# *Correction to: "Deformation of Milnor fibering"*

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In my paper [2], I have proved the following.

THEOREM 1. *Let $\{f_t(z_1, \cdots, z_{n+1})\}$, $(0 \leq t \leq 1)$ be a smooth family of analytic functions which have an isolated singular point at the origin. Assume that there exists a positive number $r$ such that the hypersurface $f_t^{-1}(0)$ and the sphere $S_{r'}=\{z \in \boldsymbol{C}^{n+1};\ \|z\|=r'\}$ meet transversely for each $0 \leq t \leq 1$ and $0<r' \leq r$. Then the Milnor fibrations of $f_0$ and $f_1$ are isomorphic.*

The proof given in [2] is apparently not sufficient because we did not show clearly that the above assumption implies the constancy of the Milnor numbers of $f_t$ for each $0 \leq t \leq 1$. Assuming it for a while, we can choose a positive number $s$ (small enough) so that $f_t^{-1}(s')$ is non singular in $D_r^{2n+2}$ and meet transversely with $S_r$ for any $s' \neq 0$, $|s'| \leq s$ and $0 \leq t \leq 1$. (Otherwise we can find a convergent sequence $(z(m), t(m)) \to (0, t_0)$ such that $\operatorname{grad} f_{t(m)}(z(m))=0$. However this implies that the Milnor number of $f_{t_0}$ is strictly bigger than that of $f_{t(m)}$ for $m$ sufficiently large, contradicting to the constancy of the Milnor numbers). Thus the rest of the proof is done exactly as in [2].

Therefore let us prove the implication. Let $K_t(r')$ be $S_{r'} \cap f_t^{-1}(0)$. By the assumption, we have an isotopy $h_t$ of $S_r$ such that $h_t(K_0(r))=K_t(r)$. Let $F_t$ be the fiber of the Milnor fibration and $\mu_t$ be its $n$-th Betti number i.e. the Milnor number of $f_t$. In the case that $n \geq 2$, $F_t$ is homotopic to the universal covering space of $S_{r(t)}-K_t(r(t))$ where $r(t)$ is sufficiently small so that $\varphi_{t,r(t)}=f_t/|f_t|\colon S_{r(t)}-K_t(r(t)) \to S^1$ is a fibration. But $S_{r(t)}-K_t(r(t))$ is diffeomorphic to $S_r-K_t(r)$ and thus diffeomorphic to $S_r-K_0(r)$. Therefore $F_t$ is homotopic to $F_0$ and $\mu_t=\mu_0$. Consider the case $n=1$. The fundamental group $\pi_1(F_t)$ is isomorphic to the kernel of $\{(\varphi_{t,r(t)})_\sharp\colon \pi_1(S_{r(t)}-K_t(r(t))) \to \pi_1(S^1)\}$ and by the assumption the latter group is isomorphic to the kernel of $\{(\varphi_{t,r})_\sharp\colon \pi_1(S_r-K_t(r)) \to \pi_1(S^1)\}$. Using the isotopy $h_t$, we can see that the last group is isomorphic to each other for each $0 \leq t \leq 1$. Thus $\pi_1(F_t)$ is isomorphic to $\pi_1(F_0)$ and in particular we get $\mu_t=\mu_0$.

## References

[1] Milnor, J., Singular points of Complex Hypersurfaces, Ann. of Math. Studies **61**, Princeton Univ. Press, 1968.

[2] Oka, M., Deformation of Milnor fibering, J. Fac. Sci. Univ. Tokyo Sect. IA, Math. **20** (1973), 397-400.

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