Generators of 2-primary components of homotopy groups of spheres, unitary groups and symplectic groups

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Introduction

This is a continuation of the previous paper [3], in which I gave the generators of the 2-primary components of $\pi_k(Sp(n))$, $\pi_k(SU(n))$ and $\pi_k(SO(n))$ for $k\leqq 13$, and proved the relations among them. To continue these calculations, we need to know more about the homotopy groups of spheres, and that is the purpose of § 1 and § 2; we shall prove the relations among the generators of the 2-primary components of $\pi_{n+k}(S^n)$ for $14\leqq k\leqq 19$, which are not included in [1]. We shall add further information about the generators of the groups π^n_{n+18} (c.f. (2.19) of § 2). Recently, M. Mimura and H. Toda calculated $\pi_k(Sp(n))$, $\pi_k(SU(3))$ and $\pi_k(SU(4))$ in [5] and [6]. So we omit the proof of this part, but we shall only define their generators by the "composition method" to study the relations among them. (§ 3–§ 5). In § 6 and § 7, we shall continue these calculations for $\pi_k(U_n)$ for $n\geqq 5$ and $k\leqq 22$ by using the results of the previous sections.

As we are interested in 2-primary components of the groups, the remarks we made in § 9 of the previous paper [3] will be carried out in this paper, that is: $\pi_k(G)$ means the 2-primary components $\pi_k(G; 2)$ of $\pi_k(G)$; and we use the terms such as *equal*, *isomorphic*, in the sense of C_2. In addition to this, if $\alpha=d\beta$ holds for some elements α and β of an infinite cyclic group and for an integer d, we are interested only in the 2-primary component ${}^{(2)}d$ of d. In this sense, we shall describe the relation as $\alpha={}^{(2)}d\beta$ for convenience(*).

In the last part of this paper, I have appended some tables of the relations, summarizing the results of [1], [3] and this paper.

§ 1. Preliminaries

Throughout this paper, we shall use the same notations as in [1]: e.g. π^n_m means 2-primary component of $\pi_m(S^n)$, G_k 2-primary component of the k-th stable homotopy group of the sphere, and $E^\infty:\pi^n_{n+k}\to G_k$ indicates the appropriate iterated suspension, etc.. First of all, we list the original generators(**) of π^n_{n+k} for $k\leqq 19$ below.

(*) I failed to make this remark in [3].

(*) Generators which are not represented by suspension, composition, nor by Whitehead product.

(1.1) (1) $\iota_n \in \pi_n^n \ (n \geqq 1)$.

(2) $\eta_n \in \pi_{n+1}^n \ (n \geqq 2)$, $H(\eta_2) = \iota_3$.

(3) $\nu' \in \pi_6^3$, $H(\nu') = \eta_5$, $2\nu' = \eta_3^3$.

$\nu_n \in \pi_{n+3}^n \ (n \geqq 4)$, $H(\nu_4) = \iota_7$, $2\nu_5 = E^2\nu'$.

(4) $\sigma''' \in \pi_{12}^5$, $\sigma'' \in \pi_{13}^6$, $\sigma' \in \pi_{14}^7$, $\sigma_n \in \pi_{n+7}^n \ (n \geqq 8)$, $H(\sigma''') = 4\nu_9$, $H(\sigma'') = \eta_{11}^2$,

$H(\sigma') = \eta_{13}$, $H(\sigma_8) = \iota_{15}$, $2\sigma'' = E\sigma'''$, $2\sigma' = E\sigma''$, $2\sigma_9 = E^2\sigma'$.

(5) $\varepsilon_n \in \pi_{n+8}^n \ (n \geqq 3)$, $H(\varepsilon_3) = \nu_5^2$, $\bar{\nu}_n \in \pi_{n+8}^n \ (n \geqq 6)$, $H(\bar{\nu}_6) = \nu_{11}$.

(6) $\mu_n \in \pi_{n+9}^n \ (n \geqq 3)$, $H(\mu_3) = \sigma'''$.

(7) $\varepsilon' \in \pi_{13}^3$, $H(\varepsilon') = \varepsilon_5$, $2\varepsilon' = \eta_3^2 \circ \varepsilon_5$, $E^2\varepsilon' = \pm 2\nu_5 \circ \varepsilon_8$.

(8) $\mu' \in \pi_{14}^3$, $H(\mu') = \mu_5$, $2\mu' = \eta_3^2 \circ \mu_5$.

$\zeta_n \in \pi_{n+11}^n \ (n \geqq 5)$, $H(\zeta_5) = 8\sigma_9$, $2\zeta_5 = \pm E^2\mu'$.

(9) $\theta' \in \pi_{23}^{11}$, $\theta \in \pi_{24}^{12}$, $H(\theta') = \eta_{21}^2$, $H(\theta) = \eta_{23}$, $E^2\theta' = 0$, $E^2\theta = 0$.

(10) $\kappa_n \in \pi_{n+14}^n \ (n \geqq 7)$, $H(\kappa_7) = \varepsilon_{13}$ or $\bar{\nu}_{13}$, $2\kappa_7 \equiv \bar{\nu}_7 \circ \nu_{15}^2 \bmod (4\sigma' \circ \sigma_{14})$.

(11) $\rho^{\mathrm{IV}} \in \pi_{20}^5$, $\rho''' \in \pi_{21}^6$, $\rho'' \in \pi_{22}^7$, $\rho' \in \pi_{24}^9$, $\rho_n \in \pi_{n+15}^n \ (n \geqq 13)$, $H(\rho^{\mathrm{IV}}) = 4\zeta_9$,

$H(\rho''') = \eta_{11} \circ \mu_{12}$, $H(\rho'') \equiv \mu_{13} \bmod \{\nu_{13}^3\} \oplus \{\eta_{13} \circ \varepsilon_{14}\}$, $H(\rho') = 8\sigma_{17}$,

$H(\rho_{13}) = 4\nu_{25}$, $2\rho''' \equiv E\rho^{\mathrm{IV}} \bmod \sigma'' \circ \pi_{21}^{13}$, $2\rho'' \equiv E\rho''' \bmod \sigma' \circ \pi_{22}^{14}$,

$2\rho' \equiv E^2\rho'' \bmod \sigma_9 \circ \pi_{24}^{16}$, $2\rho_{13} = E^4\rho'$.

$\bar{\varepsilon}_n \in \pi_{n+15}^n \ (n \geqq 3)$, $H(\bar{\varepsilon}_3) \equiv \nu_5 \circ \sigma_8 \circ \nu_{15} \bmod \nu_5 \circ \eta_8 \circ \mu_9$.

(12) $\zeta' \in \pi_{22}^6$, $H(\zeta') \equiv \zeta_{11} \bmod 2\zeta_{11}$, $E^3\zeta' = 0$.

$\omega_n \in \pi_{n+16}^n \ (n \geqq 14)$, $H(\omega_{14}) = \nu_{27}$.

$\eta^{*\prime} \in \pi_{31}^{15}$, $H(\eta^{*\prime}) = \eta_{29}^2$, $E^2\eta^{*\prime} \equiv 0 \bmod E^3\pi_{30}^{14}$.

$\eta_n^* \in \pi_{n+16}^n \ (n \geqq 16)$, $H(\eta_{16}^*) = \eta_{31}$, $\eta_{18}^* \equiv \omega_{18} \bmod \sigma_{18} \circ \mu_{25}$.

(13) $\bar{\mu}_n \in \pi_{n+17}^n \ (n \geqq 3)$, $H(\bar{\mu}_3) = \rho^{\mathrm{IV}}$.

$\bar{\varepsilon}' \in \pi_{20}^3$, $H(\bar{\varepsilon}') = \bar{\varepsilon}_5$, $2\bar{\varepsilon}' = \eta_3^2 \circ \bar{\varepsilon}_5$, $E^2\bar{\varepsilon}' = 2\nu_5 \circ \kappa_8$.

$\Delta(E\theta) \in \pi_{23}^6$, $H(\Delta(E\theta)) = \theta'$.

$\varepsilon_n^* \in \pi_{n+17}^n \ (n \geqq 12)$, $H(\varepsilon_{12}^*) = \nu_{23}$, $\varepsilon_{13}^* = \eta_{13} \circ \omega_{14}$.

(14) ξ'', $\lambda'' \in \pi_{28}^{10}$, ξ', $\lambda' \in \pi_{29}^{11}$, $\lambda \in \pi_{31}^{13}$, $\xi_n \in \pi_{n+18}^n \ (n = 12)$, $\nu_n^* \in \pi_{n+18}^n \ (n \geqq 16)$,

$H(\xi'') = \nu_{19}^3 + \eta_{19} \circ \varepsilon_{20}$, $H(\lambda'') = \eta_{19} \circ \varepsilon_{20}$ or ν_{19}^3, $H(\xi') = \varepsilon_{21} + \bar{\nu}_{21}$,

$H(\lambda') = \varepsilon_{21}$ or $\bar{\nu}_{21}$, $H(\xi_{12}) \equiv \sigma_{23} \bmod 2\sigma_{23}$, $H(\lambda) = \nu_{25}^2$, $H(\nu_{16}^*) \equiv \nu_{31} \bmod 2\nu_{31}$,

$2\xi' = E\xi''$, $2\xi_{13} = E^2\xi'$, $2\lambda' = E\lambda''$, $2\lambda = E^2\lambda'$, $2\lambda_{17}^* = E^4\lambda$,

$4(\xi_{18} + \nu_{18}^*) = 2(\xi_{19} + \nu_{19}^*) = \xi_{20} + \nu_{20}^* = 0$.

(15) $\bar{\mu}' \in \pi_{22}^3$, $H(\bar{\mu}') \equiv \bar{\mu}_5 \bmod \eta_5 \circ \mu_6 \circ \sigma_{15}$, $2\bar{\mu}' = \eta_3^2 \circ \bar{\mu}_5$.

$\bar{\zeta}_n \in \pi_{n+19}^n \ (n \geqq 5)$, $H(\bar{\zeta}_5) = 8\rho'$, $2\bar{\zeta}_5 = E^2\bar{\mu}'$.

$\bar{\sigma}_n \in \pi_{n+19} \ (n \geqq 6)$, $H(\bar{\sigma}_6) \equiv \sigma_{11}^2 \bmod 2\sigma_{11}^2$, $16\bar{\sigma}_6 = \nu_6 \circ \mu_9 \circ \sigma_{18}$.

$\omega' \in \pi_{31}^{13}$, $H(\omega') = \varepsilon_{23}$ or $\bar{\nu}_{23}$, $E^2\omega' = 2\omega_{14} \circ \nu_{30}$.

Some of these elements are obtained by the secondary compositions:

(1.2) (1) $\nu' \in \{\eta_3, 2\iota_4, \eta_4\}_1$.

(2) $\sigma''' \in \{\nu_5, 8\iota_8, \nu_8\}_t$ $(0 \leqq t \leqq 3)$.

(3) $\varepsilon_3 \in \{\eta_3, E\nu', \nu_7\}_1 = \{\nu', \nu_6, \eta_9\}$, $\varepsilon_n \in \{\eta_n, 2\iota_{n+1}, \nu_{n+1}^2\}_{n-4}$ $(n \geqq 4)$,
$\varepsilon_n \in \{\eta_n, \nu_{n+1}^2, 2\iota_{n+7}\}_t$, $\{\nu_n^2, 2\iota_{n+6}, \eta_{n+6}\}_t$,
$\{2\iota_n, \nu_n^2, \eta_{n+6}\}_t$ $(n \geqq 5)$, $(0 \leqq t \leqq n-5)$.
$\bar{\nu}_n \in \{\nu_n, \eta_{n+3}, \nu_{n+4}\}_t$ $(n \geqq 6)$, $(0 \leqq t \leqq n-2)$.

(4) $\mu_n \in \{\eta_n, 2\iota_{n+1}, E^{n-4}\sigma'''\}_{n-4} + \{\nu_n^3\}$ $(n \geqq 4)$.

(5) $\varepsilon' \in \{\nu', 2\nu_6, \nu_9\}_3$.

(6) $\mu' \in \{\eta_3, 2\iota_4, \mu_4\}_1$, $\zeta_n \in \{\nu_n, 8\iota_{n+3}, E^{n-4}\sigma'\}_{n-4}$ $(n \geqq 6)$.

(7) $\theta' \in \{\sigma_{11}, 2\nu_{18}, \eta_{21}\}_1$, $\theta \in \{\sigma_{12}, \nu_{19}, \eta_{22}\}_1$.

(8) $\bar{\varepsilon}_n \in \{\varepsilon_n, 2\iota_{n+8}, \nu_{n+8}^2\}_{n+8}$ $(n \geqq 3)$, $\rho^{\mathrm{IV}} \in \{\sigma''', 2\iota_{12}, 8\sigma_{12}\}_1$,
$\rho''' \in \{\sigma'', 4\iota_{13}, 4\sigma_{13}\}_1$, $\rho'' \in \{\sigma', 8\iota_{14}, 2\sigma_{14}\}_1$, $\rho' \in \{\sigma_9, 16\iota_{16}, \sigma_{16}\}_1$.

(9) $\zeta' \in \{\sigma'', \varepsilon_{13}, 2\iota_{21}\}_1$, $\eta^{*\prime} \in \{\sigma_{15}, 4\sigma_{22}, \eta_{29}\}_1$, $\eta_{16}^* \in \{\sigma_{16}, 2\sigma_{23}, \eta_{30}\}_1$.

(10) $\bar{\mu}_3 \in \{\mu_3, 2\iota_{12}, 8\sigma_{12}\}_1$.

(11) $\nu_{16}^* \in \{\sigma_{16}, 2\sigma_{23}, \nu_{30}\}_1$, $\xi_{12} \in \{\sigma_{12}, \nu_{19}, \sigma_{22}\}_1$.

(12) $\bar{\mu}' \in \{\mu', 4\iota_{14}, 4\sigma_{14}\}_1$, $\bar{\zeta}_5 \in \{\zeta_5, 8\iota_{16}, 2\sigma_{16}\}_1$, $\bar{\sigma}_6 \in \{\nu_6, \varepsilon_9 + \bar{\nu}_9, \varepsilon_{17}\}_1$.

Now, we list some of the Whitehead products of these generators below.

(1.3) $[\iota_1, \iota_1] = [\iota_3, \iota_3] = [\iota_7, \iota_7] = 0$, $[\iota_5, \iota_5] = \nu_5 \circ \eta_8$, $[\iota_9, \iota_9] = \sigma_9 \circ \eta_{16} + \varepsilon_9 + \bar{\nu}_9$,
$[\iota_{11}, \iota_{11}] = \sigma_{11} \circ \nu_{18}$, $[\iota_{13}, \iota_{13}] = E\theta$, $[\iota_{15}, \iota_{15}] = 2\sigma_{15}^2$, $[\iota_{17}, \iota_{17}] \equiv \eta_{17}^* + \omega_{17} \bmod \sigma_{17} \circ \mu_{24}$,
$[\iota_{19}, \iota_{19}] = \nu_{19}^* + \xi_{19} \cdot [\iota_2, \iota_2] = -2\eta_2$, $[\iota_4, \iota_4] = E\nu' - 2\nu_4$,
$[\iota_8, \iota_8] = E\sigma' - 2\sigma_8 \cdot [\iota_6, \eta_6] = 0$, $[\iota_6, \nu_6] = -2\bar{\nu}_6$, $[\iota_6, \varepsilon_6] = [\iota_6, \bar{\nu}_6] = 0$,
$[\iota_6, \mu_6] = 0$, $[\iota_6, \zeta_6] = \pm 2\zeta'$.
$[\iota_{10}, \eta_{10}] = 2\sigma_{10} \circ \nu_{17}$, $[\iota_{10}, \nu_{10}^2] = \sigma_{10} \circ \varepsilon_{17} = \sigma_{10} \circ \bar{\nu}_{17}$, $[\iota_{10}, \varepsilon_{10}] = [\iota_{10}, \bar{\nu}_{10}] = 0$,
$[\iota_{10}, \mu_{10}] = 2\sigma_{10} \circ \zeta_{17}$.
$[\iota_{12}, \eta_{12}] = E\theta'$, $[\iota_{12}, \sigma_{12}] = \pm(E\xi' - 2\xi_{12})$.
$[\iota_{14}, \eta_{14}] = 4\sigma_{14}^2$, $[\iota_{14}, \nu_{14}] = -2\omega_{14}$.
$[\iota_{16}, \eta_{16}] \equiv E\eta^{*\prime} \bmod E^2\pi_{30}^{14}$, $[\iota_{16}, \nu_{16}] = \pm(E^3\lambda - 2\nu_{16}^*)$.
$[\iota_{18}, \eta_{18}] = 2(\nu_{18}^* + \xi_{18})$.

Proofs of (1.1), (1.2) and (1.3) are found in [1](*). The Hopf invariants and Whitehead products of the other elements are calculated by the following formulae:

(1.4)
$$H(\alpha \circ E\beta) = H(\alpha) \circ E\beta \qquad \text{for } \alpha \in \pi_m^k,\ \beta \in \pi_{n-1}^{m-1}.$$
$$H(E\alpha \circ \beta) = E^k\alpha \circ E^m\alpha \circ H(\beta) \qquad \text{for } \alpha \in \pi_{m-1}^{k-1},\ \beta \in \pi_n^m.$$

(1.5) For $\alpha \in \pi_m^k$, $\beta \in \pi_n^k$,
$$[\alpha, \beta] = \begin{cases} [\iota_k, \iota_k] \circ E^{k-1}\beta \circ E^{n-1}\alpha & \text{if } k \text{ is odd.} \\ (-1)^{m+n}[\iota_k, \iota_k] \circ E^{k-1}\beta \circ E^{n-1}\alpha & \text{if } k \text{ is even.} \end{cases}$$

(*) Some of (1.3) follow directly from the results of [1].

The following two formulae will play an important role, too.

(1.6) $E^{q-1}\alpha\circ E^{m-1}\beta-(-1)^{(p+m)(q+n)}E^{p-1}\beta\circ E^{n-1}\alpha$
$=[\iota_{p+q-1},\iota_{p+q-1}]\circ E^{2p-2}H(\beta)\circ E^{n-1}H(\alpha)$ for $\alpha\in\pi^p_m$, $\beta\in\pi^q_n$. (Toda's formula)

(1.7) $(\alpha+\beta)\circ\gamma=\alpha\circ\gamma+\beta\circ\gamma+[\alpha,\beta]\circ H(\gamma)$.

(1.4) is well known (see [1]). (1.5)~(1.7) are modifications of the results of [2].

§2. Generators of homotopy groups of spheres

In this section, we shall prove some relations among the generators of 2-primary components of the homotopy groups of spheres which are not included in [1]. We shall do these calculations step by step in terms of k in the notation π^n_{n+k}. First of all, we list the relations which appear on π^n_{n+k} for $k\leqq 13$.

(2.1) $\eta_3\circ\nu_4=\nu'\circ\eta_6$, $\nu'\circ\nu_6=0$, $\eta_4\circ\sigma'''=\eta_5\circ\sigma''=\sigma'''\circ\eta_{12}=0$, $\eta_6\circ\sigma'=\sigma''\circ\eta_{13}=4\bar{\nu}_6$,
$\eta_7\circ\sigma_8=\sigma'\circ\eta_{14}+\varepsilon_7+\bar{\nu}_7$, $\eta_9\circ\sigma_{10}=\varepsilon_9+\bar{\nu}_9$, $\eta_n\circ\sigma_{n+1}=\sigma_n\circ\eta_{n+7}=\varepsilon_n+\bar{\nu}_n$ $(n\geqq 10)$,
$\varepsilon_n\circ\eta_{n+8}=\eta_n\circ\varepsilon_{n+1}$ $(n\geqq 3)$, $\eta_5\circ\bar{\nu}_6=\nu_5^3$, $\bar{\nu}_n\circ\eta_{n+8}=\eta_n\circ\bar{\nu}_{n+1}=\nu_n^3$ $(n\geqq 6)$,
$2(\nu_5\circ\sigma_8)=\nu_5\circ E\sigma'$, $\sigma'\circ\nu_{14}=x\nu_7\circ\sigma_{10}$ for some odd integer x,
$\sigma'''\circ\nu_{12}=4(\nu_5\circ\sigma_8)$, $\nu'\circ\bar{\nu}_6=\varepsilon_3\circ\nu_{11}$, $\varepsilon'\circ\eta_{13}=\nu'\circ\varepsilon_6$, $\nu_6\circ\bar{\nu}_9=\nu_6\circ\varepsilon_9=2\bar{\nu}_6\circ\nu_{14}$,
$\nu_6\circ\mu_9=8[\iota_6,\iota_6]\circ\sigma_{11}$.

The proofs are given in [1]. By using these relations, we add the followings.

PROPOSITION (2.2):

(1) $\nu'\circ\sigma''=0$, $E\nu'\circ\sigma'=2E\varepsilon'$, $\sigma''\circ\nu_{13}=\pm 2\nu_6\circ\sigma_9$.

(2) $\mu_n\circ\eta_{n+9}=\eta_n\circ\mu_{n+1}$ $(n\geqq 3)$.

(3) $\nu_5\circ\sigma_8\circ\eta_{15}=\nu_5\circ\varepsilon_8$ or $\nu_5\circ\bar{\nu}_8$.

(4) $\mu_3\circ\nu_{12}=\nu'\circ\eta_6\circ\varepsilon_7$, $\mu'\circ\eta_{14}=\nu'\circ\mu_6$.

(5) $\eta_4\circ\zeta_5\equiv E\nu'\circ\mu_7 \bmod E\nu'\circ\eta_7\circ\varepsilon_8$, $\eta_n\circ\zeta_{n+1}=0$ $(n\geqq 5)$.

(6) $\zeta_6\circ\eta_{17}=8[\iota_6,\iota_6]\circ\sigma_{11}$, $\zeta_n\circ\eta_{n+11}=0$ $(n\geqq 7)$.

(7) $\varepsilon'\circ\nu_{13}=0$.

(8) $\eta_{10}\circ\theta'=0$, $\eta_{11}\circ\theta=\sigma_{11}\circ\nu_{18}^2+\theta'\circ\eta_{23}$.

PROOF: (1) $H(E\nu'\circ\sigma')=E^4\nu'\circ E^7\nu'\circ H(\sigma')$ (by (1.4)) $=(4\nu_7^2)\circ\eta_{13}$ (by (1.1)(3), (4)) $=0$. Hence, $E\nu'\circ\sigma'\in E\pi^3_{13}$. $E(E\nu'\circ\sigma')=2\nu_5\circ E\sigma'=2(\nu_5\circ E\sigma')$ (by (1.7)) $=4(\nu_5\circ\sigma_8)$ (by (2.1)). $E(2E\varepsilon')=4(\nu_5\circ\sigma_8)$ (by (1.1)(7)). Since $E\pi^3_{13}\cap E^{-1}(0)=0$, it follows that $E\nu'\circ\sigma'=2E\varepsilon'$, where $E^{-1}(0)$ is the kernel of the suspension homomorphism $E:\pi^4_{14}\rightarrow\pi^5_{15}$.

Now, $E(\nu'\circ\sigma'')=E\nu'\circ 2\sigma'$ (by (1.1)(4)) $=4E\varepsilon'=0$. Since the homomorphism E: $\pi^3_{13}\rightarrow\pi^4_{14}$ is a monomorphism, we have $\nu'\circ\sigma''=0$.

Finally, $E(\sigma''\circ\nu_{13})=2\sigma'\circ\nu_{14}=\pm 2\nu_7\circ\sigma_{10}$ (by (2.1)). Since the homomorphism E: $\pi^6_{16}\rightarrow\pi^7_{17}$ is an isomorphism, we have $\sigma''\circ\nu_{13}=\pm 2\nu_6\circ\sigma_9$.

[Note] In the following, when the proof is made by this way, we abbreviate as follows: $E\nu'\circ\sigma'\in E\pi_{13}^3$ (H; (1.4), (1.1)(3)(4)), $E(E\nu'\circ\sigma')=E(2E\varepsilon')$ ((1.7), (2.1), (1.1)(7)), $E\pi_{13}^3\cap E^{-1}(0)=0$. etc.

(2) Substituting μ_3 for α, and η_2 for β in (1.6), we have $\mu_5\circ\eta_{14}=\eta_5\circ\mu_6$. Since the homomorphism $E^2:\pi_{13}^3\to\pi_{15}^5$ is a monomorphism, it follows that $\mu_3\circ\eta_{12}=\eta_3\circ\mu_4$.

[Note] In this case, we shall abbreviate as follows: ((1.6), $(E^2)^{-1}=0$).

(3) $\nu_6\circ[\iota_9,\iota_9]=[\nu_6,\nu_6]=[\iota_6,\nu_6]\circ\nu_{14}$ (by (1.5)) $=2\bar{\nu}_6\circ\nu_{14}$ (by (1.3)) $=\nu_6\circ\varepsilon_9$ (by (2.1)). On the other hand, $\nu_6\circ[\iota_9,\iota_9]=\nu_6\circ(\sigma_9\circ\eta_{16}+\varepsilon_9+\bar{\nu}_9)=\nu_6\circ\sigma_9\circ\eta_{16}$, because $\nu_6\circ\varepsilon_9+\nu_6\circ\bar{\nu}_9$ $=4\bar{\nu}_6\circ\nu_{14}=0$ by (2.1). Since the kernel of the homomorphism $E:\pi_{16}^5\to\pi_{17}^6$ is generated by the element $\nu_5\circ\varepsilon_8+\nu_5\circ\bar{\nu}_8$, it follows that $\nu_5\circ\sigma_8\circ\eta_{15}=\nu_5\circ\varepsilon_8$ or $\nu_5\circ\bar{\nu}_8$.

(4) $H(\mu_3\circ\nu_{12})=H(\nu'\circ\eta_6\circ\varepsilon_7)=4(\nu_5\circ\sigma_8)$ ((1.4), (1.1)(3), (4), (7), (2.1)). $H(\mu'\circ\eta_{14})=H(\nu'\circ\mu_6)=\mu_5\circ\eta_{14}$ ((1.4), (1.1)(8), (2)). $H^{-1}(0)=0$.

(5) Auxiliary calculation: $E\nu'\circ\eta_7\circ E\pi_{15}^7+\pi_9^4\circ(2\sigma_9)=\{E\nu'\circ\eta_7\circ\varepsilon_8\}$, because $2\pi_9^4=0$, $E\pi_{15}^7=\{\varepsilon_8\}\oplus\{\bar{\nu}_8\}\oplus\{E\sigma'\circ\eta_{15}\}$, and $E\nu'\circ\eta_7\circ\bar{\nu}_8=E\nu'\circ\nu_7^3=0$, $E\nu'\circ\eta_7\circ E\sigma'\circ\eta_{15}=E\sigma'\circ(4\bar{\nu}_7)\circ\eta_{15}=0$.

Now, $\eta_4\circ\zeta_5\in\{\eta_4\circ\nu_5, 8\iota_8, E\sigma'\}_1$ (by (1.2)(6)) $=\{E\nu'\circ\eta_7, 8\iota_8, E\sigma'\}_1$ (by (2.1)). $E\nu'\circ\mu_7\in\{E\nu'\circ\eta_7, 2\iota_8, 4E\sigma'\}_3$ (by (1.2)(4)) $\subset\{E\nu'\circ\eta_7, 8\iota_8, E\sigma'\}_1$. The above calculation shows that the secondary composition $\{E\nu'\circ\eta_7, 8\iota_8, E\sigma'\}_1$ is a coset of the subgroup $\{E\nu'\circ\eta_7\circ\varepsilon_8\}$. Hence, we have $\eta_4\circ\zeta_5\equiv E\nu'\circ\mu_7 \bmod E\nu'\circ\eta_7\circ\varepsilon_8$. Since $E(E\nu'\circ\mu_7)=E(E\nu'\circ\eta_7\circ\varepsilon_8)=0$, it follows that $\eta_n\circ\zeta_{n+1}=0$ $(n\geqq 5)$.

(6) (1.6), (1.1)(8), (5).

(7) $H(\varepsilon'\circ\nu_{13})=0$ ((1.4), (1.1)(7), (2.1)), $H^{-1}(0)=0$.

(8) Auxiliary calculation: $\eta_{10}\circ\sigma_{11}\circ\pi_{23}^{18}+\pi_{22}^{10}\circ\eta_{22}=0$, because $\pi_{23}^{18}=0$, and $\pi_{22}^{10}\circ\eta_{22}$ $=\{[\iota_{10},\iota_{10}]\circ\nu_{19}\circ\eta_{22}\}=0$.

In the previous paper [3], we introduced a generalization of the secondary composition, which we call the second derived composition, denoted by $\{\alpha,\beta,\gamma,\delta\}$. Now, we shall give two examples.

LEMMA (2.2): $\mu_3\in\{\eta_3, E\nu', 8\iota_7, \nu_7\}_1$, $\kappa_7\in\{\nu_7, \eta_{10}, 2\iota_{11}, \bar{\nu}_{11}\}_1$. The first set is a coset of the subgroup $\{\eta_3\circ\varepsilon_4\}$, and the second is a coset of the subgroup $\{4\sigma'\circ\sigma_{14}\}\oplus\{\bar{\nu}_7\circ\nu_{15}^2\}$. Indeed, μ_3 is chosen from the set $\{\eta_3$, Ext. $(E\nu', 8\iota_7)$, Coext. $(8\iota_7,\nu_7)\}_1$ (p. 56 of [1]), and κ_7 from the set $\{\nu_7$, Ext. $(\eta_{10}, 2\iota_{11})$, Coext. $(2\iota_{11},\bar{\nu}_{11})\}_1$ (p. 96 of [1]). From the definition of the second derived composition (p. 48 of [3]), the first half of the Lemma follows. Calculation of the moduli of these cosets is shown below: (i) $\eta_3\circ\pi_{11}^3+\pi_9^3\circ\nu_9=\{\eta_3\circ\varepsilon_4\}$ (c.f. Prop. (6.5)(i) of [3]). (ii) $\nu_7\circ\pi_{21}^{10}+\pi_{13}^7\circ\bar{\nu}_{13}=(4\sigma'\circ\sigma_{14})$, $\{\nu_7, \eta_{10}^2, \bar{\nu}_{11}\}\ni\bar{\nu}_7\circ\nu_{15}^2 \bmod 4\sigma'\circ\sigma_{14}$. (c.f. Prop. (6.5)(ii) of [3]). (i) and (ii) follow from (1.1), (1.2) and (2.1).

In [1], the following relations are proved ((10.7), Lemma (10.1) of [1]).

(2.3) $\nu_6\circ\zeta_9=2\sigma''\circ\sigma_{13},\ 2\kappa_7\equiv\bar\nu_7\circ\nu_{15}^2 \bmod 4\sigma'\circ\sigma_{14}.$

We add the followings.

PROPOSITION (2.4):

(1) $\nu'\circ\zeta_6=\mu'\circ\nu_{14}=0.$

(2) $\sigma'''\circ\sigma_{12}\equiv\nu_5\circ\zeta_8 \bmod (\nu_5\circ\bar\nu_8\circ\nu_{16}),\ \zeta_5\circ\nu_{16}\equiv\nu_5\circ\zeta_8 \bmod (\nu_5\circ\bar\nu_8\circ\nu_{16}),$
$\zeta_n\circ\nu_{n+1}=\nu_n\circ\zeta_{n+3}\ (n\geqq 6).$

(3) $\theta'\circ\eta_{23}^2=0,\ \theta\circ\eta_{24}^2\equiv 8[\iota_{12},\nu_{12}] \bmod 8\sigma_{12}^2.$

PROOF: (1) $(H;$ (1.4), (1.1)(3), (8), Prop. (2.2)(4), (5)), $H^{-1}(0)=0$.

(2) Let $E^k:\pi_{19}^5\to\pi_{19+k}^{5+k}$ be the k-fold suspension, then the kernel of E^k $(1\leqq k\leqq 8)$ is generated by $\nu_5\circ\bar\nu_8\circ\nu_{16}$. Since $E(\sigma'''\circ\sigma_{12})=2\nu_6\circ\zeta_9$ by (1.1)(4) and (2.1), and since $\zeta_9\circ\nu_{20}=\nu_9\circ\zeta_{12}$ by (1.6), the proof is complete.

(3) $E^2(\theta'\circ\eta_{23}^2)=0\ (E^2;$ (1.1)(9)), $(E^2)^{-1}(0)=0$. $\theta\circ\eta_{24}^2-2[\iota_{12},\nu_{12}]\in E\pi_{25}^{11}\ (H;$ (1.4), (1.1)(3), (9)), $E\pi_{25}^{11}\cap(E^2)^{-1}(0)=\{8\sigma_{12}^2\}$.

The following relations are proved in Lemma (10.7) of [1].

(2.5) $\varepsilon_n\circ\sigma_{n+8}=0\ (n\geqq 3),\ \sigma_n\circ\varepsilon_{n+7}=0\ (n\geqq 11),\ \bar\nu_n\circ\sigma_{n+8}=0\ (n\geqq 6),$
$\eta_n\circ\kappa_{n+1}=\bar\varepsilon_n\ (n\geqq 6).$

We add the following relations.

PROPOSITION (2.6):

(1) $\sigma'''\circ\varepsilon_{12}=\sigma'''\circ\bar\nu_{12}=0,\ \sigma''\circ\varepsilon_{13}=\sigma''\circ\bar\nu_{13}=[\iota_6,\iota_6]\circ\sigma_{11}\circ\nu_{18}=0,\ \theta'\circ\nu_{23}=0,\ \theta\circ\nu_{24}=0.$

(2) $\nu_8\circ\theta'=E\sigma'\circ\varepsilon_{15}$ or $E\sigma'\circ\bar\nu_{15}$, $\nu_9\circ\theta=\sigma_9\circ\varepsilon_{16}$ or $\sigma_9\circ\bar\nu_{16}$.

(3) $\nu_n^5=0\ (n\geqq 4).$

(4) $\kappa_7\circ\eta_{21}=\sigma'\circ\bar\nu_{14}+\bar\varepsilon_7.$

PROOF: (1) Each of the followings is a monomorphism: $E:\pi_{20}^5\to\pi_{21}^6$, $E:\pi_{21}^6\to\pi_{22}^7$, $E^2:\pi_{26}^{11}\to\pi_{28}^{13}$, and $E^2:\pi_{27}^{12}\to\pi_{29}^{14}$. Hence, (1) follows from (1.1)(4) and (9).

(2) Auxiliary calculations: (a) $F_1=E\sigma'\circ\nu_{15}\circ\pi_{23}^{18}+\pi_{22}^8\circ\eta_{22}$, then $F_1\cap E^{-1}(0)=\{E\sigma'\circ\varepsilon_{15}+E\sigma'\circ\bar\nu_{15}\}$. Indeed, $\pi_{23}^{18}=0$, $\pi_{22}^8=\{E\sigma'\circ\sigma_{15}\}\oplus\{\sigma_8^2\}\oplus\{\kappa_8\}$. The kernel of the homomorphism $E:\pi_{23}^8\to\pi_{24}^9$ is generated by $\{E\sigma'\circ\varepsilon_{15}\}\oplus\{E\sigma'\circ\bar\nu_{15}\}$. $E(\sigma_8^2\circ\eta_{22})\neq 0$, $E(\kappa_8\circ\eta_{22})\neq 0$. Hence, $F_1\cap E^{-1}(0)=\{E\sigma'\circ\sigma_{15}\circ\eta_{22}\}=\{E\sigma'\circ\varepsilon_{15}+E\sigma'\circ\bar\nu_{15}\}$ by (2.1). (b) $F_2=\nu_9\circ\sigma_{12}\circ\pi_{24}^{19}+\pi_{23}^9\circ\eta_{23}$, then $F_2\cap(E^2)^{-1}(0)=\{\sigma_9\circ\varepsilon_{16}+\sigma_9\circ\bar\nu_{16}\}$. The proof is similar with that of (a).

Now, it follows from (1.2)(7) and (5) that $\nu_8\circ\theta'\in\{\nu_8\circ\sigma_{11},2\nu_{18},\eta_{21}\}_1=\{E\sigma'\circ\nu_{15},2\nu_{18},\eta_{21}\}_1$ (by (2.1)), and $E\sigma'\circ\varepsilon_{15}\in\{E\sigma'\circ\nu_{15},2\nu_{18},\eta_{21}\}_1$. Since $E(\nu_8\circ\theta')=\nu_9\circ E\theta'=\nu_9\circ[\iota_{12},\iota_{12}]\circ\eta_{23}$ (by (1.3)) $=[\nu_9,\nu_9]\circ\eta_{23}=[\iota_9,\iota_9]\circ\nu_{17}^2\circ\eta_{23}$ (by (1.5)) $=0$, the calculation (a) shows that $\nu_8\circ\theta'=E\sigma'\circ\varepsilon_{15}$ or $E\sigma'\circ\bar\nu_{15}$. Similarly, we can prove that $\nu_9\circ\theta=\sigma_9\circ\varepsilon_{16}$ or $\sigma_9\circ\bar\nu_{16}$.

(3) $\nu_n^5=\nu_n^2\circ\eta_{n+6}\circ\bar\nu_{n+7}$ (by (2.1)) $=0$ for $n\geqq 4$.

(4) Auxiliary calculations: (a) $\nu_7\circ\pi_{22}^{10}=\{\nu_7\circ[\iota_{10},\iota_{10}]\circ\nu_{19}=\{[\nu_7,\nu_7]\circ\nu_{19}\}=0.$

(b) $\pi_{16}^7\circ\nu_{16}^2=\{\eta_7\circ\varepsilon_8\circ\nu_{16}^2\}\oplus\{\mu_7\circ\nu_{16}^2\}\oplus\{\nu_7^5\}=0$, because $\eta_7\circ\varepsilon_8\circ\nu_{16}^2=\varepsilon_7\circ\eta_{15}\circ\nu_{16}^2=0$ (by (2.1)), $\mu_7\circ\nu_{16}=E^4(\nu'\circ\eta_6\circ\varepsilon_7)$ (by Prop. (2.2)(4)) $=0$, and $\nu_7^5=0$ (by (3)). (c) $\bar\nu_7\circ\pi_{22}^{15}=\{\bar\nu_7\circ\sigma_{15}\}=0$ (by (2.5)).

These calculations (a)~(c) show that each of the first and the second derived compositions(*) below consists of a single element. Now, $\kappa_7\circ\eta_{21}=\{\nu_7,\eta_{10},2\iota_{11},\bar\nu_{11}\}_1\circ\eta_{21}=\{\nu_7,\eta_{10},2\iota_{11},\bar\nu_{11}\circ\eta_{19}\}=\{\nu_7,\eta_{10},2\iota_{11},\nu_{11}^3\}_1$ (by (2.1)) $=\{\{\nu_7,\eta_{10},\nu_{11}\},2\iota_{15},\nu_{15}^2\}_1$ (by Prop. (6.13)(ii)) $=\{\bar\nu_7,2\iota_{15},\nu_{15}^2\}_1$ (by (1.2)(3)). $\Delta(\nu_{13})=[\iota_6,\nu_6]=-2\bar\nu_6$ (by (1.3)). Hence, it follows from Proposition (7.17) of [3] that $H\{\bar\nu_7,2\iota_{15},\nu_{15}^2\}_1=\nu_{13}^3$. Since $H(\sigma'\circ\bar\nu_{14})=\nu_{13}^3$ ((1.4), (1.1)(4), (2.1)), $\kappa_7\circ\eta_{21}\equiv\sigma'\circ\bar\nu_{14} \bmod E\pi_{21}^6=\{\bar\varepsilon_7\}\oplus\{4\rho''\}$. $E^\infty(4\rho'')=16\rho\neq 0$ (by (1.1)(11)), $E^\infty(\sigma'\circ\bar\nu_{14})=0$, $E^\infty(\kappa_7\circ\eta_{21})=\eta\circ\kappa=\bar\varepsilon$. Hence, we can conclude that $\kappa_7\circ\eta_{21}=\sigma'\circ\bar\nu_{14}+\bar\varepsilon_7$.

