論文題目

Elementary computation of ramified components of Jacobi sum Hecke characters
(ヤコビ和量指標の分岐成分の初等的な計算)

氏名: 津嶋　貴弘

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Elementary computation of ramified components of the Jacobi sum Hecke characters

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Takahiro Tsushima
Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku Tokyo 153-8914, JAPAN
E-mail: tsushima@ms.u-tokyo.ac.jp

Abstract

R. Coleman and W. McCallum calculated the Jacobi sum Hecke characters explicitly using their computation of the stable reduction of the Fermat curve in [CW]. In this paper, we give an elementary proof of the main result of them without using rigid geometry or the stable model of the Fermat curve.

1 Introduction

R. Coleman and W. McCallum calculated the Jacobi sum Hecke characters explicitly using their computation of the stable reduction of the Fermat curve in [CW].

In this paper, we give an elementary proof of the main result of them about the Jacobi sum Hecke characters without using rigid geometry or the stable model of the Fermat curve. Let K denote a finite extension of $\mathbb{Q}_p$. We fix a prime number $l \neq p$. Let U_K be the projective line minus $\{0, 1, \infty\}$ over K and $\mathscr{K}_\chi$ the smooth $\bar{\mathbb{Q}}_l$-sheaf on U_K associated to a certain quotient of the Fermat curve. Our aim is to calculate explicitly the Galois action on the étale cohomology group $H^1_c(U_{\bar{K}}, \mathscr{K}_\chi)$ which realizes the Jacobi sum Hecke character as in [CW]. (Theorem 3.1.)

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2 Preliminaries

In this section, we fix some notations and show key lemmas to prove the main theorem.

Let K be a henselian discrete valuation field of mixed characteristic $(0, p)$ with $\mathcal{O}_K$ the integer ring, with $\mathfrak{m}_K$ the maximal ideal and with F the residue field. For an integer $i > 0$, we put $U_K^{(i)} := 1 + \mathfrak{m}_K^i$ and $\mathrm{Gr}_i K^\times := U_K^{(i)}/U_K^{(i+1)}$. We denote by ord_K the normalized additive valuation of K. We put $e_K := \mathrm{ord}_K(p)$. We fix a uniformizer π_K and write $p = u\pi_K^{e_K}$ for some unit $u \in \mathcal{O}_K^\times$.

Lemma 2.1. *Let the notation and the assumption be as above.*
1. Let $i > \frac{e_K}{p-1}$ be a positive integer. Then the p-th power map $K^\times \longrightarrow K^\times$ induces an isomorphism

$$\mathrm{Gr}_i K^\times \xrightarrow{\sim} \mathrm{Gr}_{i+e_K} K^\times; 1 + a \mapsto 1 + pa.$$

2. Let $i = \frac{e_K}{p-1}$ be an integer. Then the p-th power map $K^\times \longrightarrow K^\times$ induces the following map

$$\mathrm{Gr}_i K^\times \longrightarrow \mathrm{Gr}_{i+e_K} K^\times; 1 + a \mapsto 1 + pa + a^p.$$

Proof. Since we have $(1+a)^p \equiv 1 + pa + a^p \pmod{pa^2}$, the assertions follow immediately. □

Let p be an odd prime number and $n, p \not| m'$ positive integers and we put $m = p^n m'$. Let K denote a finite extension of $\mathbb{Q}_p$ containing m-th roots of unity with the residue field k. We fix ζ a primitive p-th root of unity in K. Let $\pi \in K$ be an element satisfying $\pi^{p-1} = -p$, $\frac{\pi}{1-\zeta} \equiv 1 \pmod{\pi}$. Let l be a prime number prime to p. Let $\chi : \mu_m(K) \longrightarrow \overline{\mathbb{Q}}_l^\times$ be a non-trivial character. We define $\psi_0 : \mathbb{F}_p \longrightarrow \overline{\mathbb{Q}}_l^\times$ to be the composite $\mathbb{F}_p \longrightarrow \mu_p(K) \subset \mu_m(K) \longrightarrow \overline{\mathbb{Q}}_l^\times$ where the first map has the form $1 \mapsto \zeta$ and the last map is χ. We denote by $\chi' : \mu_{m'}(k) \longrightarrow \overline{\mathbb{Q}}_l^\times$ the composite of the canonical isomorphism $\mu_{m'}(k) \simeq \mu_{m'}(K)$ and the restriction $\chi|_{\mu_{m'}(K)}$ to the subgroup $\mu_{m'}(K) \subset \mu_m(K)$. We assume that ψ_0 and χ' are non-trivial.

