.13}$$

with $\tau:\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1}\to\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times S^{d-1}$. Here $\mathcal{OOL}$ is the sheaf of smooth functions on $\boldsymbol{C}^{n-d}_w\times\boldsymbol{C}^d_z\times S^{d-1}_\xi$ with holomorphic parameters w and z.

5.6° If $H(w,\tau,z,\xi)\in\mathcal{OLOL}(\{(w,\tau,z,\xi)\in\boldsymbol{C}^{n-d}\times S^{n-d-1}\times\boldsymbol{C}^d\times S^{d-1};\ |\mathrm{Im}\,w|<\varepsilon,\ |\mathrm{Re}\,w|<\varepsilon,\ |z|<\varepsilon,\ |\tau-\tau_0|<\varepsilon,\ \langle\mathrm{Im}\,w,\tau\rangle>0,\ \langle\mathrm{Im}\,z,\xi\rangle>0\})$, then $\sigma(\sigma(Hd\sigma(\tau))d\sigma(\xi))$

defines an element of $\mathscr{B}^2_\Lambda|_{\rho_0}$.

5.7° We define a map

$$(5.14)\qquad \begin{aligned} j: \boldsymbol{R}_\theta\times S^{n-d-1}\times S^{d-1} &\longrightarrow S^{n-1} \\ (\theta,\tau,\xi) &\longmapsto (\tau\cdot\cos\theta, \xi\cdot\sin\theta). \end{aligned}$$

If we take a positive number δ small enough, then

$$(5.15)\qquad \begin{aligned} (\tau_0, 0) &\in \{(\tau\cdot\cos\theta, \xi\cdot\sin\theta)\in S^{n-1}; |\tau-\tau_0|<\varepsilon, \xi^2=1, 0\leqq\theta<\delta\} \\ &\subset\{(\tau,\xi)\in S^{n-1}; |(\tau,\xi)-(\tau_0,0)|<\varepsilon\}. \end{aligned}$$

We have by Lemma 2.3.1 of K. Kataoka [3]

$$(5.16)\qquad j^*d\sigma(\tau\cdot\cos\theta, \xi\cdot\sin\theta)=\cos^{n-d-1}\theta\,\sin^{d-1}\theta\, d\theta\wedge d\sigma(\tau)\wedge d\sigma(\xi)$$

and define

$$(5.17)\qquad H_\delta(w,\tau,z,\xi)=\int_0^\delta F(w,z,\tau\cdot\cos\theta,\xi\cdot\sin\theta)\cos^{n-d-1}\theta\,\sin^{d-1}\theta\, d\theta.$$

Then H_δ satisfies the condition in 5.6° thus $\sigma(\sigma(H_\delta d\sigma(\tau))d\sigma(\xi))$ defines an element of $\mathscr{B}^2_{\Lambda,\rho_0}$.

5.8° We prove that the correspondence above is well defined as a morphism $\mathcal{C}_M|_\Lambda\to\mathscr{B}^2_\Lambda$.

We show that the correspondence in 5.7° is independent of the choice of δ. Take positive numbers δ and δ_1 so that $\delta>\delta_1>0$. Then

$$(5.18)\qquad H_\delta-H_{\delta_1}=\int_{\delta_1}^\delta F(w,z,\tau\cdot\cos\theta,\xi\cdot\sin\theta)\cos^{n-d-1}\theta\,\sin^{d-1}\theta\, d\theta$$

extends to real points with respect to w. Thus $\sigma((H_\delta-H_{\delta_1})d\sigma(\tau))=0$ as an element of $\mathcal{COL}$.

Next we show that the correspondence above is independent of the choice of F. We prove for $\omega\in S^{n-2}$, $Fd\sigma(\tau,\xi)=d_{(\tau,\xi)}\omega$ defines 0 in $\mathscr{B}^2_\Lambda$. Because $\tau_0=(1,0,\cdots,0)$, we can take $(\tau',\xi)=(\tau_2,\cdots,\tau_{n-d},\xi_1,\cdots,\xi_d)$ [resp. $\tau'=(\tau_2,\cdots,\tau_{n-d})$] as a local chart of S^{n-1} [resp. S^{n-d-1}]. Then we can write

$$(5.19)\qquad \omega=\sum_{j=2}^{n-d} f_j d\tau^{1j}\wedge d\xi+\sum_{j=1}^{d} g_j d\tau^1\wedge d\xi^j$$

where $d\tau^{1j}=d\tau_2\wedge\cdots\overset{j}{\cdot}\cdots\wedge d\tau_{n-d}$ and $d\tau^1=d\tau_2\wedge\cdots\wedge d\tau_{n-d}$ and $d\xi^j=d\xi_1\wedge\cdots\overset{j}{\cdot}\cdots\wedge d\xi_d$. Thus it is sufficient to study the case a) $\omega=fd\tau^{12}\wedge d\xi$ and b) $\omega=gd\tau^1\wedge d\xi^1$.

a) *In the case* $\omega = f d\tau^{12} \wedge d\xi$.

(5.20) $$j^* d\omega = dj^*\omega = d_\tau(f_\tau \cos^{n-d-1}\theta \sin^{d-1}\theta \, d\tau^{12}) \wedge d\theta \wedge d\xi.$$

Thus $\int j^* F \, d\sigma(\tau, \xi) \in \operatorname{Im} d_\tau$ and $\sigma\Big(\int j^* d\sigma(\tau, \xi)\Big) = 0$ as an element of $\mathcal{COL}$.

b) *In the case* $\omega = f \, d\tau^1 \vee d\xi^1$,

(5.21) $$j^*\omega = f \cos^{n-d}\theta \sin^{d-1}\theta \, d\theta \wedge d\tau' \wedge \Omega + f \cos^{n-d-1}\theta \sin^{d-1}\theta \, \xi_1 \, d\tau' \wedge d\xi^1 + f \cos^{n-d-2}\theta \sin^d\theta \, \xi_1 \, d\theta \wedge \phi \wedge d\xi^1.$$

Here Ω is a $(d-2)$ form on S^{d-1} and ϕ is an $(n-d-2)$ form on S^{n-d-1}. We write j-th term in (5.21) as A_j. Then

(5.22) $$j^* d\omega = dj^*\omega = d_\xi A_1 + d_\theta A_2 + d_\tau A_3.$$

It is easy to see $d_\xi A_1$ and $d_\tau A_3$ define 0 in $\mathcal{B}_\Lambda^2$. On the other hand, when $d > 1$

$$\int d_\theta A_2 d\theta = [(f \cos^{n-d-1}\theta \sin^{d-1}\theta)|_{\theta=\delta} d\tau'] d\xi'$$

extends to real points with respect to w. Thus $\sigma\Big(\Big(\int d_\theta A_2 d\theta\Big) d\tau'\Big)$ defines 0 as an element of $\mathcal{COL}$.

