Nilpotent classes in Lie algebras of type F_4 over fields of characteristic 2

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Let G be a simple algebraic group of type F_4 over an algebraically closed field k of characteristic 2. This article gives a classification of nilpotent orbits in the Lie algebra $\mathfrak{g}$ of G and informations about the centralizers. Springer's correspondence and sheets are also discussed.

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1. We fix a Borel subgroup B and a maximal torus $T \subset B$. Let Φ be the root system of G with respect to T and let Φ^+ and Π be the set of positive roots and the basis corresponding to B. As in [8] we take $\Pi=\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ with $\alpha_1=\varepsilon_2-\varepsilon_3$, $\alpha_2=\varepsilon_3-\varepsilon_4$, $\alpha_3=\varepsilon_4$, $\alpha_4=1/2(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4)$ and the root $a\alpha_1+b\alpha_2+c\alpha_3+d\alpha_4$ is denoted $abcd$. For each $\lambda\in\Phi$ we choose $x_\lambda: \boldsymbol{G}_a\to G$ adapted to λ. We can do this in such a way that all the coefficients in the commutation formulae are 0 or 1 (in particular because the characteristic is 2). Let $\boldsymbol{x}_\lambda=(dx_\lambda)_0(1)$, where the Lie algebra of $\boldsymbol{G}_a$ is identified with k.

THEOREM. *There are exactly* 22 *nilpotent orbits in* $\mathfrak{g}$, *and the elements listed in table* 1 *form a complete set of representatives. If* $\boldsymbol{x}$ *is one of them, then* $\dim G_x$, G_x/G_x^0 *and the type of* $G_x^0/R_u(G_x^0)$ *are as given in the table, where* G_x *is the stabilizer of* $\boldsymbol{x}$. *Moreover the order relation given by inclusion of Zariski closures is as described in table* 2.

The classes are labelled as follows. There are some classes which "come from characteristic 0" (for example in terms of Springer's parametrization, see (4). For them we use the notation of Bala and Carter [3]. Some of these classes have also a degenerate form in $\mathfrak{g}$, and for such degenerate classes we use the Bala-Carter notation with a subscript 2. For example let L be a Levi subgroup of some parabolic subgroup of G and let $\boldsymbol{x}$ be a regular nilpotent element in the Lie algebra of L. Assume that L is of type B_2. In characteristic 0 we would have $\dim G_x=16$ but here $\dim G_x=20$. The class of $\boldsymbol{x}$ is then denoted $(B_2)_2$, and B_2 denotes the class of some other nilpotent element $\boldsymbol{x}'$ such

