On infinitesimal automorphisms of some non-degenerate Siegel domains

By Masaki SUDO

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§1. **Introduction.** The purpose of this note is to calculate infinitesimal automorphisms of non-degenerate Siegel domains of some types. Since we consider here exclusively homogeneous Siegel domains $\mathcal{D}(V, F)$ of second kind, we simply call them Siegel domains, omitting the specifications "homogeneous" and "of second kind".

Let $\mathfrak{g}(\mathcal{D})$ be the Lie algebra of all complete holomorphic vector fields (that is, all infinitesimal automorphisms) on $\mathcal{D}(V, F)$. Then we have a decomposition $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_{-1}+\mathfrak{g}_{-1/2}+\mathfrak{g}_0+\mathfrak{g}_{1/2}+\mathfrak{g}_1$ as a graded Lie algebra, and it is known that $\mathfrak{g}_a=\mathfrak{g}_{-1}+\mathfrak{g}_{-1/2}+\mathfrak{g}_0$ corresponds to the affine automorphism group $\mathfrak{g}_a$ of $\mathcal{D}(V, F)$ (cf. [3], [10]). N. Tanaka [10] and S. Murakami [4] gave a method to construct $\mathfrak{g}_{1/2}$ and $\mathfrak{g}_1$ from the affine part $\mathfrak{g}_a$. T. Tsuji [11] and K. Nakajima [6] gave the methods of determining $\mathfrak{g}_{1/2}$ and $\mathfrak{g}_1$ explicitly and calculated infinitesimal automorphisms for most of Siegel domains over self-dual cones constructed by Pjateckii-Šapiro [7]. In this note we shall treat the following two examples and determine their infinitesimal automorphisms:

$\mathrm{I}_{m,\xi}$ $(m\geq 2, 0<\xi<1)$ $\quad V=H^+(m, \boldsymbol{C})$

$$F(u, u)=\begin{pmatrix} u_1\bar{u}_1 & \xi\bar{u}_1{}^t w+\sqrt{1-\xi^2}\, u_1{}^t\bar{v} \\ \xi u_1\bar{w}+\sqrt{1-\xi^2}\,\bar{u}_1 v & v^t\bar{v}+\bar{u}^t w \end{pmatrix},$$

where $u=(u_1, v, w)\in \boldsymbol{C}\times\boldsymbol{C}^{m-1}\times\boldsymbol{C}^{m-1}$, $u_1\in\boldsymbol{C}$, ${}^t v=(u_2, u_4, \dots, u_{2m-2})\in\boldsymbol{C}^{m-1}$, ${}^t w=(u_3, u_5, \dots, u_{2m-1})\in\boldsymbol{C}^{m-1}$,

II

$$V=\left\{\begin{pmatrix} a_1 & 0 & b \\ 0 & a_2 & c \\ b & c & a_3 \end{pmatrix} \quad \begin{matrix} \begin{pmatrix} a_1 & b \\ b & a_3 \end{pmatrix}>0, \begin{pmatrix} a_2 & c \\ c & a_3 \end{pmatrix}>0 \\ a_1, a_2, a_3, b, c\in\boldsymbol{R} \end{matrix}\right\},$$

$$F(u, u)=\begin{pmatrix} u_1\bar{u}_1 & 0 & \frac{1}{2}(u_3\bar{u}_1+u_1\bar{u}_3) \\ 0 & u_2\bar{u}_2 & \frac{1}{2}(u_3\bar{u}_2+u_2\bar{u}_3) \\ \frac{1}{2}(u_3\bar{u}_1+u_1\bar{u}_3) & \frac{1}{2}(u_3\bar{u}_2+u_2\bar{u}_3) & u_3\bar{u}_3 \end{pmatrix},$$

where ${}^t u=(u_1, u_2, u_3)\in \boldsymbol{C}^3$. Our main theorem is the following

THEOREM. *Let $\mathcal{D}$ be a Siegel domain of the above type. Then* $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$.

We remark that we can get $\mathfrak{g}_a$ by applying the method of M. Takeuchi [9] and our result is not contained in the results of T. Tsuji and K. Nakajima.