After all we have proved

THEOREM 5.1. *The correspondence*

(5.23) $$\mathcal{C}_M|_{\Lambda, \rho_0} \longrightarrow \mathcal{B}^2_{\Lambda, \rho_0}$$
$$\sigma(F(w, z, \tau, \xi) \, d\sigma(\tau, \xi)) \longmapsto \sigma\Big[\sigma\Big\{\Big(\int F(w, z, \tau \cdot \cos\theta, \xi \cdot \sin\theta) d\theta\Big) d\sigma(\tau)\Big\} d\sigma(\xi)\Big]$$

is a well defined morphism.

REMARK 5.2. The morphism above will be shown injective in Noro-Tose [18].

6° Correspondence between the cohomological Radon transformation and Čech cohomology group

6.0° In this section, we give a representation of the morphism σ : $p_* \mathcal{CS}^{n-1}_g \to \mathcal{B}^2_\Lambda$ by a Čech cohomology group.

6.1° First we prepare some notation about the integration along fibers for $\mathcal{COL}$. Let $X=\boldsymbol{C}_w^{n-d}\times\boldsymbol{C}_z^d$ and $N=\boldsymbol{R}_t^{n-d}\times\boldsymbol{C}_z^d$ $(w=t+\sqrt{-1}\,s,\ z=x+\sqrt{-1}\,y)$. We take an r-dimensional smooth manifold Y and put

$$\begin{array}{ccc} X_1=\boldsymbol{C}^{n-d}\times\boldsymbol{C}^d\times Y & \longrightarrow & \boldsymbol{C}^{n-d}\times\boldsymbol{C}^d=X \\ \downarrow & & \downarrow \\ N_1=\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d\times Y & \longrightarrow & \boldsymbol{R}^{n-d}\times\boldsymbol{C}^d=N. \end{array} \tag{6.1}$$

Moreover we set

$$p: S_{N_1}^*X_1\simeq S_N^*X\times Y\longrightarrow S_N^*X. \tag{6.2}$$

Let K be an oriented compact piecewise smooth k-chain in Y. Then we have the morphism of integration along fibers

$$\int_K : \hat{p}_*(\mathcal{COL}^{(k)})\longrightarrow\mathcal{CO} \tag{6.3}$$

where $\hat{p}=p|_{S_N^*X\times K}$ and $\mathcal{COL}^{(k)}$ and $\mathcal{CO}$ are defined in (4.12) and (6.4).

We remark that the theorem of Stokes type holds for $\int_K$.

We take an r-dimensional complex manifold Z for Y. Then through $\mathcal{COO}^{(k)}\to\mathcal{COL}^{(k)}$, we can also construct

$$\int_K : \hat{p}_*(\mathcal{COO}^{(k)}|_{S_N^*X\times K})\longrightarrow\mathcal{CO}. \tag{6.4}$$

Here $\mathcal{COO}^{(k)}$ is defined in (4.5). In this case, the theorem of Poincaré type holds. Explicitly, we have

$$\int_{\partial K} F=0 \tag{6.5}$$

for a real $(r+1)$ dimensional piecewise smooth oriented compact chain K in $Z=\boldsymbol{C}^r$ and $F\in\hat{p}_*(\mathcal{COO}^{(r)}|_{S_N^*X\times K})$. Especially, we have the theorem of Cauchy type.

6.2° We follow the notation in 4.5°. We take an open proper convex set U in $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$ and an open convex set V in $\boldsymbol{R}^d$. We put

$$D=\{(z,\xi)\in\boldsymbol{C}^d\times S^{d-1}\,;\,x\in V,\ |y|<\varepsilon,\ y\cdot\xi-g(\{y^2-(y\xi)^2\}^{1/2})>0\} \tag{6.6}$$

where g is a C^2 function on $[0,\infty)$ satisfying $g(0)=g'(0)=0$ and $g, g'\geqq 0$. Then we have

$$\pi(D)=\{z\in\boldsymbol{C}^d\,;\,x\in V,\ |y|<\varepsilon\}\setminus\boldsymbol{R}^d. \tag{6.7}$$

Here $\pi : C^d \times S^{d-1} \to C^d$. By (4.23), we have

(6.8) $$H^{d-1}(\Gamma(U\times D, \mathcal{C}\mathcal{O}\mathcal{L}^{(\cdot)})) \simeq H^{d-1}(U\times\pi(D), \mathcal{C}\mathcal{O}).$$

This isomorphism was obtained as follows. Consider the diagram

(6.9)
$$\begin{array}{ccccccc}
 & & 0 & & 0 & & 0 \\
 & & \downarrow & & \downarrow & \bar\partial_z & \downarrow \\
0 & \to & \mathcal{C}\mathcal{O}(U\times\pi(D)) & \to & \mathcal{C}\mathcal{L}^{(0,0)}(U\times\pi(D)) & \to & \mathcal{C}\mathcal{L}^{(0,1)}(U\times\pi(D)) \to \cdots \\
 & & \downarrow & & \downarrow & & \downarrow \\
0 & \to & \mathcal{C}\mathcal{O}\mathcal{L}^{(0)}(U\times D) & \to & \mathcal{C}\mathcal{L}^{(0,0)}\mathcal{L}^{(0)}(U\times D) & \to & \mathcal{C}\mathcal{L}^{(0,1)}\mathcal{L}^{(0)}(U\times D) \to \cdots \\
 & & \downarrow & & \downarrow & & \downarrow \\
0 & \to & \mathcal{C}\mathcal{O}\mathcal{L}^{(1)}(U\times D) & \to & \mathcal{C}\mathcal{L}^{(0,0)}\mathcal{L}^{(1)}(U\times D) & \to & \mathcal{C}\mathcal{L}^{(0,1)}\mathcal{L}^{(1)}(U\times D) \to \cdots \\
 & & \downarrow & & \downarrow & & \downarrow
\end{array}$$

Here any column or row is exact except the first column and the first row. Thus we can find a sequence $\{\phi_{d-1}, \cdots, \phi_{-1}\}$ as follows.

(6.10) $$\phi_{d-1} = \phi \in \mathcal{C}\mathcal{O}\mathcal{L}^{(d-1)}(U\times D).$$

(6.11) $$d_\xi\phi_{d-2} = \phi_{d-1},\ \phi_{d-2} \in \mathcal{C}\mathcal{L}^{(0,0)}\mathcal{L}^{(d-2}(U\times D)$$

(6.12) $$d_\xi\phi_j = \bar\partial\phi_{j+1},\ \phi_j \in \mathcal{C}\mathcal{L}^{(0,d-2-j)}\mathcal{L}^{(j)} \quad (0\leqq j\leqq d-3).$$

(6.13) $$\phi_{-1} = \bar\partial\phi_0 \in \operatorname{Ker}(\bar\partial : \mathcal{C}\mathcal{L}^{(0,d-1)} \to \mathcal{C}\mathcal{L}^{(0,d)}).$$

Then the isomorphism (6.8) is given by $[\phi] \longmapsto [\phi_{-1}]$.

