Current-Voltage Relationship Including Plasma Flow Along the Mirror Field

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Current-Voltage Relationship Including Plasma Flow Along the Mirror Field

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Abstract

It is commonly believed that field-aligned currents (currents parallel to the magnetic field) are carried by electrons, which are more mobile than ions. Contrary to this common view, it is shown that, based on a collisionless model, even for $V_0 << v_{Te}$, where V_0 is the field-aligned plasma flow speed and v_{Te} is the electron thermal speed, plasma sheet ions can carry a net downward field-aligned current into the ionosphere when an earthward plasma flow is injected at the plasma sheet into the mirror field. This is simply due to different pitch angle distributions of ions and electrons at the plasma sheet caused by the plasma flow. Relevance of this possibility to downward field-aligned currents observed poleward of the evening auroral zone is discussed.

Introduction

The plasma sheet and the auroral ionosphere are connected by geomagnetic field lines (see Fig. 1). Along these field lines field-aligned currents (electric currents parallel to the magnetic field line) are commonly observed.[1–3] It has also been recognized recently that there is a strong field-aligned plasma flow near the plasma sheet boundary.[4,5] The purpose of this paper is to show that where there is an earthward plasma flow injected into the mirror field as shown in Fig. 1, plasma sheet ions can carry a net downward field-aligned current into the ionosphere, even if a condition $V_0 > v_{Te}$, i.e., $V_0/v_{Ti} > \sqrt{m_i/m_e}\ \sqrt{T_e/T_i} \cong 40 \sqrt{T_e/T_i}$ (V_0 is the plasma flow speed, v_{Ti} and v_{Te} are ion and electron thermal speeds, T_i and T_e are ion and electron temperatures, and m_i and m_e are ion and electron masses) is not satisfied. That condition $V_0 > v_{Te}$, is considered to be a condition for ions to carry a field-aligned current in a uniform magnetic field. That is, even for the flow speed $V_0 << v_{Te}$ or $V_0 / v_{Ti} << 40 \sqrt{T_e/T_i}$ plasma

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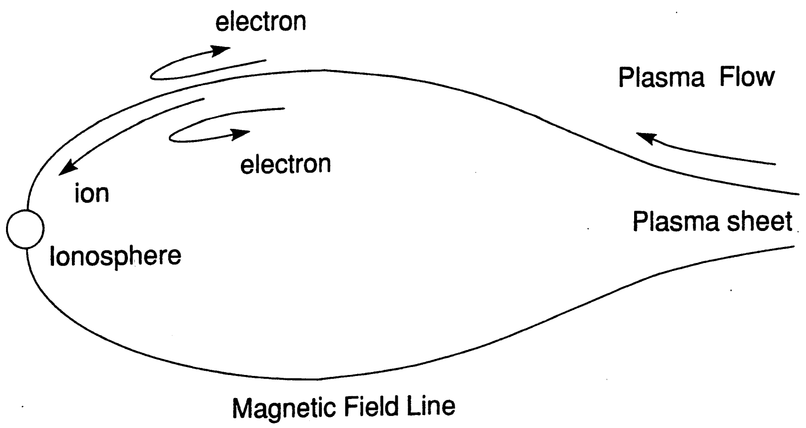


Fig. 1 An illustration showing auroral field lines along which plasma flow is injected at the plasma sheet into the mirror field.

sheet ions can carry a net downward field-aligned current into the ionosphere. This is simply due to the fact that for such a field-aligned plasma flow streaming into the mirror field, there are more ions than electrons in the loss cone at the plasma sheet. Therefore, while most of the plasma sheet electrons are reflected back toward the plasma sheet by the mirror force before reaching the ionosphere, a substantial part of the plasma sheet ions can reach the ionosphere and contribute to the net downward field-aligned current into the ionosphere.

