Rational Retractions onto Ample Divisors

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Introduction

This paper is a continuation of the studies in [S1], [S2], [F1], [F3] and [B]. Here we are interested in the following problem: *Let D be an ample effective divisor on a normal variety V. Then, does there exist a rational mapping ρ: $V\to D$ defined on an open set U containing D such that the restriction of ρ to D is the identity?*

If the answer is Yes, we can show that V must be a cone over D under certain mild conditions (see § 3). In particular, V cannot be non-singular in this case. Motivated by these observations, we will give several sufficient conditions for the affirmitive answer to the above problem (see § 2). It turns out that our method works well for various types of manifolds D (see (3.4)).

§ 1. Preliminaries

Basically we employ the notation as in [F1] and [F3].

(1.1) Throughout in this paper we will study the following situation: V is a normal complete variety of dimension $n\geqq 3$ defined over an algebraically closed field $\mathfrak{K}$ of any characteristic. L is a line bundle on V and D is a member of $|L|$. We assume that V is non-singular along D and that the restriction of L to D is ample. The formal completion of V along D will be denoted by $\hat{V}$.

(1.2) Lemma. *Let F be a line bundle on V such that $FC\geqq 0$ for every curve C in V. Then there exists an integer k such that $H^1(V, -F-tL)=0$ for any $t\geqq k$.*

Proof. $|F+sL|\neq\emptyset$ for some $s>0$ by [F5; (6.5)], since $L^n=L_D^{n-1}\{D\}>0$. Taking a member E of $|F+sL|$, we apply [F7; (7.5)].

(1.3) Theorem. *There exists a birational morphism $\pi: V\to V'$ onto a normal variety V' such that π is an isomorphism on a neighborhood of D and $D'=\pi(D)$ is an ample divisor on V'.*

For a proof, see [H; p. 110].

(1.4) Corollary. *If* char$(\mathfrak{K})=0$ *and if V is non-singular, then $H^q(V, \mathcal{O}_V)\cong H^q(D, \mathcal{O}_D)$ for any $q<n-1$.*

Indeed, $H^i(V, -L)=0$ for $i<n$ by Ramanujam's theorem [R].

(1.5) Corollary. *If* char$(\mathfrak{K})=0$ *and both V and D are smooth, then the* *natural mapping* α: $\mathrm{Alb}(D)\to\mathrm{Alb}(V)$ *and* r: $\mathrm{Pic}^0(V)\to\mathrm{Pic}^0(D)$ *are isomorphisms.*

Proof. Both are étale by (1.4). Suppose that $F\in\mathrm{Ker}(r)$. From the exact sequence $0\to\mathcal{O}_V[F-L]\to\mathcal{O}_V[F]\to\mathcal{O}_D\to 0$ we infer $h^1(V,F-tL)\leqq h^1(V,F-(t+1)L)$ for every $t\geqq 1$. Hence $h^1(V,F-L)\leqq h^1(V,F-tL)$ for any $t>1$, while the latter vanishes for $t\gg 0$ by (1.3). So $H^1(V,F-L)=0$ and $h^0(V,F)\geqq h^0(D,\mathcal{O}_D)=1$. This imples $F\cong\mathcal{O}_V$ since $F\in\mathrm{Pic}^0(V)$. Thus we see that r is injective and is an isomorphism. Hence so is α.

(1.6) Theorem. *The effective Lefschetz condition* $\mathrm{Leff}(V,D)$ *is satisfied. This means the following:*

1) *For any open set* $U\supset D$ *and any locally free sheaf* E *on* U, *there exists an open set* U' *with* $D\subset U'\subset U$ *such that the mapping* $H^0(U',E)\to H^0(\hat{V},\hat{E})$ *is bijective, where* $\hat{E}$ *is the restriction of* E *to* $\hat{V}$.

2) *For any locally free sheaf* E *on* $\hat{V}$, *there exist an open set* $U\supset D$ *and a locally free sheaf* E *on* U *such that* $\hat{E}=E$.

For a proof, see [G; Exposé X] or consult [H; § 4]. Note that we may assume that L is ample by virtue of (1.3).

§ 2. Existence of a rational retraction

(2.1) A rational mapping ρ: $V\to D$ is called a *rational retraction* of V onto D if ρ is defined on a neighborhood U of D and if the restriction of ρ to D is the identity.