Let U be a regular noetherian scheme which is flat over $\mathcal{O}_K$ and $D \subset U$ an irreducible divisor. We assume that $U \backslash D = U \otimes_{\mathcal{O}_K} K$. For an invertible function f on U, we write $\bar{f}$ for the image of f by the restriction map $\Gamma(U, \mathcal{O}_U^\times) \longrightarrow \Gamma(D, \mathcal{O}_D^\times)$. For an invertible function f on U, $\mathscr{K}_\chi(f)$ denotes the smooth $\overline{\mathbb{Q}}_l$-sheaf on $U \backslash D$ associated to the character χ and the equation $y^m = f$. For an invertible function $\bar{f}$ on D, $\mathscr{K}_{\chi'}(\bar{f})$ denotes the Kummer sheaf on D defined by the character χ' and the equation $y^{m'} = \bar{f}$. For a regular function $\bar{h}$ on D, $\mathscr{L}_{\psi_0}(\bar{h})$ denotes the Artin-Schreier sheaf on D associated to the additive character ψ_0 and the equation $a^p - a = \bar{h}$.

Corollary 2.2. *Let the notation and the assumption be as above. Let f be an invertible function on U. We assume that $f - 1$ is divisible by $\pi^p p^{n-1}$ and put $h := (f-1)/\pi^p p^{n-1}$.*
1. The sheaf $\mathscr{K}_\chi(f)$ extends to a smooth sheaf on U, which we denote by $\widetilde{\mathscr{K}}$.
2. The restiction of $\widetilde{\mathscr{K}}$ to D is the sheaf $\mathscr{L}_{\psi_0}(\frac{\bar{h}}{m'})$.

Proof. 1. Let $\mathcal{O}_{\mathbb{K}}$ be the henselization of the local ring $\mathcal{O}_{U,\eta}$ where η is the generic point of the divisor D. The sheaf $\mathscr{K}_\chi(f)$ is equal to the Kummer sheaf $\mathscr{K}$ defined by the equation $y^{pm'} = 1 + \pi^p h$ at $\mathrm{Spec}\mathbb{K}$ by the assumption and Lemma 2.1.1. Let k_1, k_2 be integers satisfying $k_1 p + k_2 m' = 1$. We have $\mathscr{K} = \mathscr{K}_1^{\otimes k_1} \otimes \mathscr{K}_2^{\otimes k_2}$ where $\mathscr{K}_1, \mathscr{K}_2$ are the Kummer sheaves defined by equations $y^{m'} = 1 + \pi^p h$ and $y^p = 1 + \pi^p h$ respectively. Obviously the sheaf $\mathscr{K}_1$ extends to a smooth sheaf whose restriction to the special fiber is a trivial sheaf. Hence, to prove 1, it is sufficient to prove that the Kummer covering of $\mathrm{Spec}\mathbb{K}$ defined by the equation $y^p = 1 + \pi^p h$ extends to a finite étale covering of $\mathrm{Spec}\mathcal{O}_{\mathbb{K}}$ by the Zariski-Nagata's purity theorem(SGA 2 X 3.4). We consider a polynominal $g = \frac{(1+\pi a)^p - (1+\pi^p h)}{\pi^p}$ where a is an indeterminant. Then g is contained in the polynominal ring $\mathcal{O}_{\mathbb{K}}[a]$ and an $\mathcal{O}_{\mathbb{K}}$-scheme $\mathrm{Spec}\mathcal{O}_{\mathbb{K}}[a]/(g)$ is finite étale over $\mathrm{Spec}\mathcal{O}_{\mathbb{K}}$ and its generic fiber is the Kummer covering $y^p = 1 + \pi^p h$. Therefore the required assertion follows.
2. The special fiber of the scheme $\mathrm{Spec}\mathcal{O}_{\mathbb{K}}[a]/(g)$ in the proof in 1 is defined by the following