The Siegel domain $\mathrm{I}_{m,\xi}$ was constructed by M. Takeuchi (cf. [9]). II was found by S. Kaneyuki - T. Tsuji [2], and its cone V is not a self-dual cone. We remark that these Siegel domains are irreducible in the sense of Takeuchi [9] and non-degenerate.

In §2 we outline how to determine $\mathfrak{g}_{1/2}$ in the non-degenerate case. We shall show $\mathfrak{g}_{1/2}=(0)$ in our cases. Then by Tsuji's result we have $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$.

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§2. Preliminaries.

Let $\mathcal{D}(V, F)$ be a Siegel domain in $R^C\times W$.

DEFINITION. A Siegel domain $\mathcal{D}(V, F)$ is called *non-degenerate* if the R-linear closure of the set $\{F(u, u);\ u\in W\}$ coincides with R.

Let $\mathrm{Aut}(\mathcal{D})$ be the automorphism group of $\mathcal{D}(V, F)$, i.e., the group of all holomorphic transformation of $\mathcal{D}(V, F)$. We identify the Lie algebra of $\mathrm{Aut}(\mathcal{D})$ with the Lie algebra $\mathfrak{g}(\mathcal{D})$ of all infinitesimal automorphisms of $\mathcal{D}(V, F)$. Since the Siegel domains in our Theorem are non-degenerate, it is enough to show $\mathfrak{g}_{1/2}=(0)$, in order to prove $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$ by a Tsuji's result [11] p. 37. On the other hand, an element of $\mathfrak{g}(\mathcal{D})$ is a polynomial vector field by a result of Kaup-Matsushima-Ochiai [3]. By Nakajima [5] p. 64, in the non-degenerate case a polynomial vector field X on $R^C\times W$ belongs to $\mathfrak{g}_{1/2}$ if and only if X has a form

$$(\sharp)\qquad X=\sum_{j,k} 2iF^k(u, c_j)z^j\frac{\partial}{\partial z^k}+\sum_{k,\alpha} c_k^\alpha z^k\frac{\partial}{\partial u^\alpha}+\sum_{\alpha,\beta,\gamma} b_{\beta\gamma}^\alpha u^\beta u^\gamma\frac{\partial}{\partial u^\alpha},$$

and satisfies

$$(*)\qquad \sum_\alpha b_{\beta\gamma}^\alpha F_{\alpha\delta}^j=i\sum_{k,\alpha}(F_{\beta\delta}^k\bar{c}_k^\alpha F_{\gamma\alpha}^j+F_{\gamma\delta}^k\bar{c}_k^\alpha F_{\beta\alpha}^j),\qquad \text{for all } \beta,\gamma,\delta,j,$$

where $F_{\alpha\beta}^k$ is a component of F^k, i.e.,

$$F^k(u, u')=\sum_{\alpha,\beta}F_{\alpha\beta}^k u^\alpha\bar{u}'^\beta.$$

By Murakami [4], we get $\mathfrak{g}_{1/2}=(0)$ if all c_k^α satisfying (*) are zero. Thus the proof of our theorem is reduced to the assertion that all c_k^α in (*) are zero, for a Siegel domain $\mathcal{D}$ in our case.

§3. **Proof of Theorem for** $\mathrm{I}_{m,\xi}$ $(m\geq 2,\ 0<\xi<1)$. In this case $V=H^+(m, \boldsymbol{C})$ and $F(u, u)$ is given by

$$F(u,u)=\begin{pmatrix} u_1\bar{u}_1 & \xi u_3\bar{u}_1+\sqrt{1-\xi^2}\,u_1\bar{u}_2 & \cdots\cdots & \xi u_{2m-1}\bar{u}_1+\sqrt{1-\xi^2}\,u_1\bar{u}_{2m-2} \\ & u_2\bar{u}_2+u_3\bar{u}_3 & \cdots\cdots & u_2\bar{u}_{2m-2}+u_{2m-1}\bar{u}_3 \\ & & & \vdots \\ & * & & u_{2m-2}\bar{u}_{2m-2}+u_{2m-1}\bar{u}_{2m-1} \end{pmatrix}.$$

