Characters of the finite general unitary group $U(5, q^2)$

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§1. Introduction

The purpose of this paper is to calculate all the complex irreducible characters of the finite general unitary group $U(5, q^2)$. The determination of the irreducible characters of the groups $U(n, q^2)$ has been given for $n=2$ and $n=3$ by V. Ennola [1], who has also tried to extend the results of J. A. Green [3] on the characters of $GL(n, q)$ to $U(n, q^2)$, and for $n=4$ by the author [4].

By a character of a finite group G we mean a rational integral combination of the complex irreducible characters of G. If χ and ϕ are complex valued class functions on G, the scalar product (χ, ϕ) and the norm $\|\chi\|$ are defined as usual. If ϕ is a character of a subgroup H of G and χ is a character of G, ϕ^G and χ_H denote the character of G induced by ϕ and the restriction of χ to H respectively. As is well known, if H is a normal subgroup of G and χ is an irreducible character of G/H, then we can extend χ to an irreducible character of G, by putting $\chi(g)=\chi(gH)$ for $g \in G$. We may denote this character also by χ.

In §2, we shall get certain irreducible characters of $U(n, q^2)$ from those of the groups $GL(k, q^2)$ and $U(n-2k, q^2)$ $\left(k=1, 2, \cdots, \left[\frac{n}{2}\right]\right)$, as the special case of the irreducibility of characters of finite groups with split (B, N)-pairs in our previous paper [5]. The conjugacy classes and the character table of $U(5, q^2)$ are given in §3. From this, we shall be able to find that the irreducible characters of $U(5, q^2)$ fall into families in a natural way, just as the conjugacy classes of $U(5, q^2)$ do. In §4, we state an outline of the determination of the irreducible characters of $U(5, q^2)$ in which Ennola's conjecture and the result in §2 play an important role.

§2.

Let $U_n=U(n, q^2)$ be the group of all non-singular $n \times n$ matrices G with elements in the Galois field $GF(q^2)$ satisfying $G^*JG=J$, where G^* is the conjugate transpose of G and J is the matrix $\begin{bmatrix} & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & \end{bmatrix}$. Let $G_n=GL(n, q^2)$ be the group of all non-singular $n\times n$ matrices G with elements in $GF(q^2)$.

For $k=1,2,\cdots,\left[\frac{n}{2}\right]$, we denote by $P_n^{(k)}$ the maximal parabolic subgroups of U_n which consist of all matrices of the forms $\begin{bmatrix} A & D & E \\ & B & F \\ & & C \end{bmatrix}$ with $A\in G_k$, $B\in U_{n-2k}$, $A^*JC=J$, and let $V_n^{(k)}$ be the subgroup of $P_n^{(k)}$ which consists of all matrices of the forms $\begin{bmatrix} I & D & E \\ & I & F \\ & & I \end{bmatrix}$ where I is the identity matrix.

For $j=0,1,\cdots,i$, we denote by $V_k^{(i,j)}$ and $H_k^{(i,j)}$ the subgroups of G_k which consist of all matrices of the forms $\begin{bmatrix} A & D & E \\ & B & \\ & F & C \end{bmatrix}$ with $A\in G_{i-j}$, $B\in G_j$, $C\in G_{k-i}$, and $\begin{bmatrix} I & D & E \\ & I & \\ & F & I \end{bmatrix}$ respectively.

It is easy to see that $V_n^{(k)}, H_k^{(i,j)}$ are normal subgroups of $P_n^{(k)}, V_k^{(i,j)}$ respectively and $P_n^{(k)}/V_n^{(k)}\cong G_k\times U_{n-2k}$.

If χ and ϕ are irreducible characters of G_k and U_{n-2k} respectively, then $\chi\phi$ is an irreducible character of $G_k\times U_{n-2k}$, and so is also an irreducible character of $P_n^{(k)}$. As the special case of the theorem in our previous paper [5], we have the following:

THEOREM. *Assume that (1) χ is not a self-conjugate and (2) no kernel of irreducible constituents of $\chi_{V_k^{(i,j)}}$ contains $H_k^{(i,j)}$, or no kernel of irreducible constituents of $\phi_{P_{n-2k}^{(k-i)}}$ contains $V_{n-2k}^{(k-i)}$. Then $(\chi\phi)^{U_n}$ is an irreducible character of U_n.*

§ 3.

Throughout this section we use the following notations. Let $K_n=GF(q^{2n})$ be the finite field with q^{2n} elements, where q is a fixed prime power. For $\alpha\in K=K_1$ we write $\bar{\alpha}=\alpha^q$, that is, the conjugate of α over K. Write $M(K_n)$ for the multiplicative group of K_n. It is clear that $M=M(K)$ is isomorphic to the cyclic group $\left\langle\begin{pmatrix}\alpha & \\ & \alpha^{-q}\end{pmatrix}\right\rangle$ of order q^2-1 for any primitive element α of K. Let κ be a generator of $M(K_{30})$, and put $\kappa_d=\kappa^{(q^{60}-1)/(q^d-1)}$. In particular, we put $\kappa_5=\theta$, $\kappa_4=\omega$, $\kappa_3=\tau$, $\omega^{q^2+1}=\sigma$, and $\sigma^{q-1}=\rho$. Choose a fixed isomorphism of $M(K_{30})$ into the multiplicative group of the field of complex numbers, and let λ,ζ,ξ,η and ε be the images of $\theta,\omega,\tau,\sigma$ and ρ respectively under this isomorphism.

For $r,s\in K$, we define

$$y_1(r)=\begin{bmatrix} 1 & r & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & -\bar{r} \\ & & & & 1 \end{bmatrix}, \quad y_2(r,s)=\begin{bmatrix} 1 & & & & \\ & 1 & r & -\bar{s} & \\ & & 1 & -\bar{r} & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \text{ where } s+\bar{s}=r\bar{r}\ ,$$

$$y_3(r,s)=\begin{bmatrix} 1 & & r & & -\bar{s} \\ & 1 & & & \\ & & 1 & & -\bar{r} \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \text{ where } s+\bar{s}=r\bar{r}\ , \quad y_4(r)=\begin{bmatrix} 1 & & & r & \\ & 1 & & & -\bar{r} \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}.$$

Then we have the following commutator relations, where the commutator $x^{-1}y^{-1}xy$ is denoted by $[x, y]$:

$$[y_1(r), y_1(s)]=[y_1(r), y_3(s, t)]=[y_2(r, s), y_4(t)]=[y_3(r, s), y_4(t)]=[y_4(r), y_4(s)]=1\ ,$$

$$[y_1(r), y_2(s, t)]=y_3(rs, r\bar{r}\bar{t})y_4(-r\bar{t})\ ,$$

$$[y_1(r), y_4(s)]=y_3(0, \bar{r}s-r\bar{s})\ ,$$

$$[y_2(r, s), y_2(t, u)]=y_2(0, \bar{r}t-r\bar{t})\ ,$$

$$[y_2(r, s), y_3(t, u)]=y_4(\bar{r}t)\ ,$$

$$[y_3(r, s), y_3(t, u)]=y_3(0, \bar{r}t-r\bar{t})\ ,$$

for $r, s, t, u \in K$. We next define

$$h(x, y, z, u, v)=\mathrm{diag}\,\{x, y, z, u, v\}\ , \quad h(x, y, z, u)=h(x, y, z, u, x^{-q})\ ,$$

$$h(x, y, z)=h(x, y, z, y^{-q}, x^{-q})\ ,$$

$$\omega_1=\begin{bmatrix} & 1 & & & \\ -1 & & & & \\ & & 1 & & \\ & & & & -1 \\ & & & 1 & \end{bmatrix}, \quad \omega_2=\begin{bmatrix} 1 & & & & \\ & & & 1 & \\ & & -1 & & \\ & 1 & & & \\ & & & & 1 \end{bmatrix}.$$

Furthermore we use the following set of parameters both for conjugacy classes and for the characters.

$$I_1=\boldsymbol{Z}/(q+1)\boldsymbol{Z}\ , \quad I_k=\{(x_1, \cdots, x_k) \in I_1^k\ ;\ x_i \neq x_j \text{ if } i \neq j\}\ , \quad J_0=\boldsymbol{Z}/(q^2-1)\boldsymbol{Z}\ ,$$

$$J_1=\{x \in J_0\ ;\ x \not\equiv 0 \bmod q-1\}\ , \quad J_2=\{(x_1, x_2) \in J_1^2\ ;\ x_1 \not\equiv x_2, -x_2 q \bmod q^2-1\}\ ,$$

Table 1. Conjugacy classes of $U(5, q^2)$

Notation	Class representative	Number of classes	Order of centralizer
A_{11}	$h(\rho^a, \rho^a, \rho^a)$	$q+1$	$q^{10}(q+1)(q^2-1)(q^3+1)(q^4-1)(q^5+1)$
A_{12}	$h(\rho^a, \rho^a, \rho^a)y_2(0, r)$	$q+1$	$q^{10}(q+1)^2(q^2-1)(q^3+1)$
A_{13}	$h(\rho^a, \rho^a, \rho^a)y_1(r)$	$q+1$	$q^9(q+1)^2(q^2-1)$
A_{14}	$h(\rho^a, \rho^a, \rho^a)y_2(r, s)$	$q+1$	$q^7(q+1)^2(q^2-1)$
A_{15}	$h(\rho^a, \rho^a, \rho^a)y_2(r, s)y_3(0, t)$	$q+1$	$q^7(q+1)^2$
A_{16}	$h(\rho^a, \rho^a, \rho^a)y_1(r)y_2(0, s)$	$q+1$	$q^5(q+1)^2$
A_{17}	$h(\rho^a, \rho^a, \rho^a)y_1(r)y_2(s, t)$	$q+1$	$q^4(q+1)$
A_{21}	$h(\rho^a, \rho^a, \rho^b)$	$q(q+1)$	$q^6(q+1)^2(q^2-1)(q^3+1)(q^4-1)$
A_{22}	$h(\rho^a, \rho^a, \rho^b)y_2(0, r)$	$q(q+1)$	$q^6(q+1)^3(q^2-1)$
A_{23}	$h(\rho^a, \rho^a, \rho^b)y_1(r)$	$q(q+1)$	$q^5(q+1)^2(q^2-1)$
A_{24}	$h(\rho^a, \rho^a, \rho^b)y_1(r)y_4(s)$	$q(q+1)$	$q^4(q+1)^3$
A_{25}	$h(\rho^a, \rho^a, \rho^b)y_1(r)y_2(0, s)$	$q(q+1)$	$q^3(q+1)^2$
A_{31}	$h(\rho^a, \rho^b, \rho^a)$	$q(q+1)$	$q^4(q+1)^2(q^2-1)^2(q^3+1)$
A_{32}	$h(\rho^a, \rho^b, \rho^a)y_2(0, r)$	$q(q+1)$	$q^4(q+1)^2(q^2-1)(q^3+1)$
A_{33}	$h(\rho^a, \rho^b, \rho^a)y_3(0, r)$	$q(q+1)$	$q^4(q+1)^3(q^2-1)$
A_{34}	$h(\rho^a, \rho^b, \rho^a)y_2(0, r)y_3(0, s)$	$q(q+1)$	$q^4(q+1)^3$
A_{35}	$h(\rho^a, \rho^b, \rho^a)y_3(r, s)$	$q(q+1)$	$q^3(q+1)^2(q^2-1)$
A_{36}	$h(\rho^a, \rho^b, \rho^a)y_2(0, r)y_3(s, t)$	$q(q+1)$	$q^3(q+1)^2$
A_{41}	$h(\rho^a, \rho^a, \rho^b, \rho^c)$	$(1/2)\cdot q(q^2-1)$	$q^3(q+1)^3(q^2-1)(q^3+1)$
A_{42}	$h(\rho^a, \rho^a, \rho^b, \rho^c)y_3(0, r)$	$(1/2)\cdot q(q^2-1)$	$q^3(q+1)^4$
A_{43}	$h(\rho^a, \rho^a, \rho^b, \rho^c)y_3(r, s)$	$(1/2)\cdot q(q^2-1)$	$q^2(q+1)^3$
A_{51}	$h(\rho^a, \rho^b, \rho^c)$	$(1/2)\cdot q(q^2-1)$	$q^2(q+1)^3(q^2-1)^2$
A_{52}	$h(\rho^a, \rho^b, \rho^c)y_2(0, r)$	$q(q^2-1)$	$q^2(q+1)^3(q^2-1)$
A_{53}	$h(\rho^a, \rho^b, \rho^c)y_2(0, r)y_3(0, s)$	$(1/2)\cdot q(q^2-1)$	$q^2(q+1)^3$
A_{61}	$h(\rho^a, \rho^b, \rho^c, \rho^d)$	$(1/6)\cdot q(q^2-1)(q-2)$	$q(q+1)^4(q^2-1)$
A_{62}	$h(\rho^a, \rho^b, \rho^c, \rho^d)y_3(0, r)$	$(1/6)\cdot q(q^2-1)(q-2)$	$q(q+1)^4$

Table 1. (Continued)

Notation	Class representative	Number of classes	Order of centralizer
A_7	$h(\rho^a, \rho^b, \rho^c, \rho^d, \rho^e)$	$(1/5!)\cdot q(q^2-1)(q-2)(q-3)$	$(q+1)^5$
B_{11}	$h(\rho^a, \sigma^b, \rho^a)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$q^3(q+1)(q^2-1)^2(q^3+1)$
B_{12}	$h(\rho^a, \sigma^b, \rho^a)y_3(0, r)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$q^3(q+1)^2(q^2-1)$
B_{13}	$h(\rho^a, \sigma^b, \rho^a)y_3(r, s)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$q^2(q+1)(q^2-1)$
B_{21}	$h(\rho^a, \sigma^b, \rho^c)$	$(1/2)\cdot q(q+1)(q^2-q-2)$	$q(q+1)^2(q^2-1)^2$
B_{22}	$h(\rho^a, \sigma^b, \rho^c)y_3(0, r)$	$(1/2)\cdot q(q+1)(q^2-q-2)$	$q(q+1)^2(q^2-1)$
B_3	$h(\sigma^d, \rho^a, \rho^b, \rho^c)$	$(1/12)\cdot q(q^2-1)(q^2-q-2)$	$(q+1)^3(q^2-1)$
B_{41}	$h(\sigma^a, \sigma^a, \rho^b)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$q^2(q+1)(q^2-1)(q^4-1)$
B_{42}	$h(\sigma^a, \sigma^a, \rho^b)y_1(r)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$q^2(q+1)(q^2-1)$
B_5	$h(\sigma^a, \sigma^b, \rho^c)$	$(1/8)\cdot(q+1)(q^2-q-2)(q^2-q-4)$	$(q+1)(q^2-1)^2$
C_{11}	$h(\rho^a, \tau^b, \tau^{-bq}, \tau^{bq^2})$	$(1/3)\cdot q(q+1)(q^2-1)$	$q(q+1)(q^2-1)(q^3+1)$
C_{12}	$h(\rho^a, \tau^b, \tau^{-bq}, \tau^{bq^2})y_3(0, r)$	$(1/3)\cdot q(q+1)(q^2-1)$	$q(q+1)(q^3+1)$
C_2	$h(\rho^a, \rho^b, \tau^c, \tau^{-cq}, \tau^{cq^2})$	$(1/6)\cdot q^2(q+1)(q^2-1)$	$(q+1)^2(q^3+1)$
D	$h(\sigma^a, \tau^b, \tau^{-bq}, \tau^{bq^2})$	$(1/6)\cdot q(q^2-1)(q^2-q-2)$	$(q^2-1)(q^3+1)$
E	$h(\omega^a, \omega^{aq^2}, \rho^b)$	$(1/4)\cdot q^2(q+1)(q^2-1)$	$(q+1)(q^4-1)$
F	$h(\theta^a, \theta^{-aq}, \theta^{aq^2}, \theta^{-aq^3}, \theta^{aq^4})$	$(1/5)\cdot q(q^4-1)$	(q^5+1)

Table 2. Characters of $U(5, q^2)$

	$A_{11}(i)$	$A_{12}(i)$	$A_{13}(i)$	$A_{14}(i)$
A_{11}	$q^{10}\varepsilon^{5ai}$	$q^6(q-1)(q^2+1)\varepsilon^{5ai}$	$q^4(q^4-q^3+q^2-q+1)\varepsilon^{5ai}$	$q^3(q^2-q+1)(q^2+1)\varepsilon^{5ai}$
A_{12}		$-q^6\varepsilon^{5ai}$	$-(q^5-q^4)\varepsilon^{5ai}$	$(q^5-q^4+q^3)\varepsilon^{5ai}$
A_{13}			$q^4\varepsilon^{5ai}$	$q^3\varepsilon^{5ai}$
A_{14}				$q^3\varepsilon^{5ai}$
A_{15}				
A_{16}				
A_{17}				
A_{21}	$q^6\varepsilon^{(4a+b)i}$	$-q^3(q-1)(q^2+1)\varepsilon^{(4a+b)i}$	$-(q^5-2q^4+q^3-q^2)\varepsilon^{(4a+b)i}$	$q(q^2-q+1)(q^2+1)\varepsilon^{(4a+b)i}$
A_{22}		$q^3\varepsilon^{(4a+b)i}$	$-(q^3-q^2)\varepsilon^{(4a+b)i}$	$(q^3-q^2+q)\varepsilon^{(4a+b)i}$
A_{23}			$q^2\varepsilon^{(4a+b)i}$	$q\varepsilon^{(4a+b)i}$
A_{24}				$q\varepsilon^{(4a+b)i}$
A_{25}				
A_{31}	$q^4\varepsilon^{(3a+2b)i}$	$-q^2(q-1)^2\varepsilon^{(3a+2b)i}$	$(q^4-q^3+2q^2-q)\varepsilon^{(3a+2b)i}$	$2q(q^2-q+1)\varepsilon^{(3a+2b)i}$
A_{32}		$q^3\varepsilon^{(3a+2b)i}$	$(q^2-q)\varepsilon^{(3a+2b)i}$	$(q^3-q^2+q)\varepsilon^{(3a+2b)i}$
A_{33}		$-q^2\varepsilon^{(3a+2b)i}$	$(q^2-q)\varepsilon^{(3a+2b)i}$	$-(q^2-2q)\varepsilon^{(3a+2b)i}$
A_{34}			$-q\varepsilon^{(3a+2b)i}$	$q\varepsilon^{(3a+2b)i}$
A_{35}				$q\varepsilon^{(3a+2b)i}$
A_{36}				
A_{41}	$-q^3\varepsilon^{(3a+b+c)i}$	$(2q^3-q^2+q)\varepsilon^{(3a+b+c)i}$	$-(q^3-2q^2+2q)\varepsilon^{(3a+b+c)i}$	$(q-1)(q^2-q+1)\varepsilon^{(3a+b+c)i}$
A_{42}		$q\varepsilon^{(3a+b+c)i}$	$-2q\varepsilon^{(3a+b+c)i}$	$(2q-1)\varepsilon^{(3a+b+c)i}$
A_{43}				$-\varepsilon^{(3a+b+c)i}$
A_{51}	$q^2\varepsilon^{(2a+2b+c)i}$	$-2q(q-1)\varepsilon^{(2a+2b+c)i}$	$(2q^2-2q+1)\varepsilon^{(2a+2b+c)i}$	$-(q^2-4q+1)\varepsilon^{(2a+2b+c)i}$
A_{52}		$q\varepsilon^{(2a+2b+c)i}$	$-(q-1)\varepsilon^{(2a+2b+c)i}$	$(2q-1)\varepsilon^{(2a+2b+c)i}$
A_{53}			$\varepsilon^{(2a+2b+c)i}$	$-\varepsilon^{(2a+2b+c)i}$
A_{61}	$-q\varepsilon^{(2a+b+c+d)i}$	$(3q-1)\varepsilon^{(2a+b+c+d)i}$	$-(3q-2)\varepsilon^{(2a+b+c+d)i}$	$3(q-1)\varepsilon^{(2a+b+c+d)i}$
A_{62}		$-\varepsilon^{(2a+b+c+d)i}$	$2\varepsilon^{(2a+b+c+d)i}$	$-3\varepsilon^{(2a+b+c+d)i}$

Table 2. (Continued)

	$A_{11}(i)$	$A_{12}(i)$	$A_{13}(i)$	$A_{14}(i)$
A_7	$\varepsilon^{(a+b+c+d+e)i}$	$-4\varepsilon^{(a+b+c+d+e)i}$	$5\varepsilon^{(a+b+c+d+e)i}$	$-6\varepsilon^{(a+b+c+d+e)i}$
B_{11}	$q^3\varepsilon^{(3a-b)i}$	$(q^2-q)\varepsilon^{(3a-b)i}$	$q^3\varepsilon^{(3a-b)i}$	$(q^3+1)\varepsilon^{(3a-b)i}$
B_{12}		$-q\varepsilon^{(3a-b)i}$		$\varepsilon^{(3a-b)i}$
B_{13}				$\varepsilon^{(3a-b)i}$
B_{21}	$q\varepsilon^{(2a-b+c)i}$	$-(q-1)\varepsilon^{(2a-b+c)i}$	$q\varepsilon^{(2a-b+c)i}$	$(q+1)\varepsilon^{(2a-b+c)i}$
B_{22}		$\varepsilon^{(2a-b+c)i}$		$\varepsilon^{(2a-b+c)i}$
B_3	$-\varepsilon^{(a+b+c-d)i}$	$2\varepsilon^{(a+b+c-d)i}$	$-\varepsilon^{(a+b+c-d)i}$	
B_{41}	$q^2\varepsilon^{(b-2a)i}$		$\varepsilon^{(b-2a)i}$	$(q^2+1)\varepsilon^{(b-2a)i}$
B_{42}			$\varepsilon^{(b-2a)i}$	$\varepsilon^{(b-2a)i}$
B_5	$\varepsilon^{(c-a-b)i}$		$\varepsilon^{(c-a-b)i}$	$2\varepsilon^{(c-a-b)i}$
C_{11}	$-q\varepsilon^{(2a+b)i}$	$-\varepsilon^{(2a+b)i}$	$-\varepsilon^{(2a+b)i}$	
C_{12}		$-\varepsilon^{(2a+b)i}$	$-\varepsilon^{(2a+b)i}$	
C_2	$\varepsilon^{(a+b+c)i}$	$-\varepsilon^{(a+b+c)i}$	$-\varepsilon^{(a+b+c)i}$	
D	$-\varepsilon^{(b-a)i}$	$-\varepsilon^{(b-a)i}$	$-\varepsilon^{(b-a)i}$	
E	$-\varepsilon^{(b-a)i}$		$\varepsilon^{(b-a)i}$	
F	ε^{ai}	ε^{ai}		$-\varepsilon^{ai}$
Number of characters	$q+1$	$q+1$	$q+1$	$q+1$

