3. Results

Results in the thesis are presented according to the main division of spatial patterns, firstly presenting findings obtained in regular and followed by findings in random and clustered point populations. Results regarding the measured population of saplings are presented in a separate chapter. Surveying forests by *c*-tree sampling gives information on both the density and spatial pattern of individual trees. On the other hand, the Mean of Angles method is only applicable to indexing spatial patterns of individual trees. Since it is known that *c*-tree sampling yields biased density estimates, with the amount of a bias being different in populations exhibiting different spatial patterns and different density estimators, results in the thesis are ordered by firstly presenting spatial pattern indices and than followed by density estimates. Density estimates are presented by plotting calculated relative errors:

Relative Error = ((estimated density – true density) $\times 100$) / true density (7)

3.1. Regular spatial patterns

Spatial pattern indices:

Arithmetic mean of 100 measurements (Equation 2) in the regular-triangle point population (Figure 2.1.a) was approximately 155° and in the regular-square point population (Figure 2.1.b) was approximately 139°. In both populations, all observed angular values were larger than 90°. In the lattice-regular point population (Figure 2.1.d), the majority of angular values were larger than 90° and the MoA was approximately 105°. The MoA may suggest that the regular-triangle point population is departing greatly from the CSR with a higher state than the regular-square point population or especially the lattice-regular point population (Figure 3.1.). Frequency of 50 angular measurements in the lattice-regular point population was not significantly different to that of the uniform distribution (Kolmogorov-Smirnov test). Increasing the number of measurements to 100, the Kolmogorov-Smirnov test has revealed a significant difference in the observed data from that of the theoretically uniform frequency distribution. In the other hand, arithmetic mean of angles in the rectangular point population (Figure 2.1.c) was approximately 74° and that could classify this point population as a clustered. Examining frequency distribution of angular measurements in this rectangular point population can also reveal the presence of the regularity in the distribution of points in plane. The majority of observed measurements was also smaller than 90° but with absence of measurements being smaller than 20° (Figure 3.1.).



Figure 3.1. Cumulative frequency distributions of measured angles in simulated theoretically most significant regular point spatial patterns

Arithmetic means of measured angles (Equation 2) in the regular-cluster point population (Figure 2.2.) were nearly equal to 90°. Increasing the sample to 200 measurements has revealed the significant difference of the simulated regular-clusters point population from that of the random point population. However, if using the Kolmogorov-Smirnov test, the sample size of 100 angular measurements may not be large enough to significantly differentiate the observed data from the uniform frequency distribution. Therefore, measuring only the angles may not give us a correct insight into exhibited spatial pattern in certain populations having regularly distributed trees inside clusters.

The L_p spatial pattern index, standing alone, may not be sufficient enough to serve as a measure of the degree of the regularity or clustering. However, it can serve to distinguish from regular and clustered populations. In contrary to the use of angles in indexing spatial patterns, the use of the L_p spatial pattern index regarded the rectangular point population as a regular. Furthermore, the L_p spatial pattern index has indexed the regular-clusters point population to as the clustered (Figure 3.2.).



Figure 3.2. Spatial pattern indices in regular point populations by the use of the L_p spatial pattern index

Density estimates:

The (c-0.5) estimator (Equation 4) and the GM estimator (Equation 6) have produced very proximate density estimates at the regular-triangle and the regular-square point population. The (c-1) estimator (Equation 3) and the Pollard estimator (Equation 5) tends to produce biased density estimates; the (c-1) estimator tends to underestimate while the Pollard estimator tends to overestimate the true density (Figure 3.3.).



Figure 3.3. Relative errors of density estimates with *c*-tree sampling at the regular-square point population

The observed relative error with the Pollard estimator when applying the 5-tree sampling procedure was insignificant in this regular-square point population (Figure 3.3.) but it overestimated the true density of the regular-triangle point population for approximately 5 percent. When applying 3-tree sampling, the (c-0.5) estimator has produced a very accurate density estimate in this regular-square point population (Figure 3.3.) but that was not the case in the regular-triangle point population where it overestimated the true density for approximately 15 percent. It is important to emphasize that applying any of evaluated estimators, the increase in the c value tend to produce density estimates with a smaller amount of a relative error. At the regular-triangle and the regular-square point population, the (c-0.5) estimator or the GM estimator was the most reliable producing smaller relative errors than being compared to the (c-1) estimator or the Pollard estimator.

At the lattice-regular point population (Figure 2.1.d), the Pollard estimator and the

(c-1) estimator have produced density estimates following the same trend as in the regular-triangle and regular-square point populations. In this point population, it is important to emphasize different trends in density estimates with the (c-0.5) estimator and the GM estimator. The (c-0.5) estimator tends to highly overestimate density of lattice-regular point populations and that was largely exhibited when applying small c values. In the lattice-regular point population, obtained density estimates with the GM estimator were the most accurate among other evaluated estimators for c-tree sampling. Observed relative errors with the GM estimator applying $c \ge 5$ were not noticeable (Figure 3.4.) and it performed unbiased properties at the lattice-regular point population.



Figure 3.4. Relative errors of density estimates with *c*-tree sampling at the lattice-regular point population

The trend exhibited in regular-triangle, regular-square and lattice-regular point populations did not applied for the rectangular point population (Figure 2.1.c). Spatial pattern indices have shown that the rectangular point population is a special case of populations having regularly distributed points and it can also be regarded as a clustered population composed by linearly distributed clusters. Here as well, increase in the c value tends to reduce the bias. The (c-1) estimator overestimated the true density with 2-tree, 3-tree and 4-tree sampling. However, when applying these small c values, the (c-1) estimator has produced the smaller amount of relative errors observed in this rectangular point population. For the same applied small c values, other evaluated estimators tend to produce greater relative errors, highly overestimating the true density (Figure 3.5.).