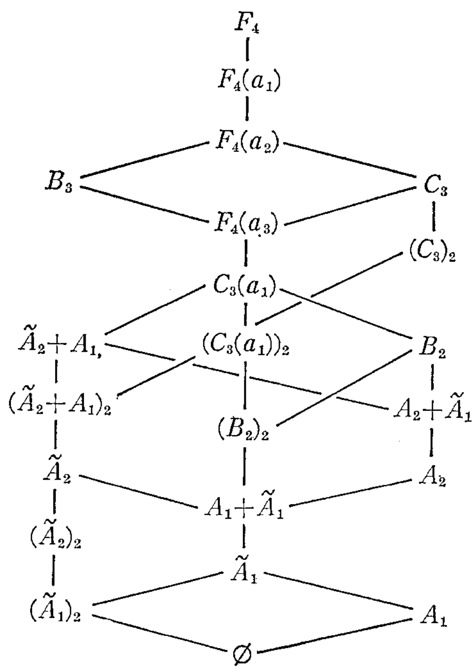
Table 1

class	representative x	dim G_x	G_x/G_x^0	type of $G_x^0/R_u(G_x^0)$
$\emptyset$	0	52	1	F_4
A_1	x_{2342}	36	1	C_3
$(\tilde{A}_1)_2$	x_{1232}	36	1	B_3
$\tilde{A}_1$	$x_{1232}+x_{2342}$	30	1	B_2
$(\tilde{A}_2)_2$	$x_{0121}+x_{1111}$	28	1	G_2
$A_1+\tilde{A}_1$	$x_{1222}+x_{1231}$	24	1	$2A_1$
A_2	$x_{1220}+x_{1122}$	22	$Z/2Z$	A_2
$\tilde{A}_2$	$x_{0121}+x_{1111}+x_{2342}$	22	1	A_1
$(B_2)_2$	$x_{1110}+x_{0122}$	20	1	B_2
$A_2+\tilde{A}_1$	$x_{1220}+x_{0122}+x_{1121}$	18	1	A_1
$(\tilde{A}_2+A_1)_2$	$x_{1110}+x_{0121}+x_{1222}$	18	1	A_1
$\tilde{A}_2+A_1$	$x_{1110}+x_{0121}+x_{1122}+x_{1220}$	16	1	$\emptyset$
B_2	$x_{1100}+x_{1120}+x_{0122}$	16	1	A_1
$(C_3(a_1))_2$	$x_{1110}+x_{0121}+x_{0122}$	16	1	A_1
$C_3(a_1)$	$x_{1100}+x_{0121}+x_{1122}+x_{1242}$	14	1	$\emptyset$
$F_4(a_3)$	$x_{1120}+x_{1110}+x_{0111}+x_{0122}$	12	$\mathfrak{S}_3$	$\emptyset$
$(C_3)_2$	$x_{0010}+x_{0001}+x_{1220}$	12	1	A_1
C_3	$x_{0001}+x_{1110}+x_{0120}+x_{1222}$	10	1	$\emptyset$
B_3	$x_{1000}+x_{0110}+x_{0122}$	10	1	A_1
$F_4(a_2)$	$x_{1000}+x_{0110}+x_{0011}+x_{0122}$	8	$Z/2Z$	$\emptyset$
$F_4(a_1)$	$x_{1000}+x_{0110}+x_{0120}+x_{0001}$	6	1	$\emptyset$
F_4	$x_{1000}+x_{0100}+x_{0010}+x_{0001}$	4	1	$\emptyset$

that dim $x'=16$.

The proof of the theorem is based on a systematic use of the commutation formulae. We can work almost entirely in the Lie algebra $\mathfrak{b}$ of B. For example if $x\in\mathfrak{b}$ the nilpotent classes contained in $\overline{G\cdot x}$ (where $g\cdot x=\mathrm{ad}(g)x$) are exactly those which meet $\overline{B\cdot x}$ since G/B is complete. Let $P\supset B$ be a proper parabolic subgroup and let $M\supset T$ be a Levi factor of P. Let $\mathfrak{m}$ be the Lie algebra of M. The nilpotent M-orbits in $\mathfrak{m}$ are known. We can find representatives y for them such that $y\in\mathfrak{m}\cap\mathfrak{b}$ and $\dim M\cdot y=2\dim(M\cap B)\cdot y$. For such elements we have $\dim G\cdot y=2\dim B\cdot y$. This allows to compute easily $\dim G_y$. We can use the same method as in [15] to compute G_y/G_y^0 and $G_y^0/R_u(G_y^0)$, using in particular reductive subgroups of G of type C_4 or B_3+A_1. For the class $A_2+\tilde{A}_1$ it is worth noting that $B\cdot x$ is closed in $G\cdot x\cap\mathfrak{b}$ (where x is the representative given

Table 2.



in table 1). This can be used in this case to compute dim $G_x^0/R_u(G_x^0)$.

The remaining classes are by definition the distinguished classes (see [3]). If x is distinguished, then G_x^0 is unipotent. It remains to compute G_x/G_x^0 for such elements. For this we use the Bruhat decomposition. For the elements in table 1 this leads to equations which are quite easy to handle. For example if x is the representative given for the orbit $F_4(a_3)$, G_x meets exactly two double cosets BwB, namely those for which w permutes the roots $\varepsilon_2+\varepsilon_4$, $\varepsilon_1-\varepsilon_2$, ε_2, $1/2(\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4)$; an easy computation shows then that G_x has two components in B and four in the other double coset). The details are omitted.