Now, the following relations are proved in [1].

(2.7) $\eta_{12}\circ\rho_{13}=\sigma_{12}\circ\mu_{19}$, $\sigma_n\circ\mu_{n+7}=\mu_n\circ\sigma_{n+9}=\eta_n\circ\rho_{n+1}=\rho_n\circ\eta_{n+15}$ $(n\geqq 13)$,
$\mu_{10}\circ\sigma_{19}=\sigma_{10}\circ\mu_{17}+8[\iota_{10},\sigma_{10}]$, $\sigma'\circ\eta_{14}\circ\varepsilon_{15}=E\zeta'$, $\nu_n\circ\sigma_{n+3}\circ\nu_{n+10}^2=\eta_n\circ\bar\varepsilon_{n+1}$ $(n\geqq 5)$,
$\varepsilon_n^2=\varepsilon_n\circ\bar\nu_{n+8}=\eta_n\circ\bar\varepsilon_{n+1}=\bar\varepsilon_n\circ\eta_{n+15}$ $(n\geqq 3)$, $\eta_n^*\equiv\omega_n \bmod \sigma_n\circ\mu_{n+7}$ $(n\geqq 18)$.

We add the followings.

PROPOSITION (2.8):

(1) $\eta_4\circ\rho^{\mathrm{IV}}=0$, $\rho^{\mathrm{IV}}\circ\eta_{20}=\eta_5\circ\rho'''=0$, $\rho'''\circ\eta_{21}=4\zeta'$.

(2) $\bar\nu_n^2=\bar\nu_n\circ\varepsilon_{n+8}=0$ $(n\geqq 6)$.

(3) $\eta_6\circ\rho''=4\zeta'$, $\rho''\circ\eta_{22}=\sigma'\circ\mu_{14}$.

(4) $\sigma'''\circ\mu_{12}=0$, $\sigma''\circ\mu_{13}=4\zeta'$.

(5) $\sigma_{10}\circ\nu_{17}^3=\sigma_{10}\circ\eta_{17}\circ\varepsilon_{18}=0$.

(6) $\rho'\circ\eta_{24}\equiv\mu_9\circ\sigma_{18}+\sigma_9\circ\mu_{16} \bmod \{\sigma_9\circ\varepsilon_{16}\circ\eta_{24}\}\oplus\{\sigma_9\circ\nu_{16}^3\}$, $E\rho'\circ\eta_{25}=8[\iota_{10},\sigma_{10}]$,
$\eta_8\circ\rho'\equiv 0 \bmod \{\eta_8\circ\bar\varepsilon_9\}\oplus\{E^2\zeta'\}\oplus\{E\sigma'\circ\mu_{15}\}$.

PROOF: (1) By Proposition (2.6)(1), $\sigma''\circ\pi_{21}^{13}=\{\sigma''\circ\varepsilon_{13}\}\oplus\{\sigma''\circ\bar\nu_{13}\}=0$. Hence we can improve (1.1)(11) as $2\rho'''=E\rho^{\mathrm{IV}}$, so that we have $E(\eta_{14}\circ\rho^{\mathrm{IV}})=0$ and $E(\rho^{\mathrm{IV}}\circ\eta_{20})=0$. $\eta_4\circ\rho^{\mathrm{IV}}\in E\pi_{19}^3$ (H; (1.4), (1.1)(11)), and $E\pi_{19}^3\cap E^{-1}(0)=0$ imply $\eta_4\circ\rho^{\mathrm{IV}}=0$.

Now, $E(\eta_5\circ\rho''')\equiv\eta_6\circ(2\rho'')=0 \bmod \eta_6\circ\sigma'\circ\pi_{22}^{14}$ (by (1.1)(11)). $\eta_6\circ\sigma'\circ\pi_{22}^{14}=(4\bar\nu_6)\circ\pi_{22}^{14}=0$ (by (2.1)). Hence, we have $E(\eta_5\circ\rho''')=0$. Thus $\rho^{\mathrm{IV}}\circ\eta_{20}=\eta_5\circ\rho'''=0$ follows from the fact that the homomorphism $E:\pi_{21}^5\to\pi_{22}^6$ is an isomorphism. Finally, $\rho'''\circ\eta_{21}-4\zeta'\in E\pi_{21}^5$ (H; (1.4), (1.1)(11), Prop. (2.2)(2), (1.1)(8)). $E(\rho'''\circ\eta_{21})=0$ ((1.6), (1.3)), $E(4\zeta')=4(E\sigma'\circ\eta_{15}\circ\varepsilon_{16})=0$ (by (2.5)). Hence, $\rho'''\circ\eta_{21}=4\zeta'$ follows from the fact that $E\pi_{21}^5\cap E^{-1}(0)=0$.

(2) $\bar\nu_6^2\in\{\nu_6,\eta_9,\nu_{10}\}_3\circ\bar\nu_{14}$ (by (1.2)(3)) $=\nu_6\circ E^3\{\eta_6,\nu_7,\bar\nu_{10}\}\subset\nu_6\circ E^3\pi_{19}^6=0$. $\bar\nu_6\circ\varepsilon_{14}=\bar\nu_6\circ(\bar\nu_{14}+\sigma_{14}\circ\eta_{21})$ (by (2.1)) $=\bar\nu_6\circ\sigma_{14}\circ\eta_{21}=0$ (by (2.5)).

(*) "First derived composition" means "secondary composition" (See [3]).

(3) Auxiliary calculations: (a) $\eta_8\circ\sigma_9\circ\varepsilon_{16}=(E\sigma'\circ\eta_{15}+\bar{\nu}_8+\varepsilon_8)\circ\varepsilon_{16}$ (by (2.1)) $=E^2\zeta'+\eta_8\circ\bar{\varepsilon}_9$ (by (2.5), (2)). (b) $\eta_8\circ\sigma_9\circ\bar{\nu}_{16}=(E\sigma'\circ\eta_{15}+\bar{\nu}_8+\varepsilon_8)\circ\bar{\nu}_{16}$ (by (2.1)) $=0$, because $\sigma'\circ\eta_{14}\circ\bar{\nu}_{15}=\sigma'\circ\nu_{14}^3=\nu_7\circ\sigma_{10}\circ\nu_{17}^2$ (by (2.1)) $=\eta_7\circ\bar{\varepsilon}_8$ (by (2.5)), $\bar{\nu}_8^2=0$ (by (2)), and $\varepsilon_8\circ\bar{\nu}_{16}=\eta_8\circ\bar{\varepsilon}_9$ (by (2.5)).

These calculations (a), (b) and (1.1)(11) imply that $E^2(\eta_6\circ\rho'')\equiv 0 \bmod \{E^2\zeta'+\eta_8\circ\bar{\varepsilon}_9\}$. However, there is no element of order 2 in π_{22}^8 whose image under E^2 is $E^2\zeta'+\eta_8\circ\bar{\varepsilon}_9$. Hence, $E^2(\eta_6\circ\rho'')$ must be 0. While, $\eta_6\circ\rho''-4\zeta'\in E\pi_{21}^5$ (H; (1.4), (1.1)(11), Prop. (2.2)(2), (1.1)(8)). Hence, the fact that $E\pi_{21}^5\cap(E^2)^{-1}(0)=0$ shows that $\eta_6\circ\rho''=4\zeta'$.

Now, $\rho''\circ\eta_{22}=\sigma'\circ\mu_{14}$ ((1.6), (1.3), $E^{-1}(0)=0$).

(4) $\sigma'''\circ\mu_{12}=0$ ((1.1)(4), $E^{-1}(0)=0$).

$\sigma''\circ\mu_{13}=4\zeta'$ (H; (1.4), (1.1)(4), (8). $E\pi_{21}^5\cap E^{-1}(0)=0$).

(5) It follows from the line 14 from the top of the page 156 in [1] that $\sigma_{10}\circ\nu_{17}^3\equiv 0 \bmod E^4\pi_{22}^6=\{\sigma_{10}\circ\mu_{17}\}$. However, $E^\infty(\sigma_{10}\circ\nu_{17}^3)=0$, while $E^\infty(\sigma_{10}\circ\mu_{17})\neq 0$. Hence, we conclude that $\sigma_{10}\circ\nu_{17}^3=0$. $\sigma_{10}\circ\eta_{17}\circ\varepsilon_{18}=(\varepsilon_{10}+\bar{\nu}_{10})\circ\varepsilon_{18}$ (by (2.1)) $=0$ (by (2.5)).

(6) Auxiliary calculations: (a) $\pi_{17}^9\circ\sigma_{17}\circ\eta_{24}\subset\{\sigma_9\circ\nu_{16}^3\}\oplus\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}\}$, because $\pi_{17}^9=\{\sigma_9\circ\eta_{16}\}\oplus\{\bar{\nu}_9\}\oplus\{\varepsilon_9\}$, $\bar{\nu}_9\circ\sigma_{17}=\varepsilon_9\circ\sigma_{17}=0$ (by (2.5)) and $\sigma_9\circ\eta_{16}\circ\sigma_{17}\circ\eta_{24}=\sigma_9\circ\nu_{16}^3+\sigma_9\circ\eta_{16}\circ\varepsilon_{17}$ (by (2.1)). (b) $\sigma_9\circ\pi_{25}^{16}=\{\sigma_9\circ\nu_{16}^3\}\oplus\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}\}\oplus\{\sigma_9\circ\mu_{16}\}$, because $\pi_{25}^{16}=\{\nu_{16}^3\}\oplus\{\eta_{16}\circ\varepsilon_{17}\}\oplus\{\mu_{16}\}$.

Now, since $\langle\eta, 2\iota, 8\sigma\rangle=\langle 8\sigma, 2\iota, \eta\rangle$ and $\mu_9\circ\nu_{18}^3=0$ (by Prop. (2.2)(4)), it follows from (1.2)(4) that $\mu_9\circ\sigma_{18}\in\{8\sigma_9, 2\iota_{16}, \eta_{16}\}\circ\sigma_{18}\subset\{\sigma_9, 16\iota_{16}, \eta_{16}\circ\sigma_{17}\}$, and it follows from (1.2)(8) that $\rho'\circ\eta_{14}\in\{\sigma_9, 16\iota_{16}, \sigma_{16}\}\circ\eta_{24}\subset\{\sigma_9\, 16\iota_{16}, \sigma_{16}\circ\eta_{23}\}$. Note that $\eta_{16}\circ\sigma_{17}=\sigma_{16}\circ\eta_{23}$, then the calculations (a), (b) show that $\mu_9\circ\sigma_{18}\equiv\sigma'\circ\eta_{14} \bmod \{\sigma_9\circ\nu_{16}^3\}\oplus\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}\}\oplus\{\sigma_9\circ\mu_{16}\}$. Now, $E^\infty(\rho'\circ\eta_{14})=(2\rho)\circ\eta=0$, $E^\infty(\sigma'\circ\eta_{14})=(2\sigma)\circ\eta=0$, $E^\infty(\sigma_9\circ\nu_{16}^3)=E^\infty(\sigma_9\circ\eta_{16}\circ\varepsilon_{17})=0$ (by (5)), however $E^\infty(\sigma_9\circ\mu_{16})\neq 0$. Hence, we conclude that $\rho'\circ\eta_{14}\equiv\mu_9\circ\sigma_{18}+\sigma_9\circ\mu_{16} \bmod \{\sigma_9\circ\nu_{16}^3\}\oplus\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}\}$.

By using the results of Propositions (2.6) and (2.8), we can improve (1.1)(11) as follows.

LEMMA (2.9): $2\rho'''=E\rho^{\mathrm{IV}}$, $2\rho''=E\rho'''$, $2\rho'=E^2\rho''$.

PROOF: The first part of this Lemma follows directly from (2.6)(1).

Now, $\sigma'\circ\pi_{22}^{14}\circ\eta_{22}=\{\sigma'\circ\varepsilon_{14}\circ\eta_{22}\}\oplus\{\sigma'\circ\bar{\nu}_{14}\circ\eta_{22}\}=\{E\zeta'\}\oplus\{\eta_7\circ\bar{\varepsilon}_8\}$, because $\sigma'\circ\varepsilon_{14}\circ\eta_{22}=E\zeta'$ (by (2.7)), and $\sigma'\circ\bar{\nu}_{14}\circ\eta_{22}=\sigma'\circ\nu_{14}^3$ (by (2.1)) $=\nu_7\circ\sigma_{10}\circ\nu_{17}^2$ (by (2.1)) $=\eta_7\circ\bar{\varepsilon}_8$ (by (2.7)). Since $E\rho'''\circ\eta_{22}\equiv 0 \bmod \sigma'\circ\pi_{22}^{14}\circ\eta_{22}$ (by (1.1)(11)), it follows that $E\rho'''\circ\eta_{22}\equiv 0 \bmod \{E\zeta'\}\oplus\{\eta_7\circ\bar{\varepsilon}_8\}$. However, $E(\rho'''\circ\eta_{21})=E(4\zeta')=0$ (by Prop. (2.8)(1)), we can conclude that $E\rho'''=2\rho''$. Similarly, we can prove the last part of the Lemma by using (1.1)(11) and Prop. (2.8)(1), (3).

By (1.3), we have that $[\iota_{16}, \eta_{16}]\equiv E\eta^{*\prime} \bmod E^2\pi_{30}^{14}=\{\sigma_{16}\circ\mu_{23}\}\oplus\{\omega_{16}\}$. However, we can improve this formula as follows.

LEMMA (2.10): $E\eta^{*\prime} \equiv [\iota_{16}, \eta_{16}] \bmod \{\sigma_{16}\circ\mu_{23}\}$.

PROOF: Auxiliary calculation: $\sigma_{15}\circ\pi_{31}^{22}+\pi_{30}^{15}\circ\eta_{30}=\{\sigma_{15}\circ\mu_{22}\}$, because $\sigma_{15}\circ\eta_{22}\circ\varepsilon_{23} =\bar{\varepsilon}_{15}\circ\eta_{30}=0$ (by (2.1). Prop. (2.8)(2), (2.7)), and $\rho_{15}\circ\eta_{30}=\sigma_{15}\circ\mu_{22}$ (by (2.7)).

Now, it follows from (1.2)(9) that $E^2\eta^{*\prime}\in\{\sigma_{17}, 4\sigma_{24}, \eta_{31}\}$ which is a coset of the subgroup $\{\sigma_{15}\circ\mu_{22}\}$ by the above calculation. While $\{\sigma_{17}, 4\sigma_{24}, \eta_{31}\}\supset 2\{\sigma_{17}, 2\sigma_{24}, \eta_{31}\} =0$, so that we conclude that $E\eta^{*\prime}\equiv[\iota_{16}, \eta_{16}] \bmod \{\sigma_{16}\circ\mu_{23}\}$.

Now, we shall add the following.

LEMMA (2.11): *The element ω_{14} is chosen from the coset*(*) $\phi[Coext.\ (E\theta, \nu_{25})]$ *of the subgroup* $\{\sigma_{14}\circ\mu_{21}\}$.

PROOF: $E\theta=[\iota_{13}, \iota_{13}]$ by (1.3). Hence the complex S_2^{13} is described as $S^{13}\cup_{E\theta} e^{26}$. Note that $E\theta\circ\nu_{25}=0$ (by Prop. (2.6) (1)). Now, let us consider a coextension $Coext.\ (E\theta, \nu_{25})\in\pi_{29}(S_2^{13})$. $H\phi[Coext.\ (E\theta, \nu_{25})]=\phi\circ i_*^{(\#)}(E\nu_{25})=E^2\nu_{25}=\nu_{27}$ by the definition of H. Hence, the element ω_{14} belongs to the coset $\phi[Coext.(E\theta, \nu_{25})]$ of the subgroup $\phi\circ i_*\circ E\pi_{28}^{12}=E^2\pi_{28}^{12}=\{\sigma_{14}\circ\mu_{21}\}$.

The following relation is proved in [1].

(2.12) $\quad E\bar{\varepsilon}'=E\nu'\circ\kappa_7,\ \ \nu_5\circ\bar{\nu}_8\circ\nu_{16}^2=2\nu_5\circ\kappa_8.$

We add the following relations which appear on π_{n+17}^n.

PROPOSITION (2.13):

(1) $\eta_{13}\circ\omega_{14}=\varepsilon_{13}^*,\ \eta_n\circ\omega_{n+1}=\omega_n\circ\eta_{n+16}=\varepsilon_n^*\ (n\geqq 14)$.

(2) $\kappa_n\circ\nu_{n+14}=\nu_n\circ\kappa_{n+3}\ (n\geqq 7)$.

(3) $\eta_{14}\circ\eta^{*\prime}\equiv 0 \bmod \sigma_{14}\circ\eta_{21}\circ\mu_{22},\ \eta_{15}\circ\eta_{16}^*\equiv\eta^{*\prime}\circ\eta_{31}+\varepsilon_{15}^* \bmod \sigma_{15}\circ\eta_{22}\circ\mu_{23}$.

(4) $\nu'\circ\bar{\nu}_6\circ\nu_{14}^2=\varepsilon_3\circ\nu_{11}^3=2\bar{\varepsilon}'$.

(5) $\zeta'\circ\eta_{22}=0,\ \eta_6\circ\zeta'=0$.

(6) $\varepsilon'\circ\sigma_{13}=2\bar{\varepsilon}'$.

(7) $\mu_3\circ\varepsilon_{12}\equiv\varepsilon_3\circ\mu_{11} \bmod 2\bar{\varepsilon}',\ \mu_3\circ\varepsilon_{12}\equiv\eta_3\circ\mu_4\circ\sigma_{13} \bmod 2\bar{\varepsilon}'$.

(8) $\mu_3\circ\bar{\nu}_{12}\equiv 0 \bmod 2\bar{\varepsilon}',\ \mu_n\circ\bar{\nu}_{n+9}=0\ (n\geqq 5),\ \bar{\nu}_n\circ\mu_{n+8}=0\ (n\geqq 6)$.

(9) $\nu_5\circ E\sigma'\circ\sigma_{15}=0,\ \nu_5\circ\sigma_8^2=2\nu_5\circ\kappa_8$ or 0.

(10) $[\iota_6, \iota_6]\circ\theta'=0$.

PROOF: (1) (1.6), $(E^3)^{-1}(0)$, where $E^3:\pi_{30}^{13}\to\pi_{33}^{16}$.

(2) $H(\kappa_7)\in\pi_{21}^{13}$ and $\pi_{21}^{13}\circ\nu_{21}=0$ imply that $H(\kappa_7\circ\nu_{21})=0$, so that $\kappa_7\circ\nu_{21}\in E\pi_{23}^6$. Since $E^\infty: E\pi_{23}^6\to G_{17}$ is a monomorphism, (2) follows from the fact that $\kappa\circ\nu=\nu\circ\kappa$ in G_{17}.

(3) $E\eta^{*\prime}\circ\eta_{32}\equiv[\iota_{16}, \iota_{16}]\circ\eta_{31}^2 \bmod \sigma_{16}\circ\mu_{23}\circ\eta_{32}$ (by Lemma (2.10)). $[\iota_{16}, \iota_{16}]\circ\eta_{31}^2= [\eta_{16}, \eta_{16}]$ (by (1.5)) $=\eta_{16}\circ[\iota_{17}, \iota_{17}]\equiv\eta_{16}\circ(\eta_{17}^*+\omega_{17})$ (by (1.3)) $=\eta_{16}\circ\eta_{17}^*+\varepsilon_{17}$ (by (1)). The

(*) ϕ indicates the canonical isomorphism $\pi_{29}(S_\infty^{13})\approx\pi_{30}(S^{14})$.

(#) i_* indicates the homomorphism induced by the inclusion $S^{26}\subset S_\infty^{26}$.

second assertion follows from the fact that the homomorphism $E:\pi_{32}^{15}\to\pi_{33}^{16}$ is a monomorphism. The first assertion holds, because $E^2(\eta_{14}\circ\eta^{*\prime})\equiv 0 \bmod \sigma_{16}\circ\mu_{23}\circ\eta_{32}$, and $E^2:\pi_{31}^{14}\to\pi_{33}^{16}$ is a monomorphism.

(4) $\nu'\circ\bar{\nu}_6\circ\nu_{14}^2=\varepsilon_3\circ\nu_{11}^3=\varepsilon_3\circ\eta_{11}\circ\bar{\nu}_{12}=\eta_3\circ\varepsilon_4\circ\bar{\nu}_{12}$ (by (2.1)) $=\eta_3^2\circ\bar{\varepsilon}_5$ (by (2.7)) $=2\bar{\varepsilon}'$ (by (1.1)(13)).

(5) Auxiliary calculations: (a) $\sigma''\circ\pi_{22}^{13}\circ\eta_{22}=0$ (by (2.1)). (b) $\eta\circ G_{16}+G_9\circ\varepsilon=\{\sigma\circ\eta\circ\mu\}\oplus\{\varepsilon^*\}$, because $G_{16}=\{\sigma\circ\mu\}\oplus\{\omega\}$, $G_9=\{\eta\circ\varepsilon\}\oplus\{\mu\}\oplus\{\nu^3\}$, $\omega\circ\eta=\varepsilon^*$, $\eta\circ\varepsilon^2=\eta^2\circ\bar{\varepsilon}=0$, $\mu\circ\varepsilon=\eta\circ\mu\circ\sigma$ (which is proved in (7)), $\nu^3\circ\varepsilon=0$. Now, $\zeta'\circ\eta_{22}\in\{\sigma'', \varepsilon_{13}, 2\iota_{21}\}_1\circ\eta_{22}=\sigma''\circ\{\varepsilon_{13}, 2\iota_{21}, \eta_{21}\}$ (by (1.2)(9)), which consists of a single element $\sigma''\circ E^{10}\varepsilon'=0$ by (a). Hence, we have $\zeta'\circ\eta_{22}=0$.

Now, $\eta_5\circ\zeta'=2\nu_5\circ\kappa_8$ or 0 (E; (2.7), (2.1), $E^{-1}(0)=\{2\nu_5\circ\kappa_8\}$.) While, $\eta_5\circ\zeta'\in\eta_5\circ\{\sigma'', \varepsilon_{13}, 2\iota_{21}\}=\{\eta_5, \sigma'', \varepsilon_{13}\}\circ(2\iota_{22})$. However, $E^\infty\{\eta_5, \sigma'', \varepsilon_{13}\}\subset\langle\eta, 4\sigma, \varepsilon\rangle$, which includes $2\langle\eta, 2\sigma, \varepsilon\rangle=0$, so that the calculation (b) implies that $\nu\circ\kappa\notin\langle\eta, 4\sigma, \varepsilon\rangle$. Hence, we can conclude that $\eta_5\circ\zeta'=0$.

To prove (6), we need the following.

LEMMA (2.14): *Secondary composition* $\{\eta_4, 2\iota_5, \nu_5\circ\sigma_8\circ\nu_{15}\}$ *consists of an element* $\bar{\varepsilon}_4$.

PROOF: Auxiliary calculations: (a) $\nu_5^2\circ\pi_{18}^{11}+\pi_{12}^5\circ\nu_{12}^2=0$. Indeed, $\pi_{18}^{11}=\{\sigma_{11}\}$, $\pi_{12}^5=\{\sigma'''\}$, and $\nu_5^2\circ\sigma_{11}=\nu_5\circ E\sigma'\circ\nu_{15}=2(\nu_5\circ\sigma_8\circ\nu_{15})=0$ (by (2.1)), $\sigma'''\circ\nu_{12}^2=(4\nu_5\circ\sigma_8)\circ\nu_{15}=0$ (by (2.1)). (b) $\eta_4\circ\pi_{19}^5+\pi_6^4\circ\nu_6\circ\sigma_9\circ\nu_{16}=0$. Indeed, $\pi_{19}^5=\{\nu_5\circ\zeta_8\}\oplus\{\nu_5\circ\bar{\nu}_8\circ\nu_{16}\}$, $\pi_6^4=\{\eta_4^2\}$, and $\eta_4\circ\nu_5\circ\zeta_8=E\nu'\circ\eta_7\circ\zeta_8=0$ (by (2.1), Prop. (2.2)(5)), $\eta_4\circ\nu_5\circ\bar{\nu}_8\circ\nu_{16}=E\nu'\circ\eta_7\circ\bar{\nu}_8\circ\nu_{16}=E\nu'\circ\nu_7^4=0$ (by (2.1)), $\eta_5\circ\nu_6=0$. Now, consider the secondary composition $\{\nu_5^2, 2\iota_{11}, \nu_{11}^2\}_1$, which consists of a single element by (a). $H\{\nu_5^2, 2\iota_{11}, \nu_{11}^2\}_1=\Delta^{-1}(2\nu_4^2)\circ\nu_{12}^2=\nu_9^3$, because $\Delta(\nu_9)=[\iota_4, \iota_4]\circ\nu_7=2\nu_4^2$ (by (1.3)). Hence $\{\nu_5^2, 2\iota_{11}, \nu_{11}^2\}_1=\nu_5\circ\sigma_8\circ\nu_{15}$ or $\nu_5\circ\sigma_8\circ\nu_{15}+\nu_5\circ\eta_8\circ\mu_9$. Now, $\bar{\varepsilon}_4\in\{\varepsilon_4, 2\iota_{12}, \nu_{12}^2\}_7=\{\{\eta_4, 2\iota_5, \nu_5^2\}, 2\iota_{12}, \nu_{12}^2\}\sim\pm\{\eta_4, 2\iota_5, \{\nu_5^2, 2\iota_{11}, \nu_{11}^2\}\}$. Note that $\{\eta_4, 2\iota_5, \nu_5\circ\eta_8\circ\mu_9\}\subset\pi_{10}^4\circ\mu_{10}=\{\nu_4^2\circ\mu_{10}\}=0$ (by (2.1)). By virtue of the calculation (b), we conclude that $\bar{\varepsilon}_4=\{\eta_4, 2\iota_5, \nu_5\circ\sigma_8\circ\nu_{15}\}$.

Proof of Proposition (2.13):

(6) $\varepsilon'\circ\sigma_{13}\in\{\nu', 2\nu_6, \nu_9\}_1\circ\sigma_{13}\subset\{\nu', 2\nu_6, \nu_9\circ\sigma_{12}\}_1$ (by (1.2)(5)), which consists of a single element $2\bar{\varepsilon}'$. Because $2\bar{\varepsilon}'=\eta_3^2\circ\bar{\varepsilon}_5$ (by (1.1)(13)) $\in\eta_3^2\circ\{\eta_5, 2\iota_6, \nu_6\circ\sigma_9\circ\nu_{16}\}_1$ (by Lemma (2.14)) $\subset\{\eta_3^3, 2\iota_6, \nu_6\circ\sigma_9\circ\nu_{16}\}_1\subset\{2\nu', 2\nu_6, \sigma_9\circ\nu_{16}\}_1\subset\{\nu', 2\nu_6, 2\sigma_9\circ\nu_{16}\}_1=\{\nu', 2\nu_6, \nu_9\circ\sigma_{12}\}_1$ (by (2.1)), and $\nu'\circ E\pi_{19}^5=\{\nu'\circ\nu_6\circ\zeta_9\}=0$, $\pi_{10}^3\circ\nu_{10}\circ\sigma_{13}=0$.

(7) $\mu_5\circ\varepsilon_{14}=\varepsilon_5\circ\mu_{13}$ ((1.6), (1.1)(6)), $\mu_3\circ\varepsilon_{12}\in E\pi_{19}^2$ (H; (1.4), (1.1)(6), Prop. (2.6)(1)). Now, $\mu_3\circ\varepsilon_{14}\circ\eta_{22}+\mu_5\circ\bar{\nu}_{14}\circ\eta_{22}=\mu_5\circ\eta_{14}\circ\sigma_{15}\circ\eta_{22}=\eta_5^2\circ\mu_7\circ\sigma_{16}\neq 0$ (by (2.1)). While, $\mu_5\circ\bar{\nu}_{14}\circ\eta_{22}=\mu_5\circ\nu_{14}^3$ (by (2.1)) $=E^2(\nu'\circ\eta_6\circ\varepsilon_7)\circ\nu_{17}^2$ (by Prop. (2.2)(4)) $=0$. Hence, $\mu_3\circ\varepsilon_{14}\circ\eta_{22}\neq 0$, so that $\mu_3\circ\varepsilon_{14}\neq 0$. Since $E^3\pi_{19}^2=\{\eta_5\circ\mu_6\circ\sigma_{15}\}$, we conclude that $\mu_5\circ\varepsilon_{14}=\eta_5\circ\mu_6\circ\sigma_{15}$. Thus, (7) follows from the fact that $E\pi_{19}^2\cap(E^2)^{-1}(0)=\{2\bar{\varepsilon}'\}$.

(8) $\mu_3\circ\bar{\nu}_{12}=\mu_3\circ\eta_{12}\circ\sigma_{13}+\mu_3\circ\varepsilon_{12}\equiv 0 \bmod 2\bar{\varepsilon}'$ (by (2.1), (7)), so that $\mu_5\circ\bar{\nu}_{14}=0$. $\bar{\nu}_6\circ\mu_{14}\in E\pi^5_{21}$ (H; (1.4), (1.1)(5), (2.1)), $\mu\circ\bar{\nu}=\bar{\nu}\circ\mu$. Hence, the last assertion of (8) follows from the fact that $E^\infty: E\pi^5_{22}\to G_{17}$ is a monomorphism.

(9) $\nu_5\circ E\sigma'\circ\sigma_{15}=2(\nu_5\circ\sigma_8^2)$ (by (2.1)) $=\pm E^2(\varepsilon'\circ\sigma_{13})$ (by (2.1)) $=\pm E^2(2\bar{\varepsilon}')$ (by (6)) $=0$. To prove the second assertion, note that $E^\infty(\nu_5\circ\sigma_8^2)=0$. The element α of π^5_{22} which satisfies the conditions $E^\infty\alpha=0$ and $2\alpha=0$ is either $2\nu_5\circ\kappa_8$ or 0. Hence, $\nu_5\circ\sigma_8^2=2\nu_5\circ\kappa_8$ or 0.

(10) $[\iota_6,\iota_6]\circ\theta'\in E\pi^5_{22}$($H$; (1.4), $2\theta'=0$), $E\pi^5_{22}\cap E^{-1}(0)=0$.

Now, we shall prove the following two lemmata.

LEMMA (2.15): (1) *We may chose ε^*_{12} from the coset $\{\theta,\nu_{24},\eta_{27}\}_1$ of the subgroup $\{\sigma_{12}\circ\eta_{19}\circ\mu_{20}\}$.*

(2) *We may chose $\bar{\varepsilon}'$ of π^3_{20} from the coset $\{\eta_3, 2\iota_4, \bar{\varepsilon}_4\}_1$ of the subgroup $\{\eta_3^2\circ\bar{\varepsilon}_5\}\oplus\{\eta_3\circ\mu_4\circ\sigma_{13}\}$. The secondary composition $\{\nu', 2\nu_6, \sigma_9\circ\nu_{16}\}_1$ consists of an element $\bar{\varepsilon}'$* or $-\bar{\varepsilon}'$.

PROOF: (1) $H\{\theta,\nu_{24},\eta_{27}\}_1\subset\{H(\theta),\nu_{24},\eta_{27}\}_1=\{\eta_{23},\nu_{24},\eta_{27}\}_1$ (by (1.1)(9)), which consists of a single element ν^2_{23}. It follows from Proposition (5.11) of [3] that the following diagram is homotopy commutative:

$$\begin{tikzcd}
S^{13} \arrow[d, "i"'] & S^{25}\cup_{\nu} e^{29} \arrow[l, "\overline{E\theta}"'] \arrow[d, "p"] & S^{30} \arrow[l, "\tilde{\eta}_{28}"'] \arrow[d, "id."] \\
S^{13}\cup_{E\theta} e^{26} & S^{29} \arrow[l, "-\tilde{\nu}_{25}"'] & S^{30} \arrow[l, "\eta_{29}"']
\end{tikzcd}$$

where $\overline{E\theta}\in Ext.(E\theta,\nu_{25})$, $\tilde{\eta}_{28}\in Coext.(\nu_{25},\eta_{28})$, $\tilde{\nu}_{25}\in Coext.(E\theta,\nu_{25})$, and i is the inclusion map, p the shrinking map. By virtue of Lemma (2.11), we have $E\{E\theta,\nu_{25},\eta_{28}\}\sim\phi\circ i_*(E\theta\circ\eta_{28})=\phi[Coext.(E\theta,\nu_{25})]\circ\eta_{30}\ni\omega_{14}\circ\eta_{30}=\eta_{14}\circ\omega_{15}$ (by Prop. (2.13)(1)). $E\{E\theta,\nu_{25},\eta_{28}\}$ and $\phi[Coext.(E\theta,\nu_{25})]\circ\eta_{30}$ are the same coset of the subgroup $\{\sigma_{14}\circ\eta_{21}\circ\mu_{22}\}$, so that we can chose an element ε^*_{12} from the secondary composition $\{\theta,\nu_{24},\eta_{27}\}_1$ such that $H(\varepsilon^*_{12})=\nu^2_{23}$ and $E\varepsilon^*_{12}=\eta_{13}\circ\omega_{14}$.

(2) $H\{\eta_3,2\iota_4,\bar{\varepsilon}_4\}_1=\Delta^{-1}(\eta_2)\circ\bar{\varepsilon}_5=\bar{\varepsilon}_5$. $2\{\eta_3,2\iota_4,\bar{\varepsilon}_4\}_1=\eta_3\circ\{2\iota_4,\bar{\varepsilon}_4,2\iota_{19}\}=\eta_3^2\circ\bar{\varepsilon}_5$. $\eta_3\circ E\pi^3_{19}+\pi^3_5\circ\bar{\varepsilon}_5=\{\eta_3^2\circ\bar{\varepsilon}_5\}\oplus\{\eta_3\circ\mu_4\circ\sigma_{13}\}$. Hence, the first assertion is proved. Now, $H\{\nu',2\nu_6,\sigma_9\circ\nu_{16}\}_1=\{\eta_5,2\nu_6,\sigma_9\circ\nu_{16}\}_1=\bar{\varepsilon}_5$ (by Lemma (2.14)). $E^\infty\{\nu',2\nu_6,\sigma_9\circ\nu_{16}\}\subset\langle 2\nu, 2\nu,\sigma\circ\nu\rangle=0$, because $\sigma\circ\nu=0$, and $2\nu\circ G_{14}=0$. $\nu'\circ E\pi^5_{19}+\pi^3_{10}\circ\sigma_{10}\circ\nu_{17}=\{\nu'\circ\nu_6\circ\zeta_9\}=0$. These calculations show that the secondary composition $\{\nu,2\nu_6,\sigma_9\circ\nu_{16}\}_1$ contains a single element $\bar{\varepsilon}'$ or $-\bar{\varepsilon}'$.

The following relations are proved in [1].

(2.16) $$\sigma'\circ\zeta_{14}=x\zeta_7\circ\sigma_{18} \text{ for some odd integer } x.$$
$$4\sigma_{10}\circ\zeta_{17}=2\sigma_{11}\circ\zeta_{18}=\sigma_{13}\circ\zeta_{20}=0.$$

We add the relations which appear on π^n_{n+18}.

PROPOSITION (2.17):

(1) $\eta^{*\prime}\circ\eta^2_{31}=4E^2\lambda$.

(2) $\eta^*_{16}\circ\eta^2_{32}\equiv 4\nu^*_{16} \bmod 4E^3\lambda$, $\eta^*_{17}\circ\eta^2_{33}=4\nu^*_{17}$.

(3) $\varepsilon^*_{13}\circ\eta_{30}=\eta_{13}\circ\varepsilon^*_{14}\equiv 4\xi_{13} \bmod 4\lambda_{13}$.

(4) $\bar{\varepsilon}_3\circ\nu_{18}=\nu'\circ\bar{\varepsilon}_6$.

(5) $\nu_5\circ\sigma_8\circ\varepsilon_{15}=\nu_5\circ\sigma_8\circ\bar{\nu}_{15}\equiv 0 \bmod \nu_5\circ\bar{\varepsilon}_8$.

(6) $\rho^{\mathrm{IV}}\circ\nu_{20}\equiv 4\zeta_5\circ\sigma_{16} \bmod \nu_5\circ\bar{\varepsilon}_8$, $\rho'''\circ\nu_{21}=+2\zeta_6\circ\sigma_{17}$,
$\rho''\circ\nu_{22}=\sigma'\circ\zeta_{14}\equiv\zeta_7\circ\sigma_{18} \bmod 2\zeta_7\circ\sigma_{18}$,
$\rho'\circ\nu_{24}\equiv -\sigma_9\circ\zeta_{16} \bmod 4\sigma_9\circ\zeta_{16}$, $\rho_{13}\circ\nu_{28}=4\lambda$.

(7) $\nu'\circ\rho'''=0$, $E\nu'\circ\rho''=0$, $\nu_5\circ E\rho''\equiv 0 \bmod 4\zeta_5\circ\sigma_{16}$, $\nu_6\circ\rho'\equiv 0 \bmod 2\zeta_6\circ\sigma_{17}$,
$\nu_{10}\circ\rho_{13}\equiv 0 \bmod 2\sigma_{10}\circ\zeta_{17}$.

(8) $\varepsilon'\circ\varepsilon_{13}=\varepsilon'\circ\bar{\nu}_{13}=\bar{\varepsilon}'\circ\eta_{20}=\nu'\circ\bar{\varepsilon}_6$.

(9) $\mu^2_3\equiv\eta_3\circ\bar{\mu}_4 \bmod 2\mu'\circ\sigma_{14}$.

(10) $\eta_n\circ\bar{\mu}_{n+1}=\bar{\mu}_n\circ\eta_{n+17}$ $(n\geqq 3)$.

(11) $\sigma'''\circ\zeta_{12}=4\zeta_5\circ\sigma_{16}$, $\sigma''\circ\zeta_{13}=\pm 2\zeta_6\circ\sigma_{17}$.

PROOF: (1) $E\eta^{*\prime}\circ\eta^2_{32}=[\iota_{16},\eta_{16}]\circ\eta^2_{32}$ (by Lemma (2.10)) $=[\iota_{16},4\nu_{16}]=4(E^3\lambda-2\nu^*_{16})$ (by (1.3)) $=4E^3\lambda$. Since $E:\pi^{15}_{33}\to\pi^{16}_{34}$ is a monomorphism, we have (1).