(2.2) Proposition. *Let* F *be a line bundle on* D *with* $\mathrm{Bs}|F|=\emptyset$ *and let* f: $D\to W\subset \boldsymbol{P}^N$ $(N=\dim|F|)$ *be the morphism defined by the linear system* $|F|$. *Suppose that* $H^q(D,F-tL_D)=0$ *for every* $t\geqq 1$, $q\leqq 1$ *and that* F *comes from* $\mathrm{Pic}(\hat{V})$. *Then there is a rational mapping* ρ *of* V *onto* $\boldsymbol{P}^N$ *defined on a neighborhood of* D *such that the restriction of* ρ *to* D *is the morphism* f: $D\to\boldsymbol{P}^N$. *Moreover, if* $\dim W<n-1$, *then* $\mathrm{Im}(\rho)=W$.

Proof. By (1.6) we have a line bundle $\tilde{F}$ on an open set U containing D such that $\tilde{F}_D=F$ and $H^0(U,\tilde{F})\cong H^0(\hat{V},\hat{F})\cong H^0(D,F)$. Hence $|\tilde{F}|$ gives a rational mapping ρ with the desired property. To prove the second assertion, we may assume that ρ is defined everywhere on V by replacing V by the normalization of the graph of ρ. Then we infer $\rho(V)=W$ by the same technique as in [F1; (2.7)].

(2.3) Corollary. *When* $D\cong\boldsymbol{P}^{n-1}$, *there exists a rational retraction of* V *onto* D *unless* $n=\mathrm{char}(\mathfrak{K})=3$ *and* $L_D=\mathcal{O}(3s)$ *for some* $s\in\boldsymbol{Z}$.

Proof. The obstruction of extending $F=\mathcal{O}_D(1)$ to $\mathrm{Pic}(\hat{V})$ lies in the $\mathfrak{K}$-vector spaces $\{H^2(D,-tL_D)\}_{t=1,2,\dots}$. On the other hand, L_D and $\omega_D=\mathcal{O}_D(-n-1)$ come from $\mathrm{Pic}(\hat{V})$ because V is non-singular along D. Hence F comes from $\mathrm{Pic}(\hat{V})$ except the case described above. So (2.2) applies if $L_D\neq F$. If $L_D=F$, use [F4; (4.3)].

(2.4) Corollary. *Let* f: $D\to S$ *be a morphism onto a projective variety* S *with* $\dim S\leqq\dim D-2=n-3$, $f_*\mathcal{O}_D=\mathcal{O}_S$. *Assume that* $\mathrm{char}(\mathfrak{K})=0$ *and* D *is non-singular. Then* f *extends to a rational mapping* ρ: $V\to S$ *defined in a neighborhood of* D.

Proof. We may assume $n\geqq 4$. So, for a very ample line bundle H on S, we infer that $F=f^*H$ come from $\mathrm{Pic}(\hat{V})$. By the technique in [F1; (2.9)], we see that (2.2) applies.

(2.5) Corollary. *Suppose that $D\cong X\times Y$ for some varieties X, Y such that $\dim X\geqq\dim Y\geqq 2$, $h^1(X_y, -tL_y)=h^1(Y_x, -tL_x)=0$ for every $t\geqq 1$ and for every general point x, y on X, Y respectively, where X_y (resp. Y_x) is the fiber over y (resp. x) of the projection $D\to Y$ (resp. X) and L_y (resp. L_x) is the restriction of L to it. Assume further that $H^2(D, -tL_D)=0$ for every $t\geqq 1$. Then there exists a rational retraction of V onto D.*

Proof. $\mathrm{Pic}(\hat{V})\to\mathrm{Pic}(D)$ is surjective by the last assumption. Let A be a very ample line bundle on X and let $p: D\to X$ be the projection. For every $t\geqq 1$ we infer $H^1(D, p^*A-tL_D)=0$ by the method in [F1; (2.5)]. Applying (2.2) to $F=p^*A$ we extend p to a rational mapping $V\to X$ defined on a neighborhood of D. Similarly we extend $D\to Y$. Combining these extensions we obtain a rational retraction, as desired.

