Comparative Concepts

PhilLogMath 2

Seiryo Keikan, Tokyo, 14-03-12

Richard Dietz
rdietz@l.u-tokyo.ac.jp

Department of Philosophy
University of Tokyo

March 14, 2012

Outline

Overview

Categorical vs comparative concepts

Categorical concepts

- type of concepts expressed by general terms in natural languages, such as “high”, “exactly 10 meters high”, “cat” or “chair”.
- rules of partitioning a set of objects.

Comparative concepts

- type of concepts expressed by comparative constructions embedding a general term, such as “is higher than”, “is less tall than” or “to look more red”.
- rules of ordering objects.

Varieties of evidence on ordering behaviour

- explicit comparisons (e.g., of the form "x is F-er than y").
- orders induced by probabilities of positive categorisation (Hampton [1998, 2007]).
- orders induced by choice probabilities assigned to ordered pairs (the probability that the one item is picked out—as an F—as compared to the other item) (Suppes et al. [1989]).

Questions

- What kind of cognitive structures underly our ability to order objects in certain ways?
- Why do we order objects in certain ways, and not in different ways?

Natural vs gerrymandered ways of ordering objects

Gruer

Suppose we examine a sample of colour patches $x_1, \ldots, x_n$, where the series is monotonically increasing in greenness. That is, we have a case where for each $0 < i \leq n$, x_i is *greener* than x_{i-1}. Suppose t designates the present point of time. It makes then extensionally no difference to say that we have a case where for each $0 < i \leq n$, x_i is *gruer* than x_{i-1}, where this relation is defined as follows: for any pair of colour patches x and y, x is gruer than y just in case either (a) x and y are examined by point t, and x is greener than y, or (b) x and y are both examined after t, and x is bluer than y.

Natural vs gerrymandered ways of ordering objects – cont.

Gruer—inductive version

Suppose we examine the colour patches $x_1, \ldots, x_n$ in the temporal order of their mention here, and that there are two other colour patches in the sequence, x_{n+1} and x_{n+2}, which are still hidden. Given n is sufficiently high, it would seem only natural to predict that x_{n+2} is greener than x_{n+1}. On the other hand, the prediction that x_{n+2} is gruer than x_{n+1} would seem quite bizarre—for it would imply that x_{n+2} is bluer than x_{n+1}.

Gradable concepts

- Gradable concepts: type of concepts expressed by gradable terms, that is, general terms such as “high” or “red” that embed in comparative constructions.
- Bridge principles: in order to have a concept of redness, it seems that we need to know that anything redder than something red must be red as well; and also that for something to be distinguishable as red from something else, the former is to be redder than the latter.
  - Put aside delineation based approaches to comparatives (Klein [1980], van Benthem [1982], van Rooij [2009]).

Aim

- Outlining a novel approach to comparative concepts that
  1. supplies means of characterising naturalness for comparative concepts, and
  2. has constraining effects on the theory of gradable concepts.
- Method: Carrying Peter Gärdenfors' conceptual spaces approach (*Conceptual Spaces* [2000]), which focusses on ungraded categorisation rules, over to comparative concepts.

Overview

Conceptual spaces

- spaces are sets $D_1, \ldots, D_n$ of quality dimensions, i.e., kinds of features with respect to which objects may be judged as more or less similar.
- a point in a space is defined by a vector $v = \langle d_1, \ldots, d_n \rangle$ where each index represents a dimension.
- each dimension has typically a geometric structure.
- objects ('stimuli') are represented as points in a space.
- concepts are represented as sets in a space.

Conceptual spaces – cont.

examples

- colours: a space with the dimensions hue, chromaticness and brightness.
- geometric figures: a space with the dimensions shape, size, and angular orientation.

Conceptual spaces – cont′

a side note

- Stalnaker's formulation of a bare particular anti-essentialism (in [1979]).
- Lambert's and van Fraassen's account of analyticity (in [1970]).
- Churchland's naturalistic approach to linguistic meaning (in [1986]).
- Bromberger's realism about types in linguistic theory (in [1992]).