(2) Auxiliary calculations: (a) $(\sigma_{16}\circ E\pi^{22}_{31}+\pi^{16}_{31}\circ\eta_{31})\circ\eta^2_{32}=\{4E^3\lambda\}$. Because, $\sigma_{16}\circ E\pi^{22}_{31}\circ\eta^2_{32}=\{\sigma_{16}\circ\mu_{23}\circ\eta^2_{32}\}=\{\sigma_{16}\circ 4\zeta_{23}\}$ (by (1.1)(8)) $=0$ (by (2.16)), and $\pi^{16}_{31}=\{\bar{\varepsilon}_{16}\}\oplus\{\rho_{16}\}\oplus\{[\iota_{16},\iota_{16}]\}$, $\bar{\varepsilon}_{16}\circ\eta^3_{31}=0$, $\rho_{16}\circ\eta^3_{31}=\sigma_{16}\circ\mu_{23}\circ\eta^2_{32}$ (by (2.7)) $=0$, $[\iota_{16},\iota_{16}]\circ\eta^3_{31}=4E^3\lambda$ (by (1)). (b) $\sigma_{16}\circ\pi^{23}_{34}+\pi^{16}_{31}\circ(4\nu_{31})=\{4E^3\lambda\}$. Because, $\sigma_{16}\circ\pi^{23}_{34}=\{\sigma_{16}\circ\zeta_{23}\}=0$ (by (2.16)). Now, $\eta^*_{16}\circ\eta^2_{32}\in\{\sigma_{16},2\sigma_{23},\eta_{30}\}_1\circ\eta^2_{32}\subset\{\sigma_{16},2\sigma_{23},4\nu_{30}\}_1$ (by (1.2)(9)). While, $4\nu^*_{16}\in\{\sigma_{16},2\sigma_{23},4\nu_{30}\}$ (by (1.2)(11)).

The calculations (a), (b) show that these two secondary compositions are cosets of the same subgroup $\{4E^3\lambda\}$, hence, $\eta^*_{16}\circ\eta^2_{32}=4\nu^*_{16} \bmod 4E^3\lambda$. The second assertion is true, because $4E^4\lambda=0$.

(3) It follows from Corollary 12.25 of [1] that $[\iota_{17},\iota_{17}]\circ\eta^2_{33}=\Delta(\eta^2_{35})=4(\nu^*_{17}+\xi_{17})$. On the other hand, $[\iota_{17},\iota_{17}]\circ\eta^2_{33}=(\eta^*_{17}+\omega_{17})\circ\eta^2_{33}$ (by (1.3)) $=4\nu^*_{17}+\varepsilon^*_{17}\circ\eta_{34}$ (by (2) and (1.1)(13)). Hence, we have $\varepsilon^*_{17}\circ\eta_{34}=4\xi_{17}$. Since the kernel of the homomorphism $E^4:\pi^{13}_{31}\to\pi^{17}_{35}$ is generated by $4\lambda_{13}$, it follows that $\varepsilon^*_{13}\circ\eta_{30}\equiv 4\xi_{13} \bmod 4\lambda_{13}$. By Applying (1.6) to the pair $(\varepsilon^*_{12},\eta_2)$, we have $\varepsilon^*_{13}\circ\eta_{30}+\eta_{13}\circ\varepsilon^*_{14}=[\iota_{13},\iota_{13}]\circ\nu^2_{25}$ (by (1.1)(13)) $=E\theta\circ\nu^2_{25}$ (by (1.3)) $=0$ (by Prop. (2.6)(1)).

(4) $\bar{\varepsilon}_3\circ\nu_{18}-\nu'\circ\bar{\varepsilon}_6\in E\pi^2_{20}$ (H; (1.4), (1.1) (3) (11), (2.7)). $E^3(\bar{\varepsilon}_3\circ\nu_{18})=\eta_6\circ\kappa_7\circ\nu_{21}$ (by (2.5)) $=\eta_6\circ\nu_7\circ\kappa_{10}$ (by Prop; (2.13) (2)) $=0$. $E^3(\nu'\circ\bar{\varepsilon}_6)=2\nu_6\circ\bar{\varepsilon}_9$ (by (1.1) (3)) $=0$. $E\pi^2_{20}\cap(E^3)^{-1}(0)=0$.

(5) $E^3(\nu_5\circ\sigma_8\circ\varepsilon_{15})=0$ (by (2.5)). Since the kernel of the homomorphism E^3: $\pi^5_{23}\to\pi^8_{26}$ is generated by $\{\nu_5\circ\bar{\varepsilon}_8\}$, it follows that $\nu_5\circ\sigma_8\circ\varepsilon_{15}\equiv 0 \bmod \nu_5\circ\bar{\varepsilon}_8$. Now,

$\nu_5\circ\sigma_8\circ\varepsilon_{15}+\nu_5\circ\sigma_8\circ\bar{\nu}_{15}=\nu_5\circ\sigma_8\circ\eta_{15}\circ\sigma_{16}$ (by (2.1)) $=\nu_5\circ\varepsilon_8\circ\sigma_{16}$ or $\nu_5\circ\bar{\nu}_8\circ\sigma_{16}$ (by Prop. (2.2)(3)). However, $\varepsilon_8\circ\sigma_{16}=\bar{\nu}_8\circ\sigma_{16}=0$ (by (2.5)). Hence, we have $\nu_5\circ\sigma_8\circ\bar{\nu}_{15}=\nu_5\circ\sigma_8\circ\varepsilon_{15}$.

(6) Applying the Hopf homomorphism, we can easily see that $\rho^{\mathrm{IV}}\circ\nu_{20}$, $\rho'''\circ\nu_{21}$, $\rho''\circ\nu_{22}$, $\rho'\circ\nu_{24}$ and $\rho_{13}\circ\nu_{28}$ are suspension elements. Since $\sigma'\circ\pi^{14}_{22}\circ\nu_{22}=\{\sigma'\circ\varepsilon_{14}\circ\nu_{22}\}\oplus\{\sigma'\circ\bar{\nu}_{14}\circ\nu_{22}\}=0$, it follows from (1.2)(8) that $\rho''\circ\nu_{22}=\{\sigma', 8\iota_{14}, 2\sigma_{14}\}_1\circ\nu_{22}=-\sigma'\circ\{8\iota_{14}, 2\sigma_{14}, \nu_{21}\}=\sigma'\circ\zeta_{14}$ (by (9.3) of [1]) $=x\zeta_7\circ\sigma_{18}$ for some odd integer x (by (2.16)). Hence, we have proved the third assertion of (6). It follows that $E(\rho'''\circ\nu_{21})=2\rho''\circ\nu_{22}$ (by Lemma (2.9)) $=\pm 2\zeta_7\circ\sigma_{18}$. Since the homomorphism $E^2: E\pi^5_{23}\to\pi^7_{25}$ is an isomorphism, we have the second assertion of (6). Similarly, we can prove the first and the fourth. Now, applying (1.6) to the pair (ρ_{13}, ν_4), we have $\nu_{16}\circ\rho_{19}+\rho_{16}\circ\nu_{31}=[\iota_{16}, \iota_{16}]\circ 4\nu_{31}$ (by (1.1)(11)) $=4E^3\lambda$ (c.f. (1)). $\nu_{10}\circ\rho_{13}\in E\pi^9_{27}=\{\sigma_{10}\circ\zeta_{17}\}$ (H; (1.4), (1.1)(11)). Since $\sigma_{13}\circ\zeta_{20}=0$, we conclude that $\rho_{16}\circ\nu_{31}=4E^3\lambda$. Since the homomorphism $E^3:\pi^{13}_{31}\to\pi^{16}_{34}$ is a monomorphism, we have $\rho_{13}\circ\nu_{28}=4\lambda$.

(7) Note that $[E\nu', E\nu']=0$. Hence, $E(\nu'\circ\rho''')=E\nu'\circ(2\rho'')=(2E\nu')\circ\rho''$ (by (1.7)) $=\eta^3_4\circ\rho''=0$ (by Prop. (2.8)(3)), so that we have $\nu'\circ\rho'''=0$. Note that $[\nu_6, \nu_6]\circ H(\rho')=\nu_6\circ\varepsilon_9\circ(8\sigma_{17})=0$ (by (1.3), (1.1)(11)). Hence, it follows from (1.7) that $4(\nu_6\circ\rho')=(4\nu_6)\circ\rho'=\eta^2_6\circ\eta_8\circ\rho'\equiv 0 \bmod \eta^2_6\circ(\{\eta_8\circ\bar{\varepsilon}_9\}\oplus\{E^2\zeta'\}\oplus\{E\sigma'\circ\mu_{15}\})$ (by Prop. (2.8)(6)). However, $\eta^3_6\circ\bar{\varepsilon}_9=0$, $\eta_7\circ E^2\zeta'=0$ (by Prop. (2.13)(5)), and $\eta_7\circ E\sigma'=4\bar{\nu}_7=0$ (by (2.1)), so that $4(\nu_6\circ\rho')=2E(\nu_5\circ E\rho'')=0$. Since $E^{-1}(0)\cap 2\pi^5_{23}=0$, we have $2(\nu_5\circ E\rho'')=0$ i.e. $\nu_5\circ E\rho''\in\{\nu_5\circ\bar{\varepsilon}_8\}\oplus\{4\zeta_5\circ\sigma_{16}\}$. On the other hand, $\nu_5\circ E\rho''\in\nu_5\circ\{E\sigma', 8\iota_{15}, 2\sigma_{15}\}_1$ (by (1.2)(8)) $\subset\{\nu_5\circ E\sigma', 8\iota_{15}, 2\sigma_{15}\}_1=\{2(\nu_5\circ\sigma_8), 8\iota_{15}, 2\sigma_{15}\}_1$ (by (2.1)) $\supset 2\{\nu_5\circ\sigma_8, 8\iota_{15}, 2\sigma_{15}\}_1$, so that we can conclude that $\nu_5\circ E\rho''\in 2\pi^5_{23}$, hence we have $\nu_5\circ E\rho''\equiv 0 \bmod 4\zeta_5\circ\sigma_{16}$, and $\nu_6\circ\rho'\equiv 0 \bmod 2\zeta_6\circ\sigma_9$. Similarly, we can prove that $E\nu'\circ\rho''\in E\pi^3_{21}\cap E^{-1}(0)\cap 2\pi^4_{22}=0$, $\nu_{10}\circ\rho_{13}\in E\pi^9_{27}\cap 2\pi^{10}_{28}=\{2\sigma_{10}\circ\zeta_{17}\}$, so that $E\nu'\circ\rho''=0$ and $\nu_{10}\circ\rho_{13}\equiv 0 \bmod 2\sigma_{10}\circ\zeta_{17}$.

(8) By using (1.4), (1.1)(3), (1.1)(7), (1.1)(13) and (2.7), we have $H(\varepsilon'\circ\varepsilon_{13})=H(\varepsilon'\circ\bar{\nu}_{13})=H(\bar{\varepsilon}'\circ\eta_{20})=H(\nu'\circ\bar{\varepsilon}_6)=\eta_5\circ\bar{\varepsilon}_6$. $E^2(\varepsilon'\circ\varepsilon_{13})=E^2(\varepsilon'\circ\bar{\nu}_{13})=E^2(\bar{\varepsilon}'\circ\eta_{20})=E^2(\nu'\circ\bar{\varepsilon}_6)=0$. Hence, (8) follows from the fact that $E\pi^2_{20}\cap(E^2)^{-1}(0)=0$.

(9) Auxiliary calculations: (a) $\eta_3\circ\mu_4\circ\pi^{13}_{21}+\pi^3_{14}\circ(8\sigma_{14})=\{2\mu'\circ\sigma_{14}\}$. Because $\pi^{13}_{21}=\{\varepsilon_{13}\}\oplus\{\bar{\nu}_{13}\}$; $\eta_3\circ\mu_4\circ\varepsilon_{13}=\eta^2_3\circ\mu_5\circ\sigma_{14}$ (by Prop. (2.13)(7)) $=2\mu'\circ\sigma_{14}$, $\eta_3\circ\mu_4\circ\bar{\nu}_{13}=0$ (by Prop. (2.13)(8)), and $8\pi^3_{14}=0$. (b) $\mu_3\circ\nu^3_{12}=\mu_3\circ\eta_{12}\circ\bar{\nu}_{13}$ (by (2.1)) $=\eta_3\circ\mu_4\circ\bar{\nu}_{13}$ (by Prop. (2.13)(8)) $=0$.

Now, it follows from (1.2)(4) and the calculation (b) that $\mu^2_3\in\{\mu_3\circ\eta_{12}, 2\iota_{13}, 8\sigma_{13}\}$. On the other hand, $\eta_3\circ\mu_4\in\{\eta_3\circ\mu_4, 2\iota_{13}, 8\sigma_{13}\}=\{\mu_3\circ\eta_{12}, 2\iota_{13}, 8\sigma_{13}\}$ (by (1.2)(10)). The calculation (a) shows that the secondary composition $\{\mu_3\circ\eta_{12}, 2\iota_{13}, 8\sigma_{13}\}$ is a coset of the subgroup $\{2\mu'\circ\sigma_{14}\}$. Hence, we have $\mu^2_3\equiv\eta_3\circ\mu_4 \bmod 2\mu'\circ\sigma_{14}$.

(10) (1.6), (1.1)(13), Lemma (2.9), (1.3); $E^{-1}(0)=0$.

(11) The proof is quite similar with that of (6).

LEMMA (2.18): (1) We can define ξ'' as an element of $\{4\sigma_{10}, \nu_{17}, \sigma_{20}\}_1$, λ'' as an element of $\phi[\text{Coext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})]$, such that $2\xi''=2\lambda''=\pm\sigma_{10}\circ\zeta_{17}$ and $H(\lambda'')=\eta_{19}\circ\varepsilon_{20}$.

(2) We can define ξ' as an element of $-\{2\sigma_{11}, \nu_{18}, \sigma_{21}\}$, λ as an element of $\phi[\text{Coext.}([\iota_{12}, \iota_{12}], \nu_{23}^2)]$.

PROOF: (1) $H\{4\sigma_{10}, \nu_{17}, \sigma_{20}\}_1 = \Delta^{-1}(4\sigma_9\circ\nu_{16})\circ\sigma_{21} = \eta_{19}^2\circ\sigma_{21} = \eta_{19}\circ\varepsilon_{20} + \eta_{19}\circ\bar{\nu}_{20} = \eta_{19}\circ\varepsilon_{20}+\nu_{19}^3$ (by (2.1)), because $\Delta(\eta_{19}^2)=\sigma_9\circ\eta_{16}^3=4\sigma_9\circ\nu_{16}$ (by (1.3)). Hence, we can chose ξ'' from the coset $\{4\sigma_{10}, \nu_{17}, \sigma_{20}\}_1$ of the subgroup $4\sigma_{10}\circ\pi_{28}^{17}+\pi_{21}^{10}\circ\sigma_{21}=\{2\sigma_{10}\circ\zeta_{17}\}$, for $\pi_{21}^{10}=\{\zeta_{10}\}$; $\zeta_{10}\circ\sigma_{21}\in\{2\sigma_{10}\circ\zeta_{17}\}$ (by (2.16)). Since $4\xi''=4(\iota_{10})\circ\{4\sigma_{10}, \nu_{17}, \sigma_{20}\}=-\{4\iota_{10}, 4\sigma_{10}, \nu_{17}\}\circ\sigma_{21}=-\zeta_{10}\circ\sigma_{21}$ (by (9.3) of [1]) $=2\sigma_{10}\circ\zeta_{17}$, ξ'' is an element of order 8.

Now, $H\phi[\mathit{Coext.}([\iota_9,\iota_9], \eta_{17}\circ\varepsilon_{18})]=E^2(\eta_{17}\circ\varepsilon_{18})$(c.f. Proof of Lemma (2.11)) $=\eta_{19}\circ\varepsilon_{20}$. It follows from (5.11) of [3] that homotopy commutativity holds in the following diagram:

$$\begin{tikzcd}
S^9 \arrow[d, "i"'] & S^{17}\cup_{\eta\circ\varepsilon} e^{27} \arrow[l, "\bar{\gamma}"'] \arrow[d, "p"] \arrow[dl, phantom, "\sim"] & S^{27} \arrow[l, "\widetilde{2\iota}"'] \arrow[d, "id."] \\
S^9\cup_{\gamma} e^{18} & S^{27} \arrow[l, "-\widetilde{\eta\circ\varepsilon}"] & S^{27} \arrow[l, "2\iota_{27}"]
\end{tikzcd}$$

where $\gamma=[\iota_9, \iota_9]$, $\bar{\gamma}\in \mathit{Ext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})$, $\widetilde{2\iota}\in \mathit{Coext.}(\eta_{17}\circ\varepsilon_{18}, 2\iota_{26})$, and $\widetilde{\eta\circ\varepsilon}\in \mathit{Coext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})$. Hence, we have $\phi[\mathit{Coext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})]\circ 2\iota_{27}\subset -\phi\circ i_*\{[\iota_9, \iota_9], \varepsilon_{17}\circ\eta_{25}, 2\iota_{26}\}=-E\{[\iota_9, \iota_9]\circ\varepsilon_{17}, \eta_{25}, 2\iota_{26}\}=-E\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}, \eta_{25}, 2\iota_{26}\}$, for $[\iota_9, \iota_9]\circ\varepsilon_{17}=(\sigma_9\circ\eta_{16}+\varepsilon_9+\bar{\nu}_9)\circ\varepsilon_{17}$; $\varepsilon_9^2=\bar{\nu}_9\circ\varepsilon_{17}=0$ (by (2.7), Prop. (2.8)(2)). While, $\pm\sigma_{10}\circ\zeta_{17}\in\sigma_{10}\circ E\{\eta_{16}\circ\varepsilon_{17}, \eta_{25}, 2\iota_{26}\}$ (by Lemma (9.1) of [1]) $\subset E\{\sigma_9\circ\eta_{16}\circ\varepsilon_{17}, \eta_{25}, 2\iota_{26}\}$, which is a coset of the subgroup $\{2\sigma_{10}\circ\zeta_{17}\}$. Hence, if we chose an element λ'' from the coset $\phi[\mathit{Coext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})]$, then $H(\lambda'')=\eta_{19}\circ\varepsilon_{20}$, and $2\lambda''=\pm\sigma_{10}\circ\zeta_{17}$, so that λ'' is also an element of order 8. Since the group $\pi_{28}^{10}=Z_8\oplus Z_2$ is generated by ξ'' and λ'', it follows that $2\lambda''=\pm 2\xi''$. Let us chose the element λ'' so that $2\lambda''=2\xi''=\pm\sigma_{10}\circ\zeta_{17}$ holds. It is possible, because $\phi[\mathit{Coext.}([\iota_9, \iota_9], \eta_{17}\circ\varepsilon_{18})]$ is a coset of the subgroup $E\pi_{27}^9=\{\sigma_{10}\circ\zeta_{17}\}$.

(2) $H\{2\sigma_{11}, \nu_{18}, \sigma_{21}\}_1=\Delta^{-1}(2\sigma_{10}\circ\nu_{17})\circ\sigma_{22}=\eta_{21}\circ\sigma_{22}=\varepsilon_{21}+\bar{\nu}_{21}$ (by (2.1)), because $\Delta(\eta_{21})=[\iota_{10}, \eta_{10}]=2\sigma_{10}\circ\nu_{17}$ (by (1.3)). $2\{2\sigma_{11}, \nu_{18}, \sigma_{21}\}_1=\{4\sigma_{11}, \nu_{18}, \sigma_{21}\}_1=-E\{4\sigma_{10}, \nu_{17}, \sigma_{20}\}_1=-E\xi''$. Hence, if we chose the element ξ' from the coset $-\{2\sigma_{11}, \nu_{18}, \sigma_{21}\}_1$, ξ' satisfies the relations: $H(\xi')=\varepsilon_{21}+\bar{\nu}_{21}$, $2\xi'=E\xi''$.

Now, consider a coextension $\phi[\mathit{Coext.}([\iota_{12}, \iota_{12}], \nu_{23}^2)]$, then $H\phi[\mathit{Coext.}([\iota_{12}, \iota_{12}], \nu_{23}^2)]=E^2\nu_{23}^2=\nu_{25}^2$, and it is a coset of the subgroup $E\pi_{30}^{12}$. Hence, it follows from (1.1)(14) that $\lambda\in\phi[\mathit{Coext.}([\iota_{12}, \iota_{12}], \nu_{23}^2)]$.

By this Lemma (2.18), we may give the generators of π_{n+18}^n ($n=10$, 11 and 12) as follows:

(2.19) $\pi_{28}^{10}=Z_8+Z_2=\{\xi''\}\oplus\{\xi''-\lambda''\}$.

$\pi_{29}^{11}=Z_8+Z_4=\{\xi'\}\oplus\{\xi'-\lambda'\}$.

$\pi_{30}^{12}=Z_{32}+Z_4+Z_4=\{\xi_{12}\}\oplus\{E\xi'-E\lambda'\}\oplus\{E\xi'+4\xi_{12}\}$.

ξ'', λ'', ξ' and λ' are the element of order 8, with the relations: $2\xi''=2\lambda''$, $4\xi'=4\lambda'$, and $4E\xi'=4E\lambda'=16\xi_{12}$.

Now, we shall prove some relations which appear on π_{n+19}^n.

PROPOSITION (2.20):

(1) $\xi''\circ\eta_{28}=\lambda''\circ\eta_{28}=\eta_{10}\circ\xi'=\eta_{10}\circ\lambda'=0$, $\eta_9\circ\lambda''=\eta_9\circ\xi''=0$.

(2) $\lambda\circ\eta_{31}\equiv E\omega' \bmod (\xi_{13}\circ\eta_{31})$.

(3) $\nu_{16}^*\circ\eta_{34}=0$, $\eta^{*\prime}\circ\nu_{31}=0$, $\eta_{16}^*\circ\nu_{32}=0$.

(4) $\varDelta(E\theta)\circ\eta_{23}^2\equiv 0 \bmod 16\bar{\sigma}_6$.

(5) $\eta_{11}\circ\xi_{12}=\xi'\circ\eta_{29}$.

(6) $\nu'\circ\zeta'=\nu'\circ\eta_6\circ\bar{\varepsilon}_7=\varepsilon_3\circ\zeta_{11}=0$, $\zeta'\circ\nu_{22}=\pm 8\bar{\sigma}_6$, $\bar{\nu}_6\circ\zeta_{14}=\pm 8\bar{\sigma}_6$,
$\nu_{11}\circ\omega_{14}\equiv\lambda'\circ\eta_{29} \bmod \xi'\circ\eta_{29}$, $\nu_{12}\circ\eta^{*\prime}\equiv 0 \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$,
$\eta_{12}\circ\lambda\equiv 0 \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$.

(7) $\eta_{17}\circ\nu_{18}^*=\omega_{17}\circ\nu_{33}$.

(8) $\nu_{13}\circ\eta_{16}^*\equiv E\omega' \bmod \xi_{13}\circ\eta_{31}$.

(9) $\theta'\circ\sigma_{23}=\xi'\circ\eta_{29}$, $\theta\circ\sigma_{24}\equiv\xi_{12}\circ\eta_{30} \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$.

(10) $\sigma'''\circ E\theta'=\sigma''\circ\theta=0$, $\sigma'\circ E\theta=0$.

(11) $\zeta_5\circ\eta_{16}\circ\sigma_{17}=\nu_5\circ\mu_8\circ\sigma_{17}$.

(12) $\varepsilon'\circ\mu_{13}=\nu'\circ\mu_6\circ\sigma_{15}$.

PROOF: (1) $E^\infty(\xi''\circ\eta_{28})=E^\infty(\lambda''\circ\eta_{28})=E^\infty(\eta_{10}\circ\xi')=E^\infty(\eta_{10}\circ\lambda')=E^\infty(\eta_9\circ\lambda'')=E^\infty(\eta_9\circ\xi'')=0$. Hence, (1) follows from the facts that the homomorphism E^∞: $\pi_{29}^{10}\to G_{19}$, and $E^\infty:\pi_{28}^9\to G_{19}$ are monomorphisms.

To prove (2), we need the following lemma:

LEMMA (2.21): *ω' belongs to the coset $\{[\iota_{12},\iota_{12}],\nu_{23}^2,\eta_{29}\}_1$ of the subgroup $\{\xi_{12}\circ\eta_{30}\}\oplus\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$.*

PROOF: $H\{[\iota_{12},\iota_{12}],\nu_{23}^2,\eta_{29}\}_1\subset\{2\iota_{23},\nu_{23}^2,\eta_{29}\}_1\ni\varepsilon_{23}$. $[\iota_{12},\iota_{12}]\circ\pi_{31}^{23}+\pi_{30}^{12}\circ\eta_{30}=\{\xi_{12}\circ\eta_{30}\}\oplus\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$. Indeed, $[\iota_{12},\iota_{12}]\circ\pi_{31}^{23}=\varDelta\pi_{33}^{25}=\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$, and $\pi_{30}^{12}\circ\eta_{30}=\{\xi_{12}\circ\eta_{30}\}\oplus\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$. Thus, the secondary composition $\{[\iota_{12},\iota_{12}],\nu_{23},\eta_{29}\}_1$ includes the element ω' or $\omega'+\bar{\sigma}_{12}$. Since $E^\infty(\omega')=0$, $E^\infty(\bar{\sigma}_{12})\neq 0$, $E^\infty\{[\iota_{12},\iota_{12}],\nu_{23}^2,\eta_{29}\}_1=0$, it follows that $\omega'\in\{[\iota_{12},\iota_{12}],\nu_{23}^2,\eta_{29}\}_1$.

Now, let us prove Proposition (2.20)(2)~(15).

(2) It follows from Lemma (2.18)(2) and Lemma (2.21) that $\lambda\circ\eta_{31}\in\phi[Coext.([\iota_{12},\iota_{12}],\nu_{23}^2)\circ\eta_{31}\subset-\phi\circ i\{[\iota_{12},\iota_{12}],\nu_{23}^2,\eta_{29}\}_1\ni E\omega'$. The last set is a coset of the subgroup $\{\xi_{13}\circ\eta_{31}\}$, hence (2) is proved.

(3) $\nu_{16}^*\circ\eta_{34}\in\{\sigma_{16},2\sigma_{23},\nu_{30}\}_1\circ\eta_{34}$ (by (1.2)(11)) $=\sigma_{16}\circ\{2\sigma_{23},\nu_{30},\eta_{33}\}\subset\sigma_{16}\circ\pi_{35}^{23}=0$.

The proofs of the other two are quite similar.

(4) It follows from Proposition (2.4)(3) that $E\theta\circ\eta_{25}^2\equiv 0 \bmod 8\sigma_{13}^2$. Since $\Delta(8\sigma_{13}^2)$ $=8[\iota_6, \iota_6]\circ\sigma_{11}^2=\nu_6\circ\mu_9\circ\sigma_{18}$ (by (2.1)) $=16\bar{\sigma}_6$ (by (1.1)(1.5)), we have $\Delta(E\theta)\circ\eta_{23}^2$ $=\Delta(E\theta\circ\eta_{25}^2)\equiv 0 \bmod 16\bar{\sigma}_6$.

(5) $H(\eta_{11}\circ\xi_{12})=H(\xi'\circ\eta_{29})$ ((1.4), (1.1)(14), (2.1)). $E^\infty(\eta_{11}\circ\xi_{12})=\eta\circ\xi=\eta\circ\nu^*$ (by (1.1)(14)) $=\nu^*\circ\eta=0$ (by (3)). $E^\infty(\xi'\circ\eta_{29})=0$. Hence, (5) follows from the fact that the homomorphism $E^\infty: E\pi_{29}^{10}\to G_{19}$ is an isomorphism.

(6) $\nu'\circ\zeta'\in\nu'\circ\{\sigma'', \varepsilon_{13}, 2\iota_{21}\}$ (by (1.2)) $=-\{\nu', \sigma'', \varepsilon_{13}\}\circ(2\iota_{22})\subset 2\pi_{22}^3=E\pi_{21}^2$. $\nu'\circ\eta_6\circ\bar{\varepsilon}_7$ $\in E\pi_{21}^2$ (H; (1.4), (1.1)(3)). $\varepsilon_3\circ\zeta_{11}\in E\pi_{21}^2$ (H; (1.4), (1.1)(5), (2.3)). $E^\infty(\nu'\circ\zeta')=E^\infty(\nu'\circ\eta_6\circ\bar{\varepsilon}_7)$ $=0$, $\varepsilon\circ\zeta\in\langle\nu^2, 2\iota, \eta\rangle\circ\zeta=\nu^2\circ\langle 2\iota, \eta, \zeta\rangle\subset\nu^2\circ G_{13}=0$. Hence $\nu'\circ\zeta'=\nu'\circ\eta_6\circ\bar{\varepsilon}_7=\varepsilon_3\circ\zeta_{11}$ $=0$, because the homomorphism $E^\infty: E\pi_{21}^2\to G_{19}$ is a monomorphism. $\zeta'\circ\nu_{22}-8\bar{\sigma}_6\in$ $E\pi_{24}^5$ (H; (1.4), (1.1)(12), Prop. (2.4)(2)). $E^\infty(\zeta'\circ\nu_{22}-8\bar{\sigma}_6)=0$. Since the kernel of the homomorphism $E^\infty: E\pi_{24}^5\to G_{19}$ is generated by $\{16\bar{\sigma}_6\}$, it follows that $\zeta'\circ\nu_{22}=\pm 8\bar{\sigma}_6$. The proof of the next assertion is quite similar. Now, $H(\nu_{11}\circ\omega_{14})=\nu_{21}^2\circ H(\omega_{14})$ (by (1.4)) $=\nu_{21}^3$. $E^\infty(\nu_{11}\circ\omega_{14})=\nu\circ\omega=\omega\circ\nu=0$. Note that $H(\lambda'\circ\eta_{29})=\varepsilon_{21}\circ\eta_{29}$ or ν_{21}^3, and $H(\xi'\circ\eta_{29})=\varepsilon_{21}\circ\eta_{29}+\nu_{21}^3$. Since the homomorphism $E^\infty: E\pi_{29}^{10}\to G_{19}$ is an isomorphism, we conclude that $\nu_{11}\circ\omega_{14}=\lambda'\circ\eta_{29}$ or $\lambda'\circ\eta_{29}+\xi'\circ\eta_{29}$. Finally, let us prove the last two assertions. $\nu_{12}\circ\eta^{*\prime}\in E\pi_{30}^{11}$ (H; (1.4), (1.1)(12)). $E^\infty(\nu_{12}\circ\eta^{*\prime})=0$, because $E^2\eta^{*\prime}\equiv 0$ mod $(\mu\circ\sigma)$ (by Lemma (2.10)), $\nu\circ\mu\circ\sigma=0$. Since the kernel of the homomorphism $E^\infty: E\pi_{30}^{11}\to G_{19}$ is generated by $\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$, it follows that $\nu_{12}\circ\eta^{*\prime}\equiv 0$ mod $\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$. The proof of the last assertion is quite similar.

(7) (1.6), (1.1)(14), (1.3), (3).

(8) $E^2\omega'=2\omega_{14}\circ\nu_{30}$ (by (1.1)(15)) $=[\iota_{14}, \nu_{14}]\circ\nu_{30}$ (by (1.3)) $=[\nu_{14}, \nu_{14}]=\nu_{14}\circ[\iota_{17}, \iota_{17}]$ $=\nu_{14}\circ(\eta_{17}^*+\omega_{17})$ (by (1.3)). Since $\nu_{14}\circ\omega_{17}\equiv E^3(\lambda'\circ\eta_{29})=0 \bmod E^3(\xi'\circ\eta_{29})=0$ (by (6)), we have $\nu_{14}\circ\eta_{17}^*=E^2\omega'$. The kernel of the homomorphism $E:\pi_{32}^{13}\to\pi_{33}^{14}$ is generated by $\{\xi_{13}\circ\eta_{31}\}$.

(9) $E\theta'\circ\sigma_{24}=[\iota_{12}, \iota_{12}]\circ\eta_{23}\circ\sigma_{24}$ (by (1.3)) $=[\iota_{12}, \iota_{12}]\circ\sigma_{23}\circ\eta_{30}$ (by (2.1)) $=(E\xi'-2\xi_{12})\circ$ η_{30} (by (1.3)) $=E\xi'\circ\eta_{30}$. Hence $\theta'\circ\sigma_{23}=\xi'\circ\eta_{29}$, because the homomorphism $E:\pi_{30}^{11}\to$ π_{31}^{12} is a monomorphism. $E\theta\circ\sigma_{25}=[\iota_{13}, \iota_{13}]\circ\sigma_{25}$ (by (1.3)) $=\eta_{13}\circ\xi_{14}+\xi_{13}\circ\eta_{31}$ (by (1.6), (1.1)(14)) $=\xi_{13}\circ\eta_{31}$, because $\eta_{13}\circ\xi_{14}=E^2(\xi'\circ\eta_{29})$ (by (5)) $=0$. Since the kernel of the homomorphism $E:\pi_{31}^{12}\to\pi_{32}^{13}$ is generated by $\{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}$, we have (9).

(10) $\sigma'''\circ E\theta'=\sigma'''\circ[\iota_{12}, \eta_{12}]$ (by (1.3)) $=[\sigma''', \sigma'''\circ\eta_{12}]=0$ (by (2.1)). $E(\sigma'''\circ\theta)=$ $(2\sigma'')\circ E\theta=0$. The homomorphism $E:\pi_{24}^5\to\pi_{25}^6$ is a monomorphism, hence we have $\sigma'''\circ\theta=0$. $\sigma''\circ E\theta=\sigma''\circ[\iota_{13}, \iota_{13}]$ (by (1.3)) $=[\sigma'', \sigma'']=[\iota_6, \iota_6]\circ(4\sigma_{11})\circ(4\sigma_{18})=16[\iota_6, \iota_6]\circ$ $\sigma_{11}^2=0$.

(11) $\zeta_6\circ\eta_{17}\circ\sigma_{18}=8[\iota_6, \iota_6]\circ\sigma_{11}^2$ (by Prop. (2.2)(6)) $=16\bar{\sigma}_6=\nu_6\circ\mu_9\circ\sigma_{18}$ (by (1.1)(15)). Since the homomorphism $E\pi_{24}^5\to\pi_{25}^6$ is a monomorphism, we have (11).

(12) $\varepsilon'\circ\mu_{13}-\nu'\circ\mu_6\circ\sigma_{15}\in E\pi^2_{21}$ (H; (1.4), (1.1)(3), (1.1)(7), Prop. (2.13)(7)). $E^2(\varepsilon'\circ\mu_{13})=E^2(\nu'\circ\mu_6\circ\sigma_{15})=0$; $E\pi^2_{21}\cap(E^2)^{-1}(0)=0$.

§3. Generators of $\pi_k(U_3)$ ($k\leqq 22$)

In the following sections, we use the same notations as in [3]. Homotopy groups of U_3, U_4 and Sp_n are calculated in [5] and [6], so that, in this section, we shall give the original generators(1) of 2-primary components of the groups, and investigate the relations among them, in terms of "composition operation".

First of all, we shall list the original generators of $\pi_k(Sp_n)$, $\pi_k(U_3)$, and $\pi_k(U_4)$(2) ($k\leqq 13$), according to [3].

(3.1) (1) $\tau''^1_3\in\pi_3(Sp_1)$.

(2) $\omega''^2_7\in\pi_7(Sp_2)$, $p''_*\omega''^2_7=4\iota_7$(3), $\omega''^2_7\in\{\tau''^1_3(2),\nu',4\iota_6\}$.

(3) $\gamma''^2_{10}\in\pi_{10}(Sp_2)$, $p''_*\gamma''^2_{10}=\nu_7$, $\gamma''^2_{10}\in\{\tau''^1_3(2),\nu',\nu_6\}$(4).

(4) $\omega''^3_{11}\in\pi_{11}(Sp_3)$, $p''_*\omega''^3_{11}=8\iota_{11}$, $\omega''^3_{11}\in\{i''^{3,2},\gamma''^2_{10},8\iota_{10}\}$.

(3.2) (1) $\tau'^1_1\in\pi_1(U_1)$.

(2) $\tau'^2_3\in\pi_3(U_2)$, $p'_*\tau'^2_3=\iota_3$, $\tau'^2_3=l^2\circ\tau''^1_3$.

(3) $\omega'^3_5\in\pi_5(U_3)$, $p'_*\omega'^3_5=2\iota_5$, $\omega'^3_5\in\{\tau'^2_3(3),\eta_3,2\iota_4\}$.

(4) $u^3_{10}\in\pi_{10}(U_3)$, $p'_*u^3_{10}=\nu_5\circ\eta^2_8$, $u^3_{10}\in\{\tau'^2_3(3),\eta_3,\nu_4\circ\eta^2_7\}$.

(5) $u^3_{11}\in\pi_{11}(U_3)$, $p'_*u^3_{11}=\nu^2_5$, $u^3_{11}\in\{\tau'^2_3(3),\eta_3,\nu^2_4\}$, $2u^3_{11}=\tau'^2_3(3)\circ\varepsilon_3$.

(6) $u^3_{12}\in\pi_{12}(U_3)$, $p'_*u^3_{12}=\sigma'''$, $u^3_{12}\in\{\omega'^3_5,4\nu_5,\nu_8\}$, $2u^3_{12}=\tau'^2_3(3)\circ\mu_3$.

(7) $\omega'^4_7\in\pi_7(U_4)$, $p'_*\omega'^4_7=2\iota_7$, $\omega'^4_7\in\{i'^{4,3},\tau'^2_3(3)\circ\nu',2\iota_6\}$.

(8) $\gamma'^4_8\in\pi_8(U_4)$, $p'_*\gamma'^4_8=\eta_7$, $\gamma'^4_8\in\{i'^{4,3},\tau'^2_3(3)\circ\nu',\eta_6\}$, $2\gamma'^4_8=\omega'^3_5(4)\circ\nu_5$.

(9) $\omega'^5_9\in\pi_9(U_5)$, $p'_*\omega'^5_9=8\iota_9$, $\omega'^5_9\in\{i'^{5,4},\gamma'^4_8,8\iota_8\}$.

(10) $u^5_{12}\in\pi_{12}(U_5)$, $p'_*u^5_{12}=4\nu_9$, $u^5_{12}\in\{i'^{5,4},\gamma'^4_8,4\nu_8\}$, $2u^5_{12}=u^3_{12}(5)$.

(11) $\omega'^6_{11}\in\pi_{11}(U_6)$, $p'_*\omega'^6_{11}=8\iota_{11}$, $\omega'^6_{11}\in\{i'^{6,5},\gamma'^5_{10},8\iota_{10}\}$(5).

(12) $\gamma'^6_{12}\in\pi_{12}(U_6)$, $p'_*\gamma'^6_{12}=\eta_{11}$, $\gamma'^6_{12}\in\{i'^{6,5},\gamma'^5_{10},\eta_{10}\}$, $2\gamma'^6_{12}=u^5_{12}(6)$.

(13) $\omega'^7_{13}\in\pi_{13}(U_7)$, $p'_*\omega'^7_{13}=16\iota_{13}$, $\omega'^7_{13}\in\{i'^{7,6},\gamma'^6_{12},16\iota_{12}\}$.