6.3° Take $\xi_1, \cdots, \xi_d \in R^d \setminus \{0\}$ so that $\xi_1, \cdots, \xi_d$ are linearly independent in R^d. We put $\xi_{j\pm} = \pm\xi^j$ and set

(6.14) $$V_{j\pm} = \{z\in\pi(D)\,;\, y\xi_{j\pm} - g(\{y^2 - (y\xi_{j\pm})^2\}^{1/2}) > 0\}.$$

Then $\mathsf{U} = \{U\times V_{j\pm}\}_{j,\pm}$ is a Leray covering of $U\times\pi(D)$ for $\mathcal{C}\mathcal{O}$ if ε is small enough. Thus we have

(6.15) $$H^{d-1}(U\times\pi(D), \mathcal{C}\mathcal{O}) \simeq H^{d-1}(C^{\cdot}(\mathsf{U}, \mathcal{C}\mathcal{O})).$$

This isomorphism is given explicitly as follows. Consider the commutative diagram

(6.16)
$$\begin{array}{ccccccc}
 & & 0 & & 0 & & 0 \\
 & & \downarrow & & \downarrow & \bar\partial_z & \downarrow \\
0 & \to & \mathcal{C}\mathcal{O}(U\times\pi(D)) & \to & \mathcal{C}\mathcal{L}^{(0,0)}(U\times\pi(D)) & \to & \mathcal{C}\mathcal{L}^{(0,1)}(U\times\pi(D)) \to \cdots \\
 & & \downarrow & & \downarrow & & \downarrow \\
0 & \to & C^0(\mathsf{U}, \mathcal{C}\mathcal{O}) & \to & C^0(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,0)}) & \to & C^0(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,1)}) \to \cdots \\
 & & \downarrow\delta & & \downarrow\delta & & \downarrow\delta \\
0 & \to & C^1(\mathsf{U}, \mathcal{C}\mathcal{O}) & \to & C^1(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,0)}) & \to & C^1(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,1)}) \to \cdots \\
 & & \downarrow & & \downarrow & & \downarrow
\end{array}$$

where any row or column is exact except the 1st row and 1st column. Thus we can choose a sequence $\{\psi_{d-1}, \cdots, \psi_{-1}\}$ so that

$$\psi_{d-1}=\phi \in C^{d-1}(\mathsf{U}, \mathcal{CO}), \tag{6.17}$$

$$\delta\psi_{d-2}=\psi_{d-1},\ \psi_{d-2} \in C^{d-2}(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,0)}) \tag{6.18}$$

$$\delta\psi_j=\bar{\partial}\omega_{j+1},\ \psi_j \in C^j(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,d-2-j)}) \qquad (0\leqq j\leqq d-3) \tag{6.19}$$

and

$$\psi_{-1}=\bar{\partial}\psi_0 \in \operatorname{Ker}(\mathcal{C}\mathcal{L}^{(0,d-1)}\rightarrow\mathcal{C}\mathcal{L}^{(0,d)}). \tag{6.20}$$

6.4° Composing (6.15) with (6.8), we obtain an isomorphism

$$h: H^{d-1}(\Gamma(U\times D, \mathcal{CO}\mathcal{L}^{(\cdot)}))\simeq H^{d-1}(C^{\cdot}(\mathsf{U}, \mathcal{CO})), \tag{6.21}$$

where h is given by

PROPOSITION 6.1. *Let $\Delta^k_s=\{(s_1, \cdots, s_k) \in \boldsymbol{R}^k; 0\leqq s_j\leqq 1\ (1\leqq j\leqq k), \sum_{j=1}^k s_j\leqq 1\}$ be a k-dimensional standard simplex with its vertexes $\{e_1, \cdots, e_{k+1}\}$. We fine an affine map*

$$[\xi_{j_1}\varepsilon_1, \cdots, \xi_{j_{k+1}}\varepsilon_{k+1}]: \Delta^k\rightarrow S^{d-1}\ (j_1<j_2<\cdots<j_k,\ \varepsilon_l=\pm) \tag{6.22}$$

satisfying

$$[\xi_{j_1}\varepsilon_1, \cdots, \xi_{j_{k+1}}\varepsilon_{k+1}]\,(e_l)=\varepsilon_l\xi_l. \tag{6.23}$$

Then for $\phi \in \Gamma(U\times D, \mathcal{CO}\mathcal{L}^{(d-1)})$, $h([\phi])=[\{\psi_{1\varepsilon_1,\dots,d\varepsilon_d}\}]=[\psi] \in H^{d-1}(C^{\cdot}(\mathsf{U}, \mathcal{CO}))$ is given by

$$\psi_{1\varepsilon_1,\cdots,d\varepsilon_d}=\int_{[\xi_1\varepsilon_1,\cdots,\xi_d\varepsilon_d]}\phi=\int_{\Delta^{d-1}}[\xi_1\varepsilon_1, \cdots, \xi_d\varepsilon_d]^*\phi. \tag{6.24}$$

PROOF. Choose a sequence $\{\phi=\phi_{d-1}, \cdots, \phi_{-1}\}$ from ϕ as in 6.2°. Then it is enough to show that there exist $\psi_{d-2}, \cdots,$ and ψ_0 $(\psi_l \in C^l(\mathsf{U}, \mathcal{C}\mathcal{L}^{(0,d-2-l)}))$ so that

$$\delta\psi_{d-2}=\psi,\quad \bar{\partial}\psi_0=\phi_{-1}\quad \text{and}\quad \delta\psi_l=\bar{\partial}\psi_{l+1}. \tag{6.25}$$

For the image of $[\phi]$ in (6.8) is $[\phi_{-1}]$ and the image of $[\phi_{-1}]$ in the isomorphism (6.15) will be $[\psi]$ by (6.25). We define ψ_k by

$$\psi_{k,(j_1\varepsilon_1,\cdots,j_{k+1}\varepsilon_{k+1})}=\int_{[\xi_{j_1}\varepsilon_1,\cdots,\xi_{j_k}\varepsilon_k]}\phi_k. \tag{6.26}$$

Then we have

$$(\delta\psi_{k-1})_{(j_1\varepsilon_1,\cdots,j_k\varepsilon_k)}=\sum_{i=1}^{k+1}\int_{[j_1\varepsilon_1,\overset{\vee}{\underset{i}{\cdots}},j_{k+1}\varepsilon_{k+1}]}\phi_{k-1} \tag{6.27}$$

$$=\int_{[\xi_{j_1}\varepsilon_1,\cdots,\xi_{j_d}\varepsilon_d]} d_\xi\phi_{k-1} \quad \text{(by Stokes' formula)}$$
$$=\int_{[\xi_{j_1}\varepsilon_1,\cdots,\xi_{j_d}\varepsilon_d]} \bar{\partial}\phi_k=\bar{\partial}\psi_{k,(j_1\varepsilon_1,\cdots,j_d\varepsilon_d)}.$$

Thus we have proved (6.25). (q.e.d.)

