

ECONOMIC GROWTH AND INTERNATIONAL TRADE

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November 1992

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Acknowledgements

I am deeply grateful to Professor Motoshige Itoh for his tremendous amount of support and Instruction for my research. I am also deeply thankful to Professor Masahiro Okuno– Fujiwara for his helpful instruction and comments. I must thank Professors Yoshitugu Kanemoto, Yoshiro Miwa, Michihiro Kandori, Katsuto Iwai, Kazuo Ueda and other faculty members of the Graduate School of Economics at the University of Tokyo for valuable guidance. Professor Grossman's instruction was also quite helpful to complete this thesis.

This thesis was completed with comments of many professors. I am thankful to Professors Ben Bernanke, Elhanan Helpman, Paolo Pesenti, Jean Tirole, Harald Uhlig, Doug Bernheim, Alan Blinder, Gene Grossman, Gilbert Metcalf, Avinash Dixit, Giuseppe Bertola, Scott Taylor, Jonathan Eaton, Motoshige Itoh, Masahiro Okuno–Fujiwara, Yoshitugu Kanemoto for valuable comments to complete this thesis. Chapter 1

Asset Bubbles and Endogenous Growth

(with Gene Grossman)

1. Introduction

Can the market price of an asset deviate from market fundamentals (i.e., the present discounted value of dividend payments) in a world populated by rational, far-sighted investors? Tirole (1982) has shown that it cannot, if the economy comprises a finite number of infinitely-lived traders, while Wallace (1980) and Tirole (1985) have shown that the same is true in a non-growing economy no matter how long are investors' trading horizons. But Tirole (1985) and Weil (1987) have established that "bubbles" sometimes can exist in the general equilibrium of a growing economy with overlapping generations.

A rational investor will only hold an asset priced differently than its fundamentals if she expects that the bubble component will yield at least a normal rate of return; i.e., that it will grow at least at the real rate of interest. But if bubbles grow at the rate of interest in every period, eventually their value will exceed the income of the young generations who must purchase these assets from the old, unless the income of these generations is growing at least as fast. Tirole (1985) investigated the conditions under which a Diamond (1965) economy with an expanding population would grow fast enough to allow for the existence of bubbles in asset prices.¹ He related the existence condition to the intertemporal efficiency of the general equilibrium without bubbles.² Of course, in the Diamond economy with a neoclassical production function and no technological progress, per capita incomes stagnate in the long run.

In this paper, we extend Tirole's (1985) results to include economies that grow in the long run at an endogenous rate. As is well known by now, long-run growth can be sustained in an economy in which real returns to whatever capital goods are being accumulated (physical, knowledge, or human) are bounded from below by a number that

[&]quot;Weil (1987) used a similar framework to study "stochastically-bursting bubbles."

 $^{^{2}\}mathrm{See}$ Tirole (1990) and Blanchard and Fischer (1989, ch.5) for excellent introductions to this literature.

exceeds the discount rate. In other words, there must be non-decreasing returns to accumulable factors in the long run. These non-decreasing returns may be inherent to the production technology [as in, e.g., Rebelo (1991) and Jones and Manuelli (1990)] or they may arise due to externalities generated in the process of capital accumulation (as in, e.g., Romer (1986, 1990) and Lucas (1988)]. We choose a simple specification that includes externalities from physical capital [following Romer (1986)] and investigate the existence conditions for bubbles and the effects that bubbles have on the growth rate of the economy and on the welfare of the various generations of agents.³ We find that the conditions under which bubbles can exist are similar to those identified by Tirole (1985), but that bubbles are not so benign in this setting as they are in the Diamond economy with an exogenous growth rate.

2. A Diamond-Romer economy without bubbles

As in Diamond (1965), agents live for two periods. They work, consume, and save when they are young, and enjoy the fruits of their savings when they are old. Each period a new generation of young is born. The young are endowed with a fixed amount of potential working time, which they supply inelastically in the labor market. They use their labor income to buy output for consumption and investment purposes and to purchase the existing capital stock from the old. We assume for now that capital goods are the only store of value. For simplicity, we assume that the economy's population is constant through time and equal to 2L.

A representative member of the generation born at time t consumes c_{yt} units of the homogeneous final good when young, and c_{ot+1} units of this good when old. She

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³ We choose this specification with capital externalities initially to bring out the similarities with the Tirole (1985) analysis of the Diamond economy. But the existence conditions for bubbles are similar in economies with other sources of endogenous long-run growth; see section 5.

chooses her consumption profile to maximize a utility function, $U(c_{yt}, c_{ot+1})$, subject to an intertemporal budget constraint. Letting r_{t+1} be the rate of return (or real interest rate) on savings invested at time t, the constraint can be written as

$$c_{yt} + \frac{c_{ot+1}}{1 + r_{t+1}} = I_t, \qquad (1)$$

where I_t is the individual's labor income earned at time t.

The consumer's optimization yields equality between the marginal rate of intertemporal substitution, U_1/U_2 , and one plus the interest rate, $1 + r_{t+1}$, as usual. This equation generates an implicit savings function, $s_t = s(I_t, r_{t+1})$. We assume henceforth that individual preferences represented by $U(\cdot)$ are homothetic. Then $s(\lambda I_t, r_{t+1}) = \lambda s(I_t, r_{t+1})$.

