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Microlocal boundary value problem for Fuchsian operators, I

——*F-mild microfunctions and uniqueness theorem*——

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Abstract. The sheaf of F-mild microfunctions is constructed as a microlocalization of the sheaf of F-mild hyperfunctions introduced previously by the author. Microlocal boundary value problem is formulated in the framework of F-mild microfunctions for microdifferential (i.e. analytic pseudodifferential) operators of Fuchsian type, and the uniqueness of solutions is proved.

Contents

Introduction

Kataoka [4] formulated the theory of microlocal boundary value problem when the boundary is non-characteristic: He introduced the notion of mild hyperfunctions, which is a sheaf of hyperfunctions defined on one side of the boundary having boundary values as hyperfunctions on the boundary. Microlocalizing the sheaf of mild hyperfunctions he defined the sheaf $\mathring{\mathscr{C}}_{N|M_+}$ (we call it the sheaf of mild microfunctions in this paper) on the purely imaginary cosphere bundle $\sqrt{-1}S^*N$ of the boundary N. Microlocal boundary value problem is neatly formulated in the framework of mild microfunctions for microdifferential operators with respect to which N is non-characteristic. Such formulation of microlocal boundary value problem is especially useful when one studies the regularity (or singularity) of solutions of boundary value problem even if the operator under consideration is a partial differential operator. In fact, applying this theory Kataoka [5] studied the propagation of regularity up to the boundary for semi-hyperbolic or diffractive operators.

On the other hand, the present author introduced in [8] the sheaf of F-mild hyperfunctions as a generalization of that of mild hyperfunctions. F-mild hyperfunctions are, roughly speaking, hyperfunctions defined on one side of the boundary which have boundary values as hyperfunctions on the boundary defined in a natural way through their defining functions. Being developed by means of the usual theory of hyperfunctions and microfunctions and of the curvilinear wave expansion (Radon transformation) for holomorphic functions, the theory of F-mild hyperfunctions is more elementary than that of mild hyperfunctions.

In this paper, microlocalizing the sheaf of F-mild hyperfunctions, we introduce the sheaf of F-mild microfunctions on $\sqrt{-1}\,S^*N$. We study properties of the sheaf of F-mild microfunctions by reducing them to microfunctions with a real analytic parameter through some (singular) coordinate transformation: The sheaf of F-mild microfunctions is a soft sheaf; suitable classes of microdifferential operators and of quantized contact transformations act on this sheaf; an F-mild microfunction $u(x)$ has the boundary value $u(+0, x')$ as a microfunction on $\sqrt{-1}\,S^*N$; the sheaf of mild microfunctions is a subsheaf of that of F-mild microfunctions.

Using the sheaf of F-mild microfunctions we formulate microlocal boundary value problem for Fuchsian microdifferential operators. We use the notation $M=\boldsymbol{R}^n \ni x=(x_1, x')$ with $x'=(x_2, \cdots, x_n)$, $N=\{x\in M;\ x_1=0\}$, $D=(D_1, D')$, $D'=(D_2, \cdots, D_n)$ with $D_j=\partial/\partial x_j$. A microdifferential operator P is called a Fuchsian operator of type (k, m) with respect to x_1 if it is written in the form

$$P=x_1^kD_1^m+A_1(x, D')x_1^{k-1}D_1^{m-1}+\cdots+A_k(x, D')D_1^{m-k}+\cdots+A_m(x, D')\,,$$

where k and m are integers such that $0\leqq k\leqq m$, A_j is an operator of order$\leqq j$ such that $A_j(0)=A_j(0, x', D')$ is of order$\leqq 0$ if $1\leqq j\leqq k$. The roots of the equation

$$\lambda(\lambda-1)\cdots(\lambda-m+1)+\sigma_0(A_1(0))(x^*)\lambda(\lambda-1)\cdots(\lambda-m+1)+\cdots \\ +\sigma_0(A_k(0))(x^*)\lambda(\lambda-1)\cdots(\lambda-m+k+1)=0$$

with respect to λ are called the characteristic exponents of P at $x^*\in\sqrt{-1}\,S^*N$, where σ_0 denotes the homogeneous part of order 0. These notions were introduced by Tahara [10] as a generalization of Fuchsian partial differential operators defined by Baouendi-Goulaouic [1]. Boundary value problem for such P is to find an F-mild microfunction u satisfying

$$Pu=f, \quad (D_1^\nu u)(+0, x')=v_\nu(x') \quad (0\leqq\nu\leqq m-k-1)$$

for a given F-mild microfunction f and given microfunctions v_ν. The case of $k=0$ corresponds to the case where N is non-characteristic with respect to P. Hence this formulation is a generalization of the microlocal non-characteristic boundary value problem.

We prove the uniqueness of solutions of this boundary value problem under the assumption that none of the characteristic exponents is an integer$\geqq m-k$. (Generally, this assumption is necessary as is seen by the equation $x_1D_1\delta(x_2)=0$.)

In order to prove this uniqueness theorem, we reduce F-mild microfunctions to mild microfunctions by a suitable change of the variable x_1, and apply the quantized Legendre transformation, which exchanges the variable x_1 and a holomorphic parameter ζ_1, in accordance with [4]: The quantized Legendre transform of P has regular singularities at $\zeta_1=\infty$. Hence we can apply the argument of canonical forms of operators with regular singularities studied previously by the author ([6]). The uniqueness theorem follows from this argument.

For Fuchsian partial differential operators, this uniqueness theorem was proved by the author ([8]) by completely different method based on analytic functionals. On the other hand, microlocal Cauchy problem (i.e. two-sided boundary value problem) was first studied by Tahara [10]; he proved the well-posedness of Cauchy problem in the framework of microfunctions with a real analytic parameter x_1 for Fuchsian microdifferential operators whose principal symbols are of the form $x_1^kD_1^m$. Uniqueness of solution of microlocal Cauchy problem was proved by the author ([7]) for general Fuchsian microdifferential operators. Our uniqueness theorem generalizes these previous results.

In subsequent papers we shall study the solvability of the microlocal boundary value problem for semihyperbolic Fuchsian microdifferential operators and also study the propagation of regularity of solutions.

§1. Sheaf of F-mild hyperfunctions and its microlocalization

1.1. F-mild hyperfunctions

First let us recall the notion of F-mild hyperfunctions introduced by

Ôaku [8]. Since we are interested in local (or microlocal) properties, we restrict our consideration to $M=\mathbf{R}^n \ni x=(x_1, x')$ with $x'=(x_2, \cdots, x_n)$ instead of general real analytic manifolds. Set $X=\mathbf{C}^n \ni z=(z_1, z')$ with $z'=(z_2, \cdots, z_n)$, $N=\{x \in M;\ x_1=0\}$, $Y=\{z \in X;\ z_1=0\}$, $M_+=\{x \in M;\ x_1 \geqq 0\}$, $\operatorname{int} M_+=\{x \in M;\ x_1>0\}$. Let $\iota\colon \operatorname{int} M_+ \to M$ be the natural embedding and consider the sheaf $\mathscr{B}_{N|M_+}=(\iota_* \iota^{-1} \mathscr{B}_M)|_N$, where $\mathscr{B}_M$ denotes the sheaf of hyperfunctions on M. The sheaf $\mathscr{B}_{N|M_+}$ is, roughly speaking, the sheaf of hyperfunctions defined on the positive side of N, and was introduced by Kataoka [4]. The sheaf of F-mild hyperfunctions is a subsheaf of $\mathscr{B}_{N|M_+}$ defined as follows.

Definition 1.1 (Ôaku [8]). Let u be a germ of $\mathscr{B}_{N|M_+}$ *at* $\mathring{x} \in N$. Then u is called *F-mild* (from the positive side of N) at $\mathring{x}$ if and only if u has an expression as a sum of boundary values of holomorphic functions as follows:

$$u(x)=\sum_{j=1}^{J} F_j(x_1, x'+\sqrt{-1}\,\Gamma_j 0) \tag{1}$$

on $\{x \in \operatorname{int} M_+;\ |x-\mathring{x}|<\varepsilon\}$, where J is a positive integer, ε is a positive number, Γ_j is an open convex cone (with vertex at the origin) in $\mathbf{R}^{n-1}$, and F_j is a holomorphic function defined on a neighborhood (in $\mathbf{C}^n$) of

$$D_+(\mathring{x}, \Gamma_j, \varepsilon)=\{z=(z_1, z') \in \mathbf{C}^n;\ |z-\mathring{x}|<\varepsilon,\ \operatorname{Re} z_1 \geqq 0,\ \operatorname{Im} z_1=0,\ \operatorname{Im} z' \in \Gamma_j\}. \tag{2}$$

For an open set U of N, $\mathscr{B}^F_{N|M_+}(U)$ denotes the set of sections of $\mathscr{B}_{N|M_+}$ over U which are F-mild at each point of U. Then $\mathscr{B}^F_{N|M_+}$ constitutes a subsheaf of $\mathscr{B}_{N|M_+}$ and its sections are called *F-mild hyperfunctions*. Note that we used the notation $\mathring{\mathscr{B}}^F_{N|M_+}$ instead of $\mathscr{B}^F_{N|M_+}$ in [8]. We sometimes abbreviate $\mathscr{B}^F_{N|M_+}$ to $\mathscr{B}^F$. The notion of F-mildness is invariant under local (real analytic) coordinate transformations which preserve N and M_+ (Proposition 1 of [8]).

Definition 1.2 ([8]). The homomorphism of 'boundary values' $b\colon \mathscr{B}^F_{N|M_+} \to \mathscr{B}_N$ is defined as follows: If $u(x)$ is an F-mild hyperfunction defined on a neighborhood of $\mathring{x} \in N$ and has an expression (1), then

$$b(u)(x')=u(+0, x')=\sum_{j=1}^{J} F_j(0, x'+\sqrt{-1}\,\Gamma_j 0) \in (\mathscr{B}_N)_{\mathring{x}}.$$

By virtue of the edge of the wedge theorem for F-mild hyperfunctions (Theorem 1 of [8]), $u(+0, x')$ is well-defined (independent of the choice of expression (1)).

We denote by $\mathscr{B}^A$ the restriction to N of the sheaf on M of hyperfunctions which have x_1 as a real analytic parameter.

LEMMA 1.3. *The sheaf homomorphism* $\alpha: \mathscr{B}^F_{N|M_+} \to \mathscr{B}^A$, *where* $\alpha(u)(x) = u(x_1^2, x')$, *is well-defined and injective, and* $\alpha(u)$ *satisfies* $\alpha(u)(-x_1, x') = \alpha(u)(x_1, x')$. *Conversely, if a section* f *of* $\mathscr{B}^A$ *over a subset* $U \subset N$ *satisfies* $f(-x_1, x') = f(x_1, x')$, *there exists a unique section* u *of* $\mathscr{B}^F_{N|M_+}$ *over* U *such that* $\alpha(u) = f$.

PROOF. The well-definedness is proved in Proposition 3 of [8] and the injectivity of α is obvious because $\mathscr{B}^F_{N|M_+}$ is a subsheaf of $\mathscr{B}_{N|M_+}$. Let us prove the latter part of the statement. Let $f \in \mathscr{B}^A(U)$ satisfy $f(-x_1, x') = f(x)$. Suppose that f has an expression

$$f(x) = \sum_{j=1}^{J} G_j(x_1, x' + \sqrt{-1}\,\Gamma_j 0)$$

on a neighborhood (in M) of $\mathring{x} \in U$, where $G_j(z)$ is holomorphic on a neighborhood of

$$D(\mathring{x}, \varepsilon, \Gamma_j) = \{z = (z_1, z') \in \boldsymbol{C}^n;\ |z - \mathring{x}| < \varepsilon,\ \mathrm{Im}\, z_1 = 0,\ \mathrm{Im}\, z' \in \Gamma_j\},$$

and Γ_j, ε, J are as in Definition 1.1. Set

$$H_j(z) = \frac{1}{2}(G_j(z_1, z') + G_j(-z_1, z')).$$

Since $f(-x_1, x') = f(x)$, we have

$$f(x) = \sum_{j=1}^{J} H_j(x_1, x' + \sqrt{-1}\,\Gamma_j 0).$$

It is easy to see that $F_j(z) = H_j(\sqrt{z_1}, z')$ is a well-defined holomorphic function on a neighborhood of $D_+(\mathring{x}, \varepsilon', \Gamma_j)$ for some $\varepsilon' > 0$. Set

$$u(x) = \sum_{j=1}^{J} F_j(x_1, x' + \sqrt{-1}\,\Gamma_j 0).$$

Then $u(x)$ is an F-mild hyperfunction defined on a neighborhood of $\mathring{x}$ and $u(x_1^2, x') = f(x)$ holds. Since α is injective, there exists a unique F-mild hyperfunction u over U such that $\alpha(u) = f$.