6.5° We make a remark about the operation of boundary values. We define the real monoidal transformation of $\tilde{\Lambda}$ along Λ by

$$\tau : \mathrm{Mon}_\Lambda(\tilde{\Lambda})=(\tilde{\Lambda}\setminus\Lambda)\cup S_\Lambda\tilde{\Lambda}\to\tilde{\Lambda}. \tag{6.28}$$

We put

$$j : \tilde{\Lambda}\setminus\Lambda \hookrightarrow \tilde{\Lambda} \tag{6.29}$$

and define

$$\tilde{\mathcal{A}}^2_\Lambda=j_*(\mathcal{CO}|_{\tilde{\Lambda}\setminus\Lambda})|_{S_\Lambda\tilde{\Lambda}}. \tag{6.30}$$

We take a coordinate of $S_\Lambda\tilde{\Lambda}$ as $(t, x, \sqrt{-1}\,\tau dt\infty, \sqrt{-1}\,v\partial/\partial x\, 0)$ or simply as $(\rho, \sqrt{-1}\,v)$ with $\rho=(t, x, \sqrt{-1}\,\tau dt\infty)$. We set

$$D_\Lambda\tilde{\Lambda}=\{(t, x, \sqrt{-1}\,v, \sqrt{-1}\,\xi)\in S_\Lambda\tilde{\Lambda}\underset{\Lambda}{\times}S^*_\Lambda\tilde{\Lambda}\,;\,\langle v, \xi\rangle\leqq 0\}. \tag{6.31}$$

We set the commutative diagram in Figure 6.1. Then we have the exact sequence constructed in Kashiwara-Laurent [7]:

$$\begin{tikzcd} & D_\Lambda\tilde{\Lambda} \arrow[dl, "\pi"'] \arrow[dr, "\tau"] & \\ S_\Lambda\tilde{\Lambda} \arrow[dr, "\tau"'] & & S^*_\Lambda\tilde{\Lambda} \arrow[dl, "\pi"] \\ & \Lambda & \end{tikzcd}$$

Fig. 6.1.

$$0\to\tilde{\mathcal{A}}^2_\Lambda\xrightarrow{b}\tau^{-1}\mathcal{B}^2_\Lambda\to\pi_*\tau^{-1}\mathcal{C}^2_\Lambda\to 0. \tag{6.32}$$

Following M. Morimoto [10] and A. Kaneko [2], we construct another boundary value operator b with the aid of Čech cohomology.

We fix an orientation of $\boldsymbol{R}^d$ and take a proper convex open set U in $S^*\boldsymbol{R}^{n-d}$ and an open subset V in $\boldsymbol{R}^d$. We take a proper convex cone Γ in $\boldsymbol{R}^d$ and a Stein neighborhood $V^{\boldsymbol{C}}$ in $\boldsymbol{C}^d_z$.

We define a boundary value of $\phi\in\mathcal{CO}(U\times((\boldsymbol{R}^d+\sqrt{-1}\,\Gamma)\cap V^c))$ as follows. Take $\xi_1,\cdots,\xi_d\in S^{d-1}$ so that

(6.33) $\{\xi_1,\cdots,\xi_d\}$ is linearly independent and has the same orientation of $\boldsymbol{R}^d$ and that

$$\{\xi_1\}^\circ\cap\cdots\cap\{\xi_d\}^\circ\subset\Gamma. \tag{6.34}$$

Here $\{\xi_j\}^\circ$ is the polar set of $\{\xi_j\}$ in $\boldsymbol{R}^d$. We put

$$\mathfrak{U}'=\{U\times(V_{j,\pm}\cap V^c)\}_{1\le j\le d,\pm} \tag{6.35}$$

where $V_{j,\pm}$ is defined in (6.14). Then image of ϕ in $H^{d-1}(C^\cdot(\mathfrak{U}',\mathcal{CO}))$ is given by

$$\phi_{1+,\cdots,d+}=\phi \tag{6.36}$$

and

$$\phi_*=0 \qquad (*\neq(1+,\cdots,d+)). \tag{6.37}$$

In this situation, we have

$$\Gamma(U\times V,\mathscr{B}_\Lambda^2)=\{\sum_{j=1}^N b(\phi_j)\,;\,\phi_j\in\mathcal{CO}(U\times(V+\sqrt{-1}\,\Gamma_j0))\}. \tag{6.38}$$

Here $(U\times(V+\sqrt{-1}\,\Gamma_j0))$ denotes an infinitesimal wedge of Γ_j type on $U\times V$ in $\tilde{\Lambda}$,

When $\phi_j\in\mathcal{CO}(U_j\times(V_j+\sqrt{-1}\,\Gamma_j0))$ $(j=1,2)$ and $\Gamma_1\cap\Gamma_2\neq\emptyset$ we have

$$b(\phi_1)+b(\phi_2)=b(\phi_1+\phi_2) \tag{6.39}$$

and

$$\phi_1+\phi_2\in\mathcal{CO}([(U_1\cap U_2)\times\{(V_1\cap V_2)+\sqrt{-1}\,(\Gamma_1\cap\Gamma_2)0\}]). \tag{6.40}$$

7° Curvilinear expansion of microfunctions with holomorphic parameters

7.1° Let M be $\boldsymbol{R}_t^{n-d}\times\boldsymbol{R}_x^d$ and X be its complexification $\boldsymbol{C}_w^{n-d}\times\boldsymbol{C}_z^d$ $(w=t+\sqrt{-1}\,s,\ z=x+\sqrt{-1}\,y)$. We set

$$\Lambda=\{(t,x\,;\sqrt{-1}\,(\tau dt+\xi dx)\infty)\in\sqrt{-1}\,S^*M\,;\,\xi=0\} \tag{7.1}$$

and

$$\tilde{\Lambda}=S_N^*X\simeq\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}\times\boldsymbol{C}^d \tag{7.2}$$

with $N=R_t^{n-d}\times C_z^d$.

We consider the curvilinear expansion of microfunctions with holomorphic parameters.

We set

$$N_\varepsilon=\{\zeta\in C^d\,;\,\zeta^2=1,\,|\operatorname{Im}\zeta|<\varepsilon\} \tag{7.3}$$

and

$$W(z,\zeta)=\frac{(d-1)!}{(-2\pi\sqrt{-1})^d}\cdot\frac{(1-\sqrt{-1}\,z\zeta)^{d-1}-(1-\sqrt{-1}\,z\zeta)^{d-2}(z^2(z\zeta))^2}{\{z\zeta+\sqrt{-1}\,(z^2-(z\zeta)^2)\}^d}. \tag{7.4}$$

We have the following facts about the domain of holomorphy of W.

(7.5) For two open sets D_0 and D_1 satisfying $D_1\subset\subset D_0$, there exists a positive number ε such that for any point $z_0\in\partial D_0+\sqrt{-1}B_\varepsilon$ ($B_\varepsilon=\{y\in R^d\,;\,|y|<\varepsilon\}$) $W(z-z_0,\zeta)$ is holomorphic on $(D_1+\sqrt{-1}\,B_\varepsilon)\times N_\varepsilon$.

