# A Characterization and the Structure of Operators with Maharam Property

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## § 1. Introduction

The purpose of this paper is to characterize and determine the structure of positive operators between AM-spaces which possess the "Maharam property". In § 2, the definition of Maharam property is recalled and three types of examples of operators with this property are described. For the sake of simplicity, we call an operator with Maharam property simply a Maharam operator. In § 3, we give a characterization of Maharam operators and show that any Maharam operator between separable AM-spaces is the composition of three types of operators described in § 2. For general terminology concerning Banach lattices, we refer to [3] or [6].

## § 2. Maharam operators

Let $E, F$ be vector lattices and $T$ be a positive linear operator from $E$ to $F$. Then $T$ is said to have the Maharam property whenever for all $0 \leqq f \in E$ and for all $0 \leqq g \in F$ satisfying $0 \leqq g \leqq Tf$ there exists an element $f_0 \in E$ such that $0 \leqq f_0 \leqq f$ and $Tf_0 = g$. Maharam property was first introduced by D. Maharam [4], [5] in the context of $F$-integrals under the name of "full-valuedness". This property plays an important role in the Radon-Nikodym type theory for positive operators developed by Luxemburg and Schep [2]. The duality between Maharam operators and lattice homomorphisms is also known, see e. g., Lotz [1].

The following is an immediate consequence of the definition.

PROPOSITION 1. *Let $E, F$ and $G$ be vector lattices and let $T: E \to F$, $S: F \to G$ be linear operators. Then the following hold.*

1) *If $T$ is a Maharam operator, the range of $T$ is an ideal of $F$. Conversely, if the range of $T$ is an ideal of $F$ and $T$ is a lattice homomorphism, then $T$ is a Maharam operator.*

2) *If $T$ and $S$ are Maharam operators, then $S\circ T$ is also a Maharam operator.*

In the rest of this paper, only the Maharam operators between AM-spaces with unit are dealt with. In other words, we study the case $T: C(X)\to C(Y)$ where $X$, $Y$ are compact Hausdorff spaces.

Three types of examples of Maharam operators $T: C(X)\to C(Y)$

($A$) *Restriction (or quotient) map*

Let $Y$ be a closed subspace of $X$. Then the mapping $T: C(X)\to C(Y)$ which maps $f\in C(X)$ to $f_{|Y}\in C(Y)$, the restriction of $f$ to $Y$, is a Maharam operator. This is immediately proved by Tietze's extension theorem. In the Banach lattice theoretic view, $C(Y)$ is isomorphic to the quotient $C(X)/I$, where $I:=\{f\in C(X); f=0 \text{ on } Y\}$, and if we identify $C(Y)$ with $C(X)/I$, $T$ is the canonical map $C(X)\to C(X)/I$ and hence a Maharam operator by Proposition 1.

($B$) *Maharam retraction*

Let $I$ be a closed sublattice of $C(X)$ containing the constant functions. Then there exists a compact Hausdorff space $Y$ for which $I$ is isometrically lattice isomorphic to $C(Y)$ by the representation theorem of S. Kakutani ([6], p. 104).

A Maharam operator $T: C(X)\to I$ satisfying $T_{|I}=\text{identity}$ is called a Maharam retraction of $C(X)$ onto $I$. If $i$ denotes the canonical injection $I\to C(X)$, them $P:=i\circ T\in\mathfrak{L}(C(X))$ (=the set of bounded linear operators on $C(X)$) is a positive projection onto $I$. Moreover, $P$ has the following property;

(*) $[0, Pf]\cap I=P[0, f]$ holds for any $0\leqq f\in C(X)$, where $[0, u]:=\{v\in C(X); 0\leqq v\leqq u\}$ for any $u\in C(X)$.

Conversely let $P\in\mathfrak{L}(C(X))$ be a positive projection with range $I$ and with the property (*). Then the operator $T: C(X)\to I$ defined by $Tf=Pf$ for $f\in C(X)$ is a Maharam retraction onto $I$ and $P=i\circ T$ holds. Thus the Maharam retractions onto $I$ and positive projections in $\mathfrak{L}(C(X))$ whose ranges are $I$ and which satisfy (*) are in one to one correspondence.

Note that by the assumption that $I$ contains the constant functions, any positive projection onto $I$ is of norm 1.