We put

$$F^{11}(u,u)=u_1\bar{u}_1\,,$$

$$F^{pp}(u,u)=u_{2p-2}\bar{u}_{2p-2}+u_{2p-1}\bar{u}_{2p-1} \qquad \text{for } 2\leq p\leq m\,,$$

$$F^{1q1}(u,u)=\frac{1}{2}\,\xi(u_{2q-1}\bar{u}_1+u_1\bar{u}_{2q-1})+\frac{1}{2}\sqrt{1-\xi^2}\,(u_1\bar{u}_{2q-2}+u_{2q-2}\bar{u}_1) \qquad \text{for } 2\leq q\leq m\,,$$

$$F^{1q2}(u,u)=-\frac{i}{2}\xi(u_{2q-1}\bar{u}_1-u_1\bar{u}_{2q-1})-\frac{i}{2}\sqrt{1-\xi^2}\,(u_1\bar{u}_{2q-2}-u_{2q-2}\bar{u}_1) \qquad \text{for } 2\leq q\leq m\,,$$

$$F^{pq1}(u,u)=\frac{1}{2}(u_{2p-2}\bar{u}_{2q-2}+u_{2q-2}\bar{u}_{2p-2})+\frac{1}{2}(u_{2q-1}\bar{u}_{2p-1}+u_{2p-1}\bar{u}_{2q-1}) \qquad \text{for } 2\leq p<q\leq m\,,$$

$$F^{pq2}(u,u)=-\frac{i}{2}(u_{2p-2}\bar{u}_{2q-2}-u_{2q-2}\bar{u}_{2p-2})-\frac{i}{2}(u_{2q-1}\bar{u}_{2p-1}-u_{2p-1}\bar{u}_{2q-1}) \qquad \text{for } 2\leq p<q\leq m\,.$$

We first consider the case $m=2$. Putting $(\beta,\gamma,\delta,j)=(1,1,2,11),(1,1,3,11)$ in (*), we have $c_{121}^1=-ic_{122}^1$, $c_{121}^1=ic_{122}^1$, so $c_{121}^1=c_{122}^1=0$. Putting $(\beta,\gamma,\delta,j)=(1,2,2,11)$, $(1,2,1,22),(1,3,1,22)$ in (*) we get

$$c_{22}^1=0\,,\qquad c_{11}^2=0\,,\qquad c_{11}^3=0\,. \tag{3.1}$$

Putting $(\beta,\gamma,\delta,j)=(2,2,1,22),(3,3,1,22),(2,3,1,22)$ we have

$$c_{121}^2=ic_{122}^2\,, \tag{3.2}$$

$$c_{121}^3=-ic_{122}^3\,, \tag{3.3}$$

$$\sqrt{1-\xi^2}\,c_{121}^3-i\sqrt{1-\xi^2}\,c_{122}^3+\xi c_{121}^2+i\xi\,c_{122}^2=0\,. \tag{3.4}$$

By (3.2), (3.3), (3.4) we get

$$\sqrt{1-\xi^2}\, c^3_{121}+\xi c^2_{121}=0 . \tag{3.5}$$

From (*) we have the following relations

$$b^1_{\beta\gamma}=i \sum_{k,\alpha} (F^k_{\beta 1}\bar{c}^\alpha_k F^{11}_{\gamma\alpha}+F^k_{\gamma 1}\bar{c}^\alpha_k F^{11}_{\beta\alpha}) , \tag{3.6}$$

$$=\frac{2i}{\sqrt{1-\xi^2}} \sum_{k,\alpha} (F^k_{\beta 2}\bar{c}^\alpha_k F^{121}_{\gamma\alpha}+F^k_{\gamma 2}\bar{c}^\alpha_k F^{121}_{\beta\alpha}) , \tag{3.7}$$

$$=\frac{2i}{\xi} \sum_{k,\alpha} (F^k_{\beta 3}\bar{c}^\alpha_k F^{121}_{\gamma\alpha}+F^k_{\gamma 3}\bar{c}^\alpha_k F^{121}_{\beta\alpha}) , \tag{3.8}$$

$$=-\frac{2}{\sqrt{1-\xi^2}} \sum_{k,\alpha} (F^k_{\beta 2}\bar{c}^\alpha_k F^{122}_{\gamma\alpha}+F^k_{\gamma 2}\bar{c}^\alpha_k F^{122}_{\beta\alpha}) . \tag{3.9}$$