A geometric approach to similarity

A metric model of distances

A two-place real-valued function d on a set M is said to be a metric iff:

D1 $d(a,b) \leq 0$ and $d(a,b) = 0$ only if $a = b$; (minimality)

D2 $d(a,b) = d(b,a)$; (symmetry)

D3 $d(a,c) \leq d(a,b) + d(b,c)$. (triangular inequality)

Similarity and distance

Similarity is inversely related to distance: linear (Tversky [1975]), exponential (Shepard [1987]), Gaussian function (Nosofski [1986]).

A geometric approach to similarity – cont.

power metric model

$$d(x, y) = [\sum_{i=1}^{n} |x_i - y_i|^r]^{\frac{1}{r}}$$

- for $r = 2$: Euclidean metric.
- for $r = 1$, city block or Manhattan metric.

Properties (Gärdenfors [2000])

- Separable dimensions: can be perceived/cognised independently from each other
    - e.g., hue, chromaticness and brightness are not separable from each other.
- Domains: sets of dimensions that are not pairwise separable, but all separable from other dimensions.
- Properties: are concepts that refer to so-called *domains*
    - e.g., compare colour concepts with *apple*, which refers to more than one domain (such as colour, shape or texture).

Criteria for naturalness (Gärdenfors [2000])

1. connectedness: A region X is said to be *connected*, if and only if, for all regions Y and Z such that $Y \cup Z = X$, it holds that $C(Y, Z)$. X is *disconnected*, if and only if X is not connected.
2. star-shapedness: A subset C of a conceptual space S is said to be *star-shaped with respect to point p*, if and only if, for all points x in C, all points between x and p are also in C.
3. convexity: A subset C of a conceptual space S is said to be *convex*, if and only if, for all points x and y in C, all points between x and y are also in C.

Convexity criterion P (Gärdenfors [2000])

A natural property is a convex region of a domain in a conceptual space.

Related discussion

- Oddie [2005] on 'natural' value properties.
- evolutionary arguments (from evolutionary psychology: Shepard [1987]; from evolutionary game theory, see Jäger [2009] and Jäger et al. [2009]).
- but see Mormann [1993] for an argument to the effect that the convexity constraint is unnecessarily strong.
- Gärdenfors' argument from prototype theory ([2000]) (sect. 3).

Overview

Modelling orders of points as orders of sets

- for any partially ordered set $\langle P, \geq \rangle$ and any subset Q of P, Q is said to be an order filter (or upward closed set) if, whenever $x \in Q$, $y \in P$ and $y \geq x$, we have $y \in Q$.
- for any arbitrary set Q of P, we define:
  - $\uparrow Q \ := \ \{y \in P \mid (\exists x \in Q)\ y \geq x\}$.
- $\uparrow$ is an isomorphism between $\langle P, \ \geq \rangle$ and $\langle \uparrow P, \ \subseteq \rangle$.

Convexity criteria for naturalness

Criterion C1

A strict partial ordering $>$ referring to one domain in a conceptual space is a natural comparative concept only if for all points x in the space, the corresponding set $\{y \mid y > x\}$ is a convex region.

E.g., criterion C1 implies that for any triple of patches x, y and z where both x and y are redder than z, any patch in between in colour shade between x and y should be redder than z as well.

Almost-connectedness

- R is almost connected: $x > y \rightarrow (z > y \ \vee \ x > z)$.
- (strict) weak orders: (strict) partial orders that are almost connected.
- indifference $(x \not> y \ \wedge \ y \not> x)$ is transitive.

Convexity criteria for naturalness – cont.

Criterion C2

A strict weak ordering $>$ referring to one domain in a conceptual space is a natural comparative concept only if for all points x in the space, the corresponding set $\{y \mid y > x \vee (x \not> y \wedge y \not> x)\}$ is a convex region.

E.g., criterion C2 implies that for any triple of patches x, y and z where both x and y are at least as red as z, any patch in between in colour shade between x and y should be at least as red as z as well.

Sivik and Taft [1994]

'Isosemantic lines' in the colour space, i.e., areas of colours that test persons tended to categorise as equally red, brown, or other, circumscribed a convex area in space.