Figure 3.5. Relative errors of density estimates with *c*-tree sampling at the rectangular point population

The regular-clusters point population (Figure 2.2.) is also a special case where a particular caution needs to be considered when obtaining density estimates with c-tree sampling. In this point population, the GM estimator was the most efficient; the GM estimator produced a smaller amount of a relative error when applying 2-tree, 3-tree, 4-tree and 5-tree sampling. Applying larger c values, the (c-1) estimator produced the most accurate density estimates; the true density was underestimated by less than 1 %

(Figure 3.6.).



Figure 3.6. Relative errors of density estimates with *c*-tree sampling at the regular-clusters point population

3.2. Random spatial patterns

Spatial pattern indices:

The *MoA* in the simulated random population from 500 random sampling points nearly equaled 90° and the frequency of angular measurements was not significantly different from that of the uniform distribution (Kolmogorov-Smirnov test; p = n.s.). This was taken as a confirmation of a spatial randomness. This also suggested that the use of random number generators is applicable in producing spatially random point populations. In all simulated random point populations, regardless of the relative density, the Mean of Angles method has proved applicable in revealing a spatial randomness.

The L_p spatial pattern index has also proved applicable in revealing the spatial randomness. In the presented random point population, applying 500 measurements, the L_p spatial pattern index was highly reliable in revealing spatial randomness, regardless of the applied c value and for $c \ge 2$ (Figure 3.7.).



Figure 3.7. Spatial pattern indices in the random point population by the use of the L_p

Density estimates:

In the simulated random point population, measuring distances from 500 random sampling points, relative errors observed with the (c-1) estimator, the GM estimator as well as in the Pollard estimator can be regarded as insignificant with $c \ge 5$ sampling; $c \ge 5$ sampling tended to underestimate the true density of the random population by approximately 2 %. The (c-0.5) estimator produced heavily biased density estimates, highly overestimating the true density. The (c-1) estimator applying smaller c values produced significantly higher relative errors of density estimates; c = 3 sampling underestimated the true density by 7.1 % and c = 2 sampling underestimated the true density by 13.2 %. The GM estimator was also more accurate than the (c-1) estimator, and for applied small c values. The Pollard estimator, thought to be unbiased in random populations, here underestimated the true density but the bias was constant and nearly equal for all applied c values. That makes the Pollard estimator preferable for the use in random populations (Figure 3.8.). It has to be emphasized that the same trend was observed in other simulated random point populations having different relative density.



Figure 3.8. Relative errors of density estimates with *c*-tree sampling at the random point population

The *c*-tree sampling method yields variable circular plot areas or distances (circular plot radiuses) in a statistically continuous measurement scale. Frequency distributions of variable circular plot samples (squared plot radiuses) in random point populations were not significantly different from the gamma frequency distribution (Kolmogorov-Smirnov test of observed data sets for $c \ge 2$ sampling; p = n.s.). Frequency distributions of variable circular plot samples applying $c \ge 5$ sampling were also significantly different from the normal frequency not distribution (Kolmogorov-Smirnov test; p = n.s.). Therefore, according to the produced density estimates at the random point population (Figure 3.8.), it is likely that (c-1) estimator, GM estimator and the Pollard estimator are all applicable if frequency distributions of variable circular plot samples (squared plot radiuses) fit the normal frequency distribution. The variance of density estimates in random populations for $c \ge 5$ sampling can be assessed by the standard deviation involved in the normal frequency distribution.

On the other hand, moments involved in the gamma frequency distribution need to be assessed (Kumar 2006).

Also interesting finding is that frequency distributions of variable plot radiuses in measured random populations and applying $c \ge 2$ sampling were not significantly different from the normal frequency distribution (Kolmogorov-Smirnov test; p = n.s.). This clearly explains the applicability of the Pollard estimator, and its smaller variance of density estimates, in populations consisted of randomly distributed individuals since it accounts for measured distances rather than variable plot areas.

3.3. Clustered spatial patterns

Spatial pattern indices:

As expected, the majority of angular values in the Matérn-clustered point population (Figure 2.4.) were smaller than 90°. Arithmetic mean, approximately amounting to 13°, may regard this clustered point population as highly departed from the CSR in the direction of clustering (Figure 3.9).

Similarly to results obtained from the simulated Matérn-clustered point population, the Mean of Angles method revealed clustering in point populations simulated by the Gap-process. At the 10mGAP point population (Figure 2.5.) the $MoA \approx$ 80° , at the 20mGAP point population (Figure 2.6.) the $MoA \approx 50^{\circ}$ and at the 30mGAP point population (Figure 2.7.) the $MoA \approx 20^{\circ}$. Therefore, we can regard the 10mGAP, the 20mGAP and the 30mGAP point populations as point populations having clustered spatial patterns. Furthermore, according to the MoA (Equation 6), clustering involved in the 30mGAP point population can be regarded as higher than that involved in the 20mGAP or in the 10mGAP point population (Figure 3.9.).



Figure 3.9. Cumulative frequency distributions of measured angles in simulated clustered point spatial patterns

The L_p spatial pattern index has also proved applicable in revealing spatial clustering. However, its use in indexing clustered spatial patterns was questionable. That was particularly exhibited in indexing clustered populations simulated by the Gap-process, where the 20mGAP population was indexed as the most highly departed population from randomness. In particular, applying *c* values smaller than 10 has regard the 30mGAP population as a population being less clustered than the 20mGAP population. Furthermore, simulated Matérn-clustered point population was assigned with indexes being less departed from the random spatial pattern than point populations simulated by the Gap-process (Figure 3.10.).



Figure 3.10. Spatial pattern indices in clustered point populations by the use of the L_p spatial pattern index

Density estimates:

The *c*-tree sampling method produced highly unreliable density estimates at the simulated Matérn-clustered point population regardless of density estimator applied. In particular, density estimates with c = 2 sampling have produced extremely high relative errors; the (c–1) estimator overestimated the true density by 135.8 %, the (c–0.5) estimator by 253.7 %. With c = 2 sampling, the GM estimator was the most accurate and overestimated the true density by 13.6 % while the Pollard estimator underestimated the true density by 15.2 %.