Putting $\beta=1$, $\gamma=1$, we have

$$(3.6)=2i\bar{c}^1_{11} ,$$

$$(3.7)=2i\sqrt{1-\xi^2}\, \bar{c}^2_{121} \qquad \text{by using (3.2), (3.3),}$$

$$(3.8)=2i\xi\bar{c}^3_{121} \qquad \text{by using (3.2), (3.3).}$$

Therefore we have by (3.6), (3.7), (3.8),

$$c^1_{11}=\sqrt{1-\xi^2}\, c^2_{121}=\xi c^3_{121} . \tag{3.10}$$

By (3.10), (3.5), we have $c^1_{11}=c^2_{121}=c^2_{122}=c^3_{121}=c^3_{122}=0$. Putting $\beta=1$, $\gamma=2$, we have

$$(3.6)=0,$$

$$(3.7)=\frac{2i}{\sqrt{1-\xi^2}}\left(\frac{\sqrt{1-\xi^2}}{2}\bar{c}^2_{22}+\frac{\xi}{2}\bar{c}^3_{22}\right),$$

$$(3.9)=-\frac{2}{\sqrt{1-\xi^2}}\left(-\frac{i}{2}\sqrt{1-\xi^2}\,\bar{c}^2_{22}+\frac{i}{2}\xi\bar{c}^3_{22}\right).$$

From these we have

$$\sqrt{1-\xi^2}\, c^2_{22}+\xi c^3_{22}=0 , \tag{3.11}$$

$$-\sqrt{1-\xi^2}\, c^2_{22}+\xi c^3_{22}=0 . \tag{3.12}$$

By (3.11), (3.12) we have $c^2_{22}=c^3_{22}=0$. Thus we have $X=0$ (cf. (#)) and $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$.

Next we consider the case $m=3$. We apply the case $m=2$ to two classes $(F^{11}, F^{22}, F^{121}, F^{122})$ and $(F^{11}, F^{33}, F^{131}, F^{132})$. We get the matrix of c^α_j's

$$\begin{pmatrix} c_{11}^1 & c_{22}^1 & c_{33}^1 & c_{121}^1 & c_{122}^1 & c_{131}^1 & c_{132}^1 & c_{231}^1 & c_{232}^1 \\ c_{11}^2 & & & & & & & & \\ c_{11}^3 & & & & & & & & \\ c_{11}^4 & & & & & & & & \\ c_{11}^5 & & & & & & & & \end{pmatrix}$$

where $c_j^\alpha=0$ enclosed by the broken lines.

Putting

$$\begin{aligned}(\beta, \gamma, \delta, j)=&\{(1,4,2,11),(1,5,3,11)\},\{(2,4,1,22),(1,2,4,22)\},\{(2,2,4,22),(2,3,5,22)\}\\ &\{(3,4,1,22),(3,5,1,22)\},\{(2,3,4,22),(3,3,5,22)\},\{(2,4,1,33),(3,4,1,33)\}\\ &\{(2,5,1,33),(3,5,1,33)\},\{(4,4,2,33),(4,5,3,33)\},\{(4,5,2,33),(5,5,3,33)\}\end{aligned}$$

in (*), we get

(3.13) $\quad c_{231}^1=ic_{231}^1,\quad c_{231}^1=-ic_{232}^1,\quad$ so $\quad c_{231}^1=c_{232}^1=0,$

(3.14) $\quad c_{131}^2=ic_{132}^2,\quad c_{131}^2=-ic_{132}^2,\quad$ so $\quad c_{131}^2=c_{132}^2=0,$

(3.15) $\quad c_{231}^2=-ic_{232}^2,\quad c_{231}^2=ic_{232}^2,\quad$ so $\quad c_{231}^2=c_{232}^2=0,$

(3.16) $\quad c_{131}^3=ic_{132}^3,\quad c_{131}^3=-ic_{132}^3,\quad$ so $\quad c_{131}^3=c_{132}^3=0,$