2. Suppose now that k is an algebraic closure of a finite field $\boldsymbol{F}_q$ (q a power of 2) and let $F\colon G\to G$ be the Frobenius morphism corresponding to some $\boldsymbol{F}_q$-structure on G. We can assume that B, T and the x_λ's are all defined over $\boldsymbol{F}_q$. The elements x in table 1 are then all in $\mathfrak{g}^F$, and we can use Lang's theorem to compute the number of nilpotent G^F-orbits in $\mathfrak{g}^F$ and the order of the centralizers. There are 26 nilpotent orbits in $\mathfrak{g}^F$. The only extra information we need to compute the order of the centralizers is the fact that for the orbit A_2 the action of G_x/G_x^0 on $G_x^0/R_u(G_x^0)$ is by outer automorphisms. This can be checked in the course of the proof of the theorem. One can also use the fact that $\mathfrak{g}^F$ has exactly q^{48} nilpotent elements [**17**]. It can also be shown that for the elements in table 1 all components of G_x are F-stable and the reductive groups $G_x^0/R_u(G_x^0)$ are split.

3. Let $\mathfrak{B}$ be the variety of all Borel subgroups of G, and for $x\in\mathfrak{g}$ let $\mathfrak{B}_x$ be the subvariety of $\mathfrak{B}$ consisting of all $B'\in\mathfrak{B}$ such that $x\in\mathfrak{b}'$, where $\mathfrak{b}'$ is the Lie algebra of B'.

PROPOSITION. *For all* $x\in\mathfrak{g}$, $\dim G_x=2\dim\mathfrak{B}_x+\dim T$.

It is enough to consider the case where x is a distinguished nilpotent element which can not be obtained by induction from a Levi factor of a proper parabolic subgroup of G [**14**]. The only case left is that of the class $\tilde{A}_2+A_1$. Let x be the representative given in table 1. We certainly have $\dim G_x=16\geqq 2\dim\mathfrak{B}_x+\dim T$ (the corresponding formula for unipotent elements is proved in [**18**]). We need therefore only to prove that $\dim\mathfrak{B}_x\geqq 6$. For each w in the Weyl group W of G the variety $\mathfrak{B}(w)=\{{}^gB\mid g\in BwB\}$ is isomorphic to an affine space, and the equations of the subvariety $\mathfrak{B}_x\cap\mathfrak{B}(w)$ are quite simple. It can be checked that $\dim(\mathfrak{B}_x\cap\mathfrak{B}(w))=6$ if we take w with matrix

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

with respect to the basis $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$.

4. We turn now to the parametrization by irreducible representations of the Weyl group W [**16**], [**6**]. The original construction given by Springer does not cover the case of F_4 in characteristic 2, but there are alternative constructions which work in all characteristics [**10**].

Let $W^\wedge$ be the set of isomorphism classes of irreducible complex representations of W. Let $x\in\mathfrak{g}$ be nilpotent and let $A(x)=G_x/G_x^0$. The finite group $A(x)$ acts on the set of irreducible components of $\mathfrak{B}_x$. Let $A(x)_0^\wedge$ be the set of all irreducible representations of $A(x)$ which occur in the corresponding permutation representation. To each $\phi\in A(x)_0^\wedge$ we can associate an irreducible representation $\rho_{x,\phi}\in W^\wedge$, and this defines a bijection from the set of conjugacy classes of such pairs to $W^\wedge$. As in [**2**] we assume that this is done in such a way that the regular class corresponds to the trivial representation and 0 corresponds to the sign representation ε. (i.e. we use [**6**] which differs from Springer's original construction by ε). The methods used in [**2**], [**13**] apply also here, in particular because of results in [7]. Using [1] we can then easily compute explicitely the map $(x, \phi) \mapsto \rho_{x,\phi}$. The only trouble is to find which of the two non-trivial characters of $\mathfrak{S}_3$ occurs. If we take x in the class $F_4(a_3)$ as in table 1 and define P to be the parabolic subgroup of G containing B and with a Levi factor of type B_2, then the Borel subgroups of P form an irreducible component of $\mathfrak{B}_x$. This component is B_x-stable, hence stabilized by an involution of $\mathfrak{S}_3$, but it is not stable under G_x as can be seen from the description of G_x given in (1). This shows that the representation of degree 2 of $\mathfrak{S}_3$ does occur.