	$A_{15}(i)$	$A_{16}(i)$	$A_{17}(i)$	$A_{21}(i, j)$
A_{11}	$q^2(q^4-q^3+q^2-q+1)\varepsilon^{5ai}$	$q(q-1)(q^2+1)\varepsilon^{5ai}$	ε^{5ai}	$q^6(q^4-q^3+q^2-q+1)\varepsilon^{a(4i+j)}$
A_{12}	$(q^4-q^3+q^2)\varepsilon^{5ai}$	$-(q^3-q^2+q)\varepsilon^{5ai}$	ε^{5ai}	$q^6\varepsilon^{a(4i+j)}$
A_{13}	$-(q^3-q^2)\varepsilon^{5ai}$	$(q^2-q)\varepsilon^{5ai}$	ε^{5ai}	
A_{14}	$q^2\varepsilon^{5ai}$	$(q^2-q)\varepsilon^{5ai}$	ε^{5ai}	
A_{15}	$q^2\varepsilon^{5ai}$	$-q\varepsilon^{5ai}$	ε^{5ai}	

Table 2. (Continued)

	$A_{15}(i)$	$A_{16}(i)$	$A_{17}(i)$	$A_{21}(i, j)$
A_{16}		$-q\varepsilon^{5ai}$	ε^{5ai}	
A_{17}			ε^{5ai}	
A_{21}	$(q^4-q^3+2q^2-q)\varepsilon^{(4a+b)i}$	$-(q-1)(q^2+1)\varepsilon^{(4a+b)i}$	$\varepsilon^{(4a+b)i}$	$q^6\varepsilon^{4ai+bj}+q^3(q-1)(q^2+1)\varepsilon^{(3a+b)i+aj}$
A_{22}	$(2q^2-q)\varepsilon^{(4a+b)i}$	$(q^2-q+1)\varepsilon^{(4a+b)i}$	$\varepsilon^{(4a+b)i}$	$-q^3\varepsilon^{(3a+b)i+aj}$
A_{23}	$(q^2-q)\varepsilon^{(4a+b)i}$	$-(q-1)\varepsilon^{(4a+b)i}$	$\varepsilon^{(4a+b)i}$	
A_{24}	$-q\varepsilon^{(4a+b)i}$	$-(q-1)\varepsilon^{(4a+b)i}$	$\varepsilon^{(4a+b)i}$	
A_{25}		$\varepsilon^{(4a+b)i}$	$\varepsilon^{(4a+b)i}$	
A_{31}	$-(q^3-2q^2+q-1)\varepsilon^{(3a+2b)i}$	$(q-1)^2\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	$q^3(q-1)\varepsilon^{(3a+b)i+bj}+q^2(q^2-q+1)\varepsilon^{2(a+b)i+aj}$
A_{32}	$(q^2-q+1)\varepsilon^{(3a+2b)i}$	$(q^2-q+1)\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	$-q^3\varepsilon^{(3a+b)i+bj}$
A_{33}	$(q^2-q+1)\varepsilon^{(3a+2b)i}$	$-(2q-1)\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	$q^2\varepsilon^{2(a+b)i+aj}$
A_{34}	$-(q-1)\varepsilon^{(3a+2b)i}$	$-(q-1)\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	
A_{35}	$\varepsilon^{(3a+2b)i}$	$-(q-1)\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	
A_{36}	$\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	$\varepsilon^{(3a+2b)i}$	
A_{41}	$(2q^2-2q+1)\varepsilon^{(3a+b+c)i}$	$(q^2-q+2)\varepsilon^{(3a+b+c)i}$	$\varepsilon^{(3a+b+c)i}$	$-q^3\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-(q^3-q^2+q)\varepsilon^{(2a+b+c)i+aj}$
A_{42}	$-(2q-1)\varepsilon^{(3a+b+c)i}$	$-(q-2)\varepsilon^{(3a+b+c)i}$	$\varepsilon^{(3a+b+c)i}$	$-q\varepsilon^{(2a+b+c)i+aj}$
A_{43}	$\varepsilon^{(3a+b+c)i}$	$2\varepsilon^{(3a+b+c)i}$	$\varepsilon^{(3a+b+c)i}$	
A_{51}	$(q^2-2q+2)\varepsilon^{(2a+2b+c)i}$	$-2(q-1)\varepsilon^{(2a+2b+c)i}$	$\varepsilon^{(2a+2b+c)i}$	$q^2\varepsilon^{2(a+b)i+cj}+(q^2-q)\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{52}	$-(q-2)\varepsilon^{(2a+2b+c)i}$	$-(q-2)\varepsilon^{(2a+2b+c)i}$	$\varepsilon^{(2a+2b+c)i}$	$-q\varepsilon^{(2a+b+c)i+bj}$
A_{53}	$2\varepsilon^{(2a+2b+c)i}$	$2\varepsilon^{(2a+2b+c)i}$	$\varepsilon^{(2a+2b+c)i}$	
A_{61}	$-(2q-3)\varepsilon^{(2a+b+c+d)i}$	$-(q-3)\varepsilon^{(2a+b+c+d)i}$	$\varepsilon^{(2a+b+c+d)i}$	$-q\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}-(q-1)\varepsilon^{(a+b+c+d)i+aj}$
A_{62}	$3\varepsilon^{(2a+b+c+d)i}$	$3\varepsilon^{(2a+b+c+d)i}$	$\varepsilon^{(2a+b+c+d)i}$	$\varepsilon^{(a+b+c+d)i+aj}$
A_7	$5\varepsilon^{(a+b+c+d+e)i}$	$4\varepsilon^{(a+b+c+d+e)i}$	$\varepsilon^{(a+b+c+d+e)i}$	$\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c+d)i+ej}$
B_{11}	$\varepsilon^{(3a-b)i}$	$(q^2-q)\varepsilon^{(3a-b)i}$	$\varepsilon^{(3a-b)i}$	$(q^3-q^2-q)\varepsilon^{(2a-b)i+aj}$
B_{12}	$\varepsilon^{(3a-b)i}$	$-q\varepsilon^{(3a-b)i}$	$\varepsilon^{(3a-b)i}$	$q\varepsilon^{(2a-b)i+aj}$
B_{13}	$\varepsilon^{(3a-b)i}$		$\varepsilon^{(3a-b)i}$	

Table 2. (Continued)

	$A_{15}(i)$	$A_{16}(i)$	$A_{17}(i)$	$A_{21}(i, j)$
B_{21}	$\varepsilon^{(2a-b+c)i}$	$-(q-1)\varepsilon^{(2a-b+c)i}$	$\varepsilon^{(2a-b+c)i}$	$q\varepsilon^{(2a-b)i+cj}+(q-1)\varepsilon^{(a-b+c)i+aj}$
B_{22}	$\varepsilon^{(2a-b+c)i}$	$\varepsilon^{(2a-b+c)i}$	$\varepsilon^{(2a-b+c)i}$	$-\varepsilon^{(a-b+c)i+aj}$
B_3	$\varepsilon^{(a+b+c-d)i}$	$2\varepsilon^{(a+b+c-d)i}$	$\varepsilon^{(a+b+c-d)i}$	$-\sum_{(a,b,c)}\varepsilon^{(a+b-d)i+cj}$
B_{41}	$q^2\varepsilon^{(b-2a)i}$		$\varepsilon^{(b-2a)i}$	$q^2\varepsilon^{-2ai+bj}$
B_{42}			$\varepsilon^{(b-2a)i}$	
B_5	$\varepsilon^{(c-a-b)i}$		$\varepsilon^{(c-a-b)i}$	$\varepsilon^{-(a+b)i+cj}$
C_{11}	$q\varepsilon^{(2a+b)i}$	$-q\varepsilon^{(2a+b)i}$	$\varepsilon^{(2a+b)i}$	$-(q-1)\varepsilon^{(a+b)i+aj}$
C_{12}			$\varepsilon^{(2a+b)i}$	$\varepsilon^{(a+b)i+aj}$
C_2	$-\varepsilon^{(a+b+c)i}$	$\varepsilon^{(a+b+c)i}$	$\varepsilon^{(a+b+c)i}$	$\sum_{(a,b)}\varepsilon^{(a+c)i+bj}$
D	$\varepsilon^{(b-a)i}$	$-\varepsilon^{(b-a)i}$	$\varepsilon^{(b-a)i}$	
E	$-\varepsilon^{(b-a)i}$		$\varepsilon^{(b-a)i}$	$-\varepsilon^{-ai+bj}$
F		$-\varepsilon^{ai}$	ε^{ai}	
Number of characters	$q+1$	$q+1$	$q+1$	$q(q+1)$

	$A_{22}(i, j)$	$A_{23}(i, j)$
A_{11}	$q^3(q^2-q+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(4i+j)}$	$q^2(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(4i+j)}$
A_{12}	$-q^3(q-1)(q^2-q+1)\varepsilon^{a(4i+j)}$	$-(q^5-2q^4+q^3-q^2)\varepsilon^{a(4i+j)}$
A_{13}	$-(q^4-q^3)\varepsilon^{a(4i+j)}$	$(q^4-q^3+q^2)\varepsilon^{a(4i+j)}$
A_{14}	$q^3\varepsilon^{a(4i+j)}$	$q^2\varepsilon^{a(4i+j)}$
A_{15}		$q^2\varepsilon^{a(4i+j)}$
A_{16}		
A_{17}		
A_{21}	$q^3(q^2-q+1)\varepsilon^{4ai+bj}-q(q-1)(q^2-q+1)(q^2+1)\varepsilon^{(3a+b)i+aj}$	$q^2(q^2+1)\varepsilon^{4ai+bj}-q(q-1)^2(q^2+1)\varepsilon^{(3a+b)i+aj}$

Table 2. (Continued)

	$A_{22}(i, j)$	$A_{23}(i, j)$
A_{22}	$q^3\varepsilon^{4ai+bj}+(2q^3-2q^2+q)\varepsilon^{(3a+b)i+aj}$	$q^2\varepsilon^{4ai+bj}-q(q-1)^2\varepsilon^{(3a+b)i+aj}$
A_{23}	$-(q^2-q)\varepsilon^{(3a+b)i+aj}$	$q^2\varepsilon^{4ai+bj}+(q^2-q)\varepsilon^{(3a+b)i+aj}$
A_{24}	$q\varepsilon^{(3a+b)i+aj}$	$-q\varepsilon^{(3a+b)i+aj}$
A_{25}		
A_{31}	$-(q^3-q^2+q)\{(q-1)\varepsilon^{(3a+b)i+bj}+(q-2)\varepsilon^{2(a+b)i+aj}\}$	$-q(q-1)^2\varepsilon^{(3a+b)i+bj}+(q^2-q+1)(q^2+1)\varepsilon^{2(a+b)i+aj}$
A_{32}	$(q^3-q^2+q)\{\varepsilon^{(3a+b)i+bj}+\varepsilon^{2(a+b)i+aj}\}$	$(q^2-q)\varepsilon^{(3a+b)i+bj}+(q^2-q+1)\varepsilon^{2(a+b)i+aj}$
A_{33}	$-(q^2-q)\{\varepsilon^{(3a+b)i+bj}+2^{2(a+b)i+aj}\}$	$(q^2-q)\varepsilon^{(3a+b)i+bj}+(q^2-q+1)\varepsilon^{2(a+b)i+aj}$
A_{34}	$q\{\varepsilon^{(3a+b)i+bj}+\varepsilon^{2(a+b)i+aj}\}$	$-q\varepsilon^{(3a+b)i+bj}-(q-1)\varepsilon^{2(a+b)i+aj}$
A_{35}	$q\varepsilon^{2(a+b)i+aj}$	$\varepsilon^{2(a+b)i+aj}$
A_{36}		$\varepsilon^{2(a+b)i+aj}$
A_{41}	$(q^2-q+1)\{q\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-\varepsilon^{(2a+b+c)i+aj}\}$	$(q^2-q)\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-(q-1)(q^2-q+1)\varepsilon^{(2a+b+c)i+aj}$
A_{42}	$q\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}+(3q-1)\varepsilon^{(2a+b+c)i+aj}$	$-q\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-(2q-1)\varepsilon^{(2a+b+c)+aj}$
A_{43}	$-\varepsilon^{(2a+b+c)i+aj}$	$\varepsilon^{(2a+b+c)i+aj}$
A_{51}	$-(q^2-2q)\varepsilon^{2(a+b)i+cj}-(q-1)(2q-1)\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$	$(q^2+1)\varepsilon^{2(a+b)i+cj}+(q-1)^2\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{52}	$q\varepsilon^{2(a+b)i+cj}+(2q-1)\varepsilon^{(2a+b+c)i+bj}+(q-1)\varepsilon^{(a+2b+c)i+aj}$	$\varepsilon^{2(a+b)i+cj}-(q-1)\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{53}	$-\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$	$\varepsilon^{2(a+b)i+cj}+\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{61}	$(2q-1)\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}+3(q-1)\varepsilon^{(a+b+c+d)i+aj}$	$-(q-1)\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}-2(q-1)\varepsilon^{(a+b+c+d)i+aj}$
A_{62}	$-\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}-3\varepsilon^{(a+b+c+d)i+aj}$	$\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}+2\varepsilon^{(a+b+c+d)i+aj}$
A_7	$-3\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c+d)i+ej}$	$2\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c+d)i+ej}$
B_{11}	$(q^2-q+1)\varepsilon^{(2a-b)i+aj}$	$(q^3+1)\varepsilon^{(2a-b)i+aj}$
B_{12}	$-(q-1)\varepsilon^{(2a-b)i+aj}$	$\varepsilon^{(2a-b)i+aj}$
B_{13}	$\varepsilon^{(2a-b)i+aj}$	$\varepsilon^{(2a-b)i+aj}$
B_{21}	$\varepsilon^{(2a-b)i+cj}-(q-1)\varepsilon^{(a-b+c)i+aj}$	$(q+1)\varepsilon^{(2a-b)i+cj}$

Table 2. (Continued)

	$A_{22}(i, j)$	$A_{23}(i, j)$
B_{22}	$\varepsilon^{(2a-b)i+cj}+\varepsilon^{(a-b+c)i+aj}$	$\varepsilon^{(2a-b)i+cj}$
B_3	$\sum_{(a,b,c)} \varepsilon^{(a+b-d)i+cj}$	
B_{41}	$q^2\varepsilon^{-2ai+bj}$	$(q^2+1)\varepsilon^{-2ai+bj}$
B_{42}		$\varepsilon^{-2ai+bj}$
B_5	$\varepsilon^{-(a+b)i+cj}$	$2\varepsilon^{-(a+b)i+cj}$
C_{11}		$(q-1)\varepsilon^{(a+b)i+aj}$
C_{12}		$-\varepsilon^{(a+b)i+aj}$
C_2		$-\sum_{(a,b)} \varepsilon^{(a+c)i+bj}$
D		
E	$-\varepsilon^{-ai+bj}$	
F		
Number of characters	$q(q+1)$	$q(q+1)$

	$A_{24}(i, j)$	$A_{25}(i, j)$
A_{11}	$q(q^2-q+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(4i+j)}$	$(q^4-q^3+q^2-q+1)\varepsilon^{a(4i+j)}$
A_{12}	$q(q^2-q+1)^2\varepsilon^{a(4i+j)}$	$-(q-1)(q^2+1)\varepsilon^{a(4i+j)}$
A_{13}	$(2q^3-2q^2+q)\varepsilon^{a(4i+j)}$	$(q^2-q+1)\varepsilon^{a(4i+j)}$
A_{14}	$q(q-1)^2\varepsilon^{a(4i+j)}$	$(q^2-q+1)\varepsilon^{a(4i+j)}$
A_{15}	$-(q^2-q)\varepsilon^{a(4i+j)}$	$-(q-1)\varepsilon^{a(4i+j)}$
A_{16}	$q\varepsilon^{a(4i+j)}$	$-(q-1)\varepsilon^{a(4i+j)}$
A_{17}		$\varepsilon^{a(4i+j)}$
A_{21}	$(q^3-q^2+q)\varepsilon^{4ai+bj}+(q-1)(q^2-q+1)(q^2+1)\varepsilon^{(3a+b)i+aj}$	$\varepsilon^{4ai+bj}-(q-1)(q^2+1)\varepsilon^{(3a+b)i+aj}$
A_{22}	$-(q^2-q)\varepsilon^{4ai+bj}+(q^3-3q^2+2q-1)\varepsilon^{(3a+b)i+aj}$	$\varepsilon^{4ai+bj}+(q^2-q+1)\varepsilon^{(3a+b)i+aj}$
A_{23}	$q\varepsilon^{4ai+bj}-(q-1)^2\varepsilon^{(3a+b)i+aj}$	$\varepsilon^{4ai+bj}-(q-1)\varepsilon^{(3a+b)i+aj}$

Table 2. (Continued)

	$A_{24}(i, j)$	$A_{25}(i, j)$
A_{24}	$q\varepsilon^{4ai+bj}+(2q-1)\varepsilon^{(3a+b)i+aj}$	$\varepsilon^{4ai+bj}-(q-1)\varepsilon^{(3a+b)i+aj}$
A_{25}	$-\varepsilon^{(3a+b)i+aj}$	$\varepsilon^{4ai+bj}+\varepsilon^{(3a+b)i+aj}$
A_{31}	$(q-1)(q^2-q+1)\varepsilon^{(3a+b)i+bj}+(2q-1)(q^2-q+1)\varepsilon^{2(a+b)i+aj}$	$-(q-1)\varepsilon^{(3a+b)i+bj}+(q^2-q+1)\varepsilon^{2(a+b)i+aj}$
A_{32}	$-(q^2-q+1)\varepsilon^{(3a+b)i+bj}+(q-1)(q^2-q+1)\varepsilon^{2(a+b)i+aj}$	$\varepsilon^{(3a+b)i+bj}+(q^2-q+1)\varepsilon^{2(a+b)i+aj}$
A_{33}	$-(q-1)^2\varepsilon^{(3a+b)i+bj}-(q^2-3q+1)\varepsilon^{2(a+b)i+aj}$	$-(q-1)\varepsilon^{(3a+b)i+bj}-(q-1)\varepsilon^{2(a+b)i+aj}$
A_{34}	$(q-1)\varepsilon^{(3a+b)i+bj}+(2q-1)\varepsilon^{2(a+b)i+aj}$	$\varepsilon^{(3a+b)i+bj}-(q-1)\varepsilon^{2(a+b)i+aj}$
A_{35}	$(q-1)\varepsilon^{(3a+b)i+bj}+(q-1)\varepsilon^{2(a+b)i+aj}$	$-(q-1)\varepsilon^{(3a+b)i+bj}+\varepsilon^{2(a+b)i+aj}$
A_{36}	$-\varepsilon^{(3a+b)i+bj}-\varepsilon^{2(a+b)i+aj}$	$\varepsilon^{(3a+b)i+bj}+\varepsilon^{2(a+b)i+aj}$
A_{41}	$-(q^2-q+1)\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}+(q-2)(q^2-q+1)\varepsilon^{(2a+b+c)i+aj}$	$\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}+(q^2-q+1)\varepsilon^{(2a+b+c)i+aj}$
A_{42}	$(q-1)\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}+(3q-2)\varepsilon^{(2a+b+c)i+aj}$	$\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-(q-1)\varepsilon^{(2a+b+c)i+aj}$
A_{43}	$-\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}-2\varepsilon^{(2a+b+c)i+aj}$	$\sum_{(b,c)}\varepsilon^{(3a+b)i+cj}+\varepsilon^{(2a+b+c)i+aj}$
A_{51}	$(2q-1)\varepsilon^{2(a+b)i+cj}-(q-1)(q-2)\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$	$\varepsilon^{2(a+b)i+cj}-(q-1)\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{52}	$(q-1)\varepsilon^{2(a+b)i+cj}+(q-2)\varepsilon^{(2a+b+c)i+bj}+2(q-1)\varepsilon^{(a+2b+c)i+aj}$	$\varepsilon^{2(a+b)i+cj}+\varepsilon^{(2a+b+c)i+bj}-(q-1)\varepsilon^{(a+2b+c)i+aj}$
A_{53}	$-\varepsilon^{2(a+b)i+cj}-2\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$	$\varepsilon^{2(a+b)i+cj}+\sum_{(a,b)}\varepsilon^{(2a+b+c)i+bj}$
A_{61}	$(q-2)\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}+3(q-1)\varepsilon^{(a+b+c+d)i+aj}$	$\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}-(q-1)\varepsilon^{(a+b+c+d)i+aj}$
A_{62}	$-2\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}-3\varepsilon^{(a+b+c+d)i+aj}$	$\sum_{(b,c,d)}\varepsilon^{(2a+b+c)i+dj}+\varepsilon^{(a+b+c+d)i+aj}$
A_{7}	$-3\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c+d)i+ej}$	$\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c+d)i+ej}$
B_{11}	$(q^3-q^2+q)\varepsilon^{(2a-b)i+aj}$	$(q^2-q+1)\varepsilon^{(2a-b)i+aj}$
B_{12}	$q\varepsilon^{(2a-b)i+aj}$	$-(q-1)\varepsilon^{(2a-b)i+aj}$
B_{13}		$\varepsilon^{(2a-b)i+aj}$
B_{21}	$q\varepsilon^{(2a-b)i+cj}+(q-1)\varepsilon^{(a-b+c)i+aj}$	$\varepsilon^{(2a-b)i+cj}-(q-1)\varepsilon^{(a-b+c)i+aj}$
B_{22}	$-\varepsilon^{(a-b+c)i+aj}$	$\varepsilon^{(2a-b)i+cj}+\varepsilon^{(a-b+c)i+aj}$
B_{3}	$-\sum_{(a,b,c)}\varepsilon^{(a+b-d)i+cj}$	$\sum_{(a,b,c)}\varepsilon^{(a+b-d)i+cj}$