Firms hire the available labor force, L (half the population, namely the young generation), and the available aggregate capital stock, K_t , and produce the homogeneous output, Y_t . A firm i that rents K_t^i units of capital from the old generation that owns it and that employs L_t^i young workers generates net output (after accounting for capital depreciation) of

$$Y_t^i = F[K_t^i, A(K_t)L_t^i],$$

where $A(\cdot)$ represents labor productivity, A' > 0. Here we have incorporated a positive spillover from the size of the aggregate capital stock to the productivity of workers in individual firms, in the manner suggested by Arrow (1962) and formalized by Sheshinski (1967) and Romer (1986).⁴ We assume that $F(\cdot, \cdot)$ exhibits constant returns to scale

⁴ As we noted in the introduction, we are not wedded to this specification of the

and that firms behave competitively. In hiring capital, the individual firm ignores its tiny influence on the aggregate capital stock and thus on the productivity of its own workers. Thus, each firm hires capital up to the point where its (private) marginal product equals the rental rate, r_t , and it hires workers until their marginal product equals the wage rate. In view of the homogeneity of degree one of $F(\cdot, \cdot)$, this gives the following relationships at the aggregate level:

$$\mathbf{r}_{t} = \mathbf{F}_{1}(\mathbf{K}_{t}, \mathbf{A}_{t}\mathbf{L}) = \mathbf{f}(\mathbf{k}_{t})$$
⁽²⁾

$$w_{t} = F(K_{t}, A_{t}L) - K_{t}F_{1}(K_{t}, A_{t}L) = f(k_{t}) - k_{t}f'(k_{t}),$$
(3)

where $A_t=A(K_t),\;k_t\equiv K_t/A_tL$ (capital per unit of efficiency labor), and $f(k_t)\equiv F(K_t/A_tL,\;1).$ Combining (2) and (3) gives a relationship between equilibrium factor prices,

$$\mathbf{w}_{t} = \phi(\mathbf{r}_{t}). \tag{4}$$

Product market equilibrium obtains when aggregate investment equals aggregate savings, i.e., the sum of desired savings by the young and desired dissavings by the old. Since the old wish to dissave their entire holdings of capital, K_t , this implies $K_{t+1} - K_t = s(w_t A_t L, r_{t+1}) - K_t$, or

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technology. Alternative formulations that preserve long-run incentives for capital accumulation would serve equally well. For a general discussion of what is needed to sustain long-run growth in a model of capital accumulation, see Grossman and Helpman (1991, ch.2). Section 5 shows how the results of the present analysis extend to economies with alternative engines of growth.

$$K_{t+1} = s(w_t A_t L, r_{t+1}).$$
 (5)

Equations (3), (4), and (5) determine the dynamic evolution of the economy (factor prices and capital stock) from any initial stock of capital, K_0 .

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In order to ensure the existence of a steady state for this economy, we take a particular functional form for the capital externality, making it linear in the aggregate capital stock; i.e.,

$$A(K_t) = K_t/a.$$
 (6)

Without further loss of generality, we normalize the size of the population to two, so that L = 1. Then $k_t = a$ for all t, and (3) and (4) imply

$$r_t = \rho$$
 for all t, (3)

$$w_t = \phi(\rho)$$
 for all t, (4')

where $\rho \equiv f'(a)$. Now since $s(\cdot, \cdot)$ is homogeneous of degree one in its first argument, $s(w_t A_t L, r_{t+1}) = A_t Ls(w_t, r_{t+1})$. Then, after substituting (6), (3'), and (4'), equation (5) becomes

$$\mathbf{K}_{t+1} = \mathbf{K}_t \mathbf{s}[\phi(\rho), \rho]/\mathbf{a}.$$

The capital stock grows at the rate

 $g_t = \bar{s}/a - 1, \tag{7}$

where $\bar{s} \equiv s[\phi(\rho), \rho]$. By (6), labor productivity $A(\cdot)$ grows at this same rate and since $F(\cdot, \cdot)$ has constant returns to scale, so does per capita income.

Before leaving this section, we note that the dynamic equilibrium without bubbles is not Pareto efficient. For suppose that at time t the old were to consume as in the above equilibrium while the young saved an additional amount ds. This would increase the capital stock at t+1 by ds and would generate additional output of $(dY_{t+1}/dK_{t+1})ds = (F_1 + F_2/a)ds > r_{t+1}ds$. If the entirety of this extra output in period t+1 were given to the (then) old, then the utility of this generation would rise (since it has set its marginal rate of intertemporal substitution equal to $1+r_{t+1}$, the extra output in the second period of life yields more utility than the loss from the consumption foregone in the first period) while no generation would lose. Of course, the inefficiency of the market equilibrium reflects the fact that (small) individual agents have no incentive to incorporate the spillover effect from capital in their private investment decisions.

3. Existence of Asset Bubbles

We now assume that the generation that is old at time 0 possesses M paper assets that are intrinsically worthless. That is, the assets produce no real output and therefore generate no dividends. The old attempt to sell these assets to the young at a positive price p_0 (in terms of goods) for each piece of paper. Would a rational, foresighted, young investor be willing to purchase one of these assets? Only if she believed that she could resell the asset when old (i.e., in period 1) to a member of the next young generation for a price that includes a real rate of return comparable to that available on other assets. The real (gross) rate of return on alternative assets is $1+r_1$ units of output in period 1. Therefore, the young investor in period 0 is willing to buy the intrinsically useless asset if she expects its price in period 1 to be at least $p_1 = (1+r_1)p_0$. Similarly,

the young generation in any period t must expect the price of the paper to be $p_t(1+r_{t+1})$ in period t+1, if it is to acquire the asset from the old generation at that time at a price p_t . If all of these expectations for capital gains on the asset can be fulfilled, then the intrinsically useless paper can be traded indefinitely; that is, there can exist a <u>bubble</u>.

