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Erratum to "Quasi-unipotent Logarithmic Riemann-Hilbert Correspondences"

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The purpose of this note is to fill in a gap in the proof of ([IKN] 7.1). In (*loc.cit.*) it is asserted, p. 38, l. -13 that "Since f is proper, it follows from 7.1.1 that the sheaves $R^q f_* \omega^p_{X/Y,\mathrm{ket}}$ (resp. $R^n f_* \omega^{\cdot}_{X/Y,\mathrm{ket}}$) on Y_{ket} are of finite presentation (as $\mathcal{O}_{Y,\mathrm{ket}}$-modules) and (ket locally) of the form $\varepsilon^* R^q f_* \omega^p_{X/Y}$ (resp. $\varepsilon^* R^n f_* \omega^{\cdot}_{X/Y}$)." Though the statement about the sheaves $R^q f_* \omega^p_{X/Y,\mathrm{ket}}$ (resp. $R^n f_* \omega^{\cdot}_{X/Y,\mathrm{ket}}$) turns out to be true, it does not follow directly from the properness of f and the calculation of stalks made in 7.1.1. The ring B of the strict log localization of Y at $\tilde{y}$ may be non noetherian, and the map from the ring of the étale localization A of Y at the image y of $\tilde{y}$ to B may be non flat, even after replacing Y by a Kummer étale neighborhood of $\tilde{y}$. So though the sheaves $\omega^p_{X/Y}$ are locally free, we have *a priori* no control on the direct images of their ket counterparts, whose stalks at $\tilde{y}$ are, by 7.1.1, the global étale cohomology groups of the base changes $B \otimes_A \omega^p_{X/Y}$.

However, one can prove 7.1 as follows. First of all, as the question is ket local on Y and f is of finite type, by A 4.3 we may and shall assume that f is *saturated.* Now, the starting point is that the ket de Rham cohomology sheaves

$$H^n_{dR,\mathrm{ket}} = R^n f^{\mathrm{ket}}_* \omega^{\cdot,\mathrm{ket}}_{X/Y}$$

are locally free on Y_{ket} (6.2 (2)). Therefore, by ket localization on Y, we may assume that they are of the form $H^n_{dR,\mathrm{ket}} = \varepsilon^* M^n$, where M^n is a free module of finite type on Y_{et}. Then, by 3.7,

$$R^n f_* \omega^{\cdot}_{X/Y} = \varepsilon_* H^n_{dR,\mathrm{ket}} = M^n$$

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is free of finite type on Y_{et} for all n. Therefore it suffices to show the following result, which complements 7.1 :

LEMMA (7.1'). *Let $f : X \to Y$ be a proper, log smooth and saturated morphism of fs log schemes of characteristic zero. Assume that the sheaves $R^n f_* \omega^\cdot_{X/Y}$ are locally free of finite type on Y_{et} for all n. Then :*

(a) The (étale) spectral sequence

$$E_1^{pq} = R^q f_* \omega^p_{X/Y} \Rightarrow R^{p+q} f_* \omega^\cdot_{X/Y} \tag{1}$$

degenerates at E_1 and has locally free initial terms.

(b) The (ket) spectral sequence

$$E_1^{pq} = R^q f_*^{\mathrm{ket}} \omega^{p,\mathrm{ket}}_{X/Y} \Rightarrow R^{p+q} f_*^{\mathrm{ket}} \omega^{\cdot,\mathrm{ket}}_{X/Y} \tag{2}$$

degenerates at E_1 and has locally free initial terms.

(c) For any interval $[a,b]$ and any integer n the natural map

$$\varepsilon^* R^n f_* \omega^{[a,b]}_{X/Y} \to R^n f_*^{\mathrm{ket}} \omega^{[a,b],\mathrm{ket}}_{X/Y}$$

is an isomorphism, in other words, the spectral sequence (2) is the inverse image by ε of the spectral sequence (1).

Let us first prove that (a) implies (c). As f is saturated, for any ket map $g : Y' \to Y$, the underlying scheme of the fs base change $X' := X \times_Y Y'$ is the schematic base change of X by g. Since the initial term of (1) is locally free, the base change maps, where $f' = f \times_Y Y'$,

$$g^* R^q f_* \omega^p_{X/Y} \to R^q f'_* \omega^p_{X'/Y'}$$

are isomorphisms. As (1) degenerates at E_1, it follows more generally that, for any interval $[a,b]$, the base change maps for the truncated complexes

$$g^* R^q f_* \omega^{[a,b]}_{X/Y} \to R^q f'_* \omega^{[a,b]}_{X'/Y'}$$

are isomorphisms of locally free sheaves on Y'_{et}. By 7.1.1 this shows that (a) implies (c), hence (b). It remains to prove (a). The particular case where the underlying scheme of Y is $\operatorname{Spec} \mathbb{C}$ is disposed of by the proof of 7.1.2 (2'). For the general case, we argue as in the rest of 7.1.2, except that we work with the étale topology instead of the Kummer étale one. We first reduce to the case where Y is of finite type over $\mathbb{C}$. As f is proper, the sheaves $R^q f_* \omega^p_{X/Y}$ are coherent. To show that they are locally free and (1) degenerates at E_1, one can further reduce, by localization and completion, and use of Grothendieck's comparison theorem, to the case where Y is the spectrum of an artinian local $\mathbb{C}$-algebra A. Then it suffices to show that, for all n, one has

$$\lg_A H^n(X, \omega^{\cdot}_{X/Y}) = \sum_{p+q=n} \lg_A H^q(X, \omega^p_{X/Y}), \tag{3}$$

where $\lg_A$ denotes the length of an A-module, and, for all p, q,

$$\lg_A H^q(X, \omega^p_{X/Y}) = (\lg A) \dim_{\mathbb{C}} H^q(X_y, \omega^p_{X_y/y}), \tag{4}$$

where y is the closed point of Y. Since the sheaves $R^n f_* \omega^{\cdot}_{X/Y}$ are free of finite type, we have

$$\lg_A H^n(X, \omega^{\cdot}_{X/Y}) = (\lg A) \dim_{\mathbb{C}} H^n(X_y, \omega^{\cdot}_{X_y/y}),$$

and therefore, by the case $Y = \operatorname{Spec} \mathbb{C}$ already treated, we have

$$\lg_A H^n(X, \omega^{\cdot}_{X/Y}) = (\lg A) \sum_{p+q=n} \dim_{\mathbb{C}} H^q(X_y, \omega^p_{X_y/y}). \tag{5}$$

Therefore we get the inequalities

$$\begin{aligned} \sum_{p+q=n} \lg_A H^q(X, \omega^p_{X/Y}) &\geq (\lg A) \sum_{p+q=n} \dim_{\mathbb{C}} H^q(X_y, \omega^p_{X_y/y}) \\ &\geq \sum_{p+q=n} \lg_A H^q(X, \omega^p_{X/Y}). \end{aligned}$$

These have to be equalities. Combining with (5), we get (3) and (4), which finishes the proof.

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References

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