The irreducible characters of W have been determined by T. Kondo [**9**]. We use the same notation as T. Shoji [**12**], namely we denote $\chi_{i,j}$ the j^{th} character of degree i in Kondo's table, except for the "isolated" characters of degree 4, 12, and 16 which are denoted χ_4, χ_{12}, χ_{16} respectively. The groups $A(x)$ are all of the form $\mathfrak{S}_r$ $(1 \leq r \leq 3)$ and we label the characters of $\mathfrak{S}_r$ by partitions of r, with (r) for the trivial character and $(1, 1, \cdots, 1)$ for the sign character.

THEOREM. *The map* $(x, \phi) \mapsto \rho_{x,\phi}$ *is as described in table* 3.

The Springer representations for F_4 have been determined by Shoji [**12**] in the case where the characteristic is not 2 or 3. We can parametrize the nilpotent classes by the $\rho_{x,\phi}$ with ϕ the trivial character of $A(x)$. In this way it makes sense to compare nilpotent classes in various characteristics. All the characteristic 0 nilpotent orbits still exist in characteristic 2, and we have some new orbits. In a similar way we can say that we have all the unipotent classes which arise

Table 3.

class of x	ϕ	$\rho_{x,\phi}$	class of x	ϕ	$\rho_{x,\phi}$
$\emptyset$	(1)	$\chi_{1,4}$	B_2	(1)	$\chi_{9,2}$
A_1	(1)	$\chi_{2,4}$	$(C_3(a_1))_2$	(1)	$\chi_{9,3}$
$(\tilde{A}_1)_2$	(1)	$\chi_{2,2}$	$C_3(a_1)$	(1)	χ_{16}
$\tilde{A}_1$	(1)	$\chi_{4,4}$	$F_4(a_3)$	(3)	χ_{12}
$(\tilde{A}_2)_2$	(1)	$\chi_{1,3}$	$F_4(a_3)$	(2,1)	$\chi_{6,2}$
$A_1+\tilde{A}_1$	(1)	$\chi_{9,4}$	$F_4(a_3)$	(1,1,1)	—
A_2	(2)	$\chi_{8,4}$	$(C_3)_2$	(1)	$\chi_{2,3}$
A_2	(1,1)	$\chi_{1,2}$	C_3	(1)	$\chi_{8,3}$
$\tilde{A}_2$	(1)	$\chi_{8,2}$	B_3	(1)	$\chi_{8,1}$
$(B_2)_2$	(1)	χ_4	$F_4(a_2)$	(2)	$\chi_{9,1}$
$A_2+\tilde{A}_1$	(1)	$\chi_{4,2}$	$F_4(a_2)$	(1,1)	$\chi_{2,1}$
$(\tilde{A}_2+A_1)_2$	(1)	$\chi_{4,3}$	$F_4(a_1)$	(1)	$\chi_{4,1}$
$\tilde{A}_2+A_1$	(1)	$\chi_{6,1}$	F_4	(1)	$\chi_{1,1}$

in characteristic 2 (they have been determined by Shinoda [11]) and two extra classes, namely $(C_3)_2$ and $(\tilde{A}_2)_2$. (When comparing with Shoji's tables one should remember to tensor by ε).

5. For every $d\geqq 0$ the elements $x\in\mathfrak{g}$ with $\dim G_x=d$ form a locally closed subvariety of $\mathfrak{g}$. The irreducible components of these varieties are called sheets.

A closely related notion is that of a packet. A packet in $\mathfrak{g}$ is an equivalence class for the following relation. Let $x=s+n$, $x'=s'+n'$ be the Jordan decompositions of x, x' (s, s' semisimple, n, n' nilpotent). Then x and x' are equivalent if there exists $g\in G$ such that $g\cdot n'=n$ and $\mathfrak{c}_{\mathfrak{g}}(g\cdot s')=\mathfrak{c}_{\mathfrak{g}}(s)$. Roughly speaking x and x' are equivalent if they have the same type of Jordan decomposition. The packet containing x is entirely determined by the conjugacy class of $\mathfrak{m}=\mathfrak{c}_{\mathfrak{g}}(s)$ in $\mathfrak{g}$ (under the action of G) and by the orbit $\mathcal{O}$ of n in $\mathfrak{m}$ (under the action of the stabilizer of $\mathfrak{m}$ in G). We denote sometimes a packet (type of $\mathfrak{m}$; $\mathcal{O}$) or ($\mathfrak{m}$; $\mathcal{O}$).