Let $B_t = p_t M$ be the aggregate value of the bubble at time t, and assume for the moment that the self-fulfilling prophecy can be realized. By the condition of no-arbitrage between bubbles and other assets, we have

$$B_{t+1} = (1 + r_{t+1})B_t.$$
(8)

We define $b_t \equiv B_t/A_tL$ as the aggregate value of the bubble per efficiency unit of labor.

The young generation must purchase the entirety of existing bubbles from the old generation in each period. The condition for goods market equilibrium becomes

$$K_{t+1} - K_t = A_t Ls(w_t, r_{t+1}) - (B_t + K_t);$$
(9)

the left-hand side is net investment, while the right-hand side is the difference between savings by the young and dissavings by the old (the term in parentheses on the far right of [9]). Note that (3') and (4') continue to describe factor prices when $A_t = K_t/a$ and L = 1. Substituting these expressions into (9), we derive $K_{t+1} = A_t(\bar{s} - b_t)$, or $K_{t+1}/K_t = (\bar{s} - b_t)/a$. Thus,

$$g_t = \frac{\overline{s} - b_t}{a} - 1. \tag{10}$$

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We now discuss whether, given an initial bubble of size B_0 such that $b_0 = B_0/A(K_0)$, the dynamics described by (8) and (10) are sustainable. If they are, then an initial bubble of size B_0 can exist in an economy that has an initial capital stock of K_0 . Note that labor productivity grows at rate g_* , so (8) and (3') imply

$$b_{t+1} = \frac{1 + \rho}{1 + g_{t}} b_{t}.$$
 (11)

Substituting (10) into (11) gives a single, recursive equation for the evolution of the value of the bubble per efficiency unit of labor.

$$b_{t+1} = a(1 + \rho) \frac{b_t}{\bar{s} - b_t}.$$
 (12)

The curve labelled BB in the top part of figure 1 depicts this relationship between the (normalized) size of the bubble in successive periods. Clearly, when this curve lies above the 45 degree line, the bubble is growing relative to the stock of efficiency labor (and, therefore, aggregate output), whereas when the curve lies below the 45 degree line, the bubble is shrinking relative to efficiency labor.

We see from the figure that, if the initial size of the bubble is such that $b_0 < b^*$, the normalized bubble shrinks monotonically over time. In this case, the assumed existence of the initial bubble does not lead to any contradiction. Asymptotically, the bubble becomes arbitrarily small in relation to the stock of efficiency labor, and the economy converges to the steady state described in section 2. If, alternatively, the initial size of the bubble is such that $b_0 > b^*$, then the normalized bubble grows monotonically over time. Eventually, at some T, $b_T > \overline{s}$. But then the aggregate savings of the young do not suffice to allow them to acquire the bubble from the old generation at the required price. The old at T would have foreseen this eventuality at T-1, and so would not have purchased the bubble from the (then) old generation at that time. The bubble unravels back to time 0; i.e., an initial bubble of the assumed size cannot be sustained. Finally, it is possible that $b_0 = b^*$. Then the bubble and the economy remain in fixed proportion to one another and the economy immediately enters a steady state.

Evidently, bubbles can exist in this economy provided their initial size is no larger than b^{*}. Notice the similarity between the existence condition for bubbles here and that described by Tirole (1985) for the Diamond economy. No bubble of any size will be possible if the BB curve is steeper at the origin than the 45 degree line. The slope of the BB curve at the origin equals $a(1+\rho)/\bar{s}$. Thus bubbles can exist if an only if $\bar{s}/a - 1 > \rho$. But $\bar{s}/a - 1$ is the economy's growth rate in the absence of any bubbles, while ρ is the real interest rate, so here, as in Tirole, the existence of bubbles requires that the growth rate exceeds the interest rate in the equilibrium of the bubbles economy.

Of course, in the model with exogenous growth, the bubble cannot affect the longrun growth rate. Here, that is not true. The bottom portion of the figure illustrates the effect that bubbles have on the growth rate. The line GG, depicting equation (10), shows the relationship between the (normalized) size of the bubble at time t and the growth rate. If the initial bubble happens to be of size B* such that $B^*/A(K_0)L = b^*$, then the bubble permanently lowers the rate of growth from $g_t = \bar{s}/a - 1$ to $g_t = \rho$. (Smaller initial bubbles reduce the rate of growth in every period, but the depression of the growth rate asymptotically approaches zero as t grows large.) The reason is straightforward. The existence of the asset bubble diverts savings away from productive investment in capital, and it is capital investment that drives long-run growth in the Diamond-Romer model.

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4. Bubbles and Intertemporal Efficiency

Tirole (1985, 1990) has shown for the Diamond economy that the asymptotically bubbly equilibrium (in which the bubble remains in fixed proportion to the size of the economy in the long run) is intertemporally efficient. The bubble eliminates the overaccumulation of capital that can arise in the Diamond model because living generations cannot trade with the as yet unborn. In fact, the bubbles can exist in that setting if and only if the bubbleless equilibrium is inefficient.

One might expect a different result in our model of externality-based endogenous growth. As we have seen at the end of section 2, the economy without bubbles has less capital formation than is required for intertemporal efficiency. Bubbles divert savings from productive use in financing capital accumulation into an unproductive store of value and thereby exacerbate the existing distortion in the market equilibrium.