(3.17) $\quad c_{231}^3=-ic_{232}^3,\quad c_{231}^3=ic_{232}^3,\quad$ so $\quad c_{231}^3=c_{232}^3=0,$

(3.18) $\quad c_{121}^4=ic_{122}^4,\quad c_{121}^4=-ic_{122}^4,\quad$ so $\quad c_{121}^4=c_{122}^4=0,$

(3.19) $\quad c_{121}^5=ic_{122}^5,\quad c_{121}^5=-ic_{122}^5,\quad$ so $\quad c_{121}^5=c_{122}^5=0,$

(3.20) $\quad c_{231}^4=ic_{232}^4,\quad c_{231}^4=-ic_{232}^4,\quad$ so $\quad c_{231}^4=c_{232}^4=0,$

(3.21) $\quad c_{231}^5=ic_{232}^5,\quad c_{231}^5=-ic_{232}^5,\quad$ so $\quad c_{231}^5=c_{232}^5=0.$

Putting $(\beta, \gamma, \delta, j)=(2,4,4,22),(3,4,4,22),(2,4,2,33),(2,5,2,33)$ in (*) we get $c_{33}^2=c_{33}^3=c_{22}^4=c_{22}^5=0$. Thus we have proved $X=0$ (cf. (#)) and so we get $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$.

Next we consider the case $m=4$. We apply the case of $m=3$ to the following three classes

$$(F^{11}, F^{22}, F^{33}, F^{121}, F^{122}, F^{131}, F^{132}, F^{231}, F^{232}),$$
$$(F^{11}, F^{22}, F^{44}, F^{121}, F^{122}, F^{141}, F^{142}, F^{241}, F^{242}),$$
$$(F^{11}, F^{33}, F^{44}, F^{131}, F^{132}, F^{141}, F^{142}, F^{341}, F^{342}).$$

We get the matrix of c_j^{α}'s

$$\begin{pmatrix} c_{11}^1 & c_{22}^1 & c_{33}^1 & c_{44}^1 & c_{121}^1 & c_{122}^1 & c_{131}^1 & c_{132}^1 & c_{141}^1 & c_{142}^1 & c_{231}^1 & c_{232}^1 & c_{241}^1 & c_{242}^1 & c_{341}^1 & c_{342}^1 \\ c_{11}^2 & & & & & & & & & & & & & & & \\ c_{11}^3 & & & & & & & & & & & & & & & \\ c_{11}^4 & & & & & & & & & & & & & & & \\ c_{11}^5 & & & & & & & & & & & & & & & \\ c_{11}^6 & & & & & & & & & & & & & & & \\ c_{11}^7 & & & & & & & & & & & & & & & \end{pmatrix}$$

where $c_j^{\alpha}=0$ enclosed by the broken lines. Putting $(\beta, \gamma, \delta, j)=\{(6,2,4,22),\ (7,2,5,22)\}$, $\{(6,3,4,22),\ (7,3,5,22)\}$ in $(*)$ we have

$$c_{341}^2=ic_{342}^2\,, \quad c_{341}^2=-ic_{342}^2\,, \qquad \text{so} \qquad c_{341}^2=c_{342}^2=0\,,$$

$$c_{341}^3=ic_{342}^3\,, \quad c_{341}^3=-ic_{342}^3\,, \qquad \text{so} \qquad c_{341}^3=c_{342}^3=0\,.$$

Putting $j=33,\ 44$ in $(*)$ we obtain analogously

$$c_{241}^4=c_{242}^4=c_{241}^5=c_{242}^5=0\,,$$

$$c_{231}^6=c_{232}^6=c_{231}^7=c_{232}^7=0\,.$$

Thus we have proved $X=0$ (cf. $(\sharp)$) and so we get $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$.