Also included in the present analysis is a potential difference between the ionosphere and the plasma sheet (the potential is higher at the ionosphere), which is an important feature of the auroral field line. Such a potential difference is responsible for downward acceleration of the plasma sheet electrons and hence for an upward field-aligned current out of the ionosphere. In this paper the above possibility is evaluated quantitatively by calculating the current-voltage relationship, i.e., the relationship between the potential difference and the field-aligned current, for a collisionless plasma based on the conservation of energy and magnetic moment of the plasma sheet electrons and ions. Such a collisionless model has been developed by Lemaire and Scherer ,[6,7] Knight,[8] and generalized by Whipple[9] and Chiu and Schulz.[10] The present analysis is considered to be a further generalization of the collisionless model intended to include plasma flow at the plasma sheet. Indeed we will see that in the limit when the plasma flow speed is zero, the resulting field-aligned current vs. the potential relationship becomes almost the same as that obtained by Knight[8] and Fridman and Lemaire,[11] because the ion current is negligible compared with the electron current in such a limit. Serizawa and Sato [12] showed that when a field-aligned plasma jet is injected at the plasma sheet into the mirror field, a large potential difference is created by the charge separation caused by the difference of ion and electron number densities in the loss cone at the plasma sheet. They obtained a potential difference of the order of ion streaming energy by assuming the plasma jet would be currentless. Although the potential difference is a parameter in the present analysis, the present analysis gives a potential difference of the same order as that obtained by Serizawa and Sato,[12] if we impose a condition that the field-aligned current is zero.

A downward field-aligned current is frequently observed by satellite observations [3] in the poleward region adjacent to the region 1 current [1] in the evening sector. Although a carrier of this downward field-aligned current has not been identified, Yamamoto, et al. [13] suggested that this downward current region is connected to the high-latitude boundary of the plasma sheet, where Elphic, et al. [2] observed field-aligned currents with the opposite direction to the region 1 current system, and also where the ion beams are frequently observed. The present analysis showing the generation of a downward field-aligned current by plasma flow along the mirror field supports their suggestion.

Static Collisionless Formulation

Let us consider a field line from the high-latitude ionosphere to the plasma sheet, along which the magnetic field strength decreases monotonically. The weakest field point (plasma sheet) is considered as the source point where field-aligned plasma flow is injected toward the ionosphere (mirror throat).

Particle Distribution Functions

At the plasma sheet, where field-aligned plasma flow is injected, the distribution function of electrons $f_e(v_\parallel,v_\perp)$ and that of ions $f_i(v_\parallel,v_\perp)$ are chosen as follows [14]

$$f_e(v_\parallel,v_\perp) = n_e(\alpha_e/\pi)^{3/2} exp\{-\alpha_e(v_\parallel^2 + v_\perp^2)\} \quad (\text{all } v_\parallel) \tag{1}$$

$$\begin{aligned} f_i(v_\parallel,v_\perp) &= n_i(\alpha_i/\pi)^{3/2} exp[-\alpha_i\{(v_\parallel - V_0)^2 + v_\perp^2\}] \quad (v_\parallel \geq 0) \\ &= n_i(\alpha_i/\pi)^{3/2} exp[-\alpha_i\{(v_\parallel + V_0)^2 + v_\perp^2\}] \quad (v_\parallel \leq 0) \end{aligned} \tag{2}$$

where $\alpha_j = m_j/(2T_j)$ (j = e for electrons and $j = i$ for ions). The parallel velocity $v_\parallel$ is defined as positive when it is directed from the plasma sheet to the ionosphere. V_0 is the parallel flow speed at the plasma sheet. The electrostatic potential ϕ is taken to be zero at the plasma sheet and is equatl to $\phi = \phi_I > 0$ at the ionosphere, where the subscript I represents the ionosphere.

