J. Math. Sci. Univ. Tokyo
14 (2007), 69–97.

Properties of Minimal Charts and Their Applications I

By Teruo Nagase and Akiko Shima

Abstract. We study surface braids using the charts with minimal complexity. We introduce several terminology to describe minimal charts and investigate properties of minimal charts. We shall show that in a minimal chart, there exist at least three white vertices in the interior of any lens.

1. Introduction

Kamada introduced a method to describe surface braids as oriented lableded graphs in a disk, called charts ([5],[6],[7]) (see Section 2 for precise definition of charts). In a chart there are three kinds of vertices; white vertices, crossings and black vertices. In this paper, we investigate properties of minimal charts which we need to prove that there is no minimal chart with exactly seven white vertices.

We proved that there is no minimal chart with exactly five vertices ([8]). Hasegawa proved that there exists a minimal chart with exactly six white vertices ([2]). This chart represents the surface braid whose closure is ambient isotopic to a 2-twist spun terfoil.

We investigated minimal charts with exactly four white vertices ([4]). In minimal charts with exactly four white vertices, there are two classes: The first class is obtained from the disjoint union of free edges, hoops and one '4-chart' where the '4-chart' represents the surface braid whose closure is a tours link. The second class is obtained from the union of one '3-chart', rings, free edges and hoops. Here a *ring* is a simple closed curve consisting of the same labeled edges which contains a crossing but does not contain any white vertices.

2000 *Mathematics Subject Classification*. Primary 57Q45; Secondary 57Q35.
The second author is partially supported by Grant-in-Aid for Scientific Research (No.16540082), Ministry of Education, Science and Culture, Japan.

Any 3-chart (with edges of label 1 and 2) is obtained from a chart without white vertices by C-moves ([5]). Hence any chart in the second class is not a minimal chart if there are neither rings, free edges nor hoops. However, Hasegawa showed that some charts in the second class are minimal charts with four white vertices, but all charts in the second class represent surface braids whose closures are ribbon surfaces ([3]).

Let Γ be a chart. For each label m, we denote by Γ_m the 'subgraph' of Γ consisting of edges of label m and their vertices. In this paper,

> *crossings are vertices of* Γ *but we do not consider crossings as vertices of* Γ_m. The vertices of Γ_m are white vertices and black vertices.

An *edge* of Γ_m is the closure of a connected component of the set obtained by taking out all white vertices from Γ_m.

Among six short arcs in a small neighborhood of a white vertex, a center arc of each three consecutive arcs oriented inward or outward is called *a middle arc* at the white vertex (see Figure 2). The other arcs are called *non-middle arcs*. There are two middle arcs in a small neighborhood of each white vertex.

Let Γ be a chart. Let D be a disk such that ∂D consists of an edge e_1 of Γ_m and an edge e_2 of Γ_{m+1} and that any edge containing a white vertex in e_1 does not intersect the open disk $Int(D)$. Let w_1 and w_2 be the white vertices in e_1. If the disk D satisfies one of the following conditions, then D is called *a lens of type* $(m, m+1)$ (see Figure 1):

(1) Neither e_1 nor e_2 contains a middle arc.

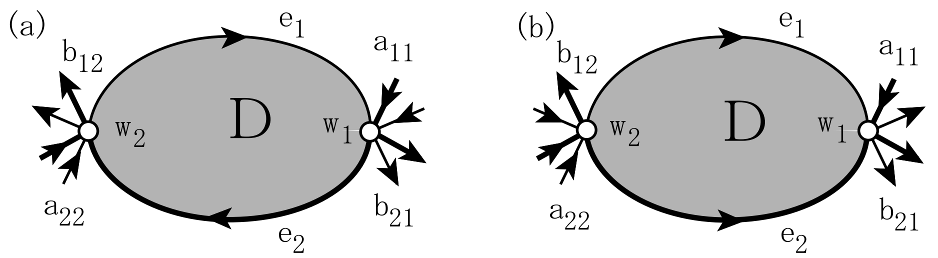

Figure 1. (a) is of type 1 and (b) is of type 2.

(2) One of the two edges e_1 and e_2 contains middle arcs at both white vertices w_1 and w_2.

If D satisfies the above condition (1) (resp. (2)), then the lens D is called *a lens of type* 1 (*resp. type of* 2). We also say that D is a *lens of* Γ. The two edges e_1 and e_2 are called *the boundary arcs* of the lens D. Note that a lens is a bigon with crossings on the boundary satisfying the above conditions (1) or (2).

The following is the main result in this paper:

THEOREM 1.1. *If there exists a lens of type* $(m, m+1)$ *in a minimal chart, then in the interior of the lens there exists a white vertex contained in an edge of label* $s \leq m-1$ *and a white vertex contained in an edge of label* $t \geq m+2$. *Furthermore there exist at least three white vertices in the interior of the lens.*

This paper is organized as follows. In Section 2, we give notations and definitions. In Section 3, we prove Disk Lemma (Lemma 3.2) which simplifies the intersection of a disk and a chart. In Section 4, we prove Shifting Lemma (Lemma 4.2) which gives conditions to move a white vertex to the other place by C-moves (see Section 2 for precise definition of C-moves). In Section 5, we examine labels of edges intersecting the boundary of a lens. In Section 6, we give the proof of Theorem 1.1 by the help of Shifting Lemma (Lemma 4.2) and Lemma 5.2.

A chart Γ is of *type* $(m; n_1, n_2, \ldots, n_k)$ or of *type* $(n_1, n_2, \ldots, n_k)$ briefly if it satisfies the following three conditions:

(1) For each $i = 1, 2, \ldots, k$, the chart Γ contains exactly n_i white vertices in $\Gamma_{m+i-1} \cap \Gamma_{m+i}$.

(2) If $i < 0$ or $i > k$, then Γ_{m+i} does not contain any white vertices.

(3) Both of the two subgraphs Γ_m and Γ_{m+k} contain at least one white vertex.

Note that $n_1 \geq 1$ and $n_k \geq 1$ by the condition (3).

We shall show that there is no minimal chart Γ with exactly seven white vertices as follows. In [9] we investigate types of minimal charts. We shall show that any minimal chart with exactly seven white vertices is of type

$(7), (5,2), (4,3), (3,2,2)$ or $(2,3,2)$ if necessary we change the label $m+i$ into $m+k-i$ for all label i. We shall show that any minimal chart with exactly seven white vertices possesses no lenses by the help of Theorem 1.1.

In [9] we investigate minimal charts with loops. Here a *loop* is a closed edge of Γ_m containing exactly one white vertex. We shall show that there does not exist any loop in a minimal chart with exactly seven white vertices, [9].

Finally we shall show that there is no minimal chart of type $(7), (5,2)$, $(4,3), (3,2,2)$ nor $(2,3,2)$ in [9].

In this paper, for a set X we denote the interior of X, the boundary of X and the closure of X by $Int(X)$, ∂X and $Cl(X)$ respectively.

2. Preliminaries

In this section, we define charts and notations.

Let n be a positive integer. An *n-chart* is an oriented labeled graph in a disk, which may be empty or have closed edges without vertices, called *hoops*, satisfying the following four conditions:

(1) Every vertex has degree 1,4, or 6.

(2) The labels of edges are in $\{1, 2, \dots, n-1\}$.

(3) In a small neighborhood of each vertex of degree 6, there are six short arcs, three consecutive arcs are oriented inward and the other three are outward, and these six are labeled i and $i+1$ alternately for some i, where the orientation and the label of each arc are inherited from the edge containing the arc.

(4) For each vertex of degree 4, diagonal edges have the same label and are oriented coherently, and the labels i and j of the diagonals satisfy $|i-j| > 1$.

A vertex of degree 1, 4, and 6 is called a *black vertex,* a *crossing*, and a *white vertex* respectively (see Figure 2).

C-moves are local modification of charts in a disk as shown in Figure 3 (see [1], [7] for the precise definition). Kamada originally defined CI-moves as follows (C-I-moves are special cases of CI-moves): A chart Γ is obtained from a chart Γ' by a *CI-move*, if there exists a disk D such that

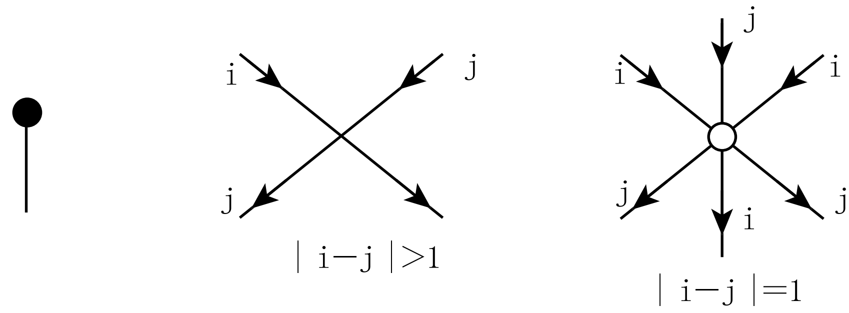

Figure 2.