Table 2. (Continued)

	$A_{24}(i, j)$	$A_{25}(i, j)$
B_{41}	$\varepsilon^{-2ai+bj}$	$\varepsilon^{-2ai+bj}$
B_{42}	$\varepsilon^{-2ai+bj}$	$\varepsilon^{-2ai+bj}$
B_5	$\varepsilon^{-(a+b)i+cj}$	$\varepsilon^{-(a+b)i+cj}$
C_{11}		$-(q-1)\varepsilon^{(a+b)i+aj}$
C_{12}		$\varepsilon^{(a+b)i+aj}$
C_2		$\sum_{(a,b)} \varepsilon^{(a+c)i+bj}$
D		
E	ε^{-ai+bj}	ε^{-ai+bj}
F		
Number of characters	$q(q+1)$	$q(q+1)$

	$A_{31}(i, j)$	$A_{32}(i, j)$
A_{11}	$q^4(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$	$q^3(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{12}	$(q^6-q^5+q^4)\varepsilon^{a(3i+2j)}$	$-q^3(q-1)(q^2+1)\varepsilon^{a(3i+2j)}$
A_{13}	$q^4\varepsilon^{a(3i+2j)}$	$q^3\varepsilon^{a(3i+2j)}$
A_{14}		$q^3\varepsilon^{a(3i+2j)}$
A_{15}		
A_{16}		
A_{17}		
A_{21}	$q^2(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}+q^3(q-1)(q^2+1)\varepsilon^{3ai+(a+b)j}$	$q(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}-q^3(q-1)(q^2+1)\varepsilon^{3ai+(a+b)j}$
A_{22}	$-(q^3-q^2)\varepsilon^{(2a+b)i+2aj}-q^3\varepsilon^{3ai+(a+b)j}$	$(q^3-q^2+q)\varepsilon^{(2a+b)i+2aj}+q^3\varepsilon^{3ai+(a+b)j}$
A_{23}	$q^2\varepsilon^{(2a+b)i+2aj}$	$q\varepsilon^{(2a+b)i+2aj}$
A_{24}		$q\varepsilon^{(2a+b)i+2aj}$
A_{25}		

Table 2. (Continued)

	$A_{31}(i, j)$	$A_{32}(i, j)$
A_{31}	$q^4\varepsilon^{3ai+2bj}+(q^4-q^3+q^2)\varepsilon^{(a+2b)i+2aj}$ $+q(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$	$q^3\varepsilon^{3ai+2bj}+(q^3-q^2+q)\varepsilon^{(a+2b)i+2aj}$ $-q(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{32}	$-(q^3-q^2+q)\varepsilon^{(2a+b)i+(a+b)j}$	$q^3\varepsilon^{3ai+2bj}+(q^3-q^2+q)\varepsilon^{(2a+b)i+(a+b)j}$
A_{33}	$q^2\varepsilon^{(a+2b)i+2aj}+(q^2-q)\varepsilon^{(2a+b)i+(a+b)j}$	$-(q^2-q)\varepsilon^{(a+2b)i+2aj}-(q^2-q)\varepsilon^{(2a+b)i+(a+b)j}$
A_{34}	$-q\varepsilon^{(2a+b)i+(a+b)j}$	$q\varepsilon^{(2a+b)i+(a+b)j}$
A_{35}		$q\varepsilon^{(a+2b)i+2aj}$
A_{36}		
A_{41}	$-q^3\varepsilon^{3ai+(b+c)j}-(q^3-q^2+q)\{\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+\varepsilon^{(a+b+c)i+2aj}\}$	$q^3\varepsilon^{3ai+(b+c)j}+(q^2-q+1)\{q\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}-\varepsilon^{(a+b+c)i+2aj}\}$
A_{42}	$-q\{\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+\varepsilon^{(a+b+c)i+2aj}\}$	$q\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+(q-1)\varepsilon^{(a+b+c)i+2aj}$
A_{43}		$-\varepsilon^{(a+b+c)i+2aj}$
A_{51}	$\sum_{(a,b)}\{q^2\varepsilon^{2ai+2bj+ci}+(q^2-q)^{(2a+b)i+(b+c)j}\}+(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\{q\varepsilon^{2ai+2bj+ci}-(q^2-q)\varepsilon^{(2a+b)i+(b+c)j}\}-(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$
A_{52}	$-q\varepsilon^{(2a+b)i+(b+c)j}-(q-1)\varepsilon^{(a+b)(i+j)+ci}$	$q\varepsilon^{2ai+2bj+ci}+q\varepsilon^{(2a+b)i+(b+c)j}+(q-1)\varepsilon^{(a+b)(i+j)+ci}$
A_{53}	$\varepsilon^{(a+b)(i+j)+ci}$	$-\varepsilon^{(a+b)(i+j)+ci}$
A_{61}	$\sum_{(b,c,d)}\{-q\varepsilon^{(2a+b)i+(c+d)j}-(q-1)\varepsilon^{(a+b+c)i+(a+d)j}\}-q\varepsilon^{2aj+(b+c+d)i}$	$\sum_{(b,c,d)}\{q\varepsilon^{(2a+b)i+(c+d)j}+(q-1)\varepsilon^{(a+b+c)i+(a+d)j}\}-\varepsilon^{2aj+(b+c+d)i}$
A_{62}	$\sum_{(b,c,d)}\varepsilon^{(a+b+c)i+(a+d)j}$	$-\sum_{(b,c,d)}\varepsilon^{(a+b+c)i+(a+d)j}-\varepsilon^{2aj+(b+c+d)i}$
A_7	$(1/12)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+(d+e)j}$	$-(1/12)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+(d+e)j}$
B_{11}	$q^3\varepsilon^{3ai-bj}+(q^3-q^2+q)\varepsilon^{(a-b)i+2aj}$	$q^3\varepsilon^{3ai-bj}+(q^2-q+1)\varepsilon^{(a-b)i+2aj}$
B_{12}	$q\varepsilon^{(a-b)i+2aj}$	$-(q-1)\varepsilon^{(a-b)i+2aj}$
B_{13}		$\varepsilon^{(a-b)i+2aj}$
B_{21}	$q\varepsilon^{(2a+c)i-bj}+q\varepsilon^{2aj+(c-b)i}+(q-1)\varepsilon^{(a-b)i+(a+c)j}$	$q\varepsilon^{(2a+c)i-bj}+\varepsilon^{2aj+(c-b)i}-(q-1)\varepsilon^{(a-b)i+(a+c)j}$
B_{22}	$-\varepsilon^{(a-b)i+(a+c)j}$	$\varepsilon^{2aj+(c-b)i}+\varepsilon^{(a-b)i+(a+c)j}$
B_3	$-\varepsilon^{(a+b+c)i-dj}\sum_{(a,b,c)}\varepsilon^{(a+b)j+(c-d)i}$	$-\varepsilon^{(a+b+c)i-dj}+\sum_{(a,b,c)}\varepsilon^{(a+b)j+(c-d)i}$
B_{41}	$(q^2+1)\varepsilon^{(b-a)i-aj}$	$(q^2+1)\varepsilon^{(b-a)i-aj}$

Table 6. (Continued)

	$A_{31}(i,j)$	$A_{32}(i,j)$
B_{42}	$\varepsilon^{(b-a)i-aj}$	$\varepsilon^{(b-a)i-aj}$
B_5	$\sum_{(a,b)} \varepsilon^{(c-a)i-bj}$	$\sum_{(a,b)} \varepsilon^{(c-a)i-bj}$
C_{11}	$-q\varepsilon^{2aj+bi}$	$-\varepsilon^{2aj+bi}$
C_{12}		$-\varepsilon^{2aj+bi}$
C_2	$\varepsilon^{(a+b)j+ci}$	$-\varepsilon^{(a+b)j+ci}$
D	$-\varepsilon^{-aj+bi}$	$-\varepsilon^{-aj+bi}$
E		
F		
Number of characters	$q(q+1)$	$q(q+1)$

	$A_{33}(i,j)$	$A_{34}(i,j)$
A_{11}	$q^2(q-1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$	$q(q-1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{12}	$-q^2(q^2-q+1)^2\varepsilon^{a(3i+2j)}$	$-(2q^5-3q^4+3q^3-2q^2+q)\varepsilon^{a(3i+2j)}$
A_{13}	$-q^2(q-1)^2\varepsilon^{a(3i+2j)}$	$q(q-1)(q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{14}	$(q^3-q^2)\varepsilon^{a(3i+2j)}$	$-q(q-1)^2\varepsilon^{a(3i+2j)}$
A_{15}	$-q^2\varepsilon^{a(3i+2j)}$	$(q^2-q)\varepsilon^{a(3i+2j)}$
A_{16}		$-q\varepsilon^{a(3i+2j)}$
A_{17}		
A_{21}	$-q(q-1)(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}$ $+q(q-1)^2(q^2+1)\varepsilon^{3ai+(a+b)j}$	$-(q-1)(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}-q(q-1)^2(q^2+1)\varepsilon^{3ai+(a+b)j}$
A_{22}	$(2q^3-2q^2+q)\varepsilon^{(2a+b)i+2aj}+q(q-1)^2\varepsilon^{3ai+(a+b)j}$	$-(q^3-3q^2+2q-1)\varepsilon^{(2a+b)i+2aj}-q(q-1)^2\varepsilon^{3ai+(a+b)j}$
A_{23}	$-(q^2-q)\varepsilon^{(2a+b)i+2aj}-(q^2-q)\varepsilon^{3ai+(a+b)j}$	$(q-1)^2\varepsilon^{(2a+b)i+2aj}+(q^2-q)\varepsilon^{3ai+(a+b)j}$
A_{24}	$q\varepsilon^{(2a+b)i+2aj}+q\varepsilon^{3ai+(a+b)j}$	$-(2q-1)\varepsilon^{(2a+b)i+2aj}-q\varepsilon^{3ai+(a+b)j}$
A_{25}		$\varepsilon^{(2a+b)i+2aj}$

Table 2. (Continued)

	$A_{33}(i,j)$	$A_{34}(i,j)$
A_{31}	$(q^3-q^2)\varepsilon^{3ai+2bj}-q(q-1)(q^2-q+1)\varepsilon^{(a+2b)i+2aj}$ $-(q-1)^2(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$	$(q^2-q)\varepsilon^{3ai+2bj}-(q-1)(q^2-q+1)\varepsilon^{(a+2b)i+2aj}$ $+(q-1)^2(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{32}	$(q^3-q^2+q)\varepsilon^{(a+2b)i+2aj}+(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$	$(q^2-q)\varepsilon^{3ai+2bj}+(q^2-q+1)\varepsilon^{(a+2b)i+2aj}$ $-(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{33}	$-q^2\varepsilon^{3ai+2bj}-(q^2-q)\varepsilon^{(a+2b)i+2aj}-(q-1)(2q-1)\varepsilon^{(2a+b)i+(a+b)j}$	$-q\varepsilon^{3ai+2bj}+(q-1)^2\varepsilon^{(a+2b)i+2aj}+(q-1)(2q-1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{34}	$q\varepsilon^{(a+2b)i+2aj}+(2q-1)\varepsilon^{(2a+b)i+(a+b)j}$	$-q\varepsilon^{3ai+2bj}-(q-1)\varepsilon^{(a+2b)i+2aj}-(2q-1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{35}	$(q-1)\varepsilon^{(2a+b)i+(a+b)j}$	$-(q-1)\varepsilon^{(a+2b)i+2aj}-(q-1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{36}	$-\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{(a+2b)i+2aj}+\varepsilon^{(2a+b)i+(a+b)j}$
A_{41}	$-(q^2-q)\varepsilon^{3ai+(b+c)j}$ $+(q^2-q+1)\{(q-1)\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+2q\varepsilon^{(a+b+c)i+2aj}\}$	$(q^2-q)\varepsilon^{3ai+(b+c)j}$ $-(q^2-q+1)\{(q-1)\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}-2\varepsilon^{(a+b+c)i+2aj}\}$
A_{42}	$q\varepsilon^{3ai+(b+c)j}+(2q-1)\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+2q\varepsilon^{(a+b+c)i+2aj}$	$-q\varepsilon^{3ai+(b+c)j}-(2q-1)\sum_{(b,a)}\varepsilon^{(2a+b)i+(a+c)j}-2(q-1)\varepsilon^{(a+b+c)i+2aj}$
A_{43}	$-\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}$	$\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+2\varepsilon^{(a+b+c)i+2aj}$
A_{51}	$\sum_{(a,b)}\{-(q^2-q)\varepsilon^{2ai+2bj+ci}-(q-1)^2\varepsilon^{(2a+b)i+(b+c)j}\}$ $-2(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\{-(q-1)\varepsilon^{2ai+2bj+ci}+(q-1)^2\varepsilon^{(2a+b)i+(b+c)j}\}$ $+2(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$
A_{52}	$q\varepsilon^{2aj+(2b+c)i}+(q-1)\sum_{(a,b)}\varepsilon^{(2a+b)i+(b+c)j}+2(q-1)\varepsilon^{(a+b)(i+j)+ci}$	$-(q-1)\varepsilon^{2ai+2bj+ci}+\varepsilon^{2aj+(2b+c)i}-(q-1)\sum_{(a,b)}\varepsilon^{(2a+b)i+(b+c)j}$ $-2(q-1)\varepsilon^{(a+b)(i+j)+ci}$
A_{53}	$-\sum_{(a,b)}\varepsilon^{(2a+b)i+(b+c)j}-2\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\{\varepsilon^{2ai+2bj+ci}+\varepsilon^{(2a+b)i+(b+c)j}\}+2\varepsilon^{(a+b)(i+j)+ci}$
A_{61}	$(q-1)\sum_{(b,c,d)}\{\varepsilon^{(2a+b)i+(c+d)j}+2\varepsilon^{(a+b+c)i+(a+d)j}\}+2q\varepsilon^{2aj+(b+c+d)i}$	$-(q-1)\sum_{(b,c,d)}\{\varepsilon^{(2a+b)i+(c+d)j}+2\varepsilon^{(a+b+c)i+(a+d)j}\}+2\varepsilon^{2aj+(b+c+d)i}$
A_{62}	$\sum_{(b,c,d)}\{-\varepsilon^{(2a+b)i+(c+d)j}-2\varepsilon^{(a+b+c)i+(a+d)j}\}$	$\sum_{(b,c,d)}\{\varepsilon^{(2a+b)i+(c+d)j}+2\varepsilon^{(a+b+c)i+(a+d)j}\}+2\varepsilon^{2aj+(b+c+d)i}$
A_7	$-(1/6)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+(d+e)j}$	$(1/6)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+(d+e)j}$
B_{11}	$(q^2-q)\varepsilon^{3ai-bj}$	$(q^2-q)\varepsilon^{3ai-bj}$
B_{12}	$-q\varepsilon^{3ai-bj}$	$-q\varepsilon^{3ai-bj}$

Table 2. (Continued)

	$A_{33}(i,j)$	$A_{34}(i,j)$
B_{13}		
B_{21}	$-(q-1)\varepsilon^{(2a+c)i-bj}$	$-(q-1)\varepsilon^{(2a+c)i-bj}$
B_{22}	$\varepsilon^{(2a+c)i-bj}$	$\varepsilon^{(2a+c)i-bj}$
B_3	$2\varepsilon^{(a+b+c)i-dj}$	$2\varepsilon^{(a+b+c)i-dj}$
B_{41}		
B_{42}		
B_5		
C_{11}	$-q\varepsilon^{2aj+bi}$	$-\varepsilon^{2aj+bi}$
C_{12}		$-\varepsilon^{2aj+bi}$
C_2	$\varepsilon^{(a+b)j+ci}$	$-\varepsilon^{(a+b)j+ci}$
D	$-\varepsilon^{-aj+bi}$	$-\varepsilon^{-aj+bi}$
E		
F		
Number of characters	$q(q+1)$	$q(q+1)$

	$A_{35}(i,j)$	$A_{36}(i,j)$
A_{11}	$q(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$	$(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{12}	$q(q^2-q+1)(q^2+1)\varepsilon^{a(3i+2j)}$	$(q^4-2q^3+2q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{13}	$(q^3-q^2+q)\varepsilon^{a(3i+2j)}$	$-(q^3-2q^2+q-1)\varepsilon^{a(3i+2j)}$
A_{14}	$(q^3-q^2+q)\varepsilon^{a(3i+2j)}$	$(2q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{15}	$q\varepsilon^{a(3i+2j)}$	$(q^2-q+1)\varepsilon^{a(3i+2j)}$
A_{16}	$q\varepsilon^{a(3i+2j)}$	$-(q-1)\varepsilon^{a(3i+2j)}$
A_{17}		$\varepsilon^{a(3i+2j)}$
A_{21}	$q(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}+(q-1)(q^2+1)\varepsilon^{3ai+(a+b)j}$	$(q^2-q+1)(q^2+1)\varepsilon^{(2a+b)i+2aj}-(q-1)(q^2+1)\varepsilon^{3ai+(a+b)j}$

Table 2. (Continued)

	$A_{35}(i,j)$	$A_{36}(i,j)$
A_{22}	$(q^3-q^2+q)\varepsilon^{(2a+b)i+2aj}-(q^2-q+1)\varepsilon^{3ai+(a+b)j}$	$(2q^2-q+1)\varepsilon^{(2a+b)i+2aj}+(q^2-q+1)\varepsilon^{3ai+(a+b)j}$
A_{23}	$q\varepsilon^{(2a+b)i+2aj}+(q-1)\varepsilon^{3ai+(a+b)j}$	$(q^2-q+1)\varepsilon^{(2a+b)i+2aj}-(q-1)\varepsilon^{3ai+(a+b)j}$
A_{24}	$q\varepsilon^{(2a+b)i+2aj}+(q-1)\varepsilon^{3ai+(a+b)j}$	$-(q-1)\varepsilon^{(2a+b)i+2aj}-(q-1)\varepsilon^{3ai+(a+b)j}$
A_{25}	$-\varepsilon^{3ai+(a+b)j}$	$\varepsilon^{(2a+b)i+2aj}+\varepsilon^{3ai+(a+b)j}$
A_{31}	$q\varepsilon^{3ai+2bj}+(q^3-q^2+q)\varepsilon^{(a+2b)i+2aj}$ $+(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}+(q^2-q+1)\varepsilon^{(a+2b)i+2aj}-(q-1)(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{32}	$(q^3-q^2+q)\varepsilon^{(a+2b)i+2aj}-(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}+(q^2-q+1)\varepsilon^{(a+2b)i+2aj}+(q^2-q+1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{33}	$q\varepsilon^{3ai+2bj}+q\varepsilon^{(a+2b)i+2aj}-(q-1)^2\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}-(q-1)\varepsilon^{(a+2b)i+2aj}+(q-1)^2\varepsilon^{(2a+b)i+(a+b)j}$
A_{34}	$q\varepsilon^{(a+2b)i+2aj}+(q-1)\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}-(q-1)\varepsilon^{(a+2b)i+2aj}-(q-1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{35}	$q\varepsilon^{3ai+2bj}+(q-1)\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}+\varepsilon^{(a+2b)i+2aj}-(q-1)\varepsilon^{(2a+b)i+(a+b)j}$
A_{36}	$-\varepsilon^{(2a+b)i+(a+b)j}$	$\varepsilon^{3ai+2bj}+\varepsilon^{(a+2b)i+2aj}+\varepsilon^{(2a+b)i+(a+b)j}$
A_{41}	$-\varepsilon^{3ai+(b+c)j}-(q^2-q+1)\{\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}-q\varepsilon^{(a+b+c)i+2aj}\}$	$\varepsilon^{3ai+(b+c)j}+(q^2-q+1)\{\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+\varepsilon^{(a+b+c)i+2aj}\}$
A_{42}	$-\varepsilon^{3ai+(b+c)j}+(q-1)\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+q\varepsilon^{(a+b+c)i+2aj}$	$\varepsilon^{3ai+(b+c)j}-(q-1)\{\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+\varepsilon^{(a+b+c)i+2aj}\}$
A_{43}	$-\varepsilon^{3ai+(b+c)j}-\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}$	$\varepsilon^{3ai+(b+c)j}+\sum_{(b,c)}\varepsilon^{(2a+b)i+(a+c)j}+\varepsilon^{(a+b+c)i+2aj}$
A_{51}	$\sum_{(a,b)}\{q\varepsilon^{2ai+2bj+ci}+(q-1)\varepsilon^{(2a+b)i+(b+c)j}\}-(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\{\varepsilon^{2ai+2bj+ci}-(q-1)\varepsilon^{(2a+b)i+(b+c)j}\}+(q-1)^2\varepsilon^{(a+b)(i+j)+ci}$
A_{52}	$q\varepsilon^{2aj+(2b+c)i}-\varepsilon^{(2a+b)i+(b+c)j}+(q-1)\varepsilon^{(a+2b)i+(a+c)j}$ $+(q-1)\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\varepsilon^{2ai+2bj+ci}+\varepsilon^{(2a+b)i+(b+c)j}-(q-1)\varepsilon^{(a+2b)(a+c)j}$ $-(q-1)\varepsilon^{(a+b)(i+j)+ci}$
A_{53}	$-\sum_{(a,b)}\varepsilon^{(2a+b)i+(b+c)j}-\varepsilon^{(a+b)(i+j)+ci}$	$\sum_{(a,b)}\{\varepsilon^{2ai+2bj+ci}+\varepsilon^{(2a+b)i+(b+c)j}\}+\varepsilon^{(a+b)(i+j)+ci}$
A_{61}	$\sum_{(b,c,d)}\{-\varepsilon^{(2a+b)i+(c+d)j}+(q-1)\varepsilon^{(a+b+c)i+(a+d)j}\}+q\varepsilon^{2aj+(b+c+d)i}$	$\sum_{(b,c,d)}\{\varepsilon^{(2a+b)i+(c+d)j}-(q-1)\varepsilon^{(a+b+c)i+(a+d)j}\}+\varepsilon^{2aj+(b+c+d)i}$
A_{62}	$-\sum_{\{b,c,d\}}\{\varepsilon^{(2a+b)i+(c+d)j}+\varepsilon^{(a+b+c)i+(a+d)j}\}$	$\sum_{(b,c,d)}\{\varepsilon^{(2a+b)i+(c+d)j}+\varepsilon^{(a+b+c)i+(a+d)j}\}+\varepsilon^{2aj+(b+c+d)i}$
A_7	$-(1/12)\cdot\sum_{(a,b,c,d,e)}\varepsilon^{(a+b+c)i+(d+e)j}$	$(1/12)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+(d+e)j}$
B_{11}	$\varepsilon^{3ai-bj}+(q^3-q^2-q)\varepsilon^{(a-b)i+2aj}$	$\varepsilon^{3ai-bj}+(q^2-q+1)\varepsilon^{(a-b)i+2aj}$
B_{12}	$\varepsilon^{3ai-bj}+q\varepsilon^{(a-b)i+2aj}$	$\varepsilon^{3ai-bj}-(q-1)\varepsilon^{(a-b)i+2aj}$