equation $a^p - a = \bar{h}$. Hence the restriction of $\widetilde{\mathscr{K}}$ to D is the Artin-Schreier sheaf defined by the equation $a^p - a = k_2\bar{h} = \frac{\bar{h}}{m'}$. □

In the following, we collect some well known facts on the cospecialization map.

Lemma 2.3. *Let K be a finite extension of $\mathbb{Q}_p$ with the integer ring $\mathcal{O}_K$. Let Y be a noetherian scheme of dimension 2 which is proper and separable over $\mathcal{O}_K$ (i.e. flat over $\mathcal{O}_K$ and its fibers are geometrically reduced.)with fibers geometrically connected. Let l be a prime number prime to p and Λ denotes a finite commutative $\mathbb{Z}_l$-algebra. Let $W \subset Y$ be an open subscheme such that the complement $Y\backslash W$ is flat over $\mathcal{O}_K$. Then the cospecialization map $H^1_c(W_{\bar{s}}, \Lambda) \longrightarrow H^1_c(W_{\bar{K}}, \Lambda)$ is injective.*

Proof. We consider the following commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & H^0(Y_{\bar{s}}, \Lambda) & \longrightarrow & H^0((Y\backslash W)_{\bar{s}}, \Lambda) & \longrightarrow & H^1_c(W_{\bar{s}}, \Lambda) & \longrightarrow & H^1(Y_{\bar{s}}, \Lambda) \\ & & \downarrow \simeq & & \downarrow & & \downarrow \text{cosp.} & & \downarrow \text{cosp.} \\ 0 & \longrightarrow & H^0(Y_{\bar{K}}, \Lambda) & \longrightarrow & H^0((Y\backslash W)_{\bar{K}}, \Lambda) & \longrightarrow & H^1_c(W_{\bar{K}}, \Lambda) & \longrightarrow & H^1(Y_{\bar{K}}, \Lambda) \end{array}$$

where the horizontal sequences are exact. Since the scheme $Y\backslash W$ is flat over $\mathcal{O}_K$, the cospecialization map $H^0((Y\backslash W)_{\bar{s}}, \Lambda) \longrightarrow H^0((Y\backslash W)_{\bar{K}}, \Lambda)$ is injective. Since the fibers of Y are geometrically connected, the map $H^0(Y_{\bar{s}}, \Lambda) \longrightarrow H^0(Y_{\bar{K}}, \Lambda)$ is an isomorphism. The cospecialization map $H^1(Y_{\bar{s}}, \Lambda) \longrightarrow H^1(Y_{\bar{K}}, \Lambda)$ is injective by the semi-continuity theorem of the fundamental group in Corollaire 2.4 in SGA1 X. Hence we have proved the required assertion by the diagram chasing. □

Corollary 2.4. *Let X be a regular noetherian scheme of dimension 2 which is proper and separable over $\mathcal{O}_K$ and $U \subset X$ an open subscheme. We assume that $X\backslash U$ is flat over $\mathcal{O}_K$. Let $f : W \longrightarrow U$ be a finite Galois étale covering of Galois group G. We assume that the fibers of W are geometrically connected. Let $\rho : G \longrightarrow \mathrm{GL}(V)$ be an l-adic representation on a finite dimensional $\overline{\mathbb{Q}}_l$-vector space V. We denote by $\mathscr{K}_\rho$ the smooth $\overline{\mathbb{Q}}_l$-sheaf on U associated to the covering $W \longrightarrow U$ and the representation ρ. Then the cospecialization map $H^1_c(U_{\bar{s}}, \mathscr{K}_\rho) \longrightarrow H^1_c(U_{\bar{K}}, \mathscr{K}_\rho)$ is injective.*