Remark. If $\mathrm{char}(\mathfrak{K})=0$ and X, Y are non-singular, then the assumptions on cohomology groups are valid by the vanishing theorem of Kodaira. This remark applies also to the following

(2.6) Corollary. *Suppose that $D\cong X\times Y\times Z$ for some varieties X, Y, Z such that $\dim X\geqq\dim Y\geqq\dim Z\geqq 1$, $h^1(D_x, -tL_x)=h^1(D_y, -tL_y)=h^1(D_z, -tL_z)=0$ for every $t\geqq 1$ and any general point x, y, z on X, Y, Z respectively, where D_x (resp. D_y, D_z) is the fiber over x (resp. y, z) of the projection $D\to X$ (resp. Y, Z) and L_x (resp. L_y, L_z) is the restriction of L to it. Assume further that $H^2(D, -tL_D)=0$ for every $t\geqq 1$. Then there exists a rational retraction of V onto D.*

Proof is similar to that in (2.5).

(2.7) Proposition. *Suppose that $D\cong G_{m,r}$, the Grassmann variety parametrizing r-dimensional vector subspaces of $\mathfrak{K}^m$. Then there exists a rational retraction of V onto D unless $r=1$, $m-r=1$ or $r=m-r=2$.*

Proof. Similarly as in [F2], we can extend the universal bundle E on D to a vector bundle $\hat{E}$ on $\hat{V}$ such that $H^0(\hat{V}, \hat{E})\cong H^0(D, E)$. By (1.6) $\hat{E}$ extends further to a vector bundle $\bar{E}$ on an open set U containing D such that $H^0(U, \bar{E})\cong H^0(\hat{V}, \hat{E})$. So, by the standard method for defining a rational mapping to $G_{m,r}$, we obtain a desired rational retraction.

(2.8) From now on, in this section, we consider the case in which $\mathrm{char}(\mathfrak{K})=0$ and D is non-singular.

Proposition. *Suppose that D is an abelian variety. Then there exists a rational retraction of V onto D.*

Proof. By the desingularization theory we may assume that V is non-singular. Then the Albanese mapping of V gives a desired retraction by (1.5).

Warning. One might think that this method works if D is birationally equivalent to an Abelian variety. But this is not true. The Albanese mapping gives a rational mapping $\rho: V\to D$, but ρ is not necessarily defined in a neighborhood of D. To construct a counterexample, take an Abelian surface A and a very ample line bundle H on A. Let $V=A\times\mathbf{P}^1$, $L=p_1^*H\otimes p_2^*\mathcal{O}_P(1)$ and let D be a general member of $|L|$. Then L is very ample on V and D is a blowing-up of A with center being H^2 points. But one easily sees that there is no rational retraction of V onto D.

(2.9) Theorem. *Suppose that* $\mathrm{char}(\mathfrak{K})=0$, *$D$ is non-singular and that* $H^1(D, T^D[-tL_D])=0$ *for every* $t\geqq 1$, *where T^D is the tangent bundle of D. Then there exists a rational retraction of V onto D.*

Proof. By the desingularization theory we may assume that V is non-singular. Let T be the tangent bundle of V. Similarly as in [F3; (1.2)], we infer $H^1(D, T[-tL]_D)=0$ for any $t\geqq 2$, which implies that $H^0(\hat{V}, \hat{T}[-\hat{L}])\to H^0(D, T[-L]_D)$ is surjective. Letting N be the normal bundle of D in V, we infer that the natural matting $H^0(D, T[-L]_D)\to H^0(D, N[-L_D])\cong H^0(D, \mathcal{O}_D)$ is surjective because $H^1(D, T^D[-L_D])=0$ by assumption. So we have $\theta\in H^0(\hat{V}, \hat{T}[-\hat{L}])$ which is mapped to $1\in H^0(D, \mathcal{O}_D)$.