Concrete example of Maharam retraction:

Let $E:=C([-1, 1])$ and $I:=\{f\in E; f(-x)=f(x) \text{ for any } x\in[-1, 1]\}$. Define $P\in\mathfrak{L}(E)$ by

$$Pf(x):=\frac{1}{2}(f(x)+f(-x)),\ x\in[-1, 1].$$

Then $P$ is a positive projection onto $I$ satisfying (*). The proof of this fact is contained in that of Theorem 1.

($C$) *Lattice isomorphism*

A lattice isomorphism $T: C(X)\to C(Y)$ is clearly a Maharam operator by

Proposition 1. The structure of such lattice isomorphisms is well known. Namely, there exists a function $p \in C(Y)$ and a homeomorphism $\psi : Y \to X$ such that $p(y) > 0$ for any $y \in Y$ and

$$Tf(y) = p(y) f \circ \psi(y)$$

holds for any $f \in C(X)$ and $y \in Y$.

*Remark.* The above examples also give typical examples for Maharam operators between general vector lattices.

## § 3. Characterization of Maharam operators

To characterize Maharam operators, we introduce the following notion. Let $T : C(X) \to C(Y)$ be a bounded linear operator, where $X$ and $Y$ are compact Hausdorff spaces. Let $T'$ be the adjoint of $T$ and $\delta_y$ denote the Dirac measure concentrated on $y \in Y$. Then, for any $y \in Y$, define the measure $\mu_y$ on $X$ by $\mu_y = T'\delta_y$ and let $S_y$ be the support of $\mu_y$. Note that $S_y = \emptyset$ if $\mu_y = 0$. Thus for any bounded linear operator $T : C(X) \to C(Y)$ there corresponds a family $\{S_y\}_{y \in Y}$ of subsets of $X$ indexed by $Y$. In general, a family $\{F_y\}_{y \in Y}$ of subsets of $X$ indexed by $Y$ is said to be upper semi-continuous if for any net $\{x_\alpha\}_{\alpha \in A}, \{y_\alpha\}_{\alpha \in A}$ such that $x_\alpha \in F_{y_\alpha}$, $x_\alpha \to x$ and $y_\alpha \to y$ for some $x \in X$ and $y \in Y$, then $x \in F_y$ holds. It is clear that $\{F_y\}_{y \in Y}$ is upper semi-continuous if and only if the set $\{(x, y);\ x \in F_y\}$ is closed in $X \times Y$. Now we have the following

PROPOSITION 2. *Let $T : C(X) \to C(Y)$ be a positive linear operator, where $X$ and $Y$ are compact Hausdorff, and let $S_y$ be the support of the measure $T'\delta_y$ for $y \in Y$. Then*

1) *if $T$ is a Maharam operator, then $S_y \cap S_{y'} = \emptyset$ for any distinct elements $y, y'$ of $Y$,*

2) *if the family $\{S_y\}_{y \in Y}$ is upper semi-continuous and $S_y \cap S_{y'} = \emptyset$ for any distinct elements $y, y'$ of $Y$, then $T$ is a Maharam operator.*

PROOF. *proof of* 1). If $T\mathbf{1}(y) = 0$, where $\mathbf{1} \in C(X)$ is the constant one, $S_y = \emptyset$ by definition. So there is nothing to be proved in this case. If $T\mathbf{1}(y) > 0$, there exists a function $g \in C(Y)$ such that $g(y') = 0$, $g(y) = T\mathbf{1}(y)$ and $0 \leqq g \leqq T\mathbf{1}$. By the assumption there exists a function $f \in C(X)$ which satisfies $Tf = g$ and $0 \leqq f \leqq \mathbf{1}$. Since

$$0 = g(y') = \int f\, d\mu_{y'},$$

$$\int \mathbf{1}\, d\mu_y = T\mathbf{1}(y) = g(y) = \int f\, d\mu_y \text{ and } 0 \leqq f \leqq \mathbf{1},$$

$f = 0$ on $S_{y'}$ and $f = 1$ on $S_y$. Hence $S_y \cap S_{y'} = \emptyset$.