Proof. We consider the following cartesian diagram

$$\begin{array}{ccc} W & \longrightarrow & Y \\ \downarrow f & & \downarrow \bar{f} \\ U & \longrightarrow & X \end{array}$$

where $\bar{f} : Y \longrightarrow X$ is the normalization of X along $f : W \longrightarrow U$. Since $W \subset Y$ satisfies the conditions in Lemma 2.3, the cospecialization map $H^1_c(W_{\bar{s}}, \overline{\mathbb{Q}}_l) \longrightarrow H^1_c(W_{\bar{K}}, \overline{\mathbb{Q}}_l)$ is injective. We consider the following commutative diagram

$$\begin{array}{ccc} H^1_c(W_{\bar{s}}, f^*\mathscr{K}_\rho) & \xrightarrow{\text{cosp.}} & H^1_c(W_{\bar{K}}, f^*\mathscr{K}_\rho) \\ f^* \uparrow & & f^* \uparrow \\ H^1_c(U_{\bar{s}}, \mathscr{K}_\rho) & \xrightarrow{\text{cosp.}} & H^1_c(U_{\bar{K}}, \mathscr{K}_\rho) \end{array}$$

where the horizontal arrows are the cospecialization maps. Since the pull-back $f^* : H^1_c(U_{\bar{s}}, \mathscr{K}_\rho) \longrightarrow H^1_c(W_{\bar{s}}, f^*\mathscr{K}_\rho)$ is injective and $f^*\mathscr{K}_\rho$ is a constant sheaf, the assertion follows from the above commutative diagram. □

3 Main theorem and its proof

Let a, b, c be integers satisfying $a+b+c=0$. We assume $a = p^r a', (p, a') = 1, (p, b) = 1, (p, c) = 1$ and $n \geq r$. We put $K = \mathbb{Q}_p(\mu_m)$. For an element $f \in K^\times$, we denote by $\mathscr{Q}(f)$ the smooth $\overline{\mathbb{Q}}_l$-sheaf on Spec$K$ defined by the quadratic Kummer extension $y^2 = f$.

Let $F^m_{a,b,c} : y^m = (-1)^c x^a (1-x)^b$ be a quotient of the Fermat curve over K. This curve is a finite Galois étale covering of $U_K := \mathbb{P}^1_K - \{0, 1, \infty\} = \mathrm{Spec}K[x^{\pm 1}, \frac{1}{1-x}]$ of Galois group $\mu_m(K)$. We simply write $\mathscr{K}_\chi$ for the smooth $\overline{\mathbb{Q}}_l$-sheaf $\mathscr{K}_\chi((-1)^c x^a (1-x)^b)$ on U_K under the notation in Corollary 2.2. We will compute the Galois action on the étale cohomology group $H^1_c(U_{\bar{K}}, \mathscr{K}_\chi)$ explicitly. We have dim $H^1_c(U_{\bar{K}}, \mathscr{K}_\chi) = 1$. Note that the Galois representation $H^1_c(U_{\bar{K}}, \mathscr{K}_\chi)$ is a direct factor of the Tate module of the Jacobian of the Fermat curve $F^m_{a,b,c}$ and the Tate module realizes the Jacobi sum Hecke character by CM theory as in [CW, Corollary 5.2]. The Galois representation $H^1_c(U_{\bar{K}}, \mathscr{K}_\chi)$ is described by using Hilbert symbols and Gauss sum as follows.

Theorem 3.1. *(Coleman and McCallum) Let the notation and the assumption be as above.*
1. We assume that $n = r$. Then we have an isomorphism

$$H^1_c(U_{\bar{K}}, \mathscr{K}_\chi) \simeq \mathscr{K}_\chi((-1)^c(\pi p^n)^a) \otimes H^1_c(\mathbb{G}_{m,\bar{k}}, \mathscr{K}_{\chi'}(s^{a'}) \otimes \mathscr{L}_{\psi_0}(\frac{b}{m'}s))$$

as a G_K-representation of degree 1.
2. We assume that $p \geq 5$ and $n > r$. We put $\gamma = -\frac{a'c}{2bm'} \in k$. Then we have an isomorphism

$$H^1_c(U_{\bar{K}}, \mathscr{K}_\chi) \simeq \mathscr{K}_\chi(a^a b^b c^c) \otimes \mathscr{Q}(\pi p^{n-r}) \otimes H^1_c(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2))$$

as a G_K-representation of degree 1.

