Vanishing of the cohomology groups in the infinite direct sum ΣC

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Introduction.

In a previous paper [1], one of the authors discussed the vanishing of the cohomology groups with coefficients in the sheaf $\mathcal{O}$ of germs of holomorphic functions over the infinite dimensional topological vector space $\Sigma \boldsymbol{C}$, the direct sum of complex planes endowed with the DFS topology. It was proved that for every $p \geqq 1$

$$H^p(U, \mathcal{O})=0$$

holds for any pseudo-convex open set U in $\Sigma \boldsymbol{C}$. In the course of the proof we employed the fine resolution of the sheaf $\mathcal{O}$:

$$0 \longrightarrow \mathcal{O} \longrightarrow \mathcal{E}^{0,0} \longrightarrow \mathcal{E}^{0,1} \longrightarrow \cdots,$$

where $\mathcal{E}^{0,p}$ represents the sheaf of germs of C^∞-differentiable $(0, p)$-forms over $\Sigma \boldsymbol{C}$.

In this paper we will define the subvarieties in $\Sigma \boldsymbol{C}$ and will study the cohomology groups of a subvariety in $\Sigma \boldsymbol{C}$. Our main result is the following:

THEOREM. *Let D be a pseudo-convex open set in $\Sigma \boldsymbol{C}$ and let V be a subvariety of D. Then we have*

$$H^p(V, {}_V\mathcal{O})=0 \qquad \text{for every } p \geqq 1,$$

where ${}_V\mathcal{O}$ denotes the sheaf of germs of holomorphic functions on V.

In Section 1, we summarize the results obtained in [1] as preliminaries. In Section 2, we prove the main theorem using a theorem of Grothendieck. Some examples of subvarieties will be given in Section 3. In Section 4, we show that the cohomology groups with coefficients in the constant sheaf $\boldsymbol{C}$ over $\Sigma \boldsymbol{R}$ can be calculated by means of a fine resolution of $\boldsymbol{C}$.

§ 1. Notations and summary of [1].

We denote by $\sum C$ the direct sum of the complex planes C endowed with the inductive limit topology of the sequence of the spaces $\{C^n; u_n^{n+1}\}$, where $u_n^{n+1}: C^n \to C^{n+1}$ is defined by $u_n^{n+1}((z_1, \cdots, z_n))=(z_1, \cdots, z_n, 0)$. Replacing C by R, we can define $\sum R$ in the same way. Hereafter, u_n denotes the canonical injection of C^n into $\sum C$ and we identify $u_n(C^n)$ with C^n.

Concerning topological properties of $\sum C$, we obtained the following propositions.

PROPOSITION 1.1. *Every open set in $\sum C$ is paracompact.*

PROPOSITION 1.2. *Every polynomially convex compact subset of $\sum C$ has a fundamental system of neighborhoods consisting of polynomially convex open subsets.*

COROLLARY 1.3. *Every point of $\sum C$ has a fundamental system of neighborhoods consisting of pseudo-convex open sets.*

As for holomorphic functions on $\sum C$, we proved the following

PROPOSITION 1.4. *Let U be an open set in $\sum C$. We set $U_n=U\cap C^n$. Then, we have the isomorphism*

$$\mathcal{O}(U) \xrightarrow{\sim} \varprojlim_n \mathcal{O}_n(U_n)$$

as topological vector spaces.

COROLLARY 1.5. *The sheaf $\mathcal{O}$ is the projective limit of the sheaves $\{(u_n)_*\mathcal{O}_n\}$ over $\sum C$, where $(u_n)_*\mathcal{O}_n$ is the direct image of the sheaf $\mathcal{O}_n$ of germs of holomorphic functions over C^n.*

PROPOSITION 1.6. *The sheaf $\mathcal{E}$ of germs of C^∞-functions over $\sum R$ is a fine sheaf.*

Combining the above results, we obtained the following result.