PROPOSITION 1.4. *The sheaf* $\mathscr{B}^F_{N|M_+}$ *of F-mild hyperfunctions is a soft sheaf on N.*

PROOF. Let Z be a closed set of N and $u(x)$ be an F-mild hyperfunction defined on a neighborhood of Z. Then $f(x) = u(x_1^2, x')$ is a section of $\mathscr{B}^A$ defined on a neighborhood of Z. Since $\mathscr{B}^A$ is a soft sheaf (cf. Lemma 1.7), there exists a section g of $\mathscr{B}^A$ over N such that $f = g$ on Z. Since $f(-x_1, x') = f(x)$

on Z, we have $f(x)=(g(x)+g(-x_1, x'))/2$ on Z. By the preceding lemma, there exists an F-mild hyperfunction v on N such that $v(x_1^2, x')=(g(x)+g(-x_1, x'))/2$. Since $\alpha(v)=f=\alpha(u)$ on Z, we get $v=u$ on Z. This completes the proof.

1.2. *F*-mild microfunctions

We shall microlocalize the sheaf $\mathscr{B}^F_{N|M_+}$. Let $\sqrt{-1}\,S^*M$ and $\sqrt{-1}\,S^*N$ be the purely imaginary cosphere bundles of M and N respectively and $\pi_M\colon \sqrt{-1}\,S^*M \to M$ and $\pi_N\colon \sqrt{-1}\,S^*N \to N$ be the canonical projections. Let

$$\rho\colon \sqrt{-1}\,S^*M|_N - \sqrt{-1}\,S^*_N M \longrightarrow \sqrt{-1}\,S^*N$$

be the canonical map; i.e. $\rho(0, x', \sqrt{-1}\,\xi\infty)=(x', \sqrt{-1}\,\xi'\infty)$ where $\xi=(\xi_1, \xi')\in \boldsymbol{R}^n$ with $\xi' \in \boldsymbol{R}^{n-1}-\{0\}$. The ρ-singular spectrum of an F-mild hyperfunction is defined as follows.

DEFINITION 1.5 ([8]). For an F-mild hyperfunction u on an open set U of N, its *ρ-singular spectrum* ρ-SS(u) is the closed subset of $\pi_N^{-1}(U)$ defined as follows: A point $x^*=(\mathring{x}, \sqrt{-1}\,\xi'\infty)$ of $\pi_N^{-1}(U)$ is not contained in ρ-SS(u) if and only if $u(x)$ has an expression (1) on a neighborhood of $\mathring{x}$ with $\xi' \notin \Gamma_j^\circ = \{\xi' \in \boldsymbol{R}^{n-1};\ \langle y', \xi'\rangle \geqq 0 \text{ for any } y' \in \Gamma_j\}$ for $j=1, \cdots, J$.

We identify $\sqrt{-1}\,S^*N$ with $N\times\sqrt{-1}\,S^{n-2}$, where S^{n-2} denotes the $(n-2)$-dimensional sphere. For open connected sets $U\subset N$ and $\Delta\subset S^{n-2}$, we associate a $\boldsymbol{C}$-vector space $\{u \in \mathscr{B}^F(U);\ \rho\text{-SS}(u)\cap(U\times\sqrt{-1}\,\Delta)=\varnothing\}$ to $U\times\sqrt{-1}\,\Delta$. This correspondence defines a presheaf on $\sqrt{-1}\,S^*N$. The sheaf associated with this presheaf is denoted by $\mathscr{A}^{*F}$.

DEFINITION 1.6. The sheaf $\mathscr{C}^F_{N|M_+}$ is defined by

$$\mathscr{C}^F_{N|M_+} = \pi_N^{-1}\mathscr{B}^F_{N|M_+}/\mathscr{A}^{*F}\,.$$

Sections of $\mathscr{C}^F_{N|M_+}$ are called *F-mild microfunctions*. Let sp: $\pi_N^{-1}\mathscr{B}^F_{N|M_+}\to\mathscr{C}^F_{N|M_+}$ be the canonical homomorphism (spectral map).

We set $\mathscr{C}^\Delta=\rho_!(\mathscr{C}_M|_Z)$ with $Z=\sqrt{-1}\,S^*M|_N-\sqrt{-1}\,S^*_N M$, where $\mathscr{C}_M$ denotes the sheaf on $\sqrt{-1}\,S^*M$ of microfunctions and $\rho_!$ denotes the direct image with proper supports. Then the following lemma follows from the flabbiness of $\mathscr{C}_M$.

LEMMA 1.7. (i) *For a closed set K of N and a compact set Δ of S^{n-2}, there is a canonical isomorphism by the spectral map,*

$$\mathscr{C}^\Delta(K\times\sqrt{-1}\,\Delta)\cong\mathscr{B}^\Delta(K)/\{u \in \mathscr{B}^\Delta(K);\ \mathrm{SS}(u)\cap\rho^{-1}(K\times\sqrt{-1}\,\Delta)=\varnothing\}\,,$$

where SS(u) *denotes the singular spectrum of* u. *In paraticular, there is an exact sequence*

$$0\longrightarrow \mathscr{A}_M|_N\longrightarrow \mathscr{B}^A\longrightarrow \pi_{N*}\mathscr{C}^A\longrightarrow 0\,,$$

where $\mathscr{A}_M$ *denotes the sheaf of real analytic functions on* M.
(ii) *The sheaf* $\mathscr{C}^A$ *is a soft sheaf on* $\sqrt{-1}\,S^*N$.

LEMMA 1.8. *There exists an injective sheaf homomorphism* $\alpha\colon \mathscr{C}^F_{N|M_+}\to\mathscr{C}^A$ *defined by* $\alpha(\mathrm{sp}(f))=\mathrm{sp}(\alpha(f))$ *for each section* f *of* $\mathscr{B}^F$. *For each section* u *of* $\mathscr{C}^F_{N|M_+}$, $\alpha(u)(-x_1, x')=\alpha(u)(x)$ *holds. Conversely, if a section* v *of* $\mathscr{C}^A$ *over* $U\subset\sqrt{-1}\,S^*N$ *satisfies* $v(-x_1, x')=v(x)$, *then there exists a unique section* u *of* $\mathscr{C}^F_{N|M_+}$ *over* U *such that* $\alpha(u)=v$.

PROOF. For a section f of $\mathscr{B}^F$, ρ-SS$(f)=\rho(\mathrm{SS}(\alpha(f)))$ holds (see Definition 3 and Proposition 4 of [8]). Hence the injective sheaf homomorphism α of $\mathscr{C}^F_{N|M_+}$ to $\mathscr{C}^A$ is well-defined by virtue of Definition 1.6 and Lemma 1.7 (i). Let $v\in\mathscr{C}^A$ satisfy $v(-x_1, x')=v(x)$. For each point x^* of U, there exists a germ g of $\mathscr{B}^A$ at $\pi(x^*)$ such that $v=\mathrm{sp}(g)$ on a neighborhood of x^*. There exists a germ f of $\mathscr{B}^F$ at $\pi(x^*)$ such that $\alpha(f)=(g(x)+g(-x_1, x'))/2$. Since $v(x)=\mathrm{sp}((g(x)+g(-x_1, x'))/2)$ holds by virtue of the assumption, sp(f) satisfies $\alpha(\mathrm{sp}(f))=v$ on a neighborhood of x^*. Since α is injective, there exists a unique section u of $\mathscr{C}^F_{N|M_+}$ over U such that $\alpha(u)=v$.

PROPOSITION 1.9. *For a closed set* K *of* N *and a compact set* Δ *of* S^{n-2},

$$\begin{aligned}&\mathscr{C}^F_{N|M_+}(K\times\sqrt{-1}\,\Delta)\\&=\mathscr{B}^F_{N|M_+}(K)/\{u\in\mathscr{B}^F_{N|M_+}(K);\ \rho\text{-SS}(u)\cap(K\times\sqrt{-1}\,\Delta)=\varnothing\}\,.\end{aligned}$$

PROOF. Since there is an exact sequence

$$\begin{aligned}&0\longrightarrow\{u\in\mathscr{B}^F(K);\ \rho\text{-SS}(u)\cap(K\times\sqrt{-1}\,\Delta)=\varnothing\}\longrightarrow\mathscr{B}^F(K)\\&\longrightarrow\mathscr{C}^F_{N|M_+}(K\times\sqrt{-1}\,\Delta)\,,\end{aligned}$$

we have only to show the surjectivity of the last homomorphism. Let u be an F-mild microfunction on $K\times\sqrt{-1}\,\Delta$. Then by Lemma 1.7 (i) there exists a section g of $\mathscr{B}^A$ over K such that $\mathrm{sp}(g)=\alpha(u)$ on $K\times\sqrt{-1}\,\Delta$. Note $\mathrm{sp}((g(x)+g(-x_1, x'))/2)=\alpha(u)$ holds on $K\times\sqrt{-1}\Delta$. There exists a section f of $\mathscr{B}^F$ over K such that $\alpha(f)=(g(x)+g(-x_1, x'))/2$ on K by virtue of Lemma 1.3. Since $\mathrm{sp}(\alpha(f))=\alpha(u)$ holds on $K\times\sqrt{-1}\,\Delta$, we get $\mathrm{sp}(f)=u$. This completes the proof.

COROLLARY 1.10. *There is a short exact sequence*

$$0 \longrightarrow \mathscr{A}_M|_N \longrightarrow \mathscr{B}^F_{N|M_+} \longrightarrow \pi_{N*}\mathscr{C}^F_{N|M_+} \longrightarrow 0 .$$

PROPOSITION 1.11. *The sheaf* $\mathscr{C}^F_{N|M_+}$ *is a soft sheaf on* $\sqrt{-1}\,S^*N$.

PROOF. Let u be an F-mild microfunction on a closed set Z of $\sqrt{-1}\,S^*N$. Since $\mathscr{C}^A$ is a soft sheaf, there exists a section v of $\mathscr{C}^A$ over $\sqrt{-1}\,S^*N$ such that $v=\alpha(u)$ on Z. There exists a section w of $\mathscr{C}^F_{N|M_+}$ over $\sqrt{-1}\,S^*N$ such that $\alpha(w)(x)=(v(x)+v(-x_1, x'))/2$. Since $\alpha(w)=\alpha(u)$ on Z, $w=u$ holds on Z. This completes the proof.

PROPOSITION 1.12. *The sheaf* $\mathscr{C}^F_{N|M_+}$ *is invariant under local coordinate transformations which preserve* N *and* M_+.

PROOF. Let $z=\varphi(w)$ be a local coordinate transformation of X defined by

$$z_1=\varphi_1(w)=\psi_1(w)w_1, \quad z_j=\varphi_j(w) \quad (2\leqq j\leqq n)$$

on a neighborhood Ω of 0 in $X=\boldsymbol{C}^n$. We assume that φ preserves N, M_+, 0, and the orientation of N; i.e. $\varphi_j(u)\in\boldsymbol{R}$ for $u\in\Omega\cap\boldsymbol{R}^n$ and $2\leqq j\leqq n$, $\psi_1(u)>0$ for $u\in\Omega\cap\boldsymbol{R}^n$, $\varphi(0)=0$, and $\det(\partial\varphi/\partial w)(0)>0$. The map φ induces a local coordinate transformation φ^* of $\sqrt{-1}\,S^*N$ so that $\varphi^*(u', \sqrt{-1}\,\eta'\infty)=(x', \sqrt{-1}\,\xi'\infty)$, where $x'=\varphi(0, u')$ and

$$\eta_i=\sum_{j=2}^{n}\frac{\partial\varphi_j}{\partial u_i}\xi_j \qquad (i=2, \cdots, n) .$$

Since $\det(\partial\varphi'/\partial u')$ does not vanish on a neighborhood of 0 ($\varphi'=(\varphi_2, \cdots, \varphi_n)$), φ^* is well-defined on a neighborhood of $\pi_N^{-1}(0)$.