Remark 3.2. If we identify a G_K-representation of degree 1 with a character of $K^\times$ by the local class field theory, the Kummer sheaf $\mathscr{K}_\chi(f)$ on Spec$K$ for an element $f \in K^\times$ is identified with the Hilbert symbol $K^\times \longrightarrow \overline{\mathbb{Q}}_l^\times; a \mapsto \chi((a, f)_m)$.

Lemma 3.3. *1. We have* dim $H^1_c(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2)) = 1$ *and the geometric Frobenius acts on this unramified representation $H^1_c(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2))$ as the multiplication by the quadratic Gauss sum $\sum_{s \in k} \psi_k(\gamma s^2)$ where ψ_k is the composite of the trace map $k \longrightarrow \mathbb{F}_p$ and the additive character ψ_0.*
2. We have dim $H^1_c(\mathbb{G}_{m,\bar{k}}, \mathscr{K}_{\chi'}(s^{a'}) \otimes \mathscr{L}_{\psi_0}(\frac{b}{m'}s)) = 1$ *and the geometric Frobenius acts on the unramified representation $H^1_c(\mathbb{G}_{m,\bar{k}}, \mathscr{K}_{\chi'}(s^{a'}) \otimes \mathscr{L}_{\psi_0}(\frac{b}{m'}s))$ as the multiplication by the Gauss sum $-\sum_{s \in k^\times} \chi'((\frac{s}{p})_{m'})^{a'} \psi_k(\frac{b}{m'}s)$ where $(\frac{\ }{p})_{m'} : k^\times \longrightarrow \mu_{m'}(k)$ is the m'-th power Legendre symbol.*

Proof. These follow from [D, Sommes Trig. (4.3)(i) and (4.3)(ii)]. (See also [AS, Lemma 4.12] for 1 and [La, Proposition (1.4.3.2)] for 2.) □

Proof of Theorem 3.1. 1. If we change a coordinate $x = \pi p^n s$, the sheaf $\mathscr{K}_\chi$ on U_K has the following form $y^m = (-1)^c (\pi p^n)^a s^a (1 - \pi p^n s)^b$. We put $f := (1 - \pi p^n s)^b$ and consider an $\mathcal{O}_K$-scheme $U := \mathrm{Spec}\mathcal{O}_K[s^{\pm 1}, \frac{1}{f}]$ whose special fiber is $D := \mathbb{G}_{m,k} = \mathrm{Spec}k[s^{\pm 1}]$. The projection formula induces an isomorphism

$$H^1_c(U_{\bar{K}}, \mathscr{K}_\chi) \simeq \mathscr{K}_\chi((-1)^c(\pi p^n)^a) \otimes H^1_c(U_{\bar{K}}, \mathscr{K}_\chi(s^a f)). \tag{3.1}$$

as a G_K-representation of degree 1. We have $\mathscr{K}_\chi(s^a f) \simeq \mathscr{K}_\chi(s^a) \otimes \mathscr{K}_\chi(f)$. By the assumption $n = r$, the sheaf $\mathscr{K}_\chi(s^a)$ extends to a smooth sheaf on U whose restriction to D is the Kummer sheaf $\mathscr{K}_{\chi'}(s^{a'})$. We have $(f-1)/\pi^p p^{n-1}|_D = bs$. Hence, by Corollary 2.2, the sheaf $\mathscr{K}_\chi(f)$ extends to a smooth sheaf on U whose restriction to the special fiber $D = \mathbb{G}_{m,k}$ is the sheaf $\mathscr{L}_{\psi_0}(\frac{b}{m'}s)$. Thereby we obtain the following cospecialization map