Table 2. (Continued)

	$A_{35}(i,j)$	$A_{36}(i,j)$
B_{13}	ε^{3ai-bj}	$\varepsilon^{3ai-bj}+\varepsilon^{(a-b)i+2aj}$
B_{21}	$\varepsilon^{(2a+c)i-bj}+q\varepsilon^{2aj+(c-b)i}+(q-1)\varepsilon^{(a-b)i+(a+c)j}$	$\varepsilon^{(2a+c)i-bj}+\varepsilon^{2aj+(c-b)i}-(q-1)\varepsilon^{(a-b)i+(a+c)j}$
B_{22}	$\varepsilon^{(2a+c)i-bj}-\varepsilon^{(a-b)i+(a+c)j}$	$\varepsilon^{(2a+c)i-bj}+\varepsilon^{2aj+(c-b)i}+\varepsilon^{(a-b)i+(a+c)j}$
B_3	$\varepsilon^{(a+b+c)i-dj}-\sum_{(a,b,c)}\varepsilon^{(a-d)i+(b+c)j}$	$\varepsilon^{(a+b+c)i-dj}+\sum_{(a,b,c)}\varepsilon^{(a-d)i+(b+c)j}$
B_{41}	$(q^2+1)\varepsilon^{(b-a)i-aj}$	$(q^2+1)\varepsilon^{(b-a)i-aj}$
B_{42}	$\varepsilon^{(b-a)i-aj}$	$\varepsilon^{(b-a)i-aj}$
B_5	$\sum_{(a,b)}\varepsilon^{-aj+(c-b)i}$	$\sum_{(a,b)}\varepsilon^{-aj+(c-b)i}$
C_{11}	$q\varepsilon^{2aj+bi}$	ε^{2aj+bi}
C_{12}		ε^{2aj+bi}
C_2	$-\varepsilon^{(a+b)j+ci}$	$\varepsilon^{(a+b)j+ci}$
D	ε^{-aj+bi}	ε^{-aj+bi}
E		
F		
Number of characters	$q(q+1)$	$q(q+1)$

	$A_{41}(i,j,k)$	$A_{42}(i,j,k)$
A_{11}	$q^3(q-1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+j+k)}$	$q(q-1)^2(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+j+k)}$
A_{12}	$(2q^6-2q^5+2q^4-q^3)\varepsilon^{a(3i+j+k)}$	$-q(q-1)^2(q^3-2q^2+q-1)\varepsilon^{a(3i+j+k)}$
A_{13}	$(q^4-q^3)\varepsilon^{a(3i+j+k)}$	$-q(q-1)(2q^2-2q+1)\varepsilon^{a(3i+j+k)}$
A_{14}	$-q^3\varepsilon^{a(3i+j+k)}$	$q(q-1)(2q-1)\varepsilon^{a(3i+j+k)}$
A_{15}		$-(2q^2-q)\varepsilon^{a(3i+j+k)}$
A_{16}		$q\varepsilon^{a(3i+j+k)}$
A_{17}		

Table 2. (Continued)

	$A_{41}(i,j,k)$	$A_{42}(i,j,k)$
A_{21}	$q^3(q-1)(q^2+1)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}$ $+q(q-1)(q^2-q+1)(q^2+1)\varepsilon^{a(2i+j+k)+bi}$	$q(q-1)^2(q^2+1)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}$ $-(q-1)^2(q^2-q+1)(q^2+1)\varepsilon^{a(2i+j+k)+bi}$
A_{22}	$-q^3\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}-(2q^3-2q^2+q)\varepsilon^{a(2i+j+k)+bi}$	$q(q-1)^2\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}+(q-1)(3q^2-2q+1)\varepsilon^{a(2i+j+k)bi}$
A_{23}	$(q^2-q)\varepsilon^{a(2i+j+k)+bi}$	$-(q^2-q)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}-(q-1)(2q-1)\varepsilon^{a(2i+j+k)+bi}$
A_{24}	$-q\varepsilon^{a(2i+j+k)+bi}$	$q\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}+(3q-1)\varepsilon^{a(2i+j+k)+bi}$
A_{25}		$-\varepsilon^{a(2i+j+k)+bi}$
A_{31}	$(q^4-q^3)\varepsilon^{3ai+b(j+k)}+q(q-1)(q^2-q+1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$	$q(q-1)^2\varepsilon^{3ai+b(j+k)}-(q-1)^2(q^2-q+1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{32}	$-q^3\varepsilon^{3ai+b(j+k)}-(q^3-q^2+q)\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}$	$-(q^2-q)\varepsilon^{3ai+b(j+k)}+(q-1)(q^2-q+1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{33}	$(q^2-q)\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}+(2q^2-q)\varepsilon^{a(i+j+k)+2bi}$	$-(q^2-q)\varepsilon^{3ai+b(j+k)}-(q-1)(2q-1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{34}	$-q\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}$	$q\varepsilon^{3ai+b(j+k)}+(2q-1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{35}	$-q\varepsilon^{a(i+j+k)+2bi}$	$(q-1)\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{36}		$-\varepsilon^{(a+b)i}\sum_{(i,j,k)}\varepsilon^{a(i+j)+bk}$
A_{41}	$-\sum_{(b,c)}\{q^3\varepsilon^{3ai+bj+ck}+(q^3-q^2+q)\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+ck}\}$ $-(q-1)(q^2-q+1)\varepsilon^{(a+b+c)i+a(j+k)}$	$-(q^2-q)\sum_{(j,k)}\varepsilon^{3ai+bj+ck}+(q-1)(q^2-q+1)$ $\times\{\sum_{(b,c)}\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+ck}+2\varepsilon^{(a+b+c)i+a(j+k)}\}$
A_{42}	$-q\sum_{(b,c)}\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+ck}-(2q-1)\varepsilon^{(a+b+c)i+a(j+k)}$	$q\sum_{(j,k)}\varepsilon^{3ai+bj+ck}+(2q-1)$ $\times\{\sum_{(b,c)}\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+ck}+2\varepsilon^{(a+b+c)i+a(j+k)}\}$
A_{43}	$\varepsilon^{(a+b+c)i+a(j+k)}$	$-\sum_{(b,c)}\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+ck}-2\varepsilon^{(a+b+c)i+a(j+k)}$
A_{51}	$\sum_{(a,b)}\{(q^2-q)\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}+(q-1)^2\varepsilon^{(a+b+c)i+aj+bk}\}$	$-(q-1)^2\sum_{(a,b)}\{\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}+2\varepsilon^{(a+b+c)i+aj+bk}\}$
A_{52}	$-q\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}-(q-1)\sum_{(a,b)}\varepsilon^{(a+b+c)i+aj+bk}$	$(q-1)\sum_{(a,b)}\{\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}+2\varepsilon^{(a+b+c)i+aj+bk}\}$
A_{53}	$\sum_{(a,b)}\varepsilon^{(a+b+c)i+aj+bk}$	$-\sum_{(a,b)}\{\varepsilon^{2ai+b(j+k)+ci}+\varepsilon^{(a+b+c)i+aj+bk}\}$

Table 2. (Continued)

	$A_{41}(i, j, k)$	$A_{42}(i, j, k)$
A_{61}	$-q\varepsilon^{2ai} \sum_{\{i,j,k\}} \varepsilon^{bi+cj+dk}$ $-(q-1)\{\sum_{(b,c,d)} \sum_{(j,k)} \varepsilon^{(a+b+c)i+aj+dk} + \varepsilon^{a(j+k)+(b+c+d)i}\}$	$(q-1) \sum_{(b,c,d)} \varepsilon^{(a+b)i} \sum_{\{i,j,k\}} \varepsilon^{ai+cj+dk} + 2(q-1)\varepsilon^{a(j+k)+(b+c+d)i}$
A_{62}	$\sum_{(b,c,d)} \sum_{(j,k)} \varepsilon^{(a+b+c)i+aj+dk} + \varepsilon^{a(j+k)+(b+c+d)i}$	$-\sum_{(b,c,d)} \varepsilon^{(a+b)i} \sum_{\{i,j,k\}} \varepsilon^{ai+cj+dk} - 2\varepsilon^{a(j+k)+(b+c+d)i}$
A_7	$(1/6) \cdot \sum_{\{a,b,c,d,e\}} \varepsilon^{(a+b+c)i+dj+ek}$	$-(1/3) \cdot \sum_{\{a,b,c,d,e\}} \varepsilon^{(a+b+c)i+dj+ek}$
B_{11}	$(q-1)(q^2-q+1)\varepsilon^{a(i+j+k)-bi}$	
B_{12}	$(2q-1)\varepsilon^{a(i+j+k)-bi}$	
B_{13}	$-\varepsilon^{a(i+j+k)-bi}$	
B_{21}	$(q-1)\varepsilon^{-bi} \sum_{(i,j,k)} \varepsilon^{a(i+j)+ck}$	
B_{22}	$-\varepsilon^{-bi} \sum_{(i,j,k)} \varepsilon^{a(i+j)+ck}$	
B_3	$-\sum_{\{a,b,c\}} \varepsilon^{ai+bj+ck-di}$	
B_{41}		
B_{42}		
B_5		
C_{11}	$-(q-1)\varepsilon^{a(j+k)+bi}$	$-(q-1)\varepsilon^{a(j+k)+bi}$
C_{12}	$\varepsilon^{a(j+k)+bi}$	$\varepsilon^{a(j+k)+bi}$
C_2	$\sum_{(j,k)} \varepsilon^{aj+bk+ci}$	$\sum_{(j,k)} \varepsilon^{aj+bk+ci}$
D		
E		
F		
Number of characters	$(1/2) \cdot q(q^2-1)$	$(1/2) \cdot q(q^2-1)$

Table 2. (Continued)

	$A_{43}(i, j, k)$	$A_{51}(i, j, k)$
A_{11}	$(q-1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(3i+j+k)}$	$q^2(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(2i+2j+k)}$
A_{12}	$(q^5-2q^4+4q^3-3q^2+2q-1)\varepsilon^{a(3i+j+k)}$	$q^2(q^2-q+1)(2q^2-q+1)\varepsilon^{a(2i+2j+k)}$
A_{13}	$(q-1)(2q^2-q+1)\varepsilon^{a(3i+j+k)}$	$(2q^4-2q^3+q^2)\varepsilon^{a(2i+2j+k)}$
A_{14}	$(q^3-3q^2+2q-1)\varepsilon^{a(3i+j+k)}$	$-(q^3-q^2)\varepsilon^{a(2i+2j+k)}$
A_{15}	$-(q-1)^2\varepsilon^{a(3i+j+k)}$	$q^2\varepsilon^{a(2i+2j+k)}$
A_{16}	$(2q-1)\varepsilon^{a(3i+j+k)}$	
A_{17}	$-\varepsilon^{a(3i+j+k)}$	
A_{21}	$(q-1)(q^2+1)\{\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}+(q^2-q+1)\varepsilon^{a(2i+j+k)+bi}\}$	$q^2(q^2-q+1)(q^2+1)\varepsilon^{2a(i+j)+bk}$ $+q(q-1)(q^2-q+1)(q^2+1)\sum_{(i,j)}\varepsilon^{a(2i+j+k)+bj}$
A_{22}	$-(q^2-q+1)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}+(q^3-3q^2+2q-1)\varepsilon^{a(2i+j+k)+bi}$	$-(q^3-q^2)\varepsilon^{2a(i+j)+bk}-(2q^3-2q^2+q)\sum_{(i,j)}\varepsilon^{a(2i+j+k)+bj}$
A_{23}	$(q-1)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}-(q-1)^2\varepsilon^{a(2i+j+k)+bi}$	$q^2\varepsilon^{2a(i+j)+bk}+(q^2-q)\sum_{(i,j)}\varepsilon^{a(2i+j+k)+bj}$
A_{24}	$(q-1)\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}+(2q-1)\varepsilon^{a(2i+j+k)+bi}$	$-q\sum_{(i,j)}\varepsilon^{a(2i+j+k)+bj}$
A_{25}	$-\sum_{(j,k)}\varepsilon^{a(3i+j)+bk}-\varepsilon^{a(2i+j+k)+bi}$	
A_{31}	$(q-1)\varepsilon^{3ai+b(j+k)}$ $+(q-1)(q^2-q+1)\{\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}+\varepsilon^{a(i+j+k)+2bi}$	$(q^2-q+1)\{q^2\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}+(q^2-q)\sum_{(i,j)}\varepsilon^{a(i+2j)+b(i+k)}$ $+(q-1)^2\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{32}	$-\varepsilon^{3ai+b(j+k)}-(q^2-q+1)\{\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}-(q-1)\varepsilon^{a(i+j+k)+2bi}\}$	$-(q^2-q+1)\{q\sum_{(i,j)}\varepsilon^{a(i+2j)+b(i+k)}+(q-1)\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{33}	$(q-1)\varepsilon^{3ai+b(j+k)}-(q-1)^2\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}+(2q-1)\varepsilon^{a(i+j+k)+2bi}$	$q^2\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}+(q^2-q)\sum_{(i,j)}\varepsilon^{a(i+2j)+b(i+k)}$ $+(q-1)(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{34}	$-\varepsilon^{3ai+b(j+k)}+(q-1)\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}+(2q-1)\varepsilon^{a(i+j+k)+2bi}$	$-q\sum_{(i,j)}\varepsilon^{a(i+2j)+b(i+k)}-(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{35}	$(q-1)\varepsilon^{3ai+b(j+k)}+(q-1)\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}-\varepsilon^{a(i+j+k)+2bi}$	$-(q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{36}	$-\varepsilon^{3ai+b(j+k)}-\sum_{(j,k)}\varepsilon^{(2a+b)i+aj+bk}-\varepsilon^{a(i+j+k)+2bi}$	$\varepsilon^{a(i+j+k)+b(i+j)}$

Table 2. (Continued)

	$A_{43}(i,j,k)$	$A_{51}(i,j,k)$
A_{41}	$-\sum_{(j,k)} \varepsilon^{3ai+bj+ck}-(q^2-q+1)\{\sum_{(b,c)}\sum_{(j,k)} \varepsilon^{(2a+b)i+aj+ck}$ $-(q-1)\varepsilon^{(a+b+c)i+a(j+k)}\}$	$-(q^2-q+1)\sum_{(i,j)}\{q\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}+(q-1)\varepsilon^{a(i+j+k)+bi+cj}\}$
A_{42}	$\sum_{(j,k)}\{-\varepsilon^{3ai+bj+ck}+(q-1)\sum_{(b,c)}\varepsilon^{(2a+b)i+aj+ck}\}$ $+(2q-1)\varepsilon^{(a+b+c)i+a(j+k)}\}$	$-\sum_{(i,j)}\{q\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}+(2q-1)\varepsilon^{a(i+j+k)+bi+cj}\}$
A_{43}	$-\varepsilon^{2ai}\sum_{\{i,j,k\}}\varepsilon^{ai+bj+ck}-\varepsilon^{(a+b+c)i+a(j+k)}$	$\sum_{(i,j)}\varepsilon^{a(i+j+k)+bi+cj}$
A_{51}	$(q-1)\sum_{(a,b)}\{\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}-(q-1)\varepsilon^{(a+b+c)i+aj+bk}\}$	$\sum_{(i,j)}\sum_{(a,b)}\{(q^2-q)\varepsilon^{2ai+b(j+k)+cj}+(q-1)^2\varepsilon^{a(i+j)+b(j+k)+ci}\}$ $+q^2\sum_{(i,j)}\varepsilon^{2ai+2bj+ck}+(q-1)^2\varepsilon^{(a+b)(i+j)+ck}$
A_{52}	$-\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}+(q-1)\varepsilon^{2bi}\sum_{(i,j,k)}\varepsilon^{a(i+j)+ck}$ $+(q-1)\sum_{(a,b)}\varepsilon^{(a+b+c)i+aj+bk}$	$-\sum_{(i,j)}\{q\varepsilon^{2ai+b(j+k)+cj}+(q-1)\sum_{(a,b)}\varepsilon^{a(i+j)+b(j+k)+ci}\}$ $-(q-1)\varepsilon^{(a+b)(i+j)+ck}$
A_{53}	$-\sum_{(a,b)}\{\varepsilon^{2ai}\sum_{(i,j,k)}\varepsilon^{b(i+j)+ck}+\varepsilon^{(a+b+c)i+aj+bk}\}$	$\sum_{(i,j)}\sum_{(a,b)}\varepsilon^{a(i+j)+b(j+k)+ci}+\varepsilon^{(a+b)(i+j)+ck}$
A_{61}	$-\varepsilon^{2ai}\sum_{\{i,j,k\}}\varepsilon^{bi+cj+dk}+(q-1)\{\sum_{(b,c,d)}\sum_{(j,k)}\varepsilon^{(a+b+c)i+aj+dk}$ $+\varepsilon^{a(j+k)+(b+c+d)i}\}$	$-\sum_{(b,c,d)}\sum_{(i,j)}\{q\varepsilon^{2ai+(b+c)j+dk}+(q-1)\varepsilon^{a(i+j)+bi+cj+dk}$ $+(q-1)\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_{62}	$-\varepsilon^{2ai}\sum_{\{i,j,k\}}\varepsilon^{bi+cj+dk}-\sum_{(b,c,d)}\sum_{(j,k)}\varepsilon^{(a+b+c)i+aj+dk}-\varepsilon^{a(j+k)+(b+c+d)i}$	$\sum_{(b,c,d)}\sum_{(i,j)}\{\varepsilon^{a(i+j)+bi+cj+dk}+\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_7	$-(1/6)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b+c)i+dj+ek}$	$(1/4)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b)i+(c+d)j+ek}$
B_{11}	$(q-1)(q^2-q+1)\varepsilon^{a(i+j+k)-bi}$	$(q^3-q^2+q)\sum_{(i,j)}\varepsilon^{a(2i+k)-bj}$
B_{12}	$(2q-1)\varepsilon^{a(i+j+k)-bi}$	$q\sum_{(i,j)}\varepsilon^{a(2i+k)-bj}$
B_{13}	$-\varepsilon^{a(i+j+k)-bi}$	
B_{21}	$(q-1)\varepsilon^{-bi}\sum_{(i,j,k)}\varepsilon^{a(i+j)+ck}$	$\sum_{(i,j)}\{q\varepsilon^{2ai-bj+ck}+(q-1)\varepsilon^{a(i+k)-bj+ci}\}$
B_{22}	$-\varepsilon^{-bi}\sum_{(i,j,k)}\varepsilon^{a(i+j)+ck}$	$-\sum_{(i,j)}\varepsilon^{a(i+k)-bj+ci}$
B_3	$-\sum_{\{a,b,c\}}\varepsilon^{ai+bj+ck-di}$	$-\sum_{(i,j)}\sum_{(a,b,c)}\varepsilon^{(a+b)i+ck-dj}$

Table 2. (Continued)

	$A_{43}(i, j, k)$	$A_{51}(i, j, k)$
B_{41}		$(q^2+1)\varepsilon^{-a(i+j)+bk}$
B_{42}		$\varepsilon^{-a(i+j)+bk}$
B_5		$\sum_{(i,j)} \varepsilon^{-ai-bj+ck}$
C_{11}	$(q-1)\varepsilon^{a(j+k)+bi}$	
C_{12}	$-\varepsilon^{a(j+k)+bi}$	
C_2	$-\sum_{(j,k)} \varepsilon^{aj+bk+ci}$	
D		
E		
F		
Number of characters	$(1/2)\cdot q(q^2-1)$	$(1/2)\cdot q(q^2-1)$

	$A_{52}(i, j, k)$
A_{11}	$q(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(2i+2j+k)}$
A_{12}	$-q(q^2-q+1)(q^3-2q^2+q-1)\varepsilon^{a(2i+2j+k)}$
A_{13}	$-(q^4-3q^3+2q^2-q)\varepsilon^{a(2i+2j+k)}$
A_{14}	$(2q^3-2q^2+q)\varepsilon^{a(2i+2j+k)}$
A_{15}	$-(q^2-q)\varepsilon^{a(2i+2j+k)}$
A_{16}	$q\varepsilon^{a(2i+2j+k)}$
A_{17}	
A_{21}	$(q^2-q+1)(q^2+1)\{q\varepsilon^{2a(i+j)+bk}-(q^2-q)\varepsilon^{a(2i+j+k)+bj}+(q-1)\varepsilon^{a(i+2j+k)+bi}\}$
A_{22}	$(q^3-q^2+q)\varepsilon^{2a(i+j)+bk}+(2q^3-2q^2+q)\varepsilon^{a(2i+j+k)+bj}+(q^3-3q^2+2q-1)\varepsilon^{a(i+2j+k)+bi}$
A_{23}	$q\varepsilon^{2a(i+j)+bk}-(q^2-q)\varepsilon^{a(2i+j+k)+bj}-(q-1)^2\varepsilon^{a(i+2j+k)+bi}$
A_{24}	$q\varepsilon^{2a(i+j)+bk}+q\varepsilon^{a(2i+j+k)+bj}+(2q-1)\varepsilon^{a(i+2j+k)+bi}$
A_{25}	$-\varepsilon^{a(i+2j+k)+bi}$