Now let $f(x)=F(x_1, x'+\sqrt{-1}\,\Gamma 0)$ be an F-mild hyperfunction, where F is holomorphic on a neighborhood of $D_+(0, \Gamma, \varepsilon)$ with an open convex cone Γ of $\boldsymbol{R}^{n-1}$ and $\varepsilon>0$. In view of Definitions 1.5 and 1.6, it suffices to show that $f(\varphi(u))$ is also F-mild at 0 and

$$\varphi^*(\rho\text{-SS}(f(\varphi(u))))\cap\pi_N^{-1}(0)\subset\{0\}\times\sqrt{-1}\,\Gamma^\circ\infty .$$

Let $\Gamma'\Subset\Gamma$ be a compactly contained subcone. By virtue of Lemma 1 of [8], F is holomorphic on

$$(3) \qquad \{z=x+\sqrt{-1}\,y\in\boldsymbol{C}^n;\ |z|<c,\ y_1^2<c|y'|^2(x_1+c|y'|^2),\ y'\in\Gamma'\}$$

with some $c>0$. Put $w=u+\sqrt{-1}\,v$ with $u, v\in\boldsymbol{R}^n$. Then we have

$$|\psi_1(u_1, w')-\psi_1(u)|\leqq C|v'| ,$$

and hence

(4) $$|\mathrm{Im}\,\varphi_1(u_1, w')| \leqq C u_1 |v'|, \qquad \mathrm{Re}\,\varphi_1(u_1, w') \geqq 0$$

for some C independent of $w=(u_1, w')$ with $|w|<\delta$ and $w_1=u_1\geqq 0$. Set

$$V=\Big\{v'=(v_2, \cdots, v_n) \in \boldsymbol{R}^{n-1};\ v_i=\sum_{j=2}^{n} \frac{\partial w_i}{\partial z_j}(0) y_j\ (i=2, \cdots, n),$$
$$y'=(y_2, \cdots, y_n) \in \Gamma'\Big\}.$$

Then for any open subcone $V' \Subset V$, there exists $\delta'>0$ such that $\mathrm{Im}\,(\varphi(w)) \in \Gamma'$ for any $w=(u_1, w')$ with $|w|<\delta'$, $u_1\geqq 0$, $\mathrm{Im}\,w' \in V'$ since

$$y_i=\sum_{j=2}^{n} \frac{\partial \varphi_i}{\partial w_j}(0) v_j + O(|u||v'|) + O(|v'|^2)$$

for $w=(u_1, w')$ with $u_1\geqq 0$ and $i=2, \cdots, n$. In view of (4) there exists $\varepsilon>0$ such that $\varphi(w)$ is contained in the set (3) if $|w|<\varepsilon$, $w_1=u_1\geqq 0$, and $\mathrm{Im}\,(w') \in V'$. Thus $f(\varphi(u))=F(\varphi(u_1, u'+\sqrt{-1}\,V'\,0))$ is also F-mild at 0 and

$$\rho\text{-SS}(f(\varphi(u))) \cap \pi_N^{-1}(0) \subset \{0\} \times \sqrt{-1}\,V^\circ \infty$$

holds since $V' \Subset V$ is arbitrary. Noting that

$$\varphi^*(\{0\} \times \sqrt{-1}\,V^\circ \infty) = \{0\} \times \sqrt{-1}\,{}^t\Big(\frac{\partial w'}{\partial z'}(0)\Big) V^\circ \infty$$
$$= \{0\} \times \sqrt{-1}\Big(\Big(\frac{\partial \varphi'}{\partial w'}(0)\Big) V\Big)^\circ \infty = \{0\} \times \sqrt{-1}\,\Gamma'^\circ \infty,$$

we get

$$\varphi^*(\rho\text{-SS}(f(\varphi(u)))) \cap \pi_N^{-1}(0) \subset \{0\} \times \Gamma^\circ \infty$$

since $\Gamma' \Subset \Gamma$ is arbitrary. This completes the proof.

PROPOSITION 1.13. *The sheaf homomorphism* b: $\mathscr{B}^F_{N|M_+} \to \mathscr{B}_N$ *defined in Definition* 1.2 *induces a sheaf homomorphism*

$$b\colon \mathscr{C}^F_{N|M_+} \longrightarrow \mathscr{C}_N$$

so that $b(\mathrm{sp}(f))=\mathrm{sp}(b(f))$ *for any F-mild hyperfunction f. Moreover, b is invariant under local coordinate transformations of M preserving N and* M_+.

PROOF. The first statement follows immediately from Definitions 1.2 and 1.6 and the fact that $\mathrm{SS}(b(f)) \subset \rho\text{-SS}(f)$ holds for any F-mild hyperfunction f. The last statement can be proved in the same way as in the proof of Proposition 1.12.

1.3. Action of microdifferential operators on F-mild microfunctions

Now we introduce a class of microdifferential operators which act on $\mathscr{C}^F_{N|M+}$. Let $\mathscr{E}_{X/Y}$ be the sheaf of microdifferential operators (of finite order) associated with the projection of X to Y; i.e. $\mathscr{E}_{X/Y}$ is the sheaf of rings of microdifferential operators $A(z_1, z', D')$ having z_1 as a holomorphic parameter (cf. [9, Chapter II]); here we write $D'=D_{z'}=(D_2, \cdots, D_n)$ with $D_j=D_{z_j}=\partial/\partial z_j$. We regard $\mathscr{E}_{X/Y}$ as a sheaf on the cotangent bundle T^*Y of Y by restricting it to a neighborhood of $z_1=0$, and also regard $\sqrt{-1}\,S^*N=N\times\sqrt{-1}\,S^{n-2}$ as a subset of T^*Y. Let $\mathscr{E}_{X/Y}[D_1]$ be the sheaf of rings of polynomials of D_1 with coefficients in $\mathscr{E}_{X/Y}$. Note that for any point x^* of $\sqrt{-1}\,S^*N$, a germ of $\mathscr{E}_{X/Y}[D_1]$ at x^* can be regarded as a microdifferential operator defined on a neighborhood of $\rho^{-1}(x^*)$. In particular, sections of $\mathscr{E}_{X/Y}[D_1]$ act on $\mathscr{C}^A$ as sheaf homomorphisms. We define the action of $\mathscr{E}_{X/Y}[D_1]$ on $\mathscr{C}^F_{N|M+}$ through its action on $\mathscr{C}^A$.

PROPOSITION 1.14. *The sheaf of rings $\mathscr{E}_{X/Y}[D_1]$ acts on $\mathscr{C}^F_{N|M+}$ so that $\alpha(A(x, D_{x'})u(x))=A(x_1^2, x', D_{x'})\alpha(u)(x)$ for a section A of $\mathscr{E}_{X/Y}$ over $U\subset\sqrt{-1}\,S^*N$ and a section u of $\mathscr{C}^F_{N|M+}$ over U.*

PROOF. Since $A((-x_1)^2, x', D_{x'})\alpha(u)(-x_1, x')=A(x_1^2, x', D_{x'})\alpha(u)(x)$ holds, there exists a unique section v of $\mathscr{C}^F_{N|M+}$ over U such that $\alpha(v)(x)=A(x_1^2, x', D_{x'})\alpha(u)(x)$ by virtue of Lemma 1.8. Hence $v=A(x, D_{x'})u$ is well-defined. If B is another section of $\mathscr{E}_{X/Y}$ over U, we have

$$\begin{aligned}\alpha(B(Au))&=B(x_1^2, x', D_{x'})\alpha(Au)=B(x_1^2, x', D_{x'})A(x_1^2, x', D_{x'})\alpha(u)\\&=(BA)(x_1^2, x', D_{x'})\alpha(u)=\alpha((BA)u)\,.\end{aligned}$$

Thus $B(Au)=(BA)u$ holds and $\mathscr{E}_{X/Y}$ acts on $\mathscr{C}^F_{N|M+}$. Since the sheaf $\mathscr{D}_X$ of partial differential operators with analytic coefficients acts on $\mathscr{C}^F_{N|M+}$, the sheaf of rings $\mathscr{E}_{X/Y}[D_1]$ acts on $\mathscr{C}^F_{N|M+}$. This completes the proof.

Next let us describe the action of $\mathscr{E}_{X/Y}[D_1]$ on $\mathscr{C}^F_{N|M+}$ concretely employing methods of Kashiwara-Kawai [3] and Bony-Schapira [2]. Let $A(x, D_{x'})=\sum_{j\leq m}A_j(x, D_{x'})$ be a germ of $\mathscr{E}_{X/Y}$ at $x^*=(0, \sqrt{-1}\,dx_n\infty)\in\sqrt{-1}\,S^*N$, where A_j is homogeneous of order j with $j, m\in\boldsymbol{Z}$. There is a constant c_0 with $0<c_0<1$ such that $A(z, D_{z'})$ is a microdifferential operator defined on a neighborhood of

$$\begin{aligned}\omega_0=\{&(z, \zeta')\in\boldsymbol{C}\times T^*\boldsymbol{C}^{n-1};\ \zeta'=(\zeta_2, \cdots, \zeta_n)\in\boldsymbol{C}^{n-1}-\{0\},\\&|z_1|\leqq c_0,\ |z'|\leqq c_0,\ |\zeta_j|\leqq c_0|\zeta_n|\ (j=2, \cdots, n-1)\}\,.\end{aligned}$$

Then A is developed formally

$$A=\sum_{\alpha'} a_{\alpha'}(z)D_{z'}^{\alpha'},$$

where the multi-index α' runs on the set $\{\alpha'=(\alpha_2, \cdots, \alpha_n)\in \boldsymbol{Z}^{n-1}; \alpha_j \geqq 0$ $(j=2, \cdots, n-1), |\alpha'|=\alpha_2+\cdots+\alpha_n \leqq m\}$. The kernel function $K(z, w')$ of A is defined by

$$K(z, w')=\sum_{\alpha'} a_{\alpha'}(z)\Phi_{\alpha'}(w'),$$

where

$$\Phi_{\alpha'}(w')=\Phi_{\alpha_2}(w_2)\cdots\Phi_{\alpha_n}(w_n)$$

with

$$\Phi_\nu(t)=\frac{1}{2\pi\sqrt{-1}}\frac{\nu!}{(-t)^{\nu+1}} \qquad (\nu=0, 1, 2, \cdots),$$

$$\Phi_\nu(t)=-\frac{1}{2\pi\sqrt{-1}}\frac{1}{(-\nu-1)!}t^{-\nu-1}\log t \qquad (\nu=-1, -2, \cdots).$$

It is easy to see that $K(z, w')$ is a (multi-valued) analytic function on

$$\Omega_0=\{(z, w')\in \boldsymbol{C}^n\times \boldsymbol{C}^{n-1}; |z_1|<c_0, |z'|<c_0, |w_n|<C^{-1}, 0<|w_n|<c_0|w_j| \\ (j=2, \cdots, n-1)\}$$

with some $C>0$. Put

$$\Gamma=\{y'\in \boldsymbol{R}^{n-1}; y_n>c_0|y_j|/2 \ \ (j=2, \cdots, n-1)\}.$$

Now let u be a germ of $\mathscr{C}_{N|M_+}^F$ at x^*. Then in view of the softness of $\mathscr{C}_{N|M_+}^F$ we can find a holomorphic function F defined on a neighborhood of $D_+(0, c_1, \Gamma)$ such that $u=\mathrm{sp}(F(x_1, x'+\sqrt{-1}\Gamma 0))$ at x^* with some c_1 such that $0<c_1<\min(c_0, C^{-1})$. Choosing a so that $0<a<c_0c_1/6n$, we put

$$\Sigma=\{z'\in \boldsymbol{C}^{n-1}; z_n=\sqrt{-1}a\},$$
$$\Omega'=\{z'\in \boldsymbol{C}^{n-1}; |z_j|+c_0^{-1}|z_n-\sqrt{-1}a|<2ac_0^{-1}\},$$
$$\Omega=\Omega'\cap(\boldsymbol{R}^{n-1}+\sqrt{-1}\Gamma).$$

For each $z'\in\Omega$, let $\gamma: \boldsymbol{T}^{n-1}\to\Omega$ (where $\boldsymbol{T}^{n-1}=(\boldsymbol{R}/\boldsymbol{Z})^{n-1}$ is the $(n-1)$-dimensional torus) be a chain such that

(i) $\gamma(s_2, \cdots, s_{n-1}, 0)\in\Sigma$ for any $(s_2, \cdots, s_{n-1})\in \boldsymbol{T}^{n-2}$,

(ii) $w'\in\gamma$ and $|z_1|<c_0$ imply $(z, w'-z')\in\Omega_0$,

(iii) $\displaystyle\frac{1}{(2\pi\sqrt{-1})^{n-1}}\int_\gamma \frac{dw'}{(w_2-z_2)\cdots(w_n-z_n)}=1.$

Then we set

$$(A)_a F(z)=\int_\gamma K(z, z'-w')F(z_1, w')dw' .$$

This depends neither on the choice of γ satisfying (i)-(iii) nor on the choice of the branch of $K(z, z'-w')$ (it is easy to see that such a γ exists). Since Ω is $c_0-\Sigma$-flat in the sense of [2] with diameter smaller than C^{-1} and since F is holomorphic in a neighborhood of $\{x_1 \in \boldsymbol{R};\ 0\leqq x_1<c_1/2\}\times\Omega$, we can verify that $(A)_a F$ is also holomorphic in a neighborhood of this set and that the germ $\mathrm{sp}((A)_a F(x_1, x'+\sqrt{-1}\,\Gamma 0))$ of $\mathscr{C}^F_{N|M+}$ at x^* is independent of a by applying the argument of [2] to the variables z'.