Field-Aligned Current

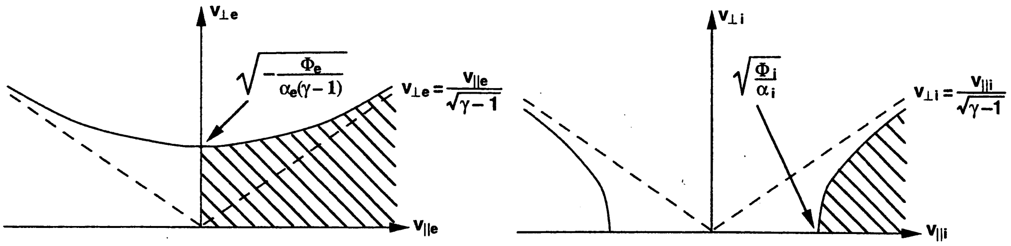
From the conservation of magnetic moment and energy we have

$$v_{\perp jI}^2 = \gamma v_{\perp j}^2 \tag{3}$$

$$v_{\parallel jI}^2 + v_{\perp jI}^2 + 2q_j\phi/m_j = v_{\parallel j}^2 + v_{\perp j}^2 \tag{4}$$

where $v_{\parallel jI}$ and $v_{\perp jI}$ are parallel and perpendicular velocities of j species at the ionosphere, $v_{\parallel j}$ and $v_{\perp j}$ are parallel and perpendicular velocities of j species at the plasma sheet, $\gamma = B_I/B_P$, B_I and B_P are the magnetic field strengths at the ionosphere and at the plasma sheet, respectively, and q_j is the charge of j species. Substitution of Eq. (3) into Eq. (4) yields

$$v_{\parallel jI}^2 = v_{\parallel j}^2 - (\gamma - 1)\, v_{\perp j}^2 - \Phi_j/\alpha_j \tag{5}$$



(a) (b)

Fig. 2 A diagram indicating areas for the straight-through particles or passing particles for electrons [the shaded area S_e in (a)] and ions [the shaded area S_i in (b)] in the velocity space at the plasma sheet.

where $\Phi_j = q_j\phi/T_j$. The particles are reflected at the ionosphere or pass through the ionosphere depending on whether $v_{\parallel jI}^2 < 0$ or $v_{\parallel jI}^2 > 0$. Since the particle distribution functions are symmetric in $v_\parallel$, those particles that are reflected at the ionosphere do not contribute to the field-aligned current. Therefore, only those particles that have straight-through trajectories or that pass through the ionosphere contribute to the field-aligned current. Fig. 2 shows a diagram indicating areas for the straight-through particles or passing particles for electrons (the shaded area S_e in Fig. 2a) and ions (the shaded area S_i in Fig. 2b) in the velocity space at the plasma sheet.

The total field-aligned current at the plasma sheet $J_{\parallel P}$ can be written as

$$J_{\parallel P} = J_{\parallel eP} + J_{\parallel iP} \tag{6}$$

where $J_{\parallel eP}$ and $J_{\parallel iP}$ are field-aligned currents at the plasma sheet carried by electrons and ions, respectively. $J_{\parallel eP}$ and $J_{\parallel iP}$ are expressed as

$$J_{\parallel eP} = -e\int_{S_e} v_\parallel f_e\, d^3v \tag{7}$$

$$J_{\parallel iP} = e\int_{S_i} v_\parallel f_i\, d^3v \tag{8}$$

Substitution of Eqs. (1) and (2) into Eqs. (7) and (8), respectively, yields

$$J_{\parallel eP} = -n_e e\sqrt{\frac{T_e}{2\pi m_e}}\{1-(1-1/\gamma)\, exp\, (-\frac{1}{\gamma-1}\frac{e\phi}{T_e})\} \tag{9}$$