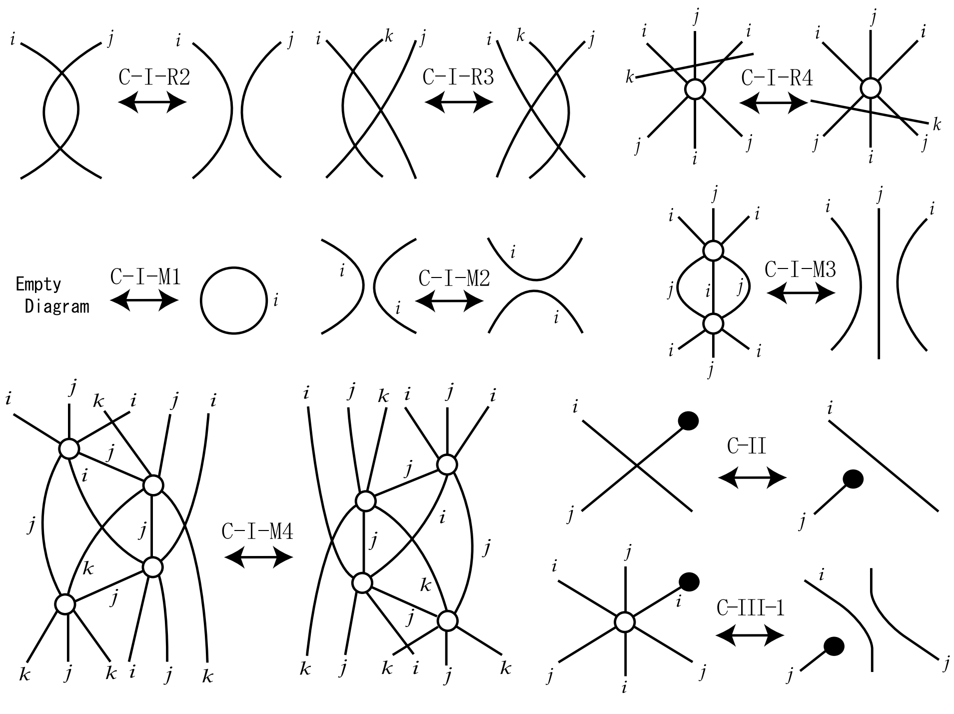

Figure 3. For the C-III-1 move, the edge containing the black vertex does not contain a middle arc in the left figure.

(1) the two charts Γ and Γ' intersect the boundary of D transversely or do not intersect the boundary of D,

(2) $\Gamma \cap D^c = \Gamma' \cap D^c$, and

(3) neither $\Gamma \cap D$ nor $\Gamma' \cap D$ contains a black vertex,

where $(\cdots)^c$ is the complement of the disk $(\cdots)$.

Two charts are *C-move equivalent* if there exists a finite sequence of C-moves which modify one of the two charts to the other.

An edge of Γ_m is called a *free edge* if it has two black vertices. An edge of Γ_m is called a *terminal edge* if it has a white vertex and a black vertex. Note that free edges, terminal edges, and loops may contain crossings of Γ.

For each chart Γ, let $w(\Gamma)$ and $f(\Gamma)$ be the number of white vertices, and the number of free edges respectively. The pair $(w(\Gamma), -f(\Gamma))$ is called the *complexity* of the chart. A chart is called a *minimal chart* if its complexity is minimal among the charts C-move equivalent to the chart with respect to the lexicographic order of pairs of integers.

In the following lemma, we investigate the difference of a chart in a disk and in a 2-sphere.

LEMMA 2.1. *Let Γ and Γ' be charts in a disk D. Suppose that Γ is ambient isotopic to Γ' in the one point compactification of the open disk $Int(D)$, i.e. the 2-sphere. Then there exist hoops $C_1, C_2, \ldots, C_k$ in $Int(D)$ such that*

(1) *the chart Γ is obtained from $\Gamma' \cup (\bigcup_{i=1}^{k} C_i)$ by C-moves in the disk,*

(2) *the chart Γ' and hoops $C_1, C_2, \ldots, C_k$ are mutually disjoint, and*

(3) *each hoop C_i bounds a disk containing the chart Γ' in the disk D.*

PROOF. Consider the one point compactification S^2 of the open disk $Int(D)$. We have the point of infinity in S^2, denoted by ∞. Suppose that the charts Γ and Γ' are ambient isotopic in S^2. We can assume that the white vertices do not pass the point ∞ by the ambient isotopy. Suppose that an edge e moves over the point ∞ by the modification of the ambient isotopy. Let α be an arc connecting two points a and b in the edge e such that

(1) $\alpha \not\ni \infty$,

(2) $\alpha \cap e = \{a, b\}$, and

(3) let e' be the subarc of e bounded by the two points a and b, then $\alpha \cup e'$ bounds a disk containing the point ∞ in its interior.

Applying a C-I-M2 move along α between the points a and b to get a new chart with a hoop around the point ∞. As a result, the edge e passes the point ∞ so that e deforms to $(e - e') \cup \alpha$. A new hoop is born each time when an edge moves over the point ∞ so that we get concentric hoops center at the point ∞ at the end of the modification. We complete the proof of Lemma 2.1. □

Lemma 2.1 says that we can move the point ∞ to the any complementary domain of the chart.

To make the argument simple, we assume that the charts lie on the 2-sphere instead of the disk. In this paper,

all charts are contained in the 2*-sphere* S^2.

We have the special point in the 2-spehre S^2, called the point at infinity, denoted by ∞. In this paper, all charts are contained in a disk such that the disk does not contain the point at infinity ∞.

A *ring* is a closed edge of Γ_m containing crossings but not containing a white vertex, and a *hoop* is a closed edge of Γ without vertices (hence without crossings, neither). A hoop is *simple* if one of the complementary domain of the hoop does not contain any white vertices. We can assume that all minimal charts Γ satisfy the following six conditions:

Assumption 1. *Any terminal edge of* Γ_m *does not contain a crossing. Hence any terminal edge of* Γ_m *is a terminal edge of* Γ *and any terminal edge of* Γ_m *contains a middle arc.*

For, using C-II moves and contracting the edge, we can eliminate the crossings of the edge. If a terminal edge does not contain a middle arc, then the white vertex of the edge can be eliminated by a C-III-1 move. This contradicts the fact that Γ is a minimal chart.

ASSUMPTION 2. *Any free edge of Γ_m does not contain a crossing. Hence any free edge of Γ_m is a free edge of Γ.*

For, using C-II moves and contracting the edge, we can eliminate the crossings of the edge. Hence any free edge of Γ_m does not contain any crossings.

ASSUMPTION 3. *All free edges and simple hoops in Γ are moved into a small neighborhood U_∞ of the point at infinity ∞.*

For, by Assumption 2 any free edge of Γ_m does not contain crossings. By using C-I-M2 moves, we can move free edges and simple hoops into the neighborhood U_∞ of the point at infinity ∞ (see Figure 4).

ASSUMPTION 4. *Each complementary domain of any ring must contain at least one white vertex.*

For, if a complementary domain D of a ring does not contain white vertices, then move all the free edges to the other complementary domain by applying C-I-M2 moves so that D does not contain white vertices nor black vertices. Hence we can change the ring to a simple hoop by a CI-move without increasing the complexity. The number of rings is reduced. Hence we assume that each complementary domain of any ring must contain at least one white vertex.

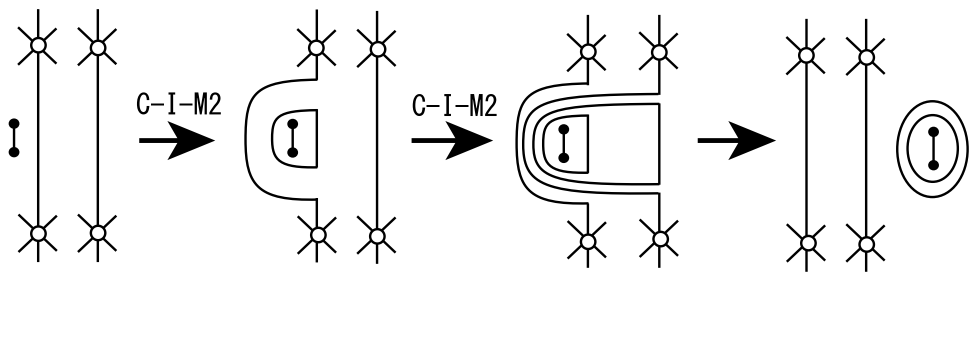

Figure 4.