Table 2. (Continued)

	$A_{52}(i, j, k)$
A_{31}	$(q^2-q+1)\{-(q^2-q)\varepsilon^{a(2i+j)+b(j+k)}+(q-1)\varepsilon^{a(i+2j)+b(i+k)}+q\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}-(q-1)^2\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{32}	$(q^2-q+1)\{q\varepsilon^{a(2i+j)+b(j+k)}-\varepsilon^{a(i+2j)+b(i+k)}+q\varepsilon^{a(2i+k)+2bj}+(q-1)\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{33}	$-(q^2-q)\varepsilon^{a(2i+j)+b(j+k)}-(q-1)^2\varepsilon^{a(i+2j)+b(i+k)}+q\varepsilon^{a(2i+k)+2bj}-(q^2-q)\varepsilon^{a(2j+k)+2bi}-(q-1)(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{34}	$q\varepsilon^{a(2i+j)+b(j+k)}+(q-1)\varepsilon^{a(i+2j)+b(i+k)}+q\varepsilon^{a(2i+k)+2bj}+(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{35}	$(q-1)\varepsilon^{a(i+2j)+b(i+k)}+q\varepsilon^{a(2j+k)+2bi}+(q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{36}	$-\varepsilon^{a(i+2j)+b(i+k)}-\varepsilon^{a(i+j+k)+b(i+j)}$
A_{41}	$(q^2-q+1)\{q\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}-\varepsilon^{2aj}\sum_{(a,b,c)}\varepsilon^{(a+b)i+ck}+(q-1)\sum_{(i,j)}\varepsilon^{a(i+j+k)+bi+cj}\}$
A_{42}	$q\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}+(q-1)\varepsilon^{2aj}\sum_{(a,b,c)}\varepsilon^{(a+b)i+ck}+(2q-1)\sum_{(i,j)}\varepsilon^{a(i+j+k)+bi+cj}$
A_{43}	$-\varepsilon^{2aj}\sum_{(a,b,c)}\varepsilon^{(a+b)i+ck}-\sum_{(i,j)}\varepsilon^{a(i+j+k)+bi+cj}$
A_{51}	$\sum_{(a,b)}\{-(q^2-q)\varepsilon^{2ai+b(j+k)+cj}+(q-1)\varepsilon^{2aj+b(i+k)+ci}+q\varepsilon^{2ai+2bj+ck}-(q-1)^2\sum_{(i,j)}\varepsilon^{a(i+j)+b(j+k)+ci}\}-(q-1)^2\varepsilon^{(a+b)(i+j)+ck}$
A_{52}	$q\varepsilon^{2ai+b(j+k)+cj}+(q-1)\varepsilon^{a(i+k)+2bj+ci}-\varepsilon^{2aj+b(i+k)+ci}+q\varepsilon^{2ai+2bj+ck}+(q-1)\sum_{(a,b)}\sum_{(i,j)}\varepsilon^{a(i+j)+b(j+k)+ci}+(q-1)\varepsilon^{(a+b)(i+j)+ck}$
A_{53}	$-\sum_{(a,b)}\{\varepsilon^{2aj+b(i+k)+ci}+\sum_{(i,j)}\varepsilon^{a(i+j)+b(j+k)+ci}\}-\varepsilon^{(a+b)(i+j)+ck}$
A_{61}	$\sum_{(b,c,d)}\{q\varepsilon^{2ai+(b+c)j+dk}-\varepsilon^{2aj+(b+c)i+dk}+(q-1)\sum_{(i,j)}\varepsilon^{a(i+j)+bi+cj+dk}+(q-1)\sum_{(i,j)}\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_{62}	$\sum_{(b,c,d)}\{-\varepsilon^{2aj+(b+c)i+dk}-\sum_{(i,j)}\varepsilon^{a(i+j)+bi+cj+dk}-\sum_{(i,j)}\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_7	$-(1/4)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b)i+(c+d)j+ek}$
B_{11}	$(q^2-q+1)\{q\varepsilon^{a(2i+k)-bj}+\varepsilon^{a(2j+k)-bi}\}$
B_{12}	$q\varepsilon^{a(2i+k)-bj}-(q-1)\varepsilon^{a(2j+k)-bi}$
B_{13}	$\varepsilon^{a(2j+k)-bi}$
B_{21}	$q\varepsilon^{2ai-bj+ck}+\varepsilon^{2aj-bi+ck}+(q-1)\varepsilon^{a(i+k)-bj+ci}-(q-1)\varepsilon^{a(j+k)-bi+cj}$
B_{22}	$\varepsilon^{2aj-bi+ck}-\varepsilon^{a(i+k)-bj+ci}+\varepsilon^{a(j+k)-bi+cj}$
B_3	$\sum_{(a,b,c)}\{-\varepsilon^{(a+b)i+ck-dj)}+\varepsilon^{(a+b)j+ck-di}\}$
B_{41}	$(q^2+1)\varepsilon^{-a(i+j)+bk}$

Table 2. (Continued)

	$A_{52}(i, j, k)$
B_{42}	$\varepsilon^{-a(i+j)+bk}$
B_5	$\sum_{(i,j)} \varepsilon^{-ai-bj+ck}$
C_{11}	
C_{12}	
C_2	
D	
E	
F	
Number of characters	$q(q^2-1)$

	$A_{53}(i, j, k)$
A_{11}	$(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(2i+2j+k)}$
A_{12}	$-(q-1)(q^2-q+1)(2q^2+1)\varepsilon^{a(2i+2j+k)}$
A_{13}	$(q^4-3q^3+4q^2-2q+1)\varepsilon^{a(2i+2j+k)}$
A_{14}	$-(q^3-4q^2+2q-1)\varepsilon^{a(2i+2j+k)}$
A_{15}	$(2q^2-2q+1)\varepsilon^{a(2i+2j+k)}$
A_{16}	$-(2q-1)\varepsilon^{a(2i+2j+k)}$
A_{17}	$\varepsilon^{a(2i+2j+k)}$
A_{21}	$(q^2-q+1)(q^2+1)\{\varepsilon^{2a(i+j)+bk}-(q-1)\sum_{(i,j)} \varepsilon^{a(2i+j+k)+bj}\}$
A_{22}	$(2q^2-q+1)\varepsilon^{2a(i+j)+bk}-(q^3-3q^2+2q-1)\sum_{(i,j)} \varepsilon^{a(2i+j+k)+bj}$
A_{23}	$(q^2-q+1)\varepsilon^{2a(i+j)+bk}+(q-1)^2 \sum_{(i,j)} \varepsilon^{a(2i+j+k)+bj}$
A_{24}	$-(q-1)\varepsilon^{2a(i+j)+bk}-(2q-1)\sum_{(i,j)} \varepsilon^{a(2i+j+k)+bj}$

Table 2. (Continued)

	$A_{53}(i,j,k)$
A_{25}	$\varepsilon^{2a(i+j)+bk}+\sum_{(i,j)}\varepsilon^{a(2i+j+k)+bj}$
A_{31}	$(q^2-q+1)\{-(q-1)\sum_{(i,j)}\varepsilon^{a(2i+j)+b(j+k)}+\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}+(q-1)^2\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{32}	$(q^2-q+1)\{\sum_{(i,j)}\varepsilon^{a(2i+j)+b(j+k)}+\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}-(q-1)\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{33}	$(q-1)\{(q-1)\sum_{(i,j)}\varepsilon^{a(2i+j)+b(j+k)}-\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}+(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}\}$
A_{34}	$-(q-1)\sum_{(i,j)}\{\varepsilon^{a(2i+j)+b(j+k)}+\varepsilon^{a(2i+k)+2bj}\}-(2q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{35}	$-(q-1)\sum_{(i,j)}\varepsilon^{a(2i+j)+b(j+k)}+\sum_{(i,j)}\varepsilon^{a(2i+k)+2bj}-(q-1)\varepsilon^{a(i+j+k)+b(i+j)}$
A_{36}	$\sum_{(i,j)}\{\varepsilon^{a(2i+j)+b(j+k)}+\varepsilon^{a(2i+k)+2bj}\}+\varepsilon^{a(i+j+k)+b(i+j)}$
A_{41}	$(q^2-q+1)\sum_{(i,j)}\{\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}-(q-1)\varepsilon^{a(i+j+k)+bi+cj}\}$
A_{42}	$-(q-1)\sum_{(i,j)}\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}-(2q-1)\sum_{(i,j)}\varepsilon^{a(i+j+k)+bi+cj}$
A_{43}	$\sum_{(i,j)}\{\varepsilon^{2ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}+\varepsilon^{a(i+j+k)+bi+cj}\}$
A_{51}	$\sum_{(a,b)}\sum_{(i,j)}\{-(q-1)\varepsilon^{2ai+b(j+k)+cj}+(q-1)^2\varepsilon^{a(i+j)+b(j+k)+ci}\}+\sum_{(i,j)}\varepsilon^{2ai+2bj+ck}+(q-1)^2\varepsilon^{(a+b)(i+j)+ck}$
A_{52}	$\sum_{(i,j)}\{\varepsilon^{2ai+b(j+k)+cj}-(q-1)\varepsilon^{a(j+k)+2bi+cj}-(q-1)\sum_{(a,b)}\varepsilon^{a(i+j)+b(j+k)+ci}+\varepsilon^{2ai+2bj+ck}\}-(q-1)\varepsilon^{(a+b)(i+j)+ck}$
A_{53}	$\sum_{(a,b)}\sum_{(i,j)}\{\varepsilon^{2ai+b(j+k)+cj}+\varepsilon^{a(i+j)+b(j+k)+ci}\}+\sum_{(i,j)}\varepsilon^{2ai+2bj+ck}+\varepsilon^{(a+b)(i+j)+ck}$
A_{61}	$\sum_{(b,c,d)}\sum_{(i,j)}\{\varepsilon^{2ai+(b+c)j+dk}-(q-1)\varepsilon^{a(i+j)+bi+cj+dk}-(q-1)\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_{62}	$\sum_{(b,c,d)}\sum_{(i,j)}\{\varepsilon^{2ai+(b+c)j+dk}+\varepsilon^{a(i+j)+bi+cj+dk}+\varepsilon^{a(i+k)+(b+c)j+di}\}$
A_7	$(1/4)\cdot\sum_{\{a,b,c,d,e\}}\varepsilon^{(a+b)i+(c+d)j+ek}$
B_{11}	$(q^2-q+1)\sum_{(i,j)}\varepsilon^{a(2i+k)-bj}$
B_{12}	$-(q-1)\sum_{(i,j)}\varepsilon^{a(2i+k)-bj}$
B_{13}	$\sum_{(i,j)}\varepsilon^{a(2i+k)-bj}$

Table 2. (Continued)

	$A_{53}(i, j, k)$
B_{21}	$\sum_{(i,j)} \{\varepsilon^{2ai-bj+ck} - (q-1)\varepsilon^{a(i+k)-bj+ci}\}$
B_{22}	$\sum_{(i,j)} \{\varepsilon^{2ai-bj+ck} + \varepsilon^{a(i+k)-bj+ci}\}$
B_3	$\sum_{(i,j)} \sum_{(a,b,c)} \varepsilon^{(a+b)i+ck-dj}$
B_{41}	$(q^2+1)\varepsilon^{-a(i+j)+bk}$
B_{42}	$\varepsilon^{-a(i+j)+bk}$
B_5	$\sum_{(i,j)} \varepsilon^{-ai-bj+ck}$
C_{11}	
C_{12}	
C_2	
D	
E	
F	
Number of characters	$(1/2)\cdot q(q^2-1)$

	$A_{61}(i, j, k, l)$
A_{11}	$q(q-1)(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(2i+j+k+l)}$
A_{12}	$q(q^2-q+1)(3q^3-3q^2+2q-1)\varepsilon^{a(2i+j+k+l)}$
A_{13}	$q(q-1)(3q^2-2q+1)\varepsilon^{a(2i+j+k+l)}$
A_{14}	$-(3q^3-3q^2+q)\varepsilon^{a(2i+j+k+l)}$
A_{15}	$(2q^2-q)\varepsilon^{a(2i+j+k+l)}$
A_{16}	$-q\varepsilon^{a(2i+j+k+l)}$
A_{17}	
A_{21}	$(q-1)(q^2-q+1)(q^2+1)\{q \sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} + (q-1)\varepsilon^{a(i+j+k+l)+bi}\}$

Table 2. (Continued)

	$A_{61}(i, j, k, l)$
A_{22}	$-(2q^3-2q^2+q)\sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl}-(q-1)(3q^2-2q+1)\varepsilon^{a(i+j+k+l)+bi}$
A_{23}	$(q-1)\{q\sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl}+(2q-1)\varepsilon^{a(i+j+k+l)+bi}\}$
A_{24}	$-q\sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl}-(3q-1)\varepsilon^{a(i+j+k+l)+bi}$
A_{25}	$\varepsilon^{a(i+j+k+l)+bi}$
A_{31}	$(q-1)(q^2-q+1)\{q\sum_{(j,k,l)} \varepsilon^{a(2i+j)+b(k+l)}+(q-1)\sum_{(j,k,l)} \varepsilon^{a(i+j+k)+b(i+l)}+q\varepsilon^{a(j+k+l)+2bi}\}$
A_{32}	$-(q^2-q+1)\sum_{(j,k,l)} \{q\varepsilon^{a(2i+j)+b(k+l)}+(q-1)\varepsilon^{a(i+j+k)+b(i+l)}\}$
A_{33}	$(q-1)\sum_{(j,k,l)} \{q\varepsilon^{a(2i+j)+b(k+l)}+(2q-1)\varepsilon^{a(i+j+k)+b(i+l)}\}+(2q^2-q)\varepsilon^{a(j+k+l)+2bi}$
A_{34}	$-\sum_{(j,k,l)} \{q\varepsilon^{a(2i+j)+b(k+l)}+(2q-1)\varepsilon^{a(i+j+k)+b(i+l)}\}$
A_{35}	$-(q-1)\sum_{(j,k,l)} \varepsilon^{a(i+j+k)+b(i+l)}-q\varepsilon^{a(j+k+l)+2bi}$
A_{36}	$\sum_{(j,k,l)} \varepsilon^{a(i+j+k)+b(i+l)}$
A_{41}	$-(q^2-q+1)\{q\sum_{\{j,k,l\}} \varepsilon^{a(2i+j)+bk+cl}+(q-1)\sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl}+(q-1)\varepsilon^{a(j+k+l)+(b+c)i}\}$
A_{42}	$-q\sum_{(j,k,l)} \varepsilon^{a(2i+j)+bk+cl}-(2q-1)\{\sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl}+\varepsilon^{a(j+k+l)+(b+c)i}\}$
A_{43}	$\sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl}+\varepsilon^{a(j+k+l)+(b+c)i}$
A_{51}	$(q-1)\sum_{(a,b)}\sum_{(j,k,l)} \{q\varepsilon^{2ai+b(j+k)+cl}+(q-1)\varepsilon^{a(i+j)+b(i+k)+cl}+(q-1)\varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{52}	$-q\sum_{\{j,k,l\}} \varepsilon^{2ai+b(j+k)+cl}-(q-1)\sum_{(a,b)}\sum_{(j,k,l)} \{\varepsilon^{a(i+j)+b(i+k)+cl}+\varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{53}	$\sum_{(a,b)}\sum_{(j,k,l)} \{\varepsilon^{a(i+j)+b(i+k)+cl}+\varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{61}	$-q\sum_{\{j,k,l\}} \varepsilon^{2ai+bj+ck+dl}-(q-1)\sum_{(b,c,d)}\{\sum_{\{j,k,l\}} \varepsilon^{a(i+j)+bi+ck+dl}+\sum_{(j,k,l)} \varepsilon^{a(j+k)+(b+c)i+dl}\}$
A_{62}	$\sum_{(b,c,d)}\{\sum_{\{j,k,l\}} \varepsilon^{a(i+j)+bi+ck+dl}+\sum_{(j,k,l)} \varepsilon^{a(j+k)+(b+c)i+dl}\}$
A_7	$(1/2)\cdot\sum_{\{a,b,c,d,e\}} \varepsilon^{(a+b)i+cj+dk+el}$

Table 2. (Continued)

	$A_{61}(i, j, k, l)$
B_{11}	$(q-1)(q^2-q+1)\varepsilon^{a(j+k+l)-bi}$
B_{12}	$(2q-1)\varepsilon^{a(j+k+l)-bi}$
B_{13}	$-\varepsilon^{a(j+k+l)-bi}$
B_{21}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(j+k)-bi+cl}$
B_{22}	$-\sum_{(j,k,l)}\varepsilon^{a(j+k)-bi+cl}$
B_3	$-\sum_{\{j,k,l\}}\varepsilon^{aj+bk+cl-di}$
B_{41}	
B_{42}	
B_5	
C_{11}	
C_{12}	
C_2	
D	
E	
F	
Number of characters	$(1/6)\cdot q(q^2-1)(q-2)$

	$A_{62}(i, j, k, l)$
A_{11}	$(q-1)(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(2i+j+k+l)}$
A_{12}	$-(q^2-q+1)(q^4-4q^3+3q^2-2q+1)\varepsilon^{a(2i+j+k+l)}$
A_{13}	$-(q-1)(2q^3-4q^2+2q-1)\varepsilon^{a(2i+j+k+l)}$
A_{14}	$(3q^3-6q^2+3q-1)\varepsilon^{a(2i+j+k+l)}$
A_{15}	$-(3q^2-3q+1)\varepsilon^{a(2i+j+k+l)}$
A_{16}	$(3q-1)\varepsilon^{a(2i+j+k+l)}$

Table 2. (Continued)

	$A_{62}(i, j, k, l)$
A_{17}	$-\varepsilon^{a(2i+j+k+l)}$
A_{21}	$(q-1)(q^2-q+1)(q^2+1)\{\sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} - (q-1)\varepsilon^{a(i+j+k+l)+bi}\}$
A_{22}	$(q^3-3q^2+2q-1) \sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} + (q-1)(3q^2-2q+1)\varepsilon^{a(i+j+k+l)+bi}$
A_{23}	$-(q-1)\{(q-1) \sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} + (2q-1)\varepsilon^{a(i+j+k+l)+bi}\}$
A_{24}	$(2q-1) \sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} + (3q-1)\varepsilon^{a(i+j+k+l)+bi}$
A_{25}	$-\sum_{(j,k,l)} \varepsilon^{a(2i+j+k)+bl} - \varepsilon^{a(i+j+k+l)+bi}$
A_{31}	$(q-1)(q^2-q+1)\{\sum_{(j,k,l)} \varepsilon^{a(2i+j)+b(k+l)} - (q-1)\sum_{(j,k,l)} \varepsilon^{a(i+j+k)+b(i+l)} + \varepsilon^{a(j+k+l)+2bi}\}$
A_{32}	$-(q^2-q+1)\{\sum_{(j,k,l)} \varepsilon^{a(2i+j)+b(k+l)} - (q-1)\sum_{(j,k,l)} \varepsilon^{a(i+j+k)+b(i+l)} - (q-1)\varepsilon^{a(j+k+l)+2bi}\}$
A_{33}	$-(q-1)\sum_{(j,k,l)} \{(q-1)\varepsilon^{a(2i+j)+b(k+l)} + (2q-1)\varepsilon^{a(i+j+k)+b(i+l)}\} + (2q-1)\varepsilon^{a(j+k+l)+2bi}$
A_{34}	$\sum_{(j,k,l)} \{(q-1)\varepsilon^{a(2i+j)+b(k+l)} + (2q-1)\varepsilon^{a(i+j+k)+b(i+l)}\} + (2q-1)\varepsilon^{a(j+k+l)+2bi}$
A_{35}	$(q-1)\sum_{(j,k,l)} \{\varepsilon^{a(2i+j)+b(k+l)} + \varepsilon^{a(i+j+k)+b(i+l)}\} - \varepsilon^{a(j+k+l)+2bi}$
A_{36}	$-\sum_{(j,k,l)} \{\varepsilon^{a(2i+j)+b(k+l)} + \varepsilon^{a(i+j+k)+b(i+l)}\} - \varepsilon^{a(j+k+l)+2bi}$
A_{41}	$(q^2-q+1)\{-\sum_{\{j,k,l\}} \varepsilon^{a(2i+j)+bk+cl} + (q-1)\sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl} + (q-1)\varepsilon^{a(j+k+l)+(b+c)i}\}$
A_{42}	$(q-1)\sum_{\{j,k,l\}} \varepsilon^{a(2i+j)+bk+cl} + (2q-1)\{\sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl} + \varepsilon^{a(j+k+l)+(b+c)i}\}$
A_{43}	$-\sum_{\{j,k,l\}} \varepsilon^{a(2i+j)+bk+cl} - \sum_{(j,k,l)}\sum_{(b,c)} \varepsilon^{a(i+j+k)+bi+cl} - \varepsilon^{a(j+k+l)+(b+c)i}$
A_{51}	$(q-1)\sum_{(a,b)}\sum_{(j,k,l)} \{\varepsilon^{2ai+b(j+k)+cl} - (q-1)\varepsilon^{a(i+j)+b(i+k)+cl} - (q-1)\varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{52}	$\sum_{(j,k,l)} \{-\varepsilon^{2ai+b(j+k)+cl} + (q-1)\varepsilon^{a(j+k)+2bi+cl}\} + (q-1)\sum_{(a,b)}\sum_{(j,k,l)} \{\varepsilon^{a(i+j)+b(i+k)+cl} + \varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{53}	$-\sum_{(a,b)}\sum_{(j,k,l)} \{\varepsilon^{2ai+b(j+k)+cl} + \varepsilon^{a(i+j)+b(i+k)+cl} + \varepsilon^{a(i+j)+b(k+l)+ci}\}$
A_{61}	$-\sum_{\{j,k,l\}} \varepsilon^{2ai+bj+ck+dl} + (q-1)\sum_{(b,c,d)} \{\sum_{\{j,k,l\}} \varepsilon^{a(i+j)+bi+ck+dl} + \sum_{(j,k,l)} \varepsilon^{a(j+k)+(b+c)i+dl}\}$
A_{62}	$-\sum_{\{j,k,l\}} \varepsilon^{2ai+bj+ck+dl} - \sum_{(b,c,d)} \{\sum_{\{j,k,l\}} \varepsilon^{a(i+j)+bi+ck+dl} + \sum_{(j,k,l)} \varepsilon^{a(j+k)+(b+c)i+dl}\}$