Overview

Associatedness

For any given comparative concept $>$ and any given categorical concept F, $>$ and F are said to be associated with each other iff they satisfy:

B1. $x > y \rightarrow (F(y) \rightarrow F(x))$.

B2. $(F(x) \wedge \neg F(y)) \rightarrow x > y$.

Note

- F may be interpreted both in terms of binary and in terms of gradable classification criteria.
- on failure of almost-connectedness, the transitive closure of indifference may include pairs of objects that should be treated differently in terms of F-ness.

The no-gap condition

- For any given strictly partially ordered set $\langle P, > \rangle$, a pair $\langle P_1, P_2 \rangle$ is said to be a cut in $\langle P, > \rangle$ iff:
  1. $\{P_1, P_2\}$ is a bipartition in P;
  2. if $x \in P_1$ and $y \in P_2$, then $x > y$.
- A strictly partially ordered set $\langle P, > \rangle$ is then said to satisfy the no-gap condition iff for every cut in the set, either $\langle T_1, > \rangle$ has a minimal element or $\langle T_2, > \rangle$ has a maximal element.

Theorem

Let $\langle P, > \rangle$ be a strict weak ordering that satifies the no-gap condition, and let F be a subset in P, where $>$ and F are associated with each other. Then for some member x of P, either

- $F = \{y \in P \mid y > x\}$, or
- $F = \{y \in P \mid (y > x) \vee (y \not> x \wedge x \not> y)\}$.

Overview

Agenda

- Approach: modelling comparative concepts in terms of conceptual space representations of prototypes.
- Focus on comparative concepts that:
  - refer to one domain (Euclidean metric).
  - are (strict) weak orderings.
  - which satisfy the no-gap condition.
  - are associated with a categorical concept.
- Optional constraint: prototype points for F-ness are maximal elements in $\langle M, >_F \rangle$ (Maximality).

Disclaimers – Open issues put aside

- Comparative concepts without prototypes? How about concepts such as *long* or *late*? (Kamp and Partee [1995] vs Hampton [2007]; Tribushinina [2008, 2009])
- Prototypes without comparative concepts? How about *dog*, *apple*, or *city*? (Schwartzchild [2008] vs Sasson [2007])
- Concepts that refer to more than one domain, e.g., plausibly, *grue*/*gruer*.
- Comparative concepts that are less precise: multi-dimensional concepts (*cleverer than*), interval orderings (*later than*), semi-orderings (*definitely larger*) (for the latter types of cases, see Suppes et al. [1989]).

Similarity, typicality, and graded membership

Naive prototype theory

- Typicality (T_F) is a strictly increasing function of similarity to a prototype.
- Graded membership (M_F) is a strictly increasing function of typicality.

Fuzzy semantics

Interpretating graded membership as similarity to the closest prototypical element (Ruspini [1991], Dubois and Prade [1997], Dubois et al. [2001]).

Similarity, typicality, and graded membership – cont.

Osherson and Smith [1997]

- $T_{bird}(robin) > T_{bird}(woodpecker)$.
- but: $M_{bird}(robin) = M_{bird}(woodpecker) = 1$.

Hampton [2007]

M_F is a cumulative normal distribution function of T_F, which has 0 as its infimum and 1 as its supremum (i.e., $M_F(x) := Prob(X \leq x)$, where the random variable X takes T_F values).

Similarity, typicality, and graded membership – cont.′

Hampton [1998]

Typicality does not always provide a good prediction of graded membership (experiments on artifact concepts).

Open questions

1 What prototypes are relevant?

  1.a non-contrastive accounts: F-er is given for a conceptual space by some prototype for F-ness in the space.

  1.b contrastive accounts: F-er is given for a conceptual space by some set of disjoint prototypes including the prototype for F-ness.

2 In what way are prototypes relevant?

  2.a distance infima (suprema): the infimum (or supremum) of distances between a particular point and any point in the prototype area.

  2.b scaling factors: the factor by which the prototype area is to be expanded/contracted in order to reach a particular point.

  ...

Working hypothesis

Combining [1.b] with [2.a].