Assumption 5. *Hence we assume that the subgraph obtained from* Γ *by omitting free edges and simple hoops does not meet the set* U_∞*. Also we assume that* Γ *does not contain free edges nor simple hoops, otherwise mentioned. Therefore we can assume that if an edge of* Γ_m *contains a black vertex, then it is not a free edge but a terminal edge and that each complementary domain of any hoops and rings of* Γ *contains a white vertex, otherwise mentioned.*

Assumption 6. *The point at infinity* ∞ *is moved in any complementary domain of* Γ*.*

For, by Lemma 2.1, we can move the point at infinity ∞ in any complementary domain of Γ increasing some simple hoops $C_1, C_2, \ldots, C_k$ in the neighborhood U_∞ of the point at infinity ∞.

In our argument, we often need a name for an unnamed edge by using a given edge and a given white vertex. For the convenience, we use the following naming: Let e', e_i, e'' be three consecutive edges containing a white vertex w_j. Here, the two edges e' and e'' are unnamed edges. There are six arcs in a neighborhood U of the white vertex w_j. If the three arcs $e' \cap U$, $e_i \cap U$, $e'' \cap U$ lie anticlockwisely around the white vertex w_j in this order, then e' and e'' are denoted by a_{ij} and b_{ij} respectively (see Figure 5). Note that, when we consider the edge e_i as the hour hand and each of a_{ij} and b_{ij} as the minute hand, the edge a_{ij} is the edge after e_i, and the edge b_{ij} is the edge before e_i. There is a possibility $a_{ij} = b_{ij}$ if they are contained in a loop.

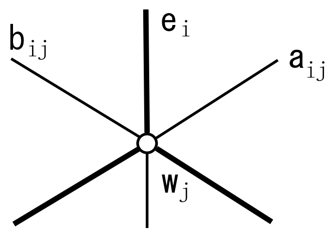

Figure 5.

3. Disk Lemma

In this section, we investigate the intersection of a disk and a chart which often appears in the future arguments.

Let Γ be a chart. If an object consists of some edges of Γ, arcs in edges of Γ and arcs around white vertices, then the object is called *a pseudo chart*.

Let Γ and Γ' be C-move equivalent charts. Suppose that a pseudo chart X of Γ is also a pseudo chart of Γ'. Then we say that Γ is modified to Γ' by *C-moves keeping X fixed*. In Figure 6, we give examples of C-moves keeping pseudo charts fixed.

Let D be a disk, α and β two arcs with $\partial D = \alpha \cup \beta$, and $\alpha \cap \beta = \partial\alpha = \partial\beta$. The pair (α, β) is called a *boundary arc pair* of the disk D.

The following lemma is a direct result of CI-moves.

LEMMA 3.1. *Let Γ be a minimal chart and α an arc in an edge e of Γ_m. Let D be a disk with a boundary arc pair (α, β). Let U be a neighbourhood of the disk D. If U does not contain any black vertices of Γ and if $Int(\beta) \cap (\Gamma_{m-1} \cup \Gamma_m \cup \Gamma_{m+1}) = \emptyset$, then without increasing the complexity of Γ we can replace the edge e by the arc $(e - \alpha) \cup \beta$ by C-moves in U.*

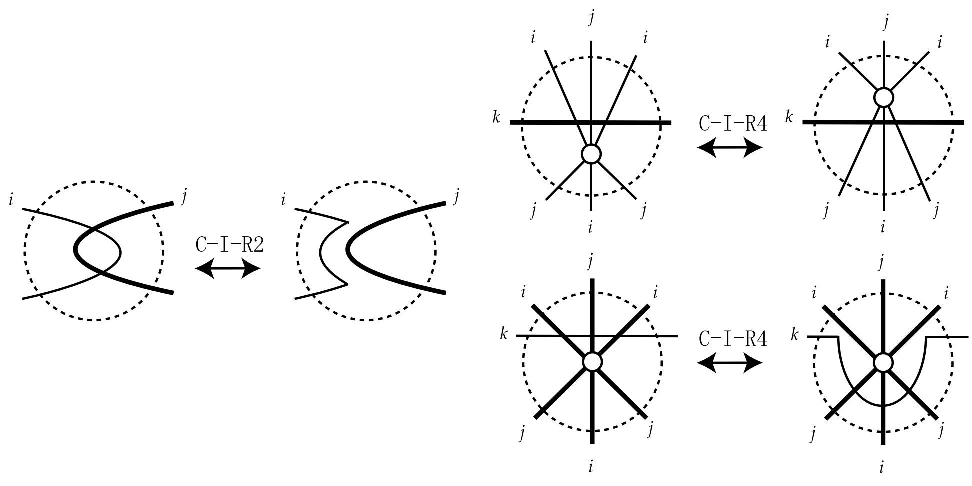

Figure 6. C-moves keeping thicken figures fixed.

Let Γ be a chart, and D the closure of an open disk U. Let α be a simple arc on $\partial U = D - U$. We call a simple arc γ in Γ_k a (D, α)-*arc* of label k provided that $\partial\gamma \subset \alpha$ and $Int(\gamma) \subset U$. If D is a disk then the arc γ separates D into two disks. One of the two disks does not meet β, where (α, β) is the boundary arc pair of D. The disk is called the α-*disk* divided by γ. If there is no (D, α)-arc in Γ, then the chart Γ is said to be (D, α)-*arc free.*

Let Γ be a chart and D the closure of an open disk U. Let α be a simple arc on ∂U. For each $k = 1, 2, \cdots$ let Σ_k be the pseudo chart which consists of all arcs in $D \cap \Gamma_k$ intersecting the set $Cl(\partial U - \alpha)$. Let $\Sigma_\alpha = \bigcup_k \Sigma_k$.

The following lemma is easy to prove. However we use often the lemma. Hence we give a proof of the lemma.

LEMMA 3.2. (*Disk Lemma*) *Let Γ be a minimal chart, and D a disk with a boundary arc pair (α, β). Suppose that the interior of α contains neither white vertices, isolated points of $D \cap \Gamma$, nor arcs of $D \cap \Gamma$. If the interior of D does not contain white vertices of Γ, then for any neighborhood V of α, there exists a (D, α)-arc free minimal chart Γ' obtained from the chart Γ by C-moves in $V \cup D$ keeping Σ_α fixed.*

PROOF. Let Γ' be a chart such that the number of (D, α)-arcs is minimal among minimal charts obtained from Γ by C-moves in $V \cup D$ keeping Σ_α fixed. Suppose that Γ' is not (D, α)-arc free. Then Γ' contains a (D, α)-arc (see Figure 7a).

Let β' be an innermost (D, α)-arc. Let D' be the α-disk divided by β'. Set $\alpha' = \alpha \cap D'$. Then Γ' does not contain (D', α')-arcs, and the pair (α', β') is a boundary pair of the disk D' (see Figure 7b). Since the disk D does not contain free edges by Assumption 5, neither does D'. Since $Int(D) \cup Int(\alpha)$ does not contain white vertices, neither does D'.

By Assumption 5, any black vertex is contained in a terminal edge of Γ'. Hence any black vertex in D' is contained in an edge intersecting α'. Thus we can contract the edge by an ambient isotopy of $D \cup V$ so that the black vertex is contained in the exterior of the disk D'. Hence we can assume that D' does not contain any black vertex.

Let e be the edge of a subgraph Γ_m containing β'. Let α'' be an arc in $V - D$ and β'' an arc in e with $\beta'' \supset \beta'$ and $\partial\alpha'' = \partial\beta''$ such that

(1) the arc α'' is almost parallel to the arc α' and very close to α, and

(2) $\alpha'' \cup \beta''$ bounds a disk D'' in $V \cup D$ (see Figure 7c).

Since Γ' does not contain (D', α')-arcs, we can assume that Γ' does not contain (D'', α'')-arcs. Hence if a proper arc γ in D'' with $\gamma \subset \Gamma'$ meets the arc α'', then the arc γ must meet the arc β''.