Suppose that a bubble of size B_0 first appears in the economy at time 0. What are the welfare implications? The generation that is old at time 0 benefits of course, as their sale of the new asset to the young enables them to consume more than otherwise. The labor income of the generation that is young at time 0 depends upon K_0 and A_0 , both of which are predetermined at time 0. Their savings accrue interest at rate $r_{t+1} = \rho$ with or without the bubble. So this generation, which has (indirect) utility given by $V[\phi(\rho)A_0, \rho]$, is not affected by the bubble. All subsequent generations are harmed, however, as growth of labor productivity is reduced by the bubble and so each generation born after time 0 earns less labor income than it would have otherwise.

It is interesting to note that the generation that is born at time 1, by itself, suffers income losses from the bubble that are sufficiently large that this generation could more than compensate the initial old for their gain from the bubble, if only there were a way to effect this intergenerational transfer. The bubble allows the initial old to increase their consumption in period 0 by B_0 . The slowdown in productivity growth causes the

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generation that is young at time 1 to lose wage income in period 1 of $(\dot{A}_1 - A_1)\phi(\rho) = A_0(\hat{g}_0 - g_0)\phi(\rho)$, where a circumflex indicates a variable in the equilibrium without bubbles. Noting the growth rates recorded in (7) and (10), we find $\hat{g}_0 - g_0 = b_0/a$. Thus, the income loss at time 1 equals $B_0\phi(\rho)/a$. Since this comes one period later than the gain to the initial old, the value of this income loss discounted back to time 0 is $B_0\phi(\rho)/a(1+\rho)$, where we have used the market interest rate (which equals also the intertemporal rate of substitution for every generation) to perform the discounting. Now we calculate the difference between the discounted loss to the young at time 1 and the gain to the initial old as

$$\frac{B_0 \phi(\rho)}{a(1+\rho)} - B_0 = \frac{B_0}{a(1+\rho)} [\phi(\rho) - a(1+\rho)]. \quad (13)$$

The existence condition for a sustainable bubble requires $\bar{s}/a - 1 > \rho$, or $\bar{s} > a(1 + \rho)$. Since the wealth constraint requires $w = \phi(\rho) > \bar{s}$, the term in square brackets on the far right-hand side of (13) is positive. This establishes our claim.⁵

But the fact that the young born at time 1 could bribe the old at time 0 to "retire"

$$\frac{A_0\phi(\rho)}{a^n(1+\rho)^n}\left[\bar{s}^n-\prod_{i=0}^{1=n}(\bar{s}-b_i)\right].$$

Since $\phi(\rho)>\bar{s}>a(1+\rho),$ the difference between the loss to the generation born at time n and the gain to the old at time 0 exceeds

$$\frac{A_0}{\bar{s}^{\,n}}\,(\bar{s}-b_0) \bigg[\bar{s}^{\,n}-\prod_{i\,=\,1}^{i\,=\,n}\,(\bar{s}-b_i^{\,})\bigg] \ > \ 0.$$

⁵In fact, it can be shown that <u>any</u> generation born after the period in which the bubble forms suffers a greater loss than the gain to the initial old generation (even after allowing for discounting). The loss to the generation born at time n, discounted to time zero, is

the bubble asset does not mean that the bubble can be made to disappear. The problem is that these two generations do not trade with one another. The transfer from the young at time 1 to the old at time 0 must be effected through the generation that is young at time 0. When this generation pays the bribe to the initial old, it will want to collect $(1 + \rho)$ times the amount that it has laid out from the young at time 1. In the interim, it has an "IOU" that is exactly like the bubble asset.⁶ The young at time 0 divert savings from capital formation in order to pay the bribe, with the result that the potential gain from the intergenerational transfer scheme disappears. The harm caused by the bubble to generations born after time 0 cannot be avoided by a simple tax/subsidy scheme that redistributes income across generations.⁷

5. Extensions

We have derived our results for a simple economy in which externalities from capital formation sustain long-run growth. In this section, we show that our results apply also to economies with other sources of sustained growth, such as endogenous innovation or human capital accumulation. Then we discuss the existence conditions for bubbles in economies with various types of rents.

5.1 Alternative Engines of Growth

We have used a simple model of growth based on capital externalities in order to highlight the similarities to the Tirole (1985) analysis. However, our results apply to a

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⁴This IOU is like a national debt. O'Connell and Zeldes (1986) and Tirole (1990) have discussed the analogy between asset bubbles and public debt in overlapping generations models. The analogy remains apt in the present context. See Alogoskouffs and van der Ploeg (1990) for an analysis of the effects of public debt in an externalities—based model of endogenous growth.

 $^{^7}$ Welfare of all generations can be improved, however, by a policy that stimulates investment and causes individuals to internalize the externality associated with capital formation.

much broader class of economies with endogenous growth. Consider for example an economy like that described in Romer (1987), where endogenous innovation sustains growth. Suppose that final output is produced by firm i from labor and differentiated intermediate goods according to the production function

$$\mathbf{Y}_{t}^{i} = (\mathbf{L}_{t}^{i})^{\alpha} \left[\boldsymbol{\Sigma}_{j=1}^{n} \left(\mathbf{x}_{jt}^{i} \right)^{1-\alpha} \right], \tag{14}$$

where L_t^i again is the labor input of firm i at time t (with an aggregate labor supply normalized to one) and x_{jt}^i is its input of variety j of the intermediate input. Only inputs numbered 1 through n_t can be used at time t, because only these inputs have previously been developed in the research lab. A research firm active in period t can invent a new variety of intermediate input by using a' units of final output.⁸ Having done so, the owners of the research firm acquire an infinitely lived, transferable patent that grants its holders the unique right to manufacture the new product. The production of one unit of any known intermediate good requires one unit of final output.