THEOREM 1.7. *Let U be a pseudo-convex open set in $\sum C$. Then, the p-th cohomology group of U with coefficients in the sheaf $\mathcal{O}$ vanishes for every $p\geqq 1$:*

$$H^p(U, \mathcal{O})=0.$$

§ 2. Vanishing of the cohomology groups.

In the sequel we refer to [3] for general results on sheaves, to [2], [5] and [6] for results on the sheaves of germs of holomorphic functions of several variables and to [8] for the general theory of holomorphic functions on infinite dimensional topological vector spaces.

First, we introduce the notion of subvarieties in $\sum C$.

DEFINITION 2.1. Let D be an open set in $\sum C$. We call a subset V of D an *analytic subvariety* if $V \cap C^n$ is an analytic subvariety of $D \cap C^n$ in the usual sense for every positive integer n.

Now, we define the sheaf of ideals of a subvariety V in $\sum C$. We put

$$\mathcal{I}_V(U)=\{f \in \mathcal{O}(U)\,;\ f \text{ vanishes on } V \cap U\}$$

for an open subset U of D. Then the presheaf $\{\mathcal{I}_V(U)\}$ constitutes a sheaf over D, which we call the sheaf of ideals of V. We denote it by $\mathcal{I}_V$. We equip $\mathcal{I}_V(U)$ with the induced topology of $\mathcal{O}(U)$. We note that $V_n=V \cap C^n$ is a subvariety of $D_n=D \cap C^n$ in the usual sense. We denote by $\mathcal{I}_{V_n}$ the sheaf of ideals of the subvariety V_n of D_n. As is well known, $\mathcal{I}_{V_n}$ is a coherent analytic sheaf over D_n.

In the sequel we will use the abbreviations as follows:

$$U_n=U \cap C^n, \qquad V_n=V \cap C^n, \qquad D_n=D \cap C^n.$$

PROPOSITION 2.2. *Let U be an open set in D. Then, we have the isomorphism*

$$\mathcal{I}_V(U) \xrightarrow{\sim} \varprojlim_n \mathcal{I}_{V_n}(U_n)$$

as topological vector spaces, the projective limit being taken with respect to the restriction mappings.

PROOF. We can easily check the conditions of Lemma 1 of 5.5 in Chapter XI in Kantrovich and Akilov [7], so that the algebraic isomorphism holds. The topology of $\mathcal{I}_V(U)$ being induced by $\mathcal{O}(U)$, the equivalence of the topologies of both sides follows from Proposition 1.4. Q.E.D.

COROLLARY 2.3. *$\mathcal{I}_V(U)$ is a Fréchet nuclear space.*

We can restate Proposition 2.2 in the following manner.

PROPOSITION 2.4. *The sheaf $\mathcal{I}_V$ over D is the projective limit of the sheaves $\{(u_n)_*\mathcal{I}_{V_n}\}$ over D, i.e.,*

$$\mathcal{I}_V=\varprojlim_n (u_n)_*\mathcal{I}_{V_n}.$$

We need the following theorem to prove Theorem 2.7 below. Let us recall the Mittag-Leffler condition for a projective system. A projective system $(A_\alpha, f_{\alpha\beta})$ is said to satisfy the Mittag-Leffler condition ((ML) for short) in the sense of

Grothendieck if the following is valid;

(ML) For any index α there exists $\beta \geqq \alpha$ such that $f_{\alpha\gamma}(A_\gamma)=f_{\alpha\beta}(A_\beta)$ for every $\gamma \geqq \beta$.

THEOREM 2.5 (Proposition 13.3.1 in A. Grothendieck [4]). *Let X be a topological space and $(\mathcal{F}_k)_{k\in N}$ a projective system of sheaves of abelian groups over X. Suppose that $\mathcal{F}=\varprojlim_k \mathcal{F}_k$ and that the following conditions hold:*

(i) *There exists a base $\mathfrak{B}$ which defines the topology of X such that for every $U\in\mathfrak{B}$ and every $p\geqq 0$ the projective system $(H^p(U, \mathcal{F}_k))_{k\in N}$ satisfies* (ML).