Thus by Lemma 3.1 we can replace the edge e by the arc $(e - \beta'') \cup \alpha''$ by C-moves in $V \cup D$ without increasing the complexity but decreasing the number of (D, α)-arcs (see Figure 7d). This contradicts the fact that the number of (D, α)-arcs of Γ' is minimal. Therefore Γ' does not contain any (D, α)-arcs. □

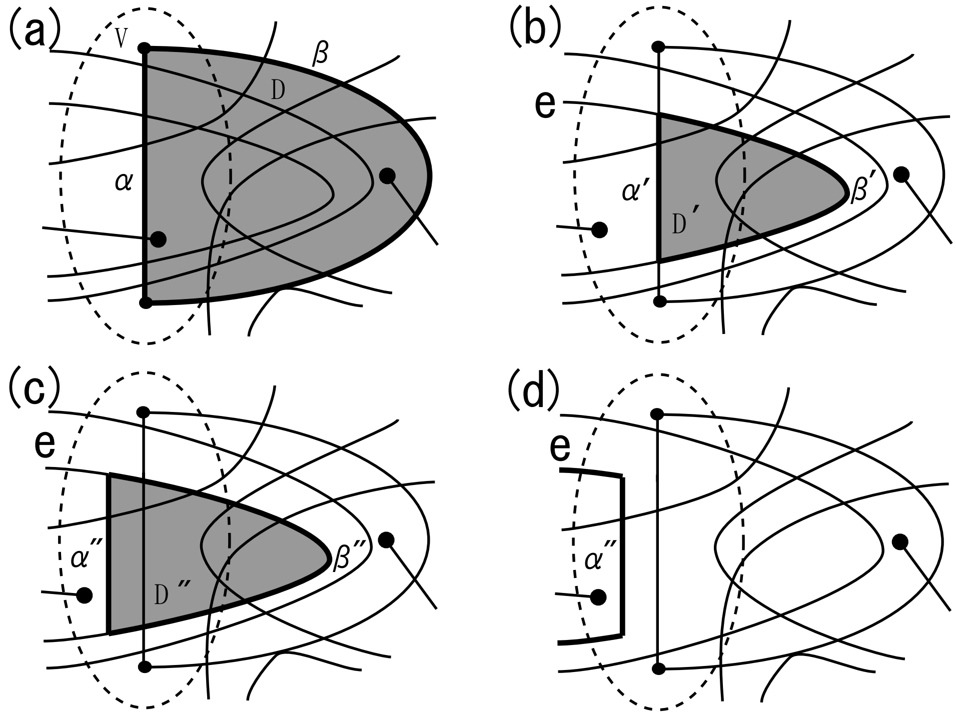

Figure 7.

We shall enrich Disk Lemma. Let Γ be a chart, and D the closure of an open disk U. A simple arc α in $\partial U = D - U$ is called an *admissible boundary arc* of D provided that $\alpha \cap Cl(\partial U - \alpha) = \partial\alpha$.

The following lemma is also called Disk Lemma.

LEMMA 3.3. (*Disk Lemma*) *Let* Γ *be a minimal chart and* D *the closure of an open disk* U. *Let* α *be an admissible boundary arc of* D. *Suppose that the interior of* α *contains neither white vertices, isolated points of* $Cl(U) \cap \Gamma$, *nor arcs of* $Cl(U) \cap \Gamma$. *If* U *does not contain white vertices of* Γ, *then for any neighborhood* V *of* α, *there exists a* (D, α)*-arc free minimal chart* Γ' *obtained from the chart* Γ *by C-moves in* $V \cup D$ *keeping* Σ_α *fixed.*

PROOF. Choose a simple arc β in $U \cup \alpha$ almost parallel to $Cl(\partial U - \alpha)$ such that $\alpha \cup \beta$ bounds a disk D' in $Cl(U)$. Apply Disk Lemma (Lemma 3.2) for the boundary arc pair (α, β) of the disk D'. $\square$

4. Shifting Lemma

In this section, we investigate conditions to move a white vertex to the other place. All lemmata in this section are special cases of CI-moves. We use the lemmata to specify situations and modifications of charts in the future arguments.

Let α be an arc, and p, q points in α. We denote by $\alpha[p, q]$ the subarc of α whose end points are p and q.

Let Γ be a chart and a, b, c mutually different three points of an arc α with $b \in \alpha[a, c]$. The arc $\alpha[a, c]$ is said to be *a bipartition arc of* Γ *with the partition point* b *with respect to the label* k provided that

(1) $\alpha[a, c] \cap Cl(\Gamma_k - \alpha[a, c]) \subset \{a, c\}$,

(2) $\alpha[a, b] \cap \Gamma_j = \emptyset$ for all j $(j > k)$, and

(3) $\alpha[b, c] \cap \Gamma_i = \emptyset$ for all i $(i < k)$.

LEMMA 4.1. (*Bipartition Lemma*) *Let* Γ *be a chart, and* D *a disk without any white vertices of* Γ. *Let* α *be a proper arc of the disk* D. *Let* a, c *be the end points of* α, *and* b *an interior point of* α. *Suppose that there exists an integer* m *with* $Cl(\Gamma_m - \alpha) \cap Int(D) = \emptyset$ *such that* $\Gamma_i \cap \alpha$ *is at most* *finitely many interior points of* α *for each* i $(i \neq m)$. *Then there exists a chart* Γ^* *obtained from* Γ *by C-I-R2 moves and C-I-R3 moves in* D *keeping* Γ_m *fixed such that*

(1) *the number of points in* $\Gamma_i \cap \alpha$ *is equal to the number of points in* $\Gamma_i^* \cap \alpha$ *for each* i, *and*

(2) *the arc* $\alpha[a, c]$ *is a bipartition arc of* Γ^* *with the partition point* b *with respect to the label* m.

PROOF. Let S be the set of charts obtained from Γ by C-I-R2 moves and C-I-R3 moves in D keeping Γ_m fixed which satisfies Condition (1).

For each element Γ' of S, let $n_1(\Gamma')$ be the number of the points in $\bigcup_{j \geq m+1} (\alpha[a, b] \cap \Gamma'_j)$, and $n_2(\Gamma')$ the number of the points in $\bigcup_{i \leq m-1} (\alpha[b, c] \cap \Gamma'_i)$. Let $n(\Gamma') = n_1(\Gamma') + n_2(\Gamma')$.

Let Γ'' be an element of S such that $n(\Gamma'')$ is minimal in S. We claim that $n(\Gamma'') = 0$.

Suppose that $n(\Gamma'') > 0$. Then $n_1(\Gamma'') > 0$ or $n_2(\Gamma'') > 0$. Suppose that $n_1(\Gamma'') > 0$. Assuming that the arc α is a line-segment, let d be the nearest point to b among the points in $\bigcup_{j \geq m+1} (\alpha[a, b] \cap \Gamma''_j)$. Then for an integer $k \geq m + 1$ the point d is in a connected component β of $\Gamma_k \cap D$. Since all the crossings between d and b are points in $\bigcup_{i \leq m-1} (\alpha[a, b] \cap \Gamma''_i)$, we can push the arc β toward $\alpha[b, c]$ by C-I-R2 moves and C-I-R3 moves in D (see Figure 8). Let Γ''' be the resulting chart. Then we have $\Gamma''' \in S$, $n_1(\Gamma''') = n_1(\Gamma'') - 1$, and $n_2(\Gamma''') = n_2(\Gamma'')$. Hence $n(\Gamma''') < n(\Gamma'')$. This contradicts the fact that $n(\Gamma'')$ is minimal among the elements in S. Hence we have $n_1(\Gamma'') = 0$. Similarly we have $n_2(\Gamma'') = 0$. Thus $n(\Gamma'') = 0$. Therefore Γ'' is a desired one. □

Note. In Lemma 4.1, if $\alpha[b, c] \cap \Gamma_i = \emptyset$ for all i $(i < m)$, then we can use C-moves keeping $\bigcup_{i \leq m} \Gamma_i$ fixed. If $\alpha[a, b] \cap \Gamma_j = \emptyset$ for all j $(j > m)$, then we can use C-moves keeping $\bigcup_{j \geq m} \Gamma_j$ fixed.

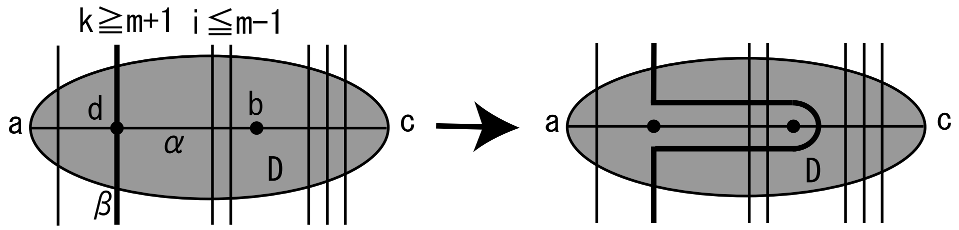

Figure 8.

Let Γ be a chart. Let α be an arc in an edge of Γ_m, and w a white vertex with $w \notin \alpha$. Suppose that there exists an arc β such that

(1) its end points are the white vertex w and an interior point p of the arc α, and

(2) the arc β is contained in Γ, or $\Gamma \cap \beta$ consists of at most finitely many points.