In this setting, the savings of the young are used (in the absence of bubbles) to acquire the existing blueprints and associated patents from the old generation and to finance new inventions. Each blueprint sells for a' units of the final good. The return on an existing blueprint consists of the monopoly rent that can be earned by a firm that manufactures the unique variety of intermediate good and sells its output to producers of final goods. As is well known, the producers of the intermediates face a constant elasticity of demand and maximize profits by pricing at a fixed mark-up over their (unit) marginal cost. Thus, the price of intermediate j is $p_{it} = 1/(1-\alpha)$, and the firm

 $^{^8}$ Or, equivalently, the research process uses a technology similar to that described in (14), so that the inputs that can produce one unit of final output can also produce $1/a^\prime$ new blueprints.

that produces this good earns a profit of $\pi_{jt} = x_{jt} \alpha/(1-\alpha)$, where x_{jt} is its total sales at time t.

It is easy to verify that sales per variety are constant over time, and thus so are the profits of any intermediate good producer.⁹ The cost of a blueprint also is constant, so the rate of return on an investment in a blueprint is constant; let this constant rate of return be ρ' . Savings of the young depend on labor income and ρ' . Using the properties of the Cobb-Douglas production function, we calculate the wage rate to be

$$\mathbf{w}_{t} = \alpha \mathbf{n}_{t} \bar{\mathbf{x}}^{1-\alpha} \equiv \phi(\bar{\mathbf{x}}) \mathbf{n}_{t} ,$$

where $\bar{\mathbf{x}}$ is the (constant) output of a typical intermediate good. Then the homotheticity of preferences implies that aggregate savings of the young are equal to $\mathbf{n}_t \bar{\mathbf{s}}'$, where $\bar{\mathbf{s}}' \equiv \mathbf{s}[\phi(\bar{\mathbf{x}}), \rho']$. Equating these savings to the cost of the new inventions plus the cost of the blueprints purchased from the old generation, we have

$$n_t \bar{s}' = a' n_t + a' (n_{t+1} - n_t),$$
 (15)

or (after dividing by n_t and rearranging terms)

$$g_{+} = \bar{s}'/a' - 1.$$
 (16)

⁹ Final good producers spend a fraction $(1-\alpha)$ of their total costs on intermediate goods. Since the final goods industry is competitive, costs equal total revenues. Therefore, $n_t x_t^{-1}/(1-\alpha) = (1-\alpha)Y_t$, where x_t is the output of a typical intermediate good at time t and Y_t is aggregate output of the final good. From the production function (14) we have $Y_t = n_t x_t^{1-\alpha}$ (recall that $L_t = 1$). Equating the two expressions for Y_t shows that x_t is constant through time. Here g_t is the rate of growth in the number of blueprints at time t. It is also the growth rate of the economy (in the absence of bubbles), since final output is proportional to the number of blueprints; i.e., $Y_t = n_t \bar{x}^{1-\alpha}$.

Notice the similarity between (16) and (7). The existence conditions for bubbles and the effects of any such bubbles on the two economies with different sources of growth also is similar. Equation (8) describes the evolution of any bubble. In place of (9) we have

$$a'(n_{t+1} - n_t) = n_t s[\phi(\bar{x}), \rho'] - (B_t + a'n_t).$$
(17)

From this we can derive

$$g_{t} = \frac{s' - b'_{t}}{a'} - 1, \qquad (18)$$

where $\mathbf{b}_t' \equiv \mathbf{B}_t/\mathbf{n}_t$. Compare this with (10) above. Bubbles slow the pace of growth in the economy with innovations just as they do in the economy with capital externalities. Any bubble eventually becomes small in relation to the size of the economy, unless the ratio of the size of the initial bubble to the initial number of differentiated inputs happens to take on a certain value. Initial bubbles larger than this critical value cannot exist, because their continued growth would eventually require that the young turn over more than 100 percent of their labor income to the old to purchase the assets, which of course is impossible.

Bubbles will retard growth in a wide class of economies besides the two examples considered here. In all economies with sustained long-run growth, the returns to whatever is being accumulated (e.g., physical, human, or knowledge capital) must eventually become constant. Then the level of savings and the productivity of investment together determine the pace of economic expansion. A bubble that diverts savings away from investments in productive assets is bound to slow growth.

Our welfare results of section 4 also extend to a wider class of economies. Consider, for example, an initial bubble of size B'_0 in the economy with growth driven by innovation. The bubble causes the generation born at time 1 to lose wage income in period 1 equal to $(\hat{n}_1 - n_1)\phi(\bar{x})$, where \hat{n}_1 is the number of intermediate goods available in period 1 in the economy with the bubble, and n_1 is the number that would have been available without the bubble. The income loss can be written as $B'_0\phi(\bar{x})/a'$, since $\hat{g}_0 - g_0 = b'_0/a'$ from (16) and (18), and $b'_0 = B'_0/n_0$ by definition. Now it is apparent that this loss in income for the generation born in period 1, discounted to time 0 using the market interest rate ρ' , must exceed the gain to the initial old generation from the formation of the bubble.¹⁰ The argument relies, as before, on the fact that wage income exceeds savings and that the bubble can exist only if the growth rate absent the bubble exceeds the interest rate (i.e., $g_0 > \rho'$). More generally, the same welfare argument can be made for any overlapping generations economy where there are constant returns to an accumulated asset and the labor share in national income is constant.