*proof of* 2). Let $0 \leqq f \in C(X)$ and $0 \leqq g \in C(Y)$ satisfy $0 \leqq g \leqq Tf$. By the upper semi-continuity of $\{S_y\}_{y \in Y}$ and the compactness of $X$ and $Y$, the set $X_0 := \bigcup_{y \in Y} S_y$ is closed in $X$. Put $Y_0 := \{y\,; Tf(y) > 0\}$. Then $O := \bigcup_{y \in Y_0} S_y$ is relatively open in $X_0$ since $\bigcup_{y \notin Y_0} S_y$ is closed. For any $x \in X_0$ define $h(x)$ by

$$h(x) = \begin{cases} \dfrac{g(y)}{Tf(y)} f(x), & \text{if } x \in S_y \text{ for some } y \in Y_0, \\ 0 \quad , & \text{if } x \in X_0 \backslash O. \end{cases}$$

$h(x)$ is well defined, since for $x \in O$ there exists a unique $y \in Y_0$ such that $x \in S_y$. To show that the function $h : x \longmapsto h(x)$ is continuous on $X_0$, let $\{x_\alpha\}_{\alpha \in A}$ be a net in $X_0$ convergent to $x \in X_0$. Suppose $x \in O$. Then we may assume that $x_\alpha \in O$ for any $\alpha \in A$ since $O$ is relatively open in $X_0$. For any $\alpha \in A$, an element $y_\alpha \in Y_0$ for which $x_\alpha \in S_{y_\alpha}$ is uniquely determined. Also an element $y \in Y_0$ such that $x \in S_y$ is uniquely determined. If $h(x_\alpha) \to h(x)$ does not hold, there exists a subnet $\{x_\beta\}_{\beta \in B}$ of $\{x_\alpha\}_{\alpha \in A}$ and $\varepsilon > 0$ for which

$$(*) \qquad |h(x_\beta) - h(x)| > \varepsilon \qquad \text{for any } \beta \in B$$

holds. By taking a suitable subnet, we may assume that the corresponding subnet $\{y_\beta\}_{\beta \in B}$ of $\{y_\alpha\}_{\alpha \in A}$ is convergent. The upper semi-continuity of $\{S_y\}_{y \in Y}$ implies that the limit of $\{y_\beta\}_{\beta \in B}$ is $y$. For these subnets

$$h(x_\beta) = \frac{g(y_\beta)}{Tf(y_\beta)} f(x_\beta) \to \frac{g(y)}{Tf(y)} f(x) = h(x)$$

holds since $Tf(y_\beta) \to Tf(y) > 0$, and hence contradicts (*).

Suppose now that $x \in X_0 \backslash O$. It suffices to show $h(x_\alpha) \to 0$ in case $x_\alpha \in O$ for any $\alpha \in A$. Since $0 \leqq h(z) \leqq f(z)$ for any $z \in X_0$, it also suffices to show that $f(x_\alpha) \to 0$. By the assumption $x \in X_0 \backslash O$, $Tf(y) = 0$ holds for the unique element $y \in Y \backslash Y_0$ such that $x \in S_y$. Therefore $\int f\, d\mu_y = 0$ and hence $f(x) = 0$ since $x \in S_y$. This implies $f(x_\alpha) \to 0$ by the continuity of $f$. Thus we have shown that $h \in C(X_0)$ and $0 \leqq h \leqq f$ on $X_0$. It is clear that $\int h\, d\mu_y = g(y)$ holds for any $y \in Y$. To obtain a $k \in C(X)$ which satisfies $0 \leqq k \leqq f$ and $Tk = g$, take any continuous, non-negative extension $h_1$ of $h$ to $X$ and let $k$ be the infimum of $f$ and $h_1$.//

If $X$ and $Y$ are metrizable, the above proposition can be improved.

THEOREM 1. *Let $X$ and $Y$ be compact metric spaces and $T : C(X) \to C(Y)$ be a positive linear operator. Let $S_y$ denote the support of the measure $\mu_y := T'\delta_y$ where $\delta_y$ is the Dirac measure concentrated on $y \in Y$. Then the following assertions are equivalent.*

(i) *$T$ is a Maharam operator.*

(ii) a) *$\{S_y\}_{y\in Y}$ is an upper semi-continuous family, and*
b) *$S_y \cap S_{y'}=\emptyset$ for any $y, y'\in Y$ such that $y\neq y'$.*