PROPOSITION 1.15. *In the above situation,* $\mathrm{sp}((A)_a F(x_1, x'+\sqrt{-1}\,\Gamma 0))$ *coincides with* $Au(x)$ *as a germ of* $\mathscr{C}^F_{N|M+}$ *at* x^*.

PROOF. Since $F(z_1^2, z')$ is holomorphic on a neighborhood of $\{z_1=x_1 \in \boldsymbol{R};\ |x_1|<\sqrt{c_1/2}\}\times\Omega$,

$$(A(z_1^2, z', D'))_a F(z_1^2, z')=((A)_a F)(z_1^2, z')$$

is also holomorphic in a neighborhood of this set. Moreover, in view of Theorem 3.10 of [3], $(A_a F)(x_1^2, x'+\sqrt{-1}\,\Gamma 0)$ coincides with $A(x_1^2, x', D_{x'})\alpha(u)$ as a germ of $\mathscr{C}^A$ at x^* (α is defined in Lemma 1.8). Hence the statement of this proposition follows from Lemma 1.8 since

$$\alpha(\mathrm{sp}((A)_a F(x_1, x'+\sqrt{-1}\,\Gamma 0)))=A(x_1^2, x', D_{x'})\alpha(u)=\alpha(Au)$$

holds.

1.4. Quantized contact transformations

Now let us introduce a class of quantized contact transformations acting on $\mathscr{C}^F_{N|M+}$ and $\mathscr{E}_{X/Y}[D_1]$ (see [9] for the definition of quantized contact transformations). Let ψ be a canonical transformation of $\sqrt{-1}\,S^*M$ defined on a neighborhood of $\rho^{-1}(x^*)$ with $x^* \in \sqrt{-1}\,S^*N$ which preserves x_1. Let (y, η) be a copy of (x, ξ). Then the transformation $(y, \eta)=\psi(x, \xi)$ is defined by

$$\begin{aligned} y_1&=\varphi_1(x, \xi)=x_1 , \\ y_j&=\varphi_j(x, \xi) \qquad (j=2, \cdots, n) , \\ \eta_k&=\psi_k(x, \xi) \qquad (k=1, \cdots, n) \end{aligned}$$

with the relations

$$\{\varphi_j, \varphi_k\}=\{\psi_j, \psi_k\}=0$$
$$\{\varphi_j, \psi_k\}=-\delta_{jk} \qquad (j, k=1, \cdots, n),$$

where δ_{jk} is Kronecker's δ and

$$\{\varphi_j, \varphi_k\}=\sum_{\nu=1}^{n}\left(\frac{\partial\varphi_j}{\partial\xi_\nu}\frac{\partial\varphi_k}{\partial x_\nu}-\frac{\partial\varphi_j}{\partial x_\nu}\frac{\partial\varphi_k}{\partial\xi_\nu}\right), \quad \text{etc.}$$

From these relations, it follows that φ_j and ψ_j $(j=2, \cdots, n)$ do not depend on ξ_1 (hence we write $\varphi_j=\varphi_j(x, \xi')$ etc.), ψ_1 is written in the form $\psi_1(x, \xi)=\xi_1+\psi_1'(x, \xi')$, and that the transformation $(y', \eta')=\psi'(x', \xi')$ defined by

$$y_j=\varphi_j(0, x', \xi'), \quad \eta_j=\psi_j(0, x', \xi') \quad (j=2, \cdots, n)$$

is a canonical transformation of $\sqrt{-1}\, S^*N$ defined on a neighborhood of x^*. Let $\mathscr{E}_X$ be the sheaf on T^*X of microdifferential operators.

There exists a quantized contact transformation

$$\Psi: (\psi^{-1}\mathscr{E}_X)|_{\rho^{-1}(U)} \xrightarrow{\sim} \mathscr{E}_X|_{\rho^{-1}(U)},$$

where U is a neighborhood of x^* in $\sqrt{-1}\, S^*N$. In fact, we can choose Ψ so that $\Psi(y_1)=x_1$ in view of the argument of the proof of Theorem 3.3.3 of [9, Chapter II]. Then the relations

$$[x_1, \Psi(D_{y_1})]=-1, \quad [x_1, \Psi(D_{y_j})]=0 \quad (j=2, \cdots, n)$$

hold, where we use the notation $[A, B]=AB-BA$ for operators A and B. Hence we can put

$$(5) \qquad \begin{cases}\Psi(y_j)=P_j(x, D_{x'}), \quad \Psi(D_{y_j})=Q_j(x, D_{x'}) \quad (j=2, \cdots, n), \\ \Psi(D_{y_1})=Q_1(x, D_x)=D_{x_1}+Q_1'(x, D_{x'}),\end{cases}$$

where $D_{x'}$ denotes $(D_{x_2}, \cdots, D_{x_n})$. In particular, we can guarantee that Ψ is defined on $\rho^{-1}(U)$ with a neighborhood U of x^* and that Ψ induces a quantized contact transformation Ψ' associated with ψ' defined by

$$\Psi'(y_j)=P_j(0, x', D_{x'}), \quad \Psi'(D_{y_j})=Q_j(0, x', D_{x'}) \quad (j=2, \cdots, n).$$

Moreover we get the following proposition easily.

PROPOSITION 1.16. *Ψ induces isomorphisms*

$$\Psi: \psi'^{-1}\mathscr{E}_{X/Y} \xrightarrow{\sim} \mathscr{E}_{X/Y}$$

and

$$\Psi: \psi'^{-1}\mathscr{E}_{X/Y}[D_1] \xrightarrow{\sim} \mathscr{E}_{X/Y}[D_1]$$

on a neighborhood U of x^*.

In order to describe the action of Ψ on $\mathscr{C}^F_{N|M_+}$, let us denote by $L(x, y)$ a kernel function of Ψ.

Proposition 1.17. *Ψ induces a sheaf isomorphism*

$$\Psi: \psi'^{-1}\mathscr{C}^F_{N|M_+} \xrightarrow{\sim} \mathscr{C}^F_{N|M_+}$$

defined on a neighborhood of x^* and compatible with the action of $\mathscr{E}_{X/Y}[D_1]$; i.e., $\Psi(Au)=\Psi(A)\Psi(u)$ for a section u of $\mathscr{C}^F_{N|M_+}$ and a section A of $\mathscr{E}_{X/Y}[D_1]$.

Proof. $L=L(x, y)$ satisfies the equations

$$\begin{aligned}
&(y_1-x_1)L=0\,,\\
&(y_j-P_j(x, D_{x'}))L=0 \qquad (j=2, \cdots, n)\,,\\
&(D_{y_1}+D_{x_1}+Q_1'(x, D_{x'}))L=0\,,\\
&(D_{y_j}+Q_j(x, D_{x'}))L=0 \qquad (j=2, \cdots, n)\,.
\end{aligned}$$

Hence L is written in the form

$$L(x, y)=\delta(y_1-x_1)L'(x, y')\,,$$

where the support of L' as a microfunction is contained in

$$\{(x, y', \sqrt{-1}(\xi, \eta')\infty);\ y_j=\varphi_j(x, \xi'),\ \eta_j=-\psi_j(x, \xi')\ (j=2, \cdots, n),\ \xi_1+\psi_1'(x, \xi')=0\}$$

and L' satisfies

$$\begin{aligned}
&(y_j-P_j(x, D_{x'}))L'(x, y')=0 \qquad (j=2, \cdots, n)\,,\\
&(D_{x_1}+Q_1'(x, D_{x'}))L'(x, y')=0\,,\\
&(D_{y_j}+Q_j(x, D_{x'}))L'(x, y')=0 \qquad (j=2, \cdots, n)\,.
\end{aligned}$$

Put

$$\alpha(L)(x, y)=\delta(y_1-x_1)L'(x_1^2, x', y')\,.$$

Then $\alpha(L)$ satisfies

$$\begin{aligned}
&(y_1-x_1)\alpha(L)(x, y)=0\,,\\
&(y_j-P_j(x_1^2, x', D_{x'}))\alpha(L)(x, y)=0 \qquad (j=2, \cdots, n)\,,\\
&(D_{y_1}+D_{x_1}+2x_1Q_1'(x_1^2, x', D_{x'}))\alpha(L)(x, y)=0\,,\\
&(D_{y_j}+Q_j(x_1^2, x', D_{x'}))\alpha(L)(x, y)=0 \qquad (j=2, \cdots, n)\,.
\end{aligned}$$

Hence $\alpha(L)$ is a kernel function of the quantized contact transformation $\alpha(\Psi)$ defined by

$$\text{(6)}\qquad \begin{cases} \alpha(\Psi)(y_1)=x_1\,, \quad \alpha(\Psi)(y_j)=P_j(x_1^2, x', D_{x'})\,, & (j=2,\cdots,n)\,,\\ \alpha(\Psi)(D_{y_1})=D_{x_1}+2x_1Q_1'(x_1^2, x', D_{x'})\,, & \\ \alpha(\Psi)(D_{y_j})=Q_j(x_1^2, x', D_{x'}) & (j=2,\cdots,n)\,. \end{cases}$$

$\alpha(\Psi)$ induces a sheaf isomorphism

$$\alpha(\Psi): \psi'^{-1}\mathscr{C}^A \xrightarrow{\sim} \mathscr{C}^A$$

on a neighborhood of x^* by

$$\text{(7)}\qquad \alpha(\Psi)v(x)=\int \alpha(L)(x, y)v(y)dy=\int L'(x_1^2, x', y')v(x_1, y')dy'\,.$$

There exists a unique sheaf homomorphism

$$\Psi: \psi'^{-1}\mathscr{C}^F_{N|M+} \longrightarrow \mathscr{C}^F_{N|M+}$$

defined on a neighborhood of x^* such that $\alpha(\Psi(u))=\alpha(\Psi)(\alpha(u))$ for any section u of $\mathscr{C}^F_{N|M+}$. Since $\alpha(\Psi)$ is an isomorphism, the injectivity of Ψ follows from that of α. Let us show that Ψ is surjective. Let u be a section of $\mathscr{C}^F_{N|M+}$ defined on a neighborhood of $\psi(x^*)$. Then there exists a section v of $\mathscr{C}^A$ defined on a neighborhood of x^* such that $\alpha(\Psi)(v)=\alpha(u)$. In view of (7), we get

$$\alpha(\Psi)(v(-x_1, x'))=\alpha(u)(-x_1, x')=\alpha(u)(x)\,.$$

By the injectivity of $\alpha(\Psi)$, $v(-x_1, x')=v(x)$ holds. Hence there exists a section w of $\mathscr{C}^F_{N|M+}$ such that $\alpha(w)=v$, and $\Psi(w)=u$ holds. Thus the sheaf homomorphism

$$\Psi: \psi'^{-1}\mathscr{C}^F_{N|M+} \longrightarrow \mathscr{C}^F_{N|M+}$$

is an isomorphism on a neighborhood of x^*.