We proved some relations among them in [3](6):

(3.3) (1) $\omega''^2_7\circ\nu_7=4\gamma''^2_{10}$.

(1) Generators which are not represented as the form of $\alpha\circ\beta$.
(2) $\pi_k(G)$ indicates only 2-primary components of the homotopy group.
(3) c.f. remark in the page 65.
(4) We denoted γ''^2_{10} by $\bar\gamma''^2_{10}$ in [3]. Note that $\triangle\iota_{11}=x\gamma''^2_{10}$ for some odd integer x.
(5) $\gamma'^5_{10}=i'^{5,4}\circ l^4\circ\gamma''^2_{10}$.
(6) There are some misprints in (10.21), (11.21)(iii)(iv) and (11.28)(ii) of [3], which should be corrected as described here.

(2) $\gamma''^{2}_{10}\circ\eta_{10}=\tau''^{1}_{3}(2)\circ\varepsilon_8$, $2\gamma''^{2}_{10}\circ\nu_{10}=\tau''^{1}_{3}(2)\circ\varepsilon'$.

(3) $\omega''^{3}_{11}\circ\eta_{11}=\tau''^{1}_{3}(3)\circ\mu_3$.

(3.4) (1) $\omega'^{3}_{5}\circ\eta_5=\tau'^{2}_{3}(3)\circ\nu'$, $\omega'^{3}_{5}\circ\nu_5^2=\tau'^{2}_{3}(3)\circ\varepsilon_3$, $\omega'^{3}_{5}\circ\sigma'''=0$, $\omega'^{3}_{5}\circ\varepsilon_5=\tau'^{2}_{3}(3)\circ\varepsilon'$.

(2) $u^3_{10}\circ\eta_{10}=0$, $u^3_{10}\circ\nu_{10}=0$.

(3) $u^3_{11}\circ\eta_{11}=0$.

(4) $u^3_{12}\circ\eta_{12}=0$.

(5) $\omega'^{4}_{7}\circ\eta_7=4\gamma'^{4}_{8}$, $\omega'^{4}_{7}\circ\nu_7^2=\tau'^{2}_{3}(4)\circ\varepsilon'$.

(6) $\gamma'^{4}_{8}\circ\nu_8=\pm u^3_{11}(4)$, $\gamma'^{4}_{8}\circ\eta_8^2=u^3_{10}(4)+4l^4\circ\gamma''^{2}_{10}$.

(7) $\omega'^{5}_{9}\circ\eta_9=4i'^{5,4}\circ l^4\circ\gamma''^{2}_{10}$, $\omega'^{5}_{9}\circ\nu_9=-2u^5_{12}$(*).

(8) $u^5_{12}\circ\eta_{12}=2i'^{5,4}\circ l^4\circ\gamma''^{2}_{10}\circ\nu_{10}$.

(9) $\omega'^{6}_{11}\circ\eta_{11}=8\gamma'^{6}_{12}$.

(3.5) $l^4\circ\omega''^{2}_{7}=2\omega'^{4}_{7}$, $l^6\circ\omega''^{3}_{11}=\omega'^{6}_{11}$,

where $l^{2n}: Sp_n\to U_{2n}$ *indicates the inclusion map.*

Now we shall add some original generators which appear on $\pi_k(U_3)$ $(14\leqq k\leqq 22)$.

LEMMA (3.6): (1) *There is an element* u^3_{16} *of the secondary composition* $\{\tau'^{2}_{3}(3), \eta_3, \nu_4\circ\bar\nu_7\}$, *which consists of a single element, such that* $p'_* u^3_{16}=\nu_5\circ\bar\nu_8$ *and* $2u^3_{16}=0$.

(2) *There is an element* u^3_{18} *of the secondary composition* $\{u^3_{10}, 2\iota_{10}, 8\sigma_{10}\}_1$, *which consists of a single element, such that* $p'_* u^3_{18}=\nu_5\circ\eta_8\circ\mu_9$ *and* $2u^3_{18}=0$.

(3) *There is an element* u^3_{20} *of the secondary composition* $\{u^3_{12}, 4\iota_{12}, 4\sigma_{12}\}_3$, *which consists of a single element, such that* $p'_* u^3_{20}=\rho^{\mathrm{IV}}$ *and* $2u^3_{20}=\tau'^{2}_{3}(3)\circ\bar\mu_3$.

PROOF: $\tau'^{2}_{3}(3)\circ\pi^3_{16}=0$, $8\pi_{11}(U_3)=0$, $4\pi_{13}(U_3)=0$. We shall prove later that $\omega'^{3}_{5}\circ\nu_5\circ\bar\nu_8=0$, $u^3_{10}\circ\varepsilon_{10}=u^3_{10}\circ\bar\nu_{10}=0$ and $u^3_{12}\circ\varepsilon_{12}=u^3_{12}\circ\bar\nu_{12}=0$. Hence, $\tau'^{2}_{3}(3)\circ\pi^3_{16}+\pi_5(U_3)\circ\nu_5\circ\bar\nu_8=0$, $u^3_{10}\circ\pi^{10}_{18}+\pi_{11}(U_3)\circ 8\sigma_{11}=0$, and $u^3_{12}\circ\pi^{12}_{20}+\pi_{13}(U_3)\circ 4\sigma_{13}=0$, which imply that each of the secondary compositions in Lemma (3.6) consists of a single element.

Since $\Delta(\iota_5)=\tau'^{2}_{3}\circ\eta_3$, it follows from Proposition (7.14) of [3] that $p'_*\{i'^{3,2}\circ\tau'^{2}_{3}, \eta_3, \nu_4\circ\bar\nu_7\}=\nu_5\circ\bar\nu_8$. $p'_*\{u^3_{10}, 2\iota_{10}, 8\sigma_{10}\}_1=\{\nu_5\circ\eta_8^2, 2\iota_{10}, 8\sigma_{10}\}_1$ (by (3.2)(4)) $=\nu_5\circ\eta_8\{\eta_9, 2\iota_{10}, 8\sigma_{10}\}_1=\nu_5\circ\eta_8\circ\mu_9$ (by (1.2)(4)). $p'_*\{u^3_{12}, 4\iota_{12}, 4\sigma_{12}\}_3=\{\sigma''', 4\iota_{12}, 4\sigma_{12}\}_3=\rho^{\mathrm{IV}}$ (by (1.2)(8)). By considering the bundle sequence of $U_3/U_2=S^5$, we see that $2u^3_{16}=0$, $2u^3_{18}=0$ and $2u^3_{20}=0$.

There are some relations among them as follows:

PROPOSITION (3.7):

(1) $\omega'^{3}_{5}\circ\nu_5\circ\bar\nu_8=\omega'^{3}_{5}\circ\nu_5\circ\varepsilon_8=0$. (2) $\omega'^{3}_{5}\circ\mu_5=\tau'^{2}_{3}(3)\circ\mu'$.

(3) $\omega'^{3}_{5}\circ\nu_5\circ\mu_8=0$. (4) $\omega'^{3}_{5}\circ\nu_5\circ\sigma_8\circ\nu_{15}=\tau'^{2}_{3}(3)\circ\bar\varepsilon_3$.

(*) There is a mistake in (11.28)(V) of [3]. In its proof,

$$\{i'^{5,4}, \omega'^{3}_{3}(4)\circ\nu_5, 4\iota_8\}\circ\nu_9=-i'^{5,3}\circ\{\omega'^{3}_{3}, 4\nu_5, \nu_8\}=-u^3_{12}(5)=-2u^5_{12}.$$

(5) $\omega'^3_5\circ\nu_5\circ\zeta_8=0$. (6) $\omega'^3_5\circ\bar{\varepsilon}_5=\tau'^2_3(3)\circ\bar{\varepsilon}'$.
(7) $\omega'^3_5\circ\rho^{\mathrm{IV}}=0$. (8) $\omega'^3_5\circ\bar{\mu}_5=\tau'^2_3(3)\circ\bar{\mu}'$.
(9) $u^3_{10}\circ\varepsilon_{10}=u^3_{10}\circ\bar{\nu}_{10}=0$. (10) $u^3_{10}\circ\mu_{10}=0$.
(11) $u^3_{10}\circ\sigma_{10}\circ\nu_{17}=0$. (12) $u^3_{10}\circ\zeta_{10}=0$.
(13) $u^3_{11}\circ\varepsilon_{11}=u^3_{11}\circ\bar{\nu}_{11}=0$. (14) $u^3_{11}\circ\mu_{11}=0$.
(15) $u^3_{11}\circ\sigma_{11}\circ\nu_{18}=0$. (16) $u^3_{11}\circ\zeta_{11}=0$.
(17) $u^3_{12}\circ\nu_{12}=2\omega'^3_5\circ\nu_5\circ\sigma_8$. (18) $u^3_{12}\circ\varepsilon_{12}=u^3_{12}\circ\bar{\nu}_{12}=0$.
(19) $u^3_{12}\circ\mu_{12}=0$. (20) $u^3_{16}\circ\eta_{16}=u^3_{11}\circ\nu^2_{11}$.
(21) $u^3_{16}\circ\nu^2_{16}=\omega'^3_5\circ\nu_5\circ\kappa_8$. (22) $u^3_{18}\circ\eta_{18}=0$.
(23) $u^3_{18}\circ\nu_{18}=0$. (24) $u^3_{20}\circ\eta_{20}=0$.

PROOF: (1) $p'_*(\omega'^3_5\circ\nu_5\circ\bar{\nu}_8)=2\nu_5\circ\bar{\nu}_8$ (by (3.2)(3)) $=0$. Similarly we have $p'_*(\omega'^3_5\circ\nu_5\circ\varepsilon_8)=2\nu_5\circ\varepsilon_8=0$. It follows from the bundle sequence of $U_3/U_2=S^5$ that $p'^{-1}_*(0)=i'^{3,2}_*\pi_{16}(U_2)=0$. so that $\omega'^3_5\circ\nu_5\circ\bar{\nu}_8=\omega'^3_5\circ\nu_5\circ\varepsilon_8=0$.

(2) $\omega'^3_5\circ\mu_5\in\{\tau'^2_3(3),\eta_3,2\iota_4\}\circ\mu_5=\tau'^2_3(3)\circ\{\eta_3,2\iota_4,\mu_4\}\ni\tau'^2_3(3)\circ\mu'$ (by (1.2)(6)). Since $\tau'^2_3(3)\circ\pi^3_5\circ\mu_5=\{\tau'^2_3(3)\circ\eta^2_3\circ\mu_5\}=i'^{3,2}_*\circ\Delta(\eta_5\circ\mu_6)=0$, the secondary composition $\{\tau'^2_3(3),\eta_3,2\iota_4\}$ consists of a single element, so that $\omega'^3_5\circ\mu_5=\tau'^2_3(3)\circ\mu'$.

(3) $p'_*(\omega'^3_5\circ\nu_5\circ\mu_8)=2\nu_5\circ\mu_8=0$. By considering the bundle sequence of $U_3/U_2=S^5$, $p'^{-1}_*(0)=i'^{3,2}_*\pi_{17}(U_2)=0$. Hence we have $\omega'^3_5\circ\nu_5\circ\mu_8=0$.

To prove (4), we need the following lemma.

LEMMA (3.8): *The secondary composition* $\{\nu^2_5,2\iota_{11},\nu^2_{11}\}_6$ *consists of an element* $\nu_5\circ\sigma_8\circ\nu_{15}$ or $\nu_5\circ\sigma_8\circ\nu_{15}+\nu_5\circ\eta_8\circ\mu_9$.

PROOF: $\nu^2_5\circ E^6\pi^5_{12}=\{\nu^2_5\circ 8\sigma_{11}\}=0$, $\pi^5_{12}\circ\nu^2_{12}=\{\sigma'''\circ\nu^2_{12}\}=\{4\nu_5\circ\sigma_8\circ\nu_{15}\}=0$. Hence the secondary composition $\{\nu^2_5,2\iota_{11},\nu^2_{11}\}_6$ consists of a single element. Since $\Delta(\nu_9)=[\iota_4,\iota_4]\circ\nu_7=(E\nu'-2\nu_4)\circ\nu_7$ (by (1.3)) $=-2\nu^2_4$, it follows that $H\{\nu^2_5,2\iota_{11},\nu^2_{11}\}_6=\Delta^{-1}(2\nu^2_4)\circ\nu^2_{12}=\nu^3_9$. On the other hand, $H(\nu_5\circ\sigma_8\circ\nu_{15})=\nu^2_9\circ H(\sigma_8)\circ\nu_{15}$ (by (1.4)) $=\nu^3_9$ (by (1.1)(4)). Since $H^{-1}(0)=E\pi^4_{17}=\{\nu_5\circ\eta_8\circ\mu_9\}$, we have Lemma (3.8).

Proof of (4) Note that $\omega'^3_5\circ\nu_5\circ\eta_8\circ\mu_9=\omega'^3_5\circ\nu_5\circ\mu_8\circ\eta_{17}=0$ (by (3)). Hence it follows from Lemma (3.8) thae $\omega'^3_5\circ\nu_5\circ\sigma_8\circ\nu_{15}\in\{\omega'^3_5\circ\nu^2_5,2\iota_{11},\nu^2_{11}\}_6$. On the other hand, $\tau'^2_3(3)\circ\bar{\varepsilon}_3\in\{\tau'^2_3(3)\circ\varepsilon_3,2\iota_{11},\nu^2_{11}\}_6$ (by (1.2) (8)) $=\{\omega'^3_5\circ\nu^2_5,2\iota_{11},\nu^2_{11}\}_6$ (by (3.4) (1)). Since $\pi_{12}(U_3)\circ\nu^2_{12}=\{u^3_{12}\circ\nu^2_{12}\}=\{2\omega'^3_5\circ\nu_5\circ\sigma_8\circ\nu_{15}\}$ (see (17)) $=0$, we conclude that $\omega'^3_5\circ\nu_5\circ\sigma_8\circ\nu_{15}=\tau'^2_3(3)\circ\bar{\varepsilon}_3$.

(5) In Proposition (2.4)(2), we have proved that $\nu_5\circ\zeta_8\equiv\sigma'''\circ\sigma_{12}\bmod\nu_5\circ\bar{\nu}_8\circ\nu_{16}$. Since $\omega'^3_5\circ\sigma'''=0$ (by (3.4)(1)), and $\omega'^3_5\circ\nu_5\circ\bar{\nu}_8=0$ (by (1)), it follows that $\omega'^3_5\circ\nu_5\circ\zeta_8=0$.

(6) $\omega'^3_5\circ\bar{\varepsilon}_5\in\{\tau'^2_3(3),\eta_3,2\iota_4\}\circ\bar{\varepsilon}_5=\tau'^2_3(3)\circ\{\eta_3,2\iota_4,\bar{\varepsilon}_4\}\ni\tau'^2_3(3)\circ\bar{\varepsilon}'$ (by Lemma (2.15)(2)). Since $\tau'^2_3(3)\circ\pi^3_5\circ\bar{\varepsilon}_5=\{\tau'^2_3(3)\circ\eta^2_3\circ\bar{\varepsilon}_5\}=0$, we have $\omega'^3_5\circ\bar{\varepsilon}_5=\tau'^2_3(3)\circ\bar{\varepsilon}'$.

(7) It follows from (1.2)(8) that $\omega'^3_5\circ\rho^{\mathrm{IV}}=\omega'^3_5\circ\{\sigma''',2\iota_{12},8\sigma_{12}\}_1=\{\omega'^3_5,\sigma''',2\iota_{12}\}\circ 8\sigma_{13}\subset\pi_{13}(U_3)\circ 8\sigma_{13}=0$.

(8) Note that $4\pi_{15}(U_3)=0$, and $\omega'^3_5\circ\mu_5\circ E\pi^{13}_{21}=0$. Indeed, $\omega'^3_5\circ\mu_5\circ\varepsilon_{14}=\omega'^3_5\circ\eta_5\circ\mu_6\circ\sigma_{15}$ (by Prop. (2.13)(7)) $=\tau'^2_3(3)\circ\nu'\circ\mu_6\circ\sigma_{15}$ (by (3.4)(1)) $=0$, $\omega'^3_5\circ\mu_5\circ\bar{\nu}_{14}=0$ (by Prop. (2.13)(8)). Hence the secondary composition $\{\omega'^3_5\circ\mu_5, 4\iota_{14}, 4\sigma_{14}\}_1$ consists of a single element. Now, $\omega'^3_5\circ\bar{\mu}_5=\{\omega'^3_5\circ\mu_5, 2\iota_{14}, 8\sigma_{14}\}_1$ (by (1.2)(10)) $=\{\tau'^2_3(3)\circ\mu', 4\iota_{14}, 4\sigma_{14}\}_1$ (by (2)) $=\tau'^2_3(3)\circ\bar{\mu}'$ (by (1.2)(12)).

(9) $u^3_{10}\circ\varepsilon_{10}\in u^3_{10}\circ\{\eta_{10}, \nu^2_{11}, 2\iota_{17}\}$ (by (1.2)(3)) $=\{u^3_{10}, \eta_{10}, \nu^2_{11}\}\circ 2\iota_{18}\subset 2\pi_{18}(U_3)=0$. $u^3_{10}\circ\bar{\nu}_{10}=u^3_{10}\circ\varepsilon_{10}+u^3_{10}\circ\eta_{10}\circ\sigma_{11}$ (by (2.1)) $=0$.

(10) Since $u^3_{10}\circ\nu^3_{10}=u^3_{10}\circ\eta_{10}\circ\bar{\nu}_{11}=0$, it follows from (1.2)(4) that $u^3_{10}\circ\mu_{10}\in u^3_{10}\circ\{\eta_{10}, 2\iota_{11}, 8\sigma_{11}\}=\{u^3_{10}, \eta_{10}, 2\iota_{11}\}\circ 8\sigma_{12}\subset\pi_{12}(U_3)\circ 8\sigma_{12}=0$.

(11) It follows from (3.2)(4) that $u^3_{10}\circ\sigma_{10}\circ\nu_{17}\in\{\tau'^2_3(3), \eta_3, \nu_4\circ\eta^2_7\}\circ\sigma_{10}\circ\nu_{17}$ $=-\tau'^2_3(3)\circ\{\eta_3, \nu_4\circ\eta^2_7, \sigma_9\circ\nu_{16}\}=-\tau'^2_3(3)\circ\{\nu'\circ\eta_6, \eta^2_7, \sigma_9\circ\nu_{16}\}\subset-\tau'^2_3(3)\circ\{\nu', 4\nu_6, \sigma_9\circ\nu_{16}\}$ $=-\tau'^2_3(3)\circ\{\nu', \nu_6, 4\sigma_9\circ\nu_{16}\}=-\tau'^2_3(3)\circ\{\nu', \nu_6, 2\nu_9\circ\sigma_{12}\}$, which contains $\tau'^2_3(3)\circ\varepsilon'\circ\sigma_{13}$ $=2\tau'^2_3(3)\circ\bar{\varepsilon}'=0$ (by (2.1) and Prop. (2.13)(6)), and is a coset of the subgroup $\tau'^2_3(3)\circ\nu'\circ\pi^6_{20}$ $+\tau'^2_3(3)\circ\pi^3_{10}\circ\sigma_{10}\circ\nu_{17}=\{2\tau'^2_3(3)\circ\bar{\varepsilon}'\}=0$, because $\pi^3_{10}=0$, $\nu'\circ\pi^6_{20}=\{\nu'\circ\sigma''\circ\sigma_{13}\}\oplus\{\nu'\circ\bar{\nu}_6\circ\nu^2_{14}\}$ $=2\bar{\varepsilon}'$ (by Prop. (2.2)(1) and Prop. (2.13)(4)).

(12) It follows from (1.2)(6) that $u^3_{10}\circ\zeta_{10}\in u^3_{10}\circ\{\nu_{10}, 8\iota_{13}, 2\sigma_{13}\}_6=\{u^3_{10}, \nu_{10}, 8\iota_{13}\}\circ 2\sigma_{14}$ $\subset\pi_{14}(U_3)\circ 2\sigma_{14}=2u^3_{11}\circ\nu_{11}\circ\sigma_{14}=0$.

(13) It follows from (1.2)(3) that $u^3_{11}\circ\varepsilon_{11}\in u^3_{11}\circ\{\eta_{11}, 2\iota_{12}, \nu^2_{12}\}=-\{u^3_{11}, \eta_{11}, 2\iota_{12}\}\circ\nu^2_{13}$ $\subset\pi_{13}(U_3)\circ\nu^2_{13}=\{\tau'^2_3(3)\circ\varepsilon'\circ\nu^2_{13}\}=0$ (by Prop. (2.2)(7)). $u^3_{11}\circ\bar{\nu}_{11}=u^3_{11}\circ\varepsilon_{11}+u^3_{11}\circ\eta_{11}\circ\sigma_{12}=0$.

(14) The proof is quite similar with that of (10).

(15) $u^3_{11}\circ\sigma_{11}\circ\nu_{18}=u^3_{11}\circ[\iota_{11}, \iota_{11}]$ (by (1.3)) $=[u^3_{11}, u^3_{11}]=0$.

(16) It follows from (9.3) of [1] that $x\zeta_{11}\in\{\nu_{11}, \sigma_{14}, 16\iota_{21}\}$ for some odd integer x. Hence, $u^3_{11}\circ\zeta_{11}\in u^3_{11}\{\nu_{11}, \sigma_{14}, 16\iota_{21}\}=-\{u^3_{11}, \nu_{11}, \sigma_{14}\}\circ 16\iota_{22}\subset 16\pi_{22}(U_3)=0$.

(17) $p'_*(u^3_{12}\circ\nu_{12})=\sigma'''\circ\nu_{12}$ (by (3.2)(6)) $=4(\nu_5\circ\sigma_8)$ (by (2.1)). $p'_*(2\omega'^3_5\circ\nu_5\circ\sigma_8)$ $=4(\nu_5\circ\sigma_8)$. Since $p'^{-1}_*(0)=i'^{3,2}_*\pi_{15}(U_2)=0$, it follows that $u^3_{12}\circ\nu_{12}=2\omega'^3_5\circ\nu_5\circ\sigma_8$.

(18) It follows from (1.2)(3) that $u^3_{12}\circ\varepsilon_{12}\in u^3_{12}\circ\{\eta_{12}, \nu_{13}, 2\nu_{16}\}=-\{u^3_{12}, \eta_{12}, \nu_{13}\}\circ 2\nu_{17}$ $\subset\pi_{17}(U_3)\circ 2\nu_{17}=0$. $u^3_{12}\circ\bar{\nu}_{12}=u^3_{12}\circ\varepsilon_{12}+u^3_{12}\circ\eta_{12}\circ\sigma_{13}=0$.

(19) The proof is quite similar with that of (10).

(20) The proof is quite similar with that of (17).

(21) $p'_*(u^3_{16}\circ\nu^2_{16})=\nu_5\circ\bar{\nu}_8\circ\nu^2_{16}$ (by Lemma (3.6)(1)) $=2\nu_5\circ\kappa_8$ (by (2.12)). $p'_*(\omega'^3_5\circ\nu_5\circ\kappa_8)$ $=2\nu_5\circ\kappa_8$. By the bundle sequence of $U_3/U_2=S^5$, we see that $p'^{-1}_*(0)=i'^{3,2}_*\pi_{22}(U_2)$ $=\{\tau'^2_3(3)\circ\mu'\}$, hence we have (21).

(22) $u^3_{18}\circ\eta_{18}=\{u^3_{10}, 2\iota_{10}, 8\sigma_{10}\}\circ\eta_{18}$ (by Lemma (3.6)(2)) $=u^3_{10}\circ\{2\iota_{10}, 8\sigma_{10}, \eta_{17}\}\subset u^3_{10}\circ\pi^{10}_{19}$ $=\{u^3_{10}\circ\eta_{10}\circ\varepsilon_{11}\}\oplus\{u^3_{10}\circ\nu^3_{10}\}\oplus\{u^3_{10}\circ\mu_{10}\}=0$ (by (3.4)(2), (10)).

(23) The proof is quite similar with that of (22).

(24) $u^3_{20}\circ\eta_{20}=\{u^3_{12}, 4\iota_{12}, 4\sigma_{12}\}_3\circ\eta_{20}$ (by Lemma (3.6)(3)) $=u^3_{12}\circ\{4\iota_{12}, 4\sigma_{12}, \eta_{19}\}\subset u^3_{12}\circ\pi^{12}_{21}$ $=\{u^3_{12}\circ\eta_{12}\circ\varepsilon_{13}\}\oplus\{u^3_{12}\circ\nu^3_{12}\}\oplus\{u^3_{12}\circ\mu_{12}\}=0$ (by (3.4)(4), (19)).

§4. Generators of $\pi_k(Sp_2)$, $\pi_k(U_4)$, $\pi_k(R_5)$ and $\pi_k(R_6)$ $(k\leqq 22)$

In (9.1) and (9.2) of [3], we defined the original generators of $\pi_k(Rn)$ $(k\leqq 7$, $n=1, 3, 4, 7, 8)$:

(4.1) (1) $\tau_1^2\in\pi_1(R_2)$, $p_*\tau_1^2=\iota_1$, $\tau_1^2=k^2\circ\tau'^1_1$

(2) $\lambda_3^3\in\pi_3(R_3)$, $p_*\lambda_3^3=\eta_2$, $\lambda_3^3(5)=2\tau_3^4(5)$

(3) $\tau_3^4\in\pi_3(R_4)$, $p_*\tau_3^4=\iota_3$, $\tau_3^4=k^4\circ\tau'^2_3$

Let $q: Sp_2\to R_5$ and $q': U_4\to R_6$ be the projections of the well known coverings, then the original generators of $\pi_k(R_5)$ and $\pi_k(R_6)$ are given by these projections ((12.7) of [3]):

(4.2) (1) $q_*\omega''^2_7=r_7^5\in\pi_7(R_5)$, $p_*r_7^5=4\nu_4$, $r_7^5\in\{\tau_3^4(5), \nu', 4\iota_6\}$.

(2) $q_*\gamma''^2_{10}=r_{10}^5\in\pi_{10}(R_5)$, $p_*r_{10}^5=\nu_4^2$, $r_{10}^5\in\{\tau_3^4(5), \nu', \nu_6\}$.

(3) $q'_*\omega'^4_7=r_7^6\in\pi_7(R_6)$, $p_*r_7^6=\eta_5^2$, $2r_7^6=r_7^5(6)$, $r_7^6\in\{i^{6,5}, \tau_3^4(5)\circ\eta_3, \eta_4^2\}$.

(4) $q'_*\gamma'^4_8=r_8^6\in\pi_8(R_6)$, $p_*r_8^6=\nu_5$, $r_8^6\in\{i^{6,5}, \tau_3^4(5)\circ\eta_3, \nu_4\}$.

Relations between these generators and other elements are obtained from (3.3), (3.4) and (3.5) by applying q_* and q'_*.

Now, we shall define some original generators of $\pi_k(Sp_2)$, $\pi_k(U_4)$, $\pi_k(R_5)$ and $\pi_k(R_6)$ for $14\leqq k\leqq 22$. Let

$$p_1: U_4\to S^5\times S^7=U_4/U_2,\ p_2: S^5\times S^7\to S^5,\ p_3: S^5\times S^7\to S^7$$

be the projections, then commutativity holds in the diagram

(4.3)
$$\begin{tikzcd}
\pi_k(Sp_2) \arrow[rr, "q_*"] \arrow[d, "l^4_*"'] & & \pi_k(R_5) \arrow[d, "i_*"] \\
\pi_k(U_4) \arrow[rr, "q'_*"] \arrow[d, "p'_*"'] \arrow[dr, "p_{1*}"] & & \pi_k(R_6) \arrow[d, "p_*"] \\
\pi_k(S^7) & \pi_k(S^5\times S^7) \arrow[l, "p_{3*}"] \arrow[r, "p_{2*}"'] & \pi_k(S^5)
\end{tikzcd}$$

LEMMA (4.4): (1) *There is an element* $s_{14}^2\in\pi_{14}(Sp_2)$ *such that* $l^4\circ s_{14}^2=\omega'^4_7\circ\sigma'$, $2s_{14}^2=\omega''^2_7\circ\sigma'$, $4s_{14}^2=\pm\tau''^1_3(2)\circ\mu'$ *and* $p''_*s_{14}^2=2\sigma'$. *Denote* $q_*s_{14}^2$ *by* r_{14}^5.

(2) *Each of the secondary compositions* $\{\tau''^1_3(2), 2\varepsilon', 2\iota_{13}\}_1$ *and* $\{\tau''^1_3(2), \nu', \sigma''\}$ *contains* s_{14}^2.

(3) *There is an element* r_{14}^6 *of the secondary composition* $\{i^{6,5}, \tau_3^4(5)\circ\eta_3, \eta_4\circ\varepsilon_5\}$ $\subset\pi_{14}(R_6)$ *such that* $p_*r_{14}^6=\eta_5\circ\varepsilon_6$, $2r_{14}^6=r_{14}^5(6)$. *Denote* $(q'_*)^{-1}r_{14}^6$ *by* u_{14}^4. *Then,* $p'_*u_{14}^4=\sigma'$, $2u_{14}^4=\omega'^4_7\circ\sigma'$ *and* $8u_{14}^4=\pm\tau'^2_3(4)\circ\mu'$.

PROOF: (1) According to [5], $p''_*:\pi_{14}(Sp_2)\to Z_4=\{2\sigma'\}$ is an epimorphism. Note that $p'_*(\omega'^4_7\circ\sigma')=2\sigma'$, and that $p_*(r_7^6\circ\sigma')=\eta_5^2\circ\sigma'=0$ (by (2.1)). Hence $r_7^6\circ\sigma'\in i_*\pi_{14}(R_5)$, which implies that $\omega'^4_7\circ\sigma'\in l^4_*\pi_{14}(Sp_2)$ by (4.3), that is, there exists an element s^2_{14} such that $l^4\circ s^2_{14}=\omega'^4_7\circ\sigma'$ and $p''_*s^2_{14}=2\sigma'$. Since $l^4\circ\omega''^2_7=2\omega'^4_7$ (by (3.5)), we can easily see that $2s^2_{14}=\omega''^2_7\circ\sigma'$. Since $\pi_{14}(Sp_2)$ is a cyclic group, it follows from the bundle sequence of $Sp_2/Sp_1=S^7$ that $4s^2_{14}=\pm\tau''^1_3(2)\circ\mu'$.

(2) Note that $\Delta(\sigma')=2\tau''^1_3\circ\varepsilon'$ (according to [5]). Applying (7.14) of [3] by substituting (Sp_2, Sp_1) for (Y, A) and (S^7, e^0) for (Z, z^0), we have $p''_*\{\tau''^1_3(2), 2\varepsilon', 2\iota_{13}\}_1=\Delta^{-1}(2\tau''^1_3\circ\varepsilon')\circ 2\iota_{14}=2\sigma'$. Similarly, we have $p''_*\{\tau''^1_3(2), \nu', \sigma''\}=E\sigma''=2\sigma'$. Hence, s^2_{14} belongs to each of these secondary compositions.

(3) Applying (7.14) of [3] by substituting (R_6, R_5) for (Y, A) and (S^5, e^0) for (Z, z^0), we have $p_*\{i^{6,5}, \tau^4_3(5)\circ\eta_3, \eta_4\circ\varepsilon_5\}=\eta_5\circ\varepsilon_6$. Next, $2\{i^{6,5}, \tau^4_3(5)\circ\eta_3, \eta_4\circ\varepsilon_5\}=-i^{6,5}\circ\{\tau^4_3(5)\circ\eta_3, \eta_4\circ\varepsilon_5, 2\iota_{13}\}\subset -i^{6,5}\circ\{\tau^4_3(5), 2\varepsilon', 2\iota_{13}\}\ni -r^5_{14}(6)$ (by (2)), which is a coset of the subgroup $2i^{6,5}_*\pi_{14}(R_5)$. Hence we can chose an element r^6_{14} from the secondary composition $\{i^{6,5}, \tau^4_3(5)\circ\eta_3, \eta_4\circ\varepsilon_5\}$ such that $p_*r^6_{14}=\eta_5\circ\varepsilon_6$ and $2r^6_{14}=r^5_{14}(6)$. Denote $(q'_*)^{-1}r^6_{14}$ by u^4_{14}, then it follows from (4.3) that $2u^4_{14}=l^4\circ s^2_{14}=\omega'^4_7\circ\sigma'$, so that $p'_*u^4_{14}=\sigma'$ or $5\sigma'$. It is easily observed that we can chose r^6_{14} such that $p'_*u^4_{14}=\sigma'$ without any change of other relations. $8u^4_{14}=4\omega'^4_7\circ\sigma'=l^4\circ(2\omega''^2_7\circ\sigma')=\pm l^4\circ\tau''^1_3(2)\circ\mu'=\pm\tau'^2_3(4)\circ\mu'$.

LEMMA (4.5): (1) *There is an element* s^2_{15} *of the secondary composition* $\{\tau''^1_3(2), 2\varepsilon', \eta_{13}\}_1$ *such that* $p''_*s^2_{15}=\sigma'\circ\eta_{14}$, $2s^2_{15}=0$, *and* $l^4\circ s^2_{15}=u^4_{14}\circ\eta_{14}+4\gamma'^4_8\circ\sigma_8$.

(2) *There is an element* u^4_{16} *of the secondary composition* $\{\gamma'^4_8, 8\iota_8, E\sigma'\}_1$ *such that* $p'_*u^4_{16}=\mu_7$ *and* $2u^4_{16}=\omega'^3_5(4)\circ\zeta_5$. *Denote* $q'_*u^4_{16}$ *by* r^6_{16}, *then* $p_*r^6_{16}\equiv\zeta_5 \bmod \nu_5\circ\bar\nu_8$ *and* $2r^6_{16}=k^6\circ\omega'^3_5\circ\zeta_5$.

(3) *There is an element* s^2_{18} *of the secondary composition* $\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3$ *such that* $p''_*s^2_{18}=\zeta_7$, $4s^2_{18}=\omega''^2_7\circ\zeta_7$ *and* $8s^2_{18}=0$.

(4) *There is an element* s^2_{21} *of the secondary composition* $\{\tau''^1_3(2), 2\varepsilon', \sigma_{13}\}_1$ *such that* $p''_*s^2_{21}=\sigma'\circ\sigma_{14}$, $2s^2_{21}=s^2_{14}\circ\sigma_{14}$, *and* $l^4\circ s^2_{21}\equiv u^4_{14}\circ\sigma_{14} \bmod 8u^4_{14}\circ\sigma_{14}$.

(5) *There is an element* $\hat{s}^2_{22}$ *of the secondary composition* $\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1$ *such that* $p''_*\hat{s}^2_{22}=\sigma'\circ\varepsilon_{14}$, $2\hat{s}^2_{22}=0$ *and* $l^4\circ\hat{s}^2_{22}=u^4_{14}\circ\varepsilon_{14}$.

PROOF: (1) Considering the bundle sequence of $R_6/R_5=S^5$, we have the following exact sequence:

$$0 \longleftarrow Z_8 \xleftarrow{p_*} \pi_{15}(R_6) \xleftarrow{i_*} \pi_{15}(R_5) \longleftarrow 0$$

where $Z_8=\{\nu_5\circ\sigma_8\}\subset\pi^5_{15}$. $\pi_{15}(R_6)=Z_8\oplus Z_2=\{r^6_8\circ\sigma_8\}\oplus\{r^6_{14}\circ\eta_{14}\}$ and $\pi_{15}(R_5)=Z_2$. Note that $p_*(r^6_8\circ\sigma_8)=\nu_5\circ\sigma_8$ and $p_*(r^6_{14}\circ\eta_{14})=\eta_5\circ\varepsilon_6\circ\eta_{14}=\eta^2_5\circ\varepsilon_7=4\nu_5\circ\sigma_8$ (by 2.1)). Hence it follows that $4r^6_8\circ\sigma_8+r^6_{14}\circ\eta_{14}\in i_*\pi_{15}(R_5)$, which implies that there exists an element s^2_{15} such that $l^4\circ s^2_{15}=4\gamma'^4_8\circ\sigma_8+u^4_{14}\circ\eta_{14}$. Then $p''_*s^2_{15}=p'_*(l^4\circ s^2_{15})=\sigma'\circ\eta_{14}$. Since l^4_*: $\pi_{15}(Sp_2)\to\pi_{15}(U_4)$ is a monomorphism, we have $2s^2_{15}=0$. Now, consider a secondary composition $\{\tau''^1_3(2), 2\varepsilon', \eta_{13}\}_1$, then $p''_*\{\tau''^1_3(2), 2\varepsilon', \eta_{13}\}_1=\sigma'\circ\eta_{14}$. (c.f. Lemma (4.4)(2)). Hence it follows that $s^2_{15}\in\{\tau''^1_3(2), 2\varepsilon', \eta_{13}\}$.

(2) Consider a secondary composition $\{\gamma'^4_8, 8\iota_8, E\sigma'\}_1$. $p'_*\{\gamma'^4_8, 8\iota_8, E\sigma'\}_1\subset$

$\{\eta_7, 8\iota_8, E\sigma'\}_1$, which contains μ_7. Considering the bundle sequence of $U_4/U_2=S^5\times S^7$, there exists an element α such that $p_{1*}\alpha=\zeta_5\oplus\mu_7\in\pi^5_{16}\oplus\pi^7_{16}$ (by Lemma 6.2(ii) of [5]). Note that $\pi_{16}(U_3)=Z_2\oplus Z_4=\{u^3_{16}\}\oplus\{\omega'^3_5\circ\zeta_5\}$ (according to [5]). Hence $\alpha+xu^3_{16}(4)+y\omega'^3_5(4)\circ\zeta_5\in\{\gamma'^4_8, 8\iota_8, E\alpha'\}_1$ for some integers x and y. Let us show that $y=0$. Indeed, $p_*(q'_*\omega'^3_5(4)\circ\zeta_5)=p_*(k^6\circ\omega'^3_5\circ\zeta_5)=p'_*(\omega'^3_5\circ\zeta_5)=2\zeta_5$. However, $p_*\circ q'_*\{\gamma'^4_8, 8\iota_8, E\sigma'\}\subset\{\nu_5, 8\iota_8, E\sigma'\}_1$, which does not contain $2\zeta_5$. Now we define $u^4_{16}=\alpha+xu^3_{16}(4)$ ($x=0$ or 1). Then $p'_*u^4_{16}=\mu_7$, but $p_*(q'_*u^4_{16})\equiv\zeta_5 \bmod p'_*u^3_{16}=\nu_5\circ\bar{\nu}_8$ (by Lemma (3.6)(1)). Since $2u^3_{16}(4)=0$, $2u^4_{16}\in\{2\gamma'^4_8, 8\iota_8, E\sigma'\}_1=\{\omega'^3_5(4)\circ\nu_5, 8\iota_8, E\sigma'\}_1$ (by (3.2)(8)). It is easy to see that the last set consists of a single element $\omega'^3_5(4)\circ\zeta_5$.