Then we say that *the white vertex w connects with the point p of α by the arc β.*

LEMMA 4.2. (*Shifting Lemma*) *Let Γ be a chart and α an arc in an edge of Γ_m. Let w be a white vertex of $\Gamma_k \cap \Gamma_h$ where $h = k+\varepsilon, \varepsilon \in \{+1, -1\}$. Suppose that the white vertex w connects with a point r of the arc α by an arc in an edge e of Γ_k. Suppose that one of the following two conditions is satisfied:*

(1) *$h > k > m$.*

(2) *$h < k < m$.*

Then for any neighborhood V of the arc $e[w, r]$ we can shift the white vertex w to $e - e[w, r]$ along the edge e by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in V keeping $\bigcup_{i<0} \Gamma_{k+i\varepsilon}$ fixed.

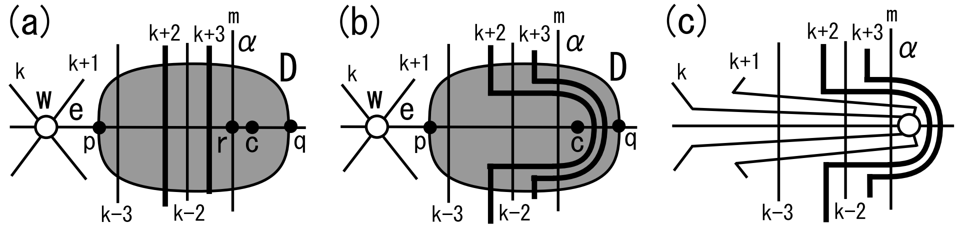

Figure 9. Lemma 4.2, Case (1): $k > m$ and $\varepsilon = +1$.

PROOF. Suppose the condition (1) $h > k > m$. Then we have $h = k+1$ and $\varepsilon = +1$. Since the edge e of Γ_k intersects the arc α of Γ_m, we have $k \geq m + 2$. Let p' be a point of the edge e near the point w such that $e[w, p']$ does not contain any crossings. Let D be a regular neighborhood of $e[p', r]$ in V. Let $\partial D \cap e = \{p, q\}$ where $p \in e[w, p']$. We can assume that $e[r, q]$ does not contain any crossing except the point r. Let c be a point in $Int(e[r, q])$ (see Figure 9a).

Applying Bipartition Lemma (Lemma 4.1) for the disk D, the arc $e[p, q]$ and the point c, we have a chart Γ' such that

(i) the chart Γ' is obtained from Γ by C-I-R2 moves and C-I-R3 moves in V keeping $\bigcup_{i \leq k} \Gamma_i$ fixed, and

(ii) the arc $e[p, q]$ is a bipartition arc of Γ' with the partition point c with respect to the label k (see Figure 9b).

Since $Int(e[w, q]) \cap (\Gamma_{k-1} \cup Cl(\Gamma_k - e[w, q]) \cup \Gamma_{k+1}) = \emptyset$, we have $\bigcup_{j \geq k-1} (Int(e[w, q]) \cap Cl(\Gamma'_j - e[w, q])) = \bigcup_{j \geq k+2} (Int(e[c, q]) \cap \Gamma'_j) \subset e[c, q]$. Since $h - i > k - i \geq 2$ for each $i \leq k - 2$ and since $h - m > k - m \geq 2$, we can shift the white vertex w to $e - e[w, r]$ by C-I-R2 moves, C-I-R3 moves, and C-I-R4 moves keeping $\bigcup_{i < k} \Gamma'_i$ fixed (see Figure 9c).

If the condition (2) $h < k < m$ is satisfied, then we can show the lemma by using a method similar to that in the case where $h > k > m$. □

Note. In Lemma 4.2, for any neighborhood V of the arc $e[w,r]$ we can shift the white vertex w to $e - e[w,r]$ along the edge e by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in V keeping $\bigcup_{i\neq h,k} \Gamma_i$ fixed if one of the following two conditions is satisfied:

(1) $h = k+1, k < m$, and $e[w,r] \cap \Gamma_{k+2} = \emptyset$.

(2) $h = k-1, k > m$, and $e[w,r] \cap \Gamma_{k-2} = \emptyset$.

In Lemma 4.2, we can move the arc α instead of the white vertex.

Corollary 4.3. *Let Γ be a chart and α an arc in an edge of Γ_m. Let w be a white vertex of $\Gamma_k \cap \Gamma_h$ where $h = k + \varepsilon, \varepsilon \in \{+1, -1\}$. Suppose that the white vertex w connects with a point r of the arc α by an arc in an edge e of Γ_k. Suppose that one of the following two conditions is satisfied:*

(1) *$h > k > m$.*

(2) *$h < k < m$.*

Then for any neighborhood V of the arc $e[w,r]$ we can push α to the other side of the white vertex w along e by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in V keeping $\bigcup_{-1\leq i} \Gamma_{k+i\varepsilon}$ fixed (*see Figure 10*).

Corollary 4.4. *Let Γ be a chart and α an arc in an edge of Γ_m. Let w be a white vertex of $\Gamma_k \cap \Gamma_h$ where $h = k + \varepsilon, \varepsilon \in \{+1, -1\}$. Suppose that*

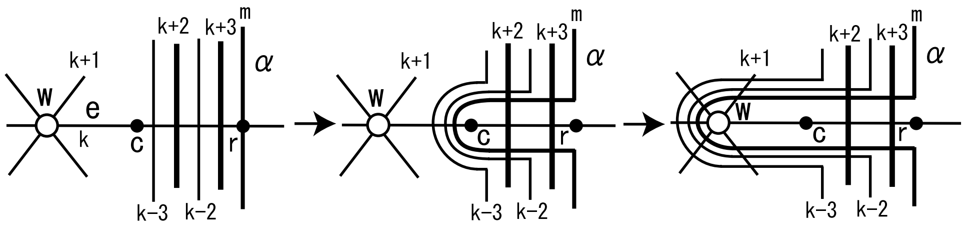

Figure 10. Corollary 4.3, Case (1): $h > k > m$.

the white vertex w *connects with a point* r *of the arc* α *by an arc in an edge* e *of* Γ_k*. Suppose that one of the following two conditions is satisfied:*

(1) $h = k+1, k < m,$ *and* $e[w,r] \cap \Gamma_{k+2} = \emptyset$.

(2) $h = k-1, k > m,$ *and* $e[w,r] \cap \Gamma_{k-2} = \emptyset$.

Then for any neighborhood V *of the arc* $e[w,r]$ *we can push* α *to the other side of the white vertex* w *along* e *by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in* V *keeping* $\bigcup_{i\leq 2} \Gamma_{k+i\varepsilon}$ *fixed* (*see Figure 11*).

PROOF. Case (1): Let c be a point of the edge e near the white vertex w so that $Int(\alpha[w,c])$ does not contain any crossings. Let D be a regular neighborhood of $e[c,r]$ in V. Let $\partial D \cap e = \{p,q\}$ where $p \in e[w,c]$.

Since r is a crossing in $\Gamma_m \cap \Gamma_k$, we have $m \geq k+2$. And $e[w,r] \cap \Gamma_{k+2} = \emptyset$ implies that $r \notin \Gamma_{k+2}$. Hence we have $m \geq k+3$.

By Bipartition Lemma (Lemma 4.1) we can assume that $e[q,p]$ is a bipartition arc of Γ with the partition point c with respect to the label $k+2$ where we can keep $\bigcup_{i\leq k+2} \Gamma_i$ fixed. Let Γ' be the resulting chart. Now the arc α is deformed so that the arc intersects $e[w,c]$. Since $\bigcup_{i<k+3} (Int(e[w,q]) \cap Cl(\Gamma'_i - e[w,q]))= \bigcup_{i\leq k-2} (Int(e[c,q]) \cap \Gamma'_i) \subset e[c,q]$, and since $j-k > j-h \geq (k+3)-(k+1) = 2$ for each $k+3 \leq j$, we can push all the arcs intersecting $Int(e[w,c])$ to the other side of the white vertex w.

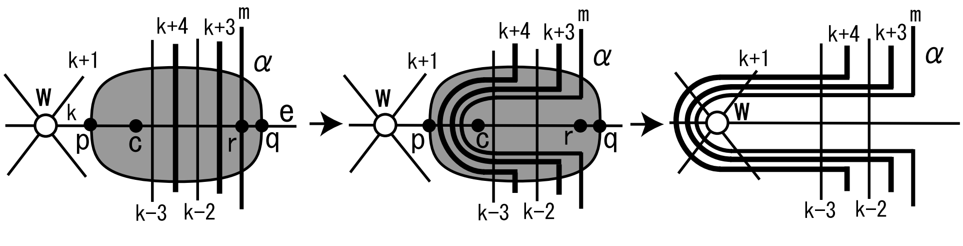

Figure 11. Corollary 4.4, Case $k+2 < m$.