The (c-0.5) estimator with c = 3, c = 5, c = 7 and c = 9 sampling has produced relatively accurate density estimates at this Matérn-clustered point population. Relatively high errors produced by the (c-1) estimator tend to reduce with the increase in *c* value, where an approaching trend to the true density was observed (Figure 3.11.). That can be explained by the regular nature of the simulated Matérn-clustered point population (each of clusters containing two points) as well as exhibited frequency distributions of the distances (plot radiuses) and variable circular plot areas involved in *c*-tree sampling. The distances (plot-radiuses) in this simulated point population fitted the normal frequency distribution ($c \ge 2$) but the distribution of variable circular plot areas with the *c* = 2 sampling procedure well fitted the exponential frequency distribution, being the reason of produced highly biased density estimates with the (c-1) estimator and the (c-0.5) estimator. Sampling procedures applying $c \ge 3$ produced variable circular plot areas fitting the gamma frequency distribution (Kolmogorov-Smirnov test; p = n.s.). It is also likely that increase in the *c* value would produce variable circular plot areas fitting the normal frequency distribution and, theoretically, produce less biased density estimates with *c*-tree sampling.



Figure 3.11. Relative errors of density estimates with *c*-tree sampling at the Matérn-clustered point population

At simulated clustered point populations applying the Gap-process, the true density in the 10mGAP population was 736.8 points/ha, in the 20mGAP it was 293.2 points/ha and in the 30mGAP, 65.6 points/ha. The Pollard estimator as well as the GM estimator tends to grossly underestimate while the (c-0.5) estimator overestimated the true density similarly like in here presented results obtained in 30mGAP point population (Figure 3.12.).



Figure 3.12. Relative errors of density estimates with the (c-0.5) estimator, the GM estimator and the Pollard estimator at the 30mGAP point population

The (c-1) estimator, when applying small c values like c = 2 or c = 3 sampling, has produced density estimates with the smaller amount of relative error when compared to other evaluated estimators. In the simulated 10mGAP population and measuring distances from 500 random sampling points, c = 3 sampling was the most accurate underestimating the true density by only 1.7 %. The c = 2 sampling procedure underestimated the true density by 10.0 %, and c = 4 sampling overestimated the true density by 1.8 %, while $c \ge 5$ sampling overestimated the true density by approximately 5 % (Figure 3.13.). In simulated 20mGAP population the c = 2 sampling procedure was the most accurate, underestimating the true density by only 3.1 %. The c = 3 sampling procedure overestimated the true density by 5.1 %. The amount of bias tended to increase with the increase in the c value and it was higher with c = 10 sampling where the true density was overestimated by 21.5 % (Figure 3.13.). In the simulated 30mGAP population, measuring distances from 500 random sampling points, the c = 2 sampling procedure was also the most accurate but here it overestimated the true density by 5.0 %. In a similar way to the density estimates obtained in the simulated 20mGAP population, the bias in the simulated 30mGAP population tended to increase with the increase in the c value. The size of the bias was highest when applying the c = 10 sampling procedure, where the true density was overestimated by 31.0 % (Figure 3.13.).



Figure 3.13. Relative errors of density estimates with the (c–1) estimator in clustered point populations created by the Gap-process

In the highly clustered 30mGAP population, and when applying $c \ge 5$ sampling, observed frequency distributions of samples were not significantly different from the exponential frequency distribution (Kolmogorov-Smirnov test; p = n.s.). Observed frequency distributions in clustered populations when setting small c values, such as c =2 or c = 3, seems to follow the Pareto principle; the Pareto principle states that 20% of the population would hold approximately 80% of the sample wealth. The c = 2 or c = 3sampling procedures also tend to produce smaller bias in clustered populations than setting some higher c values (Figure 3.13.). Observed frequency distributions are characterized by a long tail, which also characterizes the Pareto frequency distribution. However, our observed data sets were significantly different from that of the Pareto frequency distribution (Kolmogorov-Smirnov test; p < 0.01), which is usually characterized by a much longer tail. Observed frequency distributions in these simulated clustered point populations fitted the generalized Pareto frequency distribution (for example, Choulakian and Stephens 2001). Estimating the variance involved in the generalized Pareto distribution, defined by the function $F(x) = 1 - (1 - kx/\sigma)^{1/k}$ where σ is a positive scale parameter and k is a shape parameter, is influenced by the accuracy of estimated parameter values and the method used; the variance reported by Choulakian and Stephens (2001) is $s^2 = \sigma^2 / \{(1+k)^2 (1+2k)\}$. Furthermore, k = 0 yields the exponential distribution, k = 1 yields the uniform distribution and k < 0 yields the Pareto distribution. This suggests that the shape parameter should be accurately estimated which may not be feasible in all theoretical cases (Luceño 2006).

Using the bootstrap statistical technique, estimating density for 100 times by a randomization of variables, have shown that applying smaller c values has produced a higher variance. The reduction of the variance can be achieved by increasing the sample. The variance involved in highly clustered populations was also likely to be higher that that involved in moderately clustered populations (Table 3.1.).

Table 3.1. The variance of density estimates per hectare in simulated clustered point populations obtained by the bootstrap for c = 2 and c = 10 sampling. Each value of the squared root variance^{*} is obtained from 100 density estimates; where λ_i is the *i*th density estimate with the (c–1) estimator (Equation 3) and *m* is a number of density estimates and here equals to 100.