At last we consider the case $m\geq 5$. We prove $c_j^{\alpha}=0$ for all α, j by induction on m. We assume $c_j^{\alpha}=0$ up to $n=m-1$. If we change the variables $(u_1, \dots, u_{2m-3})$ in the case $n=m-1$ to $(u_1, u_2, u_3, \dots, \widehat{u_{2k}, u_{2k+1}}, \dots, u_{2m-4}, u_{2m-3}, u_{2m-2}, u_{2m-1})$ ($\widehat{\ \ }$ means omission) for $1\leq k\leq m-1$, we have $m-1$ classes. We can select these $m-1$ classes from $F^{pq}(u,u)$ in the case $n=m$. Apply the result in the case of $n=m-1$ to these $m-1$ classes, we get $c_j^{\alpha}=0$ for all α, j, because each $F^{pq}(u,u)$ $(1\leq p\leq q\leq m)$ appears at least twice in each class of the above $m-1$ classes as $m\geq 5$. Therefore we get $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$. q.e.d.

§ 4. Proof of Theorem for II. In this case we put

$$F^p(u,u)=u_p\bar{u}_p \qquad \text{for } 1\leq p\leq 3\,,$$

$$F^4(u,u)=\frac{1}{2}(u_3\bar{u}_1+u_1\bar{u}_3)\,,$$

$$F^5(u, u)=\frac{1}{2}(u_3\bar{u}_2+u_2\bar{u}_3).$$

Putting $(\beta, \gamma, \delta, j)=(2,1,2,1), (3,1,2,1), (3,1,3,1), (1,1,3,1)$ in $(*)$, we have

$$c_2^1=0, \quad c_5^1=0, \quad c_3^1=0, \quad c_4^1=0. \tag{4.1}$$

Putting $(\beta, \gamma, \delta, j)=(1,2,1,2), (3,2,1,2), (3,2,3,2), (2,2,3,2)$ in $(*)$, we have

$$c_1^2=0, \quad c_4^2=0, \quad c_3^2=0, \quad c_5^2=0. \tag{4.2}$$

Putting $(\beta, \gamma, \delta, j)=(1,3,1,3), (3,3,1,3), (2,3,2,3), (3,3,2,3)$ in $(*)$, we have

$$c_1^3=0, \quad c_4^3=0, \quad c_2^3=0, \quad c_5^3=0. \tag{4.3}$$

From $(*)$ we have the following relations

$$b_{\beta\gamma}^3=i\sum_k (F_{\beta 3}^k\bar{c}_k^\gamma F_{\gamma\gamma}^3+F_{\gamma 3}^k\bar{c}_k^\beta F_{\beta\beta}^3) \tag{4.4}$$

$$=2i\sum_{k,\alpha} (F_{\beta 1}^k\bar{c}_k^\alpha F_{\gamma\alpha}^4+F_{\gamma 1}^k\bar{c}_k^\alpha F_{\beta\alpha}^4). \tag{4.5}$$

Putting $\beta=3$, $\gamma=3$, we have by (4.1), (4.2), (4.3),

$$(4.4)=2i\bar{c}_3^3,$$

$$(4.5)=0.$$

Therefore we have $c_3^3=0$. Putting $\beta=1$, $\gamma=3$, we have by (4.1), (4.2), (4.3),

$$(4.4)=0,$$

$$(4.5)=i\bar{c}_1^1.$$

Therefore we have $c_1^1=0$. From $(*)$ we have also the following relations

$$b_{\beta\gamma}^2=2i\sum_{k,\alpha} (F_{\beta 3}^k\bar{c}_k^\alpha F_{\gamma\alpha}^5+F_{\gamma 3}^k\bar{c}_k^\alpha F_{\beta\alpha}^5) \tag{4.6}$$

$$=i\sum_k (F_{\beta 2}^k\bar{c}_k^\gamma F_{\gamma\gamma}^2+F_{\gamma 2}^k\bar{c}_k^\beta F_{\beta\beta}^2).$$

Putting $\beta=\gamma=2$, we have

$$(4.6)=0,$$

$$(4.7)=2i\bar{c}_2^2.$$

Therefore we have $c_2^2=0$. Thus we have $c_j^\alpha=0$ for all α, j. Then as is remarked at the end of §2, we get $\mathfrak{g}(\mathcal{D})=\mathfrak{g}_a$. q.e.d.

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Department of Mathematics
Tokyo Metropolitan University
Fukazawa, Setagaya-ku, Tokyo
158 Japan