$$\begin{aligned} J_{\parallel iP} = -n_i e\sqrt{\frac{2T_i}{\pi m_i}}[& -\frac{1}{2\gamma}\, exp\{-\alpha_i\, (V_0-\sqrt{\frac{2e\phi}{m_i}})^2\} \\ & -\sqrt{\alpha_i}\, V_0\, erfc\{-\sqrt{\alpha_i}\, (V_0-\sqrt{\frac{2e\phi}{m_i}})\} \\ & +\sqrt{\alpha_i}\, V_0\, (\frac{1}{1-1/\gamma})^{-3/2}\, exp\{-\frac{\alpha_i}{\gamma}(V_0^2-\frac{1}{1-1/\gamma}\frac{2e\phi}{m_i})\} \\ & \times erfc\{-\sqrt{\frac{\alpha_i}{1-1/\gamma}}((1-1/\gamma)V_0-\sqrt{\frac{2e\phi}{m_i}})\}] \end{aligned} \tag{10}$$

where

$$erfc\,(x) \equiv \int_x^\infty e^{-t^2} dt \tag{11}$$

Quasineutrality

To have a relationship between n_e and n_i the quasineutrality condition is imposed at the plasma sheet. That is, we assume $N_e = N_i$, where N_e and N_i are total number densities of electrons and ions at the plasma sheet, respectively. Fig. 3 shows a diagram indicating areas for particles [(a) for electrons and (b) for ions] existing in the velocity space at the plasma sheet. Only particles except those in the shaded areas exist at the plasma sheet. Those particles in the shaded areas do not contribute to the particle densities at the plasma sheet because they are lost at the ionosphere and can not be reflected back to the plasma sheet. Solid curves in Fig. 3 showing parts of a circle (Fig. 3a) and two combined circles (Fig. 3b) represent equi-contours of the distribution functions.

We obtain

$$N_e = \frac{n_e}{2}\{1 + \sqrt{1 - 1/\gamma}\, exp\,(-\frac{1}{\gamma - 1}\frac{e\phi}{T_e})\} \tag{12}$$

$$\begin{aligned} N_i = \frac{n_i}{\sqrt{\pi}}\ [\,& 2\, erfc\,(-\sqrt{\alpha_i}\, V_0) - erfc\,\{-\sqrt{\alpha_i}\,(V_0 - \sqrt{\frac{2e\phi}{m_i}}\,)\} \\ & + \sqrt{1 - 1/\gamma}\, exp\{-\frac{\alpha_i}{\gamma}\,(V_0^2 - \frac{1}{1 - 1/\gamma}\frac{2e\phi}{m_i})\} \\ & \times erfc\{-\sqrt{\frac{\alpha_i}{1 - 1/\gamma}}((1 - 1/\gamma)V_0 - \sqrt{\frac{2e\phi}{m_i}})\}\,] \end{aligned} \tag{13}$$

From the quasineutrality at the plasma sheet we have

$$N_e = N_i \tag{14}$$

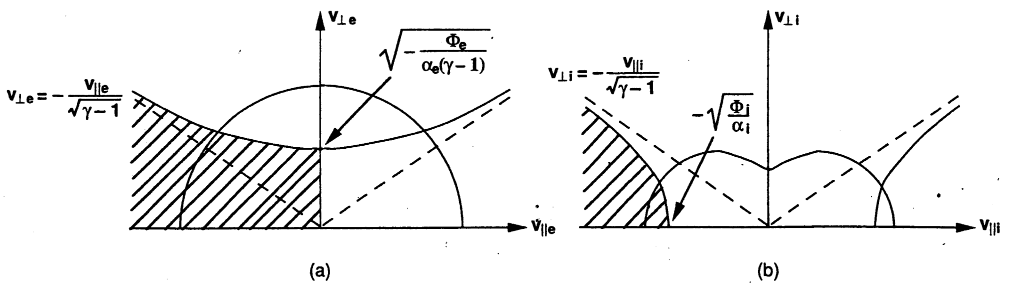


Fig. 3 A diagram indicating areas for particles [(a) for electrons and (b) for ions] existing in the velocity space at the plasma sheet. Only particles except those in shaded areas exist at the plasma sheet. Solid circular curves represent equi-contours of the distribution functions.