	<i>c</i> value	2	10	
	<u>n</u>			
10mGAP	50	118.0	62.2	
	100	73.3	40.7	
True density =	200	43.5	30.6	
736.8 points/ha	300	27.4	17.5	
	400	18.8	10.3	
20mGAP	50	99.1	51.5	
	100	58.1	35.7	
True density =	200	34.7	25.5	
293.2 points/ha	300	22.8	14.8	
	400	13.9	9.1	
30mGAP	50	51.3	25.7	
	100	30.8	16.4	
True density =	200	17.6	11.0	
65.6 points/ha	300	11.8	6.9	
	400	7.6	3.8	
* Sq.r. Variance = $\sqrt{s^2}$; $s^2 = \left[\sum_{i=1}^m (\lambda_i - \overline{\lambda})^2\right] / (m-1)$				

3.4. Study case of naturally regenerated Chamaecyparis spp. saplings

Spatial pattern indices:

Arithmetic mean of angles measured from randomly distributed 500 sampling points amounts to 60.4° and systematically distributed 115 sampling points amounts to 65.3°. Those indicate that saplings in a measured forest stand exhibited moderately clustered pattern. Observing frequency distribution of measured angles may also indicate a presence of regularity or randomness in certain parts of the forest stand. However, clustering prevails in this population (Figure 3.14.).



Figure 3.14. Frequency distribution of 500 measured angles from randomly distributed sampling points at the saplings population

The L_p spatial pattern index has also revealed spatial clustering of saplings, either accounting for measurements from 500 randomly distributed sampling points (Figure 3.15.) or from systematically distributed sampling points. The L_p spatial pattern index when applying c = 2 sampling from 115 systematically distributed sampling points was 2.5. When measuring distances from 99 systematically distributed sampling points, the L_p spatial pattern index with c = 3 sampling was 2.8 while with c = 4 sampling it was 3.3.



Figure 3.15. Spatial pattern indices in the saplings populations by the use of the L_p spatial pattern index randomly distributing 500 sampling points

Density estimates:

The Pollard estimator as well as the GM estimator grossly underestimated the true density of the saplings population; either applying random (Figure 3.16.) or systematical sampling procedure (Figure 3.17.). Applying the random sampling procedure, the (c-1) estimator was the most accurate when compared to the other evaluated density estimators. Observed relative errors of density estimates with the (c-1) estimator at the saplings population (Figure 3.16.) were proximate to those obtained in moderately

clustered point populations created by the Gap-process (Figure 3.13.) Here as well, c = 2 sampling underestimated the true density (Relative Error = -10.9 %), c = 3 sampling slightly overestimated the true density (Relative Error = 2.7 %) while c = 4 sampling (Relative Error = 11.0 %) has significantly overestimated the true density (Figure 3.16.).



Figure 3.16. Relative errors of density estimates with *c*-tree sampling from 500 randomly distributed sampling points at the saplings point population

Applying the systematical sampling procedure, the (c–0.5) estimator overestimated the true density with c = 2 and c = 3 sampling while the c = 4 sampling procedure produced very close estimate to the true density. The (c–1) estimator underestimated the true density and the density estimate with c = 3 sampling was the most accurate. Applying c = 2 sampling, the (c–1) estimator has also produced the most accurate estimate of density compared to other evaluated density estimators (Figure 3.17.).



Figure 3.17. Relative errors of density estimates with *c*-tree sampling from systematically distributed sampling points at the saplings point population

Applying *c*-tree sampling along a systematical sampling design has also a greater potential that a simple random sampling design in stratifying populations of interest. In particular, stratifying the saplings population by its relative density was highly practical with c = 2 sampling (Figure 3.18.).



Figure 3.18. Distribution of distances with 2-tree sampling from systematically distributed sampling points at the saplings population

Fixed-area plot sampling applying circular plots having 1 m and 2 m radii have underestimated the true density of saplings, either from randomly or systematically distributed sampling points. Applying random sampling and 500 plots, only 70 saplings were counted on 1 m radii circular fixed-area sampling plots and the true density was grossly underestimated by 16.7 %. The 2 m radii circular fixed-area plot sampling design enumerated 315 saplings in total and it was more accurate, underestimating the true density by 6.2 %. Both fixed-area plot sampling designs applying the systematical sampling procedure, using circular plots of 1 m (Figure 3.19.) and 2 m (Figure 3.20.) radii, grossly underestimated the true density. Estimated density on 115 systematically distributed 1 m radii circular fixed-area plots was 359.8, underestimating the true density by 40.4 %. Estimated density on 115 systematically distributed 2 m radii circular fixed-area plots was 422.1, underestimating the true density by 30.1 %.



Figure 3.19. Tree counts on 1 m radii circular fixed-area plots



Figure 3.20. Tree counts on 2 m radii circular fixed-area plots

The sample of distances measured from 500 randomly distributed sampling points was not significantly different to the gamma frequency distribution while variable plot areas (squared radiuses) fitted the generalized Pareto distribution. Observed frequency distributions of 115 variable circular plot area samples with c = 2 sampling and observed frequency distributions of 99 variable circular plot area samples with c = 3 and c = 4 sampling respectively were not significantly different from the exponential frequency distribution (Kolmogorov-Smirnov test; p = n.s.). On the other hand, measured distances (plot radiuses) were not significantly different from the normal frequency distribution (Kolmogorov-Smirnov test; p = n.s.).

Using the bootstrap statistical technique, estimating density for 100 times by a randomization of variables being obtained by conducting systematical sampling, have shown that increasing the sample size leads to the reduction of the variance. However,

observed variance in 100 density estimates with c = 3 sampling was even greater than that performed by the c = 2 sampling procedure (Table 3.2.).

Table 3.2. The variance of density estimates per hectare with the (c-1) estimator in the saplings population obtained by the bootstrap of variables; c = 2, c = 3 and c = 4 sampling from systematically distributed sampling points.