Let $A=A(x, D_{x'})$ be a section of $\mathscr{E}_{X/Y}$ on a neighborhood of x^*. Then we have

$$\text{(8)}\qquad \begin{aligned} \alpha(\Psi(Au)) &= \alpha(\Psi)(A(y_1^2, y', D_{y'})\alpha(u))\\ &= \alpha(\Psi)(A(y_1^2, y', D_{y'}))\alpha(\Psi)(\alpha(u))\\ &= \Psi(A)(x_1^2, x', D_{x'})\alpha(\Psi(u))\\ &= \alpha(\Psi(A)\Psi(u))\,. \end{aligned}$$

In fact we can verify $\alpha(\Psi)(A(y_1^2, y', D_{y'}))=\Psi(A)(x_1^2, x', D_{x'})$ using the relations (5) and (6). From (8) we get $\Psi(Au)=\Psi(A)\Psi(u)$.

On the other hand, we get from (6)

$$\begin{aligned}2x_1\alpha(\Psi(D_{y_1}u(y)))&=2x_1\alpha(\Psi)((D_{y_1}u)(y_1^2, y'))\\&=\alpha(\Psi)(2y_1(D_{y_1}u)(y_1^2, y'))=\alpha(\Psi)(D_{y_1}\alpha(u))=\alpha(\Psi)(D_{y_1})\alpha(\Psi)(\alpha(u))\\&=(D_{x_1}+2x_1Q_1'(x_1^2, x', D_{x'}))\alpha(\Psi(u))(x)\\&=2x_1(D_{x_1}\Psi(u))(x_1^2, x')+2x_1Q_1'(x_1^2, x', D_{x'})(\Psi(u))(x_1^2, x')\\&=2x_1\alpha((D_{x_1}+Q_1'(x, D_{x'}))\Psi(u)(x))=2x_1\alpha(\Psi(D_{y_1})\Psi(u)).\end{aligned}$$

Hence setting

$$v=\alpha(\Psi(D_{y_1}u))-\alpha(\Psi(D_{y_1})\Psi(u))$$

we get $x_1v(x)=0$. Since bicharacteristics of the operator x_1 are fibers of ρ, the support of v is a union of fibers of ρ. Thus we get $v=0$ and hence

$$\Psi(D_{y_1}u)=\Psi(D_{y_1})\Psi(u)$$

because the support of v as a section of $\mathscr{C}_M$ is proper with respect to ρ. Hence Ψ is compatible with the action of $\mathscr{E}_{X/Y}[D_1]$. This completes the proof.

Since $L'(0, x', y')$ satisfies

$$\begin{aligned}&(y_j-P_j(0, x', D_{x'}))L'(0, x', y')=0\,,\\&(D_{y_j}+Q_j(0, x', D_{x'}))L'(0, x', y')=0 \qquad (j=2, \cdots, n)\,,\end{aligned}$$

it is a kernel function of the quantized contact transformation Ψ'. We define a sheaf isomorphism

$$\Psi': \psi'^{-1}\mathscr{C}_N \xrightarrow{\sim} \mathscr{C}_N$$

so that

$$\Psi'(v)(x')=\int L'(0, x', y')v(y')dy'$$

for a section v of $\mathscr{C}_N$ on a neighborhood of $\psi'(x^*)$.

Proposition 1.18. *The homomorphism b: $\mathscr{C}_{N|M_+}^F \to \mathscr{C}_N$ is compatible with Ψ and Ψ'; i.e., $b(\Psi(u))=\Psi'(b(u))$ holds for any section u of $\mathscr{C}_{N|M_+}^F$ on a neighborhood of $\psi'(x^*)$.*

Proof. Since $b(\alpha(u))=b(u)$ holds, we get

$$b(\Psi(u))=b(\alpha(\Psi(u)))=b\Big(\int L'(x_1^2, x', y')\alpha(u)(x_1^2, y')dy'\Big)$$
$$=\int L'(0, x', y')\alpha(u)(0, y')dy'=\Psi'(b(\alpha(u)))=\Psi'(b(u)).$$

This completes the proof.

1.5. Relation between F-mild microfunctions and mild microfunctions

We shall relate the sheaf of F-mild microfunctions to that of mild microfunctions of Kataoka [4]. This subsection is a preliminary part of the proof of the uniqueness theorem of Section 2. First let us recall the definition of the sheaf $\mathring{\mathscr{C}}_{N|M_+}$ of mild microfunctions in accordance with [4]. Put

$$S_N^*X-S_Y^*X=\{(0, x', (\zeta_1, \sqrt{-1}\,\xi')\infty);\ x'\in \boldsymbol{R}^{n-1}, \zeta_1\in \boldsymbol{C}, \xi'\in \boldsymbol{R}^{n-1}-\{0\}\},$$
$$S_N^*X^\infty=(S_N^*X-S_Y^*X)\cup(\sqrt{-1}\,S^*N\times\{\infty\})$$
$$=\{(0, x', (\zeta_1, \sqrt{-1}\,\xi')\infty);\ x'\in \boldsymbol{R}^{n-1}, \zeta_1\in \boldsymbol{C}\cup\{\infty\}, \xi'\in \boldsymbol{R}^{n-1}-\{0\}\}$$

and let

$$\iota\colon S_N^*X-S_Y^*X\longrightarrow\sqrt{-1}\,S^*N,$$
$$\iota^\infty\colon S_N^*X^\infty\longrightarrow\sqrt{-1}\,S^*N$$

be the canonical projections. The sheaf $\mathscr{C}_{N|X}^\infty$ is defined by

$$\mathscr{C}_{N|X}^\infty=i_*(\mathscr{C}_{N|X}|_{S_N^*X-S_Y^*X}),$$

where $i\colon S_N^*X-S_Y^*X\to S_N^*X^\infty$ is the natural embedding. Let

$$\iota^+\colon \bar{G}_+-S_Y^*X=\{(0, x', (\zeta_1, \sqrt{-1}\,\xi')\infty)\in S_N^*X-S_Y^*X;\ \mathrm{Re}\,\zeta_1\geqq 0\}\longrightarrow\sqrt{-1}\,S_N^*Y$$

be the canonical projection. Then the sheaf $\mathring{\mathscr{C}}_{N|M_+}$ of mild microfunctions is defined by

$$\mathring{\mathscr{C}}_{N|M_+}=((\iota^+)_*(\mathscr{C}_{M_+|X})\cap(\mathscr{C}_{N|X}^\infty|_{\sqrt{-1}S^*N\times\{\infty\}}))/\iota_*\mathscr{C}_{N|X},$$

where $\sqrt{-1}\,S^*N\times\{\infty\}$ is identified with $\sqrt{-1}\,S^*N$. Note that $\mathring{\mathscr{C}}_{N|M_+}$ is an $\mathscr{E}_{X/Y}[D_1]$-module.

Let $\mathring{\mathscr{B}}_{N|M_+}$ be the sheaf of mild hyperfunctions (on the positive side of N) and let $\mathring{\mathscr{A}}_{N|M_+}^*$ be the sheaf associated with the presheaf

$$U\times\sqrt{-1}\,\Delta\longmapsto\{f\in\mathring{\mathscr{B}}_{N|M_+}(U);\ \rho\text{-SS}(f)\cap(U\times\sqrt{-1}\,\Delta)=\emptyset\}$$

for open connected sets U of N and Δ of S^{n-2}. It is easy to see that ρ-SS(f)

coincides with ι-SS(f) defined in [4] for mild hyperfunctions f in view of the proof of Lemma 2 of [8]. Hence by virtue of Proposition 2.1.21 of [4], there exists an exact sequence

$$0 \longrightarrow \mathring{\mathscr{A}}^*_{N|M+} \longrightarrow \pi_N^{-1}\mathring{\mathscr{B}}_{N|M+} \longrightarrow \mathring{\mathscr{C}}_{N|M+} \longrightarrow 0, \tag{9}$$

where the last homomorphism is defined by sp(ext(f)) for sections f of $\mathring{\mathscr{B}}_{N|M+}$ and ext$(f)=Y(x_1)f(x)$ is the canonical extension of f defined in [4] (note that ext(f) is a section of $\mathscr{B}_M$ with support contained in M_+). From this exact sequence, we get the following proposition immediately.

PROPOSITION 1.19. *There exists a natural injective sheaf homomorphism* $j: \mathring{\mathscr{C}}_{N|M+} \rightarrow \mathscr{C}^F_{N|M+}$ *such that* $j(\mathrm{sp}(\mathrm{ext}(f)))=\mathrm{sp}(f)$ *for any mild hyperfunction* f, *where* sp(f) *denotes the F-mild microfunction corresponding to F-mild hyperfunction* f.

PROPOSITION 1.20. *For any integer* $q \geqq 2$, *there exists a unique sheaf homomorphism*

$$\beta': \mathscr{C}^F_{N|M+} \longrightarrow \mathring{\mathscr{C}}_{N|M+}$$

such that $\beta'(\mathrm{sp}(f(x)))=\mathrm{sp}(\mathrm{ext}(f(x_1^q, x')))$ *holds for any F-mild hyperfunction* f. *Moreover,* β' *is injective, and*

$$A(x_1^q, x', D_{x'})\beta'(u)=\beta'(A(x, D_{x'})u) \tag{10}$$

holds for a section A *of* $\mathscr{E}_{X/Y}$ *over* $U \subset \sqrt{-1}\,S^*N$ *and a section* u *of* $\mathscr{C}^F_{N|M+}$ *over* U.

PROOF. In view of the exact sequence (9), it is easy to see that β' is uniquely determined and injective (see Proposition 2 of [8]). Let us verify (10). We may assume that U is a neighborhood of $x^*=(0, \sqrt{-1}\,dx_n\infty) \in \sqrt{-1}\,S^*N$ and that A is defined on a neighborhood of ω_0 in the notation in Subsection 1.3. Hereafter we use the notation in Subsection 1.3. There exists a holomorphic function F defined on a neighborhood of $D_+(0, c_1, \Gamma)$ with $0<c_1<c_0$ and

$$\Gamma=\{y' \in \boldsymbol{R}^{n-1};\, y_n>c_0|y_j|/2 \ \ (j=2, \cdots, n)\}$$

such that $\mathrm{sp}(F(x_1, x'+\sqrt{-1}\,\Gamma 0))=u$ at x^*. Then from the definition of the canonical extension, we get

$$\beta'(u)=\mathrm{sp}(Y(x_1)F(x_1^q, x'+\sqrt{-1}\,\Gamma 0))\,;$$

here Y denotes the Heaviside function, and $Y(x_1)F(x_1^q, x'+\sqrt{-1}\,\Gamma 0)$ denotes the boundary value of the hyperfunction $Y(x_1)F(x_1^q, z')$ with respect to holomorphic parameters z'. Note that $Y(x_1)F(x_1^q, z')$ is well-defined on

$$\{(x_1, z') \in \boldsymbol{R}\times\boldsymbol{C}^{n-1};\ |x_1|^q \leqq c_0/2,\ |z'|<c_0/2,\ \operatorname{Im} z' \in \Gamma\}\ .$$

Let Ω_0, Σ, Ω', K, γ be as in Subsection 1.3 and set

$$G(x_1, z')=\int_\gamma K(x_1^q, z', z'-w')(Y(x_1)F(x_1^q, w'))dw'\ ,$$
$$H(z)=\int_\gamma K(z, z'-w')F(z_1, w')dw'\ .$$

Then $G(x_1, z')$ is a hyperfunction with holomorphic parameters z' defined on

$$\{x_1 \in \boldsymbol{R};\ |x_1|^q<c_0/2\}\times\Omega$$

and $H(z)$ is holomorphic on $\{z_1 \in \boldsymbol{C};\ |z_1|<c_0/2\}\times\Omega$. By virtue of Theorem 3.10 of [3], we get

$$\begin{aligned} A(x_1^q, x', D_{x'})\beta'(u)&=A(x_1^q, x', D_{x'})\operatorname{sp}(Y(x_1)F(x_1^q, x'+\sqrt{-1}\,\Gamma 0))\\ &=\operatorname{sp}(G(x_1, x'+\sqrt{-1}\,\Gamma 0))\ . \end{aligned}$$

On the other hand it follows from Proposition 1.15 that

$$\beta'(A(x, D_{x'})u)=\operatorname{sp}(Y(x_1)H(x_1^q, x'+\sqrt{-1}\,\Gamma 0))\ .$$

Since $Y(x_1)H(x_1^q, z')=G(x_1, z')$, we get

$$A(x_1^q, x', D_{x'})\beta'(u)=\beta'(Au)\ .$$

This completes the proof.