Substitution of Eqs. (12) and (13) into Eq. (14) yields

$$n_i = \nu n_e \tag{15}$$

where ν is a function of ϕ and V_0.

Let us consider a trivial case where $\phi = V_0 = 0$. In this case Eqs. (12) and (13) give

$$N_e = \frac{n_e}{2}(1 + \sqrt{1 - 1/\gamma}) \tag{16}$$

$$N_i = \frac{n_i}{2}(1 + \sqrt{1 - 1/\gamma}) \tag{17}$$

Therefore, the quasineutrality condition (14) simply gives $n_e = n_i$ or $\nu = 1$.

Normalization of the Field-aligned Current

In the actual calculation the field-aligned current is normalized by an appropriate normalization unit. By assuming $\phi = 0$ in Eq. (9), a field-aligned current by the electron thermal flux at the plasma sheet can be obtained as $\gamma^{-1} n_e e \sqrt{T_e/(2\pi m_e)}$. If we assume the conservation of the field-aligned current along the flux tube, the field-aligned current at the ionosphere by the plasma sheet electron thermal flux can be written as $n_e e \sqrt{T_e/(2\pi m_e)}$. For convenience we choose the normalization unit J_0 of the field-aligned current at the ionosphere as follows

$$J_0 = -n_e e \sqrt{T_e/(2\pi m_e)}$$

Field-aligned currents at the ionosphere carried by electrons ($J_{\parallel eI}$) and ions ($J_{\parallel iI}$) can be calculated by multiplying Eqs. (9) and (10) by γ. In the following we drop the subscript I representing the ionosphere from the total field-aligned current $J_{\parallel I}$, $J_{\parallel eI}$, and $J_{\parallel iI}$ and express them simply by $J_\parallel$, $J_{\parallel e}$, and $J_{\parallel i}$.

Numerical Results

Fig. 4 shows field-aligned currents carried by electrons ($J_{\parallel e}$) and ions ($J_{\parallel i}$), and the total field-aligned current ($J_\parallel$), which is the sum of $J_{\parallel e}$ and $J_{\parallel i}$ as a function of the normalized potential difference $e\phi/T_i$ for $\gamma = 10^4$, $T_e/T_i = 0.2$, which is a typical temperature ratio at the plasma sheet, and $V_0/v_{Ti} = 10$, where $v_{Ti} = \sqrt{T_i/m_i}$. These field-aligned currents are calculated at the ionosphere and are normalized by J_0. Notice that J_0 is negative. Therefore, the normalized field-aligned current is positive when it is directed upward out of the ionosphere. In a range of $e\phi/T_i$ shown in Fig. 4 the field-aligned current carried by electrons [Eq. (9)] is almost proportional to the potential difference ϕ between the ionosphere and the plasma sheet and is almost equal to the field-aligned current calculated by Knight[8] and Fridman and Lemaire.[11] The field-aligned current carried by ions shown by the dotted curve in Fig. 4 is peaked at $\phi = 0$ and decreases monotonically with increasing ϕ, because the upward electric field by $\phi > 0$ is decelerating the ion flow from the plasma sheet into the ionosphere. This field-aligned current carried by ions is generated by field-aligned plasma flow at the plasma sheet. The total field-aligned current $J_{\|}$ is shown by the solid curve. At $\phi = 0$ the total downward field-aligned current normalized by J_0 is nearly equal to -33.0 and it becomes zero near $e\phi/T_i \sim 6.0$. Since the electric field along the magnetic field is upward and the field-aligned current is downward into the ionosphere for $e\phi/T_i < 6.0$, the mirror configuration is a dynamo for this potential range. This means that plasma flow along the mirror field is decelerated and it generates a downward field-aligned current or a transverse magnetic field component. For $e\phi/T_i > 6.0$ the mirror field is a load, wherein electrons are accelerated downward by the upward electric field. If the potential difference at which the field-aligned current vanishes is expressed by ϕ_0, Fig. 4 shows that $e\phi_0 \sim 6T_i \sim 0.12 \times m_i V_0^2/2$. This potential difference is about 70% of the potential difference generated by a currentless plasma jet, which was obtained by Serizawa and Sato .[12]