It is clear that each packet is contained in at least one sheet, and every sheet is a union of packets. As general references on sheets we refer to the articles by Borho and Kraft [5] and Borho [4]; for the case of bad characteristic see also [14].

THEOREM. *Let $\mathfrak{N}$ be the nilpotent variety of* $\mathfrak{g}$, *let $\mathcal{S}$ be a sheet in* $\mathfrak{g}$ *and let* ($\mathfrak{m}$; $\mathcal{O}$) *be the packet dense in $\mathcal{S}$. The packets occuring in this way are exactly*

Table 4.

d	$\mathfrak{m}$	$\mathcal{O}$	$\tilde{\mathcal{O}}$	d	$\mathfrak{m}$	$\mathcal{O}$	$\tilde{\mathcal{O}}$
4	$\emptyset$	$\emptyset$	F_4	16	D_4	$2A_1$	$C_3(a_1)_2$
6	$\tilde{A}_1$	$\emptyset$	$F_4(a_1)$	16	F_4	A_2+A_1	$\tilde{A}_2+A_1$
6	$4A_1$	$3A_1$	$F_4(a_1)$	18	C_3+A_1	A_1; $\emptyset$	$A_2+\tilde{A}_1$
8	$4A_1$	$2A_1$	$F_4(a_2)$	18	D_4	A_1	$(\tilde{A}_2+A_1)_2$
8	B_2+2A_1	A_1; A_1; A_1	$F_4(a_2)$	20	B_4	$2A_1$	$(B_2)_2$
10	$\tilde{A}_2$	$\emptyset$	B_3	22	C_3+A_1	$\emptyset$; A_1	A_2
10	$4A_1$	A_1	C_3	22	B_4	$\tilde{A}_1^{(*)}$	$\tilde{A}_2$
10	B_2+2A_1	A_1; A_1; $\emptyset$	B_3	24	C_3+A_1	$\emptyset$; $\emptyset$	$A_1+\tilde{A}_1$
12	$4A_1$	$\emptyset$	$(C_3)_2$	24	B_4	A_1	$A_1+\tilde{A}_1$
12	B_2+2A_1	$\emptyset$; A_1; A_1	$F_4(a_3)$	28	D_4	$\emptyset$	$(\tilde{A}_2)_2$
12	B_2+2A_1	A_1; $\emptyset$; $\emptyset$	$F_4(a_3)$	30	F_4	$\tilde{A}_1$	$\tilde{A}_1$
14	B_2+2A_1	$\emptyset$; A_1; $\emptyset$	$C_3(a_1)$	36	B_4	$\emptyset$	$(\tilde{A}_1)_2$
16	B_2+2A_1	$\emptyset$; $\emptyset$; $\emptyset$	$C_3(a_1)_2$	36	F_4	A_1	A_1
16	C_3+A_1	A_1; A_1	B_2	52	F_4	$\emptyset$	$\emptyset$

those listed in table 4. Moreover $\bar{\mathfrak{S}}\cap\mathfrak{N}$ is irreducible and is the closure of the nilpotent orbit $\tilde{\mathcal{O}}$ of $\mathfrak{g}$ given in the table.

In the table $d=\dim G_x$ for $x\in(\mathfrak{m}\,;\mathcal{O})$. The notations for the nilpotent orbits in $\mathfrak{m}$ are similar to those used for $\mathfrak{g}$. In particular for the class marked (*), $\mathfrak{m}$ is of type B_4 and $\tilde{A}_1$ denotes the Richardson orbit corresponding to the subalgebras of type B_3 in $\mathfrak{m}$.