REMARK. In the same way as in the proof of this proposition, we can show that the inclusion $\mathscr{C}_{N|M_+}\subset\mathscr{C}^F_{N|M_+}$ is compatible with the action of $\mathscr{E}_{X/Y}[D_1]$.

§ 2. Fuchsian microdifferential operators and uniqueness theorem

2.1. Definitions and statement of the uniqueness theorem

Tahara [10] introduced Fuchsian microdifferential operators as a generalization of Fuchsian partial differential operators defined by Baouendi-Goulaouic [1]. In this section we give a uniqueness theorem in the microlocal boundary value problem for Fuchsian microdifferential operators. This generalizes our previous result for Fuchsian partial differential operators (Theorem 2 of [8]).

Let us begin with definitions: A section P of $\mathscr{E}_{X/Y}[D_1]$ over an open subset U of $\sqrt{-1}\,S^*N$ is called a *Fuchsian operator of type* (k, m) *with respect to* x_1 if it is written in the form:

$$P=Q(x, D_{x'})(x_1^k D_{x_1}^m + A_1(x, D_{x'})x_1^{k-1}D_{x_1}^{m-1}+\cdots + A_k(x, D_{x'})D_{x_1}^{m-k}+A_{k+1}(x, D_{x'})D_{x_1}^{m-k-1}+\cdots+A_m(x, D_{x'})) ; \tag{11}$$

here k, m are integers, Q and A_j are sections of $\mathscr{E}_{X/Y}$ over U satisfying the following conditions:

(i) $0\leqq k\leqq m$,

(ii) the order of $A_j(x, D_{x'})$ is at most j for $j=1, \cdots, m$,

(iii) the order of $A_j(0, x', D_{x'})$ is at most 0 for $j=1, \cdots, k$,

(iv) Q is elliptic on $\rho^{-1}(U)$ when it is regarded as a microdifferential operator on $\rho^{-1}(U)$.

For a point x^* of U, we define the *indicial polynomial* $e(\lambda, P, x^*)$ of P at x^* by

$$\begin{aligned} e(\lambda, P, x^*)=\lambda(\lambda-1)\cdots(\lambda-m+1) \\ +\sigma_0(A_1(0, x', D_{x'}))(x^*)\lambda(\lambda-1)\cdots(\lambda-m+2)+\cdots \\ +\sigma_0(A_k(0, x', D_{x'}))(x^*)\lambda(\lambda-1)\cdots(\lambda-m+k+1)\,, \end{aligned}$$

where σ_j denotes the homogeneous part of order j.

LEMMA 2.1. *Let P be a Fuchsian operator of type (k, m) with respect to x_1. Then P is written in the form* (11) *in a unique way.*

PROOF. P is a polynomial of degree m with respect to D_1, and the coefficient of D_1^m is $Q(x, D_{x'})x_1^k$. Since the principal symbol $\sigma(Q)$ of Q does not vanish, k and Q are uniquely determined. Since the coefficient of D_1^j in P is $QA_jx_1^p$ with $p=\max(0, j-m+k)$, A_j is also determined uniquely. This completes the proof.

PROPOSITION 2.2. *Let P be given by* (11) *with the conditions* (i)-(iv). *Let $(y, \eta)=\psi(x, \xi)$ be a contact transformation of $\sqrt{-1}\,S^*M$ defined on a neighborhood of $\rho^{-1}(\psi'^{-1}(x^*))$ preserving x_1, where ψ' is the canonical transformation of $\sqrt{-1}S^*N$ induced by ψ, and let Ψ be a quantized contact transformation associated with ψ such that $\Psi(y_1)=x_1$. Then $\Psi(P)$ is slso a Fuchsian operator of type (k, m), and the indicial polynomial $e(\lambda, \Psi(P), \psi'^{-1}(x^*))$ satisfies*

$$e(\lambda, \Psi(P), \psi'^{-1}(x^*))=e(\lambda, P, x^*)\,.$$

PROOF. We may assume that Ψ is defined by the relations (5) in Subsection 1.4 with x^* replaced by $\psi'^{-1}(x^*)$. Then $\Psi(P)$ is written in the form

$$\Psi(P)=\Psi(Q)(x_1^k(D_{x_1}+Q_1'(x, D_{x'}))^m+\Psi(A_1)x_1^{k-1}(D_{x_1}+Q_1'(x, D_{x'}))^{m-1} + \cdots + \Psi(A_k)(D_{x_1}+Q_1'(x, D_{x'}))^{m-k}+\cdots+\Psi(A_m)).$$

It is easy to see that $\Psi(Q)$ is elliptic on $\rho^{-1}(\psi'^{-1}(x^*))$ and that $\Psi(A_j)$ satisfies conditions (ii) and (iii). Since D_{x_1} and Q_1' commute, we have

$$(D_{x_1}+Q_1')^j=\sum_{\nu=0}^{j}\binom{j}{\nu}Q_1'^{j-\nu}D_{x_1}^{\nu}.$$

Hence $\Psi(P)$ is written in the form

$$\Psi(P)=\Psi(Q)(x_1^kD_{x_1}^m+B_1x_1^{k-1}D_{x_1}^{m-1}+\cdots+B_kD_{x_1}^{m-k}+\cdots+B_m),$$

where B_j is a section of $\mathscr{E}_{X/Y}$ on a neighborhood of $\psi'^{-1}(x^*)$ of order at most j such that

$$B_j(0, x', D_{x'})=A_j(0, x', D_{x'}) \qquad (j=1, \cdots, k).$$

Thus $\Psi(P)$ is also a Fuchsian operator of type (k, m) and its indicial polynomial at $\psi'^{-1}(x^*)$ coincides with that of P at x^*. This completes the proof.

Now let us state the uniqueness theorem.

THEOREM 2.3. *Let P be a Fuchsian microdifferential operator of type (k, m) with respect to x_1 defined on a neighborhood of $x^*\in\sqrt{-1}S^*N$. Suppose that $e(\nu, P, x^*)\neq 0$ for any integer $\nu\geqq m-k$. Under these assumptions, if u is an F-mild microfunction defined on a neighborhood of x^* satisfying*

$$Pu=0, \quad b(D_{x_1}^{\nu}u)=0 \quad (0\leqq\nu\leqq m-k-1)$$

on a neighborhood of x^* (here b denotes the homomorphism defined in Proposition 1.13), then u vanishes on a neighborhood of x^*.

We shall prove this theorem in the next subsection.

REMARK. We have proved this theorem in [8] when P is a Fuchsian partial differential operator, and have proved in [7] the uniqueness theorem of the Cauchy problem (in the framework of microfunctions with a real analytic parameter x_1) for Fuchsian microdifferential operators. Theorem 2.3 generalizes these two results.

2.2. Proof of the uniqueness theorem

Now we begin the proof of Theorem 2.3. Let P and u satisfy the assumptions of Theorem 2.3. We assume, without loss of generality, that $x^*=(0, \sqrt{-1}\,dx_n)$. By virtue of Lemma 3 of [8], there exists a germ u' of $\mathscr{C}^F_{N|M+}$ at x^* such that $u(x)=x_1^{m-k}u'(x)$, and $Px_1^{m-k}u'=0$ holds. It is easy to see that Px_1^{m-k} is a Fuchsian operator of type (m, m) with the indicial polynomial

$$e(\lambda, Px_1^{m-k}, x^*)=e(\lambda+m-k, P, x^*)\,.$$

Thus Px_1^{m-k} satisfies the assumptions of the theorem with $k=m$.

Next, let q be an integer such that $q\geqq \max(m, 2)$ and that

$$e(\nu/q, Px_1^{m-k}, x^*)\neq 0$$

for any integer $\nu\geqq 0$. Put $v(x)=\beta'(u')$, where β' is the homomorphism defined in Proposition 1.20 with this q. Then v is a germ of $\mathring{\mathscr{C}}_{N|M+}$ at x^* and satisfies an equation

$$((x_1D_{x_1})^m-B_1(x, D_{x'})(x_1D_{x_1})^{m-1}-\cdots-B_m(x, D_{x'}))v(x)=0\,,$$

with

$$B_j(x, D_{x'})=B_{j,0}(x', D_{x'})+x_1^qB_{j,1}(x, D_{x'})\qquad (j=1, \cdots, m)\,,$$

where $B_{j,0}$ is a germ of $\mathscr{E}_{X/Y}$ at x^* of order at msot 0, $B_{j,1}$ is a germ of $\mathscr{E}_{X/Y}$ at x^* of order at most j, and

$$\nu^m-\sigma_0(B_{j,0})(x^*)\nu^{m-1}-\cdots-\sigma_0(B_{m,0})(x^*)\neq 0$$

for any integer $\nu\geqq 0$.

By virtue of the above argument and Proposition 1.20, we have only to prove the following proposition:

PROPOSITION 2.4. *Assume that a germ u of $\mathring{\mathscr{C}}_{N|M+}$ at $x^*=(0, \sqrt{-1}\,dx_n)\in\sqrt{-1}\,S^*N$ satisfies an equation $Pu=0$ with*

$$P=(x_1D_{x_1})^m-A_1(x, D_{x'})(x_1D_{x_1})^{m-1}-\cdots-A_m(x, D_{x'})\,;$$

here A_j is a germ of $\mathscr{E}_{X/Y}$ at x^* such that

$$A_j(x, D_{x'})=\sum_{\kappa=0}^{\infty}A_{j,\kappa}(x', D_{x'})x_1^{\kappa}$$

with order $A_{j,\kappa}\leqq\min(j, \kappa)$ for $j=1, \cdots, m$. Assume moreover that

$$e(\nu, P, x^*)=\nu^m-\sigma_0(A_{1,0})(x^*)\nu^{m-1}-\cdots-\sigma_0(A_{m,0})(x^*)\neq 0$$

for any integer $\nu \geqq 0$. *Then* u *vanishes on a neighborhood of* x^*.

PROOF. By the definition of the sheaf $\mathring{\mathscr{C}}_{N|M_+}$, u is the equivalence class represented by a germ $\tilde{u}$ of $(\iota^+)_* \mathscr{C}_{M_+|X}$ at x^* in the notation in Subsection 1.5. Note that $\tilde{u}$ is also a germ of $\mathscr{C}^\infty_{N|X}$ at $\{x^*\} \times \{\infty\}$. Then $P\tilde{u}$ is a germ of $\iota_* \mathscr{C}_{N|X}$ at x^*.