(3) Consider a secondary composition $\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_1$. $p''_*\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_1\subset\{\nu_7, 8\iota_{10}, 2\sigma_{10}\}_1$, which consists of a single element ζ_7 (by 1.2)(6)). $4\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_1\subset\{4\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_1=\{\omega''^2_7\circ\nu_7, 8\iota_{10}, 2\sigma_{10}\}_1$ (by (3.3)(1)), which consists of a single element $\omega''^2_7\circ\zeta_7$. Chose an element s^2_{18} from the secondary composition $\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_1$, then $p''_*s^2_{18}=\zeta_7$ and $4s^2_{18}=\omega''^2_7\circ\zeta_7$. It follows from the bundle sequence of Sp_2/Sp_1 that s^2_{18} is an element of order 8.

(4) Note that $p_*(r^6_{14}\circ\sigma_{14})=\eta_5\circ\varepsilon_6\circ\sigma_{14}=0$ (by (2.5)). Hence by the diagram (4.3), $u^4_{14}\circ\sigma_{14}\in l^4_*\pi_{21}(Sp_2)$, that is, there exists an element s^2_{21} such that $l^4\circ s^2_{21}=u^4_{14}\circ\sigma_{14}$. Then $p''_*s^2_{21}=p'_*(u^4_{14}\circ\sigma_{14})=\sigma'\circ\sigma_{14}$ and $l^4(2s^2_{21})=(2u^4_{14})\circ\sigma_{14}=l^4(s^2_{14}\circ\sigma_{14})$ (by Lemma (4.4)). Considering the bundle sequence of $R_6/R_5=S^5$, we see that the kernel of l^4_* is generated by $2\tau''^1_3(2)\circ\mu'\circ\sigma_{14}=16s^2_{21}$. Hence, if we chose the element s^2_{21} so that $2s^2_{21}=s^2_{14}\circ\sigma_{14}$ holds, then $l^4\circ s^2_{21}\equiv u^4_{14}\circ\sigma_{14} \bmod 8u^4_{14}\circ\sigma_{14}$. It is easily observed that $s^2_{21}\in\{\tau''^1_3(2), 2\varepsilon', \sigma_{13}\}_1$.

(5) Note that $p'_*(u^4_{14}\circ\varepsilon_{14})=\sigma'\circ\varepsilon_{14}$ and $p_*(r^6_{14}\circ\varepsilon_{14})=\eta_5\circ\varepsilon^2_6=\eta_5\circ\bar{\varepsilon}_7=0$ (by 2.7)). Hence, there exists an element $\alpha\in\pi_{22}(Sp_2)$ such that $l^4\circ\alpha=u^4_{14}\circ\varepsilon_{14}$ and $p''_*\alpha=\sigma'\circ\varepsilon_{14}$. Now consider a secondary composition $\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1$. $p''_*\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1=\sigma'\circ\varepsilon_{14}$ (c.f. Lemma (4.4)(2)), and $2\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1=-\tau''^1_3(2)\circ\{2\varepsilon', \varepsilon_{13}, 2\iota_{21}\}=-\tau''^2_3(2)\circ\varepsilon'\circ\{2\iota_{13}, \varepsilon_{13}, 2\iota_{21}\}=\tau''^1_3(2)\circ\varepsilon'\circ\eta_{13}\circ\varepsilon_{14}=\tau''^1_3(2)\circ\nu'\circ\varepsilon^2_6=0$ (by (2.1)). Note that $i''^{2,1}_*\pi_{22}(Sp_1)=Z_4=\{\tau''^1_3(2)\circ\mu'\}$. Hence, either α or $\alpha+2\tau''^1_3(2)\circ\mu'$ belongs to $\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1$. However, $l^4_*(2\tau''^1_3(2)\circ\mu')=2\tau'^2_3(4)\circ\mu'=0$. Let us define either $\hat{s}^2_{22}=\alpha$ or $\hat{s}^2_{22}=\alpha+2\tau''^1_3(2)\circ\mu'$ so that $\hat{s}^2_{22}$ is contained in $\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1$. But any way, $p''_*\hat{s}^2_{22}=\sigma'\circ\varepsilon_{14}$, $2\hat{s}^2_{22}=0$ and $l^4\circ\hat{s}^2_{22}=u^4_{14}\circ\varepsilon_{14}$.

LEMMA (4.6): (1) *There is an element* u^4_{22} *of the secondary composition* $\{u^4_{14}, 8\iota_4, 2\sigma_{14}\}_5$ *such that* $p'_*u^4_{22}=\rho''$, $2u^4_{22}=\omega'^4_7\circ\rho''$ *and* $8u^4_{22}=\tau'^2_3(4)\circ\bar{\mu}'$.

(2) *There is an element* $s^2_{22}\in\pi_{22}(Sp_2)$ *such that* $p''_*s^2_{22}=\rho''$, $4s^2_{22}=\omega''^2_7\circ\rho''$, $8s^2_{22}=\tau''^1_3(2)\circ\bar{\mu}'$ *and* $l^4\circ s^2_{22}\equiv u^4_{22} \bmod 2\gamma'^4_8\circ\kappa_8$.

PROOF: Considering the bundle sequence of $U_4/U_2=S^5\times S^7$, there is an element α of $\pi_{22}(U_4)$ such that $p_{1*}\alpha=0\oplus\rho''\in\pi^5_{22}\oplus\pi^7_{22}$ (by Lemma (6.2)(ii) of [5]) and

$2\alpha=\omega'^4_7\circ\rho''$. Note that $\alpha\in l^4_*\pi_{22}(Sp_2)$. Now consider a secondary composition $\{u^4_{14}, 16\iota_{14}, \sigma_{14}\}_5$. $p'_*\{u^4_{14}, 16\iota_{14}, \sigma_{14}\}_5\subset\{\sigma', 16\iota_{14}, \sigma_{14}\}_5$, which contains ρ''. $8\{u^4_{14}, 16\iota_{14}, \sigma_{14}\}_5 = \{4u^4_{14}, 16\iota_{14}, 4\sigma_{14}\}_5=\{8u^4_{14}, 4\iota_{14}, 4\sigma_{14}\}=\{\pm\tau'^2_3(4)\circ\mu', 4\iota_{14}, 4\sigma_{14}\}$ (by Lemma (4.4) (3)), which consists of a single element $\tau'^2_3(4)\circ\bar{\mu}'$ (by (1.2)(12)). Next we note that $\pi_{22}(U_3) = Z_2\oplus Z_2=\{\omega'^3_5\circ\nu_5\circ\kappa_8\}\oplus\{\tau'^2_3(3)\circ\bar{\mu}'\}$. Hence we can assert that $\alpha+x\omega'^3_5(4)\circ\nu_5\circ\kappa_8$ or $9\alpha+x\omega'^3_5(4)\circ\nu_5\circ\kappa_8$ is contained in $\{u^4_{14}, 16\iota_{14}, \sigma_{14}\}_5$ for $x=0$ or 1. Define $s^2_{22}\in\pi_{22}(Sp_2)$ and $u^4_{22}\in\pi_{22}(U_4)$ so that $l^4\circ s^2_{22}=\alpha$ or 9α, $u^4_{22}=l^4\circ s^2_{22}+x\omega'^3_5(4)\circ\nu_5\circ\kappa_8\in\{u^4_{14}, 16\iota_{14}, \sigma_{14}\}_5$ holds. If we note that $\omega'^3_5(4)\circ\nu_5=2\gamma'^4_8$, we can easily observe that the Lemma (4.6) holds.

Now we shall prove the following relations:

PROPOSITION (4.7):

(1) $\omega''^2_7\circ\varepsilon_7=\omega''^2_7\circ\bar{\nu}_7=0$,
(2) $\omega''^2_7\circ\mu_7=0$,
(3) $\omega''^2_7\circ\kappa_7\equiv 0 \bmod \tau''^1_3(2)\circ\mu'\circ\sigma_{14}$,
(4) $\gamma''^2_{10}\circ\varepsilon_{10}=\gamma''^2_{10}\circ\bar{\nu}_{10}=\tau''^1_3(2)\circ\bar{\varepsilon}_3$,
(5) $\gamma''^2_{10}\circ\mu_{10}\equiv\tau''^1_3(2)\circ\mu_3\circ\sigma_{12} \bmod \tau''^1_3(2)\circ\eta_3\circ\bar{\varepsilon}_4$,
(6) $\gamma''^2_{10}\circ\zeta_{10}\equiv 4s^2_{21} \bmod \tau''^1_3(2)\circ\mu'\circ\sigma_{14}$,
(7) $s^2_{14}\circ\eta_{14}=0$,
(8) $s^2_{14}\circ\nu_{14}=\pm 2\gamma''^2_{10}\circ\sigma_{10}$,
(9) $s^2_{14}\circ\varepsilon_{14}=s^2_{14}\circ\bar{\nu}_{14}=0$,
(10) $s^2_{15}\circ\eta^2_{15}=4\gamma''^2_{10}\circ\sigma_{10}$,
(11) $s^2_{15}\circ\nu_{15}=0$,
(12) $s^2_{18}\circ\eta_{18}\equiv\tau''^1_3(2)\circ\mu_3\circ\sigma_{12} \bmod \tau''^1_3(2)\circ\eta_3\circ\bar{\varepsilon}_4$,
(13) $s^2_{18}\circ\nu_{18}\equiv 4s^2_{21} \bmod \tau''^1_3(2)\circ\mu'\circ\sigma_{14}$,
(14) $s^2_{21}\circ\eta_{21}\equiv s^2_{15}\circ\sigma_{15} \bmod 2\tau''^1_3(2)\circ\bar{\mu}'$.

PROOF: (1) $p''_*(\omega''^2_7\circ\varepsilon_7)=4\varepsilon_7$ (by (3.1)(2)) $=0$. Since $p''^{-1}_*(0)=i''_*\pi_{15}(Sp_1)=0$, it follows that $\omega''^2_7\circ\varepsilon_7=0$. Similarly we have $\omega''^2_7\circ\bar{\nu}_7=0$. In the same way, we can prove (2), (6), (7), (8), (10), (13) and (14) by using the relations: $\nu_7\circ\zeta_{10}=\zeta_7\circ\nu_{18} =4\sigma'\circ\sigma_{14}$ (by (2.3) and Prop. (2.4)(2)), $2\sigma'\circ\nu_{14}=\pm 2\nu_7\circ\sigma_{10}$ (by (2.1)).

(3) $2\omega''^2_7\circ\kappa_7\equiv\omega''^2_7\circ\bar{\nu}_7\circ\nu^2_{15} \bmod 4\omega''^2_7\circ\sigma'\circ\sigma_{14}$ (by (1.1)(10)). $\omega''^2_7\circ\bar{\nu}_7=0$ (by (1)), $4\omega''^2_7\circ\sigma'\circ\sigma_{14}=2\tau''^1_3(2)\circ\mu'\circ\sigma_{14}$ (by Lemma (4.4)(1)). Hence $\omega''^2_7\circ\kappa_7\equiv 0 \bmod \{\tau''^1_3(2)\circ\mu'\circ\sigma_{14}\}\oplus\{\tau''^1_3(2)\circ\eta_3\circ\mu_4\}$. Now, $\omega''^2_7\circ\kappa_7\circ\eta_{21}=\omega''^2_7\circ\sigma'\circ\bar{\nu}_{14}+\omega''^2_7\circ\bar{\varepsilon}_7$ (by Prop. (2.6)(4)) $=0$. Indeed, $\omega''^2_7\circ\sigma'\circ\bar{\nu}_{14}=0$, and $\omega''^2_7\circ\bar{\varepsilon}_7\in\omega''^2_7\circ\{\varepsilon_7, 2\iota_{15}, \nu^2_{15}\}_{10}=-\{\omega''^2_7, \varepsilon_7, 2\iota_{15}\}\circ\nu^2_{16}\subset\pi_{16}(Sp_2)\circ\nu^2_{16}= \{\gamma''^2_{10}\circ\nu^4_{10}\}\oplus\{s^2_{15}\circ\eta_{15}\circ\nu^2_{16}\}=0$. However $\tau''^1_3(2)\circ\eta_3\circ\bar{\mu}_4\circ\eta_{21}=\tau''^1_3(2)\circ\eta^2_3\circ\mu_5=2\tau''^1_2(2)\circ\bar{\mu}'\neq 0$. Hence we conclude that $\omega''^2_7\circ\kappa_7\equiv 0 \bmod \tau''^1_3(2)\circ\mu'\circ\sigma_{14}$.

(4) $\gamma''^2_{10}\circ\varepsilon_{10}\in\{\gamma''^2_{10}\circ\eta_{10}, 2\iota_{11}, \nu^2_{11}\}_6$ (by (1.2) (3)) $=\{\tau''^1_3(2)\circ\varepsilon_3, 2\iota_{11}, \nu^2_{11}\}_6$ (by (3.3) (2)), which consists of a single element $\tau''^1_3(2)\circ\bar{\varepsilon}_3$, because $\varepsilon_3\circ E^6\pi^5_{12}=\varepsilon_3\circ(8\sigma_{11})=0$, $\pi_{12}(Sp_2)\circ\nu^2_{12}=i''^{2,1}_*\pi_{12}(Sp_1)\circ\nu^2_{12}\subset i''^{2,1}_*\pi_{15}(Sp_2)\circ\nu_{15}=0$. Hence $\gamma''^2_{10}\circ\varepsilon_{10}=0$. Then, $\gamma''^2_{10}\circ\bar{\nu}_{10}=\gamma''^2_{10}\circ\varepsilon_{10}+\gamma''^2_{10}\circ\eta_{10}\circ\sigma_{11}=\tau''^1_3(2)\circ\varepsilon_3\circ\sigma_{11}=0$ (by (2.1), (2.5)).

(5) $\gamma''^2_{10}\circ\mu_{10}\in\pi_{19}(Sp_2)=Z_2+Z_2=\{\tau''^1_3(2)\circ\eta_3\circ\bar{\varepsilon}_4\}\oplus\{\tau''^1_3(2)\circ\mu_3\circ\sigma_{12}\}$ (according to [5]). $\gamma''^2_{10}\circ\mu_{10}\circ\eta_{19}=\gamma''^2_{10}\circ\eta_{10}\circ\mu_{11}$ (by Prop. (2.2)(2)) $=\tau''^1_3(2)\circ\varepsilon_3\circ\mu_{11}$ (by (3.3)(2)) $=\tau''^1_3(2)\circ\eta_3\circ\mu_4\circ\sigma_{13}$ (by Prop. (2.13)(7)). $\tau''^1_3(2)\circ\eta_3\circ\bar{\varepsilon}_4\circ\eta_{19}=2\tau''^1_3(2)\circ\bar{\varepsilon}'=0$. Hence we have $\gamma''^2_{10}\circ\mu_{10}\equiv\tau''^1_3(2)\circ\mu_3\circ\sigma_{12} \bmod \tau''^1_3(2)\circ\eta_3\circ\bar{\varepsilon}_4$.

(9) $s^2_{14} \circ \varepsilon_{14} \in s^2_{14} \circ \{\eta_{14}, \nu_{15}, 2\nu_{18}\}$ (by (1.2)(3)) $= -\{s^2_{14}, \eta_{14}, \nu_{15}\} \circ 2\nu_{19} \subset 2\pi_{19}(Sp_2) \circ \nu_{19} = 0$. $s^2_{14} \circ \bar{\nu}_{14} = s^2_{14} \circ \varepsilon_{14} + s^2_{14} \circ \eta_{14} \circ \sigma_{15} = 0$ (by (2.1)).

(11) $s^2_{15} \circ \nu_{15} \in \{\tau''^1_3(2) \circ \varepsilon', 2\iota_{13}, \eta_{13}\} \circ \nu_{15}$ (by Lemma (4.5) (1)) $= -\tau''^1_3(2) \circ \varepsilon' \circ \{2\iota_{13}, \eta_{13}, \nu_{14}\} \subset \tau''^1_3(2) \circ \varepsilon' \circ \pi^{13}_{19} = 0$.

(12) $s^2_{18} \circ \eta_{18} \in \{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3 \circ \eta_{18} = \gamma''^2_{10} \circ \{8\iota_{10}, 2\sigma_{10}, \eta_{17}\}$, which contains $\gamma''^2_{10} \circ \mu_{10} + \gamma''^2_{10} \circ \nu^3_{10}$ (by (1.2)(4)). $\gamma''^2_{10} \circ \mu_{10} \equiv \tau''^1_3(2) \circ \mu_3 \circ \sigma_{12} \bmod \tau''^1_3(2) \circ \eta_3 \circ \bar{\varepsilon}_4$, $\gamma''^2_{10} \circ \nu^3_{10} = \gamma''^2_{10} \circ \bar{\nu}_{10} \circ \eta_{17}$ (by (2.1)) $= \tau''^1_3(2) \circ \bar{\varepsilon}_3 \circ \eta_{17}$ (by (4)) $= \tau''^1_3(2) \circ \eta_3 \circ \bar{\varepsilon}_4$, and the set $\gamma''^2_{10} \circ \{8\iota_{10}, 2\sigma_{10}, \eta_{17}\}$ is a coset of the subgroup $\gamma''^2_{10} \circ \pi^{10}_{18} \circ \eta_{18} = \{\tau''^1_3(2) \circ \eta_3 \circ \bar{\varepsilon}_4\}$ (by (4)). Hence we conclude that $s^2_{18} \circ \eta_{18} \equiv \tau''^1_3(2) \circ \mu_3 \circ \sigma_{12} \bmod \tau''^1_3(2) \circ \eta_3 \circ \bar{\varepsilon}_4$.

PROPOSITION (4.8):

(1) $\omega'^4_7 \circ \varepsilon_7 = 4\gamma'^4_8 \circ \sigma_8$,

(2) $\omega'^4_7 \circ \bar{\nu}_7 = 0$,

(3) $\omega'^4_7 \circ \mu_7 = 4u^4_{16}$,

(4) $\omega'^4_7 \circ \nu_7 \circ \sigma_{10} = \pm 2u^4_{14} \circ \nu_{14}$,

(5) $\omega'^4_7 \circ \nu_7 = 2l^4 \circ \gamma''^2_{10}$,

(6) $\omega'^4_7 \circ \zeta_7 = 2l^4 \circ s^2_{18}$,

(7) $\omega'^4_7 \circ \kappa_7 \equiv \gamma'^4_8 \circ \sigma_8 \circ \nu^2_{15} \bmod 4u^4_{14} \circ \sigma_{14}$,

(8) $\omega'^4_7 \circ \bar{\varepsilon}_7 = 0$,

(9) $\gamma'^4_8 \circ E\sigma' = 2\gamma'^4_8 \circ \sigma_8$,

(10) $\gamma'^4_8 \circ \sigma_8 \circ \eta_{15} \equiv u^4_{14} \circ \eta^2_{14} + \gamma'^4_8 \circ \varepsilon_8 + \gamma'^4_8 \circ \bar{\nu}_8 \bmod i'_* \pi_{16}(U_3)$,

(11) $\gamma'^4_8 \circ \eta_8 \circ \varepsilon_9 = u^3_{10}(4) \circ \sigma_{10} + u^3_{11}(4) \circ \nu^2_{11} + 4u^4_{14} \circ \nu_{14}$,

(12) $\gamma'^4_8 \circ \sigma_8 \circ \eta^2_{15} \equiv u^3_{10}(4) \circ \sigma_{10} \bmod u^3_{11}(4) \circ \nu^2_{11}$,

(13) $\gamma'^4_8 \circ \mu_8 \circ \eta_{17} = 4l^4 \circ s^2_{18} + u^3_{18}(4)$,

(14) $\gamma'^4_8 \circ \zeta_8 \equiv \pm u^3_{12}(4) \circ \sigma_{12} \bmod u^3_{16}(4) \circ \nu_{16}$,

(15) $\gamma'^4_8 \circ \bar{\nu}_8 \circ \nu_{16} \equiv u^3_{16}(4) \circ \nu_{16} \bmod 2u^3_{12}(4) \circ \sigma_{12}$,

(16) $\gamma'^4_8 \circ \sigma^2_8 \equiv u^4_{14} \circ \varepsilon_{14} + u^4_{14} \circ \bar{\nu}_{14} \bmod \{\tau'^2_3(4) \circ \mu'\} \oplus \{2\gamma'^4_8 \circ \kappa_8\}$,

(17) $u^4_{16} \circ \eta_{16} \equiv \gamma'^4_8 \circ \mu_8 \bmod \{u^3_{11}(4) \circ \nu^2_{11}\} \oplus \{u^3_{10}(4) \circ \sigma_{10}\}$,

(18) $u^4_{16} \circ \nu^2_{16} \equiv 0 \bmod 2\gamma'^4_8 \circ \kappa_8$,

(19) $l^4 \circ \gamma''^2_{10} \circ \nu^2_{10} \equiv u^3_{16}(4) + \gamma'^4_8 \circ \bar{\nu}_8 \bmod u^4_{14} \circ \eta^2_{14}$,

(20) $l^4 \circ \gamma''^2_{10} \circ \sigma_{10} = x u^4_{14} \circ \nu_{14}$ for some odd integer x.

PROOF: (1) In the diagram (4.3), $p_{1*} \circ p_{2*}(\omega'^4_7 \circ \varepsilon_7) = \eta^2_5 \circ \varepsilon_7$ (by (4.2)(3)) $= 4\nu_5 \circ \sigma_8$ (by (2.1)), and $p_{2*} \circ p_{1*}(4\gamma'^4_8 \circ \sigma_8) = 4\nu_5 \circ \sigma_8$ (by (4.2)(4)). Considering the bundle sequence of $R_6/R_5 = S^5$, it follows that $l^4_* : \pi_{15}(Sp_2) \to \pi_{15}(U_4)$ is a monomorphism. Hence $\omega'^4_7 \circ \varepsilon_7 = 4\gamma'^4_8 \circ \sigma_8$. Similarly, we can prove (3) by using the relation: $\eta^2_5 \circ \mu_7 = 4\zeta_5$ (by (1.1)(8)).

(2) $\omega'^4_7 \circ \bar{\nu}_7 = \omega'^4_7 \circ \eta_7 \circ \sigma_8 + \omega'^4_7 \circ \sigma' \circ \eta_{14} + \omega'_7 \circ \varepsilon_7$ (by (2.1)). $\omega'^4_7 \circ \eta_7 \circ \sigma_8 = 4\gamma'^4_8 \circ \sigma_8$ (by (3.4)(5)). $\omega'^4_7 \circ \sigma' \circ \eta_{14} = 2u^4_{14} \circ \eta_{14} = 0$ (by Lemma (4.4)(3)). $\omega'^4_7 \circ \varepsilon_7 = 4\gamma'^4_8 \circ \sigma_8$ (by (1)). Hence

we have $\omega'^4_7\circ\bar\nu_7=0$.

(4) $\omega'^4_7\circ\nu_7\circ\sigma_{10}=\omega'^4_7\circ(x\sigma'\circ\nu_{14})$ for some odd integer x (by (2.1)) $=\pm 2u^2_{14}\circ\nu_{14}$ (by Lemma (4.4)(3)).

(5) $p'_*(l^4\circ\gamma''^2_{10}\circ\sigma_{10})=\nu_7\circ\sigma_{10}$ (by (3.1)(3)), hence $l^4\circ\gamma''^2_{10}\equiv xu^4_{14}\circ\nu_{14}$ mod $i'^{4,3}_*\pi_{17}(U_3)$ for some odd integer x. Since $2\pi_{17}(U_3)=0$, we have $2l^4\circ\gamma''^2_{10}\circ\sigma_{10}=\pm 2u^4_{14}\circ\nu_{14}$. Now $p'_*(\omega'^4_7\circ\nu_7)=2\nu_7$, so that $\omega'^4_7\circ\nu_7\equiv 2l^4\circ\gamma''^2_{10}$ mod $i'^{4,3}_*\pi_{10}(U_3)=\{u^3_{10}(4)\}$. However, if $\omega'^4_7\circ\nu_7=2l^4\circ\gamma''^2_{10}+u^3_{10}(4)$, then $\omega'^4_7\circ\nu_7\circ\sigma_{10}=2l^4\circ\gamma''^2_{10}\circ\sigma_{10}+u^2_{10}(4)\circ\sigma_{10}$, which contradicts to (4).

(6) $\omega'^4_7\circ\zeta_7\in\{\omega'^4_7\circ\nu_7, 8\iota_{10}, 2\sigma_{10}\}_3$ (by (1.2)(6)) $=\{2l^4\circ\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3$ (by (5)), which consists of a single element $2s^2_{18}$ (by Lemma (4.5)(3)), because $2l^4\circ\gamma''^2_{10}\circ\pi^{10}_{18}=0$, and $2\pi_{11}(U_4)\circ\sigma_{11}\subset 2i'^{4,3}_*\pi_{18}(U_3)=0$. Hence we have $\omega'^4_7\circ\zeta_7=2s^2_{18}$.

(7) $p'_*(\omega'^4_7\circ\kappa_7)=2\kappa_7\equiv\bar\nu_7\circ\nu^2_{15}$ mod $4\sigma'\circ\sigma_{14}$ (by Prop. (2.5)(4)). $p'^{-1}_*(0)=i'_*\pi_{21}(U_3)=\{\tau'^2_3(4)\circ\mu'\circ\sigma_{14}\}=\{8u^4_{14}\circ\sigma_{14}\}$. $p'_*(u^4_{14}\circ\sigma_{14})=\sigma'\circ\sigma_{14}$, $p'_*(\gamma'^4_8\circ\sigma_8\circ\nu^2_{15})=\eta_7\circ\sigma_8\circ\nu^2_{15}=(\sigma'\circ\eta_{14}+\varepsilon_7+\bar\nu_7)\circ\nu^2_{15}=\bar\nu_7\circ\nu^2_{15}$ (by (2.1)). Hence it follows that $\omega'^4_7\circ\kappa_7\equiv\gamma'^4_8\circ\sigma_8\circ\nu^2_{15}$ mod $4u^4_{14}\circ\sigma_{14}$. In the same way, we can prove (10) and (17).

(8) It follows from Proposition (2.6)(4) that $\bar\varepsilon_7=\kappa_7\circ\eta_{21}+\sigma'\circ\bar\nu_{14}$. Hence $\omega'^4_7\circ\bar\varepsilon_7=\omega'^4_7\circ\sigma'\circ\bar\nu_{14}$ (by (7)) $=(2u^4_{14})\circ\bar\nu_{14}$ (by Lemma (4.4)(3)) $=0$.

(9) $\gamma'^4_8\circ E\sigma'-2\gamma'^4_8\circ\sigma_8=\gamma'^4_8\circ(E\sigma'-2\sigma_8)=\gamma'^4_8\circ[\iota_8, \iota_8]=0$ (by (1.3)).

(11) $\gamma'^4_8\circ\eta_8\circ\varepsilon_9=\gamma'^4_8\circ\eta_8\circ\bar\nu_9+\gamma'^4_8\circ\eta^2_8\circ\sigma_{10}$ (by (2.1)). It follows from (3.4)(6) that $\gamma'^4_8\circ\eta_8\circ\bar\nu_9=\gamma'^4_8\circ\nu^3_8=u^3_{11}(4)\circ\nu^2_{11}$, and $\gamma'^4_8\circ\eta^2_8\circ\sigma_{10}=u^3_{10}(4)\circ\sigma_{10}+4l^4\circ\gamma''^2_{10}\circ\sigma_{10}$. $4l^4\circ\gamma''^2_{10}\circ\sigma_{10}=4u^4_{14}\circ\nu_{14}$ (c.f. Proof of (5)). Summarizing, we have $\gamma'^4_8\circ\eta_8\circ\varepsilon_9=u^3_{10}(4)\circ\sigma_{10}+u^3_{11}(4)\circ\nu^2_{11}+4u^4_{14}\circ\nu_{14}$.

(12) Note that $i'_*\pi_{16}(U_3)\circ\eta_{16}=\{u^3_{16}(4)\circ\eta_{16}\}\oplus\{2u^4_{16}\circ\eta_{16}\}=\{u^3_{11}(4)\circ\nu^2_{11}\}$ (by Prop. (3.7)(20)). It follows from (10) that $\gamma'^4_8\circ\sigma_8\circ\eta^2_{15}\equiv 4u^4_{14}\circ\nu_{14}+\gamma'^4_8\circ\varepsilon_8\circ\eta_{16}+\gamma'^4_8\circ\bar\nu_8\circ\eta_{16}$ mod $u^3_{11}(4)\circ\nu^2_{11}$, so that $\gamma'^4_8\circ\sigma_8\circ\eta^2_{15}\equiv u^3_{10}(4)\circ\sigma_{10}$ mod $u^3_{11}(4)\circ\nu^2_{11}$ (by (11)).

(13) Consider the secondary compositions $\{4l^4\circ\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3$, $\{u^3_{10}, 8\iota_{10}, 2\sigma_{10}\}_3$ and $\{\gamma'^4_8\circ\eta^2_8, 2\iota_{10}, 2\sigma_{10}\}_3$. Each of them consists of a single element. Indeed, $4l^4\circ\gamma''^2_{10}\circ\pi^{10}_{18}=u^3_{10}\circ\pi^{10}_{18}=\gamma'^4_8\circ\eta^2_8\circ\pi^{10}_{18}=0$ (Prop. (3.7)(9)), $\pi_{11}(U_4)\circ 2\sigma_{11}\subset 2i'_*\pi_{18}(U_3)=0$. Hence it follows that $4l^4\circ s^2_{18}=\{4l^4\circ\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3=\{u^3_{10}(4), 8\iota_{10}, 2\sigma_{10}\}_3+\{\gamma'^4_8\circ\eta^2_8, 8\iota_{10}, 2\sigma_{10}\}_3$ (by (3.4)(6)) $=u^3_{18}(4)+\gamma'^4_8\circ\eta_8\circ\mu_9$ (by (3.5)(2), (1.2)(4)).

(14) $\gamma'^4_8\circ\zeta_8\in\{i'^{4,3}, \tau'^2_3(3)\circ\nu', \eta_6\}\circ\eta_8=-i'^{4,3}\{\tau'^2_3(3)\circ\nu', \eta_6, \zeta_7\}=-i'^{4,3}\circ\{\tau'^2_3(3), \eta_3\circ\nu_4, \zeta_7\}=-i'^{4,3}\circ\{\tau'^2_3(3), \eta_3, \nu_4\circ\zeta_7\}$, which is a coset of the subgroup $i'^{4,3}\circ\pi_5(U_3)\circ\nu_5\circ\zeta_8+\tau'^2_3(4)\circ\pi^3_{19}=\{2u^3_{12}(4)\circ\sigma_{12}\}$ (by Prop. (3.7)(5) and (3.2)(6)). Now, $p'_*\{\tau'^2_3(3), \eta_3, \nu_4\circ\zeta_7\}=\nu_5\circ\zeta_8$, $p'_*(u^3_{12}\circ\sigma_{12})=\sigma'''\circ\sigma_{12}\equiv\nu_5\circ\zeta_8$ mod $\nu_5\circ\bar\nu_8\circ\nu_{16}$ (by (3.2)(6), Prop. (2.4)(2)), and $p'_*(u^3_{16}\circ\nu_{16})=\nu_5\circ\bar\nu_8\circ\nu_{16}$ (by Lemma (3.6)(1)). Note that $p'^{-1}_*(0)=i'_*\pi_{19}(U_2)=\{2u^3_{12}\circ\sigma_{12}\}$. Hence we can conclude that $\gamma'^4_8\circ\zeta_8\equiv\pm u^3_{12}\circ\sigma_{12}$ mod $u^3_{16}\circ\nu_{16}$. Similarly, we can prove (15).

(16) Operating the projection p'_*, we obtain $\gamma'^4_8\circ\sigma^2_8\equiv u^4_{14}\circ\varepsilon_{14}+u^4_{14}\circ\bar\nu_{14}$ mod $i'_*\pi_{22}(U_3)=\{\tau'^2_3(4)\circ\bar\mu'\}\oplus\{2\gamma'^4_8\circ\kappa_8\}$. Note that $i'^{5,4}_*\gamma'^4_8=0$. We shall show later that $u^4_{14}(8)=0$, but $\tau'^2_3(8)\circ\bar\mu'\neq 0$. Hence we have $\gamma'^4_8\circ\sigma^2_8\equiv u^4_{14}\circ\varepsilon_{14}+u^4_{14}\circ\bar\nu_{14}$ mod $2\gamma'^4_8\circ\kappa_8$.

(18) $u^4_{16}\circ\nu^2_{16}\in\{\gamma'^4_8, 8\iota_8, E\sigma'\circ\nu^2_{15}\}_1\subset\{\gamma'^4_8, 8\iota_8, \nu_8\circ\sigma_{11}\circ\nu_{18}\}_1$, which is a coset of the subgroup $\gamma'^4_8\circ E\pi^7_{21}=\{\gamma'^4_8\circ\kappa_8\}$. While, $0=\omega'^4_7\circ\bar{\varepsilon}_7$ (by (8)) $\in\{\omega'^4_7\circ\eta_7, 2\iota_8, \nu_8\circ\sigma_{11}\circ\nu_{18}\}_3$ (by Lemma (2.14)) $=\{4\gamma'^4_8, 2\iota_8, \nu_8\circ\sigma_{11}\circ\nu_{18}\}_3$ (by (3.4)(5)) $\subset\{\gamma'^4_8, 8\iota_8, \nu_8\circ\sigma_{11}\circ\nu_{18}\}_3$. Hence it follows that $u^4_{16}\circ\nu^2_{16}\equiv 0 \bmod 2\gamma'^4_8\circ\kappa_8\}$.

(19) Considering the bundle sequence of $R_6/R_5=S^5$, we see that the kernel of the homomorphism $p_{2*}\circ p_{1*}:\pi_{16}(U_4)\to\pi^5_{16}$ is generated by $u^3_{16}(4)+\gamma'^4_8\circ\bar{\nu}_8$ and $u^4_{14}\circ\eta^2_{14}$, that is, $l^4_*\pi_{16}(Sp_2)=\{u^3_{16}(4)+\gamma'^4_8\circ\bar{\nu}_8\}\oplus\{u^4_{14}\circ\eta^2_{14}\}$. Note that $\pi_{16}(Sp_2)=Z_2\oplus Z_2=\{\gamma''^2_{10}\circ\nu^2_{10}\}\oplus\{s^2_{15}\circ\eta_{15}\}$, and that $l^4\circ s^2_{15}\circ\eta_{15}=u^4_{14}\circ\eta^2_{14}$ (by Lemma (4.5)(1)). Hence we have $l^4\circ\gamma''^2_{10}\circ\nu^2_{10}\equiv u^3_{16}(4)+\gamma'^4_8\circ\bar{\nu}_8 \bmod u^4_{14}\circ\eta^2_{14}$.

(20) This is proved by the method of (7) and (19).

§5. Generators of $\pi_k(Sp_n)$ ($n\geqq 3$, $14\leqq k\leqq 22$)

Let us define original generators of (2-primary components of) $\pi_k(Sp_3)$, $\pi_k(Sp_4)$ and $\pi_k(Sp_5)$ for $14\leqq k\leqq 22$, and prove the relations among them.

LEMMA (5.1): (1) *There is an element γ''^3_{14} of the secondary composition $\{i''^{3,2}, \gamma''^2_{10}, 4\nu_{10}\}$ such that $p''_*\gamma''^3_{14}=4\nu_{11}$ and $2\gamma''^3_{14}=s^2_{14}(3)$.*

(2) *There is an element s^3_{19} of the secondary composition $\{i''^{3,2}, \gamma''^2_{10}, \eta_{10}\circ\sigma_{11}\}$ such that $p''_*s^3_{19}=\eta_{11}\circ\sigma_{12}$ and $2s^3_{19}=0$.*

PROOF: Note that $4\gamma''^2_{10}\circ\nu_{10}=2\tau''^1_3(2)\circ\varepsilon'=0$ (by (3.3)(2)), and $\gamma''^2_{10}\circ\eta_{10}\circ\sigma_{11}=\gamma''^2_{10}\circ\varepsilon_{10}+\gamma''^2_{10}\circ\bar{\nu}_{10}=0$ (by Prop. (4.7)(4)). It is obvious that $p''_*\gamma''^3_{14}=4\nu_{11}$ and $p''_*s^3_{19}=\eta_{11}\circ\sigma_{12}$. γ''^3_{14} is a generator of the cyclic group of $\pi_{14}(Sp_2)$ by (10.1) of [3], hence by the bundle sequence of $Sp_3/Sp_2=S^{11}$, $2\gamma''^3_{14}\in i''^{3,2}\pi_{14}(Sp_2)=\{s^2_{14}(3)\}$. We can chose γ''^3_{14} so that $2\gamma''^3_{14}=s^2_{14}(3)$ holds. Since $\pi_{19}(Sp_3)=Z_2$, it follows that $2s^3_{19}=0$.

Now the following relations hold:

PROPOSITION (5.2):

(1) $\omega''^3_{11}\circ\nu_{11}=\pm 4\gamma''^3_{14}$,	(2) $2\omega''^3_{11}\circ\sigma_{11}=-s^2_{18}(3)$,
(3) $\omega''^3_{11}\circ\varepsilon_{11}=\omega''^3_{11}\circ\bar{\nu}_{11}=0$,	(4) $\omega''^3_{11}\circ\mu_{11}=\tau''^1_3(3)\circ\bar{\mu}_3$,
(5) $\omega''^3_{11}\circ\zeta_{11}=\pm 4s^2_{22}(3)$,	(6) $\gamma''^3_{14}\circ\eta_{14}=s^2_{15}(3)$,
(7) $\gamma''^3_{14}\circ\nu_{14}=0$,	(8) $\gamma''^3_{14}\circ\sigma_{14}=\pm s^2_{21}(3)$,
(9) $\gamma''^3_{14}\circ\varepsilon_{14}=\hat{s}^2_{22}(3)=0$,	(10) $\gamma''^3_{14}\circ\bar{\nu}_{14}=s^2_{15}(3)\circ\sigma_{15}$,
(11) $s^3_{19}\circ\eta^2_{19}=2s^2_{21}(3)$,	(12) $s^3_{19}\circ\nu_{19}=s^2_{15}(3)\circ\sigma_{15}$.

PROOF: (1) $\omega''^3_{11}\circ\nu_{11}\in\{i''^{3,2}, \gamma''^2_{10}, 8\iota_{10}\}\circ\nu_{11}=-i''^{3,2}\circ\{\gamma''^2_{10}, 8\iota_{10}, \nu_{10}\}$, which consists of a single element, because $i''^{3,2}_*\pi_{11}(Sp_2)=0$. Now $p''_*\{\gamma''^2_{10}, 8\iota_{10}, \nu_{10}\}=\{\nu_7, 8\iota_{10}, \nu_{10}\}=E^2\sigma'''$ (by (1.2)(2)) $=4\sigma'$, hence $2s^2_{14}$ or $-2s^2_{14}$ belongs to $\{\gamma''^2_{10}, 8\iota_{10}, \nu_{10}\}$, that is, $\omega''^3_{11}\circ\nu_{11}=\pm 2s^2_{14}(3)$.

(2) $2\omega''^3_{11}\circ\sigma_{11}\in\{i''^{3,2}, \gamma''^2_{10}, 8\iota_{10}\}\circ 2\sigma_{11}$ (by (3.1)(4)) $=-i''^{3,2}\circ\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}$, which consists of a single element $-s^2_{18}(3)$ (by Lemma (4.5)(3)), because $i''^{3,2}_*\pi_{11}(Sp_2)=0$.