(7.6) For a bounded open subset $\tilde{D}$ in C^d, there exists a positive number K such that $W(z,\zeta)$ is holomorphic on $\{(z,\zeta)\in\tilde{D}\times N_\varepsilon\,;\,g(y,\xi):=y\xi-(y^2-(y\xi)^2)>K|\eta|\}$ where $y=\operatorname{Im}z$, $\xi=\operatorname{Re}\zeta$ and $\eta=\operatorname{Im}\zeta$.

PROPOSITION 7.1 (cf. K. Kataoka [3], A. Kaneko [2]). *Let U be an open subset in $\sqrt{-1}\,S^*R^{n-d}$ and take D_0, D_1 and ε as above in* (7.5). *We assume ∂D_0 is piecewise real analytic. Take an open subset D in R^d satisfying $D\supset D_0$ and an open convex cone Γ in R^d. We set*

$$D_{\Gamma,\varepsilon'}=D+\sqrt{-1}(\Gamma\cap B_{\varepsilon'}) \tag{7.7}$$

for $\varepsilon'>\varepsilon$. We take $a\in\Gamma\cap B_\varepsilon$ and put

$$F(t,z,\zeta)=\int_{D_0+\sqrt{-1}a}f(t,\tilde{z})\,W(z-\tilde{z},\zeta)\,d\tilde{z} \tag{7.8}$$

for $f(t,z)\in\mathcal{C}_{\tilde{\Lambda}}(U\times D_{\Gamma,\varepsilon})$. Then $F\in\mathcal{C}_{\tilde{\Lambda}_0}(\tilde{E})$ where

$$\tilde{\Lambda}_0=S^*_{R^{n-d}\times C^d\times N_\varepsilon}(X\times N_\varepsilon)\simeq\sqrt{-1}\,S^*R^{n-d}\times C^d\times N_\varepsilon \tag{7.9}$$

and $\tilde{E}$ is a neighborhood of

$$E:=U\times\bigcup_{y_0\in B_\varepsilon\cap\Gamma}\{(z,\xi)\in(D_1+\sqrt{-1}\,B_\varepsilon)\times S_\xi\,;\,g(y-y_0,\xi)>0\}$$

in $\tilde{\Lambda}_0$. Moreover, for any proper convex subset Δ in S^{d-1}, we have

$$F\in\mathcal{C}_{\tilde{\Lambda}_0}(U\times(D_1+\sqrt{-1}\,(\Gamma+\Delta))0\times\Delta^\circ). \tag{7.10}$$

Especially,

(7.11) $$F\in \mathcal{C}_{\tilde{\Lambda}_0}(U\times(D_1)_{\Gamma,\varepsilon}\times S^{d-1})$$

and

(7.12) $$\int_{S^{d-1}} F(t,z,\xi)\,d\sigma(\xi)=f(t,z)$$

on $U\times(D_1)_{\Gamma,\varepsilon}$. *Here* $(D_1)_{\Gamma,\varepsilon}=D_1+\sqrt{-1}(\Gamma\cap B_\varepsilon)$.

PROOF. For any point $y_0\in\Gamma\cap B_\varepsilon$, we define a d-chain in $D+\sqrt{-1}(\Gamma\cap B_\varepsilon)$ by

(7.13) $$\gamma_{y_0}=\bigcup_{x\in\partial D_0}\{x+\sqrt{-1}\,y\,;\,y\in\overline{ay_0}\}\cup\{x+\sqrt{-1}\,y_0\,;\,x\in D_0\}.$$

(See Figure 7. 1.) We set

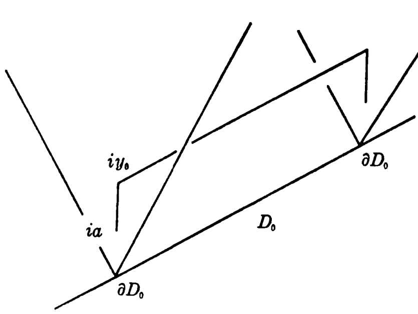

Fig. 7.1.

(7.14) $$F_{y_0}(t,z,\zeta)=\int_{\gamma_{y_0}} f(t,\tilde{z})\,W(z-\tilde{z},\zeta)d\tilde{z}.$$

By (7.5) and (7.6), we have

(7.15) $$F_{y_0}\in\mathcal{C}_{\tilde{\Lambda}_0}(U\times\tilde{E}_{y_0})$$

where

(7.16) $$\tilde{E}_{y_0}=\{(z,\zeta)\in(D_1+\sqrt{-1}\,B_\varepsilon)\times N_\varepsilon\,;\,g(y-y_0,\xi)>K|\eta|\}$$

for some $K>0$. We set

(7.17) $$E_{y_0}=\{(z,\xi)\in(D_1+\sqrt{-1}\,B_\varepsilon)\times S_\xi^{d-1}\,;\,g(y-y_0,\xi)>0\}.$$

Remarking that $-g(y, \varepsilon)$ is a convex function of y for $\xi \in S^{d-1}$, we find

$$E_{y_0} \cap E_{y_1} = \bigcap_{0 \leq t \leq 1} E_{ty_0+(1-t)y_1}. \tag{7.18}$$

We take a $(d+1)$ chain in $D+\sqrt{-1}\,(\Gamma \cap B_\varepsilon)$:

$$K_{y_0y_1} = \bigcup_{x \in D_0} \{x+\sqrt{-1}\,y\,;\, y \in \overline{y_0y_1}\} \cup \bigcup_{x \in \partial D_0} \{x+\sqrt{-1}\,y\,;\, y \in \Delta a y_0 y_1\}. \tag{7.19}$$

(See Figure 7.2.) Then for any $(z, \xi) \in E_{y_0} \cap E_{y_1}$, there exists an open subset

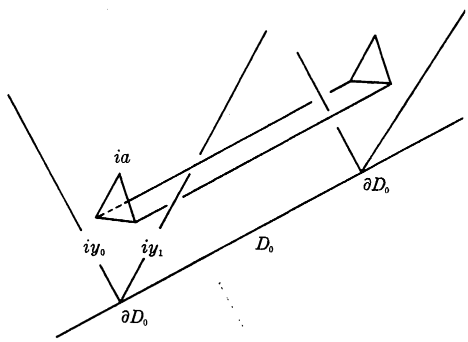

Fig. 7.2.

V in $\bigcap_{0 \leq t \leq 1} \tilde{E}_{ty_0+(1-t)y_1}$ such that $f(t, \tilde{z}) \cdot K(z-\tilde{z}, \zeta)$ is defined in $U \times (K_{y_0y_1})_{\tilde{z}} \times (V)_z$. Thus

$$F_{y_0} = F_{y_1} \tag{7.20}$$

on $U \times (E_{y_0} \cap E_{y_1})$ follows by applying the Poincaré's theorem on $K_{y_0y_1}$. Here the assertion that $F \in \mathcal{C}_{\tilde{\Lambda}_0}(\tilde{E})$ is verified.