PROOF. The implication (ii)$\Rightarrow$(i) is already proved in Proposition 2. To see that (i) implies (ii), it suffices to show that (i) and b) imply a), since the implication (i)$\Rightarrow$b) is proved in Proposition 2. Suppose a) does not hold. Then there exists a sequence $\{x_n\}_{n\in N}$ [resp. $\{y_n\}_{n\in N}$] of points of $X$ [resp. $Y$] with the following properties:

$x_n\in S_{y_n}$, $\{x_n\}_{n\in N}$ [resp. $\{y_n\}_{n\in N}$] is convergent to $x_0\in X$ [resp. $y_0\in Y$], and $x_0\notin S_{y_0}$.

Let $d$ be the metric on $Y$. By choosing an appropriate subsequence if necessary, we may assume that

$$(**) \qquad 0<d(y_0, y_{n+1})<\frac{1}{3}d(y_0, y_n)$$

holds for any $n\in N$. Then it is easily proved that there exists a sequence $\{f_n\}_{n\in N}$ of continuous functions on $Y$ such that

$$0\leqq f_n\leqq 1,\ f_n(y_n)=1, \quad \text{and} \quad \operatorname{supp} f_n\subset\{y\,;\,d(y, y_n)<\frac{1}{2}d(y_0, y_n)\}$$

holds for any $n\in N$, where $\operatorname{supp} f_n$ denotes the support of $f_n$. Then a simple computation shows that $\operatorname{supp} f_n\cap\operatorname{supp} f_m=\emptyset$ if $n\neq m$. Put $g(y)=\sum_{n=1}^{\infty} f_{2n}(y)$ for each $y\in Y$. It is easy to see that this definition is valid and $0\leqq g(y)\leqq 1$ for any $y\in Y$. The function $g$ is continuous on $Y\backslash\{y_0\}$ since the sum is locally finite except at $y_0$. On the other hand, there exists a function $f\in C(X)$ such that $0\leqq f\leqq 1$, $f(x_0)=1$ and $\operatorname{supp} f\cap S_{y_0}=\emptyset$. The last condition implies

$$Tf(x_0)=\int f\,d\mu_{y_0}=0.$$

Hence the function

$$g_1:=Tf\cdot g,$$

defined by pointwise multiplication, belongs to $C(Y)$ and satisfies $0\leqq g_1\leqq Tf$.

By the assumption (i), there exists a function $h\in C(X)$ such that $0\leqq h\leqq f$ and $Th=g_1$. Then for odd $n\in N$,

$$0=g_1(y_n)=\int h\,d\mu_{y_n}, \text{ hence } h(x_n)=0. \tag{1}$$

On the other hand, if $n\in N$ is even

$$\int f\,d\mu_{y_n}=Tf(y_n)=g_1(y_n)=\int h\,d\mu_{y_n},$$

which implies

$$h(x_n)=f(x_n). \tag{2}$$

(1) and (2) contradicts the continuity of $h$ at $x_0$.//

REMARK. Theorem 1 still holds if the assumption of the metrizability of $X$ and $Y$ is relaxed to the assumption that they satisfy the first axiom of countability.

In case $Y$ is connected, we have the following

PROPOSITION 3. *Let $X, Y$ be compact metric spaces and suppose that $Y$ is connected. Let $T: C(X)\to C(Y)$ be a Maharam operator which is not identically zero. Then $T$ is surjective.*

PROOF. It suffices to show that $T\mathbf{1}(y)>0$ for any $y\in Y$, since it implies the surjectivity by Proposition 1, (1). Put

$$Y_0:=\{y\in Y;\ T\mathbf{1}(y)>0\}.$$

Then $Y_0$ is clearly a non-void open subset of $Y$. We will see that $Y_0$ is also closed and hence equals $Y$. In fact, let $y_0\in \bar{Y}_0\backslash Y_0$. Then there exists a sequence $\{y_n\}_{n\in \boldsymbol{N}}$ of points of $Y_0$ convergent to $y_0$. Choose $x_n\in X$ such that $x_n\in S_{y_n}$ for each $n\in \boldsymbol{N}$, where $S_{y_n}$ is defined as in Theorem 1. (Note that $S_{y_n}\neq\emptyset$ since $y_n\in Y_0$.) By choosing a suitable subsequence if necessary, we may assume that $\{x_n\}_{n\in \boldsymbol{N}}$ converges to $x_0\in X$. Moreover, we may also assume that $\{y_n\}_{n\in \boldsymbol{N}}$ satisfies the condition (**) in the proof of Theorem 1. Then in exactly the same way as in the proof of Theorem 1, we can construct a function $g: Y\to \boldsymbol{R}$ such that