10	2*	<i>c</i> -tree	/**
n	Ζ.	2	4
30	212.8	293.4	124.3
50	151.9	170.7	84.5
70	101.6	112.1	58.8
90	59.7	71.0	30.7

*True density = 603.7 saplings/ha

**True density = 690.1 saplings/ha

Estimating density for 100 times by a randomization of variables has also shown that the variance of density estimates with the use of fixed-area plot sampling can be reduced by increasing the sample size (Table 3.3.). Systematically distributing 30 circular fixed-area plots having 1 m radii, 8 % of samples did not spot any sapling sized from 1.5 to 5.0 meters in height in this forest stand. However, that was not the case when increasing the sample to 50 or more or when applying circular fixed-area plots having 2 m radii and for the sample sized to 30 or more.

Table 3.3. The variance of density estimates per hectare with the 1 m and 2 m radii circular fixed-area plot sampling in the saplings population obtained by the bootstrap of variables taken from systematically distributed sampling points.

	circular fixed-area	
	plot sampling*	
n	1 m radii	2 m radii
30	193.7	121.1
50	146.8	78.3
70	101.0	55.7
90	62.6	40.1

*True density = 603.7 saplings/ha

Systematically distributing 115 circular fixed-area plots having 1 m radii, only 13 saplings were counted in total. The total number of saplings being included in the sample of 115 circular fixed-area plots having 2 m radii was 61. Average number of counted saplings on 1 m and 2 m radii circular fixed-area plot samples, being obtained by the bootstrap from 100 estimates of density (Table 3.4.), was relatively low.

Table 3.4. The mean count of saplings on 1 m and 2 m radii circular fixed-area plot samples; 100 estimates obtained by the bootstrap of variables taken from systematically distributed sampling points.

	circular fixed-area plot sampling		
	l m radii	2 m radii	
п	(mean count)	(mean count)	
30	3.1	16.3	
50	5.6	26.4	
70	7.8	37.1	
90	10.2	47.6	

4. Discussion

4.1. Spatial patterns of trees

Trees in forests can exhibit a large variety of spatial distributions. It is rather difficult task to accurately mimic all feasible varieties in spatial distributions of trees in forests or to easily assess them all. Particularly that is difficult in naturally regenerated or in largely disturbed forests. In many cases, trees may not be uniformly distributed or spatial patterns and relative densities of trees can differ according to sizes of the trees, ecological properties of tree species, species compositions, respective forest types, scale, terrain conditions and many other factors.

The difficulty to indexing spatial patterns of trees is further increased by the fact that spatial patterns of certain forest stands can exhibit both regular and clustered properties, such as the case in the rectangular population (Figure 2.1.). Forest plantations are usually established by planting trees in regular spacing. It is usually preferred to maintaining regular spatial patterns in order to increase benefits yielded from a timber. Such planting practices, with later on thinning, are projected to minimize unnecessary competition between neighboring trees. Maintaining a regular spatial distribution of planted trees in forest stands throughout their maturing gives an equal amount of available light to each individual tree and that is likely to maximize a yield in a most wanted and a highly priced timber. It is shown that a disturbance, such as harvest or a natural disaster, can lead to spatial distribution of trees exhibiting clustering. That is feasible also in forest plantations and foresters should be concerned to reducing such events which highly drives spatial patterns toward clustering.

Supplemental planting inside naturally regenerated forests, being as well conducted in regular spacing, is also often aimed to accelerate a natural process which drives a change in spatial distributions of trees toward regularity. Supplemental planting is also aimed to mitigate disturbances, either artificially or naturally caused, and prevent growth of unwanted species. In such cases, along to assessing relative density of naturally regenerated juvenile trees, it is also necessary to indexing their spatial patterns.

In some naturally regenerated stands, seeds can be randomly dispersed under the mother-trees' cover. Spatial patterns of mother-trees, influence of a wind, water, gravity, animals and possibly other factors or combination of these factors can also lead to some form of dispersed seed pattern exhibiting properties of clustering (Bigwood and Inouye, 1988). Spatial distribution of germinated seeds can greatly depend on relief and other site conditions or it can be influenced by grazing or pathogens and succeeded naturally regenerated trees can exhibit highly variable spatial pattern distributions. Naturally regenerated juvenile trees are likely to exhibiting clustered spatial patterns; clusters being irregular in size and shape. As trees grow, it is widely accepted opinion that a competition between individual trees drives their distribution toward regular spatial pattern (Moeur 1993). That is a slow process but large sized trees in naturally regenerated forests can exhibit regularity in their spatial pattern (Ishibashi et al. 1989). The process is also likely to last longer in the case of larger disturbances which can increase the degree of clustering.

Here proposed the "Gap-process" can be used to simulate clustered spatial patterns and it attempts to mimic disturbances in forests. In ecological terms, the Gap-process can be seen as a disturbance; artificial (for example harvest) or natural (for example damage caused by strong winds, forest fires etc.). Similar to the Gibbs process (Stoyan and Stoyan, 1994), a host point population, as a starting point in creating clustered populations by the Gap-process, can be not only the random point population as presented in the thesis but also any other point spatial pattern. A host point population, which could be either regular, clustered or some combined pattern, could potentially mimic a spatial pattern of trees before the disturbances while changing patterns and intensity of disturbances can lead into different spatial patterns. Taking into example presented point populations clustered by the Gap-process, the increase in the plot area as well as the increase in the number of plots is likely to increase the level of clustering until the great majority of points are erased.

4.2. Spatial pattern indices

Among large number of methods being proposed in the past for the use in indexing spatial patterns of trees, *c*-tree sampling and the Mean of Angles method are particularly suitable. Both are based on measurements from sampling points, positions of trees do not need to be mapped and required measurements are simple to obtain.

Both the Mean of Angles and the L_p spatial pattern index can serve to testing whether the trees in forests are distributed randomly or not. For a purpose of testing a spatial randomness, statistical power involved in the Mean of Angles method may not be as high as that involved in the L_p spatial pattern index. For example, the L_p spatial pattern index may not require a large sample to reveal a significant difference between lattice-regular point population and a population following a complete spatial randomness. The L_p spatial pattern index has also performed a high statistical power in revealing clustering being exhibited in the regular-clusters point population (Figure 3.2.). The Mean of Angles method may need a relatively large sample in order to reveal a difference between regular-clusters point population from the theoretically random population. Moreover, distinguishing between lattice-regular and random populations by the use of the Mean of Angles method may also require obtaining a relatively large sample.