(ii) *For every $x\in X$ and every $p>0$, we have $\varinjlim_U(\varprojlim_k H^p(U, \mathcal{F}_k))=0$, where U runs over the neighborhoods of x belonging to $\mathfrak{B}$.*

(iii) *The homomorphisms $v_{hk}: \mathcal{F}_k \to \mathcal{F}_h$ $(k\geqq h)$ defining the projective system $(\mathcal{F}_k)$ are surjective.*

Then, if the projective system $(H^{p-1}(X, \mathcal{F}_k))_{k\in N}$ satisfies (ML), *theh the canonical homomorphism*

$$h_p : H^p(X, \mathcal{F}) \longrightarrow \varprojlim_k H^p(X, \mathcal{F}_k)$$

is bijective.

LEMMA 2.6. *Let D be a pseudo-convex open set in $\sum C$ and let V be a subvariety of D. Let U be a pseudo-convex open set in D. Then, the restriction mapping of $\mathcal{J}_{V_{n+1}}(U_{n+1})$ into $\mathcal{J}_{V_n}(U_n)$ is surjective.*

PROOF. Since it is easy to see that the sheaf homomorphism $f: \mathcal{J}_{V_{n+1}} \to (u^n_{n+1})_*\mathcal{J}_{V_n}$ induced by the restriction mappings is surjective, we consider the following short exact sequence of the sheaves on D_{n+1}:

$$0 \longrightarrow \operatorname{Ker} f \longrightarrow \mathcal{J}_{V_{n+1}} \longrightarrow (u^n_{n+1})_*\mathcal{J}_{V_n} \longrightarrow 0\,,$$

where $\operatorname{Ker} f$ denotes the kernel of the homomorphism f and $(u^n_{n+1})_*\mathcal{J}_{V_n}$ denotes the direct image of $\mathcal{J}_{V_n}$. As the sheaf $(u^n_{n+1})_*\mathcal{J}_{V_n}$ is a coherent sheaf of $\mathcal{O}_{n+1}$-modules by Theorem 8 in Chapter IV, D in Gunning and Rossi [5], $\operatorname{Ker} f$ is a coherent sheaf of $\mathcal{O}_{n+1}$-modules. Thus, we have $H^1(U_{n+1}, \operatorname{Ker} f)=0$. Therefore, the restriction mapping of $\mathcal{J}_{V_{n+1}}(U_{n+1})$ into $\mathcal{J}_{V_n}(U_n)$ is surjective. Q.E.D.

Under the above preparation, we can show the following

THEOREM 2.7. *Let D be a pseudo-convex open set in $\sum C$ and let V be a subvariety of D. Then we have*

$$H^p(D, \mathcal{J}_V)=0 \qquad \text{for every} \quad p\geqq 1\,.$$

PROOF. By Proposition 2.4, $\mathcal{J}_V$ is the projective limit of the sheaves $\{(u_n)_*\mathcal{J}_{V_n}\}$ over D. Every point z in D has a fundamental system $\mathfrak{B}_z$ of neighborhoods consisting of pseudo-convex open sets by Corollary 1.3. We have $H^p(U, (u_n)_*\mathcal{I}_{V_n}) = H^p(U_n, \mathcal{I}_{V_n}) = 0$ $(p \geqq 1)$ for any $U \in \mathfrak{B}_z$, because U_n is also a pseudo-convex open set in C^n and that $\mathcal{I}_{V_n}$ is a coherent sheaf. Therefore, the conditions (i) for $p>0$ and (ii) in Theorem 2.5 are satisfied. The homomorphism of $H^0(U, (u_{n+1})_*\mathcal{I}_{V_{n+1}}) = \mathcal{I}_{V_{n+1}}(U_{n+1})$ into $H^0(U, (u_n)_*\mathcal{I}_{V_n}) = \mathcal{I}_{V_n}(U_n)$ is surjective for any $U \in \mathfrak{B}_z$ by Lemma 2.6. Thus, the conditions (i) for $p=0$ and (iii) are satisfied. If we take $U=D$ in the above discussion, the projective system $(H^{p-1}(D, (u_n)_*\mathcal{I}_{V_n}))$ satisfies the condition (ML) for any $p>0$. Thus, the theorem results from Theorem 2.5.