Overview

Voronoi diagrams – standard

Two variants

A Given a set of 'prototypical' points in a metric space, a *Voronoi diagram* divides the space into subsets, where each subset contains one and only one 'prototypical' point p and consists of all points with respect to which there is no closer 'prototypical' point than p (Okabe et al. [1992 [2000]]).

B Given a set of 'prototypical' points in a metric space, a *Voronoi diagram* divides the space into subsets, where each subset contains one and only one 'prototypical' point p and consists of all points with respect to which p is closer than any other 'prototypical' point (Aurenhammer and Klein [2000]).

Convexity result (Okabe et al. [2000: 85])

For Euclidean n-spaces, Voronoi regions are convex.

Let $\{M, d\}$ be a Euclidean metric space and P be a subset (in that space) of points $p_1, \ldots, p_n$. Then for each p_i in P, the Voronoi region associated with p_i relative to P is convex.

How to deal with prototype *areas*?

Generalised Voronoi Categorisation (for 2D-spaces)

An object represented as a point in a conceptual pace belongs to the category for which the corresponding prototypical circle is the closest (Gärdenfors [2000]).

Nearest Neighbour Categorisation (for finite sets of prototype points)

An object represented as a point x in a conceptual space belongs to the category for which the prototype instance that is closest to x is included (cf. Reed [1972]).

How to deal with prototype *areas*? – cont.

Average Distance Categorisation

An object represented as a point x in a conceptual space belongs to the category to which x has the smallest average distance (Nosofski [1988]).

How to deal with prototype *areas*? – cont.′

Collated Voronoi Categorisation

An object represented as a point x in a conceptual space belongs to the category for which, for each prototype instance y, x at least as close to y as to any prototype instance of any 'competing' category.

Igor Douven, Lieven Decock, Richard Dietz, Paul Egré: "Vagueness: A Conceptual Spaces Approach", *Journal of Philosophical Logic*, forthcoming.

Collated Voronoi categorisation

Let $R = \{r_1, \ldots, r_n\}$ be a distribution of disjoint prototype areas. The set of prototype point distributions for R is defined as:

$$\Pi(R) := \{P = \langle p_1, \ldots, p_n \rangle \mid p_i \in r_i\}.$$

The *Voronoi region associated with a point p relative to P*, where $P \in \Pi(R)$ and $p \in P$ is defined as

$$v(p, P) := \{q \mid d(q, p) \leq d(q, p'), \text{ with } p' \in P \text{ and } p' \neq p\}.$$

Accordingly, the *Voronoi region associated with a set r_i relative to* R comes to

$$u(r_i, R) := \bigcap_{p \in P \in \Pi(R)} \{v(p, P) \mid p \in r_i\},$$

Convexity result (Douven et al. [forthcoming: sect. 3])

Let $\{M, d\}$ a Euclidean metric space and P be a subset (in that space) of points $p_1, \ldots, p_n$. Then for each p_i in P, the collated Voronoi region associated with p_i relative to P is convex.

How to deal with *comparative* concepts which are associated with a prototype area?

Collated Voronoi Categorisation Generalised

Richard Dietz: "Comparative Concepts", *Synthese*, forthcoming.

Overview

An equivalence result on collated Voronoi categorisation

Theorem (T1)

Let $\langle M, d \rangle$ be a metric space and R be a set of disjoint subsets $r_1, \ldots, r_n$ in M. Then Voronoi region associated with a set r_i relative to R, $u(r_i, R)$, is given by:

$$\{p \in M \mid \sup\{d(p,x) \mid x \in r_i\} \leq \inf\{d(p,y) \mid y \in r_j \in R, j \neq i\}\}.$$

informally $\cdots$

Collated Voronoi Categorisation′: An object represented as a point x in a conceptual space belongs to the category F for which the supremum of distances between x and any point in the prototype area of F is no greater than the infimum of distances between x and any point in any prototype area for any 'competing' category.

Collated Voronoi categorisation generalised

Graded Collated Voronoi Categorisation

- For any λ where $0 \leq \lambda \leq 1$, call distances scaled by λ λ-distances.
- For any λ where $0 \leq \lambda \leq 1$, an object represented as a point x in a conceptual space belongs relative to λ to the category for which, for each prototype instance y, the λ-distance between x and y is no greater than the $(1 - \lambda)$-distance between x and any prototype instance of any 'competing' category.