Table 2. (Continued)

	$A_{62}(i, j, k, l)$
A_7	$-(1/2)\cdot \sum_{\{a,b,c,d,e\}} \varepsilon^{(a+b)i+cj+dk+el}$
B_{11}	$(q-1)(q^2-q+1)\varepsilon^{a(j+k+l)-bi}$
B_{12}	$(2q-1)\varepsilon^{a(j+k+l)-bi}$
B_{13}	$-\varepsilon^{a(j+k+l)-bi}$
B_{21}	$(q-1) \sum_{(j,k,l)} \varepsilon^{a(j+k)-bi+cl}$
B_{22}	$-\sum_{(j,k,l)} \varepsilon^{a(j+k)-bi+cl}$
B_3	$-\sum_{\{j,k,l\}} \varepsilon^{aj+bk+cl-di}$
B_{41}	
B_{42}	
B_5	
C_{11}	
C_{12}	
C_2	
D	
E	
F	
Number of characters	$(1/6)\cdot q(q^2-1)(q-2)$

	$A_7(i, j, k, l, m)$	$B_{11}(i, j)$
A_{11}	$(q-1)^2(q^2-q+1)(q^2+1)(q^4-q^3+q^2-q+1)\varepsilon^{a(i+j+k+l+m)}$	$q^3(q^2+1)(q^5+1)\varepsilon^{a(i+3j)}$
A_{12}	$(q-1)(q^2-q+1)(4q^3-3q^2+2q-1)\varepsilon^{a(i+j+k+l+m)}$	$q^3\varepsilon^{a(i+3j)}$
A_{13}	$(q-1)(5q^3-6q^2+3q-1)\varepsilon^{a(i+j+k+l+m)}$	$(q^4+q^3)\varepsilon^{a(i+3j)}$
A_{14}	$-(q-1)(6q^2-3q+1)\varepsilon^{a(i+j+k+l+m)}$	$q^3\varepsilon^{a(i+3j)}$

Table 2. (Continued)

	$A_7(i,j,k,l,m)$	$B_{11}(i,j)$
A_{15}	$(5q^2-4q+1)\varepsilon^{a(i+j+k+l+m)}$	
A_{16}	$-(4q-1)\varepsilon^{a(i+j+k+l+m)}$	
A_{17}	$\varepsilon^{a(i+j+k+l+m)}$	
A_{21}	$(q-1)^2(q^2-q+1)(q^2+1)\sum_{(i,j,k,l,m)}\varepsilon^{a(i+j+k+l)+bm}$	$q(q^2+1)(q^3+1)\varepsilon^{a(i+2j)+bj}$
A_{22}	$-(q-1)(3q^2-2q+1)\sum_{(i,j,k,l,m)}\varepsilon^{a(i+j+k+l)+bm}$	$q\varepsilon^{a(i+2j)+bj}$
A_{23}	$(q-1)(2q-1)\sum_{(i,j,k,l,m)}\varepsilon^{a(i+j+k+l)+bm}$	$(q^2+q)\varepsilon^{a(i+2j)+bj}$
A_{24}	$-(3q-1)\sum_{(i,j,k,l,m)}\varepsilon^{a(i+j+k+l)+bm}$	$q\varepsilon^{a(i+2j)+bj}$
A_{25}	$\sum_{(i,j,k,l,m)}\varepsilon^{a(i+j+k+l)+bm}$	
A_{31}	$(1/12)\cdot(q-1)^2(q^2-q+1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	$(q^4+q)\varepsilon^{a(i+j)+2bj}+(q^4+q^3)\varepsilon^{3aj+bi}$
A_{32}	$-(1/12)\cdot(q-1)(q^2-q+1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	$q^3\varepsilon^{3aj+bi}$
A_{33}	$(1/12)\cdot(q-1)(2q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	$q\varepsilon^{a(i+j)+2bj}$
A_{34}	$-(1/12)\cdot(2q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	
A_{35}	$-(1/12)\cdot(q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	$q\varepsilon^{a(i+j)+2bj}$
A_{36}	$(1/12)\cdot\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+b(l+m)}$	
A_{41}	$-(1/6)\cdot(q-1)(q^2-q+1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+bl+cm}$	$-(q^3+1)\varepsilon^{ai+(a+b+c)j}$
A_{42}	$-(1/6)\cdot(2q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+bl+cm}$	$-\varepsilon^{ai+(a+b+c)j}$
A_{43}	$(1/6)\cdot\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j+k)+bl+cm}$	$-\varepsilon^{ai+(a+b+c)j}$
A_{51}	$(1/4)\cdot(q-1)^2\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j)+b(k+l)+cm}$	$(q^2+q)\sum_{(a,b)}\varepsilon^{ai+(2b+c)j}$
A_{52}	$-(1/4)\cdot(q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j)+b(k+l)+cm}$	$q\varepsilon^{bi+(2a+c)j}$
A_{53}	$(1/4)\cdot\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j)+b(k+l)+cm}$	

Table 2. (Continued)

	$A_7(i, j, k, l, m)$	$B_{11}(i, j)$
A_{01}	$-(1/2)\cdot(q-1)\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j)+bk+cl+dm}$	$-(q+1)\varepsilon^{ai+(b+c+d)j}$
A_{02}	$(1/2)\cdot\sum_{\{i,j,k,l,m\}}\varepsilon^{a(i+j)+bk+cl+dm}$	$-\varepsilon^{ai+(b+c+d)j}$
A_7	$\sum_{\{i,j,k,l,m\}}\varepsilon^{ai+bj+ck+dl+em}$	
B_{11}		$(q^3+1)\varepsilon^{ai+(a-b)j}+q^3\varepsilon^{3aj}\tilde{\eta}^{bi}$
B_{12}		$\varepsilon^{ai+(a-b)j}$
B_{13}		$\varepsilon^{ai+(a-b)j}$
B_{21}		$(q+1)\varepsilon^{ai+(c-b)j}+q\varepsilon^{(2a+c)j}\tilde{\eta}^{bi}$
B_{22}		$\varepsilon^{ai+(c-b)j}$
B_3		$-\varepsilon^{(a+b+c)j}\tilde{\eta}^{di}$
B_{41}		$(q^2+1)\varepsilon^{(b-a)j}\tilde{\eta}^{ai}$
B_{42}		$\varepsilon^{(b-a)j}\tilde{\eta}^{ai}$
B_5		$\sum_{(a,b)}\varepsilon^{(c-b)j}\tilde{\eta}^{ai}$
C_{11}		$-(q+1)\varepsilon^{ai+bj}$
C_{12}		$-\varepsilon^{ai+bj}$
C_2		
D		$-\varepsilon^{bj}\tilde{\eta}^{ai}$
E		
F		
Number of characters	$(1/5!)\cdot q(q^2-1)(q-2)(q-3)$	$(1/2)\cdot(q+1)(q^2-q-2)$

Table 2. (Continued)

	$B_{12}(i, j)$	$B_{13}(i, j)$
A_{11}	$q(q-1)(q^2+1)(q^5+1)\varepsilon^{a(i+3j)}$	$(q^2+1)(q^5+1)\varepsilon^{a(i+3j)}$
A_{12}	$-(q^6+q^3-q^2+q)\varepsilon^{a(i+3j)}$	$(q^5+q^2+1)\varepsilon^{a(i+3j)}$
A_{13}	$(q^2-q)\varepsilon^{a(i+3j)}$	$(q^2+1)\varepsilon^{a(i+3j)}$
A_{14}	$(q^2-q)\varepsilon^{a(i+3j)}$	$(q^3+q^2+1)\varepsilon^{a(i+3j)}$
A_{15}	$-q\varepsilon^{a(i+3j)}$	$(q^2+1)\varepsilon^{a(i+3j)}$
A_{16}	$-q\varepsilon^{a(i+3j)}$	$\varepsilon^{a(i+3j)}$
A_{17}		$\varepsilon^{a(i+3j)}$
A_{21}	$-(q-1)(q^2+1)(q^3+1)\varepsilon^{a(i+2j)+bj}$	$(q^2+1)(q^3+1)\varepsilon^{a(i+2j)+bj}$
A_{22}	$(q^3+q^2-q+1)\varepsilon^{a(i+2j)+bj}$	$(q^3+q^2+1)\varepsilon^{a(i+2j)+bj}$
A_{23}	$-(q-1)\varepsilon^{a(i+2j)+bj}$	$(q^2+1)\varepsilon^{a(i+2j)+bj}$
A_{24}	$-(q-1)\varepsilon^{a(i+2j)+bj}$	$\varepsilon^{a(i+2j)+bj}$
A_{25}	$\varepsilon^{a(i+2j)+bj}$	$\varepsilon^{a(i+2j)+bj}$
A_{31}	$-(q-1)(q^3+1)\varepsilon^{a(i+j)+2bj}+(q^3-q)\varepsilon^{3aj+bi}$	$(q^3+1)\varepsilon^{a(i+j)+2bj}+(q+1)\varepsilon^{3aj+bi}$
A_{32}	$(q^3+1)\varepsilon^{a(i+j)+2bj}+(q^2-q)\varepsilon^{3aj+bi}$	$(q^3+1)\varepsilon^{a(i+j)+2bj}+\varepsilon^{3aj+bi}$
A_{33}	$-(q-1)\varepsilon^{a(i+j)+2bj}-(q^2+q)\varepsilon^{3aj+bi}$	$\varepsilon^{a(i+j)+2bj}+(q+1)\varepsilon^{3aj+bi}$
A_{34}	$\varepsilon^{a(i+j)+2bj}-q\varepsilon^{3aj+bi}$	$\varepsilon^{a(i+j)+2bj}+\varepsilon^{3aj+bi}$
A_{35}	$-(q-1)\varepsilon^{a(i+j)+2bj}$	$\varepsilon^{a(i+j)+2bj}+(q+1)\varepsilon^{3aj+bi}$
A_{36}	$\varepsilon^{a(i+j)+2bj}$	$\varepsilon^{a(i+j)+2bj}+\varepsilon^{3aj+bi}$
A_{41}	$2(q^3+1)\varepsilon^{ai+(a+b+c)j}$	$(q^3+1)\varepsilon^{ai+(a+b+c)j}$
A_{42}	$2\varepsilon^{ai+(a+b+c)j}$	$\varepsilon^{ai+(a+b+c)j}$
A_{43}	$2\varepsilon^{ai+(a+b+c)j}$	$\varepsilon^{ai+(a+b+c)j}$
A_{51}	$-(q^2-1)\sum_{(a,b)}\varepsilon^{ai+(2b+c)j}$	$(q+1)\sum_{(a,b)}\varepsilon^{ai+(2b+c)j}$
A_{52}	$(q+1)\varepsilon^{ai+(2b+c)j}-(q-1)\varepsilon^{bi+(2a+c)j}$	$(q+1)\varepsilon^{ai+(2b+c)j}+\varepsilon^{bi+(2a+c)j}$
A_{53}	$\sum_{(a,b)}\varepsilon^{ai+(2b+c)j}$	$\sum_{(a,b)}\varepsilon^{ai+(2b+c)j}$
A_{61}	$2(q+1)\varepsilon^{ai+(b+c+d)j}$	$(q+1)\varepsilon^{ai+(b+c+d)j}$

Table 2. (Continued)

	$B_{12}(i, j)$	$B_{13}(i, j)$
A_{62}	$2\varepsilon^{ai+(b+c+d)j}$	$\varepsilon^{ai+(b+c+d)j}$
A_7		
B_{11}	$(q^2-q)\varepsilon^{3aj}\tilde{\eta}^{bi}$	$(q^3+1)\varepsilon^{ai+(a-b)j}+\varepsilon^{3aj}\tilde{\eta}^{bi}$
B_{12}	$-q\varepsilon^{3aj}\tilde{\eta}^{bi}$	$\varepsilon^{ai+(a-b)j}+\varepsilon^{3aj}\tilde{\eta}^{bi}$
B_{13}		$\varepsilon^{ai+(a-b)j}+\varepsilon^{3aj}\tilde{\eta}^{bi}$
B_{21}	$-(q-1)\varepsilon^{(2a+c)j}\tilde{\eta}^{bi}$	$(q+1)\varepsilon^{ai(c-b)j}+\varepsilon^{(2a+c)j}\tilde{\eta}^{bi}$
B_{22}	$\varepsilon^{(2a+c)j}\tilde{\eta}^{bi}$	$\varepsilon^{ai+(c-b)j}+\varepsilon^{(2a+c)j}\tilde{\eta}^{bi}$
B_3	$2\varepsilon^{(a+b+c)j}\tilde{\eta}^{di}$	$\varepsilon^{(a+b+c)j}\tilde{\eta}^{di}$
B_{41}		$(q^2+1)\varepsilon^{(b-a)j}\tilde{\eta}^{ai}$
B_{42}		$\varepsilon^{(b-a)j}\tilde{\eta}^{ai}$
B_5		$\sum_{(a,b)}\varepsilon^{(c-b)j}\tilde{\eta}^{ai}$
C_{11}	$-(q+1)\varepsilon^{ai+bj}$	$(q+1)\varepsilon^{ai+bj}$
C_{12}	$-\varepsilon^{ai+bj}$	ε^{ai+bj}
C_2		
D	$-\varepsilon^{bj}\tilde{\eta}^{ai}$	$\varepsilon^{bj}\tilde{\eta}^{ai}$
E		
F		
Number of characters	$(1/2)\cdot(q+1)(q^2-q-2)$	$(1/2)\cdot(q+1)(q^2-q-2)$

	$B_{21}(i, j, k)$	$B_{22}(i, j, k)$
A_{11}	$q(q^2-q+1)(q^2+1)(q^5+1)\varepsilon^{a(i+2j+k)}$	$(q^2-q+1)(q^2+1)(q^5+1)\varepsilon^{a(i+2j+k)}$
A_{12}	$q(q^2-q+1)(q^3+q^2+1)\varepsilon^{a(i+2j+k)}$	$-(q^2-q+1)(q^4-q^2-1)\varepsilon^{a(i+2j+k)}$
A_{13}	$(q^4+q^3-q^2+q)\varepsilon^{a(i+2j+k)}$	$(2q^2-q+1)\varepsilon^{a(i+2j+k)}$
A_{14}	$(q^3-q^2+q)\varepsilon^{a(i+2j+k)}$	$(q^3+2q^2-q+1)\varepsilon^{a(i+2j+k)}$

Table 2. (Continued)

	$B_{21}(i, j, k)$	$B_{22}(i, j, k)$
A_{15}	$q\varepsilon^{a(i+2j+k)}$	$(q^2-q+1)\varepsilon^{a(i+2j+k)}$
A_{16}	$q\varepsilon^{a(i+2j+k)}$	$-(q-1)\varepsilon^{a(i+2j+k)}$
A_{17}		$\varepsilon^{a(i+2j+k)}$
A_{21}	$(q^2+1)(q^3+1)\{(q-1)\varepsilon^{a(i+j+k)+bj}+q\varepsilon^{a(i+2j)+bk}\}$	$(q^2+1)(q^3+1)\{-(q-1)\varepsilon^{a(i+j+k)+bj}+\varepsilon^{a(i+2j)+bk}\}$
A_{22}	$-(q^3+q^2-q+1)\varepsilon^{a(i+j+k)+bj}+q\varepsilon^{a(i+2j)+bk}$	$(q^3+q^2-q+1)\varepsilon^{a(i+j+k)+bj}+(q^3+q^2+1)\varepsilon^{a(i+2j)+bk}$
A_{23}	$(q-1)\varepsilon^{a(i+j+k)+bj}+(q^2+q)\varepsilon^{a(i+2j)+bk}$	$-(q-1)\varepsilon^{a(i+j+k)+bj}+(q^2+1)\varepsilon^{a(i+2j)+bk}$
A_{24}	$(q-1)\varepsilon^{a(i+j+k)+bj}+q\varepsilon^{a(i+2j)+bk}$	$-(q-1)\varepsilon^{a(i+j+k)+bj}+\varepsilon^{a(i+2j)+bk}$
A_{25}	$-\varepsilon^{a(i+j+k)+bj}$	$\varepsilon^{a(i+j+k)+bj}+\varepsilon^{a(i+2j)+bk}$
A_{31}	$(q^3+1)\{(q-1)\varepsilon^{a(i+j)+b(j+k)}+q\varepsilon^{a(i+k)+2bj}+q\varepsilon^{a(2j+k)+bi}\}$	$(q^3+1)\{-(q-1)\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(i+k)+2bj}+\varepsilon^{a(2j+k)+bi}\}$
A_{32}	$-(q^3+1)\varepsilon^{a(i+j)+b(j+k)}+(q^3-q^2+q)\varepsilon^{a(2j+k)+bi}$	$(q^3+1)\{\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(i+k)+2bj}\}+(q^2-q+1)\varepsilon^{a(2j+k)+bi}$
A_{33}	$(q-1)\varepsilon^{a(i+j)+b(j+k)}+q\varepsilon^{a(i+k)+2bj}+(q^2+q)\varepsilon^{a(2j+k)+bi}$	$-(q-1)\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(i+k)+2bj}-(q^2-1)\varepsilon^{a(2j+k)+bi}$
A_{34}	$-\varepsilon^{a(i+j)+b(j+k)}+q\varepsilon^{a(2j+k)+bi}$	$\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(j+k)+2bj}-(q-1)\varepsilon^{a(2j+k)+bi}$
A_{35}	$(q-1)\varepsilon^{a(i+j)+b(j+k)}+q\varepsilon^{a(i+k)+2bj}$	$-(q-1)\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(i+k)+2bj}+(q+1)\varepsilon^{a(2j+k)+bi}$
A_{36}	$-\varepsilon^{a(i+j)+b(j+k)}$	$\varepsilon^{a(i+j)+b(j+k)}+\varepsilon^{a(i+k)+2bj}+\varepsilon^{a(2j+k)+bi}$
A_{41}	$-(q^3+1)\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$	$(q^3+1)\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$
A_{42}	$-\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$	$\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$
A_{43}	$-\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$	$\varepsilon^{ai}\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}$
A_{51}	$(q+1)\sum_{(a,b)}\{(q-1)\varepsilon^{ai+b(j+k)+cj}+q\varepsilon^{ai+2bj+ck}\}$	$(q+1)\sum_{(a,b)}\{-(q-1)\varepsilon^{ai+b(j+k)+cj}+\varepsilon^{ai+2bj+ck}\}$
A_{52}	$-(q+1)\varepsilon^{ai+b(j+k)+cj}+(q-1)\varepsilon^{a(j+k)+bi+cj}+q\varepsilon^{2aj+bi+ck}$	$(q+1)\{\varepsilon^{ai+b(j+k)+cj}+\varepsilon^{ai+2bj+ck}\}-(q-1)\varepsilon^{a(j+k)+bi+cj}+\varepsilon^{2aj+bi+ck}$
A_{53}	$-\sum_{(a,b)}\varepsilon^{ai+b(j+k)+cj}$	$\sum_{(a,b)}\{\varepsilon^{ai+b(j+k)+cj}+\varepsilon^{ai+2bj+ck}\}$
A_{61}	$-(q+1)\sum_{(b,c,d)}\varepsilon^{ai+(b+c)j+dk}$	$(q+1)\sum_{(b,c,d)}\varepsilon^{ai+(b+c)j+dk}$
A_{62}	$-\sum_{(b,c,d)}\varepsilon^{ai+(b+c)j+dk}$	$\sum_{(b,c,d)}\varepsilon^{ai+(b+c)j+dk}$
A_7		

Table 2. (Continued)

	$B_{21}(i, j, k)$	$B_{22}(i, j, k)$
B_{11}	$(q^3+1)\varepsilon^{a(i+k)-bj}+(q^3-q^2+q)\varepsilon^{a(2j+k)}\tilde{\eta}^{bi}$	$(q^3+1)\varepsilon^{a(i+k)-bj}+(q^2-q+1)\varepsilon^{a(2j+k)}\tilde{\eta}^{bi}$
B_{12}	$\varepsilon^{a(i+k)-bj}+q\varepsilon^{a(2j+k)}\tilde{\eta}^{bi}$	$\varepsilon^{a(i+k)-bj}-(q-1)\varepsilon^{a(2j+k)}\tilde{\eta}^{bi}$
B_{13}	$\varepsilon^{a(i+k)-bj}$	$\varepsilon^{a(i+k)-bj}+\varepsilon^{a(2j+k)}\tilde{\eta}^{bi}$
B_{21}	$(q+1)\varepsilon^{ai-bj+ck}+(q-1)\varepsilon^{a(j+k)+cj}\tilde{\eta}^{bi}+q\varepsilon^{2aj+ck}\tilde{\eta}^{bi}$	$(q+1)\varepsilon^{ai-bj+ck}-(q-1)\varepsilon^{a(j+k)+cj}\tilde{\eta}^{bi}+\varepsilon^{2aj+ck}\tilde{\eta}^{bi}$
B_{22}	$\varepsilon^{ai-bj+ck}-\varepsilon^{a(j+k)+cj}\tilde{\eta}^{bi}$	$\varepsilon^{ai-bj+ck}+\varepsilon^{a(j+k)+cj}\tilde{\eta}^{bi}+\varepsilon^{2aj+ck}\tilde{\eta}^{bi}$
B_3	$-\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}\tilde{\eta}^{di}$	$\sum_{(a,b,c)}\varepsilon^{(a+b)j+ck}\tilde{\eta}^{di}$
B_{41}	$(q^2+1)\varepsilon^{bk-aj}\tilde{\eta}^{ai}$	$(q^2+1)\varepsilon^{bk-aj}\tilde{\eta}^{ai}$
B_{42}	$\varepsilon^{bk-aj}\tilde{\eta}^{ai}$	$\varepsilon^{bk-aj}\tilde{\eta}^{ai}$
B_5	$\sum_{(a,b)}\varepsilon^{ck-bj}\tilde{\eta}^{ai}$	$\sum_{(a,b)}\varepsilon^{ck-bj}\tilde{\eta}^{ai}$
C_{11}		
C_{12}		
C_2		
D		
E		
F		
Number of characters	$(1/2)\cdot q(q+1)(q^2-q-2)$	$(1/2)\cdot q(q+1)(q^2-q-2)$