Take an open proper convex subset Δ in S_ξ^{d-1}. We have

$$F_{y_0} \in \mathcal{C}_{\tilde{\Lambda}_0}(U \times [D_1 + \sqrt{-1} \bigcap_{\xi \in \Delta} \{y \in \boldsymbol{R}^d\,;\, g(y-y_0, \xi) > 0\}] \times \Delta). \tag{7.21}$$

When we move y_0 in $\Gamma \cap B_\varepsilon$, then the 2nd assertion follows.

To prove the 3rd assertion, it is enough to show (7.12) locally on $U \times (D_1)_{\Gamma, \varepsilon}$. We take a point $\rho_0 = ((t_0, \sqrt{-1}\,\tau_0\, dt\infty), z_0) \in U \times (D_1)_{\Gamma, \varepsilon}$. Then there exists a neighborhood $U_0 = \{(t, \sqrt{-1}\,\tau\infty)\,;\, |t-t_0| < \delta, |\tau-\tau_0| < \delta\}$ of $(t_0, \sqrt{-1}\,\tau_0)$ and a neighborhood $W_0 = \{z = x+\sqrt{-1}\,y \in \boldsymbol{C}^d\,;\, |x-x_0| < \delta, |y-y_0| < \delta\}$ of $z_0 = x_0 + \sqrt{-1}\,y_0$ and a wedge $\Sigma\ (\subset \boldsymbol{C}_w^{n-d})$ on $\{t \in \boldsymbol{R}^{n-d}\,;\, |t-t_0| < \delta\}$ and $F(w, z) \in \mathcal{O}_{\boldsymbol{C}^n}(\Sigma \times W_0)$ such that

(7.22) $$f(t, z) = sp\,[F(w, z)].$$

We set

(7.23) $$V_0 = \{x \in \boldsymbol{R}^d\,;\ |x - x_0| < \varepsilon\} \text{ and } I_0 = \{y \in \boldsymbol{R}^d_y\,;\ |y| < \varepsilon\}$$

and take open subsets V_1 and V_2 in V_0 so that

(7.24) $$V_2 \Subset V_1 \Subset V_0.$$

Then there exists a positive number h such that for any point $\hat{z} \in \bigcup_{\tilde{x} \in \partial V_1} \{x + \sqrt{-1}\, y_0\,;\ |\tilde{x} - x| < h\}$, $W(z - \hat{z}, \zeta)$ is holomorphic on $\{x + \sqrt{-1}\, y\,;\ x \in V_2,\ |y - y_0| < h\} \times N_h$. We set for $\sigma = (\pm 1, \cdots, \pm 1) \in \boldsymbol{R}^{n-d}$

(7.25) $$b_\sigma = y_0 - \rho\sigma \qquad (\text{with } \rho > 0 \text{ such that } |\rho\sigma| < h)$$

and define a d-chain γ_σ in $(D_1)_{\Gamma, \varepsilon}$ as in Figure 7.3. We put

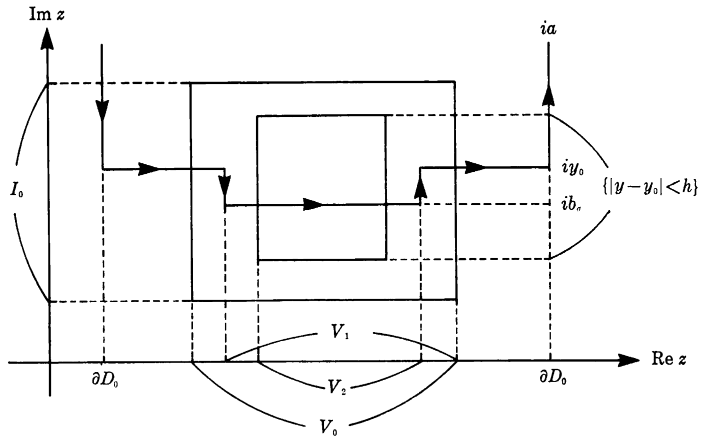

Fig. 7.3.

(7.26) $$\tilde{\Gamma}_\sigma = \{\xi \in \boldsymbol{R}^d \setminus \{0\}\,;\ \sigma_i \xi_i \geqq 0\}$$

and

(7.27) $$\Gamma_\sigma = \tilde{\Gamma}_\sigma \cap S^{d-1}.$$

There exists an infinitesimal cone $\sqrt{-1}\, B_\sigma$ in $\sqrt{-1}\, \boldsymbol{R}^d_y$ summited at $\sqrt{-1}\, b_\sigma$ and contained in $\sqrt{-1}\,(b_\sigma + \tilde{\Gamma}_\sigma)$ such that

$$F(t, z, \zeta) \in \mathcal{C}_{\tilde{A}_0}(U_0 \times (V_2 + \sqrt{-1}\, B_\sigma) \times \Gamma_\sigma). \tag{7.28}$$

Moreover when we take ρ small enough, there exists an open subset I_1 in $\sqrt{-1}\, \boldsymbol{R}_y^d$ such that

$$\bigcap_\sigma B_\sigma \supset I_1 \ni y_0. \tag{7.29}$$

Thus we have on $U_0 \times (V_2 + \sqrt{-1}\, I_1)$

$$\begin{aligned} \int_{S^{d-1}} F(t, z, \xi) d\sigma(\xi) &= \sum_\sigma \int_{\Gamma_\sigma} F(t, z, \xi) d\sigma(\xi) \\ &= \sum_\sigma \int_{\Gamma_\sigma} d\sigma(\xi) \int_{\gamma_\sigma} f(t, \tilde{z}) W(z - \tilde{z}, \xi) d\tilde{z}. \end{aligned} \tag{7.30}$$

We divide γ_σ into two parts so that

$$\gamma_\sigma = \gamma_\sigma^1 \cup \gamma_\sigma^2 \quad \text{(disjoint union)} \tag{7.31}$$

and

$$\gamma_\sigma^1 = \gamma_\sigma \cap (\bar{V}_1 + \sqrt{-1}\, \{y\,;\, |y - y_0| < h\}). \tag{7.32}$$

We set

$$f_i = \sum_\sigma \int_{\Gamma_\sigma} d\sigma(\xi) \int_{\gamma_\sigma^i} d\tilde{z}\, f(t, \tilde{z}) \cdot W(z - \tilde{z}, \xi) \qquad (i = 1, 2). \tag{7.33}$$

Then

$$\begin{aligned} f_1(t, z) &= sp\left[\sum_\sigma \int_{\Gamma_\sigma} d\sigma(\xi) \int_{\gamma_\sigma^1} d\tilde{z}\, F(w, \tilde{z}) \cdot W(z - \tilde{z}, \xi) \right] \\ &= sp\,[F(w, z)] \end{aligned} \tag{7.34}$$

on $U_0 \times (V_2 + \sqrt{-1}\, I_1)$. Moreover we have

$$f_2(t, z) = \int_{\gamma_\sigma^2} d\tilde{z}\, f(t, \tilde{z}) \int_{S^{d-1}} d\sigma(\xi)\, W(z - \tilde{z}, \xi), \tag{7.35}$$

where $\int_{S^{d-1}} W(z - \tilde{z}, \xi)\, d\sigma(\xi) = 0$. Thus

$$f_2(t, z) = 0 \tag{7.36}$$

follows. After all we have proved (7.12) on $U_0 \times (V_2 + \sqrt{-1}\, I_1)$. (q.e.d.)