Q. E. D.

REMARK. Theorem 1.7 can also be proved in the same way as above by using Theorem 2.5. More generally, we can prove a similar theorem for a sheaf over $\sum C$ defined as the projective limit of coherent analytic sheaves over finite dimensional subspaces.

Next, we consider the quotient sheaf ${}_V\tilde{\mathcal{O}}_D = \mathcal{O}_D/\mathcal{I}_V$. Since $({}_V\tilde{\mathcal{O}}_D)_z = 0$ for every $z \in D-V$, we put

$$ {}_V\mathcal{O} = {}_V\tilde{\mathcal{O}}_D|_V . $$

DEFINITION 2.8. The sheaf ${}_V\mathcal{O}$ is called the sheaf of germs of holomorphic functions on the subvariety V.

We recall a lemma to prove Proposition 2.10 below.

LEMMA 2.9 (Proposition 13.2.2 in A. Grothendieck [4]). *Suppose that I is a filtering ordered set having a countable cofinal subset and that the following is an exact sequence of a projective system of abelian groups for $\alpha \in I$:*

$$ 0 \longrightarrow A_\alpha \xrightarrow{u_\alpha} B_\alpha \xrightarrow{v_\alpha} C_\alpha \longrightarrow 0 . $$

If (A_α) satisfies the condition (ML), *the following sequence is exact*:

$$ 0 \longrightarrow \varprojlim A_\alpha \longrightarrow \varprojlim B_\alpha \longrightarrow \varprojlim C_\alpha \longrightarrow 0 . $$

PROPOSITION 2.10. *Let U be a pseudo-convex open set in D and let V be a subvariety of D. Then, we have*

$$ \Gamma(U, {}_V\tilde{\mathcal{O}}_D) \xrightarrow{\sim} \varprojlim_n \Gamma(U_n, \mathcal{O}_{D_n}/\mathcal{I}_{V_n}) , $$

the projective limit being taken with respect to the restriction mappings. Here, $\Gamma(W, \cdot)$ denotes the section module over W.

PROOF. Since $\mathcal{I}_{V_n}$ is coherent, the following sequence is exact:

$$ 0 \longrightarrow \mathcal{I}_{V_n}(U_n) \longrightarrow \mathcal{O}_{D_n}(U_n) \longrightarrow \Gamma(U_n, \mathcal{O}_{D_n}/\mathcal{I}_{V_n}) \longrightarrow 0 . $$

By Lemma 2.6 and Lemma 2.9, we obtain

$$0 \longrightarrow \varprojlim_n \mathcal{J}_{V_n}(U_n) \longrightarrow \varprojlim_n \mathcal{O}_{D_n}(U_n) \longrightarrow \varprojlim_n \Gamma(U_n, \mathcal{O}_{D_n}/\mathcal{J}_{V_n}) \longrightarrow 0\,.$$

On the other hand $H^1(U, \mathcal{J}_V)=0$ holds by Theorem 2.7. Therefore, we have

$$0 \longrightarrow \mathcal{J}_V(U) \longrightarrow \mathcal{O}_D(U) \longrightarrow \Gamma(U, {}_V\tilde{\mathcal{O}}_D) \longrightarrow 0\,.$$

By Proposition 1.4 and Proposition 2.2, we have the required isomorphism.

Q. E. D.

This proposition implies that ${}_V\tilde{\mathcal{O}}_D$ is the sheaf associated with the presheaf $\left\{\varprojlim_n \Gamma(U_n, \mathcal{O}_{D_n}/\mathcal{J}_{V_n})\right\}$.