5.2 Economies with Rents

Until now, we have considered economies with only two types of assets: one type is is productive and accumulable while the other is nonproductive and nonaccumulable. A third type of asset contributes to the production of output but cannot be accumulated and so cannot serve as an engine of growth. Such assets — of which land, natural

¹⁰ It can also be shown that all future generations, not just the one born in period 1, suffer income losses that, when discounted back to period 0, exceed the initial value of the bubble.

resources, and paintings by old masters are examples — generate real *rents*. We now consider how the presence of such assets affects the conditions for the existence of bubbles in economies with endogenous growth.

We follow Tirole (1985) in distinguishing two cases. In the first case, the total amount of rents in the economy is fixed in units of the final good. In this case, endogenous growth makes the rents decreasingly important in relation to the size of the economy. The second case we consider is one in which rents expand to keep pace with the growth of output.

Consider once again a Diamond-Romer economy with capital externalities, but introduce now an asset that generates a fixed quantity of rents per unit time. In particular, let the aggregate production function be

$$Y_{t} = F[K_{t}, A(K_{t})L] + qT,$$
 (19)

where $A(K_t) = K_t/a$ and T is the fixed stock of an asset that produces q units of output per period. Let V_t be the market price of a unit of the asset T at time t. All units of T, like the bubble, are owned by the old generation at the beginning of any period. This generation collects the rents generated by the asset during the period and then sells the asset to the young generation after production has taken place. Then the accumulation equation (9) must be replaced by

$$K_{t+1} - K_t = A_t Ls(w_t, r_{t+1}) - (B_t + K_t + V_t T).$$
(20)

Arbitrage ensures that the rent-generating asset yields a normal rate of return, or that

$$\mathbf{r}_{t+1}\mathbf{V}_t = \mathbf{V}_{t+1} - \mathbf{V}_t + \mathbf{q}.$$
 (21)

It is fairly clear that the existence of such a rent-generating asset does not change our main arguments. The capital externality fixes the marginal product of capital at ρ ; so (3') still applies. Then (4') gives the wage rate. If the rent-generating asset is priced at its fundamental value, then $V_t = q/\rho$ for all t. We can define $v_t \equiv V_t/A_t$, so that $v_{t+1}/v_t = A_t/A_{t+1} = 1/(1 + g_t)$. Clearly, v_t goes to zero if capital accumulation is sustained. From (20) we derive the growth rate at time t,

$$g_t = \frac{\bar{s} - b_t - v_t}{a} - 1.$$
 (22)

Then substituting this expression into (11) gives

$$b_{t+1} = a(1 + \rho) \frac{b_t}{\bar{s} - b_t - v_t}$$
 (23)

while

$$\mathbf{v}_{t+1} = \mathbf{a} \frac{\mathbf{v}_t}{\bar{\mathbf{s}} - \mathbf{b}_t - \mathbf{v}_t}.$$
(24)

From these dynamics for v_t and b_t we can conclude the following. First, a bubble that is not too large initially can exist here, just as in section 3. For almost all values of b_0 the bubble eventually becomes small in relation to the size of the economy. But now there may be an initial stage during which the bubble grows faster than output (i.e., a range of t such that $b_{t+1} > b_t$). As before, there does exist a unique value for b_0 (that

depends on v_0 and therefore on q) such that the bubble comes to absorb a fixed share of savings in the long run. If the initial bubble is of this size, it grows faster than the economy at all t, but asymptotes to the same steady-state b^* as in the economy without rents.

Now consider the second case where total rents grow as the economy does. This would happen, for example, if the aggregate production function took the form,

$$Y_{t} = F[K_{t}, A(K_{t})G(L,T)]$$
 (25)

for $A(K_t) = K_t/a$ and $G(\cdot)$ homogeneous of degree one. Here, the capital externality serves to raise not only the productivity of labor, but also the productivity of the rentgenerating asset.¹¹ For concreteness, let us call this asset "land," and let its market value once again be denoted by V_t . It is easy to show that (normalized) factor prices again will be constant in this case, with rents of (say) \bar{q} units of final output per unit of effective land. Thus, total rents, $\bar{q}A_tT$, grow at same rate as final output; both grow at the rate g_t given in equation (22).

We will now show that no bubble can exist in this economy. We do so by establishing a contradiction after first assuming that a bubble does exist. In the event, the size of the bubble must evolve according to equation (23), while from (20), (3'), and (22) we derive a similar equation for the normalized value of a unit of land, namely

¹¹ Tirole (1985) considers an economy where new rent-generating assets are created in every period. There, claims to future rents cannot be sold prior to the appearance of the rents in the economy. Tirole shows that bubbles can exist in such an economy where prices of existing assets do not capitalize the value of future rents. In our case, land prices at time t do capitalize the value of all rents that will be generated subsequently. Were we to assume the opposite, we would find like Tirole that bubbles can exist in an economy where rents grow at the same rate as final output.

$$v_{t+1} = a(1 + \rho) \frac{v_t}{\bar{s} - b_t - v_t} - \bar{q}.$$
 (26)

We know from our previous discussions that b_t cannot grow without bound. Suppose b_t were to approach a constant $\bar{b} > 0$ as $t \to \infty$. Then, from (23) we would have $\bar{s} - \bar{b} - v_t = a(1 + \rho)$ in the long run, so that (26) would imply $v_{t+1} = v_t - \bar{q}$. But this is impossible, because the value of land cannot become negative. The remaining possibility is that $b_t \to 0$ as $t \to \infty$. The normalized value of land must approach some constant, for if it were always increasing then land purchases eventually would absorb more than 100% of available savings and if it were always decreasing it would eventually become negative. Let \bar{v} denote the long-run (normalized) value of land. Note that we can solve for this land value by setting $v_{t+1} = v_t$ and $b_t = 0$ in (26).