Fig. 5 shows the field-aligned current vs. the potential difference characteristics for four different V_0 and for $\gamma = 10^4$ and $T_e/T_i = 0.2$. When V_0 is 0, the ion current is negligible compared with the electron current and the total field-aligned current is upward out of the ionosphere for all ϕ. As the plasma sheet flow speed V_0 increases the downward ion current increases and the total field-aligned current becomes downward for small ϕ. For all V_0 the upward field-aligned currents increase with increasing ϕ and they approach asymptotically the field-aligned current for $V_0 = 0$.

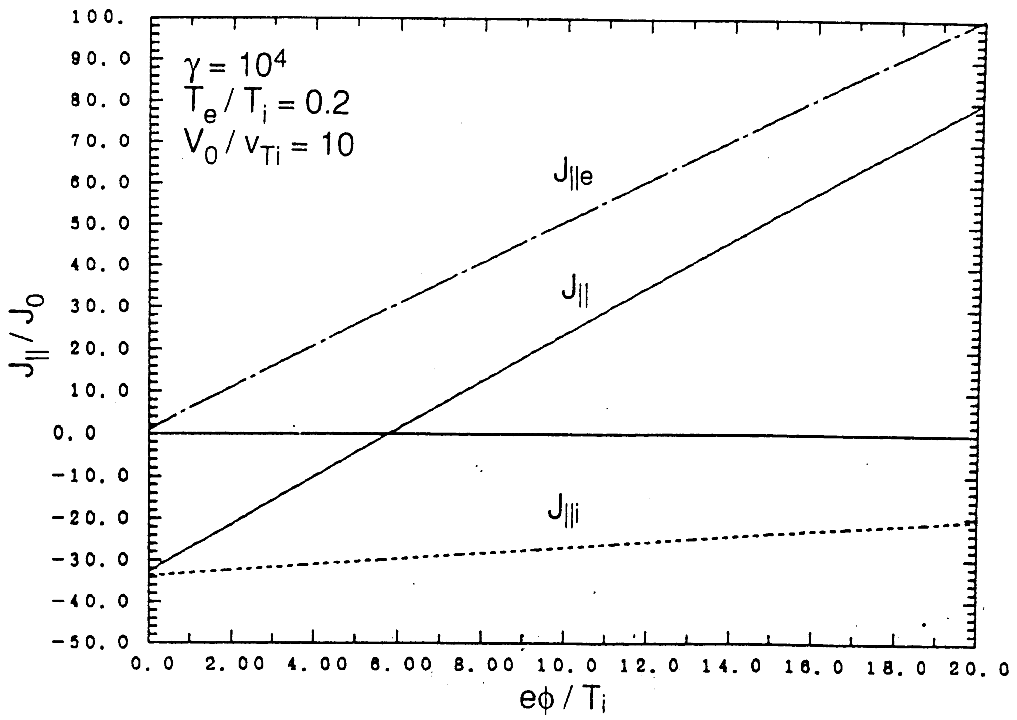


Fig. 4 Field-aligned currents carried by electrons ($J_{\|e}$) and ions ($J_{\|i}$), and the total field-aligned current ($J_{\|}$), which is the sum of $J_{\|e}$ and $J_{\|i}$, as a function of the normalized potential difference $e\phi/T_i$ for $\gamma = 10^4$, $T_e/T_i = 0.2$, and $V_0/v_{Ti} = 10$.