$$0\leqq g(y)\leqq 1 \text{ for any } y\in Y,$$

$$g(y_{2n})=1,\ g(y_{2n-1})=0 \text{ for any } n\in \boldsymbol{N}$$

and

$$g \text{ is continuous except at } y_0.$$

Since $T\mathbf{1}\cdot g$ is continuous on $Y$ and $0\leqq T\mathbf{1}\cdot g\leqq T\mathbf{1}$, there exists a function $h\in C(X)$ such that $0\leqq h\leqq \mathbf{1}$ and $Th=T\mathbf{1}\cdot g$. The same argument as in the proof of Theorem 1 shows that $h(x_{2n})=1$ and $h(x_{2n-1})=0$ hold for any $n\in \boldsymbol{N}$. This contradicts the continuity of $h$.//

REMARK. The assumption of metrizability cannot be dropped as the following example shows.

EXAMPLE. Let $E:=C_b((0,1])$ (=the set of bounded continuous functions on $(0,1]$), and $F:=C([0,1])$. $E$ is isometrically lattice isomorphic to $C(\beta(0,1])$, where $\beta(0,1]$ is the Čech compactification of $(0,1]$, which is not metrizable. Define a positive operator $T: E \to F$ by

$$Tf(x) = \begin{cases} xf(x), & \text{if } 0 < x \leqq 1 \\ 0 \ , & \text{if } x = 0 \end{cases}$$

for $f \in E$. Then it is easy to see that $T$ is a Maharam operator not identically zero, although it is not surjective.

As a preparation for the structure theorem for Maharam operators, we state the following apparently known lemma with a proof.

LEMMA. *Let $X, Y$ be compact Hausdorff spaces, and let $\{S_y\}_{y \in Y}$ be an upper semi-continuous family of non-void closed subsets of $X$ which satisfies $S_y \cap S_{y'} = \emptyset$ if $y$ and $y'$ are distinct. Put $X_0 := \bigcup_{y \in Y} S_y$ and define an equivalence relation $\sim$ by $x_1 \sim x_2$ if and only if $\exists y \in Y [x_1 \in S_y$ and $x_2 \in S_y]$. Then $Y$ is homeomorphic to the quotient space $X_0/\sim$.*

PROOF. Let $\phi: X_0 \to Y$ be the mapping which maps $x \in X$ to the unique element $y \in Y$ such that $x \in S_y$. Let $\pi$ denote the canonical surjection $X_0 \to X_0/\sim$. Then there exists a unique bijection $\psi: X_0/\sim \to Y$ for which $\phi = \psi \circ \pi$ holds. It is easy to see that $\phi$ is continuous, and hence $\psi$ is continuous. On the other hand $\psi$ is a closed mapping since $X_0/\sim$ is compact. Hence $\psi$ is a homeomorphism.//

Combining the above results, we have the following

THEOREM 2. *Let $X, Y$ be compact metric spaces and let $Y$ be connected. Suppose $T: E \to F$ be a non-zero Maharam operator where $E = C(X)$ and $F = C(Y)$. Then there exists a factorization*

$$E \xrightarrow{T_1} E/I \xrightarrow{T_2} J \xrightarrow{T_3} F$$

*of $T$ (i.e., $T = T_3 \circ T_2 \circ T_1$) with the following properties:*

a) *$I$ is a closed ideal of $E$ and $T_1$ is the canonical surjection. (cf. § 2 example (A))*
b) *$J$ is a closed sublattice of $E/I$ and $T_2$ is a Maharam retraction onto $J$, and $T_2$ is strictly positive, i.e., $T_2 f = 0$ and $0 \leqq f \in E/I$ imply $f = 0$. (cf. § 2 example (B))*
c) *$T_3$ is a lattice isomorphism. (cf. § 2 example (C))*