Let $\mathcal{J}$ be the ideal of D in V and set $\mathcal{O}_m=\mathcal{O}_V/\mathcal{J}^{m+1}$ for every $m\geqq 0$. Then, of course, $\hat{\mathcal{O}}=\mathrm{proj.}\lim\mathcal{O}_m$ is the structure sheaf of $\hat{V}$. Note that $\hat{\mathcal{J}}=\mathcal{J}\hat{\mathcal{O}}$ is an $\hat{\mathcal{O}}$-ideal and $\mathcal{O}_m\cong\hat{\mathcal{O}}/\hat{\mathcal{J}}^{m+1}$. Since $\theta\in H^0(\hat{V}, \hat{T}[-\hat{L}])\cong H^0(\hat{V}, \hat{\mathcal{J}}[\hat{T}])$, θ can be viewed as a $\mathfrak{K}$-derivation of the sheaf $\hat{\mathcal{O}}$ of $\mathfrak{K}$-algebras such that $\theta(\hat{\mathcal{O}})\subset\hat{\mathcal{J}}$. So $\theta(\hat{\mathcal{J}}^k)\subset\hat{\mathcal{J}}^k$ for every k and θ induces a derivation θ_m of $\mathcal{O}_m$ and an endomorphism θ'_m of $\hat{\mathcal{J}}^m/\hat{\mathcal{J}}^{m+1}$ for every $m\geqq 1$. We easily check that θ'_m is actually $\mathcal{O}_D$-linear. Moreover, $\theta'_1\in\mathrm{End}(\hat{\mathcal{J}}/\hat{\mathcal{J}}^2)\cong H^0(D, \mathcal{O}_D)$ is nothing but the image of θ of by the foregoing natural surjective mapping. Hence θ'_1 is an isomorphism. Since $\mathrm{char}(\mathfrak{K})=0$, we also see that θ'_m is an isomorphim for every $m\geqq 1$. On the other hand, $\theta'_0=0$ is clear.

Now, for any affine open set U in D and any $\psi\in H^0(U, \mathcal{O}_{m-1})$ with $\theta_{m-1}(\psi)=0$, we claim, there exists one and only one element ψ_m of $H^0(U, \mathcal{O}_m)$ such that $\theta_m(\psi_m)=0$ and ψ_m is mapped to ψ by the natural homomorphism $\mathcal{O}_m\to\mathcal{O}_{m-1}$. To see this, take $\psi'\in H^0(U, \mathcal{O}_m)$ which is mapped to ψ. Then $\theta_m(\psi')\in\hat{\mathcal{J}}^m/\hat{\mathcal{J}}^{m+1}\subset\mathcal{O}_m$ since $\theta_{m-1}(\psi)=0$. By the surjectivity of θ'_m we have $\delta\in H^0(U, \hat{\mathcal{J}}^m/\hat{\mathcal{J}}^{m+1})$ such that $\theta'_m(\delta)=\theta_m(\psi')$. Then $\psi_m=\psi'-\delta$ has the desired property. The uniqueness of ψ_m follows from the injectivity of θ'_m.

Given any $\varphi\in H^0(U, \mathcal{O}_D)$, we apply the above claim repeatedly to obtain $\{\varphi_m\}\in H^0(U, \hat{\mathcal{O}})$ such that $\varphi_0=\varphi$ and $\theta_m(\varphi_m)=0$ for every $m\geqq 0$. By the uniqueness we infer that this construction gives rise to a homomorphism $\theta^*: \mathcal{O}_D\to\hat{\mathcal{O}}$ of sheaf of rings.

Take a very ample line bundle H on D. We define $\theta^*H\in\mathrm{Pic}(\hat{V})$ in the obvious way. Moreover, we have a mapping $\theta^*: H^0(D, H)\to H^0(\hat{V}, \theta^*H)$ such that $\theta^*\zeta_D=\zeta$ for any $\zeta\in H^0(D, H)$. Using (1.6), we take a neighhorhood V_0 of D in V and a line bundle $\tilde{H}$ on V_0 such that the restriction of $\tilde{H}$ to $\hat{V}$ is θ^*H and $H^0(V_0, \tilde{H})\cong H^0(\hat{V}, \theta^*H)$. Take a basis $\zeta_0, \zeta_1, \cdots, \zeta_M$ of $H^0(D, H)$ and let $\tilde{\zeta}_j\in H^0(V_0, \tilde{H})$ be the extension of $\theta^*\zeta_j\in H^0(\hat{V}, \theta^*H)$. Then $\tilde{\zeta}_j$'s define a linear system $\tilde{\Lambda}$ on V_0 such that $\tilde{\Lambda}_D=|H|$. Moreover, for any relation $R(\zeta_0, \cdots, \zeta_M)=0$ in $\oplus_{t\geqq 0}H^0(D, tH)$, we have $R(\theta^*\zeta_0, \cdots, \theta^*\zeta_M)=0$ on $\hat{V}$ and hence $R(\tilde{\zeta}_0, \cdots, \tilde{\zeta}_M)=0$ in $\oplus_{t\geqq 0}H^0(V_0, t\tilde{H})$. Therefore $\tilde{\Lambda}$ gives a rational mapping onto $D\subset\mathbf{P}^M$, which is a desired rational retraction.