In accordance with [4], we apply the quantized Legendre transformation to P and $\tilde{u}$. Put

$$U_\varepsilon = \{(x', \sqrt{-1}\,\xi'\infty) \in \sqrt{-1}\,S^*N;\ |x'| < \varepsilon,\ |\xi_j| < \varepsilon \xi_n\ (j=2, \cdots, n-1)\}$$

with $\varepsilon > 0$ and let

$$\varphi: \{\zeta_1 \in \boldsymbol{C}\} \times U_\varepsilon \longrightarrow \{(0, x', (\zeta_1, \sqrt{-1}\,\xi')\infty) \in S^*_N X;\ (x', \sqrt{-1}\,\xi'\infty) \in U_\varepsilon\}$$

be the natural map. Let $\mathscr{E}_X$ and $\mathscr{E}_{X'}$ be the sheaves of microdifferential operators with the variables $x=(x_1, x')$ and (ζ_1, x') respectively, and $\mathscr{C}\mathscr{O}$ be the sheaf on $\{\zeta_1 \in \boldsymbol{C}\} \times U_\varepsilon$ of microfunctions with holomorphic parameter ζ_1. Then there exists a quantized contact transformation

$$\Phi: \varphi^{-1}\mathscr{E}_X \xrightarrow{\sim} \mathscr{E}_{X'}, \qquad \Phi: \varphi^{-1}\mathscr{C}_{N|X} \xrightarrow{\sim} \mathscr{C}\mathscr{O}$$

defined on $\{\zeta_1 \in \boldsymbol{C}\} \times U_\varepsilon$ such that

$$\begin{aligned}
&\Phi(x_1) = -\sqrt{-1}\,D_{\zeta_1} D_{x_n}^{-1}, \quad \Phi(x_j) = x_j \quad (j=2, \cdots, n-1),\\
&\Phi(x_n) = x_n + D_{\zeta_1}\zeta_1 D_{x_n}^{-1},\\
&\Phi(D_{x_1}) = -\sqrt{-1}\,\zeta_1 D_{x_n}, \quad \Phi(D_{x_j}) = D_{x_j} \quad (j=2, \cdots, n).
\end{aligned}$$

Set

$$v(\zeta_1, x') = \begin{bmatrix} \Phi(\tilde{u}) \\ \Phi(x_1 D_{x_1}\tilde{u}) \\ \vdots \\ \Phi((x_1 D_{x_1})^{m-1}\tilde{u}) \end{bmatrix},$$

$$Q = -D_{\zeta_1}\zeta_1 I_m - \begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ \Phi(A_m) & \cdots & \Phi(A_2) & \Phi(A_1) \end{bmatrix},$$

where I_m denotes the unit matrix of degree m. Since $\tilde{u}$ represents a mild microfunction, v is a section of $(\mathscr{C}\mathscr{O})^m$ defined on $\{\zeta_1 \in \boldsymbol{C};\ \mathrm{Re}\,\zeta_1 > 0 \text{ or } |\zeta_1| > \varepsilon^{-1}\} \times U_\varepsilon$ with sufficiently small ε. Since

$$Qv=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \Phi(P\tilde{u}) \end{bmatrix}$$

and $P\tilde{u}$ is a germ of $\iota_* \mathscr{C}_{N|X}$ at x^*, $w=Qv$ is a section of $(\mathscr{C}\mathscr{O})^m$ defined on $\{\zeta_1 \in \boldsymbol{C}\} \times U_\varepsilon$ if ε is small enough.

Our aim is to show that v is a section of $(\mathscr{C}\mathscr{O})^m$ defined on a neighborhood of $\boldsymbol{C}\times\{x^*\}$. Hereafter we use the notation $D'=(D_2, \cdots, D_n)$ with $D_j=D_{x_j}$ for $j=2, \cdots, n$, $x''=(x_2, \cdots, x_{n-1})$, and $\zeta=\zeta_1$. Set

$$B(x, D')=\begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ A_m & \cdots & A_2 & A_1 \end{bmatrix}$$

and develop B into the form

$$B(x, D')=\sum_{\kappa=0}^{\infty} B_\kappa(x', D')x_1^\kappa, \qquad B_\kappa(x', D')=\sum_{\nu=0}^{\infty} B_{\kappa,\nu}(x'', D')x_n^\nu .$$

Then each element of $B_{\kappa,\nu}$ is of order $\leq \min(\kappa, m)$, and Q is written in the form

$$Q=-D_\zeta \zeta I_m - \sum_{\kappa,\nu\geq 0} B_{\kappa,\nu}(x'', D')(x_n+D_\zeta \zeta D_n^{-1})^\nu(-\sqrt{-1}\,D_\zeta D_n^{-1})^\kappa . \tag{12}$$

Set

$$v_0(\zeta, x')=\frac{1}{2\pi\sqrt{-1}}\int_{|\tau|=r}\frac{v(\tau, x')}{\tau-\zeta}d\tau$$

when $|\zeta|<r$ with $r>\varepsilon^{-1}$, and

$$v_1(\zeta, x')=-\frac{1}{2\pi\sqrt{-1}}\int_{|\tau|=r}\frac{v(\tau, x')}{\tau-\zeta}d\tau$$

when $|\zeta|>r$ with $r>\varepsilon^{-1}$. Then by virtue of Cauchy's integral formula for sections of $\mathscr{C}\mathscr{O}$, we can ensure that $v=v_0+v_1$, that v_0 and v_1 are sections of $(\mathscr{C}\mathscr{O})^m$ (independent of r) defined on $\boldsymbol{C}\times U_\varepsilon$ and on $\{\zeta \in \boldsymbol{P};\ \mathrm{Re}\,\zeta>0 \text{ or } |\zeta|>\varepsilon^{-1}\}\times U_\varepsilon$ respectively and that $v_1(\infty, x')=0$, where $\boldsymbol{P}=\boldsymbol{C}\cup\{\infty\}$. Setting $\tau=\zeta^{-1}$, we can write

$$Q=(\tau D_\tau-1)I_m-\sum_{\kappa,\nu\geq 0} B_{\kappa,\nu}(x'', D')(x_n-(\tau D_\tau-1)D_n^{-1})^\nu(\sqrt{-1}\,\tau^2 D_\tau D_n^{-1})^\kappa . \tag{13}$$

Hence Q is well-defined on a neighborhood of $\tau=0$ (i.e. $\zeta=\infty$). It follows from (13) that Qv_1 is a section of $(\mathscr{C}\mathscr{O})^m$ on $\{\zeta \in \boldsymbol{P};\ \mathrm{Re}\,\zeta>0 \text{ or } |\zeta|>\varepsilon^{-1}\}\times U_\varepsilon$ and satisfies $(Qv_1)(\infty, x')=0$. On the other hand,

$$Qv_1 = w - Qv_0$$

is a section of $(\mathscr{C}\mathscr{O})^m$ on $\boldsymbol{C}\times U_\varepsilon$. Hence Qv_1 is a section of $(\mathscr{C}\mathscr{O})^m$ on $\boldsymbol{P}\times U_\varepsilon$ with $(Qv_1)(\infty, x')=0$. This implies $Qv_1=0$. In order to show that v is a section of $(\mathscr{C}\mathscr{O})^m$ on a neighborhood of $\boldsymbol{C}\times\{x^*\}$, we have only to show that v_1 is a section of $(\mathscr{C}\mathscr{O})^m$ defined there. Hence, from now on, we assume that $v=v_1$.

Now set

$$Q' = -D_\zeta \zeta I_m - \sum_{\kappa,\nu\geqq 0} B_{\kappa,\nu}(x'', D')(x_n + D_\zeta \zeta D_n^{-1})^\nu (-\sqrt{-1}\,D_n^{-1}\zeta^{-1})^\kappa$$

with the same $B_{\kappa,\nu}$ that appeared in (12). Setting $\tau=\zeta^{-1}$, we can write

$$Q' = (\tau D_\tau - 1)I_m - \sum_{\kappa,\nu\geqq 0} B_{\kappa,\nu}(x'', D')(x_n - (\tau D_\tau - 1)D_n^{-1})^\nu (-\sqrt{-1}\,D_n^{-1}\tau)^\kappa .$$

Since

$$\mathrm{order}(B_{\kappa,\nu}(x'', D')(x_n - (\tau D_\tau - 1)D_n^{-1})^\nu(-\sqrt{-1}\,D_n^{-1}\tau)^\kappa) \leqq \min(0, m-\kappa),$$

Q' has regular singularities along $\tau D_\tau=0$ in the sense of [6].

Let $\lambda_1, \cdots, \lambda_m$ be the roots of the equation $e(\lambda, P, x^*)=0$ with respect to λ. We define an equivalence relation in the set $J=\{1, 2, \cdots, m\}$ as follows: $i\sim j$ if and only if $\lambda_i-\lambda_j \in \boldsymbol{Z}$. Let

$$J = J_1 \cup \cdots \cup J_r$$

be the classification of J with respect to this relation with $\# J_s = m_s$ for $s=1, \cdots, r$. Set

$$\mu_s = \min\{\lambda_i + 1;\ i \in J_s\} \qquad (s=1, \cdots, r).$$

We can assume by rearranging $\lambda_1, \cdots, \lambda_m$ appropriately that $\mu_1 \in \boldsymbol{Z}$ and that there are integers $n(s, i)\geqq 0$ $(s=1, \cdots, r,\ i=1, \cdots, m_s)$ such that

$$0 = n(s, 1) \leqq n(s, 2) \leqq \cdots \leqq n(s, m_s),$$
$$\lambda_1 + 1 = \mu_1,\ \lambda_2 + 1 = \mu_1 + n(1, 2),\ \cdots,\ \lambda_{m_1} + 1 = \mu_1 + n(1, m_1),$$
$$\lambda_{m_1+1} + 1 = \mu_2,\ \lambda_{m_1+2} + 1 = \mu_2 + n(2, 2),\ \cdots,\ \lambda_m + 1 = \mu_r + n(r, m_r).$$

Note that $\mu_1 \leqq \mu_1 + n(1, m_1) \leqq 0$ follows from the assumption that $e(\nu, P, x^*)\neq 0$ for any $\nu \in \boldsymbol{N}=\{0, 1, 2, \cdots\}$.

Using the argument of the proof of Proposition 2.6 of [6], we can find $m\times m$ matrices $S(\tau, x', D')$, $S'(\tau, x', D_\tau, D')$, $B'(\tau, x', D')$ of microdifferential operators of order $\leqq 0$ defined on a neighborhood of $\tau=0$, $D_\tau=0$, $(x', \sqrt{-1}\,\xi'\infty)=x^*$ such that $\sigma_0(S)$ and $\sigma_0(S')$ are invertible there and that

$$
(14)\qquad
\begin{aligned}
&Q'=S'(\tau D_\tau I_m-B'(\tau, x', D'))S^{-1}, \\
&B'_{ij}(\tau, x', D')=B''_{ij}(x', D')\tau^{\lambda_i-\lambda_j} && \text{if } \lambda_i-\lambda_j\in N, \\
&B'_{ij}(\tau, x', D')=0 && \text{if } \lambda_i-\lambda_j\notin N, \\
&\sigma_0(B')(0, x^*)=\begin{bmatrix} \lambda_1+1 & b_{1,2}\cdots b_{1,m} \\ 0 & \ddots \quad \vdots \\ \vdots \quad \ddots & b_{m-1,m} \\ 0\cdots 0 & \lambda_m+1 \end{bmatrix}
\end{aligned}
$$

with operators B''_{ij} and complex numbers b_{ij}; here B'_{ij} denotes the (i, j) element of the matrix B'. Thus B' is decomposed into a direct sum

$$
B'=B'_1\oplus\cdots\oplus B'_r=\begin{bmatrix} B'_1 & & \\ & \ddots & \\ & & B'_r \end{bmatrix},
$$

where B'_s is an $m_s\times m_s$ matrix of operators. The (i, j) element of B'_s is written in the form

$$
\begin{aligned}
&(B'_s)_{ij}=(B''_s)_{ij}(x', D')\tau^{n(s,i)-n(s,j)} && \text{if } n(s, i)\geqq n(s, j), \\
&(B'_s)_{ij}=0 && \text{if } n(s, i)<n(s, j)
\end{aligned}
$$

with some $(B''_s)_{ij}$.