*Moreover, such factorization is unique up to lattice isomorphism.*

PROOF. *Existence of a factorization.* Proposition 3 implies that the function $p := T\mathbf{1}$ is everywhere positive and hence $S_y \neq \emptyset$ for any $y \in Y$. Then by Theorem 1 and Lemma 1, there exists a homeomorphism

$$\psi : X_0/\sim \to Y,$$

where $X_0, \sim, \psi$ are the same as in the proof of Lemma 1. Let

$$I := \{f \in E ;\ T|f| = 0\}.$$

Then $I$ is a closed ideal of $E$ and $I = \{f \in E ;\ f|_{X_0} = 0\}$. Hence $E/I$ is isometrically lattice isomorphic to $C(X_0)$. Let $T_1$ be the canonical mapping $E \to E/I$. Since $I \subset \operatorname{Ker} T$, there exists a factorization

$$E \xrightarrow{T_1} E/I \xrightarrow{\tilde{T}} F, \quad T = \tilde{T} \circ T_1.$$

Note that $\tilde{T}$ is also a Maharam operator and strictly positive. It is easy to see that the ideal

$$J := \{f \in C(X_0) ;\ f|_{S_y} = \text{const. for any } y \in Y\}$$

is a closed sublattice of $C(X_0)$ containing the constant functions, and is isometrically isomorphic to $C(X_0/\sim)$. The isomorphism $\pi^* : C(X_0/\sim) \to J$ is given by $\pi^*(f) = f \circ \pi$ where $\pi : X_0 \to X_0/\sim$ is the canonical surjection. Now an operator $T_2 : E/I \to J$ is defined by the composition of the following mappings:

$$E/I \xrightarrow{\tilde{T}} F \xrightarrow{i} C(X_0/\sim) \xrightarrow{\pi^*} J,$$

where $i$ is the isomorphism defined by $i(f) = \left(\frac{1}{p} f\right) \circ \psi$ for $f \in C(Y)$. It is clear that $T_2$ is a Maharam operator and $T_2|_J$ is the identity. The strict positivity of $T_2$ follows from that of $\tilde{T}$. Lastly let $T_3 : J \to F$ be defined by $T_3 = i^{-1} \circ \pi^{*-1}$. Then $T_3$ is a lattice isomorphism and $T = T_3 \circ T_2 \circ T_1$ holds. Thus the existence of a factorization is proved.

*Uniqueness of factorizations.* To see the uniqueness let

$$E \xrightarrow{\tilde{T}} E/\tilde{I} \xrightarrow{\tilde{T}} \tilde{J} \xrightarrow{\tilde{T}} F$$

be another factorization satisfying a), b) and c). Then

$$\operatorname{Ker} \tilde{T}_1 = \{f ;\ \tilde{T}_1|f| = 0\} = \{f ;\ T|f| = 0\} = \operatorname{Ker} T_1$$

holds since $\tilde{T}_3 \circ \tilde{T}_2$ is strictly positive. Hence $\tilde{T}_1 = T_1$. The equality $T = T_3 \circ T_2 \circ T_1 = \tilde{T}_3 \circ \tilde{J}_2 \circ \tilde{T}_1$ and the surjectivity of $T_1 = \tilde{T}_1$ imply $T_3 \circ T_2 = \tilde{T}_3 \circ \tilde{T}_2$, and hence $\tilde{T}_2 = \tilde{T}_3^{-1} \circ T_3 \circ T_2$. This proves the theorem since $\tilde{T}_3^{-1} \circ T_3$ is a lattice isomorphism from $J$ onto $\tilde{J}$.//

The factorization $E \xrightarrow{T_1} E/I \xrightarrow{T_2} J \xrightarrow{T_3} F$ described in the first half of the proof of Theorem 2 is called the canonical factorization of $T$. Then we have the following

PROPOSITION 3. *Let $X, Y$ be compact metric spaces and let $Y$ be connected. Suppose*

$$E \xrightarrow{T_1} E/I \xrightarrow{T_2} J \xrightarrow{T_3} F$$

*be the canonical factorization of a non-zero Maharam operator $T: E \to F$, where $E=C(X)$ and $F=C(Y)$. Then*

i) *$T$ is strictly positive if and only if $T_1$ equals the identity, and*
ii) *$T$ is lattice homomorphic if and only if $T_2$ equals the identity.*

PROOF. Both assertions follow immediately from Theorem 2.//

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