(2.10) Remark. There are various types of polarized manifolds (D, L_D) which satisfy the condition in (2.9) (see [F3]). For example we have:

a) Abelian varieties of dimension $\geqq 2$.

b) Kummer manifolds of dimension $\geqq 3$.
c) Grassmann varieties $G_{m,r}$ unless $r=1$, $m-r=1$ or $r=m-r=2$.
d) Non-trivial product $D=D_1\times D_2\times D_3$.
e) $D=D_1\times D_2$ with $\dim D_1\geqq\dim D_2\geqq 2$.
f) $D=D_1\times D_2$, the cotangent bundles of both D_1 and D_2 are generated by global sections.
g) $D=D_1\times D_2$, $\dim D_1\geqq 2$ and the cotangent bundle of D_1 is generated by global sections.
h) Fiber bundle $D\to S$, with fiber satisfying the condition (2.9).
i) Manifolds which are isogenous to a manifold satisfying the condition (2.9). Here, M and M' are said to be isogenous to each other if they are dominated by a common manifold via étale morphisms.
j) Blowing-ups of manifolds of dimension $\geqq 3$ as above with center being finitely many points.
k) Any small deformation of polarized manifolds (D, L_D) satisfying the condition (2.9).

In particular, (2.9) gives a new proof of (2.8).

(2.11) Remark. The argument in (2.9) proves also the following: *If the natural mapping* $H^0(\hat{V}, \hat{T}[-\hat{L}])\to H^0(D, N[-L_D])\cong H^0(D, \mathcal{O}_D)$ *is surjective, then there exists a rational retraction of V onto D.*

(2.12) Proposition. *Suppose that* $\operatorname{char}(\mathfrak{K})=0$, *$D$ is non-singular and that the cotangent bundle Ω^D of D is a direct sum of two vector bundles E, F of positive rank which are generated by global sections. Then there exists a rational retraction of V onto D.*

Proof. The Albanese mapping $\alpha_D\colon D\to\operatorname{Alb}(D)=A$ is étale over the normalization W of the image $\alpha_D(D)$ because $\alpha_D^*\Omega^A\to\Omega^D$ is surjective by assumption. In particular W is smooth and $T^D\cong\alpha_D^*T^W$. To prove the assertion, we may assume that V is non-singular. Then $\operatorname{Alb}(V)\cong A$ by (1.5). Moreover, using (1.4), we infer $\alpha_V(V)=\alpha_D(D)$ by the same method in [S1; Proposition I]. Hence we obtain a morphism $f\colon V\to W$ which is an extension of $D\to W$. Restricting the homomorphism $f_*\colon T^V\to f^*T^W$ to D, we get a splitting of the exact sequence $0\to T^D\to T^V_D\to N\to 0$, since $T^D\cong T^W_D$. Therefore $F_D\cong N\cong L_D$ for $F=\operatorname{Ker}(f_*)$. Since $H^1(D, [F-tL]_D)=0$ for every $t\geqq 2$, we infer that $H^0(\hat{V}, \hat{F}-\hat{L})\to H^0(D, \mathcal{O}_D)$ is surjective. This implies $\hat{F}=\hat{L}$ in $\operatorname{Pic}(\hat{V})$. Now, the inclusion $F\to T^V$ gives $\theta\in H^0(\hat{V}, \hat{T}[-\hat{L}])$ as in (2.9). So, by (2.11), we obtain the conclusion.

§ 3. Characterization of cones

(3.1) Theorem. *Suppose that there exists a rational retraction ρ of V onto D and that $H^1(D, -tL_D)=0$ for every $t\geqq 1$. Then the graded algebra $G(V, L)=\bigoplus_{t\geqq 0}H^0(V, tL)$ is isomorphic to the polynomial algebra with one variable of degree one over the graded algebra $G(D, L_D)=\bigoplus_{t\geqq 0}H^0(D, tL_D)$.*

Proof. Replacing V by the normalization of the graph of the rational mapping ρ, we may assume that $\rho\colon V\to D$ is a morphism. Then we claim $H^1(V, -\rho^*L_D)=0$.