For the proof we consider the various packets in $\mathfrak{g}$. Using the method of induction for nilpotent orbits and sheets [14] we can deal with all packets except those for which $\mathfrak{m}$ is of semisimple rank 4 and $\mathcal{O}$ can not be obtained by induction. Moreover if $\mathfrak{P}$, $\mathfrak{P}'$ are two packets corresponding to the same dimension of centralizers and $\mathfrak{P}\subset\bar{\mathfrak{P}}'$, it is sufficient to look at $\mathfrak{P}'$. In some cases we can use the fact that there is only one nilpotent orbit of dimension d in $\mathfrak{g}$.

Suppose that we know the result for the packet $(4A_1\,;\emptyset)$. Let $\mathfrak{P}=(\mathfrak{m}\,;\mathcal{O})$ be one of the remaining packets. We may assume that $\mathfrak{m}\neq\mathfrak{g}$. Define $\mathfrak{P}'=(\mathfrak{m}\,;\mathcal{O}')$ as follows.

$\mathfrak{m}$	B_2+2A_1	B_2+2A_1	C_3+A_1	D_4	B_4
$\mathcal{O}$	$A_1\,;\emptyset\,;\emptyset$	$\emptyset\,;\emptyset\,;\emptyset$	$A_1\,;\emptyset$	A_1	$\emptyset$
$\mathcal{O}'$	$A_1\,;A_1\,;\emptyset$	$(\tilde{A}_1)_2\,;\emptyset\,;\emptyset$	$A_1\,;A_1$	$2A_1$	$(\tilde{A}_1)_2$

(here $(\tilde{A}_1)_2$ means a short root element). In each case $\mathfrak{P}\subset\bar{\mathfrak{P}}'$ and $\tilde{\mathcal{O}}$ is the unique orbit of dimension d in $\mathfrak{N}\cap\bar{\mathfrak{P}}'$.

It remains therefore only to consider the case where $\mathfrak{m}$ is of type $4A_1$ and $\mathcal{O}=\{0\}$. Let $\mathfrak{P}$ be this packet.

Let $\mathfrak{t}$ be the Lie algebra of T and let $Z=\{s\in\mathfrak{t}\,|\,\alpha_2(s)=0\}$.

Claim. $\overline{B\cdot Z}=\{s+\sum_{\lambda>0}c_\lambda x_\lambda\,|\,s\in Z,\ c_{0100}=c_{0120}=c_{0122}=c_{2342}-\alpha_1(s)(c_{1000}c_{1342}+c_{1100}c_{1242}+c_{1120}c_{1222}+c_{1220}c_{1122})=0\}$.

Granting the claim, we see that $\overline{B\cdot Z}\cap\mathfrak{N}$ is irreducible and contains $x_{1100}+x_{0010}+x_{0001}$. But for all $x\in\overline{B\cdot Z}$ we have $\dim G_x\geqq 12$. By semicontinuity this implies that the nilpotent orbit $(C_3)_2$ contains a dense open subset of $\overline{B\cdot Z}\cap\mathfrak{N}$. Since G/B is complete, $\bar{\mathfrak{P}}\cap\mathfrak{N}$ is the closure of $(C_3)_2$.

The claim follows from the commutation formulae. Let $s\in Z$ and let $a=\alpha_1(s)$, $c=\alpha_3(s)$, $d=\alpha_4(s)$. Assume that none of a, c, d, $a+c$, $a+d$, $c+d$, $a+c+d$ vanishes. We need only to show that the B-orbit of s contains all $s+\sum_{\lambda>0}c_\lambda x_\lambda$ with the c_λ's satisfying the conditions given in the claim. This can be achieved by acting successively by elements of the form $x_\lambda(d_\lambda)$ with suitable $d_\lambda\in k$. We can take the λ's in the following order: 0010, 0001, 0011, 0110, 0111, 0121, 1000, 1100, 1120, 1220, 1122, 1222, 1242, 1342, 1110, 1111, 1121, 1221, 1231, 1232. At each step we can arrange to get any specified value for c_λ without changing the previous ones, and at the end we have $c_{0100}=c_{0120}=c_{0122}=0$, $c_{2342}=\alpha_1(s)(c_{1000}c_{1342}+c_{1100}c_{1242}+c_{1120}c_{1222}+c_{1220}c_{1122})$. This proves the claim.

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