In the situation of Proposition 7.1, we have

COROLLARY 7.2. *Take a sub-cone Δ $(\subset \Gamma)$ and put*

$$F(t, z, \Delta^{\circ}) = \int_{\Delta^{\circ} \cap S^{d-1}} F(t, z, \xi)\, d\sigma(\xi) \in \mathcal{C}_{\tilde{\Lambda}}(U \times (K + \sqrt{-1}\,\Delta 0)). \tag{7.37}$$

Here Δ° is a polar set of Δ and K is a compact subset in D. Then we have

$$f(t, z) - F(t, z, \Delta^{\circ}) \in \mathcal{A}^2_{\Lambda}(U \times K). \tag{7.38}$$

In the situation above, we give

COROLLARY 7.3. *The morphism*

$$\tilde{\mathcal{A}}^2_{\Lambda} \to \tau^{-1}(p_* \mathcal{C}S^{d-1}/d_{\xi} p_* \mathcal{C}S^{d-2}) \simeq \tau^{-1} \mathcal{B}^2_{\Lambda}$$

is given by

$$f(t, z) \longmapsto \int_{S^{d-1}} \int_{D + \sqrt{-1}\,a} f(t, \tilde{z})\, W(z - \tilde{z}, \xi) d\tilde{z} d\sigma(\xi). \tag{7.39}$$

The proof of Corollary 7.2 can be given in the same manner as A. Kaneko [2]. Corollary 7.3 is obtained by Proposition 7.1 and the results in 6°. The morphism

$$\tilde{\mathcal{A}}^2_{\Lambda} \to \tau^{-1}(p_* \mathcal{C}S^{d-1}/d_{\xi} p_* \mathcal{C}S^{d-2})$$

is given by

$$[F(t, x, \xi)] = \left[\int_{D + \sqrt{-1}\,a} f(t, \tilde{z}) W(z - \tilde{z}, \xi) d\tilde{z} \right].$$

Then Proposition 7.1 assures that $[F(t, x, \xi)]$ coincides with $b(f)$ through the correspondence between the Čech cohomology group and the cohomological Radon transformation in 6°.

Let U be an open subset in $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$ and V be an open subset in $\boldsymbol{R}^d_x$. Take a proper convex cone Γ in $\boldsymbol{R}^d_y$. Then for $F(t, x) \in \mathcal{C}_{\tilde{\Lambda}}(U \times (V + \sqrt{-1}\,\Gamma 0))$, we set

$$b(F) = b_{\Gamma}(F) = F(t, x + \sqrt{-1}\,\Gamma 0). \tag{7.40}$$

We give a proposition about a criterion for the 2-singular spectrum of a 2-hyperfunction.

PROPOSITION 7.4. *The proper convex cones $\Gamma_1, \cdots, \Gamma_N$ in $\boldsymbol{R}^d$ and $F_j \in \mathcal{C}_{\tilde{\Lambda}}(U + (V + \sqrt{-1}\,\Gamma_j 0))$ $(1 \leq j \leq N)$. For $u = \sum_{j=1}^{N} b_{\Gamma_j}(F_j) \in \mathcal{B}^2_{\Lambda}(U \times V)$ and $\rho_0 \in U$, we have*

$$(\rho_0 ; x_0, \sqrt{-1}\,\xi_0 dx\infty) \notin SS^2_{\Lambda}(u) \tag{7.41}$$

if and only if

$$F(t, z, \zeta)=\sum_{j}\int_{V_0+\sqrt{-1}\,a_j} F_j(t, \tilde{z})W(z-\tilde{z}, \zeta)d\tilde{z}\in \mathcal{C}_{\tilde{\Lambda}_0}\Big|_{(\rho_0, x_0, \xi_0)}. \tag{7.42}$$

Here $a_j\in\Gamma_j$ ($|a_j|\ll 1$) *and* $V_0\subset V$.

PROOF. We assume (7.42). Let $u(t, z)=\sigma(F(z, \xi)d\sigma(\xi))$. By the de Rham's Theorem for $\mathcal{COL}$, there exists $\omega\in\mathcal{COL}^{(d-2)}_{(\rho_0, x_0, \xi_0)}$ such that $Fd\sigma(\xi)=d_\xi\omega$. Thus $Sp^2_\Lambda(u)=\sigma(d\omega)=0$ at $(\rho_0, x_0, \sqrt{-1}\,\xi_0)$.

Conversely, we assume (7.41). We put

$$G_j=\int_{V_0+\sqrt{-1}\,a_j} F_j(t, \tilde{z})W(z-\tilde{z}, \zeta)\, d\tilde{z}. \tag{7.43}$$

Then we have

$$F_j(t, z)=\int_{S^{d-1}} G_j(t, z, \xi)\, d\sigma(\xi) \tag{7.44}$$

on $U\times(V_1+\sqrt{-1}\,\Gamma_j 0)$ for an open subset V_1 in V_0 satisfying $V_1\subset V_0$. Thus on $U\times V_3\times S^{d-1}$

$$F(t, z, \zeta)=\sum_{j}\int_{V_2+\sqrt{-1}\,b_j} d\tilde{z}\int_{S^{d-1}} d\sigma(\tilde{\xi})G_j(t, \tilde{z}, \tilde{\xi})\, W(z-\tilde{z}, \zeta) \tag{7.45}$$

modulo $\mathcal{C}_{\tilde{\Lambda}_0}\Big|_{\sqrt{-1}\,S^*R^{n-d}\times R^d\times S^{d-1}}$ when we take $b_j\in\Gamma_j$ with $|b_j|$ small enough and open subsets V_2 and V_3 in V_0 so that $V_3\subset V_2\subset V_1$. We set

$$D_\varepsilon=\{(z, \xi)\in C^d\times S^{d-1}\,;\, |z-x_0|<\varepsilon,\ |\xi-\xi_0|<\varepsilon,\ y\xi-\{y^2-(y\xi)^2\}>0\}. \tag{7.46}$$