Case (2): In this case, the lemma is shown by an argument similar to the case (1). □

Further we can arrange Lemma 4.2 by using an arc instead of the edge e and adding conditions as follows.

Corollary 4.5. *Let Γ be a chart and α an arc in an edge of Γ_m. Let w be a white vertex of $\Gamma_k \cap \Gamma_h$ where $h = k + \varepsilon, \varepsilon \in \{+1, -1\}$. Suppose that the white vertex w connects with a point r of the arc α by an arc β such that $Int(\beta)$ intersects Γ transversely. Further suppose that one of the following two conditions is satisfied:*

(1) *$h > k > m$ and $\Gamma_s \cap \beta[w, r] = \emptyset$ for some integer s with $k > s > m$.*

(2) *$h < k < m$ and $\Gamma_s \cap \beta[w, r] = \emptyset$ for some integer s with $k < s < m$.*

Then for any neighborhood V of the arc $\beta[w, r]$ we can shift the white vertex w to the other side of the arc α along the arc β by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in V keeping $\bigcup_{i \le 0} \Gamma_{s+i\varepsilon}$ fixed (see Figure 12). Also we can push the arc α to the other side of white vertex w along the arc β by C-I-R2 moves, C-I-R3 moves and C-I-R4 moves in V keeping $\bigcup_{0 \le i} \Gamma_{s+i\varepsilon}$ fixed (see Figure 13).

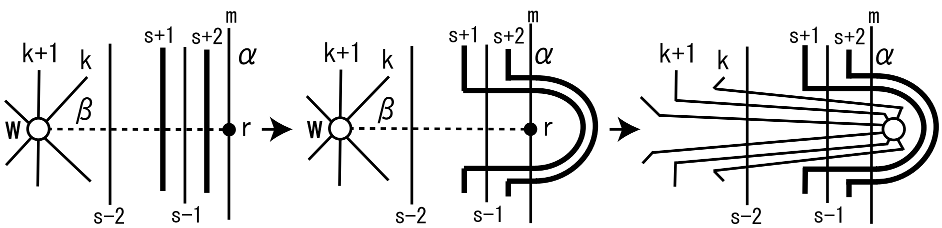

Figure 12. Corollary 4.5, Case (1) $k > s > m$. Shifting the white vertex w.

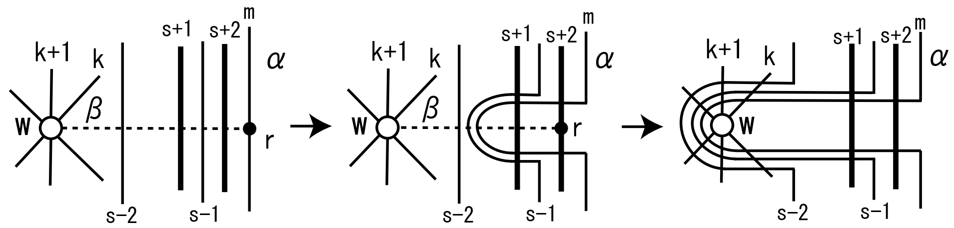

Figure 13. Corollary 4.5, Case (1) $k > s > m$. Pushing the arc α.

5. Lenses

In this section we investigate arcs intersecting the boundary of a lens.

Let Γ be a chart, and v a vertex. Let α be a short arc of Γ in a small neighborhood of v with $v \in \partial\alpha$. If the arc α is oriented to v, then α is called *an inward arc*, and otherwise α is called *an outward arc*.

The following lemma is shown in [5, Lemma 18.24 (E)]. Thus we omit the proof.

LEMMA 5.1. (*Cut Edge Lemma*) *Let* Γ *and* Γ' *be charts, and* D *a disk with* $\Gamma \cap D^c = \Gamma' \cap D^c$. *If* $\Gamma \cap D$ *and* $\Gamma' \cap D$ *are pseudo charts as shown in Figure 14, and if both* $\Gamma_{m+1} \cap D$ *and* $\Gamma'_m \cap D$ *consist of an inward arc and an outward arc, then* Γ *is C-move equivalent to* Γ'.

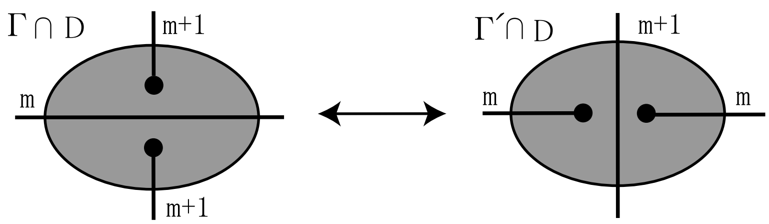

Figure 14.

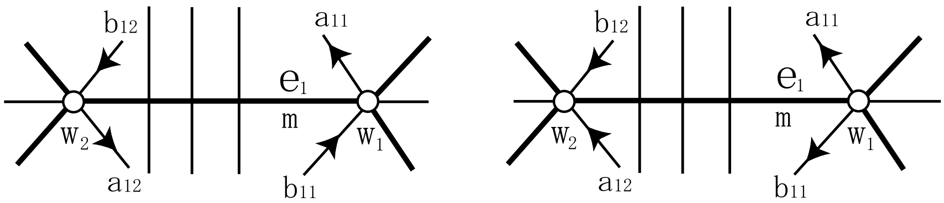

Figure 15.

Lemma 5.2. *Let* Γ *be a minimal chart. Let* e_1 *be an edge of* Γ_m *with* $\partial e_1 \subset \Gamma_{m+\varepsilon}$ ($\varepsilon \in \{+1, -1\}$)*. Let* w_1 *and* w_2 *be the white vertices of the edge* e_1*. Suppose that*

(1) *one of the two edges* a_{11} *and* b_{12} *contains an inward arc and the other contains an outward arc, and*

(2) *one of the two edges* a_{12} *and* b_{11} *contains an inward arc and the other contains an outward arc* (*see Figure 15*)*.*

Then the edge e_1 *contains at least one crossing in* $\Gamma_m \cap \Gamma_{m+2\varepsilon}$*. In particular if both edges* a_{11} *and* b_{12} *are terminal edges, or if both edges* a_{12} *and* b_{11} *are terminal edges, then* e_1 *contains at least two crossings in* $\Gamma_m \cap \Gamma_{m+2\varepsilon}$*.*

Proof. Suppose that $e_1 \cap \Gamma_{m+2\varepsilon} = \emptyset$. Since $w_1, w_2 \in \Gamma_m \cap \Gamma_{m+\varepsilon}$ and since $Int(e_1)$ does not intersect other edges of label $m, m \pm \varepsilon$ nor $m + 2\varepsilon$, all the edges intersecting e_1 are pushed out from e_1 through the white vertex w_1 by Corollary 4.3 or 4.4. Hence we can assume that $Int(e_1)$ does not intersect other edges of Γ. (see Figure 16). Applying a C-I-M2 move between a_{11} and b_{12} and a C-I-M2 move between a_{12} and b_{11} in a small neighborhood of e_1, we obtain three edges connecting the two white vertices w_1 and w_2. We can eliminate the two white vertices by a C-I-M3 move. This contradicts that Γ is minimal. Hence $e_1 \cap \Gamma_{m+2\varepsilon} \neq \emptyset$.

Suppose that the two edges a_{11} and b_{12} are terminal edges. If $e_1 \cap \Gamma_{m+2\varepsilon}$ contains only one point, then there exists an arc α connects the black vertices of a_{11} and b_{12} such that $Int(\alpha) \cap \Gamma_{m+2\varepsilon}$ contains only one point. By using C-II moves between one of the terminal edges and the edges intersecting

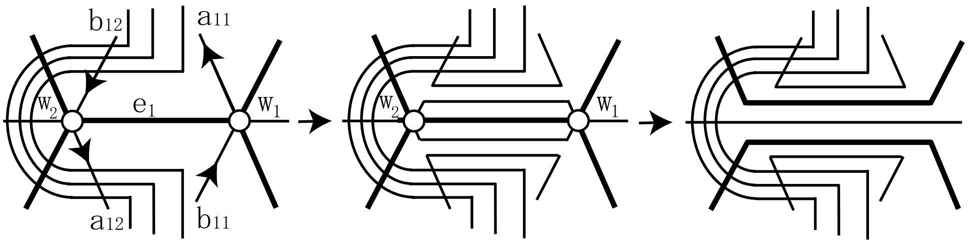

Figure 16.