The use of *c*-tree sampling in testing randomness is also burden by the variance as it is its use in estimating a density of trees. However, that shall not pose any constraints for the use of *c*-tree sampling and the L_p spatial pattern index in testing whether trees in forests are distributed at random or exhibiting some form of regular or clustered spatial pattern. The L_p (Equation 5) value being not significantly different to 1 could suggest that the trees in forest are distributed at random. We need to bear in mind that increase in the number of measurements would give us a higher confidence. Results of the study have shown that the L_p spatial pattern index can serve to distinguish between regular and clustered populations. The L_p spatial pattern index was also reliable in revealing a spatial randomness and for any applied c value larger or equal to two (Figure 3.7.). However, being proposed by Liu (2001) to serve as a measure of the degree of regularity or clustering, this study have shown that it can not be completely relied on indices provided only by using the L_p spatial pattern index. In particular, the L_p spatial pattern index could not reveal a presence of clustering in the rectangular point population nor it could be reliable enough to serve as a measure of the degree of regularity or clustering (Figure 3.2., Figure 3.10.).

It may not be much important in forestry and in forest research to assessing whether trees in forests are distributed at random or not. It is much important to indexing the degree of regularity or clustering of trees in forest stands. The Mean of Angles method has revealed a presence of clustering at the rectangular point population and it has proved reliable in indexing the degree of regularity (Figure 3.1.) or clustering (Figure 3.9.). Its ability to distinguishing between moderately and highly clustered point populations (Figure 3.9.) also holds a great practical value. According to the results, the *MoA* (Equation 6) can serve as a practical index of the degree of regularity or clustering. However, it can not be completely relied on it in describing all feasible spatial pattern distributions of trees in forests unless taking relatively large sample of angles and examining their frequency distributions. This later constraint is shown by the rectangular (Figure 2.1.) or the regular-clusters point population (Figure 2.2.).

The rectangular and the regular-clusters point populations are emphasized in the thesis because the practice of planning roads or timber extraction routes in forest plantations can form clusters where each cluster can be composed by trees being planted at regular spacing. For example, the rectangular point population is a population consisted of regularly distributed points in a plane, but it also can be considered as a population consisted of regularly distributed linear clusters. In this point population, the L_p spatial pattern index as well as the arithmetic mean of angles were both correct and in spite of their completely different indices; the L_p spatial pattern index has indicated the rectangular point population as a regular while the arithmetic mean of angles to as a clustered. Moreover, the L_p spatial pattern index has revealed a presence of clustering in the regular-clusters point population (Figure 3.2.). Therefore, the use of the Mean of Angles method along with the L_p spatial pattern index can give us a further insight regarding a spatial distribution of individual trees.

4.3. C-tree sampling density estimators

The two main statistical approaches can be applied to estimate a relative density of trees with the use of *c*-tree sampling. The first consider relation between the tree density and distances measured from sampling points to constant number of nearest trees; longer distances would imply a lower tree density. The second approach considering that circular plots are statistical variables along to setting assumptions about an average number of trees being included in sampled areas.

The first approach is widely acknowledged in the past as the least-variable (Picard et al. 2005) but its statistical advantage in estimating the tree density is only in the case if the trees in measured forests are distributed at random (Pollard 1971). Otherwise it can produce large bias in estimating the density of trees, with the bias largely driven by spatial pattern distributions of trees. Its practical use is limited to the only case of uniformly random populations and when distances (plot radiuses) as well as variable circular plot areas fitting the normal frequency distribution. In other cases, observed frequency distributions of distances or variable circular plots different than the normal frequency distribution could induce a presence of a relatively large bias in density estimates. It can be claimed that forest stands with the trees exhibiting a spatial randomness do not represent a significantly large share of the World Forest. Therefore, density estimators such as the Pollard estimator hold almost only a theoretical value.

Picard et al. (2005) have proposed an approach to estimating density of trees exhibiting clustering by firstly assessing the density of individual clusters and further multiplying it to an average number of trees inside those clusters. Picard et al. (2005) suggested that such an approach could be used to estimating density of coppice forests and in the case if the clusters are distributed at random. In such a case, the density of clusters formed by the sprouts can be assessed by the use of c-tree sampling and it can be further multiplied with an average number of stems inside the clusters. Such information can again be biased as it requires investigating spatial pattern distributions of clusters, since it can not be assumed that the clusters are randomly distributed over the coppice forest area. However, that is still an interesting approach for the use in coppice forests where a high precision of estimates is usually not required in a practical forestry. In the case when a spatial distribution of clusters exhibits regularity or even possibly clustering, it is feasible to using a more appropriate density estimator such as the GM estimator or the (c-1) estimator. However, such an approach proposed by Picard et al. (2005) is clearly not practical in forests being regenerated from naturally dispersed seeds where defining individual clusters being irregular in size and shape can pose a great difficulty.

Merits of using the second approach which considers averaging variable circular plots is often argued by statisticians because of its higher variance involved; a sample of variable circular plot areas would much vary when compared to the sample of distances taken from the same population. However, in spite of a larger variance, these density estimators may not involve as large bias of density estimates as the approach which accounts for the distances. Increasing a sample size would reduce a variance and that would not pose a risk to increase a bias when using these density estimators. The use of this statistical approach is particularly emphasized in populations exhibiting a regular spatial pattern since here the variation in circular plots is not significantly high. In such cases, when spatial patterns are indexed by both L_p spatial pattern index and Mean of Angles to as a regular, the GM estimator can increase a reliability of density estimates.