	$B_3(i, j, k, l)$	$B_{41}(i, j)$	$B_{42}(i, j)$
A_{11}	$(q-1)(q^2-q+1)(q^2+1)(q^5+1)\varepsilon^{a(i+j+k+l)}$	$q^2(q^3+1)(q^5+1)\varepsilon^{a(2i+j)}$	$(q^3+1)(q^5+1)\varepsilon^{a(2i+j)}$
A_{12}	$(q^2-q+1)(2q^4+q^3-q^2+q-1)\varepsilon^{a(i+j+k+l)}$	$q^2(q^3+1)\varepsilon^{a(2i+j)}$	$(q^3+1)\varepsilon^{a(2i+j)}$
A_{13}	$(q-1)(q^3+2q^2-q+1)\varepsilon^{a(i+j+k+l)}$	$q^2\varepsilon^{a(2i+j)}$	$(q^4+q^3+1)\varepsilon^{a(2i+j)}$
A_{14}	$-(3q^2-2q+1)\varepsilon^{a(i+j+k+l)}$	$(q^3+q^2)\varepsilon^{a(2i+j)}$	$(q^3+1)\varepsilon^{a(2i+j)}$
A_{15}	$-(q-1)^2\varepsilon^{a(i+j+k+l)}$	$q^2\varepsilon^{a(2i+j)}$	$\varepsilon^{a(2i+j)}$
A_{16}	$(2q-1)\varepsilon^{a(i+j+k+l)}$		$\varepsilon^{a(2i+j)}$

Table 2. (Continued)

	$B_3(i,j,k,l)$	$B_{41}(i,j)$	$B_{42}(i,j)$
A_{17}	$-\varepsilon^{a(i+j+k+l)}$		$\varepsilon^{a(2i+j)}$
A_{21}	$(q-1)(q^2+1)(q^3+1)\sum_{(j,k,l)}\varepsilon^{a(i+j+k)+bl}$	$q^2(q+1)(q^3+1)\varepsilon^{2ai+bj}$	$(q+1)(q^3+1)\varepsilon^{2ai+bj}$
A_{22}	$-(q^3+q^2-q+1)\sum_{(j,k,l)}\varepsilon^{a(i+j+k)+bl}$	$(q^3+q^2)\varepsilon^{2ai+bj}$	$(q+1)\varepsilon^{2ai+bj}$
A_{23}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(i+j+k)+bl}$	$q^2\varepsilon^{2ai+bj}$	$(q^2+q+1)\varepsilon^{2ai+bj}$
A_{24}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(i+j+k)+bl}$		$(q+1)\varepsilon^{2ai+bj}$
A_{25}	$-\sum_{(j,k,l)}\varepsilon^{a(i+j+k)+bl}$		ε^{2ai+bj}
A_{31}	$(q-1)(q^3+1)\{\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}+\varepsilon^{a(j+k+l)+bi}\}$	$(q+1)(q^3+1)\varepsilon^{(a+b)i+aj}$	$(q+1)(q^3+1)\varepsilon^{(a+b)i+aj}$
A_{32}	$-(q^3+1)\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}+(q-1)(q^2-q+1)\varepsilon^{a(j+k+l)+bi}$	$(q^3+1)\varepsilon^{(a+b)i+aj}$	$(q^3+1)\varepsilon^{(a+b)i+aj}$
A_{33}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}+(q+1)(2q-1)\varepsilon^{a(j+k+l)+bi}$	$(q+1)\varepsilon^{(a+b)i+aj}$	$(q+1)\varepsilon^{(a+b)i+aj}$
A_{34}	$-\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}+(2q-1)\varepsilon^{a(j+k+l)+bi}$	$\varepsilon^{(a+b)i+aj}$	$\varepsilon^{(a+b)i+aj}$
A_{35}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}-(q+1)\varepsilon^{a(j+k+l)+bi}$	$(q+1)\varepsilon^{(a+b)i+aj}$	$(q+1)\varepsilon^{(a+b)i+aj}$
A_{36}	$-\sum_{(j,k,l)}\varepsilon^{a(i+j)+b(k+l)}-\varepsilon^{a(j+k+l)+bi}$	$\varepsilon^{(a+b)i+aj}$	$\varepsilon^{(a+b)i+aj}$
A_{41}	$-(q^3+1)\sum_{\{j,k,l\}}\varepsilon^{a(i+j)+bk+cl}$		
A_{42}	$-\sum_{\{j,k,l\}}\varepsilon^{a(i+j)+bk+cl}$		
A_{43}	$-\sum_{\{j,k,l\}}\varepsilon^{a(i+j)+bk+cl}$		
A_{51}	$(q^2-1)\sum_{(a,b)}\sum_{(j,k,l)}\varepsilon^{ai+b(j+k)+cl}$	$(q+1)^2\varepsilon^{(a+b)i+cj}$	$(q+1)^2\varepsilon^{(a+b)i+cj}$
A_{52}	$\sum_{(j,k,l)}\{-(q+1)\varepsilon^{ai+b(j+k)+cl}+(q-1)\varepsilon^{a(j+k)+bi+cl}\}$	$(q+1)\varepsilon^{(a+b)i+cj}$	$(q+1)\varepsilon^{(a+b)i+cj}$
A_{53}	$-\sum_{(a,b)}\sum_{(j,k,l)}\varepsilon^{ai+b(j+k)+cl}$	$\varepsilon^{(a+b)i+cj}$	$\varepsilon^{(a+b)i+cj}$
A_{61}	$-(q+1)\sum_{\{j,k,l\}}\varepsilon^{ai+bj+ck+dl}$		
A_{62}	$-\sum_{\{j,k,l\}}\varepsilon^{ai+bj+ck+dl}$		

Table 2. (Continued)

	$B_8(i,j,k,l)$	$B_{41}(i,j)$	$B_{42}(i,j)$
A_7			
B_{11}	$(q-1)(q^2-q+1)\varepsilon^{a(j+k+l)}\tilde{\eta}^{bi}$	$(q^3+1)\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$	$(q^3+1)\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$
B_{12}	$(2q-1)\varepsilon^{a(j+k+l)}\tilde{\eta}^{bi}$	$\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$	$\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$
B_{13}	$-\varepsilon^{a(j+k+l)}\tilde{\eta}^{bi}$	$\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$	$\varepsilon^{a(i+j)}\tilde{\eta}^{bi}$
B_{21}	$(q-1)\sum_{(j,k,l)}\varepsilon^{a(j+k)+c}\tilde{\eta}^{lbi}$	$(q+1)\varepsilon^{ai+cj}\tilde{\eta}^{bi}$	$(q+1)\varepsilon^{ai+cj}\tilde{\eta}^{bi}$
B_{22}	$-\sum_{(j,k,l)}\varepsilon^{a(j+k)+cl}\tilde{\eta}^{bi}$	$\varepsilon^{ai+cj}\tilde{\eta}^{ai}$	$\varepsilon^{ai+cj}\tilde{\eta}^{bi}$
B_{23}	$-\sum_{(j,k,l)}\varepsilon^{aj+bk+cl}\tilde{\eta}^{di}$		
B_{41}		$q^2\varepsilon^{bj}\tilde{\eta}^{2ai}+(q^2+1)\varepsilon^{bj-ai}$	$\varepsilon^{bj}\tilde{\eta}^{2ai}+(q^2+1)\varepsilon^{bj-ai}$
B_{42}		ε^{bj-ai}	$\varepsilon^{bj}\tilde{\eta}^{2ai}+\varepsilon^{bj-ai}$
B_5		$\varepsilon^{bj}\tilde{\eta}^{ai}\tilde{\eta}^{bi}$	$\varepsilon^{cj}\tilde{\eta}^{ai}\tilde{\eta}^{bi}$
C_{11}			
C_{12}			
C_2			
D			
E		$-\varepsilon^{bj}\tilde{\eta}^{ai}$	$\varepsilon^{bj}\tilde{\eta}^{ai}$
F			
Number of characters	$(1/12)\cdot q(q^2-1)(q^2-q-2)$	$(1/2)\cdot(q+1)(q^2-q-2)$	$(1/2)\cdot(q+1)(q^2-q-2)$

	$B_5(i,j,k)$	$C_{11}(i,j)$	$C_{12}(i,j)$	$C_2(i,j,k)$
A_{11}	$(q^2+1)(q^3+1)(q^5+1)\varepsilon^{a(i+j+k)}$	$q(q^4-1)(q^5+1)\varepsilon^{a(2i+j)}$	$(q^4-1)(q^5+1)\varepsilon^{a(2i+j)}$	$(q-1)(q^4-1)(q^5+1)\varepsilon^{a(i+j+k)}$
A_{12}	$(q^2+1)(q^3+1)\varepsilon^{a(i+j+k)}$	$-(q^4+q)\varepsilon^{a(2i+j)}$	$-(q+1)$ $\times(q^5-q^3+q^2-q+1)\varepsilon^{a(2i+j)}$	(q^2-1) $\times(q^4+q^3-q^2+q-1)\varepsilon^{a(i+j+k)}$
A_{13}	$(q^4+q^3+q^2+1)\varepsilon^{a(i+j+k)}$	$(q^3-q)\varepsilon^{a(2i+j)}$	$(q^4-1)\varepsilon^{a(2i+j)}$	$-(q-1)(q^3+1)\varepsilon^{a(i+j+k)}$

Table 2. (Continued)

	$B_5(i,j,k)$	$C_{11}(i,j)$	$C_{12}(i,j)$	$C_2(i,j,k)$
A_{14}	$(q+1)(2q^2-q+1)\varepsilon^{a(i+j+k)}$	$-q\varepsilon^{a(2i+j)}$	$-\varepsilon^{a(2i+j)}$	$-(q-1)\varepsilon^{a(i+j+k)}$
A_{15}	$(q^2+1)\varepsilon^{a(i+j+k)}$	$-(q^2+q)\varepsilon^{a(2i+j)}$	$-\varepsilon^{a(2i+j)}$	$-(q^2+q-1)\varepsilon^{a(i+j+k)}$
A_{16}	$\varepsilon^{a(i+j+k)}$	$-q\varepsilon^{a(2i+j)}$	$-\varepsilon^{a(2i+j)}$	$-(q-1)\varepsilon^{a(i+j+k)}$
A_{17}	$\varepsilon^{a(i+j+k)}$		$-\varepsilon^{a(2i+j)}$	$\varepsilon^{a(i+j+k)}$
A_{21}	$(q+1)(q^2+1)(q^3+1)\varepsilon^{a(i+j)+bk}$	$(q^2-1)(q^4-1)\varepsilon^{a(i+j)+bi}$	$-(q^2-1)(q^4-1)\varepsilon^{a(i+j)+bi}$	$(q^2-1)(q^4-1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$
A_{22}	$(q+1)(q^2+1)\varepsilon^{a(i+j)+bk}$		$(q^2-1)\varepsilon^{a(i+j)+bi}$	$-(q^2-1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$
A_{23}	$(2q^2+q+1)\varepsilon^{a(i+j)+bk}$		$(q^2-1)\varepsilon^{a(i+j)+bi}$	$-(q^2-1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$
A_{24}	$(q+1)\varepsilon^{a(i+j)+bk}$		$-\varepsilon^{a(i+j)+bi}$	$\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$
A_{25}	$\varepsilon^{a(i+j)+bk}$		$-\varepsilon^{a(i+j)+bi}$	$\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$
A_{31}	$(q+1)(q^3+1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$	$q(q+1)(q^2-1)\varepsilon^{aj+2bi}$	$(q+1)(q^2-1)\varepsilon^{aj+2bi}$	$(q^2-1)^2\varepsilon^{ak+b(i+j)}$
A_{32}	$(q^3+1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$		$(q+1)(q^2-1)\varepsilon^{aj+2bi}$	$-(q+1)(q^2-1)\varepsilon^{ak+b(i+j)}$
A_{33}	$(q+1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$	$-(q^2+q)\varepsilon^{aj+2bi}$	$-(q+1)\varepsilon^{aj+2bi}$	$-(q^2-1)\varepsilon^{ak+b(i+j)}$
A_{34}	$\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$		$-(q+1)\varepsilon^{aj+2bi}$	$(q+1)\varepsilon^{ak+b(i+j)}$
A_{35}	$(q+1)\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$	$-q\varepsilon^{aj+2bi}$	$-\varepsilon^{aj+2bi}$	$-(q-1)\varepsilon^{ak+b(i+j)}$
A_{36}	$\sum_{(i,j)}\varepsilon^{a(i+k)+bj}$		$-\varepsilon^{aj+2bi}$	$\varepsilon^{ak+b(i+j)}$
A_{41}		$-(q+1)(q^2-1)\varepsilon^{aj+(b+c)i}$	$(q+1)(q^2-1)\varepsilon^{aj+(b+c)i}$	$-(q+1)(q^2-1)\sum_{(i,j)}\varepsilon^{ak+bi+cj}$
A_{42}			$-(q+1)\varepsilon^{aj+(b+c)i}$	$(q+1)\sum_{(i,j)}\varepsilon^{ak+bi+cj}$
A_{43}			$-\varepsilon^{aj+(b+c)i}$	$\sum_{(i,j)}\varepsilon^{ak+bi+cj}$
A_{51}	$(q+1)^2\sum_{(i,j)}\varepsilon^{ai+bj+ck}$			
A_{52}	$(q+1)\sum_{(i,j)}\varepsilon^{ai+bj+ck}$			

Table 2. (Continued)

	$B_5(i,j,k)$	$C_{11}(i,j)$	$C_{12}(i,j)$	$C_2(i,j,k)$
A_{53}	$\sum_{(i,j)} \varepsilon^{ai+bj+ck}$			
A_{61}				
A_{62}				
A_7				
B_{11}	$(q^3+1)\sum_{(i,j)} \varepsilon^{a(i+k)}\tilde{\eta}^{bj}$	$(q+1)(q^2-1)\varepsilon^{aj-bi}$	$(q+1)(q^2-1)\varepsilon^{aj-bi}$	
B_{12}	$\sum_{(i,j)} \varepsilon^{a(i+k)}\tilde{\eta}^{bj}$	$-(q+1)\varepsilon^{aj-bi}$	$-(q+1)\varepsilon^{aj-bi}$	
B_{13}	$\sum_{(i,j)} \varepsilon^{a(i+k)}\tilde{\eta}^{bj}$	$-\varepsilon^{aj-bi}$	$-\varepsilon^{aj-bi}$	
B_{21}	$(q+1)\sum_{(i,j)} \varepsilon^{ai+ck}\tilde{\eta}^{bj}$			
B_{22}	$\sum_{(i,j)} \varepsilon^{ai+ck}\tilde{\eta}^{bj}$			
B_3				
B_{41}	$(q^2+1)\{\varepsilon^{bk}\tilde{\eta}^{a(i+j)}+\varepsilon^{bk}\tilde{\eta}^{a(i-jq)}\}$			
B_{42}	$\varepsilon^{bk}\tilde{\eta}^{a(i+j)}+\varepsilon^{bk}\tilde{\eta}^{a(i-jq)}$			
B_5	$\sum_{(i,j)} \varepsilon^{ck}\tilde{\eta}^{ai}\tilde{\eta}^{bj}$			
C_{11}		$-q\varepsilon^{2a}\tilde{\xi}^{ibj}$	$-\varepsilon^{2ai}\tilde{\xi}^{bj}$	$-(q-1)\varepsilon^{a(i+j)}\tilde{\xi}^{bk}$
C_{12}			$-\varepsilon^{2ai}\tilde{\xi}^{bj}$	$\varepsilon^{a(i+j)}\tilde{\xi}^{bk}$
C_2		$\varepsilon^{(a+b)i}\tilde{\xi}^{cj}$	$-\varepsilon^{(a+b)i}\tilde{\xi}^{cj}$	$\sum_{(i,j)} \varepsilon^{ai+bj}\tilde{\xi}^{ck}$
D		$-\varepsilon^{-ai}\tilde{\xi}^{bj}$	$-\varepsilon^{-ai}\tilde{\xi}^{bj}$	
E				
F				
Number of characters	$(1/8)\cdot(q+1)(q^2-q-2)(q^2-q-4)$	$(1/3)\cdot q(q+1)(q^2-1)$	$(1/3)\cdot q(q+1)(q^2-1)$	$(1/6)\cdot q^2(q+1)(q^2-1)$

Table 2. (Continued)

	$D(i, j)$	$E(i, j)$	$F(i)$
A_{11}	$(q+1)(q^4-1)(q^5+1)\varepsilon^{a(i+j)}$	$(q^2-1)(q^3+1)(q^5+1)\varepsilon^{a(i+j)}$	$(q+1)(q^2-1)(q^3+1)(q^4-1)\varepsilon^{ai}$
A_{12}	$-(q+1)(q^5+1)\varepsilon^{a(i+j)}$	$(q^2-1)(q^3+1)\varepsilon^{a(i+j)}$	$-(q+1)(q^2-1)(q^3+1)\varepsilon^{ai}$
A_{13}	$(q+1)(q^3-1)\varepsilon^{a(i+j)}$	$-(q+1)(q^3-q+1)\varepsilon^{a(i+j)}$	$-(q+1)(q^2-1)\varepsilon^{ai}$
A_{14}	$-(q+1)\varepsilon^{a(i+j)}$	$(q^2-1)\varepsilon^{a(i+j)}$	$-(q+1)(q^2-1)\varepsilon^{ai}$
A_{15}	$-(q^2+q+1)\varepsilon^{a(i+j)}$	$(q^2-1)\varepsilon^{a(i+j)}$	$(q+1)\varepsilon^{ai}$
A_{16}	$-(q+1)\varepsilon^{a(i+j)}$	$-\varepsilon^{a(i+j)}$	$(q+1)\varepsilon^{ai}$
A_{17}	$-\varepsilon^{a(i+j)}$	$-\varepsilon^{a(i+j)}$	ε^{ai}
A_{21}		$(q+1)(q^2-1)(q^3+1)\varepsilon^{ai+bj}$	
A_{22}		$(q+1)(q^2-1)\varepsilon^{ai+bj}$	
A_{23}		$-(q+1)\varepsilon^{ai+bj}$	
A_{24}		$-(q+1)\varepsilon^{ai+bj}$	
A_{25}		$-\varepsilon^{ai+bj}$	
A_{31}	$(q+1)^2(q^2-1)\varepsilon^{aj+bi}$		
A_{32}	$(q+1)(q^2-1)\varepsilon^{aj+bi}$		
A_{33}	$-(q+1)^2\varepsilon^{aj+bi}$		
A_{34}	$-(q+1)\varepsilon^{aj+bi}$		
A_{35}	$-(q+1)\varepsilon^{aj+bi}$		
A_{36}	$-\varepsilon^{aj+bi}$		
A_{41}			
A_{42}			
A_{43}			
A_{51}			
A_{52}			
A_{53}			
A_{61}			
A_{62}			

Table 2. (Continued)

	$D(i, j)$	$E(i, j)$	$F(i)$
A_7			
B_{11}	$(q+1)(q^2-1)\varepsilon^{aj}\tilde{\eta}^{bi}$		
B_{12}	$-(q+1)\varepsilon^{aj}\tilde{\eta}^{bi}$		
B_{13}	$-\varepsilon^{aj}\tilde{\eta}^{bi}$		
B_{21}			
B_{22}			
B_3			
B_{41}		$(q^2-1)\varepsilon^{bj}\tilde{\eta}^{ai}$	
B_{42}		$-\varepsilon^{bj}\tilde{\eta}^{ai}$	
B_5			
C_{11}	$-(q+1)\varepsilon^{ai}\tilde{\xi}^{bj}$		
C_{12}	$-\varepsilon^{ai}\tilde{\xi}^{bj}$		
C_2			
D	$-\tilde{\eta}^{ai}\tilde{\xi}^{bj}$		
E		$-\varepsilon^{bj}\tilde{\zeta}^{ai}$	
F			$\tilde{\lambda}^{ai}$
Number of characters	$(1/6)\cdot q(q^2-1)(q^2-q-2)$	$(1/4)\cdot q^2(q+1)(q^2-1)$	$(1/5)\cdot q(q^4-1)$

$R_0=\mathbf{Z}/(q^3+1)\mathbf{Z}$, $R_1=\{x\in R_0 ;\ x\not\equiv 0 \bmod q^2-q+1\}$,

$S_0=\mathbf{Z}/(q^4-1)\mathbf{Z}$, $S_1=\{x\in S_0 ;\ x\not\equiv 0 \bmod q^2+1\}$,

$T_0=\mathbf{Z}/(q^5+1)\mathbf{Z}$, $T_1=\{x\in T_0 ;\ x\not\equiv 0 \bmod q^4-q^3+q^2-q+1\}$.