Collated Voronoi categorisation generalised

Graded Collated Voronoi Categorisation – more formally

The *Voronoi region associated with a point p relative to P and a factor* λ where $P \in \Pi(R)$, $p \in P$, and $0 \leq \lambda \leq 1$ is defined as

$$v(p, P, \lambda) := \{q \mid \lambda \cdot \ d(q, p) \leq (1 - \lambda) \cdot \ d(q, p'), \text{ with } p' \in P \text{ and } p' \neq p\}.$$

The *Voronoi region associated with a set* r_i *relative to a set R and factor* λ is defined as

$$u(r_i, R, \lambda) := \bigcap\{v(p, P, \lambda) \mid p \in r_i\}.$$

Collated Voronoi categorisation generalised – cont.

limiting case

For $\lambda = .5$, graded collated Voronoi categorisation amounts to collated Voronoi categorisation.

An equivalence result on graded collated Voronoi categorisation

Theorem (T2)

Let $\langle M, d \rangle$ be a metric space and R be a set of disjoint subsets $r_1, \ldots, r_n$ in M. Then for any $0 \leq \lambda \leq 1$, the Voronoi region corresponding with r_i, R and λ, $u(r_i, R, \lambda)$, is given by:

$$\{p \mid \sup\{\lambda d(p,x) \mid x \in r_i\} \leq \inf\{(1-\lambda)d(p,y) \mid y \in r_j \in R, j \neq i\}\}$$

An equivalence result – cont.

informally $\cdots$

For any λ with $0 \leq \lambda \leq 1$, an object represented as a point x in a conceptual space belongs relative to λ to the category for which the supremum of λ-distances between x and prototype instances is no greater than the infimum of $(1-\lambda)$-distances between x and any prototype instances of any 'competing' category.

A restricted convexity result

Theorem (T3)

Let $\langle M, d \rangle$ be a Euclidean n-space, with a prototype set distribution $R := \{r_1, \ldots, r_n\}$. For any r_i from R then, the graded collated Voronoi region $u(r_i, R, \lambda)$ is convex if $\lambda \geq .5$.

A restricted convexity result – cont.'

Let $\langle M, d\rangle$ be a metric space and R be a set of disjoint subsets $r_1, \ldots, r_n$ in M. Then for any $0 \leq \lambda \leq 1$, for any pair of distinct 'prototypical points' x and y (where for some $P \in \Pi(R)$, x, $y \in P$), the Voronoi diagram for x, y and λ is given by the equation:

$$\Sigma_{1\leq i\leq n}(\lambda p_i - \lambda x_i)^2 \quad = \quad \Sigma_{1\leq i\leq n}((1-\lambda)p_i - (1-\lambda)y_i)^2$$

$\lambda = .5$

Equation of a hyperplane that separates the space into two half-spaces:

$$\Sigma_{1\leq i\leq n}(a_i \times p_i) + b_i \leq 0, \text{ where } p_i \text{ is the only variable,}$$

The half-spaces are (assuming a Euclidean metric) convex.

A restricted convexity result – cont.″

$\lambda \neq .5$

- equation of a hypersphere centred on a_i, with the radius being $\sqrt{c_i}$:

  $\Sigma_i (p_i - a_i)^2 \leq c_i$, where p_i is the only variable and $c_i > 0$,

  The area circumscribed by the hypersphere is (assuming a Euclidean metric) a convex area, whereas the complement is not convex.
- for $\lambda > .5$ ($\lambda < .5$), the hypersphere is centred on x (y).

Nestedness Lemma

For any metric space $\langle M, d \rangle$, with a prototype set distribution $R := \{r_1, \ldots, r_n\}$, for any $\lambda \in [0,1]$ and $\lambda' \in [0,1]$, if $\lambda \geq \lambda'$, then $u(r_i, R, \lambda) \subseteq u(r_i, R, \lambda')$.