It is important to emphasize that the increase in a c value when using c-tree sampling density estimators in regular and random populations would reduce an amount of a bias. That would also result into reduced variance in density estimates and thus the reliability of estimates would be higher. However, instead of increasing the c value what is likely to make surveys much difficult, with applying the Pollard estimator in random and the GM estimator in regular populations it may not be necessary to increase the cvalue to more than three. Of course, that depends upon objectives of surveys, sample size and a required precision of density estimates.

The (c-1) estimator, being originally proposed by Eberhardt (1967), accounts for variable circular plot areas and the results of the thesis have demonstrated its applicability to assessing a relative density of random or clustered populations. The results have shown that the use of the (c-1) estimator is particularly applicable in estimating relative density of clustered populations such are populations of naturally regenerated juvenile trees.

According to the results, in populations exhibiting random spatial patterns and applying $c \leq 4$ sampling, the variable circular plots fitted the gamma frequency distribution and not the normal frequency distribution and the true density is likely to be underestimated. It is also clear that the bias involved in the (c-1) estimator will be relatively small, and density estimates can be regarded as reliable, if observed variable circular plots fit the normal frequency distribution. That is certain when setting $c \geq 5$ in random populations (Figure 3.8.) and that clearly explains findings of Lessard et al. (1994) regarding the applicability of the (c-1) estimator in forests having nearly random distribution of individual trees in forests.

Eberhardt (1967) also proposed that the (c-1) estimator can be used in forests where the distribution of counted trees on fixed-area plot samples yields the negative binomial frequency distribution; fixed-area plot sampling producing a counts of trees fitting the negative binomial frequency distribution usually indicate a clustered spatial pattern. Eberhardt (1967) suggested that increase in the c value would give more reliable estimates of density in clustered populations and result to a reduced variance of density estimates. The results of this study have confirmed that the (c-1) estimator is a robust density estimator and it can be used in populations exhibiting random or clustered spatial patterns. However, applying small c values such as c = 2 in highly clustered or c = 3 sampling in moderately clustered populations could be a more appropriate sampling procedure than applying some higher c values. That was certain in point populations clustered by the Gap-process in randomly distributed host population (Figure 3.13.) as well as in the saplings population (Figure 3.16.) but not in the Matérn-clustered point population. The answer to the applicability of *c*-tree sampling and the (c-1) estimator in the clustered populations lies in the exhibited frequency distributions of variable circular plot areas (squared plot radiuses). In the Matérn-clustered point population, distributions of variable circular plot areas fitted the exponential frequency distribution or the gamma frequency distribution. According to the results obtained from the other clustered populations, it is likely that observed frequency distributions being not significantly different from the exponential frequency distribution would also hold with the higher bias in density estimates with the (c-1) estimator. Applying the smallest applicable c value was the most appropriate sampling procedure because it holds a highest probability that variable circular plot areas would fit the generalized Pareto frequency distribution.

The distribution of variable circular plot areas in simulated clustered point populations, being uniform in structure, is likely to change with the increase in the c value. The change in the distribution is likely to starting from the generalized Pareto frequency distribution for the small c values through the exponential frequency distribution and the gamma frequency distribution to the normal frequency distribution in clustered populations may require increasing the c value to infinity and thus it is clearly impractical. At least that is impractical in a field surveys and using a conventional field surveying equipment.

4.4. Forest regeneration surveys

In most of forest regeneration surveys which attempts assessing relative density of juvenile trees, assessing relative density of trees by the use of fixed-area plot sampling requires setting a size of plots prior to conducting the survey. Setting a size of plots is a difficult decision to make as even a small increase in a plot size can largely influence later on efforts in the field. The usual practice in forest regeneration surveys is to setting relatively small sized plots, what is irrefutably very fast surveying approach. The conventional approach of counting juvenile trees on small sized plots is much faster than *c*-tree sampling which requires conducting a search for nearest trees from each sampling point. However, setting a plot size to some small value is likely to fail in statistically representing populations of interest unless a sample size is large enough. Setting a plot size to some small value may also induce that a majority of plots would contain no trees to count and it was shown that such a sampling design may also indicate an absence of saplings at the forest stand in the Kiso area. It was also shown that setting a plot size to some large value may not be practically justified; for example, setting a plot size to 0.04 ha would require an average count of 26.9 saplings per plot or that could even require a count of as much as 171 saplings per plot at the forest stand in Kiso area. On the other hand, fixed-area plot sampling is an unbiased approach to assess relative density or other forest or forest stand parameters. It is likely that the increase of the sample size, and applying the random sampling procedure, would produce unbiased density estimates on the forest or forest stand level.

Results of this study have shown that *c*-tree sampling can produce reliable estimates of density of trees in forests and that is at least applicable in forest regeneration surveys where a relatively small bias can be neglected in obtained estimates of a tree density. It is shown above that the bias can be effectively reduced if indexing spatial pattern distributions of trees, stratifying forest area and choosing an appropriate density estimator. Several new findings regarding the use of c-tree sampling in estimating density of trees in clustered populations are probably the most significant contribution of the thesis. These findings defend a c-tree sampling approach, being widely regarded to as unreliable, and promote its use in forest regeneration surveys.

Indexing spatial pattern distributions of individual trees plays a crucial role to the choice of an appropriate density estimator. The L_p spatial pattern index (Liu 2001) was derived for the *c*-tree sampling procedure and thus measuring distances, or defining variable plot areas, can also serve in assessing information regarding a relative density of trees in forests. Moreover, measuring distances, applying *c*-tree sampling, can serve in assessing specific relative densities of juvenile trees, being more site-specific than estimates on the forest stand level. Those can serve to delineate more specific areas of forest stands with no regeneration occurred, with moderate density to those having abundant number of juvenile trees. Furthermore, measurements of distances do not need to be performed with high precision to bring a practical value to a collected data being presented in a form of maps (Figure 3.18.). In cases where remote sensing can not detect juvenile trees, forest stand area can be stratified by the use of these distances and those can give a higher confidence than the use of small-sized fixed-area plots (Figure 3.19.).