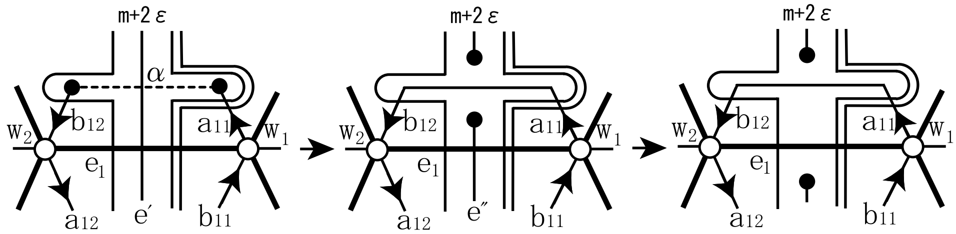

Figure 17.

$Int(\alpha)$ one by one, we can assume that $Int(\alpha) \cap \Gamma$ contains only one point in $\alpha \cap \Gamma_{m+2\varepsilon}$ (see Figure 17). Let e' be the edge in $\Gamma_{m+2\varepsilon}$ which intersects the edge e_1. Apply Cut Edge Lemma (Lemma 5.1) for the two edges a_{11} and b_{12} and the edge e'. Then the edge e' splits into two edges. One of the edges intersects the edge e_1, say e''. Shrink the edge e'' so that e_1 does not intersect any edges of label $m + 2\varepsilon$. This contradicts the first half of our lemma. □

LEMMA 5.3. *Let* Γ *be a minimal chart and* D *a lens of type* $(m, m+1)$ *with the boundary arc pair* (e_m, e_{m+1}) *where* $e_m \subset \Gamma_m$ *and* $e_{m+1} \subset \Gamma_{m+1}$. *Then* $e_m \cap \Gamma_{m+2} \neq \emptyset$ *and* $e_{m+1} \cap \Gamma_{m-1} \neq \emptyset$.

PROOF. Looking at the edge e_m, we have $e_m \cap \Gamma_{m+2} \neq \emptyset$ by Lemma

5.2. Similarly by looking at the edge e_{m+1} we can get $e_{m+1} \cap \Gamma_{m-1} \neq \emptyset$. □

The following lemma shows how to use Lemma 5.2 and Disk lemma (Lemma 3.2).

LEMMA 5.4. *If a minimal chart* Γ *contains the pseudo chart as shown in Figure 18a, then the interior of the disk* D *contains at least one white vertex, where* D *is the disk with the boundary* $e_3 \cup e_4 \cup e$.

PROOF. We give a proof for the case that the edge e_1 is oriented outward at the white vertex w_1. Hence edges are oriented as shown in Figure 18b. We use notations as shown in Figure 18b. Suppose that the interior of D does not contain any white vertices.

Let e' be the edge of $\Gamma_{m+\varepsilon}$ which contains the white vertex w_3 and is different from the two edges e_3 and e_4. Since $e' \cap \partial D = \{w_3\}$, we have that $e' \subset D$ or $e' \cap D = \{w_3\}$. Since e_3 is oriented from w_3 to w_1, and since e_4 is oriented from w_2 to w_3, the edge e' does not contain a middle arc at the white vertex w_3. Thus e' is not a terminal edge by Assumption 1.

Since there exists no white vertex in the interior of D, we have $e' \not\subset D$. Thus $e' \cap D = \{w_3\}$. Hence there exists exactly one edge e'' of Γ_m or $\Gamma_{m+2\varepsilon}$ with $w_3 \in e''$ and $e'' \cap Int(D) \neq \emptyset$ (see Figure 18c). Hence for a neighborhood V of w_3 there exists at most one arc of label $m + 2\varepsilon$ in $V \cap Int(D)$.

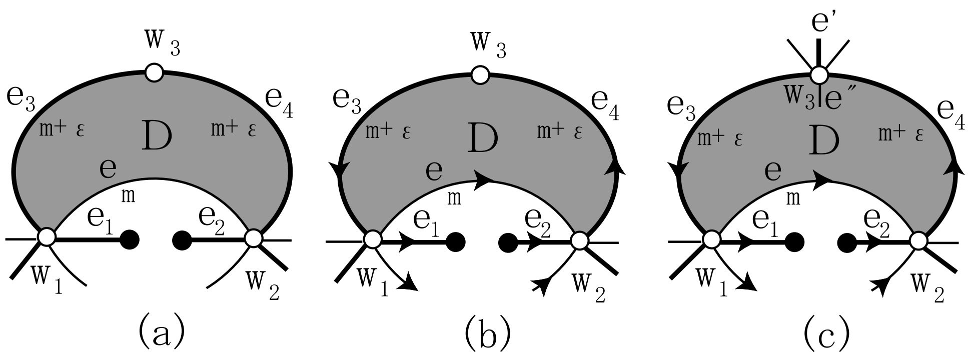

Figure 18.

On the other hand, by Lemma 5.2 the edge e contains at least two crossings in $\Gamma_m \cap \Gamma_{m+2\varepsilon}$. Since there is no white vertex in $Int(D)$, there is no (D, e)-arc by Disk Lemma (Lemma 3.2). Now any arc containing one of the crossings does not intersect $Int(e_3) \cup Int(e_4)$. Thus any arc containing one of the crossings must contain the white vertex w_3. Hence there must exist at least two arcs of label $m+2\varepsilon$ in $V \cap Int(D)$. This is a contradiction. □

6. Proof of Theorem 1.1

Let Γ be a chart and D a disk. A disk D' in D with a boundary arc pair (α, β) is called a *contact lens of label m in D* provided that

(1) $D' \cap \partial D = \alpha$, and

(2) the arc β is contained in an edge of label m.

The arc α (resp. β) is called an *outer* (*resp. inner*) *arc* of the contact lens D'.

Let D_1 and D_2 be contact lenses in a disk D. The contact lens D_1 is *smaller* than the contact lens D_2 if D_1 is also a contact lens in D_2.

A contact lens of label m in a disk D is *locally inner-most* if it does not contain any smaller contact lens of label $m-1, m$, nor $m+1$ in D.

LEMMA 6.1. *Let D be a lens of a chart Γ. Let $\varepsilon \in \{+1, -1\}$. If there is a contact lens D' of label m in D such that no contact lens of label $m-\varepsilon$ in D is smaller than D', then for some non-negative integer s, there exists a locally inner-most contact lens D'' of label $m+\varepsilon s$ in D with $D'' \subset D'$.*

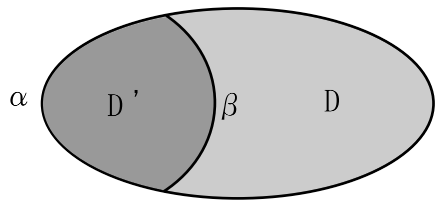

Figure 19.

Proof. Suppose that $\varepsilon = +1$. If D' does not contain any smaller contact lens whose label is greater than or equal to m, then the contact lens D' is locally inner-most.

If D' contains a smaller contact lens whose label is greater than or equal to m, then an inner-most contact lens among these smaller contact lenses is locally inner-most. If the condition $\varepsilon = -1$ is satisfied, then the lemma is shown by an argument similar to the case $\varepsilon = +1$. □

Let Γ be a chart, and e_1 and e_2 edges in Γ_m (possibly $e_1 = e_2$). Let α be an arc such that

(1) $\partial\alpha$ consists of a point in e_1 and a point in e_2, and

(2) $Int(\alpha)$ transversely intersects Γ (see Figure 20a).

Let D be a regular neighborhood of the arc α. let $\gamma_1 = e_1 \cap D$ and $\gamma_2 = e_2 \cap D$. Then γ_1 and γ_2 are proper arcs of D and they split the disk D into three disks. Let E be the one of the three disks with $E \supset \alpha$. A chart Γ' *is obtained from* Γ *by a surgery along* α provided that

(1) $\Gamma'_m = (\Gamma_m - (\gamma_1 \cup \gamma_2)) \cup Cl(\partial E - (\gamma_1 \cup \gamma_2))$, and

(2) $\Gamma'_i = \Gamma_i$ $(i \neq m)$ (see Figure 20b).