We have identified a steady state, which is the only possible long-run equilibrium for the growing economy. Our final task is to show that this steady state cannot be approached along a trajectory with $b_t > 0$ for some t. We note that, in the neighborhood of the steady state, $v_{t+1}/v_t \approx 1$. But notice from (23) and (26) that $b_{t+1}/b_t > v_{t+1}/v_t$ for all t. This means that, as the normalized value of land approaches \bar{v} , the bubble must be growing faster than output. This is impossible, because the size of the normalized bubble must approach zero in the long run. It follows that the unique equilibrium trajectory has $b_t = 0$ and $v_t = \bar{v}$ for all t.

6. Conclusions

In settings where long-run growth is driven by investments in physical, human, or knowledge capital, the existence of an unproductive asset — one that yields a financial return but does not contribute to the production of real output —can be harmful to growth. The unproductive asset, or bubble, attracts savings away from more productive

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uses. Each new generation purchases the asset at least partly at the expense of investment in growth-promoting capital.

In this paper, we have examined the conditions under which asset bubbles can exist in economies with endogenous growth. As in the neoclassical growth setting, a bubble can survive only if the equilibrium growth rate exceeds the interest rate in the bubbleless economy. Here, however, the equilibrium growth rate, like the interest rate, is determined by parameters of tastes and technology. Bubbles are more likely to be possible when households are patient (i.e., savings propensities are high for a given interest rate) and when investments in accumulable assets are very productive. The existence in the economy of assets that produce rents does not rule out the formation of bubbles, unless the aggregate rents grow at the same rate as the economy and the price of the rent-generating assets in any period fully capitalizes the value of all subsequent dividends. When bubbles do exist, they retard economic growth along the transition path to the steady state and possibly even in the long run. The bubbles also harm all generations born after the period in which the asset first appears, and to an extent that exceeds the gain to the generation that benefits from the bubble.

In our models, bubbles can exist only on nonaccumulable useless assets. That is, there cannot be any bubble in the price of capital or blueprints. This is because new units of an asset must have the same price as old, and it is always possible to create new units of capital or new blueprints at a constant cost in terms of final output.¹² Thus, competition from potential new supply prevents exponential growth in the price of the accumulable assets.

¹² This argument assumes that different units of capital (or different blueprints), which are economically indistinguishable in our model, also are physically indistinguishable. Otherwise, there could be bubbles in the prices of specific units of capital. One might say, in such a case, that the capital is priced at its fundamental value, but that the "names" of specific pieces of equipment (which are intrinsically useless assets) acquire value as bubbles.

This raises the fundamental question about asset bubbles: what determines their supply? Every individual is willing to exchange a worthless piece of paper for a positive amount of goods. If asset bubbles do appear in the economy, is there any way to predict ahead of time how many and which ones? Is there any way to prevent their formation in situations where their existence will retard growth? These questions remain to be answered.

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Chapter 2

Increasing the Capital Income Tax Leads to Faster Growth

(with Harald Uhlig)

1 Introduction

Most economists, when asked about their deep beliefs, would probably agree that a lower capital income tax is desirable, at least for efficiency reasons, see e.g. Lucas (1990). The usual argument runs, that a lower capital income tax increases the private return to capital, thus encouraging investment and growth¹.

However, this is not necessarily so. The argument above implicitely assumes that the capital income tax can be lowered costlessly. The government, however, is faced with tradeoffs: lower capital income taxation means either lower government expenditures or higher debt financing or higher labor income taxes². Keep the level of government expenditure and debt financing fixed for the sake of the argument. If we think of labor income being paid mostly to the young and capital income accruing mostly to the old, a lower capital income tax and thus a higher labor income tax means that the younger people in an economy are left with less income out of which to save and to buy the capital stock. If savings decisions are not too elastic with respect to long term interest rates, this will lead to lower savings and thereby to slower growth rather than faster growth. The issue becomes clearer when thinking about lump-sum taxes instead: if a given amount of revenue has to be raised, taxing the old rather than the young will lead to faster growth, since agents compensate for the tax shift through higher savings. With proportional taxes, the question simply is whether the adverse effect on savings

 $^2 This argument in turn, of course, ignores Laffer curve type effects, see Sargent (1987). The experience with Reagonomics indicates, however, that this may not be a worrisome mistake.$

¹There are also two arguments in favor of capital income taxation. The first stresses its progressivity and the tradeoff between some kind of "fair" income distribution and efficiency. The second argues, that it may be sensible to highly tax capital aiready in place, since it as a fixed factor, but tax capital little or not at all in the more distant future, see Jones, Manuelli and Rossi (1991). Obviously, the issue of time consistency is not trivial here, see Chari, Kehoe and Precotot (1989)

through lower interest rates is enough to undo the growth effect of a taxburden shift towards the old. We argue that measured savings elasticities are indeed low enough for the described effect to take place. We therefore claim that higher capital income taxation means faster growth. Note, that the effect on welfare will be ambiguous in general, since the initially old will always prefer less to more capital income taxation and the generations in the far distant future will always prefer faster growth.