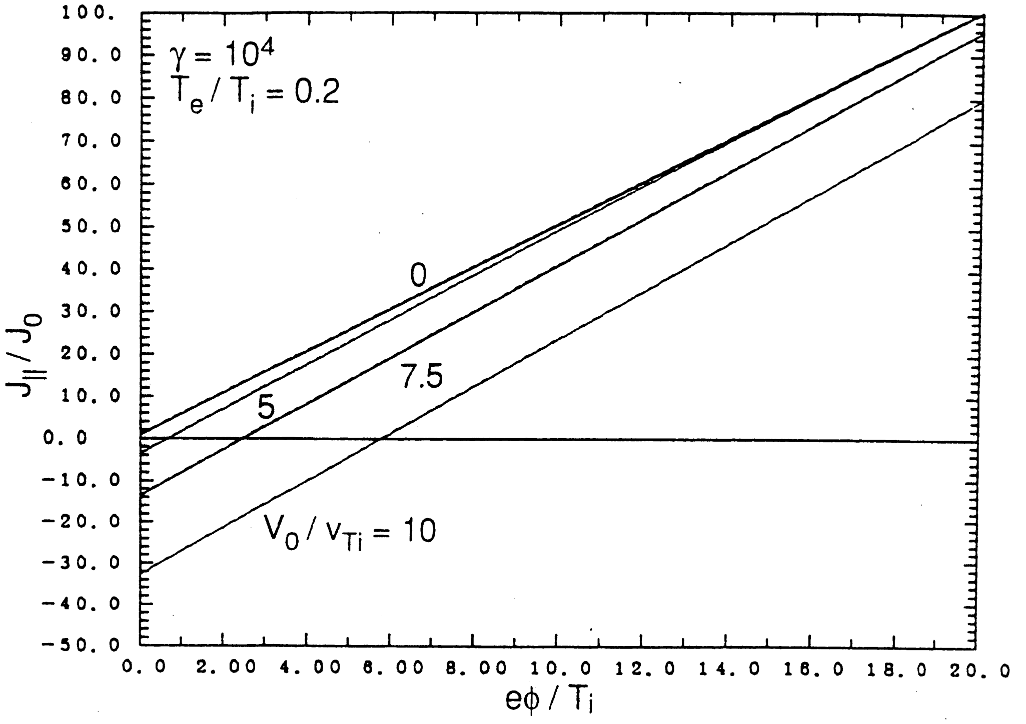


Fig. 5 **The field-aligned current vs the potential difference characteristics for four different V_0 and for $\gamma = 10^4$ and $T_e/T_i = 0.2$.**

Fig. 6 shows the normalized total field-aligned current for $\phi = 0$ as a function of the mirror ratio γ for three different flow speeds. For all flow speeds the normalized field-aligned current is positive for $\gamma \sim 1$, but it becomes negative for $\gamma \gg 1$ and for $\gamma > 100$ the field-aligned current is almost constant. This striking dependence of the field-aligned current on γ is due to the fact that in the present calculation the downward current is generated by flowing ions which fall into the loss cone at the plasma sheet.

Summary and Discussion

The present analysis shows that even for $V_0 \ll v_{Te}$, i.e., $V_0/v_{Ti} \ll 40 \sqrt{T_e/T_i}$, a net downward current into the ionosphere can be carried by plasma sheet ions when an earthward plasma flow is injected at the plasma sheet into the mirror field. But in order for this downward field-aligned current to be generated the potential difference (higher at the ionosphere) must be smaller than ϕ_0, because the upward electric field due to this potential difference decelerates precipitating ions and accelerates precipitating electrons, thus contributing to the upward field-aligned current out of the ionosphere.