Now, we can prove our main theorem:

THEOREM 2.11. *Let D be a pseudo-convex open set in $\sum C$ and let V be a subvariety of D. Then, we have*

$$H^p(V, {}_V\mathcal{O})=0 \qquad \textit{for every } p \geqq 1\,.$$

PROOF. Because of the exactness of the sequence

$$0 \longrightarrow \mathcal{J}_V \longrightarrow \mathcal{O}_D \longrightarrow {}_V\tilde{\mathcal{O}}_D \longrightarrow 0\,,$$

we have the following long exact sequence:

$$\begin{aligned} 0 \longrightarrow \mathcal{J}_V(D) &\longrightarrow \mathcal{O}_D(D) \longrightarrow \Gamma(D, {}_V\tilde{\mathcal{O}}_D) \\ &\longrightarrow H^1(D, \mathcal{J}_V) \longrightarrow H^1(D, \mathcal{O}_D) \longrightarrow H^1(D, {}_V\tilde{\mathcal{O}}_D) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\longrightarrow H^k(D, \mathcal{J}_V) \longrightarrow H^k(D, \mathcal{O}_D) \longrightarrow H^k(D, {}_V\tilde{\mathcal{O}}_D) \longrightarrow \cdots\,. \end{aligned}$$

By Theorem 1.7 and Theorem 2.2 we have $H^p(D, {}_V\tilde{\mathcal{O}}_D)=0$. Therefore, $H^p(V, {}_V\mathcal{O}) =H^p(D, {}_V\tilde{\mathcal{O}}_D)=0$ for every $p\geqq 1$.

Q. E. D.

§ 3. Some examples.

In this section we will give some simple examples of subvarieties of a pseudo-convex open set D in $\sum C$.

1. Let $f(z)$ be a holomorphic function on $\sum C$ independent of z_1. Put

$$V=\{(z_1, z_2, z_3, \cdots)\in D\,;\ z_1=f(z)\}\,.$$

Then, V is a subvariety of codimension one.

2. Let V' be a subvariety of D_{n_0} in the usual sense for some positive integer n_0. Then, $u_{n_0}(V')$ is a finite dimensional subvariety of D. Here, u_{n_0} is the

canonical injection of C^{n_0} into $\sum C$.

3. We put

$$V=\{z\in D\,;\ z_{2k}=0\ (k=1,\,2,\,\cdots)\}.$$

Then, V is a subvariety of D and its dimension and codimension are both infinite.

4. Let us consider the case $V=D_n$. In this case we can show the following

PROPOSITION 3.1. *We have the isomorphism*

$$_{D_n}\mathcal{O}\cong\mathcal{O}_{D_n}\,,$$

where $\mathcal{O}_{D_n}$ *denotes the sheaf of germs of holomorphic functions over* D_n *in the usual sense.*

PROOF. Let z be an arbitrary point of D. Then, we have for any $k\geqq n$

$$\Gamma(U_K,\,\mathcal{O}_{D_k}/\mathcal{I}_{V_k})\cong\Gamma(U_n,\,\mathcal{O}_{D_n}/\mathcal{I}_{V_n})\cong\Gamma(U_n,\,\mathcal{O}_{D_n})$$

for any pseudo-convex open neighborhood U of z in D. By Proposition 2.10 we have

$$\Gamma(U,\,_V\tilde{\mathcal{O}}_D)\cong\Gamma(U_n,\,\mathcal{O}_{D_n})\,.$$

Thus, we obtain the isomorphism $_V\mathcal{O}\cong\mathcal{O}_{D_n}$. Q.E.D.

§4. The cohomology groups with coefficients in the constant sheaf C.

We study in this section the cohomology groups with coefficients in the constant sheaf C over $\sum R$ by analogy with the de Rham theorem in the case of finite dimensions.