$\sum_{(x,y,\dots,z)}$ means a sum over the cyclic permutations of $x, y, \cdots, z$ and $\sum_{\{x,y,\dots,z\}}$ means a sum over all permutations of $x, y, \cdots, z$. When the value of a character is zero, the corresponding entry in the character table will be left vacant. We also use the abbreviations:

$$\tilde{\eta}^a=\eta^a+\eta^{-aq}\ ,\quad \tilde{\xi}^a=\xi^a+\xi^{-aq}+\xi^{aq^2}\ ,\quad \tilde{\zeta}^a=\zeta^a+\zeta^{-aq}+\zeta^{aq^2}+\zeta^{-aq^3}\ ,$$

$$\tilde{\lambda}^a=\lambda^a+\lambda^{-aq}+\lambda^{aq^2}+\lambda^{-aq^3}+\lambda^{aq^4}\ .$$

We note that $U_5=U(5, q^2)$, $P_5^{(2)}$ and $P_5^{(1)}$ are generated by $B\cup\{\omega_1, \omega_2\}$, $B\cup\{\omega_1\}$ and $B\cup\{\omega_2\}$ respectively, where B is the Borel subgroup of $U(5, q^2)$ generated by $\{h(\sigma^x, \sigma^y, \rho^z), y_1(r), y_2(s, t), y_3(u, v), y_4(w) ;\ (x, y, z)\in J_0^2\times I_1,\ r, s, t, u, v, w\in K\}$.

From the results of G. E. Wall [7], we see that the conjugacy classes of U_5 are parametrized in the table 1[1].

Our assertion is that the character table of $U(5, q^2)$ is as given in table 2.

§4.

It is clear that $A_{17}(i)$ $(i\in I_1)$ is the family of linear characters which correspond to the family of conjugacy class A_{17}. By the theorem in §2 for $n=5$ and the group $P_5^{(1)}$ we can find that eight families

$$B_{1m}(i, j)\quad (m=1, 2, 3)\ ,$$

$$B_{2m}(i, j, k)\quad (m=1, 2)\ ,$$

$$B_3(i, j, k, l)\ ,$$

$$B_5(i, j, k)\ ,$$

and $D(i, j)$

are irreducible characters induced from $U(3, q^2)$ which correspond to the families of conjugacy classes B_{1m} $(m=1, 2, 3)$, B_{2m} $(m=1, 2)$, B_3, B_5 and D respectively, if and only if $(i, j)\in J_1\times I_1$, $(i, j, k)\in J_1\times I_2$, $(i, j, k, l)\in J_1\times I_3$, $(i, j, k)\in J_2\times I_1$ and $(i, j)\in J_1\times R_1$ respectively. By the theorem in §2 for $n=5$ and the group $P_5^{(2)}$ we can also find that three families

[1] The class representatives which we shall use in this table are not necessarily the elements of $U(5, q^2)$, but their canonical forms in an extension field of K.

$$B_{4m}(i, j) \quad (m=1, 2)$$

$$\text{and} \quad E(i, j)$$

are irreducible characters induced from $GL(2, q^2)$ which correspond to the families of conjugacy classes B_{4m} $(m=1, 2)$ and E respectively, if and only if $(i, j) \in J_1 \times I_1$ and $(i, j) \in S_1 \times I_1$ respectively. We may note that $B_5(i, j, k)$ are also induced from $GL(2, q^2)$. Moreover we get the family $A_{11}(i)$ of Steinberg characters corresponding to the family of conjugacy class A_{11} as follows:

$$A_{11}(i) = B_{11}((1-q)i, i) - B_{42}((1-q)i, i) + A_{17}(i) .$$

Next we have to prove that four families

$$A_{43}(i, j, k) , \quad (i, j, k) \in I_1^3 ,$$

$$A_{62}(i, j, k, l) , \quad (i, j, k, l) \in I_1^4 ,$$

$$C_{12}(i, j) , \quad (i, j) \in I_1 \times R_0$$

$$\text{and} \quad C_2(i, j, k) , \quad (i, j, k) \in I_1^2 \times R_0$$

which correspond to the families of conjugacy classes A_{43}, A_{62}, C_{12}, and C_2 respectively are characters of U_5. To do this, we make use of the following Brauer's fundamental theorem of characters of finite groups, that is,

A classfunction χ on a finite group G is a character of G if and only if the restriction of χ to H is a character of H for every elementary subgroup H of G.

Hence, in order to show that these four families of classfunctions on U_5 are characters of U_5, it is enough to show that these classfunctions are characters of a system of subgroups of U_5, which has the property that this system together with all its conjugates covers every elementary subgroup of U_5. In our case we now have

LEMMA (Ennola [1]). *This property is possessed by the following system of subgroups*

1) $\{S_p \times Z\}$, *where* S_p *is the Sylow p-subgroup of* U_5 (q=*power of a prime* p) *and* Z *is the center of* U_5,

2) $\{U_i \times U_{5-i}\}$ $(i=1, 2)$, *where* $U_i \times U_{5-i}$ *means the subgroup of* U_5 *which consists of all matrices of the forms* $diag\{x, y\}$ *with* $x \in U_i$ *and* $y \in U_{5-i}$,

3) $\{C_{U_5}(F)\}$, *where* $C_{U_5}(F)$ *is the centralizer of conjugacy class* F,

4) $\{S_5 Z\}$, *where* S_5 *is the Sylow* 5-*subgroup of* U_5.

We omit the straightforward verification, that this is the case. Calculating the scalar product, we see that $A_{43}(i, j, k)$, $A_{62}(i, j, k, l)$, $C_{12}(i, j)$ and $C_2(i, j, k)$ are irreducible characters which correspond to the families of conjugacy classes A_{43}, A_{62}, C_{12} and C_2 respectively if and only if $(i,j,k)\in I_3$, $(i,j,k,l)\in I_4$, $(i,j)\in I_1\times R_1$ and $(i,j,k)\in I_2\times R_1$ respectively. We also get four families $C_{11}(i,j)((i,j)\in I_1\times R_1)$, $A_{61}(i,j,k,l)$ $((i,j,k,l)\in I_4)$, $A_{41}(i,j,k)$ $((i,j,k)\in I_3)$ and $A_{42}(i,j,k)$ $((i,j,k)\in I_3)$ of irreducible characters corresponding to the families of conjugacy classes C_{11}, A_{61}, A_{41} and A_{42} respectively as follows.

$$
\begin{aligned}
&C_{11}(i,j)=C_{12}(i,j)+C_2(i,i,j)\ ,\\
&A_{61}(i,j,k,l)=B_3((1-q)i,j,k,l)-A_{62}(i,j,k,l)\ ,\\
&A_{41}(i,j,k)=B_3((1-q)i,i,j,k)-A_{43}(i,j,k)\ ,\\
&A_{42}(i,j,k)=A_{62}(i,i,j,k)-A_{43}(i,j,k)\ .
\end{aligned}
$$

We can now construct the remaining irreducible characters of U_5 as a biproduct of the families of characters which we have already obtained. We first consider three families of irreducible characters of U_5 corresponding to the families of conjugacy classes A_{51} to A_{53}. Calculating the scalar product, we see that each of three families $A_{62}(j,i,i,k)$, $B_{21}((1-q)j,i,k)$ and $B_{22}((1-q)i,j,k)$ is the sum or the difference of two irreducible characters, which are not among those that we have already obtained, for $(i,j,k)\in I_3$. Calculating again the scalar product, we see that

$$
\|A_{62}(j,i,i,k)-B_{21}((1-q)r,s,k)\|=\begin{cases}2 & \text{if } (r,s)=(j,i)\\ 4 & \text{otherwise}\end{cases}
$$

$$
\|B_{22}((1-q)i,j,k)-B_{21}((1-q)r,s,k)\|=\begin{cases}2 & \text{if } (r,s)=(j,i)\\ 4 & \text{otherwise}\end{cases}
$$

$$
\|B_{22}((1-q)i,j,k)-A_{62}(j,i,i,k)\|=4
$$

$$
\|B_{21}((1-q)j,i,k)-B_{21}((1-q)i,j,k)\|=2\ .
$$

Hence we can assume that $B_{21}((1-q)j,i,k)=\varphi_1+\varphi_2$ and $B_{22}((1-q)i,j,k)=\varphi_2+\varphi_3$ where φ_i $(i=1,2,3)$ are characters distinct from each other, and either φ_i or $-\varphi_i$ is irreducible $(i=1,2,3)$. It then follows that $A_{62}(j,i,i,k)$ must be of the form $\varphi_1+\varphi_4$ or $\varphi_2-\varphi_3$ where φ_4 is a character distinct from the φ_i, and either φ_4 or $-\varphi_4$ is irreducible. We now suppose that $A_{62}(j,i,i,k)=\varphi_1+\varphi_4$. Then we see that $B_{21}((1-q)i,j,k)$ must be of the form $\varphi_1-\varphi_4$ or $\varphi_2-\varphi_3$. If $B_{21}((1-q)i,j,k)=\varphi_1-\varphi_4$, then

$$
\frac{1}{2}\{A_{62}(j,i,i,k)+B_{21}((1-q)i,j,k)\}=\varphi_1\ .
$$

But since the value of the left-hand side at the conjugacy class A_{17} of U_5 is equal to $-\frac{1}{2}\varepsilon^{a(2i+2j+k)}$, this is impossible. By a similar argument, the case $B_{21}((1-q)i, j, k)=\varphi_2-\varphi_3$ is also impossible. Hence we get $A_{62}(j, i, i, k)=\varphi_2-\varphi_3$. This shows that

$$\varphi_3=\frac{1}{2}\{B_{22}((1-q)i, j, k)-A_{62}(j, i, i, k)\} \quad (i, j, k)\in I_3 ,$$

$$\varphi_2=\frac{1}{2}\{B_{22}((1-q)i, j, k)+A_{62}(j, i, i, k)\} \quad (i, j, k)\in I_3 ,$$

$$\varphi_1=B_{21}((1-q)j, i, k)-\varphi_2 \quad (i, j, k)\in I_3 ,$$

are irreducible characters of U_5, which are denoted by $A_{53}(i, j, k)$, $A_{52}(i, j, k)$ and $A_{51}(i, j, k)$ respectively.

Secondly we construct six families of irreducible characters of U_5 corresponding to the families of conjugacy classes A_{31} to A_{36}. It is easy to see that each of three families $B_{13}((1-q)j, i)$, $A_{53}(j, i, i)$ and $A_{43}(i, j, j)$ is the sum or the difference of two irreducible characters, which are not among those that we have already obtained, for $(i, j)\in I_2$. Calculating the scalar product, we can show that

$$\|B_{22}((1-q)i, j, i)-B_{11}((1-q)j, i)\|=2$$

$$\|B_{22}((1-q)i, j, i)-B_{13}((1-q)j, i)\|=2$$

$$\|B_{11}((1-q)j, i)-B_{13}((1-q)j, i)\|=4 .$$

Hence we can assume that $B_{22}((1-q)i, j, i)=\phi_1+\phi_2$, $B_{11}((1-q)j, i)=\phi_1+\phi_3$ and $B_{13}((1-q)j, i)=\phi_2+\phi_4$ where ϕ_i $(i=1, 2, 3, 4)$ are characters distinct from each other, and either ϕ_i or $-\phi_i$ is irreducible. Then we see that $A_{43}(i, j, j)$ must be of the form $\phi_1-\phi_3$ or $\phi_2-\phi_4$, because we can show that

$$\|B_{22}((1-q)i, j, i)+A_{43}(i, j, j)\|=2$$

$$\|B_{13}((1-q)j, i)-A_{43}(i, j, j)\|=4$$

$$\|B_{11}((1-q)j, i)-A_{43}(i, j, j)\|=4 .$$

Suppose that $A_{43}(i, j, j)=\phi_1-\phi_3$. Then we get $\frac{1}{2}\{A_{43}(i, j, j)+B_{11}((1-q)j, i)\}=\phi_1$, but this is impossible, since the value of $\frac{1}{2}\{A_{43}(i, j, j)+B_{11}((1-q)j, i)\}$ at the conjugacy class A_{17} of U is not integral. Hence we get $A_{43}(i, j, j)=\phi_2-\phi_4$. Thus we have irreducible characters

$$\phi_2=\frac{1}{2}\{A_{43}(i, j, j)+B_{13}((1-q)j, i)\} , \quad \phi_4=B_{13}((1-q)j, i)-\phi_2$$

$$\phi_1=B_{22}((1-q)i, j, i)-\phi_2 , \qquad \phi_3=B_{11}((1-q)j, i)-\phi_1$$

for $(i, j) \in I_2$ which are denoted by $A_{36}(i, j)$, $A_{35}(i, j)$, $A_{32}(i, j)$ and $A_{31}(i, j)$ respectively. Moreover we get the remaining two families $A_{34}(i, j)$ $((i, j) \in I_2)$ and $A_{33}(i, j)$ $((i, j) \in I_2)$ of irreducible characters as follows:

$$A_{34}(i, j)=C_{12}(j, (1-q+q^2)i)-A_{32}(i, j)+A_{36}(i, j)$$

$$A_{33}(i, j)=B_{12}((1-q)j, i)-A_{34}(i, j) .$$

Thirdly we construct five families of irreducible characters of U_5 corresponding to the families of conjugacy classes A_{21} to A_{25}. It is easy to see that each of two families $A_{41}(i, i, j)$ and $A_{43}(i, i, j)$ is the sum or the difference of two irreducible characters which are not among those that we have already obtained. Since $\|A_{41}(i, i, j)+A_{43}(i, i, j)\|=4$ and the value of $\frac{1}{2}\{A_{41}(i, i, j)+A_{43}(i, i, j)\}$ at the conjugacy class A_{17} is not integral, we can assume that $A_{41}(i, i, j)=\psi_1+\psi_2$ and $A_{43}(i, i, j)=\psi_3+\psi_4$ where ψ_i $(i=1, 2, 3, 4)$ are characters distinct from each other, and either ψ_i or $-\psi_i$ is irreducible. It follows again from the calculations of the scalar product that each of three families $A_{42}(i, i, j)$, $B_{22}((1-q)i, i, j)$ and $B_{41}((1-q)i, j)$ is the sum or the difference of three irreducible characters which are not among those that we have already obtained. Since

$$\|A_{42}(i, i, j)+A_{41}(i, i, j)\|=\|A_{42}(i, i, j)-A_{43}(i, i, j)\|=3 ,$$

we can assume that $A_{42}(i, i, j)=-\psi_1+\psi_3+\psi_5$ where ψ_5 is a character distinct from the ψ_i, and either ψ_5 or $-\psi_5$ is irreducible. We also have

$$\|B_{22}((1-q)i, i, j)+A_{41}(i, i, j)\|=\|B_{22}((1-q)i, i, j)+A_{43}(i, i, j)\|=3 ,$$

$$\|B_{22}((1-q)i, i, j)\pm A_{42}(i, i, j)\|=6 .$$

It then follows that $B_{22}((1-q)i, i, j)$ must be of the form $-\psi_2-\psi_3+\psi_5$, $-\psi_1-\psi_3+\psi_6$ or $-\psi_1-\psi_4-\psi_5$ where ψ_6 is a character distinct from the ψ_i, and either ψ_6 or $-\psi_6$ is irreducible. Now we suppose that $B_{22}((1-q)i, i, j)=-\psi_2-\psi_3+\psi_5$. Then we get $\frac{1}{2}\{A_{41}(i, i, j)+A_{42}(i, i, j)+B_{22}((1-q)i, i, j)\}=\psi_5$. But the value of $\frac{1}{2}\{A_{41}(i, i, j)+A_{42}(i, i, j)+B_{22}((1-q)i, i, j)\}$ at the conjugacy class A_{17} is not integral, so this is impossible. Since

$$\|B_{41}((1-q)i, j)-B_{22}((1-q)i, i, j)\|=2$$

$$\|B_{41}((1-q)i, j)+A_{41}(i, i, j)\|=\|B_{41}((1-q)i, j)+A_{43}(i, i, j)\|=5$$

$$\|B_{41}((1-q)i, j)\pm A_{42}(i, i, j)\|=6 ,$$

the case $B_{22}((1-q)i, i, j)=-\psi_1-\psi_3+\psi_6$ is also impossible. Hence we get $B_{22}((1-q)i, i, j)=-\psi_1-\psi_4-\psi_5$. Thus we have an irreducible character

$$-\phi_5=\frac{1}{2}\{B_{22}((1-q)i, i, j)+A_{43}(i, i, j)-A_{42}(i, i, j)\} \quad (i, j)\in I_2$$

which is denoted by $A_{23}(i, j)$. Moreover it follows from the previous calculations of the scalar product that $B_{41}((1-q)i, j)$ must be of the form $\phi_3-\phi_4-\phi_5$ or $-\phi_1+\phi_2-\phi_5$. If $B_{41}((1-q)i, j)=\phi_3-\phi_4-\phi_5$, then we have $\frac{1}{2}\{A_{43}(i, i, j)+B_{41}((1-q)i, j)-A_{23}(i, j)\}=\phi_3$. But this is impossible, since the value of $\frac{1}{2}\{A_{43}(i, i, j)+B_{41}((1-q)i, j)-A_{23}(i, j)\}$ at the conjugacy class A_{17} is not integral. Hence we get $B_{41}((1-q)i, j)=-\phi_1+\phi_2+A_{23}(i, j)$. This shows that

$$\phi_2=\frac{1}{2}\{A_{41}(i, i, j)+B_{41}((1-q)i, j)-A_{23}(i, j)\}, \quad -\phi_1=\phi_2-A_{41}(i, i, j)$$

$$\phi_3=A_{42}(i, i, j)+\phi_1+A_{23}(i, j), \quad -\phi_4=\phi_3-A_{43}(i, i, j)$$

are irreducible characters for $(i, j)\in I_2$, which are denoted by $A_{21}(i, j)$, $A_{22}(i, j)$, $A_{24}(i, j)$ and $A_{25}(i, j)$ respectively.

Next we consider two families of irreducible characters corresponding to the families of conjugacy classes F and A_7. If $x_1, x_2, \cdots, x_5$ are the latent roots of $x\in U_5$, then we have by Theorem 1 of Green ([3], p. 403) that the classfunctions

$$\phi^{(i)}(x)=x_1^i+x_2^i+\cdots+x_5^i,$$

$$\phi^{(i,j,k,l,m)}(x)=\sum_{\{i,j,k,l,m\}} x_1^i x_2^j x_3^k x_4^l x_5^m$$

are characters of U_5, for any integers i, j, k, l, m. Then making use of these characters, together with irreducible characters already obtained, we get

$$F(i)=\phi^{(i)}+E(i, 0)+C_{12}(0, i)-B_{13}(i, 0)-A_{25}(0, i),$$

$$\begin{aligned}A_7(i, j, k, l, m)=\phi^{(i,j,k,l,m)}+\sum_{\{i,j,k,l,m\}}\Big\{&\frac{1}{12}B_3(i-jq, k, l, m)\\ &-\frac{1}{8}B_5(i-jq, k-lq, m)-\frac{1}{6}C_2(i, j, k-lq+mq^2)\\ &+\frac{1}{6}D(i-jq, k-lq+mq^2)+\frac{1}{4}E(i-jq+kq^2-lq^3, m)\\ &-\frac{1}{5}F(i-jq+kq^2-lq^3+mq^4)\Big\}.\end{aligned}$$

These are irreducible if and only if $i\in T_1$ and $(i, j, k, l, m)\in I_5$ respectively.

Finally we construct five families of irreducible characters of U_5 corresponding to the families of conjugacy classes A_{12} to A_{16}. If we repeat the similar argument for characters $B_{11}((1-q)i, i)$, $B_{12}((1-q)i, i)$, $B_{13}((1-q)i, i)$, $C_{12}(i, (1-q+q^2)i)$,

$A_{43}(i, i, i)$ and note that the number of irreducible characters equals the number of conjugacy classes, then it is easy to see that

$$A_{13}(i)=\frac{1}{4}\{2B_{11}((1-q)i, i)-B_{13}((1-q)i, i)+2C_{12}(i, (1-q+q^2)i) -A_{43}(i, i, i)-2A_{11}(i)-2A_{17}(i)\}$$

$$A_{14}(i)=B_{11}((1-q)i, i)-A_{11}(i)-A_{13}(i)$$

$$A_{15}(i)=B_{13}((1-q)i, i)-A_{17}(i)-A_{14}(i)$$

$$A_{12}(i)=C_{12}(i, (1-q+q^2)i)-A_{17}(i)-A_{13}(i)$$

$$A_{16}(i)=B_{12}((1-q)i, i)-A_{12}(i)$$

are irreducible characters. Thus we have obtained all the irreducible characters of $U(5, q^2)$, since we have as many as there are conjugacy classes of $U(5, q^2)$.

REMARK. Let $x_1, x_2, \cdots, x_n$ be the latent roots of $x \in U_n$. Let $\phi^{(i)}$ and $\phi^{(i_1,\cdots,i_n)}$ be classfunctions on U_n defined by

$$\phi^{(i)}(x)=x_1^i+x_2^i+\cdots+x_n^i ,$$

$$\phi^{(i_1,\cdots,i_n)}(x)=\sum_{\{i_1,\cdots,i_n\}} x_1^{i_1}x_2^{i_2}\cdots x_n^{i_n} .$$

Of course, $\phi^{(i)}$ and $\phi^{(i_1,\cdots,i_n)}$ are characters of U_n by Theorem of Green. Then, in general, it seems that $\phi^{(i)}$ is a linear combination of n irreducible characters of U_n corresponding to the conjugacy classes $(f^{\{1\}}, g^{\{d\}})$ where f is a U-irreducible polynomial of degree $n-d$ over K, g is a U-irreducible polynomial of degree 1 over K distinct from f $(d=0, 1, \cdots, n-1)$, and $\phi^{(i_1,\cdots,i_n)}$ is a linear combination of $n!$ irreducible characters of U_n corresponding to principal classes (that is, conjugacy classes of which all latent roots are distinct).

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