(3) $\omega''^3_{11}\circ\varepsilon_{11}\in\{i''^{3,2}, \gamma''^2_{10}, 8\iota_{10}\}\circ\varepsilon_{11}$ (by (3.1)(4)) $=i''^{3,2}\circ\{\gamma''^2_{10}, 8\iota_{10}, \varepsilon_{10}\}\subset i''^{3,2}_*\pi_{19}(Sp_2)=0$. Similarly we have $\omega''^3_{11}\circ\bar{\nu}_{11}=0$.

(4) Note that $\omega''^3_{11}\circ\nu^3_{11}=\omega''^3_{11}\circ\bar{\nu}_{11}\circ\eta_{19}=0$, and $\omega''^3_{11}\circ\eta_{11}=\tau''^1_3(3)\circ\mu_3$ (by (3.3)(3)). Hence by (1.2)(4), $\omega''^3_{11}\circ\mu_{11}\in\{\omega''^3_{11}\circ\eta_{11}, 2\iota_{12}, 8\sigma_{12}\}_7=\{\tau''^1_3(3)\circ\mu_3, 2\iota_{12}, 8\sigma_{12}\}_7$, which consists of a single element $\tau''^1_3(3)\circ\bar{\mu}_3$ (by (1.2)(10)), because $\pi_{13}(Sp_3)=0$.

(5) $\omega''^3_{11}\circ\zeta_{11}\in\{\omega''^3_{11}\circ\nu_{11}, 8\iota_{14}, 2\sigma_{14}\}_5$ (by (1.2)(6)) $=\{\pm 2s^2_{14}(3), 8\iota_{14}, 2\sigma_{14}\}_5$ (by Lemma (5.1)(1) and Prop. (5.2)(1)) $=\{\pm\omega''^2_7(3)\circ\sigma', 8\iota_{14}, 2\sigma_{14}\}_5$ (by Lemma (4.4)(1)), which consists of a single element $\pm\omega''^2_7\circ\rho''$ (by (1.2)(8)), because $2s^2_{14}(3)\circ\pi^{14}_{22}=0$, $2\pi_{15}(Sp_2)\circ\sigma_{15}=0$. Since $\omega''^2_7\circ\rho''=4s^2_{22}$ (by Lemma (4.6)(2)), we have $\omega''^3_{11}\circ\zeta_{11}=\pm 4s^2_{22}(3)$.

(6) It follows from the bundle sequence of $Sp_3/Sp_2=S^{11}$ that $i''_*:\pi_{15}(Sp_2)\to\pi_{15}(Sp_3)$ is an isomorphism. $\pi_{15}(Sp_2)=Z_2=\{s^2_{15}\}$. $\pi_{15}(Sp_3)=Z_2=\{\gamma''^3_{14}\circ\eta_{14}\}$ (by (10.7) of [3]). Hence we have $s^2_{15}(3)=\gamma''^3_{14}\circ\eta_{14}$.

(7) This is obvious, because $\gamma''^3_{14}\circ\nu_{14}\in\pi_{17}(Sp_3)=0$.

(8) $\gamma''^3_{14}\circ\sigma_{14}\in\{i''^{3,2}, \gamma''^2_{10}, 4\nu_{10}\}\circ\sigma_{14}$ (by Lemma (5.1)(1)) $=-i''^{3,2}\circ\{\gamma''^2_{10}, 4\nu_{10}, \sigma_{13}\}=-i''^{3,2}\circ\{\{\tau''^1_3(2), \nu', \nu_6\}, 4\nu_{10}, \sigma_{13}\}$ (by (3.1)(3)). Now $H\{\nu_6, 4\nu_9, \sigma_{12}\}\supset H\{\nu_6, \eta_9, \eta^2_{10}\circ\sigma_{12}\}=\Delta^{-1}(\nu_5\circ\eta_8)\circ\eta^2_{11}\circ\sigma_{13}=\eta^2_{11}\circ\sigma_{13}$ (by (1.3)). $H(\sigma''\circ\sigma_{13})=\eta^2_{11}\circ\sigma_{13}$. Since $\{\nu_6, 4\nu_9, \sigma_{12}\}$ is a coset of the subgroup $\nu_6\circ\pi^9_{20}+\pi^6_{13}\circ\sigma_{13}=\{\nu_6\circ\zeta_9\}+\{\sigma''\circ\sigma_{13}\}=\{\sigma''\circ\sigma_{13}\}$, it follows that $\{\nu_6, 4\nu_9, \sigma_{12}\}$ is the subgroup generated by $\sigma''\circ\sigma_{13}$. $i''^{3,2}\circ\{\tau''^1_3(2), \nu', \sigma''\circ\sigma_{13}\}$ consists of an element $s^2_{14}(3)\circ\sigma_{14}=2s^2_{21}(3)$ (by Lemma (4.4)(2) and Lemma (4.5)(4)), because $i''^{3,2}_*\pi_7(Sp_2)\circ(2\sigma'\circ\sigma_{14})=\{2\omega''^2_7(3)\circ\sigma'\circ\sigma_{14}\}=\{4s^2_{14}(3)\circ\sigma_{14}\}$ (by Lemma (4.4)(1)) $=0$. Now $\{\nu', \nu_6, 4\nu_9\}$ consists of a single element $2\varepsilon'$ (by (1.2)(5)). Finally, note that $i''^{3,2}\circ\{\gamma''^2_{10}, 4\nu_{10}, \sigma_{13}\}$ is a coset of the subgroup $i''^{3,2}\circ\pi_{14}(Sp_2)\circ\sigma_{14}=\{s^2_{14}(3)\circ\sigma_{14}\}=\{2s^2_{21}(3)\}$ (by Lemma (4.5)(4)). Summarizing these facts, we conclude that $-i''^{3,2}\circ\{\gamma''^2_{10}, 4\nu_{10}, \sigma_{13}\}=-i''^{3,2}\circ\{\tau''^1_3(2), 2\varepsilon', \sigma_{13}\}$, which consists of $s^2_{21}(3)$ and $-s^2_{21}(3)$, so that we have $\gamma''^3_{14}\circ\sigma_{14}=\pm s^2_{21}(3)$.

(9) It follows from (10.10) of [3] that there exists an element $s^4_{23}\in\pi_{23}(Sp_4)$ such that $p''_*s^4_{23}=\varepsilon_{15}$. Hence $\gamma''^3_{14}\circ\varepsilon_{14}=\Delta(\varepsilon_{15})=\Delta\circ p''_*(s^4_{23})=0$. Now, $\gamma''^3_{14}\circ\varepsilon_{14}\in\{i''^{3,2}, \gamma''^2_{10}, 4\nu_{10}\}\circ\varepsilon_{14}=i''^{3,2}\circ\{\gamma''^2_{10}, 4\nu_{10}, \varepsilon_{13}\}=i''^{3,2}\circ\{2\gamma''^2_{10}\circ\nu_{10}, 2\iota_{13}, \varepsilon_{13}\}=i''^{3,2}\circ\{\tau''^1_3(2)\circ\varepsilon', 2\iota_{13}, \varepsilon_{13}\}$ (by (3.3)(2)), which consists of a single element $\hat{s}^2_{22}(3)$ (by Lemma (4.5)(5)), because $i''^{3,2}_*\pi_{14}(Sp_2)\circ\varepsilon_{14}=\{s^2_{14}(3)\circ\varepsilon_{14}\}=0$ (by Prop. (4.7)(9)).

(10) $\gamma''^3_{14}\circ\bar{\nu}_{14}=\gamma''^3_{14}\circ\varepsilon_{14}+\gamma''^3_{14}\circ\eta_{14}\circ\sigma_{15}=s^2_{15}(3)\circ\sigma_{15}$ (by (6)).

(11) $s^3_{19}\circ\eta^2_{19}\in\{i''^{3,2}, \gamma''^2_{10}, \eta_{10}\circ\sigma_{11}\}\circ\eta^2_{19}=i''^{3,2}\circ\{\gamma''^2_{10}, \eta_{10}\circ\sigma_{11}, \eta^2_{18}\}$, which consists of a single element, because $i''^{3,2}_*\pi_{19}(Sp_2)=0$. Now $p''_*\{\gamma''^2_{10}, 4\nu_{10}, \sigma_{13}\}\subset\{\nu_7, 4\nu_{10}, \sigma_{13}\}$, which is the subgroup generated by $2\sigma'\circ\sigma_{14}$ (c.f. Proof of (8)). $p''^{-1}_*(0)=i''_*\pi_{21}(Sp_1)=\{\tau''^1_3(2)\circ\eta_3\circ\bar{\mu}_4\}\oplus\{\tau''^1_3(2)\circ\mu'\circ\sigma_{14}\}$. However $\tau''^1_3(3)\circ\mu'\circ\sigma_{14}=0$, so that $2s^2_{21}(3)$ or $2s^2_{21}(3)+\tau''^1_3(3)\circ\eta_3\circ\bar{\mu}_4$ belongs to $i''^{3,2}\circ\{\gamma''^2_{10}, \eta_{10}\circ\sigma_{11}, \eta^2_{18}\}$, that is, $s^3_{19}\circ\eta^2_{19}\equiv 2s^2_{21}(3) \bmod \tau''^1_3(3)\circ\eta_3\circ\bar{\mu}_4$. $s^3_{19}(5)=0$, $s^2_{21}(4)=0$, but $\tau''^1_3(5)\circ\eta_3\circ\bar{\mu}_4\neq 0$. Hence we can conclude that $s^3_{19}\circ\eta^2_{19}=2s^2_{21}(3)$.

(12) $s^3_{19} \in \{i''^{3,2}, \gamma''^2_{10} \circ \sigma_{10}, \eta_{17}\}$ (by Lemma (5.1)(2)), so that $s^3_{19} \circ \nu_{19} \in \{i''^{3,2}, \gamma''^2_{10} \circ \sigma_{10}, \eta_{17}\} \circ \nu_{19} = -i''^{3,2} \circ \{\gamma''^2_{10} \circ \sigma_{10}, \eta_{17}, \nu_{18}\}$, which consists of a single element, because $i''^{3,2}_* \pi_{19}(Sp_2) = 0$. Now $p''_* \{\gamma''^2_{10} \circ \sigma_{10}, \eta_{17}, \nu_{18}\} \subset \{\nu_7 \circ \sigma_{10}, \eta_{17}, \nu_{18}\} = \{\pm\sigma' \circ \nu_{14}, \eta_{17}, \nu_{18}\}$, which consists of a single element $\sigma' \circ \bar{\varepsilon}_{14}$ (by (1.2)(3)). Note that $p''_*(\hat{s}^2_{22} + s^2_{15} \circ \sigma_{15}) = \sigma' \circ \varepsilon_{14} + \sigma' \circ \eta_{14} \circ \sigma_{15}$ (by Lemma (4.5)) $= \sigma' \circ \bar{\varepsilon}_{14}$ (by (2.1)), $\hat{s}^2_{22}(3) = 0$ (by (9)), and $p''^{-1}_*(0) = \tau''^1_3(2) \circ \mu$. Hence we have $s^3_{19} \circ \nu_{19} \equiv s^2_{15}(3) \circ \sigma_{15} \bmod (\tau''^1_3(3) \circ \mu')$. $s^3_{19}(5) = 0$, $s^2_{15}(4) = 0$, but $\tau''^1_3(5) \circ \mu' \neq 0$. Hence we conclude that $s^3_{19} \circ \nu_{19} = s^2_{15}(3) \circ \sigma_{15}$.

LEMMA (5.3): (1) *There is an element ω''^4_{15} of the secondary composition $\{i''^{4,3}, \gamma''^3_{14}, 32\iota_{14}\}$ such that $p''_*\omega''^4_{15} = 32\iota_{15}$.*

(2) *There is an element γ''^4_{18} of the secondary composition $\{i''^{4,3}, \gamma''^3_{14}, \nu_{14}\}$ such that $p''_*\gamma''^4_{18} = \nu_{15}$ and $8\gamma''^4_{18} = x\omega''^3_{11}(4) \circ \sigma_{11}$ for some odd integer x.*

(3) *There is an element s^4_{22} of the secondary composition $\{i''^{4,3}, \gamma''^3_{14}, 4\sigma_{14}\}$ such that $p''_* s^4_{22} = 4\sigma_{15}$ and $4s^4_{22} = \pm s^2_{22}(4)$.*

PROOF: Note that $32\gamma''^3_{14} = 0$, $\gamma''^3_{14} \circ \nu_{14} = 0$ and $4\gamma''^3_{14} \circ \sigma_{14} = 0$. Indeed, $32\gamma''^3_{14} = 16s^2_{14}(3) = 4\tau''^1_3(3) \circ \bar{\mu}' = 0$, $\gamma''^3_{14} \circ \nu_{14} = 0$ (by Prop. (5.2)(7)), and $4\gamma''^3_{14} \circ \sigma_{14} = 4s^2_{21}(3) = 0$ (by Prop. (4.7)(6)). $\pi_{18}(Sp_4)$ is a cyclic group generated by γ''^4_{18}, and $\pi_{22}(Sp_5)$ is also a cyclic group by [6], so we can chose γ''^4_{18} and s^4_{22} so that (2) and (3) hold.

PROPOSITION (5.4): *The following relations hold.*

(1) $\omega''^4_{15} \circ \eta_{15} = 0$,	(2) $\omega''^4_{15} \circ \nu_{15} = 64\gamma''^4_{18}$,
(3) $\omega''^4_{15} \circ \sigma_{15} = \pm 8s^4_{22}$,	(4) $\gamma''^4_{18} \circ \eta_{18} = s^3_{19}(4)$.

PROOF: (1) $\omega''^4_{15} \circ \eta_{15} \in \pi_{16}(Sp_4) = 0$.

(2) $\omega''^4_{15} \in \{i''^{4,3}, 4\gamma''^3_{14}, 8\iota_{14}\} = \{i''^{4,3}, \omega''^2_7(3) \circ \sigma', 8\iota_{14}\}$ (by Lemma (5.1)(1) and Lemma (4.4)(1)). Hence $\omega''^4_{15} \circ \nu_{15} \in \{i''^{4,3}, \omega''^2_7(3) \circ \sigma', 8\iota_{14}\} \circ \nu_{15} = -i''^{4,3} \circ \{\omega''^2_7(3) \circ \sigma', 8\iota_{14}, \nu_{14}\}$, which consists of a single element $\pm\omega''^2_7(4) \circ \zeta_7$ (by (9.2) of 1), because $i''^{4,3}_* \pi_{15}(Sp_3) = 0$. Now $\pm\omega''^2_7(4) \circ \zeta_7 = 4s^2_{21}(4) = 64\gamma''^4_{18}$ (by Lemma (4.5)(3), Prop. (5.2)(2) and Lemma (5.3)(2)).

(3) Note that the secondary composition $\{s^2_{14}, 8\iota_{14}, \sigma_{14}\}$ contains the element $2s^2_{22}$ by Lemma (4.6), and that $\omega''^4_{15} \in \{i''^{4,3}, 2\gamma''^3_{14}, 16\iota_{14}\} = \{i''^{4,3}, s^2_{14}(3), 16\iota_{14}\}$. Hence $\omega''^4_{15} \circ \sigma_{15} \in \{i''^{4,3}, s^2_{14}(3), 16\iota_{14}\} \circ \sigma_{15} = -i''^{4,3} \circ \{s^2_{14}(3), 16\iota_{14}, \sigma_{14}\}$, which consists of a single element $-2s^2_{22}(4) = \pm 8s^4_{22}$ (by Lemma (5.3)(3)), because $i''^{4,3}_* \pi_{15}(Sp_3) = 0$.

(4) It follows from the bundle sequence $Sp_5/Sp_4 = S^{19}$ that $\Delta : \pi^{19}_{20} \to \pi_{19}(Sp_4)$ is an isomorphism. Since $\pi_{19}(Sp_4) = \{s^3_{19}(4)\}$, it follows that $s^3_{19}(4) = \Delta(\eta_{19}) = \gamma''^4_{18} \circ \eta_{18}$.

The following Lemma obviously holds.

LEMMA (5.5): (1) *There is an element ω''^5_{19} of the secondary composition $\{i''^{5,4}, \gamma''^4_{18}, 128\iota_{18}\}$ such that $p''_*\omega''^5_{19} = 128\iota_{19}$.*

(2) *There is an element γ''^5_{22} of the secondary composition $\{i''^{5,4}, \gamma''^4_{18}, 2\nu_{18}\}$ such that $p''\gamma''^5_{22} = 2\nu_{19}$ and $4\gamma''^5_{22} = \pm s^4_{22}(5)$.*

PROPOSITION (5.6): *The following relations hold.*

(1) $\omega''^{5}_{19}\circ\eta_{19}=\tau''^{1}_{3}(5)\circ\bar{\bar{\mu}}_3$, (2) $\omega''^{5}_{19}\circ\nu_{19}=\pm 128\gamma''^{5}_{22}$.

PROOF: Note that $\omega''^{3}_{11}\circ\mu_{11}=\tau''^{1}_{3}(3)\circ\bar{\mu}_3$ (by Prop. (5.2)(4)), and $\omega''^{3}_{11}\circ\zeta_{11}=\pm 4s^{2}_{22}(3)$ (by Prop. (5.2)(5)). Then each of the proofs of (1) and (2) is quite similar with that of Proposition (5.4)(2).

§ 6. $\pi_k(U_5)$ and $\pi_k(U_6)$ ($14\leqq k\leqq 22$)

First of all, I must correct some mistakes in the previous paper [3]. There is a misunderstanding about $\pi_{2n+7}(U_n)$ in (11.10) of [3], which should be corrected as follows:

$$(6.1) \qquad \pi_{2n+7}(U_n)=\begin{cases} Z_2+Z_{t(n)} & \text{for } n\equiv 2 \bmod 2,\ n\equiv 3 \bmod 8, \\ Z_{t(n)} & \text{for } n\equiv 1,\ 5,\ 7 \bmod 8. \end{cases}$$

where $t(n)$ is the integer defined as follows:

$$t(n)=\begin{cases} 16 & \text{for } n\equiv 0,\ 1 \bmod 8,\ n\equiv 10 \bmod 16,\ n\equiv 59 \bmod 64, \\ 8 & \text{for } n\equiv 4,\ 5 \bmod 8,\ n\equiv 2 \bmod 16,\ n\equiv 27 \bmod 64, \\ 4 & \text{for } n\equiv 6 \quad \bmod 8,\ n\equiv 11 \bmod 32, \\ 2 & \text{for } n\equiv 7 \quad \bmod 8,\ n\equiv 3 \bmod 16. \end{cases}$$

Hence, there are some changes about (11.16), (11.17) and (11.18), so that from Line 9 of the page 69 to Line 11 of the page 70 of [3] should be corrected as follows.

Let $n\equiv 2 \bmod 4$ ($n\geqq 6$), then $\gamma'^{n}_{2n}\circ\nu^{2}_{2n}=0$ (c.f. the proof of (11.5) of [3]). Consider a secondary composition $\{\gamma'^{n}_{2n},\nu^{2}_{2n},2\iota_{2n+6}\}$. $p'_*\{\gamma'^{n}_{2n},\nu^{2}_{2n},2\iota_{2n+6}\}\subset\{\eta_{2n-1},\nu^{n}_{2n},2\iota_{2n+6}\}$, which contains ε_{2n-1}. $i'^{n+1,n}\circ\{\gamma'^{2}_{2n},\nu^{2}_{2n},2\iota_{2n+6}\}=-\{i'^{n+1,n},\gamma'^{2}_{2n},\nu^{2}_{2n}\}\circ(2\iota_{2n+7})$, which contains $-2u^{n+1}_{2n+7}$. In this case, the bundle sequence of $U_{n+1}/U_n=S^{2n+1}$ becomes as follows:

$$\pi^{2n+1}_{2n+7} \xleftarrow{p'_*} \pi_{2n+1}(U_{n+1}) \xleftarrow{i'_*} \pi_{2n+7}(U_n) \xleftarrow{\varDelta} \pi^{2n+1}_{2n+8}$$

where $\pi^{2n+1}_{2n+7}=Z_2=\{\nu^{2}_{2n+1}\}$, $\pi_{2n+1}(U_{n+1})=Z_{d(n+2)}=\{u^{n+1}_{2n+1}\}$, $\pi_{2n+7}(U_n)=Z_2\oplus Z_{t(n)}$ and $\pi^{2n+1}_{2n+8}=Z_{16}=\{\sigma_{2n+1}\}$. If $n\equiv 2 \bmod 8$ ($n\geqq 10$), then $d(n+2)=4$ and $t(n)=8$ or 16, and if $n\equiv 6 \bmod 8$, then $d(n+2)=8$ and $t(n)=4$. Hence we conclude that

(6.2) *If* $n\equiv 2 \bmod 8$ ($n\geqq 10$), $\pi_{2n+7}(U_n)=Z_2\oplus Z_{t(n)}=\{u^{n}_{2n+7}\}\oplus\{\gamma'^{n}_{2n}\circ\sigma_{2n}\}$,

and if $n\equiv 6 \bmod 8$ ($n\geqq 6$), $\pi_{2n+7}(U_n)=Z_2\oplus Z_4=\{\gamma'^{n}_{2n}\circ\sigma_{2n}\}\oplus\{u^{n}_{2n+7}\}$,

where $u^{n}_{2n+7}\in\{\gamma'^{n}_{2n},\nu^{2}_{2n},2\iota_{2n+6}\}$ *such that* $p'_*u^{n}_{2n+6}=\varepsilon_{2n-1}$ *and* $u^{n}_{2n+6}(n+1)=2u^{n+1}_{2n+7}$.

Next, considering the bundle sequence of U_n/U_{n-1}, we have the following (c.f. (11.7) of [3]).

(6.3) *For values of* $n \geqq 7^{(*)}$,

$$\pi_{2n+6}(U_n)=\begin{cases} {}^{(2)}Z_{(n+3)!\times d(n+1)/4}\oplus Z_2=\{u^n_{2n+6}\}\oplus\{\gamma'^n_{2n}\circ\nu^2_{2n}\} & \text{for } n\equiv 0 \bmod 4,\ n\not\equiv 60 \bmod 64, \\ {}^{(2)}Z_{(n+3)!\times d(n+1)/4}=\{u^n_{2n+6}\} & \text{for } n\equiv 6 \bmod 8, \\ {}^{(2)}Z_{(n+3)!\times d(n+1)/16}\oplus Z_2=\{u^n_{2n+6}\}\oplus\{\gamma'^n_{2n}\circ\nu^2_{2n}\} & \text{for } n\equiv 5 \bmod 8, \\ {}^{(2)}Z_{(n+3)!\times d(n+1)/16}=\{u^n_{2n+6}\} & \text{for } n\equiv 3 \bmod 4,\ n\not\equiv 11 \bmod 8, \end{cases}$$

where $u^n_{2n+6}\in\{i'^{n,n-1}, \gamma'^{n-1}_{2n-2}, t(n-1)\sigma_{2n-2}\}$ *except for* $n\equiv 7 \bmod 8$,

and $u^n_{2n+6}\in\{i'^{n,n-1}, \gamma'^{n-1}_{2n-2}, 2\sigma_{2n-2}\}$ *for* $n\equiv 7 \bmod 8$.

Note that for $n=5$, $\omega'^5_9\circ\sigma_9\in\pi_{16}(U_5)$ and $p'_*\omega'^5_9\circ\sigma_9=8\sigma_9$.

Now let us define original generators of the 2-primary components of $\pi_{19}(U_5)$ and $\pi_{20}(U_5)$.

LEMMA (6.4): (1) *There is an element* u^5_{19} *of the secondary composition* $\{i'^{5,4}, \gamma'^4_8, 4\sigma_8\circ\nu_{15}\}$ *such that* $p'_*u^5_{19}=4\sigma_9\circ\nu_{16}$ *and* $2u^5_{19}=0$.

(2) *There is an element* u^5_{20} *of the secondary composition* $\{u^5_{12}, 2\sigma_{12}, 8\iota_{19}\}_3$ *such that* $p'_*u^5_{20}=4\zeta_9$ *and* $2u^5_{20}=\pm u^3_{20}(5)$.

PROOF: (1) Note that $\pi_{19}(U_4)=Z_2\oplus Z_2=\{u^3_{12}(4)\circ\sigma_{12}\}\oplus\{u^3_{16}(4)\circ\nu_{16}\}$. it follows from Proposition (4.8)(14) and (15) that the boundary homomorphism $\Delta:\pi^9_{20}\to\pi_{19}(U_4)$ of the bundle sequence of $U_5/U_4=S^9$ is an epimorphism, that is, $i'_*:\pi_{19}(U_4)\to\pi_{19}(U_5)$ is a zero-homomorphism. Hence $2\{i'^{5,4}, \gamma'^4_8, 4\sigma_8\circ\nu_{15}\}=\{i'^{5,4}, \gamma'^4_8, 4\sigma_8\circ\nu_{15}\}=-i'^{5,4}\{\gamma'^4_8, 4\sigma_8\circ\nu_{15}, 2\iota_{18}\}\subset i'^{5,4}_*\pi_{19}(U_4)=0$, so that $2u^5_{19}=0$. It is evident that $p'_*\{i'^{5,4}, \gamma'^4_8, 4\sigma_8\circ\nu_{15}\}=4\sigma_9\circ\nu_{16}$.

(2) $2u^5_{12}\circ\sigma_{12}=u^3_{12}(5)\circ\sigma_{12}$ (by (3.2)(10)) $\in i'^{5,4}_*\pi_{19}(U_4)=0$. $p'_*\{u^5_{12}, 2\sigma_{12}, 8\iota_{19}\}_3\subset\{4\nu_9, 2\sigma_{12}, 8\iota_{19}\}_3$ (by (3.2)(10)) $=\eta^2_9\circ\{\eta_{11}, 2\sigma_{12}, 8\iota_{19}\}_3$, which consists of a single element $\eta^2_9\circ\mu_{11}$ (by (1.2)(4)) $=4\zeta_9$. $2u^5_{20}\in\{2u^5_{12}, 2\sigma_{12}, 8\iota_{19}\}_3\subset\{u^3_{12}(5), 4\sigma_{12}, 4\iota_{19}\}_3$, which contains $u^3_{20}(5)$ (by Lemma (3.6)(3)), and is a coset of the subgroup $u^3_{12}(5)\circ\pi^{12}_{20}+4\pi_{20}(U_5)=\{2u^3_{20}(5)\}=\{\tau'^2_3(5)\circ\bar{\mu}_3\}$, Hence we have $2u^5_{20}=\pm u^3_{20}(5)$.

LEMMA (6.5): $2\omega'^5_9\circ\sigma_9=-u^4_{16}(5)$.

PROOF: $2\omega'^5_9\circ\sigma_9\in\{i'^{5,4}, \gamma'^4_8, 8\iota_8\}\circ E^2\sigma'=-i'^{5,4}\circ\{\gamma'^4_8, 8\iota_8, E\sigma'\}$, which consists of a single element $-u^4_{16}(5)$ (by Lemma (4.5)(2)), because $i'^{5,4}_*\pi_{19}(U_4)\circ 2\sigma_9=0$. Hence we have $2\omega'^5_9\circ\sigma_9=-u^4_{16}(5)$.

Now, (3.4)(6) and Proposition (4.8)(9)-(16) determine the boundary homomorphism $\Delta:\pi^9_{k+1}\to\pi_k(U_4)$ of the bundle sequence of $U_5/U_4=S^9$ for every $14\leqq k\leqq 22$. Lemma (6.4) and Lemma (6.5) together with the results above completely determine (2-primary components of) $\pi_k(U_5)$ for $14\leqq k\leqq 22$, which are listed as follows:

THEOREM (6.6):

$\pi_{14}(U_5)=Z_{16}=\{u^4_{14}(5)\}$,

(*) For $n=6$, see the footnote of the next page.

$\pi_{15}(U_5)=Z_2=\{u^4_{14}(5)\circ\eta_{14}\}$,
$\pi_{16}(U_5)=Z_2\oplus Z_{16}=\{u^3_{16}(5)\}\oplus\{\omega'^5_9\circ\sigma_9\}$,
$\pi_{17}(U_5)=Z_4{}^{(*)}=\{u^4_{14}(5)\circ\nu_{14}\}$,
$\pi_{18}(U_5)=Z_8=\{i'^{5,4}\circ l^4\circ s^2_{18}\}$,
$\pi_{19}(U_5)=Z_2=\{u^5_{19}\}$,
$\pi_{20}(U_5)=Z_2\oplus Z_8=\{u^4_{14}(5)\circ\nu^2_{14}\}\oplus\{u^5_{20}\}$,
$\pi_{21}(U_5)=Z_{16}=\{u^4_{14}(5)\circ\sigma_{14}\}$,
$\pi_{22}(U_5)=Z_{16}\oplus Z_2=\{u^4_{22}(5)\}\oplus\{u^4_{14}(5)\circ\varepsilon_{14}\}$.

PROPOSITION (6.7): *The following relations hold.*

(1) $\omega'^5_9\circ\sigma_9\circ\eta_{16}=\omega'^5_9\circ\varepsilon_9=\omega'^5_9\circ\bar{\nu}_9=0$.
(2) $\omega'^5_9\circ\mu_9=4i'^{5,4}\circ l^4\circ s^2_{18}$.
(3) $\omega'^5_9\circ\sigma_9\circ\nu_{16}=0$.
(4) $\omega'^5_9\circ\zeta_9=-2u^5_{20}$.
(5) $u^5_{12}\circ\nu_{12}=0$.
(6) $u^5_{12}\circ\sigma_{12}=0$.
(7) $u^5_{12}\circ\varepsilon_{12}=u^5_{12}\circ\bar{\nu}_{12}=0$.
(8) $u^5_{12}\circ\mu_{12}=8u^4_{14}(5)\circ\sigma_{14}$.
(9) $u^5_{19}\circ\eta_{19}=0$.
(10) $u^5_{19}\circ\nu_{19}=0$.
(11) $u^5_{20}\circ\eta_{20}=8u^4_{14}(5)\circ\sigma_{14}$.

PROOF: (1) Let $E\alpha$ be any element of $\pi^9_{17}=E\pi^8_{16}$, so that $2\alpha=0$. Note that $\omega'^5_9\in\{i'^{5,4},2\gamma'^4_8,4\iota_8\}=\{i'^{5,4},\omega'^3_5(4)\circ\nu_5,4\iota_8\}$ (by (3.2)(8)), and that $i'^{5,4}_*\pi_9(U_4)=0$. Hence $\omega'^5_9\circ E\alpha=\{i'^{5,4},\omega'^3_5(4)\circ\nu_5,4\iota_8\}\circ E\alpha=i'^{5,3}\circ\{\omega'^3_5\circ\nu_5,4\iota_8,\alpha\}\subset i'^{5,3}_*\pi_{17}(U_3)=0$, so that $\omega'^5_9\circ E\alpha=0$.

(2) It follows from (1.2)(4) that $\omega'^5_9\circ\mu_9\in\omega'^5_9\circ\{\eta_9,2\iota_{10},8\sigma_{10}\}_3\subset\{\omega'^5_9\circ\eta_9,2\iota_{10},8\sigma_{10}\}_3=\{4i'^{5,4}\circ l^4\circ\gamma''^2_{10},8\iota_{10},2\sigma_{10}\}_3$ (by (3.4)(7)), which consists of a single element $4i'^{5,4}\circ l^4\circ s^2_{18}$ (by Lemma (4.5)(3)), because $4\pi^{10}_{18}=0$ and $\pi_{11}(U_5)=0$.

(3) $p'_*(\omega'^5_9\circ\sigma_9\circ\nu_{16})=8\sigma_9\circ\nu_{16}$ (by (3.2)(9)) $=0$. $p'^{-1}_*(0)=i'_*\pi_{19}(U_4)=0$. Hence we have $\omega'^5_9\circ\sigma_9\circ\nu_{16}=0$.

(4) $\omega'^5_9\circ\zeta_9\in\{\omega'^5_9\circ\nu_9,8\iota_{12},2\sigma_{12}\}_3$ (by (1.2)(6)) $=\{-2u^5_{12},8\iota_{12},2\sigma_{12}\}_3$ (by (3.4)(7)), which consists of a single element $-2u^5_{20}$ (by Lemma (6.4)(2)), because $2\pi^{12}_{19}=0$, and $2\pi_{13}(U_5)\circ\sigma_{13}\subset\{\tau'^2_3(5)\circ\varepsilon'\circ\sigma_{13}\}=\{\tau'^2_3(5)\circ 2\bar{\varepsilon}'\}$ (by Prop. (2.15)(6)) $=0$.

(5) Note that $u^5_{12}\in\{i'^{5,4},2\gamma'^4_8,2\nu_8\}=\{i'^{5,4},\omega'^3_5(4)\circ\nu_5,2\nu_8\}$ (by (3.2)(10) and (3.2)(8)). Hence $u^5_{12}\circ\nu_{12}\in\{i'^{5,4},\omega'^3_5(4)\circ\nu_5,2\nu_8\}\circ\nu_{12}=-i'^{5,4}\circ\{\omega'^3_5(4)\circ\nu_5,2\nu_8,\nu_{11}\}$, which consists of a single element, because $i'^{5,4}_*\pi_{12}(U_4)\circ\nu_{12}=\{i'^{5,4}\circ u^3_{12}(4)\circ\nu_{12}\}\subset i'^{5,3}_*\pi_{15}(U_3)=0$. Hence, $-i'^{5,4}\circ\{\omega'^3_5(4)\circ\nu_5,2\nu_8,\nu_{11}\}=-i'^{5,3}\{\omega'^3_5\circ\nu_5,2\nu_8,\nu_{11}\}\subset i'^{5,3}_*\circ\pi_{15}(U_3)=0$, that is, $u^5_{12}\circ\nu_{12}=0$.

(6) The proof is quite similar with that of (3).

(7) In the same way as in (5), we can prove that $u^5_{12}\circ\varepsilon_{12}$ and $u^5_{12}\circ\bar{\nu}_{12}$ belong to $\omega'^3_5(5)\circ\pi^5_{20}=\{\omega'^3_5(5)\circ\rho^{IV}\}\oplus\{\omega'^3_5(5)\circ\bar{\varepsilon}_5\}=0$ (by Prop. (3.7)(6) and Prop. (3.7)(7)).

(*) This result does not agree with (6.1). It seemed to me that (6.1) is valid for $n\geqq 6$, so that $u^6_{18}\in\{i'^{6,5},\gamma'^5_{10},4\sigma_{10}\}$ (by the exact sequence of the bundle $U_6/U_5=S^{11}$).

(8) Note that $u^5_{12} \in \{i'^{5,4}, 2\gamma'^4_8 \circ \nu_8, 2\iota_{11}\} = \{i'^{5,4}, \tau'^2_3(4) \circ \varepsilon_3, 2\iota_{11}\}$ (by (3.2)(8) and (3.4)(1)). Hence $u^5_{12} \circ \mu_2 \in \{i'^{5,4}, \tau'^2_3(4) \circ \varepsilon_3, 2\iota_{11}\} \circ \mu_{12} = i'^{5,4} \circ \{\tau'^2_3(4) \circ \varepsilon_3, 2\iota_{11}, \mu_{11}\}$, which consists of a single element, because $i'^{5,4} \circ \pi_{12}(U_4) \circ \mu_{12} = \{u^8_{12}(5) \circ \mu_{12}\} = 0$ (by Prop. (3.7)(19)). Now, $i'^{5,4} \circ \{\tau'^2_3(4) \circ \varepsilon_3, 2\iota_{11}, \mu_{11}\} = \tau'^2_3(5) \circ \{\varepsilon_3, 2\iota_{11}, \mu_{11}\}$. $2\{\varepsilon_3, 2\iota_{11}, \mu_{11}\} = \varepsilon_3 \circ \{2\iota_{11}, \mu_{11}, 2\iota_{20}\} = \varepsilon_3 \circ \mu_{11} \circ \eta_{20} = \eta_3 \circ \mu_4 \circ \sigma_{13} \circ \eta_{20}$ (by Prop. (2.13)(7)) $= \eta^2_3 \circ \mu_5 \circ \sigma_{14} = 2\mu' \circ \sigma_{14}$ (by (1.1)(8)), so that $\{\varepsilon_3, 2\iota_{11}, \mu_{11}\}$ contains $\mu' \circ \sigma_{14} + \alpha$ for some $\alpha \in \pi^3_{21}$ such that $2\alpha = 0$. However, we can see that $\tau'^2_3(5) \circ \alpha = 0$ for such α. Hence we conclude that $u^5_{12} \circ \mu_{12} = \tau'^2_3(5) \circ \mu' \circ \sigma_{14} = 8u^4_{14} \circ \sigma_{14}$ (by Lemma (4.4)(3)).

(9) The proof is quite similar with that of (7).

To prove (10), we need the following.

LEMMA (6.8): *The secondary composition* $\{u^4_{14}(5), 4\nu_{14}, \eta_{17}\}_1$ *consists of a single element* u^5_{19}.

PROOF: $\pi_{18}(U_5) \circ \eta_{18} \subset i'^{5,4}_* \pi_{19}(U_4)$ (by Th. 6.6) $= 0$. $u^4_{14}(5) \circ \pi^{14}_{19} = 0$. Hence the secondary composition consists of a single element. Now it follows from Proposition (4.8)(11) and (12) that $\Delta(\sigma_9 \circ \eta^2_{16} + \varepsilon_9 \circ \eta_{17} + x\nu^3_9) = 4u^4_{14} \circ \nu_{14}$ for $x = 0$ or 1. Hence $p'_* \{u^4_{14}(5), 4\nu_{14}, \eta_{17}\}_1 = \Delta^{-1}(4u^4_{14} \circ \nu_{14}) \circ \eta_{18} = \sigma_9 \circ \eta^3_{16} = 4\sigma_9 \circ \nu_{16}$, because $\varepsilon_{19} \circ \eta^2_{17} = \nu^3_9 \circ \eta_{18} = 0$. Since $p'_* u^5_{19} = 4\sigma_9 \circ \nu_{16}$, and since $p'^{-1}_*(0) = i'^{5,4}_* \pi_{19}(U_4) = 0$ (c.f. the proof of Lemma (6.4)(1)), it follows that the secondary composition $\{u^4_{14}(5), 4\nu_{14}, \eta_{17}\}_1$ contains u^5_{19}.

Proof of (10). $u^5_{19} \circ \nu_{19} = \{u^4_{14}(5), 4\nu_{14}, \eta_{17}\} \circ \nu_{19} = -u^4_{14}(5) \circ \{4\nu_{14}, \eta_{17}, \nu_{18}\} = 4u^4_{14}(5) \circ \bar{\nu}_{14}$ (by (1.2)(3)) $= 0$.

(11) Note that $u^5_{12} \circ \pi^{12}_{20} = 0$. Hence $u^5_{20} \circ \eta_{20} = \{u^5_{12}, 2\sigma_{12}, 8\iota_{19}\}_3 \circ \eta_{20}$ (by Lemma (6.4)(2)) $= u^5_{12} \circ \{2\sigma_{12}, 8\iota_{19}, \eta_{19}\} = u^5_{12} \circ \mu_{12} = 8u^4_{14}(5) \circ \sigma_{14}$ (by (8)).