Let U be an open set in $\sum R$. $\mathcal{E}^q(U)$ is, by definition, the set of the following differential q-forms:

$$f=\sum_{i_1<\cdots<i_q} f_{i_1\cdots i_q}dx_{i_1}\wedge\cdots\wedge dx_{i_q}\,,$$

where $f_{i_1\cdots i_q}\in\mathcal{E}(U)$. We put $\mathcal{E}^0(U)=\mathcal{E}(U)$. We define the operator d as follows.

$$df=\sum_{i_1<\cdots<i_q} d(f_{i_1\cdots i_q}dx_{i_1}\wedge\cdots\wedge dx_{i_q})$$

where

$$\begin{aligned}
&d(f_{i_1\cdots i_q}dx_{i_1}\wedge\cdots\wedge dx_{i_q})\\
&=\sum_{j=1}^{i_1-1}\frac{\partial f_{i_1\cdots i_q}}{\partial x_j}dx_j\wedge dx_{i_1}\wedge\cdots\wedge dx_{i_q}\\
&\quad+\sum_{k=1}^{q-1}\sum_{i_k<j<i_{k+1}}(-1)^k\frac{\partial f_{i_1\cdots i_q}}{\partial x_j}dx_{i_1}\wedge\cdots\wedge dx_{i_k}\wedge dx_j\wedge dx_{i_{k+1}}\wedge\cdots\wedge dx_{i_q}\\
&\quad+(-1)^q\sum_{j>i_q}\frac{\partial f_{i_1\cdots i_q}}{\partial x_j}dx_{i_1}\wedge\cdots\wedge dx_{i_q}\wedge dx_j\,.
\end{aligned}$$

$\mathcal{E}^q(U_n)$ denotes the set of C^∞-differentiable q-forms on U_n in the usual sense, where $U_n = U \cap \boldsymbol{R}^n$.

LEMMA 4.1. *Let U be an open set in $\sum\boldsymbol{R}$. Then the following sequence over U is exact:*

$$0 \longrightarrow \boldsymbol{C}_U \longrightarrow \mathcal{E}_U^0 \xrightarrow{d} \mathcal{E}_U^1 \xrightarrow{d} \cdots,$$

i.e., the above sequence is a fine resolution of the sheaf $\boldsymbol{C}$.

PROOF. For any x in U, all the open sets of the following kind

$$W = \Delta_x(r) = \{(y_1, y_2, \cdots) \in \sum\boldsymbol{R}\,;\, |y_j - x_j| < r_j \ (j=1, 2, \cdots)\} \subset U \quad (r_j > 0)$$

form a fundamental system $\mathfrak{B}_x$ of neighborhoods of x. By the Poincaré lemma the following sequence is exact:

$$0 \longrightarrow \Gamma(W_n, \boldsymbol{C}) \longrightarrow \mathcal{E}^0(W_n) \longrightarrow \cdots \longrightarrow \mathcal{E}^n(W_n) \longrightarrow 0\,;$$

i.e.,

$$(4.1) \qquad \begin{cases} 0 \longrightarrow \Gamma(W_n, \boldsymbol{C}) \longrightarrow \mathcal{E}^0(W_n) \longrightarrow (\operatorname{Im} d)_{n,0} \longrightarrow 0\,, \\ 0 \longrightarrow (\operatorname{Ker} d)_{n,k} \longrightarrow \mathcal{E}^k(W_n) \longrightarrow (\operatorname{Im} d)_{n,k} \longrightarrow 0 \quad (n-1 \geqq k \geqq 1)\,, \\ (\operatorname{Im} d)_{n,k-1} \cong (\operatorname{Ker} d)_{n,k} \quad (n-1 \geqq k \geqq 1)\,, \end{cases}$$

where $(\operatorname{Im} d)_{n,k}$ is the image of $\{d : \mathcal{E}^k(W_n) \to \mathcal{E}^{k+1}(W_n)\}$ and $(\operatorname{Ker} d)_{n,k}$ is the kernel of $\{d : \mathcal{E}^k(W_n) \to \mathcal{E}^{k+1}(W_n)\}$. On the other hand, we have

$$(4.2) \qquad \begin{cases} \Gamma(W, \boldsymbol{C}) \cong \varprojlim_n \Gamma(W_n, \boldsymbol{C})\,, \\ \mathcal{E}^k(W) \cong \varprojlim_n \mathcal{E}^k(W_n) \qquad (k \geqq 0)\,. \end{cases}$$