We define $m\times m$ matrices $T=T(\tau)$ and $E=E(x', D')$ by

$$
\begin{aligned}
&T=T_1\oplus\cdots\oplus T_r, \qquad T_s=\begin{bmatrix} \tau^{n(s,1)} & & 0 \\ & \ddots & \\ 0 & & \tau^{n(s,m_s)} \end{bmatrix}, \\
&E=E_1\oplus\cdots\oplus E_r, \\
&(E_s)_{ij}=(B''_s)_{ij}(x', D')-\delta_{ij}n(s, i) && \text{if } n(s, i)\geqq n(s, j), \\
&(E_s)_{ij}=0 && \text{if } n(s, i)<n(s, j)
\end{aligned}
$$

for $s=1, \cdots, r$ and $1\leqq i, j\leqq m_s$. Then

$$
(15)\qquad T^{-1}(\tau D_\tau I_m-B'(\tau, x', D'))T=\tau D_\tau I_m-E(x', D')
$$

holds, and the eigenvalues of $\sigma_0(E_s)(x^*)$ are all μ_s for $s=1, \cdots, r$. From (14) and (15) we get

(16) $$Q'=S'T(\tau D_\tau I_m-E(x',D')T^{-1}S^{-1}.$$

We define $\tilde{S}$ and $\tilde{S}_\kappa$ by

$$S(\tau,x',D')T(\tau)=\tilde{S}(\tau,x',D')=\sum_{\kappa=0}^{\infty}\tilde{S}_\kappa(x',D')\tau^\kappa.$$

Then it follows from (16) that

$$\begin{aligned}&Q'(\tau,x',\partial_\tau,D')(\tilde{S}(\tau,x',D')\tau^{E(x',D')})\\&=S'(\tau,x',\partial_\tau,D')T(\tau)(\tau\partial_\tau I_m-E(x',D'))(\tau^{E(x',D')})=0\,;\end{aligned}$$

here ∂_τ acts on sections $A(\tau,x',D')$ of $\mathscr{E}_{X''/Y}$ with $X''=\{\tau\in \boldsymbol{C}\}\times Y$ by

$$\partial_\tau A(\tau,x',D')=[D_\tau,A(\tau,x',D')].$$

Replacing the variable τ by ζ, we get

(17) $$Q'(\zeta,x',\partial_\zeta,D')(\tilde{S}(\zeta^{-1},x',D')\zeta^{-E(x',D')})=0.$$

We can develop Q and Q' into the form

$$\begin{aligned}Q&=\sum_{\kappa,\nu\geqq 0}Q_{\kappa,\nu}(x',D')(\zeta D_\zeta)^\nu D_\zeta^\kappa,\\Q'&=\sum_{\kappa,\nu\geqq 0}Q_{\kappa,\nu}(x',D')(\zeta D_\zeta)^\nu\zeta^{-\kappa}\end{aligned}$$

using the same $Q_{\kappa,\nu}$ both for Q and for Q'. The equation (17) is equivalent to the relations

(18) $$\sum_{\nu\geqq 0}Q_{0,\nu}\tilde{S}_\kappa(-E-\kappa I_m)^\nu=-\sum_{\lambda=1}^{\kappa}\sum_{\mu\geqq 0}Q_{\lambda,\mu}\tilde{S}_{\kappa-\lambda}(-E-\kappa I_m)^\mu\qquad(\kappa\in \boldsymbol{N}).$$

Here sums with respect to ν and μ converge. We define a formal power series $\tilde{R}(\tau,x',D')$ in τ whose coefficients are microdifferential operators with variables x' by

$$\begin{aligned}&\tilde{R}(\tau,x',D')=\sum_{\kappa=0}^{\infty}\tilde{R}_\kappa(x',D')\tau^\kappa,\\&\tilde{R}_\kappa(x',D')=\tilde{S}_\kappa(x',D')(-E)\cdots(-E-(\kappa-1)I_m).\end{aligned}$$

Then from (18) we get

$$\sum_{\nu\geqq 0}Q_{0,\nu}\tilde{R}_\kappa(-E-\kappa)^\nu=-\sum_{\lambda=1}^{\kappa}\sum_{\mu\geqq 0}Q_{\lambda,\mu}\tilde{R}_{\kappa-\lambda}(-E-\kappa)^\mu(-E-\kappa+1)\cdots(-E-\kappa+\lambda)$$

for $\kappa=0,1,2,\cdots$. These relations are equivalent to the equation

(19) $$Q(\zeta,x',\partial_\zeta,D')(\tilde{R}(\zeta^{-1},x',D')\zeta^{-E(x',D')})=0$$

as a formal power series in ζ^{-1}. Note that Q acts on formal power series in ζ^{-1} whose coefficients are microdifferential operators since Q is written in the form (12).

We shall show that $\tilde{R}$ converges in a neighborhood of $\tau=0$ and defines a germ of $\mathscr{E}_{X''/Y}$ at x^*. Setting

$$\tilde{R}(\tau, x', D')\tau^{E(x',D')}=F(\tau, x', D')=(F_{ij}(\tau, x', D'))_{1\leq i,j\leq m},$$

we get from (19)

$$(20)\qquad \begin{cases}\Phi(P)(\zeta, x', \partial_\zeta, D')F_{1,j}(\zeta^{-1}, x', D')=0 & (j=1, \cdots, m), \\ F_{ij}(\zeta^{-1}, x', D')=(-\partial_\zeta\zeta)^{i-1}F_{1,j}(\zeta^{-1}, x', D') & (i, j=1, \cdots, m).\end{cases}$$

Developing $A_{j,\kappa}$ into the form

$$A_{j,\kappa}(x', D')=\sum_{\nu\geq 0} A_{j\kappa\nu}(x'', D')x_n^\nu,$$

we can write

$$\Phi(A_j)=\sum_{\kappa,\nu\geq 0} A_{j\kappa\nu}(x'', D')(x_n+D_\zeta\zeta D_n^{-1})^\nu(-\sqrt{-1}\,D_\zeta D_n^{-1})^\kappa.$$

Set $\tau=\zeta^{-1}$ again and put

$$C_{j,k}=\sum_{\nu\geq 0} A_{jk\nu}(x'', D')(x_n+(1-\tau D_\tau)D_n^{-1})^\nu(\sqrt{-1}\,D_n^{-1})^k$$

for $0\leq k<j\leq m$ and

$$C_{j,j}=\sum_{\nu\geq 0}\sum_{\kappa\geq j} A_{j\kappa\nu}(x'', D')(x_n+(1-\tau D_\tau)D_n^{-1})^\nu(\sqrt{-1}\,\tau^2 D_\tau D_n^{-1})^{\kappa-j}(\sqrt{-1}\,D_n^{-1})^j$$

for $1\leq j\leq m$. Then $C_{j,k}$ is of order ≤ 0 since $A_{j,\kappa}$ is of order $\leq \min(\kappa, j)$, and $\Phi(P)$ is written in the form

$$\Phi(P)=(\tau D_\tau-1)^m-\sum_{j=1}^{m}\sum_{k=0}^{j} C_{j,k}(\tau, x', D_\tau, D')(\tau^2 D_\tau)^k(\tau D_\tau-1)^{m-j}.$$

Thus we can find microdifferential operators $V=V(\tau, x', D_\tau, D')$ and $C_j=C_j(\tau, x', D_\tau, D')$ $(j=1, \cdots, m)$ defined on a neighborhood of $\{(0, 0d\tau)\}\times\{x^*\}$ such that V is invertible, C_j are of order ≤ 0 and that

$$\Phi(P)=V((\tau D_\tau-1)^m-C_1(\tau D_\tau-1)^{m-1}-\cdots-C_m).$$

From this and (20) we get

$$(21)\qquad ((\tau\partial_\tau-1)I_m-C(\tau, x', \partial_\tau, D'))F(\tau, x', D')=0,$$

where

$$C(\tau, x', D_\tau, D') = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & 0 & 1 \\ C_m & \cdots & C_2 & C_1 \end{bmatrix}.$$

Using the argument of the proof of Proposition 2.4 of [6], we can find $m \times m$ matrices $V' = V'(\tau, x', D_\tau, D')$ and $C'(\tau, x', D')$ of microdifferential operators of order $\leqq 0$ defined on a neighborhood of $\{(0, 0d\tau)\} \times \{x^*\}$ such that V' is invertible and

$$(22) \quad (\tau D_\tau - 1)I_m - C(\tau, x', D_\tau, D') = V'(\tau, x', D_\tau, D')(\tau D_\tau I_m - C'(\tau, x', D')).$$

From this and (21) we get

$$(23) \qquad (\tau \partial_\tau I_m - C'(\tau, x', D'))F(\tau, x', D') = 0.$$

Hence it follows immediately that $\tilde{R}(\tau, x', D')$ converges and defines a section of $\mathscr{E}_{X''/Y}$ on a neighborhood of x^*.

Next let us show that $\tilde{R}(\tau, x', D')$ is invertible on $\{0 < |\tau| < \varepsilon'\} \times U_{\varepsilon'}$ with some $\varepsilon' > 0$. For this purpose, we define R and R_κ by

$$R(\tau, x', D') = \sum_{\kappa=0}^{\infty} R_\kappa(x', D')\tau^\kappa = \tilde{R}(\tau, x', D')T(\tau)^{-1}.$$

Then we get by easy calculation

$$\det(\sigma_0(R_0)(x^*)) = \det(\sigma_0(S_0)(x^*)) \prod_{s=1}^{r} \prod_{i=1}^{m_s} ((-\mu_s) \cdots (-\mu_s - n(s, i) + 1)),$$

where $S_0 = S(0, x', D')$. We get $\det(\sigma_0(R_0)(x^*)) \neq 0$ since $\sigma_0(S_0)(x^*)$ is invertible, $\mu_s \notin \boldsymbol{Z}$ for $s \geqq 2$, and $\mu_1 + n(1, i) \leqq 0$ for $s = 1, \cdots, m_1$. Hence R is invertible on $\{|\tau| < \varepsilon'\} \times U_{\varepsilon'}$ with some ε'. Consequently $\tilde{R}$ is invertible on $\{0 < |\tau| < \varepsilon'\} \times U_{\varepsilon'}$.

Since $Qv = 0$, it follows from (22) that

$$(24) \qquad (\tau D_\tau I_m - C'(\tau, x', D'))v(\tau^{-1}, x') = 0$$

holds on $\{|\tau| < \varepsilon\} \times U_\varepsilon$ with sufficiently small ε. On the other hand, we get from (23)

$$(\tau D_\tau I_m - C'(\tau, x', D'))\tilde{R}(\tau, x', D')\tau^{E(x', D')} = \tilde{R}(\tau, x', D')\tau^{E(x', D')}\tau D_\tau.$$

From this and (24) we get

$$\begin{aligned} &\tau D_\tau((\tilde{R}(\tau, x', D')\tau^{E(x', D')})^{-1}v(\tau^{-1}, x')) \\ &= \tau^{-E(x', D')}\tilde{R}(\tau, x', D')^{-1}(\tau D_\tau I_m - C'(\tau, x', D'))v(\tau^{-1}, x') = 0 \end{aligned}$$

on a neighborhood of $\{0\}\times\{x^*\}$. Hence there exists a column vector $a(x')$ of m microfunctions on U_ε with sufficiently small ε such that

$$v(\tau^{-1}, x')=\tilde{R}(\tau, x', D')\tau^{E(x', D')}a(x').$$

Since v and $\tilde{R}$ are single-valued with respect to τ, so is $\tau^E a$, and

$$(\exp(2\pi\sqrt{-1}\,E(x', D'))-I_m)a(x')=0$$

holds. We write $a(x')$ in the form

$$a(x')=\begin{bmatrix} a_1(x') \\ \vdots \\ a_r(x') \end{bmatrix}$$

with column vectors $a_s(x')$ of m_s microfunctions on U_ε for $s=1, \cdots, r$. Then we have

$$(25)\qquad (\exp(2\pi\sqrt{-1}\,E_s(x', D'))-I_{m_s})a_s(x')=0 \qquad (s=1, \cdots, r).$$

Since $\exp(2\pi\sqrt{-1}\,E_s)-I_{m_s}$ is invertible for $s=2, \cdots, r$, we get $a_s=0$ for $s=2, \cdots, r$. Since the eigenvalues of $\sigma_0(E_1)(x^*)$ are all μ_1, which is a non-positive integer, it follows from (25)

$$(26)\qquad (E_1(x', D')-\mu_1 I_{m_1})a_1(x')=0,$$

and consequently

$$\tau^{E_1(x', D')}a_1(x')=\tau^{\mu_1}\exp((E_1(x', D')-\mu_1)\log\tau)a_1(x')=\tau^{\mu_1}a_1(x').$$

Hence we get from (26)

$$\begin{aligned} v(\zeta, x')&=\sum_{\kappa=0}^{\infty}\tilde{R}_\kappa(x', D')\zeta^{-\kappa-\mu_1}\begin{bmatrix} a_1(x') \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ &=\sum_{\kappa=0}^{\infty}\tilde{S}_\kappa(x', D')(-E)\cdots(-E-\kappa+1)\zeta^{-\kappa-\mu_1}\begin{bmatrix} a_1(x') \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ &=\sum_{\kappa=0}^{-\mu_1}\tilde{S}_\kappa(x', D')(-E)\cdots(-E-\kappa+1)\zeta^{-\kappa-\mu_1}\begin{bmatrix} a_1(x') \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \end{aligned}$$

Thus $v(\zeta, x')$ is continued to a section of $(\mathscr{C}\mathscr{O})^m$ on $C\times U_\varepsilon$ with sufficiently small ε. Consequently, $\tilde{u}$ is a germ of $\iota_*\mathscr{C}_{N|X}$ at x^*, and $u=0$ holds on a neighborhood of x^*. This completes the proof of Proposition 2.4 and, at the same time, completes the proof of Theorem 2.3.

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