Collated Voronoi orderings: definition

For any n-space with a metric d, $\langle M, d \rangle$, with a prototype area distribution $R := \{r_1, \ldots, r_n\}$, for any λ where $0 \leq \lambda \leq 1$, let $u(r_i, R, \lambda)$ be the category corresponding to r_i, R and λ. For any set $r \in R$, for any x and y in $\langle M, d \rangle$ then:

$$x >^{c\mathcal{V}}_{\langle R, r \rangle} y \Leftrightarrow_{df}$$

$$(\exists \lambda : 0 \leq \lambda \leq 1)\ (x \in u(r_i, R, \lambda)\ \wedge y \notin u(r_i, R, \lambda)).$$

Collated Voronoi orderings: features

- If the metric is Euclidean, then $>^{c\mathcal{V}}_{\langle R,r\rangle}$ validates C_1 and C_2 only restrictedly—with respect to any Voronoi region $u(r_i, R, \lambda)$, where $\lambda \geq .5$.
- $>^{c\mathcal{V}}_{\langle R,r\rangle}$ is a strict weak ordering.
- If the metric is Euclidean, then any categorical concept that is associated with $>^{c\mathcal{V}}_{\langle R,r\rangle}$ is convex, if it is identical with $\{y \mid y >^{c\mathcal{V}}_{\langle R,r\rangle} x\}$, or identical with $\{y \mid (y >^{c\mathcal{V}}_{\langle R,r\rangle} x) \vee (y \not>^{c\mathcal{V}}_{\langle R,r\rangle} x \wedge x \not>^{c\mathcal{V}}_{\langle R,r\rangle} y)\}$, for some member x of $u(r, R, \lambda)$ where $\lambda \geq .5$.

Collated Voronoi orderings: features – cont.

- $>^{c\mathcal{V}}_{\langle R,r\rangle}$ does not satify Maximality. E.g.:
  - Suppose $R = \{p_1, \cdots p_3\}$,
    where $p_1 = [0,1], p_2 = [2,3], p_3 = [5,6]$.
    Then for $x = 2$ and $y = 3$, $x, y \in p_2$,
    but $y >^{c\mathcal{V}}_{\langle R,p_2\rangle} x$.
  - Suppose $R = \{q_1, \cdots q_3\}$,
    where $q_1 = [0,2] \times [0,1], q_2 = [3,5] \times [0,1], q_3 = [0,2] \times [2,3]$.
    Then for $x = \{2,0\}$ and $y = \{1,1\}$, $x, y \in q_2$,
    but $y >^{c\mathcal{V}}_{\langle R,q_2\rangle} x$.

Conclusion

- The collated Voronoi tesselation method in Douven et al. [2009], which accommodates prototype areas, can be furthermore generalised for graded cases of categorisation.
- Gärdenfors' convexity criterion P for natural properties may be recovered in terms of the convexity criteria C1 and C2 for order filters.
- C1 and C2 supply even more sufficient means of motivating a generalisation of the convexity criterion P for graded categorisation.
- The criteria C1 and C2 are logically independent from P, and they have intuitive force of their own.
- Food for thought: More general models which still have some psychological reality (concepts more than one domain; doing without prototypes; doing without geometric criteria in the first instance).

References

Aurenhammer, F. and Klein, R. [2000] "Voronoi diagrams", in J.-R. Sack and J. Urrutia (eds.) *Handbook of Computational Geometry*, Amsterdam: Elsevier, pp. 201–90.

Benthem, J. van [1982] "Later than late: On the logical origin of the temporal order", *Pacific Philosophical Quarterly* 63: 193–203.

Bromberger, S. [1992]] "Types and tokens in linguistic theory", in S. Bromberger *On What We Know We Don't Know: Explanation, Theory, Linguistics, and How Questions Shape Them*, Chicago: Chicago University Press, pp. 170–208.

Churchland, P. M. [1986] "Some reductive strategies in cognitive neurobiology", *Mind* 95: 279–309.

Douven, I., Decock, L., Dietz, R. and Egré, P. [forthcoming] "Vagueness: A conceptual spaces approach", *Journal of Philosophical Logic*.

Dubois, D. and Prade, H. [1997] "The three semantics of fuzzy sets", *Fuzzy Sets and Systems* 90: 141-50.