$$H_c^1(\mathbb{G}_{m,\bar{k}}, \mathscr{K}_{\chi'}(s^{a'}) \otimes \mathscr{L}_{\psi_0}(\frac{b}{m'}s)) \longrightarrow H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(s^a f)) \tag{3.2}$$

and this map is injective by Corollary 2.4. We have dim $H_c^1(\mathbb{G}_{m,\bar{k}}, \mathscr{K}_{\chi'}(s^{a'}) \otimes \mathscr{L}_{\psi_0}(\frac{b}{m'}s)) = 1$ by Lemma 3.3.2 and obviously dim $H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(s^a f)) =$ dim $H_c^1(U_{\bar{K}}, \overline{\mathbb{Q}}_l) = 1$. Hence the map (3.2) induces an isomorphism. Therefore the required assertion 1 follows from (3.1).

2. If we change a coordinate $t = 1 + \frac{c}{a}x$, the sheaf $\mathscr{K}_\chi$ is associated to the following equation $y^m = a^a b^b c^c (1-t)^a (1+\frac{a}{b}t)^b$. We put $f := (1-t)^a(1+\frac{a}{b}t)^b$. In the same way as (3.1), the projection formula induces an isomorphism as a G_K-representation of degree 1

$$H_c^1(U_{\bar{K}}, \mathscr{K}_\chi) \simeq \mathscr{K}_\chi(a^a b^b c^c) \otimes H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f)). \tag{3.3}$$

We consider the quadratic extension $K_1 := K((\pi p^{n-r})^{1/2})/K$ and again change a coordinate $t = (\pi p^{n-r})^{1/2}s$. Let U denote an $\mathcal{O}_{K_1}$-scheme Spec$\mathcal{O}_{K_1}[s, \frac{1}{f}]$. Then the generic fiber of U is canonically isomorphic to $U_{K_1} = \mathbb{P}^1_{K_1} - \{0, 1, \infty\}$ and the special fiber $D := U_s$ is isomorphic to $\mathbb{A}^1_k =$ Spec$k[s]$.

We calculate the Galois action on $H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f))$. Since we assume $p \geq 5, n > r$, we have $(f-1)/\pi^p p^{n-1}|_D = -\frac{a'c}{2b}s^2$. Hence, by Corollary 2.2, the sheaf $\mathscr{K}_\chi(f)$ extends to a smooth sheaf on U whose restriction to the special fiber $\mathbb{A}^1_k$ is the Artin-Schreier sheaf $\mathscr{L}_{\psi_0}(\gamma s^2)$. Hence we acquire the following cospecialization map $H_c^1(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2)) \longrightarrow H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f))$. We have dim $H_c^1(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2)) = 1$ by Lemma 3.3.1 and obviously we have dim $H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f)) = 1$. Since the cospecialization map is injective by Corollary 2.4, the map induces an isomorphism

$$H_c^1(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2)) \simeq H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f)).$$

as a G_{K_1}-representation of degree 1. We write $\mathscr{Q}(\pi p^{n-r})$ for the character $G_K \longrightarrow \{\pm 1\}; \sigma \mapsto ((\pi p^{n-r})^{1/2})^{\sigma - 1}$. Then the Galois group G_K acts on $\mathbb{A}^1_{\bar{k}}$ as $s \mapsto \mathscr{Q}(\pi p^{n-r})(\sigma)s$. The inertia group I_{K_1} acts on $H_c^1(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2))$ trivially and the quotient I_K/I_{K_1} acts on this group as $\mathscr{Q}(\pi p^{n-r})$. Thereby we obtain the following isomorphism

$$H_c^1(\mathbb{A}^1_{\bar{k}}, \mathscr{L}_{\psi_0}(\gamma s^2)) \otimes \mathscr{Q}(\pi p^{n-r}) \simeq H_c^1(U_{\bar{K}}, \mathscr{K}_\chi(f)) \tag{3.4}$$

as a G_K-representation. By the isomorphisms (3.3) and (3.4), the required assertion 2 follows. Hence we have proved the main theorem. □

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