Now that $(\rho_0, x_0, \sqrt{-1}\,\xi_0\infty)\notin SS^2_\Lambda(u)$, there exist a positive number ε and an open subset U_0 in U and $\omega\in\mathcal{COL}^{(d-2)}(U_0\times D_\varepsilon)$ such that

$$F(t, z, \xi)d\sigma(\xi)=d_\xi\omega. \tag{7.47}$$

Again we take an open subset V_4 ($\subset V_3$) small enough so that

$$V_4\subset\{x\in V_3\,;\, |x-x_0|<\varepsilon\} \tag{7.48}$$

and take $\tilde{b}_j\in\Gamma_j$ with $|\tilde{b}_j|$ small enough so that we can integrate $\left(\int_{S^{d-1}} d\sigma(\tilde{\xi})G_j(t, \tilde{z}, \tilde{\xi})\right)W(z-\tilde{z}, \zeta)$ with respect to $\tilde{z}$ on $V_4+\sqrt{-1}\,\tilde{b}_j$. Then we have

$$F(t, z, \zeta)\equiv\int_{V_4+\sqrt{-1}\,\tilde{b}_j} d\tilde{z}\left\{\int_{S^{d-1}} d\sigma(\tilde{\xi})G_j(t, \tilde{z}, \tilde{\xi})\right\}W(z-\tilde{z}, \zeta) \tag{7.49}$$

modulo $\mathcal{C}_{\tilde{\Lambda}_0}\Big|_{\sqrt{-1}S^*R^{n-d}\times R^d\times S^{d-1}}$. We take a proper convex open neighborhood Δ_0 of ξ_0 in S^{d-1} and divide $S^{d-1}\backslash\Delta_0$ as

(7.50) $$S^{d-1}\backslash\Delta_0=\bigcup_{j=1}^{L}\Delta_j.$$

Then

(7.51) $$F(t,z,\zeta)\equiv\sum_{j,k\geqq 1}\int_{V_4+\sqrt{-1}\tilde{b}_j}d\tilde{z}\Big\{\int_{\Delta_k}G_j(t,\tilde{z},\tilde{\xi})d\sigma(\xi)\Big\}W(z-\tilde{z},\zeta)$$
$$+\sum_{j=1}^{N}\int_{V_4+\sqrt{-1}\tilde{b}_j}d\tilde{z}\Big\{\int_{\Delta_0}G_j(t,\tilde{z},\tilde{\xi})d\sigma(\xi)\Big\}W(z-\tilde{z},\zeta).$$

Because $\int_{\Delta_k}G_j(t,z,\xi)d\sigma(\xi)\in\mathcal{C}_{\tilde{\Lambda}}(U\times\{V_1+\sqrt{-1}(\Gamma_j+\Delta_k^\circ)\}0)$, we have

(7.52) $$F(t,z,\zeta)=\sum_{j,k\geqq 1}\int_{V_4+\sqrt{-1}c_k}d\tilde{z}\Big\{\int_{\Delta_k}d\sigma(\tilde{\xi})G_j(t,\tilde{z},\tilde{\xi})\Big\}W(z-\tilde{z},\zeta)$$
$$+\sum_{j=1}^{N}\int_{V_4+\sqrt{-1}c_0}\Big\{\int_{\Delta_0}d\sigma(\tilde{\xi})G_j(t,\tilde{z},\tilde{\xi})\Big\}W(z-\tilde{z},\zeta)$$

with $c_k\in\Delta_k^\circ$ $(k=0,1,\cdots,L)$.

We remark that

(7.53) $$\int_{\Delta_k}G_j(t,\tilde{z},\tilde{\xi})d\sigma(\xi)\in\mathcal{C}_{\tilde{\Lambda}}(U\times(V_1+\sqrt{-1}\Delta_k)0)$$

and that

(7.54) $$\int_{V_4+\sqrt{-1}c_k}d\tilde{z}\Big\{\int_{\Delta_k}G_j(t,\tilde{z},\tilde{\xi})d\sigma(\tilde{\xi})\Big\}W(z-\tilde{z},\zeta)\in\mathcal{C}_{\tilde{\Lambda}_0}\Big|_{(\rho_0,x_0,\xi_0)}\quad\text{for } k=1,\cdots,L.$$

On the other hand, we have

(7.55) $$\int_{\Delta_0}G_j(t,\tilde{z},\tilde{\xi})d\sigma(\tilde{\xi})=\int_{\Delta_0}d_\xi\omega=\int_{\partial\Delta_0}\omega.$$

We decompose $\partial\Delta_0$ into $(d-1)$ dimensional simplexes as

(7.56) $$\partial\Delta_0=\bigcup_l B_l.$$

Then

(7.57) $$\int_{\partial\Delta_0}\omega=\sum_l\int_{B_l}\omega.$$

Here we have

(7.58) $$\int_{B_l}\omega\in\mathcal{C}_{\tilde{\Lambda}}(U_0\times(\{x\in R^d\,;\,|x-x_0|<\varepsilon\}+\sqrt{-1}B_l^\circ 0)).$$

Thus

$$\int_{V_4+\sqrt{-1}c_0} W(z-\tilde{z}, \zeta)d\tilde{z}\int_{B_l} \omega \in \mathcal{C}_{\tilde{\Lambda}_0}\Big|_{(\rho_0,\, x_0,\, \xi_0)} \tag{7.59}$$

follows. After all, we have proved

$$F(t, z, \zeta) \in \mathcal{C}_{\tilde{\Lambda}_0}\Big|_{(\rho_0, x_0, \xi_0)}. \tag{7.60}$$

(q.e.d.)

Using the propositions above in this section, we can prove the following two theorems in the same way as A. Kaneko [2].

THEOREM 7.5. *Let U be a proper convex subset in $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$ and V be an open subset in $\boldsymbol{R}^d_x$. Take an open subset $V_0 \Subset V$ [resp. $U_0 \Subset U$]. Let Γ_j $(j=1, \cdots, N)$ be a proper convex cone in S^{d-1}. Then for $f(t, x) \in \mathcal{B}^2_\Lambda(U\times V)$ satisfying*

$$SS^2_\Lambda(f) \subset U\times V\times\sqrt{-1}\bigcup_{j=1}^{N} \Gamma_j^\circ, \tag{7.61}$$

there exists $F_j \in \mathcal{C}_{\tilde{\Lambda}}(U_0\times(V_0+\sqrt{-1}\Gamma_j)0)$ $(j=1, \cdots, N)$ such that

$$f=\sum_{j=1}^{N} b_{\Gamma_j}(F_j). \tag{7.62}$$

THEOREM 7.6. *Let U be a proper convex open subset in $\sqrt{-1}\,S^*\boldsymbol{R}^{n-d}$ and V be an open subset in $\boldsymbol{R}^d$. For $F_j \in \mathcal{C}_{\tilde{\Lambda}}((U\times(V+\sqrt{-1}\Gamma_j))0)$ $(j=1, \cdots, N)$, we set $f=\sum_{j=1}^{N} b_{\Gamma_j}(F_j) \in \mathcal{B}^2_\Lambda(U\times V)$. Here Γ_j's are proper convex cones in $\boldsymbol{R}^d$. If $f=0$ on $U\times V$, then for an open set U_0 $(\Subset U)$ and an open set V_0 $(\Subset V)$ and cones Δ_{jk} $(j, k=1, \cdots, N)$ satisfying $\Delta_{jk} \Subset \Gamma_j+\Gamma_k$ there exist $H_{jk} \in \mathcal{C}_{\tilde{\Lambda}}((U_0\times(V_0+\sqrt{-1}\,\Delta_{jk}))0)$ such that*

$$H_{jk}=-H_{kj} \tag{7.63}$$

and

$$F_j=\sum_k H_{jk}. \tag{7.64}$$

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