References – cont.

Dubois, D., Godo, L., Prade, H. and Esteva, F. [2005] "An information-based discussion of vagueness", in H. Cohen and C. Lefebvre (eds.) *Handbook of Categorization in Cognitive Science*, Amsterdam: Elsevier, pp. 892–913.

Gärdenfors, P. [2000] *Conceptual Spaces: The Geometry of Thought*, Cambridge, MA: MIT Press.

Hampton, J. A. (1998) "Similarity-based categorization and fuzziness of natural categories", *Cognition* 65: 137–65.

Hampton, J. A. [2007] "Typicality, graded membership, and vagueness", *Cognitive Science* 31: 355–83.

Jäger, G. [2009] "Natural color categories are convex sets", manuscript.

Jäger, G., Koch-Metzger, L. and Riedel, F. [2009] "Voronoi languages", manuscript.

Kamp, H. and Partee, B. [1995] "Prototype theory and compositionality", *Cognition* 57: 129–191.

References – cont.′

Klein, E. [1980] "The semantics of positive and comparative adjectives", *Linguistics and Philosophy* 4: 1–45.

Klein, E. and Rovatsos, M. [2009] "Modelling vagueness in computational semantics", slide show presented at the ESSLLI, Bordeaux, July 24, 2009.

Lambert, K. and van Fraassen, B. C. [1970] "Meaning relations, possible objects, and possible worlds", in K. Lambert (ed.) *Philosophical Problems in Logic: Some Recent Developments*, Dordrecht: Reidel, pp. 1–19.

Mormann, T. [1993] "Natural predicates and topological structures of conceptual spaces", *Synthese* 95: 219–40.

Nosofski, R. [1986] "Attention, similarity, and the identification-categorization relationship", *Journal of Experimental Psychology: General* 115: 39–57.

Nosofski, R. [1988] "Exemplar-based accounts of relations between classification, recognition, and typicality", *Journal of Experimental Psychology: Learning, Memory, and Cognition* 14: 700–8.

Oddie, G. [2005] *Value, Reality, and Desire*, Oxford: Clarendon Press.

References – cont.''

Okabe, A., Boots, B., Sugihara, K., and Chiu, S. N. [1992] *Spatial Tesselations: Concepts and Applications of Voronoi Diagrams* (1st. ed.), New York: Wiley.

Okabe, A., Boots, B., Sugihara, K., and Chiu, S. N. [2000] *Spatial Tesselations: Concepts and Applications of Voronoi Diagrams* (2nd rev. ed.), New York: Wiley.

Osherson, D. N. and Smith, E. E. [1997] "On typicality and vagueness", *Cognition* 64: 189–206.

Reed, S. K. [1972] "Pattern recognition and categorization", *Cognitive Psychology* 3: 382–407.

Rooij, R. van [2009] "Up and down the scale: Adjectives, comparisons, and measurement", manuscript.

Ruspini, E. H. [1991] "On the semantics of fuzzy logic", *International Journal of Approximate Reasoning* 5: 45-88.

Sassoon, G. W. [2007] *Vagueness, Gradability and Typicality: A Comprehensive Semantic Analysis*, doctoral dissertation, Tel Aviv University.

References – cont.'''

Schwarzschild, R. [2008] "The semantics of comparatives and other degree constructions", *Language and Linguistics Compass* 2: 308–31.
Sivik, L. and Taft, C. [1994] "Color naming: A mapping in the NCS of common color terms", *Scandinavian Journal of Psychology* 35: 144–64.
Stalnaker, R. [1979] "Anti-essentialism", *Midwest Studies in Philosophy* 4: 434–55.
Suppes, P., Krantz D. H., R. D. Luce and Tversky, A. [1989] *Foundations of Measurement. Vol. II: Geometrical, Threshold, and Probabilistic Representations*, San Diego: Academic Press.
Tribushinina, E. [2008] *Cognitive Reference Points: Semantics Beyond the Prototypes in Adjectives of Space and Colour*, doctoral dissertation, University of Leiden, (LOT Dissertation Series 192) Utrecht: LOT.

Thank you!