Now, let us calculate (2-primary components) of $\pi_k(U_6)$.
In the exact sequence

$$\pi^{11}_k \xleftarrow{p'_*} \pi_k(U_6) \xleftarrow{i'_*} \pi_k(U_5) \xleftarrow{\Delta} \pi^{11}_{k+1}$$

$\Delta(\iota_{11}) = xi'^{5,4} \circ l^4 \circ \gamma''^2_{10}$ for some odd integer x, which we denote by γ'^5_{10} for abbreviation. Now the boundary homomorphism Δ is determined by (3.3)(2), Proposition (4.7)(4)-(6) and Proposition (4.8)(19), (20) for each $14 \leqq k \leqq 21$. We need:

PROPOSITION (6.9):

(1) $u^4_{14}(6) = xl^6 \circ \gamma''^3_{14} + \gamma'^6_{12} \circ \eta^2_{12}$ *for some odd integer* x.

(2) $\Delta(\iota') = u^4_{14}(5) \circ \varepsilon_{14}$.

PROOF: (1) The bundle sequence above becomes as follows:

$$0 \longleftarrow Z_2 \xleftarrow{p'_*} \pi_{14}(U_6) \xleftarrow{i'_*} \pi_{14}(U_5) \xleftarrow{\Delta} \pi^{11}_{15}$$

where $Z_2 = \{4\nu_{11}\} \subset \pi^{11}_{14}$, $\pi^{11}_{15} = 0$, $\pi_{14}(U_5) = Z_{16} = \{u^4_{14}(5)\}$. It follows from (11.8) of [3] that $\pi_{14}(U_6) = Z_{16} \oplus Z_2 = \{l^6 \circ \gamma''^3_{14}\} \oplus \{\gamma'^6_{12} \circ \eta^2_{12}\}$. Since $p'_*(l^6 \circ \gamma''^3_{14}) = p'_*(\gamma'^6_{12} \circ \eta^2_{12}) = 4\nu_{11}$, we have $l^6\circ\gamma''^3_{14}+\gamma'^6_{12}\circ\eta^2_{12}\in i'_*\pi_{14}(U_5)$, that is $u^4_{14}(6)=xl^6\circ\gamma''^3_{14}+\gamma'^6_{12}\circ\eta^2_{12}$ for some odd integer x.

(2) Since $\gamma''^3_{14}\circ\varepsilon_{14}=0$ (by Prop. (5.2)(9)) and $\eta^2_{12}\circ\varepsilon_{14}=0$, we have $u^4_{14}(6)\circ\varepsilon_{14}=0$. Hence $\Delta(\theta')=u^4_{14}(5)\circ\varepsilon_{14}$, because $\pi^{11}_{23}=Z_2=\{\theta'\}$.

LEMMA (6.10): (1) *There is an element* u^6_{18} *of the secondary composition* $\{i'^{6,5}, \gamma'^5_{10}, 4\sigma_{10}\}$ *such that* $p'_*u^6_{18}=4\sigma_{11}$, $2u^6_{18}=\pm\omega'^6_{11}\circ\sigma_{11}$ *and* $4u^6_{18}=\pm l^6\circ s^2_{18}(3)$.

(2) *There is an element* u^6_{19} *of the secondary composition* $\{\gamma'^6_{12}, \nu^2_{12}, 2\iota_{18}\}_1$ *such that* $p'_*u^6_{19}=\varepsilon_{11}$ *and* $2u^6_{19}=u^5_{19}(6)$.

(3) *There is an element* u^6_{20} *of the secondary composition* $\{\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}_3$ *such that* $p'_*u^6_{20}=\mu_{11}$ *and* $2u^6_{20}\equiv u^5_{20}(6) \bmod \tau'^2_3(6)\circ\bar{\mu}_3$.

PROOF: (1) and (2) follow from (6.2) and (6.3).

(3) It follows from (6.2) that $2\gamma'^6_{12}\circ\sigma_{12}=0$. Now, $p'_*\{\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}_3\subset\{\eta_{11}, 2\sigma_{12}, 8\iota_{19}\}_3$, which contains μ_{11} (by (1.2)(4)). $2u^6_{20}\in\{2\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}_3=\{u^5_{12}(6), 2\sigma_{12}, 8\iota_{19}\}_3$ (by (3.2)(12)), which contains $u^5_{20}(6)$ (by Lemma (6.4)(2)), and is a coset of the subgroup $8\pi_{20}(U_6)=\{\tau'^2_3(6)\circ\bar{\mu}_3\}$, because $u^5_{12}\circ\pi^{12}_{20}=0$ (by Prop. (5.7)(7)). Hence we can chose an element u^6_{20} from the coset $\{\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}_3$ such that $p'_*u^6_{20}=\mu_{11}$ and $2u^6_{20}\equiv u^5_{20}(6)$ mod $\tau''^2_3(6)\circ\bar{\mu}_3$.

Thus, Proposition (6.9), Lemma (6.10), (11.8) and (11.10) of [3], (6.2) and (6.3) completely determine (2-primary components of) $\pi_k(U_6)$ for $14\leqq k\leqq 22$ and their generators as follows.

THEOREM (6.11):

$\pi_{14}(U_6)=Z_{16}\oplus Z_2=\{l^6\circ\gamma''^3_{14}\}\oplus\{\gamma'^6_{12}\circ\eta^2_{12}\}$,

$\pi_{15}(U_6)=Z_2=\{\gamma'^6_{12}\circ\nu_{12}\}$,

$\pi_{16}(U_6)=Z_{16}=\{\omega'^5_9(6)\circ\sigma_9\}$,

$\pi_{17}(U_6)=0$,

$\pi_{18}(U_6)=Z_{32}=\{u^6_{18}\}$,

$\pi_{19}(U_6)=Z_4\oplus Z_2=\{u^6_{19}\}\oplus\{\gamma'^6_{12}\circ\sigma_{12}\}$,

$\pi_{20}(U_6)=Z_{16}\oplus Z_2\oplus Z_2=\{u^6_{20}\}\oplus\{\gamma'^6_{12}\circ\varepsilon_{12}\}\oplus\{\gamma'^6_{12}\circ\bar{\nu}_{12}\}$,

$\pi_{21}(U_6)=Z_4\oplus Z_2=\{u^4_{14}(6)\circ\sigma_{14}\}\oplus\{\gamma'_{12}\circ\mu_{12}\}$,

$\pi_{22}(U_6)=Z_{16}\oplus Z_2=\{u^4_{22}(6)\}\oplus\{\gamma'_{12}\circ\eta_{12}\circ\mu_{13}\}$.

While we are doing these calculations, we obtain the following relations.

PROPOSITION (6.12):

(1) $u^4_{14}(6)\circ\eta_{14}=\gamma'^6_{12}\circ\nu_{12}$. (2) $\gamma'^5_{10}\circ\nu^2_{10}=u^3_{16}(5)$ (c.f. Prop. (4.8)(19)).

Note that $l^6\circ\omega''^3_{11}=\omega'^6_{11}$ (by (3.5)). Hence Proposition (5.2)(1), (3), (4) and (5) together with Proposition (6.9) and (6.10) show the following relations:

PROPOSITION (6.13):

(1) $\omega'^6_{11}\circ\nu_{11}=\pm 4l^6\circ\gamma''^3_{14}=\pm 4u^4_{14}(6)$. (2) $\omega'^6_{11}\circ\varepsilon_{11}=\omega'^6_{11}\circ\bar{\nu}_{11}=0$.

(3) $\omega'^6_{11}\circ\mu_{11}=8u^6_{20}$. (4) $\omega'^6_{11}\circ\zeta_{11}=4u^4_{22}(6)$.

Now we shall add the followings.

PROPOSITION (6.14):

(1) $l^6\circ s^3_{19}\equiv\gamma'^6_{12}\circ\sigma_{12} \bmod 2u^6_{19}$. (2) $u^6_{18}\circ\eta_{18}=u^5_{19}(6)$.

(3) $u^6_{18}\circ\nu_{18}\equiv 0 \bmod 2u^4_{14}(6)\circ\sigma_{14}$. (4) $u^6_{19}\circ\eta_{19}=\gamma'^6_{12}\circ\varepsilon_{12}$.

(5) $u^6_{19}\circ\nu_{19}=0$. (6) $\gamma'^6_{12}\circ\eta_{12}\circ\varepsilon_{13}=2u^4_{14}(6)\circ\sigma_{14}$.

(7) $u^6_{20}\circ\eta_{20}\equiv\gamma'^6_{12}\circ\mu_{12} \bmod 2u^4_{14}(6)\circ\sigma_{14}$.

PROOF: (1) $p'_*(l^6\circ s^3_{19})=p''_*s^3_{19}=\eta_{11}\circ\sigma_{12}$ (by Lemma (5.1)(2)). $p'_*(\gamma'^6_{12}\circ\sigma_{12})=\eta_{11}\circ\sigma_{12}$. $p'^{-1}_*(0)=i'_*\pi_{19}(U_5)=\{u^5_{19}(6)\}=\{2u^6_{19}\}$. Hence we have (1).

(2) Note that $\gamma'^5_{10}\circ(2\sigma_{10})=2u^4_{14}(5)\circ\nu_{14}$ (by Prop. (4.8)(20)), so that $u^6_{18}\in\{i'^{6,5}, 2u^4_{14}(5)\circ\nu_{14}, 2\iota_{17}\}$. Hence $u^6_{18}\eta_{18}\in\{i'^{6,5}, 2u^4_{14}(5)\circ\nu_{14}, 2\iota_{17}\}\circ\eta_{18}=i'^{6,5}\circ\{2u^4_{14}(5)\circ\nu_{14}, 2\iota_{17}, \eta_{17}\}$, which consists of a single element, because $i'^{6,5}_*\pi_{18}(U_5)\circ\eta_{18}=\{4u^6_{18}\circ\eta_{18}\}=0$. Now $i'^{6,5}\circ\{2u^4_{14}(5)\circ\nu_{14}, 2\iota_{17}, \eta_{17}\}=i'^{6,5}\circ\{u^4_{14}(5), 4\nu_{14}, \eta_{17}\}=u^5_{19}(6)$ (by Lemma (6.8)).

(3) $p'_*(u^6_{18}\circ\nu_{18})=4\sigma_{11}\circ\nu_{18}=0$. $2u^6_{18}\circ\nu_{18}=\pm\omega'^6_{11}\circ\sigma_{11}\circ\nu_{18}=\pm\omega'^6_{11}\circ[\iota_{11}, \iota_{11}]$ (by (1.3)) $=0$. The element of order 2 which belongs to $p'^{-1}_*(0)=i'_*\pi_{21}(U_5)$ is $2u^4_{14}(6)\circ\sigma_{14}$. Hence we have $u^6_{18}\circ\nu_{18}\equiv 0 \bmod 2u^4_{14}(6)\circ\sigma_{14}$.

(4) $u^6_{19}\circ\eta_{19}\in\{\gamma'^6_{12}, \nu^2_{12}, 2\iota_{18}\}_1\circ\eta_{19}=\gamma'^6_{12}\circ\{\nu^2_{12}, 2\iota_{18}, \eta_{18}\}$, which contains $\gamma'^6_{12}\circ\varepsilon_{12}$ (by (1.2)(3)), and is a coset of the subgroup $\gamma'^6_{12}\circ\pi^{12}_{19}\circ\eta_{19}=\{\gamma'^6_{12}\circ\sigma_{12}\circ\eta_{19}\}=\{\gamma'^6_{12}\circ\varepsilon_{12}+\gamma'^6_{12}\circ\bar{\nu}_{12}\}$. Hence $u^6_{19}\circ\eta_{19}=\gamma'^6_{12}\circ\varepsilon_{12}$ or $\gamma'^6_{12}\circ\bar{\nu}_{12}$. However $p'_*(u^6_{19}\circ\eta_{19})=\varepsilon_{11}\circ\eta_{19}$ (by Lemma (6.10)(2)), which implies that $u^6_{19}\circ\eta_{19}=\gamma'^6_{12}\circ\varepsilon_{12}$.

(5) $u^6_{19}\circ\nu_{19}\in\{\gamma'^6_{12}, \nu^2_{12}, 2\iota_{18}\}\circ\nu_{19}\subset\{\gamma'^6_{12}, \nu^2_{12}, 2\nu_{18}\}$. Note that $u^4_{14}(6)\circ\eta_{14}=\gamma'^6_{12}\circ\nu_{12}$ (by Prop. (6.12)(1)). Hence $0=u^4_{14}(6)\circ\varepsilon_{14}$ (by Prop. (6.9)(2)) $\in\{u^4_{14}(6)\circ\eta_{14}, \nu_{15}, 2\nu_{18}\}$ (by (1.2)(3)) $=\{\gamma'^6_{12}\circ\nu_{12}, \nu_{15}, 2\nu_{18}\}\subset\{\gamma'^6_{12}, \nu^2_{12}, 2\nu_{18}\}$, which is (a coset of) the subgroup $\gamma'^6_{12}\circ\pi^{12}_{22}+\pi_{16}(U_6)\circ\nu^2_{16}=\{\gamma'^6_{12}\circ\eta_{12}\circ\mu_{13}\}+\{\omega'^5_9\circ\sigma_9\circ\nu^2_{16}\}=\{\gamma'^6_{12}\circ\eta_{12}\circ\mu_{13}\}$ (by Prop. (5.7)(3)). $p'_*(u^6_{19}\circ\nu_{19})=\varepsilon_{11}\circ\nu_{19}=0$. $p'_*(\gamma'^6_{12}\circ\eta_{12}\circ\mu_{13})=\eta^2_{11}\circ\mu_{13}=4\zeta_{11}\neq 0$. Hence we conclude that $u^6_{19}\circ\nu_{19}=0$.

(6) Consider a secondary composition $\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}$. $2\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}=\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}\circ 2\iota_{21}=-\gamma''^2_{10}\circ\{\eta^2_{10}, \varepsilon_{12}, 2\iota_{20}\}$, which contains $\pm\gamma''^2_{10}\circ\zeta_{10}$ (by Lemma 9.1 of [1]) $\equiv -4xs^2_{21}$ for some odd integer x. Hence $\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}$ contains $2xs^2_{21}+y\tau''^1_3(2)\circ\eta_3\circ\bar{\mu}_4$ for some odd integers x and y. Note that $\tau'^2_3(4)\circ\eta_3\circ\bar{\mu}_4=0$, and that $2i'^{6,4}\circ l^4\circ s^2_{21}=2u^4_{14}(6)\circ\sigma_{14}$, which is an element of order 2. Hence the secondary composition $i'^{6,4}\circ l^4\circ\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}$ consists of a single element $2u^4_{14}(6)\circ\sigma_{14}$, because $i'^{6,4}_*\circ l^4\circ\pi_{13}(Sp_2)=\{i'^{6,4}\circ l^4\circ\gamma''^2_{10}\circ\nu_{10}\}=0$. Now $\gamma'^6_{12}\circ\eta_{12}\circ\varepsilon_{13}=\{i'^{6,5}, \gamma'^5_{10}, \eta_{10}\}\circ\eta_{12}\circ\varepsilon_{13}=i'^{6,5}\circ\{\gamma'^5_{10}, \eta_{10}, \eta_{11}\circ\varepsilon_{12}\}=i'^{6,5}\circ\{\gamma'^5_{10}, \eta^2_{10}, \varepsilon_{12}\}=i'^{6,4}\circ l^4\circ\{\gamma''^2_{10}, \eta^2_{10}, \varepsilon_{12}\}=2u^4_{14}(6)\circ\sigma_{14}$, because $i'^{6,5}\circ\pi_{13}(U_5)\circ\varepsilon_{13}=0$.

(7) $p'_*(u^6_{20}\circ\eta_{20})=\mu_{11}\circ\eta_{20}$ (by Lemma (6.10)(3)). $p'^{-1}_*(0)=i'_*\pi_{21}(U_5)=\{u^4_{14}(6)\circ\sigma_{14}\}$. Hence $u^6_{20}\circ\eta_{20}\equiv\gamma'^6_{12}\circ\mu_{12} \bmod 2u^4_{14}(6)\circ\sigma_{14}$.

§7. $\pi_k(U_n)$ $(14\leqq k\leqq 22,\ n\geqq 7)$

The bundle sequence of $U_7/U_6=S^{13}$ is automatically determined by virtue of (11.10), (11.12) and (11.15) of [3], and of (6.1), (6.3) and Proposition (6.14)(6). We obtain the followings.

LEMMA (7.1): *There are original generators as follows*:

(1) $u^7_{16}\in\{i'^{7,6}, \gamma'^6_{12}, 2\nu_{12}\}$ *such that* $p'_* u^7_{16}=2\nu_{13}$, *and* $4u^7_{16}=\pm\omega'^5_9(7)\circ\sigma_9$.

(2) $u^7_{19}\in\{i'^{7,6}, \gamma'^6_{12}, \nu^2_{12}\}$ *such that* $p'_* u^7_{19}=\nu^2_{13}$ *and* $2u^7_{19}=u^6_{19}(7)$.

(3) $u^7_{20}\in\{i'^{7,6}, \gamma'^6_{12}, 2\sigma_{12}\}$ *such that* $p'_* u^7_{20}=2\sigma_{13}$ *and* $8u^7_{20}\equiv -u^6_{20}(7) \bmod \tau'^2_3(7)\circ\bar{\mu}_3$.

We must prove the last assertion of (3). Indeed, $8u^7_{20}\in\{i'^{7,6}, \gamma'^6_{12}, 2\sigma_{12}\}\circ 8\iota_{20}= -i'^{7,6}\circ\{\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}$, which contains $-u^6_{20}(7)$ (by Lemma (6.10)(3)), and is a coset of the subgroup $i'^{7,6}_*(8\pi_{20}(U_6))=\{\tau'^2_3(7)\circ\bar{\mu}_3\}$.

THEOREM (7.2): *(2-primary components of)* $\pi_k(U_7)$ $(14\leqq k\leqq 22)$ *and their generators are listed below*:

$\pi_{14}(U_7)=Z_{16}=\{\gamma'^7_{14}\}$ (*where* $\gamma'^7_{14}=i'^{7,6}\circ l^6\circ\gamma''^3_{14}$),
$\pi_{15}(U_7)=0$,
$\pi_{16}(U_7)=Z_{64}=\{u^7_{16}\}$,
$\pi_{17}(U_7)=0$,
$\pi_{18}(U_7)=Z_{32}=\{u^6_{18}(7)\}$,
$\pi_{19}(U_7)=Z_8=\{u^7_{19}\}$,
$\pi_{20}(U_7)=Z_{128}=\{u^7_{20}\}$,
$\pi_{21}(U_7)=Z_2=\{u^4_{14}(7)\circ\sigma_{14}\}$,
$\pi_{22}(U_7)=Z_{16}\oplus Z_2=\{u^4_{22}(7)\}\oplus\{u^7_{19}\circ\nu_{19}\}$.

Note that

(7.3) $u^4_{14}(7)=x\gamma'^7_{14}$ *for some odd integer* x. (c.f. Prop. (6.9)(1))

PROPOSITION (7.4): *The following relations hold*:

(1) $\omega'^7_{13}\circ\eta_{13}=8\gamma'^7_{14}$.
(2) $\omega'^7_{13}\circ\nu_{13}=\pm 16u^7_{16}$.
(3) $\omega'^7_{13}\circ\sigma_{13}\equiv 8u^7_{20} \bmod 16u^7_{20}$.
(4) $\omega'^7_{13}\circ\varepsilon_{13}=\omega'^7_{13}\circ\bar{\nu}_{13}=0$.
(5) $\omega'^7_{13}\circ\mu_{13}=8u^4_{22}(7)$.
(6) $u^7_{16}\circ\eta_{16}=0$, $u^7_{16}\circ\nu_{16}=\pm 2u^7_{19}$.
(7) $u^7_{19}\circ\eta_{19}=0$.
(8) $u^7_{20}\circ\eta_{20}=u^4_{14}(7)\circ\sigma_{14}$.

PROOF: (1) Note that $\omega'^7_{13}\in\{i'^{7,6}, \gamma'^6_{12}, 16\iota_{12}\}\subset\{i'^{7,6}, 8\gamma'^6_{12}, 2\iota_{12}\}=\{i'^{7,6}, \tau'^2_3(6)\circ\mu_3, 2\iota_{12}\}=\{\tau'^2_3(7), \mu_3, 2\iota_{12}\}$. Hence $\omega'^7_{13}\circ\eta_{13}\in\{\tau'^2_3(7), \mu_3, 2\iota_{12}\}\circ\eta_{13}=\tau'^2_3(7)\circ\{\mu_3, 2\iota_{12}, \eta_{12}\}$, which consists of a single element $\tau'^2_3(7)\circ\mu'$ (by (1.2)(6)[(*)]) $=8u^4_{14}(7)=8\gamma'^7_{14}$ (by Lemma (4.4)(3) and (7.3)), because $\tau'^2_3(7)\circ\pi^3_{13}\circ\eta_{13}=\{\tau'^2_3(7)\circ\eta_3\circ\mu_4\circ\eta_{13}\}=0$.

(*) $\langle\mu, 2\iota, \eta\rangle=\langle\eta, 2\iota, \mu\rangle$. Hence $\{\mu_3, 2\iota_{12}, \eta_{12}\}$ also contains μ'.

(2) $2\omega'^7_{13}\circ\nu_{13}\in\{\omega'^7_{13}\circ\eta_{13}, 2\iota_{14}, \eta_{14}\}$ (by (1.2)(1)) $=\{8u^4_{14}(7), 2\iota_{14}, \eta_{14}\}$ (by (1)) $=\{4\omega'^4_7(7)\circ\sigma', 2\iota_{14}, \eta_{14}\}$ (by Lemma (4.4)(3)), which consists of a single element $\omega'^4_7(7)\circ\mu_7$ (by (1.2)(4)) $=4u^4_{16}(7)$ (by Prop. (4.8)(3)) $=32u^7_{16}$ (by Lemma (6.5) and Lemma (7.1)(1)), because $\pi_{15}(U_7)\circ\eta_{15}=0$. Hence $2\omega'^7_{13}\circ\nu_{13}=32u^7_{16}$, so that $\omega'_{13}\circ\nu^7_{13}=\pm 16u^7_{16}$.

(3) $\omega'^7_{13}\circ\sigma_{13}\in\{i'^{7,6}, \gamma'^6_{12}, 16\iota_{12}\}\circ\sigma_{13}=-i'^{7,6}\circ\{\gamma'^6_{12}, 16\iota_{12}, \sigma_{12}\}$, which consists of a single element, because $i'^{7,6}\circ\pi_{13}(U_6)=0$. Now $p'_*\{\gamma'^6_{12}, 16\iota_{12}, \sigma_{12}\}\subset\{\eta_{11}, 16\iota_{12}, \sigma_{12}\}$, which contains μ_{11}. Hence $i'^{7,6}\circ\{\gamma'^6_{12}, 16\iota_{12}, \sigma_{12}\}=xu^6_{20}$ for some odd integer x (by Lemma (6.10)(3)), that is, $\omega'^7_{13}\circ\sigma_{13}\equiv 8u^7_{20} \bmod 16u^7_{20}$.

(4) In the same way as in (1), we can prove that $\omega'^7_{13}\circ\varepsilon_{13}$ and $\omega'^7_{13}\circ\bar{\nu}_{13}$ belong to $\tau'^3_3(7)\circ\pi^3_{21}=0$.

(5) $\omega'^7_{13}\circ\mu_{13}\in\{\omega'^7_{13}\circ\eta_{13}, 2\iota_{14}, 8\sigma_{14}\}$ (by (1.2)(4)) $=\{8u^4_{14}(7), 2\iota_{14}, 8\sigma_{14}\}$, which consists of a single element $8u^4_{22}(7)$ (by Lemma (4.6)(1)), because $8\pi^{14}_{22}=0$ and $\pi_{15}(U_7)=0$.

(6) $u^7_{16}\circ\eta_{16}\in\pi_{17}(U_7)=0$. Next $u^7_{16}\circ\nu_{16}\in\{i'^{7,6}, \gamma'^6_{12}, 2\nu_{12}\}\circ\nu_{16}=-i'^{7,6}\circ\{\gamma'^6_{12}, 2\nu_{12}, \nu_{15}\}$, which consists of a single element, because $i'^{7,6}\circ\pi_{16}(U_6)\circ\nu_{16}=\{\omega'^5_9(7)\circ\sigma_9\circ\nu_{16}\}=0$ (by Prop. (5.7)(3)). Now $p'_*\{\gamma'^6_{12}, 2\nu_{12}, \nu_{15}\}\subset\{\eta_{11}, 2\nu_{12}, \nu_{15}\}$, which contains ε_{11} (by (1.2)(3)). Hence $\{\gamma'^6_{12}, 2\nu_{12}, \nu_{15}\}$ contains u^6_{19} or $-u^6_{19}$, that is, $u^7_{16}\circ\nu_{16}=\pm u^6_{19}(7)=\pm 2u^7_{19}$ (by Lemma (7.1)(2)).

(7) $u^7_{19}\circ\eta_{19}\in\{i'^{7,6}, \gamma'^6_{12}, \nu^2_{12}\}\circ\eta_{19}=i'^{7,6}\circ\{\gamma'^6_{12}, \nu^2_{12}, \eta_{18}\}=i'^{7,6}\circ\{\gamma'^6_{12}\circ\nu_{12}, \nu_{15}, \eta_{18}\}=i'^{7,6}\circ\{u^4_{14}(6)\circ\eta_{14}, \nu_{15}, \eta_{18}\}$, which consists of a single element because $i'^{7,6}\circ\pi_{19}(U_6)\circ\eta_{19}=\{2u^7_{19}\circ\eta_{19}\}=0$. Now $i'^{7,6}\circ\{u^4_{14}(6)\circ\eta_{14}, \nu_{15}, \eta_{18}\}=u^4_{14}(7)\circ\nu^2_{14}$ (by Lemma (5.12) of [1]) $=0$ (by Prop. (4.8)(20)).

(8) $u^7_{20}\circ\eta_{20}\in\{i'^{7,6}, \gamma'^6_{12}, 2\sigma_{12}\}\circ\eta_{20}=i'^{7,6}\circ\{\gamma'^6_{12}, 2\sigma_{12}, \eta_{19}\}$, which consists of a single element, because $i'^{7,6}\circ\pi_{20}(U_6)\circ\eta_{20}=\{8u^7_{20}\circ\eta_{20}\}=0$. Now $2\{\gamma'^6_{12}, 2\sigma_{12}, \eta_{19}\}=\gamma'^6_{12}\circ\{2\sigma_{12}, \eta_{19}, 2\iota_{20}\}=\gamma'^6_{12}\circ\sigma_{12}\circ\eta^2_{19}=\gamma'^6_{12}\circ\varepsilon_{12}\circ\eta_{20}=2u^4_{14}(6)\circ\sigma_{14}$ (by Prop. (6.14)(6)). Since $\pi_{21}(U_7)=i'_*\pi_{21}(U_6)=Z_2=\{u^4_{14}(7)\circ\sigma_{14}\}$, we conclude that $u^7_{20}\circ\eta_{20}=u^4_{14}(7)\circ\sigma_{14}$.

$\pi_k(U_8)$ $(14\leqq k\leqq 22)$ and their generators are obtained by the results of §11 of [3], some of which are corrected in (6.1)$\sim$(6.3).

LEMMA (7.5): *There are original generators of* $\pi_k(U_8)$ $(14\leqq k\leqq 22)$, *which are listed below:*

(1) $\omega'^8_{15}\in\{i'^{8,7}, \gamma'^7_{14}, 16\iota_{14}\}$ *such that* $p'_*\omega'^8_{15}=16\iota_{15}$.

(2) $\gamma'^8_{16}\in\{i'^{8,7}, \gamma'^7_{14}, \eta_{14}\}$ *such that* $p'_*\gamma'^8_{16}=\eta_{15}$ *and* $2\gamma'^8_{16}=u^7_{16}(8)$.

(3) $u^8_{22}\in\{i'^{8,7}, \gamma'^7_{14}, 2\sigma_{14}\}$ *such that* $p'_*u^8_{22}=2\sigma_{15}$ *and* $8u^8_{22}=xu^4_{22}(8)$ *for some odd integer* x.

THEOREM (7.6): *(2-primary components of)* $\pi_k(U_8)$ $(14\leqq k\leqq 22)$ *are listed below:*

$\pi_{14}(U_n)=0$ $(n\geqq 18)$.

$\pi_{15}(U_8)=Z=\{\omega'^8_{15}\}$.

$\pi_{16}(U_8)=Z_{128}=\{\gamma'^8_{16}\}$.

$\pi_{17}(U_8)=Z_2=\{\gamma'^8_{16}\circ\eta_{16}\}$.

$\pi_{18}(U_8)=Z_{128}\oplus Z_2=\{l^8\circ\gamma''^4_{18}\}\oplus\{\gamma'^8_{16}\circ\eta^2_{16}\}$.

$\pi_{19}(U_8)=Z_8=\{\gamma'^8_{16}\circ\nu_{16}\}$,

$\pi_{20}(U_8)=Z_{128}=\{u^7_{20}(8)\}$,

$\pi_{21}(U_8)=Z_2=\{l^8\circ\gamma''^4_{18}\circ\nu_{18}\}$,

$\pi_{22}(U_8)=Z_{128}\oplus Z_2=\{u^8_{22}\}\oplus\{\gamma'^8_{16}\circ\nu^2_{16}\}$.

While we are doing these calculations, we obtain the following relations:

PROPOSITION (7.7):

(1) $4l^8\circ\gamma''^4_{18}+\gamma'^8_{16}\circ\eta^2_{16}=xu^6_{18}(8)$ *for some odd integer* $x^{(*)}$.

(2) $\gamma'^8_{16}\circ\nu_{16}=xu^7_{19}(8)$ *for some odd integer* x.

Now we shall add the relations as follows.

PROPOSITION (7.8):

(1) $\omega'^8_{15}\circ\eta_{15}=64\gamma'^8_{16}$.

(2) $\omega'^8_{15}\circ\nu_{15}=\pm 32l^8\circ\gamma''^4_{18}$.

(3) $\omega'^8_{15}\circ\sigma_{15}\equiv 8u^8_{22} \bmod 16u^8_{22}$.

(4) $l^8\circ\omega''^4_{15}=2\omega'^8_{15}$.

(5) $l^8\circ s^4_{22}\equiv 2u^8_{22} \bmod 8u^8_{22}$.

(6) $l^8\circ\gamma''^4_{18}\circ\eta_{18}\equiv 0 \bmod 4u^7_{19}(8)$.

PROOF: (1) The proof is quite similar with that of Proposition (7.4)(1).

(2) The proof is quite similar with that of Proposition (7.4)(2).

(3) $\omega'^8_{15}\circ\sigma_{15}\in\{i'^{8,7},\gamma'^7_{14},16\iota_{14}\}\circ\sigma_{15}=-i'^{8,7}\circ\{\gamma'^7_{14},16\iota_{14},\sigma_{14}\}$, which consists of a single element $xu^4_{22}(8)$ for some odd integer x (by (7.3), Lemma (4.6)(1)), because $i'^{8,7}_*\pi_{15}(U_7)=0$.

(4) $p'_*(l^8\circ\omega''^4_{15})=32\iota_{15}$. $p'^{-1}_*(0)=0$. Hence $l^8\circ\omega''^4_{15}=2\omega'^8_{15}$.

(5) $l^8\circ s^4_{22}\in\{l^8\circ i''^{4,3},\gamma''^3_{14},4\sigma_{14}\}\subset\{i'^{8,7},\gamma'^7_{14},4\sigma_{14}\}$, which contains $2u^8_{22}$ (by Lemma (7.5)(3)), and is a coset of the subgroup $i'^{8,7}\circ\pi_{22}(U_7)+\pi_{15}(U_8)\circ 4\sigma_{15}=\{8u^8_{22}\}+\{4\omega'^8_{15}\circ\sigma_{15}\}=\{8u^8_{22}\}$ (by (3)). Hence, we have $l^8\circ s^4_{22}\equiv 2u^8_{22} \bmod 8u^8_{22}$.

(6) $l^8\circ\gamma''^4_{18}\circ\eta_{18}=l^8\circ s^3_{19}(4)$ (by Prop. (5.4)(4)) $=i'^{8,6}\circ l^6\circ s^3_{19}\equiv i'^{8,6}\circ\gamma'^6_{12}\circ\sigma_{12}=0 \bmod 2u^6_{19}(8)=4u^7_{19}(8)$ (by Prop. (6.14)(1)).

$\pi_k(U_n)$ $(15\leqq k\leqq 22,\ n\geqq 9)$ and their generators are calculated by §11 of [3] as follows:

LEMMA (7.9):

(1) $\omega'^9_{17}\in\{i'^{9,8},\gamma'^8_{16},128\iota_{16}\}$ *such that* $p'_*\omega'^9_{17}=128\iota_{17}$.

(2) $\omega'^{10}_{19}\in\{i'^{10,9},\gamma'^9_{18},128\iota_{18}\}$ *such that* $p'_*\omega'^{10}_{19}=128\iota_{19}$, *where* $\gamma'^9_{18}=i'^{9,8}\circ l^8\circ\gamma''^4_{18}$.

(3) $\gamma'^{10}_{20}\in\{i'^{10,9},\gamma'^9_{18},\eta_{18}\}$ *such that* $p'_*\gamma'^{10}_{20}=\eta_{19}$ *and* $2\gamma'^{10}_{20}=u^7_{20}(10)$.

(4) $\omega'^{11}_{21}\in\{i'^{11,10},\gamma'^{10}_{20},256\iota_{20}\}$ *such that* $p'_*\omega'^{11}_{21}=256\iota_{21}$.

THEOREM (7.10): *(2-primary components of)* $\pi_k(U_n)$ $(15\leqq k\leqq 22,\ n\geqq 9)$ *are listed below:*

(*) Actually we can see that $x\equiv 1 \bmod 16$.

(1) $\pi_{15}(U_n)=Z=\{\omega'^{8}_{15}(n)\}$ $(n\geqq 9)$,
$\pi_{16}(U_n)=0$ $(n\geqq 9)$,
$\pi_{17}(U_n)=Z=\{\omega'^{9}_{17}(n)\}$ $(n\geqq 9)$,
$\pi_{18}(U_9)=Z_{128}=\{\gamma'^{9}_{18}\}$ (*where* $\gamma'^{9}_{18}=i'^{9,8}\circ l^8\circ\gamma''^{4}_{18}$),
$\pi_{19}(U_9)=0$
$\pi_{20}(U_9)=Z_{128}=\{u^{7}_{20}(9)\}$,
$\pi_{21}(U_9)=Z_2=\{\gamma'^{9}_{18}\circ\nu_{18}\}$,
$\pi_{22}(U_9)=Z_{128}=\{u^{8}_{22}(9)\}$,

(2) $\pi_{18}(U_n)=0$ $(n\geqq 10)$,
$\pi_{19}(U_n)=Z=\{\omega'^{10}_{19}(n)\}$ $(n\geqq 10)$,
$\pi_{20}(U_{10})=Z_{256}=\{\gamma'^{10}_{20}\}$,
$\pi_{21}(U_{10})=Z_2=\{\gamma'^{10}_{20}\circ\eta_{20}\}$,
$\pi_{22}(U_{10})=Z_{256}\oplus Z_2=\{l^{10}\circ\gamma''^{5}_{22}\}\oplus\{\gamma'^{10}_{20}\circ\eta^{2}_{20}\}$.

(3) $\pi_{20}(U_n)=0$ $(n\geqq 11)$,
$\pi_{21}(U_n)=Z=\{\omega'^{11}_{21}(n)\}$ $(n\geqq 11)$,
$\pi_{22}(U_n)=0$ $(n\geqq 11)$.

PROPOSITION (7.11): *The following relations hold*:

(1) $\omega'^{9}_{17}\circ\eta_{17}=64\gamma'^{9}_{18}$.
(2) $\omega'^{7}_{17}\circ\nu_{17}=\pm 32u^{7}_{20}(9)$.
(3) $\pm 2l^{10}\circ\gamma''^{5}_{22}+\gamma'^{10}_{20}\circ\eta^{2}_{20}=xu^{8}_{22}(10)$ *for some odd integer* x.
(4) $l^{10}\circ\omega''^{5}_{19}=\omega'^{10}_{19}$.
(5) $\omega'^{10}_{19}\circ\eta_{19}=128\gamma'^{10}_{20}$.
(6) $\omega'^{10}_{19}\circ\nu_{19}=\pm 64l^{10}\circ\gamma''^{5}_{22}$.
(7) $\omega'^{11}_{21}\circ\eta_{21}=128\gamma'^{11}_{22}$.

Proofs of (1), (2), (5), (6) and (7) are quite similar with that of Proposition (7.4)(1). Proofs of (3) and (4) are quite similar with those of Proposition (7.8)(1) and (4), respectively.