Let D be a lens of a minimal chart Γ. Let $w(D)$ be the number of white vertices contained in $Int(D)$ and $c(D)$ the number of crossings on ∂D. The pair of integers $(w(D), c(D))$ is called the *local complexity with respect to D*,

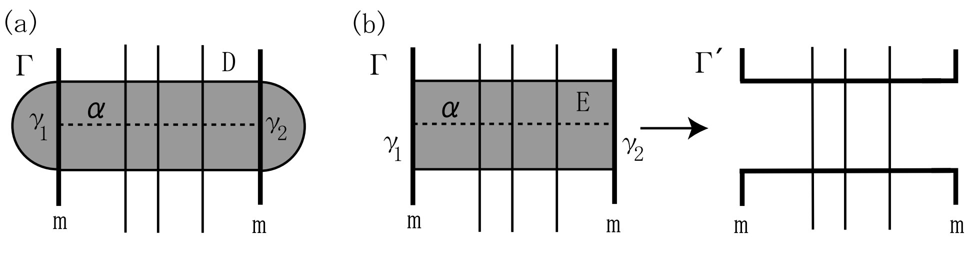

Figure 20.

denoted by $\ell c(D;\Gamma)$. A chart is said to be *locally minimal with respect to D* if its local complexity with respect to D is minimal among the minimal charts C-move equivalent to the chart keeping ∂D fixed with respect to the lexicographic order.

LEMMA 6.2. *Let D be a lens of type $(m, m+1)$ with a boundary pair (e_m, e_{m+1}) in a minimal chart Γ where $e_m \subset \Gamma_m$ and $e_{m+1} \subset \Gamma_{m+1}$. Suppose that Γ is locally minimal with respect to D. Then the following hold:*

(1) *The lens D contains a white vertex which connects with a point in e_{m+1} by an arc of label less than or equal to $m-1$.*

(2) *The lens D contains a white vertex which connects with a point in e_m by an arc of label greater than or equal to $m+2$.*

PROOF. We show the statement (2). By Lemma 5.3 there exists an edge e of label $m+2$ with $e \cap e_m \neq \emptyset$. If the lens D contains a white vertex of e, then we have done for (2).

Suppose that the lens D does not contain any white vertex of e. Then any connected component of $e \cap D$ is a proper arc γ of D of label $m+2$. Since any edge of label $m+2$ does not have a crossing with e_{m+1}, we have $\partial\gamma \subset e_m$. Hence the lens D contains a contact lens D' of label $m+2$ whose outer arc is contained in e_m. Since any edge of label $m+1$ does not have a crossing with e_m, the lens D does not contain any contact lens of label $m+1$ smaller than D'. Hence for some integer $t \geq m+2$ there exists a locally inner-most contact lens D'' of label $t \geq m+2$ in D with $D'' \subset D'$ by Lemma 6.1.

If there is no edge of label $t-1, t$, or $t+1$ which intersects the interior of the outer arc of D'', we can apply a surgery along the outer arc of D''. This reduces the number of crossings on ∂D. This contradicts that Γ is locally minimal with respect to D.

Hence there exists an edge e' of label $t-1, t$, or $t+1$ which intersects the interior of the outer arc of $D'' \subset D'$. If $Int(D'')$ does not contain any white vertex of the edge e', then the lens D'' contains a proper arc of label $t-1, t$, or $t+1$. This contradicts that D'' is locally inner-most. Hence $Int(D'')$ contains a white vertex of e', and so does $Int(D)$. Since $t \geq m+2$ and since any edge of label $m+1$ does not contain a crossing with e_m, the label of the edge e' is greater than or equal to $m+2$. Hence the result follows. The statement (1) is shown in an argument similar to that we did for the statement (2). □

The following Corollary is a direct result of Lemma 6.2.

Corollary 6.3. *Let Γ be a minimal chart. Then the following hold:*

(1) *If D is a lens of type $(m, m+1)$, then there exist two integers s and t with $s \leq m-1, m+2 \leq t$ such that there exist a white vertex of Γ_s and a white vertex of Γ_t in the interior of D.*

(2) *If Γ is a minimal chart of type $(m; n_1, n_2, \ldots, n_k)$, then there does not exist any lens of type $(m, m+1)$ nor $(m+k-1, m+k)$.*

(3) *There does not exist any lens of a minimal chart of type $(m; n_1)$ nor $(m; n_1, n_2)$.*

Proof. Now (1) is shown in Lemma 6.2.

We show the statement (2). If there exists a lens of type $(m, m+1)$, then the lens contains one white vertex of Γ_h in its interior for some $h \leq m-1$ by (1). However this contradicts the fact that the label m is the lowest number among the labels of edges containing white vertices.

Similarly there does not exist any lens of type $(m+k-1, m+k)$. The statement (3) is a direct result of (2). □

Proof of Theorem 1.1. Let D be a lens of type $(m, m+1)$ in a minimal chart Γ. Let (e_m, e_{m+1}) be the boundary arc pair of the lens D with $e_m \subset \Gamma_m$ and $e_{m+1} \subset \Gamma_{m+1}$. We can assume that Γ is locally minimal with respect to the lens D. By Lemma 6.2 the lens D contains two white vertices w_1 and w_2 such that the vertex w_1 connects with a point in e_m by an arc γ_1 of label $t \geq m+2$ and the vertex w_2 connects with a point in e_{m+1} by an arc γ_2 of label $s \leq m-1$. Hence $Int(D)$ contains at least two white vertices.

We show the theorem by contradiction. Suppose that the lens D contains only two white vertices w_1 and w_2. Let e_h be an edge of label h containing w_1, where $h = t \pm 1$. Since $h - (s+1) \geq ((m+2)-1) - ((m-1)+1) = 1$, no edge contains the two vertices w_1 and w_2 simultaneously. Hence e_h can not contain the vertex w_2. Thus the vertex w_1 connects with a point in e_m or e_{m+1} by an arc γ of e_h. There are three cases:

(1) $h = t + 1$.

(2) $h = t - 1$ and $\gamma \cap e_m \neq \emptyset$.

(3) $h = t - 1$ and $\gamma \cap e_{m+1} \neq \emptyset$.

Case (1): Since $m < t < h$, we can shift the vertex w_1 to the outside of D along the arc γ_1 by Shifting Lemma (Lemma 4.2). This contradicts that Γ is locally minimal with respect to D.

Case (2): Since $m < h < t$, we can shift the vertex w_1 to the outside of D along the arc γ by Shifting Lemma (Lemma 4.2). This contradicts that Γ is locally minimal with respect to D.

Case (3): Since $m + 2 \leq t$, we have $h = t - 1 \geq m + 1$. Further $e_{m+1} \cap e_h \neq \emptyset$, implies that $h \geq m + 3$. Hence $m + 1 < h < t$. Thus we can

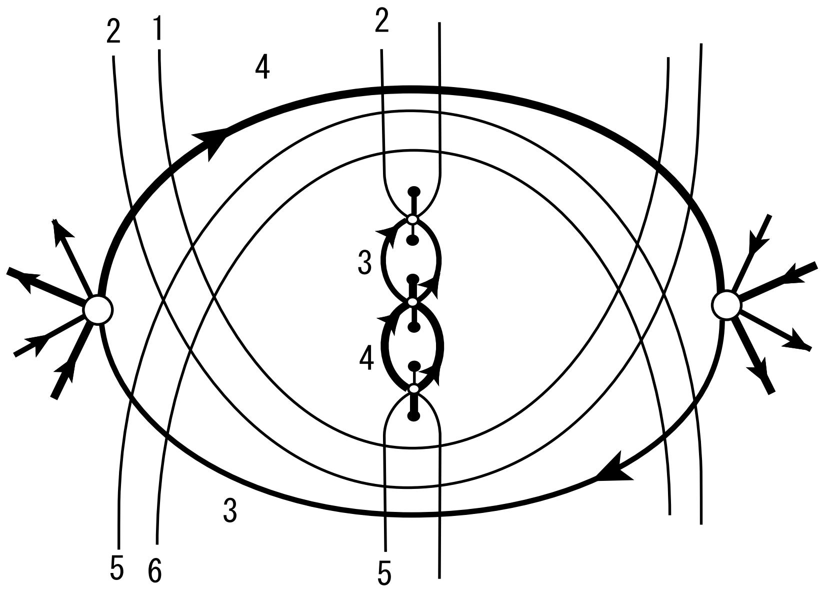

Figure 21.

shift the vertex w_1 to the outside of D along the arc γ by Shifting Lemma (Lemma 4.2). This contradicts that Γ is locally minimal with respect to D. We get a contradiction for all cases. □

Finally we give an example of a lens of type $(3, 4)$ which contains exactly three white vertices (see Figure 21).

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(Received September 26, 2006)
(Revised January 15, 2007)

Department of Mathematics
Tokai University
1117 Kitakaname, Hiratuka
Kanagawa 259-1292, Japan
E-mail: nagase@keyaki.cc.u-tokai.ac.jp
shima@keyaki.cc.u-tokai.ac.jp