The downward current generated by plasma flow along the mirror field may explain the observed downward field-aligned current in the poleward region of the region 1 current in the evening side.[3] Although such a downward field-aligned current is usually ascribed to upward-flowing ionospheric thermal electrons, the present results suggest a completely different interpretation of the carrier of this

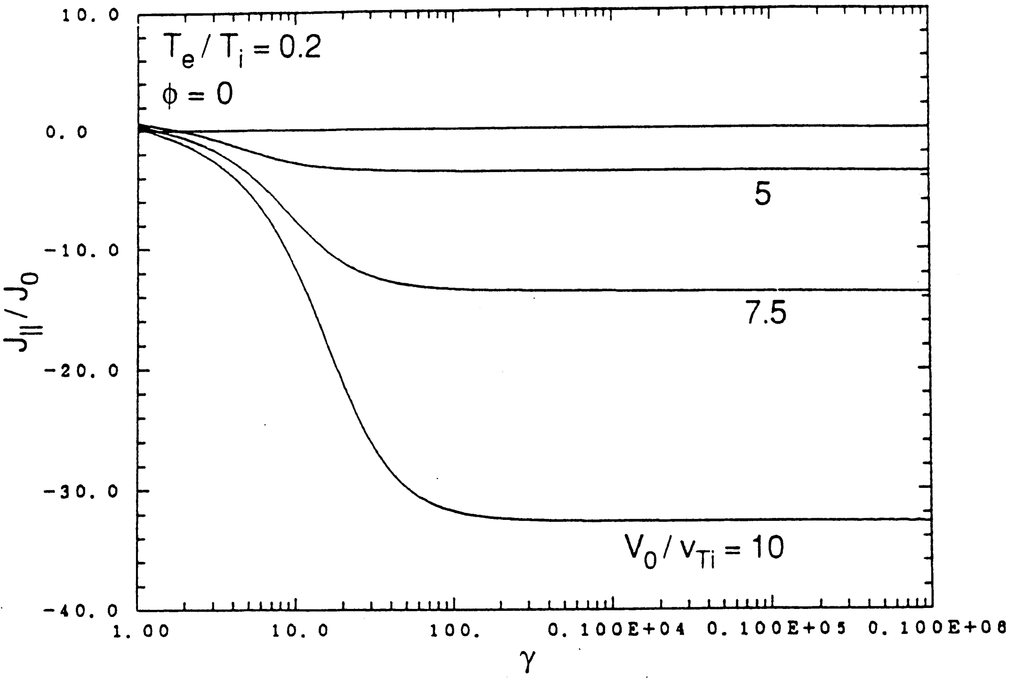


Fig. 6 The normalized total field-aligned current for $\phi = 0$ as a function of the mirror ratio γ for three different flow speeds.

field-aligned current. The importance of existence of such a field-aligned current carried by ions is that in such a region the mirror field configuration becomes a dynamo region, wherein directions of the field-aligned current and the electric field are opposite. For $n_e = 1.0$ cm^{-3} and $T_e = 200$ eV at the plasma sheet, the normalization unit J_0 of the field-aligned current becomes $|J_0| = 0.378$ μA/m^2. Fig. 6 shows that for $3 < V_0/v_{Ti} < 5$, the downward field-aligned current has a magnitude of $< 4\ |J_0| = 1.51$ μA/m^2 for $T_e/T_i = 0.2$. Such a field-aligned current intensity is comparable to that observed in the auroral zone. Here, it should be pointed out, however, that observation by the Akebono satellite shows that the downward field-aligned current region does not necessarily coincide with the region of the ion precipitation. Therefore the above possibility, i.e., that plasma sheet ions can contribute to a net downward field-aligned current when the plasma flow is injected at the plasma sheet into the mirror field, still remains a hypothesis to be checked carefully by observations in the future.

The formation of a large, field-aligned potential difference by high-speed collisionless plasma flow was observed in a laboratory experiment by Sato, et al.[15] They suggested that a large potential difference is created by different pitch angle distributions of ions and electrons caused by the plasma flow as demonstrated by Serizawa and Sato [12] and Washimi and Katanuma .[14] In their experiment they also observed an electric current, although it was directed parallel to the field-aligned electric field. It is presumed that in their experiment $\phi > \phi_0$ was satisfied and therefore the parallel electric current antiparallel to the field-aligned electric field was not realized.

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