As W_n is connected for every $n > 0$,

$$(4.3) \qquad \Gamma(W_{n+1}, \boldsymbol{C}) \longrightarrow \Gamma(W_n, \boldsymbol{C})$$

is surjective. In view of (4.1), (4.2) and (4.3), by Lemma 2.9 and Lemma 2.21 in [1] we obtain the exact sequence

$$0 \longrightarrow \Gamma(W, \boldsymbol{C}) \longrightarrow \mathcal{E}^0(W) \longrightarrow \mathcal{E}^1(W) \longrightarrow \cdots.$$

Taking the inductive limit as W runs over $\mathfrak{B}_x$, we have the required exact sequence. Q.E.D.

Thus, we have the following

PROPOSITION 4.2. *Let U be an open set in $\sum\boldsymbol{R}$. Then, we have*

$$H^p(U, \boldsymbol{C}) \cong \frac{\{f\,;\, f\in\Gamma(U, \mathcal{E}_U^p),\ df=0\}}{\{dg\,;\, g\in\Gamma(U, \mathcal{E}_U^{p-1})\}}$$

for every $p\geqq 1$.

Applying this proposition to an convex set in $\sum\boldsymbol{R}$, we obtain the following

PROPOSITION 4.3. *Let U be a convex open set in $\sum\boldsymbol{R}$. Then, we have*

$$H^p(U, \boldsymbol{C})=0$$

for every $p\geqq 1$.

PROOF. Proposition 4.2 implies that it is sufficient to show that for any $g\in\mathcal{E}^q(U)$ such that $dg=0$, there exists an element $f\in\mathcal{E}^{q-1}(U)$ such that $df=g$. We consider the following diagram:

$$\begin{array}{ccccccccc}
\cdots\longrightarrow & \mathcal{E}_{n+1}^{q-2}(U_{n+1}) & \longrightarrow & \mathcal{E}_{n+1}^{q-1}(U_{n+1}) & \longrightarrow & \mathcal{E}_{n+1}^{q}(U_{n+1}) & \longrightarrow\cdots \\
 & \downarrow & & \downarrow & & \downarrow & \\
\cdots\longrightarrow & \mathcal{E}_{n}^{q-2}(U_{n}) & \longrightarrow & \mathcal{E}_{n}^{q-1}(U_{n}) & \longrightarrow & \mathcal{E}_{n}^{q}(U_{n}) & \longrightarrow\cdots
\end{array}$$

As U is convex, each row is exact. Put $g_n=g|_{R^n}$. Then, there exists $f_n\in\mathcal{E}_n^{q-1}(U_n)$ such that $df_n=g_n$. Since $d(f_{n+1}|_{R^n}-f_n)=0$ holds, there exists $h_n\in\mathcal{E}_n^{q-2}(U_n)$ for $q>1$ and $h_n\in\Gamma(U_n, \boldsymbol{C})$ for $q=1$ such that $dh_n=f_{n+1}|_{R^n}-f_n$ for $q>1$ and $h_n=f_{n+1}|_{R^n}-f_n$ for $q=1$, respectively. By Lemma 2.21 in [1], there exists $h_{n+1}\in\mathcal{E}_{n+1}^{q-2}(U_{n+1})$ such that $h_{n+1}|_{R^n}=h_n$ for $q>1$. Obviously, there exists $h_{n+1}\in\Gamma(U_{n+1}, \boldsymbol{C})$ such that $h_{n+1}|_{R^n}=h_n$ for $q=1$. Put $f'_{n+1}=f_{n+1}-dh_{n+1}$ for $q>1$ and $f'_{n+1}=f_{n+1}-h_{n+1}$ for $q=1$. Thus, we assume without loss of generality that we have the sequence $\{f_n\}$ such that $f_{n+1}|_{R^n}=f_n$ and $df_n=g_n$. Therefore, the sequence determines an element $f\in\mathcal{E}^q(U)$ such that $df=g$. Q.E.D.

COROLLARY 3.4. *We have*

$$H^p(\sum\boldsymbol{R}, \boldsymbol{C})=0$$

for every $p\geqq 1$.

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