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Table I Compositions (on π^n_{n+k} $k\leqq 19$) (2-primary components)

π^n_{n+3}	$\eta^3_3=2\nu'$. $E^2\nu'=2\nu_5$.
π^n_{n+4}	$\eta_3\circ\nu_4=\nu'\circ\eta_6$.
π^n_{n+6}	$\nu'\circ\nu_6=0$.
π^n_{n+7}	$E\sigma'''=2\sigma''$, $E\sigma''=2\sigma'$, $E^2\sigma'=2\sigma_9$.
π^n_{n+8}	$\eta_4\circ\sigma'''=\eta_5\circ\sigma''=\sigma'''\circ\eta_{12}=0$, $\eta_6\circ\sigma'=\sigma''\circ\eta_{13}=4\bar{\nu}_6$, $\eta_7\circ\sigma_8=\sigma'\circ\eta_{14}+\varepsilon_7+\bar{\nu}_7$, $\eta_9\circ\sigma_{10}=\varepsilon_9+\bar{\nu}_9$, $\eta_n\circ\sigma_{n+1}=\sigma_n\circ\eta_{n+7}=\varepsilon_n+\bar{\nu}_n$ $(n\geqq 10)$

Table I Continued

π_{n+9}^n	$\varepsilon_n\circ\eta_{n+8}=\eta_n\circ\varepsilon_{n+1}$ $(n\geqq 3)$. $\eta_5\circ\bar{\nu}_6=\nu_5^3$, $\bar{\nu}_n\circ\eta_{n+8}=\eta_n\circ\bar{\nu}_{n+1}=\nu_n^3$ $(n\geqq 6)$.
π_{n+10}^n	$\eta_3^2\circ\varepsilon_5=2\varepsilon'$. $2(\nu_5\circ\sigma_8)=\nu_5\circ E\sigma'=\pm E^2\varepsilon'$. $\nu'\circ\sigma''=0$, $E\nu'\circ\sigma'=2E\varepsilon'$, $\sigma'''\circ\nu_{12}=4(\nu_5\circ\sigma_8)$, $\sigma''\circ\nu_{13}=\pm 2\nu_6\circ\sigma_9$, $\sigma'\circ\nu_{14}=x\nu_7\circ\sigma_{10}$ for some odd integer x, $\nu_9\circ\sigma_{12}=\pm 2\sigma_9\nu_{16}$ $\mu_n\circ\eta_{n+9}=\eta_n\circ\mu_{n+1}$ $(n\geqq 3)$.
π_{n+11}^n	$\eta_3^2\circ\mu_5=2\mu'$, $E^2\mu'=\pm 2\zeta_5$. $\nu'\circ\bar{\nu}_6=\varepsilon_3\circ\nu_{11}$, $\nu_6\circ\bar{\nu}_9=\nu_6\circ\varepsilon_9=2\bar{\nu}_6\circ\nu_{14}$. $\varepsilon'\circ\eta_{13}=\nu'\circ\varepsilon_6$. $\nu_5\circ\sigma_8\circ\eta_{15}=\nu_5\circ\varepsilon_8$ or $\nu_5\circ\bar{\nu}_8$.
π_{n+12}^n	$\eta_4\circ\zeta_5\equiv E\nu'\circ\mu_7$ mod $E\nu'\circ\eta_7\circ\varepsilon_8$, $\eta_n\circ\zeta_{n+1}=0$ $(n\geqq 5)$. $\zeta_6\circ\eta_{17}=8[\iota_6,\iota_6]\circ\sigma_{11}$, $\zeta_n\circ\eta_{n+11}=0$ $(n\geqq 7)$. $\mu'\circ\eta_{14}=\nu'\circ\mu_6$. $\mu_3\circ\nu_{12}=\nu'\circ\eta_6\circ\varepsilon_7$, $\nu_6\circ\mu_9=8[\iota_6,\iota_6]\circ\sigma_{11}$.
π_{n+13}^n	$\varepsilon'\circ\nu_{13}=0$. $\eta_{10}\circ\theta'=0$, $\eta_{11}\circ\theta=\sigma_{11}\circ\nu_{18}^2+\theta'\circ\eta_{23}$.
π_{n+14}^n	$2\kappa_7\equiv\bar{\nu}_7\circ\nu_{15}^2$ mod $4\sigma'\circ\sigma_{14}$. $\theta'\circ\eta_{23}^2=0$, $\theta\circ\eta_{24}^2\equiv 2[\iota_{12},\nu_{12}]$ mod $8\sigma_{12}^2$. $\nu'\circ\zeta_6=\mu'\circ\nu_{14}=0$, $\sigma'''\circ\sigma_{12}\equiv\nu_5\circ\zeta_8$ mod $\nu_5\circ\bar{\nu}_8\circ\nu_{16}$, $\zeta_5\circ\nu_{16}\equiv\nu_5\circ\zeta_8$ mod $\nu_5\circ\bar{\nu}_8\circ\nu_{16}$, $\zeta_n\circ\nu_{n+11}=\nu_n\circ\zeta_{n+3}$ $(n\geqq 6)$, $\nu_6\circ\zeta_9=2\sigma''\circ\sigma_{13}$.
π_{n+15}^n	$E\rho^{\mathrm{IV}}=2\rho'''$, $E\rho'''=2\rho''$, $E^2\rho''=2\rho'$, $E^4\rho'=2\rho_{13}$. $\eta_n\circ\kappa_{n+1}=\bar{\varepsilon}_n$ $(n\geqq 6)$, $\kappa_7\circ\eta_{21}=\sigma'\circ\bar{\nu}_{14}+\bar{\varepsilon}_7$, $\kappa_n\circ\eta_{n+14}=\bar{\varepsilon}_n$ $(n\geqq 9)$. $\nu_n^5=0$. $\nu_8\circ\theta'=E\sigma'\circ\varepsilon_{15}$ or $E\sigma'\circ\bar{\nu}_{15}$, $\nu_9\circ\theta=\sigma_9\circ\varepsilon_{16}$ or $\sigma_9\circ\bar{\nu}_{16}$, $\theta'\circ\nu_{23}=\theta\circ\nu_{24}=0$. $\sigma'''\circ\varepsilon_{12}=\sigma'''\circ\bar{\nu}_{12}=\sigma''\circ\varepsilon_{13}=\sigma''\circ\bar{\nu}_{13}=[\iota_6,\iota_6]\circ\sigma_{11}\circ\nu_{18}=0$. $\varepsilon_n\circ\sigma_{n+8}=0$ $(n\geqq 3)$, $\bar{\nu}_n\circ\sigma_{n+8}=0$ $(n\geqq 6)$, $\sigma_{10}\circ\varepsilon_{17}=\sigma_{10}\circ\bar{\nu}_{17}=[\iota_{10},\nu_{10}^2]$.
π_{n+16}^n	$\eta_4\circ\rho^{\mathrm{IV}}=\rho^{\mathrm{IV}}\circ\eta_{20}=\eta_5\circ\rho'''=0$, $\rho'''\circ\eta_{21}=\eta_6\circ\rho''=4\zeta'$, $\rho''\circ\eta_{22}=\sigma'\circ\mu_{14}$. $\rho'\circ\eta_{24}\equiv\sigma_9\circ\mu_{16}+\mu_9\circ\sigma_{18}$ mod $\{\sigma_9\circ\varepsilon_{16}\circ\eta_{24}\}\oplus\{\sigma_9\circ\nu_{16}^3\}$, $\eta_{12}\circ\rho_{13}=\sigma_{12}\circ\mu_{19}$, $\eta_n\circ\rho_{n+1}=\rho_n\circ\eta_{n+15}=\mu_n\circ\sigma_{n+9}=\sigma_n\circ\mu_{n+7}$ $(n\geqq 13)$. $\sigma'''\circ\mu_{12}=0$, $\sigma''\circ\mu_{13}=4\zeta'$, $\mu_{10}\circ\sigma_{19}=\sigma_{10}\circ\mu_{17}+8[\iota_{10},\sigma_{10}]$. $\sigma'\circ\eta_{14}\circ\varepsilon_{15}=E\zeta'$. $\nu_n\circ\sigma_{n+3}\circ\nu_{n+10}^2=\eta_n\circ\bar{\varepsilon}_{n+1}$ $(n\geqq 5)$, $\sigma_{10}\circ\nu_{17}^3=\sigma_{10}\circ\eta_{17}\circ\varepsilon_{18}=0$. $\varepsilon_n^2=\varepsilon_n\circ\bar{\nu}_{n+8}=\eta_n\circ\bar{\varepsilon}_{n+1}=\bar{\varepsilon}_n\circ\eta_{n+15}$ $(n\geqq 3)$. $\bar{\nu}_n^2=\bar{\nu}_n\circ\varepsilon_{n+8}=0$ $(n\geqq 6)$. $\eta_n^*\equiv\omega_n$ mod $\sigma_n\circ\mu_{n+7}$ $(n\geqq 18)$.

Table I Continued

	$E\bar{\bar{\varepsilon}}'=E\nu'\circ\kappa_7,\ \eta_3^2\circ\bar{\bar{\varepsilon}}_5=2\bar{\bar{\varepsilon}}'.$
	$\eta_{13}\circ\omega_{14}=\varepsilon_{13}^*,\ \eta_n\circ\omega_{n+1}=\omega_n\circ\eta_{n+16}=\varepsilon_n^*\ (n\geqq 14).$
	$\eta_n^*\circ\eta_{n+16}\equiv\varepsilon_n^* \bmod \sigma_n\circ\eta_{n+7}\circ\mu_{n+8}\ (n\geqq 18),\ E^2\eta^{*\prime}\circ\eta_{33}\equiv 0 \bmod \sigma_{17}\circ\eta_{24}\circ\mu_{25},$
	$\eta_{14}\circ\eta^{*\prime}\equiv 0 \bmod \sigma_{14}\circ\eta_{21}\circ\mu_{22},\ \eta_{15}\circ\eta_{16}^*\equiv\eta^{*\prime}\circ\eta_{31}+\varepsilon_{15}^* \bmod \sigma_{15}\circ\eta_{22}\circ\mu_{23}.$
	$\zeta'\circ\eta_{22}=\eta_5\circ\zeta'=0.$
π_{n+17}^n	$\kappa_n\circ\nu_{n+14}=\nu_n\circ\kappa_{n+3}\ (n\geqq 7).$
	$\nu'\circ\bar{\nu}_6\circ\nu_{14}^2=\varepsilon'\circ\sigma_{13}=\varepsilon_3\circ\nu_{11}^2=2\bar{\bar{\varepsilon}}',\ \nu_5\circ\bar{\nu}_8\circ\nu_{16}^2=2(\nu_5\circ\kappa_8).$
	$\nu_5\circ E\sigma'\circ\sigma_{15}=0,\ \nu_5\circ\sigma_8^2=2\nu_5\circ\kappa_8$ or $0.$
	$\mu_3\circ\varepsilon_{12}\equiv\varepsilon_3\circ\mu_{11} \bmod 2\bar{\bar{\varepsilon}}',\ \mu_3\circ\varepsilon_{12}\equiv\eta_3\circ\mu_4\circ\sigma_{13} \bmod 2\bar{\bar{\varepsilon}}'.$
	$\mu_3\circ\bar{\nu}_{12}\equiv 0 \bmod 2\bar{\bar{\varepsilon}}',\ \mu_n\circ\bar{\nu}_{n+9}=\bar{\nu}_n\circ\mu_{n+8}=0\ (n\geqq 6).$
	$[\iota_6,\iota_6]\circ\theta'=0.$
	$E\xi''=2\xi',\ E^2\xi'=2\xi_{13},\ E\lambda''=2\lambda',\ E^2\lambda'=2\lambda,\ E^4\lambda=2\nu_{17}^*,\ \nu_n^*=\xi_n\ (n\geqq 14),$
	$2\xi''=2\lambda''=\pm\sigma_{10}\circ\zeta_{17},\ 4\xi'=4\lambda'=\sigma_{11}\circ\zeta_{18},\ 4E\xi'=16\xi_{12}=\sigma_{12}\circ\zeta_{19}.$
	$\eta_n\circ\bar{\mu}_{n+1}=\bar{\mu}_n\circ\eta_{n+17}\ (n\geqq 3).$
	$\bar{\varepsilon}'\circ\eta_{20}=\bar{\varepsilon}_3\circ\nu_{18}=\nu'\circ\bar{\varepsilon}_6.$
	$\eta^{*\prime}\circ\eta_{31}^2=4E^2\lambda,\ \eta_{16}^*\circ\eta_{32}^2\equiv 4\nu_{16}^* \bmod 4E^3\lambda,\ \eta_{17}^*\circ\eta_{33}^2=4\nu_{17}^*,$
	$\varepsilon_{13}^*\circ\eta_{30}=\eta_{13}\circ\varepsilon_{14}^*\equiv 4\xi_{13} \bmod 4\lambda_{13}.$
	$\nu'\circ\rho'''=E\nu'\circ\rho''=0,\ \nu_5\circ E\rho''\equiv 0 \bmod 4\zeta_5\circ\sigma_{16},\ \nu_6\circ\rho'\equiv 0 \bmod 2\zeta_6\circ\sigma_{17},$
π_{n+18}^n	$\nu_{10}\circ\rho_{13}\equiv 0 \bmod 2\sigma_{10}\circ\zeta_{17}.$
	$\rho^{IV}\circ\nu_{20}\equiv 4\zeta_5\circ\sigma_{16} \bmod \nu_5\circ\bar{\varepsilon}_8,\ \rho'''\circ\nu_{21}=\pm 2\zeta_6\circ\sigma_{17},$
	$\rho''\circ\nu_{22}=\sigma'\circ\zeta_{14}=x\zeta_7\circ\sigma_{18}$ for some odd inter x,
	$\rho'\circ\nu_{24}\equiv -\sigma_9\circ\zeta_{16} \bmod 4\sigma_9\circ\zeta_{16},\ \rho_{13}\circ\nu_{28}=4\lambda.$
	$\nu_5\circ\sigma_8\circ\varepsilon_{15}=\nu_5\circ\sigma_8\circ\bar{\nu}_{15}\equiv 0 \bmod \nu_5\circ\bar{\varepsilon}_8.$
	$\varepsilon'\circ\varepsilon_{13}=\varepsilon'\circ\bar{\nu}_{13}=\nu'\circ\bar{\varepsilon}_6.$
	$4\sigma_{10}\circ\zeta_{17}=2\sigma_{11}\circ\zeta_{18}=\sigma_{13}\circ\zeta_{20}=0,\ \sigma'''\circ\zeta_{12}=4\zeta_5\circ\sigma_{16},\ \sigma''\circ\zeta_{13}=\pm 2\zeta_6\circ\sigma_{17}.$
	$\mu_3^2\equiv\eta_3\circ\bar{\mu}_4 \bmod 2\mu'\circ\sigma_{14}.$
	$\eta_3^2\circ\mu_5=2\bar{\mu}',\ E^2\bar{\mu}'=2\bar{\zeta}_5.$
	$E^2\omega'=2\omega_{14}\circ\nu_{30},\ \nu_6\circ\mu_9\circ\sigma_{18}=16\bar{\sigma}_6.$
	$\xi''\circ\eta_{28}=\lambda''\circ\eta_{28}=\eta_{10}\circ\xi'=\eta_{10}\circ\lambda'=0,\ \eta_9\circ\lambda''=\eta_9\circ\xi''=0,\ \eta_{11}\circ\xi_{12}=\xi'\circ\eta_{29},$
	$\nu_{16}^*\circ\eta_{34}=0,\ \eta_{17}\circ\nu_{18}^*=\omega_{17}\circ\nu_{33},\ \lambda\circ\eta_{31}\equiv E\omega' \bmod \xi_{13}\circ\eta_{31},$
	$\eta_{12}\circ\lambda\equiv 0 \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}.$
	$\Delta(E\theta)\circ\eta_{23}^2\equiv 0 \bmod 16\bar{\sigma}_6.$
	$\nu'\circ\zeta'=\nu'\circ\eta_6\circ\bar{\varepsilon}_7=\varepsilon_3\circ\zeta_{11}=0,\ \zeta'\circ\nu_{22}=\pm 8\bar{\sigma}_6,\ \nu_{11}\circ\omega_{14}\equiv\lambda'\circ\eta_{29} \bmod \xi'\circ\eta_{29},$
π_{n+19}^n	$\eta^{*\prime}\circ\nu_{31}=\eta_{16}^*\circ\nu_{32}=0,\ \nu_{12}\circ\eta^{*\prime}\equiv 0 \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\},$
	$\nu_{13}\circ\eta_{16}^*\equiv E\omega' \bmod \xi_{13}\circ\eta_{31}.$
	$\sigma'''\circ E\theta'=\sigma''\circ\theta=\sigma'\circ E\theta=0,\ \theta'\circ\sigma_{23}=\xi'\circ\eta_{29},$
	$\theta\circ\sigma_{24}\equiv\xi_{12}\circ\eta_{30} \bmod \{E\xi'\circ\eta_{30}\}\oplus\{E\lambda'\circ\eta_{30}\}.$
	$\bar{\nu}_6\circ\zeta_{14}=\pm 8\bar{\sigma}_6.$
	$\zeta_5\circ\eta_{16}\circ\sigma_{17}=\nu_5\circ\mu_8\circ\sigma_{17}.$
	$\varepsilon'\circ\mu_{13}=\nu'\circ\mu_6\circ\sigma_{15}.$
	$[\iota_6,\iota_6]\circ\sigma_{11}^2\equiv 2\bar{\sigma}_6 \bmod 4\bar{\sigma}_6.$

Table II Original generators of $\pi_k(Sp_n)$ $(k\leq 22)$ (2-primary components)

n	α	$\alpha\in\{\beta,\gamma,\delta\}$	$p''_*\alpha$	$l^{2n}_*\alpha$	$d\alpha$
1	τ''^1_3			τ'^2_3	
2	ω''^2_7	$\{\tau''^1_3(2), \nu', 4\iota_6\}$	$4\iota_7$	$2\omega'^4_7$	
	γ''^2_{10}	$\{\tau''^1_3(2), \nu', \nu_6\}$	ν_7		$4\gamma''^2_{10}=\omega''^2_7\circ\nu_7$
	s^3_{14}	$\{\tau''^1_3(2), 2\varepsilon', 2\iota_{13}\}_1$ $\{\tau''^1_3(2), \nu', \sigma''\}$	$2\sigma'$	$\omega'^4_7\circ\sigma'$	$2s^2_{14}=\omega''^2_7\circ\sigma'$, $4s^2_{14}=\pm\tau''^1_3(2)\circ\mu'$
	s^2_{15}	$\{\tau''^1_3(2), 2\varepsilon', \eta_{13}\}_1$	$\sigma'\circ\eta^4_{14}$	$u_{14}\circ\eta_{14}+4\gamma'^4_8\circ\sigma_8$	$2s^2_{15}=0$
	s^2_{18}	$\{\gamma''^2_{10}, 8\iota_{10}, 2\sigma_{10}\}_3$	ζ_7		$4s^2_{18}=\omega''^2_7\circ\zeta_7$, $8s^2_{18}=0$
	s^2_{21}	$\{\tau''^1_3(2), 2\varepsilon', \sigma_{13}\}_1$	$\sigma'\circ\sigma_{14}$	$u^4_{14}\circ\sigma_{14}$ (mod $8u^4_{14}\circ\sigma_{14}$)	$2s^2_{21}=s^2_{14}\circ\sigma_{14}$
	$\hat{s}^2_{22}$	$\{\tau''^1_3(2), 2\varepsilon', \varepsilon_{13}\}_1$	$\sigma'\circ\varepsilon_{14}$	$u^4_{14}\circ\varepsilon_{14}$	$2\hat{s}^2_{22}=0$
	s^2_{22}		ρ''	u^4_{22} (mod $2\gamma'^4_8\circ\kappa_8$)	$4s^2_{22}=\omega''^2_7\circ\rho''$, $8s^2_{22}=\tau_3{}''^1_3(2)\circ\bar{\mu}'$
3	ω''^3_{11}	$\{i''^{3,2}, \gamma''^2_{10}, 8\iota_{10}\}$	$8\iota_{11}$	ω'^6_{11}	
	γ''^3_{14}	$\{i''^{3,2}, \gamma''^2_{10}, 4\nu_{10}\}$	$4\nu_{11}$	$xu^4_{14}(6)+\gamma'^6_{12}\circ\eta^2_{12}$ for some odd integer x	$2\gamma''^2_{14}=s^2_{14}(3)$
	s^3_{19}	$\{i''^{3,2}, \gamma''^2_{10}, \eta_{10}\circ\sigma_{11}\}$	$\eta_{11}\circ\sigma_{12}$	$\gamma'^6_{12}\circ\sigma_{12}$ (mod $2u^6_{19}$)	$2s^3_{19}=0$
4	ω''^4_{15}	$\{i''^{4,3}, \gamma''^3_{14}, 32\iota_{14}\}$	$32\iota_{15}$	$2\omega'^8_{15}$	
	γ''^4_{18}	$\{i''^{4,3}, \gamma''^3_{14}, \nu_{14}\}$	ν_{15}		$8\gamma''^4_{18}=x\omega''^3_{11}(4)\circ\sigma_{11}$ for some odd integer x.
	s^4_{32}	$\{i''^{4,3}, \gamma''^3_{14}, 4\sigma_{14}\}$	$4\sigma_{15}$	$2u^8_{22}$ (mod $8u^8_{22}$)	$4s^4_{22}=\pm s^2_{22}(4)$
5	ω''^5_{19}	$\{i''^{5,4}, \gamma''^4_{18}, 128\iota_{18}\}$	$128\iota_{19}$	ω'^{10}_{19}	
	γ''^5_{32}	$\{i''^{5,4}, \gamma''^4_{18}, 2\nu_{18}\}$	$2\nu_{19}$		$4\gamma''^5_{22}=\pm s^4_{22}(5)$

Table III Compositions (on $\pi_k(Sp_n)$ $(k\leqq 22)$) (2-primary components)

α	$\omega''^{2}_{7}\circ\alpha$	$\gamma''^{2}_{10}\circ\alpha$	$\omega''^{3}_{11}\circ\alpha$	$\gamma''^{3}_{14}\circ\alpha$	$\omega''^{4}_{15}\circ\alpha$
η	0	$\tau''^{1}_{3}(2)\circ\varepsilon_3$	$\tau''^{1}_{3}(3)\circ\mu_3$	$s^{2}_{15}(3)$	0
ν	$4\gamma''^{2}_{10}$	(1)	$\pm 4\gamma''^{3}_{14}$	0	$64\gamma''^{4}_{18}$
σ'	$2s^{2}_{14}$				
σ			(2)	$\pm s^{2}_{21}(3)$	$\pm 8s^{4}_{22}$
ε	0	$\tau''^{1}_{3}(2)\circ\bar{\varepsilon}_3$	0	0	
$\bar{\nu}$	0	$\tau''^{1}_{3}(2)\circ\bar{\bar{\varepsilon}}_3$	0	$s^{2}_{15}(3)\circ\sigma_{15}$	
μ	0	$\tau''^{1}_{3}(2)\circ\mu_3\circ\sigma_{12}$ mod $\tau''^{1}_{3}(2)\circ\eta_3\circ\bar{\varepsilon}_4$	$\tau''^{1}_{3}(3)\circ\bar{\mu}_3$		
ζ	$4s^{2}_{18}$	$4s^{2}_{21}$ mod $\tau''^{1}_{3}(2)\circ\mu'\circ\sigma_{14}$	$\pm 4s^{2}_{22}(3)$		
κ	0 mod $\tau''^{1}_{3}(2)\circ\mu'\circ\sigma_{14}$				
ρ''	$4s^{2}_{22}$				

α	$\gamma''^{4}_{18}\circ\alpha$	$\omega''^{5}_{19}\circ\alpha$
η	$s^{3}_{19}(4)$	$\tau''^{1}_{3}(5)\circ\bar{\mu}_3$
ν		$\pm 128\gamma''^{5}_{22}$

(1) $2\gamma''^{2}_{10}\circ\nu_{10}=\tau''^{1}_{3}(2)\circ\varepsilon'$

(2) $2\omega''^{3}_{11}\circ\sigma_{11}=-s^{2}_{18}(3)$.

α	$s^{2}_{14}\circ\alpha$	$s^{2}_{15}\circ\alpha$	$s^{2}_{18}\circ\alpha$	$s^{2}_{21}\circ\alpha$	$s^{3}_{19}\circ\alpha$
η	0		$\tau''^{1}_{3}(2)\circ\mu_3\circ\sigma_{12}$ mod $\tau''^{1}_{3}(2)\circ\eta_3\circ\bar{\varepsilon}_4$	$s^{2}_{15}\circ\sigma_{15}$ mod $2\tau''^{1}_{3}(2)\circ\bar{\mu}'$	
η^2		$4\gamma''^{2}_{10}\circ\sigma_{10}$			$2s^{3}_{21}(3)$
ν	$\pm 2\gamma''^{2}_{10}\circ\sigma_{10}$	0	$4s^{2}_{21}$ mod $\tau''^{1}_{3}(2)\circ\mu'\circ\sigma_{14}$		$s^{2}_{15}(3)\circ\sigma_{15}$
σ	$2s^{3}_{21}$				
ε	0				
ν	0				

Table IV Original generators of $\pi_k(U_n)$ ($k \leqq 22$) (2-primary components)

n	α	$\alpha \in \{\beta, \gamma, \delta\}$	$p'_*\alpha$	$d\alpha$
1	τ'^1_1		ι_1	
2	τ'^2_3		ι_3	
3	ω'^3_5	$\{\tau'^2_3(3), \eta_3, 2\iota_4\}$	$2\iota_5$	
	u^3_{11}	$\{\tau'^2_3(3), \eta_3, \nu^2_4\}$	ν^2_5	$2u^3_{11}=\tau'^2_3(3)\circ\varepsilon_3$
	u^3_{12}	$\{\omega'^3_5, 4\nu_5, \nu_8\}$	σ'''	$2u^3_{12}=\tau'^2_3(3)\circ\mu_3$
	u^3_{16}	$\{\tau'^2_3(3), \eta_3, \nu_4\circ\bar{\nu}_7\}$	$\nu_5\circ\bar{\nu}_8$	$2u^3_{16}=0$
	u^3_{18}	$\{u^3_{10}, 2\iota_{10}, 8\sigma_{10}\}_1$	$\nu_5\circ\eta_8\circ\mu_9$	$2u^3_{18}=0$
	u^3_{20}	$\{u^3_{12}, 4\iota_{12}, 4\sigma_{12}\}_3$	ρ^{IV}	$2u^3_{10}=\tau'^2_3(3)\circ\bar{\mu}_3$
4	ω'^4_7	$\{i'^{4,3}, \tau'^2_3(3)\circ\nu', 2\iota_6\}$	$2\iota_7$	
	γ'^4_8	$\{i'^{4,3}, \tau'^2_3(3)\circ\nu', \eta_6\}$	η_7	$2\gamma'^4_8=\omega'^3_5(4)\circ\nu_5$
	u^4_{14}		σ'	$2u^4_{14}=\omega'^4_7\circ\sigma'$, $8u^4_{14}=\tau'^2_3(4)\circ\mu'$
	u^4_{16}	$\{\gamma'^4_8, 8\iota_8, E\sigma'\}_1$	μ_7	$2u^4_{16}=\omega'^3_5(4)\circ\zeta_5$
	u^4_{22}	$\{u^4_{14}, 8\iota_{14}, 2\sigma_{14}\}_5$	ρ''	$2u^4_{22}=\omega'^4_7\circ\rho''$, $8u^4_{22}=\tau'^2_3(4)\circ\bar{\mu}'$
5	ω'^5_9	$\{i'^{5,4}, \gamma'^4_8, 8\iota_8\}$	$8\iota_9$	
	u^5_{12}	$\{i'^{5,4}, \gamma'^4_8, 4\nu_8\}$	$4\nu_9$	$2u^5_{12}=u^3_{12}(5)$
	u^5_{19}	$\{i'^{5,4}, \gamma'^4_8, 4\sigma_8\circ\nu_{15}\}$ $\{u^4_{14}(5), 4\nu_{14}, \eta_{17}\}_1$	$4\sigma_9\circ\nu_{16}$	$2u^5_{19}=0.$
	u^5_{20}	$\{u^5_{12}, 2\sigma_{12}, 8\iota_{19}\}_3$	$4\zeta_9$	$2u^5_{20}=\pm u^3_{20}(5)$
6	ω'^6_{11}	$\{i'^{6,5}, \gamma'^5_{10}, 8\iota_{10}\}^{(*)}$	$8\iota_{11}$	
	γ'^6_{12}	$\{i'^{6,5}, \gamma'^5_{10}, \eta_{10}\}$	η_{11}	$2\gamma'^6_{12}=u^5_{12}(6)$
	u^6_{18}	$\{i'^{6,5}, \gamma'^5_{10}, 4\sigma_{10}\}$	$4\sigma_{11}$	$2u^6_{18}=\pm\omega'^6_{11}\circ\sigma_{11}$, $4u^6_{18}=\pm l^6\circ s^2_{18}(3)$
	u^6_{19}	$\{\gamma'^6_{12}, \nu^2_{12}, 2\iota_{18}\}_1$	ε_{11}	$2u^6_{19}=u^5_{19}(6)$
	u^6_{20}	$\{\gamma'^6_{12}, 2\sigma_{12}, 8\iota_{19}\}_3$	μ_{11}	$2u^6_{20}\equiv u^5_{20}(6)$ mod $\tau'^2_3(6)\circ\bar{\mu}_3$
7	ω'^7_{13}	$\{i'^{7,6}, \gamma'^6_{12}, 16\iota_{12}\}$	$16\iota_{13}$	
	u^7_{16}	$\{i'^{7,6}, \gamma'^6_{12}, 2\nu_{12}\}$	$2\nu_{13}$	$4u^7_{16}=\pm\omega'^5_9(7)\circ\sigma_9$
	u^7_{19}	$\{i'^{7,6}, \gamma'^6_{12}, \nu^2_{12}\}$	ν^2_{13}	$2u^7_{19}=u^6_{19}(7)$
	u^7_{20}	$\{i'^{7,6}, \gamma'^6_{12}, 2\sigma_{12}\}$	$2\sigma_{13}$	$8u^7_{20}\equiv -u^6_{20}(7)$ mod $\tau'^2_3(7)\circ\bar{\mu}_3$

Table IV Continued

n	α	$\alpha\in\{\beta,\gamma,\delta\}$	$p'_*\alpha$	$d\alpha$
	ω'^{8}_{15}	$\{i'^{8,7},\ \gamma'^{7}_{14},\ 16\iota_{14}\}$	$16\iota_{15}$	
8	γ'^{8}_{16}	$\{i'^{8,7},\ \gamma'^{7}_{14},\ \eta_{14}\}$	η_{15}	$2\gamma'^{8}_{16}=u^{7}_{16}(8)$
	u^{8}_{22}	$\{i'^{8,7},\ \gamma'^{7}_{14},\ 2\sigma_{14}\}$	$2\sigma_{15}$	$8u^{8}_{22}=xu^{4}_{22}(8)$ for some odd integer x.
9	ω'^{9}_{17}	$\{i'^{9,8},\ \gamma'^{8}_{16},\ 128\iota_{16}\}$	$128\iota_{17}$	
10	ω'^{10}_{19}	$\{i'^{10,9},\ \gamma'^{9}_{18},\ 128\iota_{18}\}^{(*)}$	$128\iota_{19}$	
	γ'^{10}_{20}	$\{i'^{10,9},\ \gamma'^{9}_{18},\ \eta_{18}\}$	η_{19}	$2\gamma'^{10}_{20}=u^{7}_{16}(10)$
11	ω'^{11}_{21}	$\{i'^{11,10},\ \gamma'^{10}_{20},\ 256\iota_{20}\}$	$256\iota_{21}$	

(*) $\gamma'^{5}_{10}=i'^{5,4}\circ l^4\circ\gamma''^{2}_{10},\ \gamma'^{9}_{18}=i'^{9,8}\circ l^8\circ\gamma''^{4}_{18}$.

Table V Compositions (on $\pi_k(U_n)$ $(k\leqq 22)$) (2-primary components)

α	$\omega'^{3}_{5}\circ\alpha$	$\omega'^{4}_{7}\circ\alpha$	$\omega'^{5}_{9}\circ\alpha$	$\omega'^{6}_{11}\circ\alpha$	$\omega'^{7}_{13}\circ\alpha$
η	$\tau'^{2}_{3}(3)\circ\nu'$	$4\gamma'^{4}_{8}$	$4i'^{5,4}\circ l^4\circ\gamma''^{2}_{10}$	$4u^{5}_{12}(6)$	$8u^{4}_{14}(7)$
ν		$2l^4\circ\gamma''^{2}_{10}$	$-2u^{5}_{12}$	$\pm 4u^{4}_{14}(6)$	$\pm 16u^{7}_{16}$
σ'''	0				
σ'		$2u^{4}_{14}$			
σ			(*)	$\pm 2u^{6}_{18}$	$8u^{7}_{20}$ mod $16u^{7}_{20}$
$\sigma\circ\eta$			0		
ε	$\tau'^{2}_{3}(3)\circ\varepsilon'$	$4\gamma'^{4}_{8}\circ\sigma_8$	0	0	0
$\bar{\nu}$		0	0	0	0
μ	$\tau'^{2}_{3}(3)\circ\mu'$	$4u^{4}_{16}$	$4i'^{5,4}\circ l^4\circ s^{2}_{18}$	$8u^{6}_{20}$	$8u^{4}_{22}(7)$
$\sigma\circ\nu$			0		
$\nu\circ\bar{\nu}$	0				
$\nu\circ\mu$	0				
ζ		$2l^4\circ s^{2}_{18}$	$-2u^{5}_{20}$	$\pm 4u^{4}_{22}(6)$	
$\nu\circ\sigma\circ\nu$	$\tau'^{2}_{3}(3)\circ\bar{\varepsilon}_3$				
κ		$\gamma'^{4}_{8}\circ\sigma_8\circ\nu^{2}_{15}$ mod $u^{4}_{14}\circ\sigma_{14}$			
$\nu\circ\zeta$	0				
$\bar{\varepsilon}$	$\tau'^{2}_{3}(3)\circ\bar{\varepsilon}'$	0			
ρ^{IV}	0				
ρ''		$2u^{4}_{22}$			
$\bar{\mu}$	$\tau'^{2}_{3}(3)\circ\bar{\mu}'$				

(*) $2\omega'^{5}_{9}\circ\sigma_9=-u^{4}_{16}(5)$.

Table V Continued

α	$\omega'^{8}_{15}\circ\alpha$	$\omega'^{9}_{17}\circ\alpha$	$\omega'^{10}_{19}\circ\alpha$	$\omega'^{11}_{21}\circ\alpha$
η	$64\gamma'^{8}_{16}$	$64\gamma'^{9}_{18}$	$128\gamma'^{10}_{20}$	$128\gamma'^{11}_{22}$
ν	$\pm 32 l^{8}\circ\gamma''^{4}_{18}$	$\pm 32 u^{7}_{20}(9)$	$\pm 64 l^{10}\circ\gamma''^{5}_{22}$	
σ	$8u^{8}_{22}$ mod $16u^{8}_{22}$			

α	$\gamma'^{4}_{8}\circ\alpha$	$\gamma'^{6}_{12}\circ\alpha$	$\gamma'^{8}_{16}\circ\alpha$	$\gamma'^{10}_{20}\circ\alpha$
η^2	$u^{3}_{10}(4)+4l^{4}\circ\gamma''^{2}_{10}$	$u^{4}_{14}(6)+xl^{6}\circ\gamma''^{3}_{10}$	$yu^{6}_{18}(8)+4l^{8}\circ\gamma''^{4}_{18}$	$mu^{8}_{22}(10)\pm 2l^{10}\circ\gamma''^{5}_{22}$
ν	$\pm u^{3}_{11}(4)$	$u^{4}_{14}(6)\circ\eta_{14}$	$zu^{7}_{19}(8)^{(*)}$	
$E\sigma'$	$2\gamma'^{4}_{8}\circ\sigma_8$			
$\sigma\circ\eta$	$u^{4}_{14}\circ\eta^{2}_{14}+\gamma'^{4}_{8}\circ\varepsilon_8+\gamma'^{4}_{8}\circ\bar{\nu}_8$ mod $u^{3}_{10}(4)$			
$\varepsilon\circ\eta$	$u^{3}_{11}(4)\circ\nu^{2}_{11}+u^{3}_{10}(4)\circ\sigma_{10}+4u^{4}_{14}\circ\nu_{14}$	$2u^{4}_{14}(6)\circ\sigma_{14}$		
$\mu\circ\eta$	$4l^{4}\circ s^{2}_{18}+u^{3}_{18}(4)$			
ζ	$\pm u^{3}_{12}(4)\circ\sigma_{12}$ mod $u^{3}_{16}(4)\circ\nu_{16}$			
$\bar{\nu}\circ\nu$	$u^{3}_{16}(4)\circ\nu_{16}$ mod $2u^{3}_{12}(4)\circ\sigma_{12}$			
σ^2	$u^{4}_{14}\circ\varepsilon_{14}+u^{4}_{14}\circ\bar{\nu}_{14}$ mod $\{\tau'^{2}_{3}(4)\circ\bar{\mu}'\}$ $\oplus\{2\gamma'^{4}_{8}\circ\kappa_8\}$			

(*) x, y, z and m are some odd integers.

α	$u^{3}_{10}\circ\alpha$	$u^{3}_{11}\circ\alpha$	$u^{3}_{12}\circ\alpha$	$u^{3}_{16}\circ\alpha$	$u^{3}_{18}\circ\alpha$	$u^{3}_{20}\circ\alpha$
η	0	0	0	$u^{3}_{11}\circ\nu^{2}_{11}$	0	0
ν	0		$2\omega'^{3}_{5}\circ\nu_5\circ\sigma_8$		0	
ν^2		$u^{3}_{10}\circ\eta_{16}$		$\omega'^{3}_{5}\circ\nu_5\circ\kappa_8$		
$\varepsilon,\ \bar{\nu}$	0	0	0			
μ	0	0	0			
$\sigma\circ\nu$	0	0				
ζ	0	0				

α	$u^{6}_{18}\circ\alpha$	$u^{6}_{19}\circ\alpha$	$u^{6}_{20}\circ\alpha$	$u^{7}_{16}\circ\alpha$	$u^{7}_{19}\circ\alpha$	$u^{7}_{20}\circ\alpha$
η	$u^{6}_{19}(6)$	$\gamma'^{6}_{12}\circ\varepsilon_{12}$	$\gamma'^{6}_{12}\circ\mu_{12}$ mod $2u^{6}_{14}\circ\sigma_{14}$	0	0	$u^{4}_{14}(7)\circ\sigma_{14}$
ν	0 mod $2u^{4}_{14}(6)\circ\sigma_{14}$	0		$\pm u^{6}_{19}(7)$		

Table V Continued

α	$u^4_{16}\circ\alpha$	$l^4\circ\gamma''^2_{10}\circ\alpha$	$u^5_{13}\circ\alpha$	$u^5_{19}\circ\alpha$	$u^5_{20}\circ\alpha$
η	$\gamma'^4_8\circ\mu_8 \bmod \{u^3_{11}(4)\circ\nu^2_{11}\} \oplus\{u^3_{10}(4)\circ\sigma_{10}\}$	$\tau'^2_3(4)\circ\varepsilon_3$	$2i'^{5,4}\circ l^4\circ\gamma''^2_{10}\circ\nu_{10}$	0	$8u^4_{14}(5)\circ\sigma_{14}$
ν			0	0	
ν^2	$0 \bmod 2\gamma'^4_8\circ\kappa_8$	$u^3_{10}(4)+\gamma'^4_8\circ\bar{\nu}_8 \bmod u^4_{14}\circ\eta^2_{14}$			
σ		$xu^4_{14}\circ\nu_{14}$ (*)	0		
ε, $\bar{\nu}$		$\tau'^2_3(4)\circ\bar{\varepsilon}_3$	0		
μ		$2u^3_{12}(4)\circ\sigma_{12}$	$8u^4_{14}(5)\circ\sigma_{14}$		
ζ		$\pm 4u^4_{14}\circ\sigma_{14}$			

(*) x is an odd integer.

The corrigenda of the previous paper [3]

Page	Line	Instead of	Read
51	23	(A_1, A_2, A_3)	(A_2, A_3, A_4)
63	24	$\pi_{4n+6}(Sp_{n+1})$	$\pi_{4n+6}(Sp_{n+1})/Z_8$
67	22-23		See (6.1) of this paper
69	3	$\gamma'^{n+1}_{2n+2}\circ\nu_{2n+2}$	$x\gamma'^{n+1}_{2n+2}\circ\nu_{2n+2}$ *for some odd integer* x.
Page 69, Line 9—Page 70, Line 11			See the first paragraph of §6 of this paper
70	24-27		See (3.2)(5) and (3.2)(6) of this paper
72	4	η'_9	η_9
72	6	$2u^5_{12}$	$-2u^5_{12}$
75	26	$\pm 2r^5_{10}$	$\pm 2r^5_{10}(6)$
76	24	$+4r^6_{10}$	$\bmod 4r^6_{10}$
76	31	$k^6\circ\omega'^3_6$	$k^6\circ\omega'^3_5$

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