The use of the Mean of Angles method have a potential to increase a reliability of density estimates and give a more reliable insight into the degree of regularity or clustering. However, in cases when a cost of forest regeneration surveys is crucial, the angles do not necessarily need to be measured. Even if measuring the angles is a very fast procedure, its introduction to forest regeneration surveys would add an additional cost. Therefore, its use can be limited to only surveying highly priced forest resources where obtaining much reliable information can be economically justified. In such cases, other sources of information could be also considered such as the use of fixed-area plots,

the use of remotely sensed data and subjective appraisements of surveyors as well. All these information appropriately spatially assigned and stored to an information system hold a great potential to benefit a management of any particular forest resource.

Random sampling is theoretically more appropriate than systematical sampling since it can statistically better represent measured populations. However, systematical sampling designs can be preferred procedures for practical applications in forestry. Along to easier search for systematically distributed sampling points in the field, systematical sampling designs are also practical in stratifying forest area; for example, separating areas being not regenerated from those with abundant number of juvenile trees. Fixed-area plot sampling can also be used in stratifying forest stand areas (Figure 3.19. and Figure 3.20.). However, c-tree sampling has proved more reliable in representing and stratifying the saplings population (Figure 3.18.) and thus it has a great practical advantage over applying fixed-area plot sampling. Applying *c*-tree sampling assumes that a search for nearest trees needs to be conducted once a sampling point is defined. That also applies for the Mean of Angles method. For example, setting the c to two would mean that the distance to the second nearest tree from each sampling point need to be measured. That may not be necessary in forest regeneration surveys and when applying systematical sampling designs. Large distances to nearest juvenile trees induce that there is no regeneration occurred and such spatially based information can serve to improve managerial decisions such as those regarding a harvest or supplemental planting.

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5. Conclusion

Many factors can influence spatial distribution of trees in forests. Among the most significant factors is a competition between individual trees and a disturbance. Competition between individual trees tend to naturally drive a change in spatial patterns of trees toward regularity while disturbances backward it toward clustering. Acquiring information regarding spatial patterns of trees can give us insight into a level of competitiveness between individual trees as well as into a level of disturbances. Such information could serve to expand an insight into past management practices or ecological processes and serve in projecting a future development and growth of trees associated in respective forest types.

It is not necessary to mapping positions of individual trees in order to reveal a great variability of spatial patterns. A simple statistical methodology designed to measuring angles between lines of sight from sampling points to their nearest two neighboring trees can serve in acquiring a reliable information regarding spatial patterns of trees. That is also a practical approach to apply in forest regeneration surveys since the method is simple and robust enough. Moreover, the measurements of angles do not need to be performed with a high precision. An arithmetic mean of angles can serve as a simple spatial pattern index and it is applicable to indexing the degree of regularity or clustering of trees in forests. On the other hand, more complex arrangements of individual trees can be assessed by analyzing frequency distributions of measured angles.

Spatial distributions of trees could be a complex association of different tree species and tree sizes. Spatial pattern indices based on measured angles between lines of sight from sampling points to their nearest two neighboring trees are not dependent upon a relative density of trees. The relative density could also largely differ inside respective forest stands. Therefore, a spatial variety can be further assessed by combining measurements of angles with other methods such as conventional fixed-area plot sampling or to introducing measurement of distances between the trees or between sampling points and trees.

Measuring distances between sampling points and trees may not give us a reliable insight into the degree of regularity or clustering. An extensive statistical expertise is necessary in order to look beyond the indices, what is clearly not appropriate in supporting practitioners in the field. Furthermore, its use in assessing relative densities of trees can still produce biased estimates. The choice to use *c*-tree sampling in forest inventories should be only in the case when relatively high variance of density estimates can be accepted, when we can afford an increase in the number of measurements in order to reduce the variance and, most of all, when we can accept the risk of a bias. The bias is the biggest limiting factor for the use of distance sampling to estimating density of trees in forests. Furthermore, the use of *c*-tree sampling in clustered populations implies that individual trees being located on outskirts of clusters would have a greater probability to be included in a sample than those being located inside the clusters. Therefore, assessing information regarding individual trees with *c*-tree sampling in clustered populations can be a biased approach since a selection of individual trees is not equal. It is not recommended to use c-tree sampling to assessing information regarding individual trees, unless in forests where trees exhibiting regular or random spatial pattern. The use of *c*-tree sampling should not be preferred in cases where precision is demanded and in cases where biased estimates may lead to a heavy damage of managed resources. For example, it is clear that the use of a biased method is not applicable to inventorying highly valued forest resources designed to forest policy and management decisions on regional or national levels.

The c-tree sampling approach has a remarkable potential use in forest regeneration

surveys where it can be worthwhile to obtaining measurements of distances between sampling points and their second nearest trees. That could be the most practical approach to testing whether trees in forests are distributed at random since, in such a case, the Mean of Angles method may require a slightly larger sample. Moreover, such an approach can serve to distinguishing between clustered and regular populations. That can be used to choose an appropriate density estimator. Applying the GM estimator to assess density of trees when they exhibit regular spatial pattern, the Pollard estimator in random and the (c-1) estimator in clustered populations is likely to reduce a bias to an acceptable value and to fulfill required precision of most forest regeneration surveys. In general, a bias would not increase with the increase in the number of measurements (sample size) and thus increasing the sample size would increase a confidence. It is feasible to derive new estimators being statistically more efficient and the bias could be further reduced. Applying systematical sampling designs can additionally add to a practical applicability of *c*-tree sampling and serve in stratifying forest stand areas by a relative density of juvenile trees, such as to those abundant, having moderate number or no regeneration occurred. However, it should be bear in mind that a combined use of c-tree sampling with remotely sensed data, fixed-area plots and angles can further contribute to assessing a potential of juvenile trees to mature into an ecologically sound and an economically worthwhile forest.

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