Indeed, by the exact sequence $H^1(V, -\rho^*L_D-tL)\to H^1(V, -\rho^*L_D-(t-1)L)\to H^1(D, -tL_D)=0$ for $t\geqq 1$, we obtain $h^1(V, -\rho^*L_D)\leqq h^1(V, -\rho^*L_D-tL)$ for every $t\geqq 0$. The last term vanishes for $t\gg 0$ by (1.2). So $H^1(V, -\rho^*L_D)=0$.

This claim implies that $H^0(V, L-\rho^*L_D)\to H^0(D, \mathcal{O}_D)$ is surjective. So we have $\varepsilon\in H^0(V, L-\rho^*L_D)$ such that ε_D induces the isomorphism $L_D\cong(\rho^*L_D)_D$. Now, setting $\Phi_t(\varphi)=\varepsilon^t\otimes\rho^*\varphi$ for $\varphi\in H^0(D, tL_D)$, we obtain a $\mathfrak{K}$-algebra homomorphism Φ: $G(D, L_D)\to G(V, L)$. Clearly $r_t\cdot\Phi_t$ is the identity for the restriction mapping r_t: $H^0(V, tL)\to H^0(D, tL_D)$. Therefore $H^0(V, tL)\cong \mathrm{Im}\,(\Phi_t)\oplus\mathrm{Ker}\,(r_t)$. Take $\delta\in H^0(V, L)$ such that D is the zero divisor of δ. Then $\mathrm{Ker}\,(r_t)=\delta H^0(V, (t-1)L)$. From these observations we infer that $G(V, L)$ is the polynomial algebra over $\mathrm{Im}\,(\Phi)\cong G(D, L_D)$ generated by δ. Thus we complete the proof.

(3.2) Corollary. *If in addition L is ample on V, then V is isomorphic to the cone obtained by contracting a section of* $\mathrm{P}=\boldsymbol{P}_D(L_D\oplus\mathcal{O}_D)$ *with normal bundle* $\cong[-L_D]$ *to a normal point. In particular, V is not smooth unless* $(D, L_D)\cong(\boldsymbol{P}^{n-1}, \mathcal{O}(1))$.

(3.3) Corollary. *If L is very ample, then V is a projective cone over D. In particular, (D, L_D) is projectively normal since V is normal.*

Proof. Let $\zeta_1, \cdots, \zeta_M$ be a basis of $H^0(D, L_D)$ and set $\xi_j=\Phi(\zeta_j)$ for $j=1, \cdots, M$. Then $\delta, \xi_1, \cdots, \xi_M$ is a basis of $H^0(V, L)$. For any relation $R(\zeta_1, \cdots, \zeta_M)=0$ in $G(D, L_D)$, we have $R(\xi_1, \cdots, \xi_M)=0$ in $G(V, L)$. Hence V is a cone over D.

(3.4) By virtue of results in §2, (3.1) applies if (D, L_D) is a polarized manifold of the following types:

a) $\mathbf{P}^{n-1}$ unless $n=\mathrm{char}\,(\mathfrak{K})=3$.

b) Grassmann variety $G_{m,r}$ unless $r=m-r=2$, $r=1$ or $m-r=1$.

c) Product of them.

d) Those of the types in (2.10) and (2.12), when $\mathrm{char}\,(\mathfrak{K})=0$.

(3.5) Remark. Combining (2.11) and (3.1), we obtain the following result in case $\mathfrak{K}=\mathbf{C}$.

Suppose that D is non-singular, L is ample and that $H^0(\hat{V}, \hat{T}[-\hat{L}])\to H^0(D, \mathcal{O}_D)$ is surjective. Then V is the cone over D as in (3.2).

This follows also from J. Wahl's result [W; Theorem 2]. Indeed, we have $\theta\in H^0(V, \hat{T}[\hat{L}])$ as in (2.9). Let Θ be the sheaf of $\mathfrak{K}$-derivations of $\mathcal{O}_V$. By (1.6), Θ extends to a section $\tilde{\theta}$ of Θ on an open neighborhood U of D in V. $V-U$ is a finite set since D is ample. So $\tilde{\theta}$ extends to a section of Θ on the whole space V because Θ is reflexive and has depth $\geqq 2$. Hence J. Wahl's theorem applies.

It is amusing to note that the same phenomenon can be interpreted in different ways.

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