We demonstrate the claim in an overlapping generations model with endogenous growth and show its robustness to three possible objections. Related arguments have been brought forward by Feldstein (1978) in a twoperiod model, Auerbach (1979) in an overlapping generations economy with neoclassical growth and in particular Jones and Manuelli (1992), who also consider an overlapping generations economy with endogeneous growth. By using an externality-driven AK model³ here rather than a concave production function as in the tax analysis in Jones and Manuelli, the growth effects are immediate rather than asymptotic and transfers to the young are not necessary for sustainable growth. In addition to their paper we show that reversing our claim often requires negative savings, see section 4.3. In contrast to Auerbach (1979), the endogeneous growth structure here simplifies as well as amplifies the analysis: heuristically, the economy is always in the first period of the transition phase to a new steady state of a comparable neoclassical growth model. Thus, the effect on the capital stock is greater and no intricate transitional dynamics need to be considered.

The second section introduces the model. The third section demonstrates our claim for interest inelastic, logarithmic utilities. We then examine three possible objections against that claim in the forth section of the paper and argue that none of these objections is serious enough to undo our claim. The first objection concerns the potential effect of positively interest elastic savings. We argue, that savings decisions are sufficiently interest inelastic in

³The term AK model refers to equation (3).

the US economy for our effect to hold. The second objection raises the issue of a grandfather clause for initial capital. We show, that even then our claim holds, as long as the labor income tax is lowered only in those periods in which additional revenue from higher capital income taxes is created. The third objection asks whether possible labor income of the old could undo our result. We show, that the parameter ranges which reverse our claim are either extreme or fragile. We thus conclude in the fifth and final section, that a higher capital income tax leads indeed to faster growth.

2 The Model

Growth is about the long term and thereby necessarily about the tradeoff between generations⁴. To illustrate our argument, we will consider a fairly standard, deterministic overlapping generations model with endogenous growth, where the productivity-augmented labor input contains an externality term which relates to aggregate capital.

Specifically, agents in this economy live two periods. We assume that there is no population growth and that there is one representative agent per generation. When young, the agent is endowed with $0 < \lambda \leq 1$ units of time and when old, his or her time endowment is $1 - \lambda$. There is one consumption good per period and an agent born in period t is assumed to enjoy consumption according to the utility function

$u(c_{y,t}; c_{o,t+1}),$

where $c_{y,t} \ge 0$ is the consumption when young and $c_{o,t+1} \ge 0$ is the consumption when old. We assume that u is homothetic and satisfies the usual list of conditions. In particular then, there is a continuously differentiable

⁴Unless, of course, one believes in infinitely lived dynasties linked by bequests, see Barro (1974), Kotlikoff and Summers (1979), Abel and Bernheim (1991) and the related literature.

consumption rule C(R) for R > 0, so that the utility function above, subject to the constraint

$$c_{\mathbf{v},t}+\frac{c_{o,t+1}}{R}\leq W,$$

is uniquely maximized at consumption

$$c_{y,t} = C(R)W$$

for any value of the endowment W > 0 in terms of consumption at date t and any (after-tax) interest factor R = 1 + r > 0 (r is the after-tax interest rate). It is then easy to calculate savings as

$$S_{t} = S(R; \frac{W_{y}}{W})W = (1 - C(R))W_{y} - C(R)W_{o}/R,$$
(1)

where W_y is the value of the time endowment in consumption goods when young, W_o is the value of the time endowment in consumption goods when old and $W = W_y + W_o/R$ is the total endowment in terms of present consumption. The agents supply their time endowment inelastically as labor, so that the total labor supply per period is unity⁵. Below, it will turn out, that wages when young per unit of time are given by $w(K_t/\alpha)$, growing at some rate g per period. We can then use the formulas above with $W_y = \lambda w(K_t/\alpha)$ and $W_o = g(1 - \lambda)w(K_t/\alpha)$.

There are many competing firms in this economy. The production function for the individual firm i is given by

$$y_{i,t} = k_{i,t}^{\rho} (n_{i,t} \frac{K_t}{\alpha})^{1-\rho},$$
 (2)

where $k_{i,t}$ is the firm-specific capital, $n_{i,t}$ is the labor hired by that firm and $K_t = \sum k_{i,t}$ is the aggregate capital stock. The capital share is given by $0 < \rho < 1$ and the size of the externality effect is given by $\alpha > 0$: labor

⁵We could assume a preference for leisure as well in the utility function. This would only strengthen our argument, since a lower labor income tax will mean less distortion in the labor market on top of simply leaving more income after taxes.
input is augmented by the factor K_t/α , which generates externalities of the kind often used in theories of endogenous growth, see e.g. Romer (1986) or Grossman and Helpman (1991). Since all firms will have the same capitallabor ratio in equilibrium, dividends accruing to the holders of all capital in firm i are given by $dk_{i,t} = \rho y_{i,t}$, whereas labor income paid to $n_{i,t}$ will equal $w_{n,t} = (1 - \rho)y_{i,t}$. Aggregating, we find that total production is given by

$$Y_t = aK_t, \tag{3}$$

where $a = \alpha^{\rho-1}$: a high value for *a* means a large spillover effect and thus higher output⁶. Dividends *d* per unit of capital are given by

$$d_t = \rho \frac{Y_t}{K_t} = \rho a, \tag{4}$$

independently of t. Wages per unit of time are likewise given by $(1 - \rho)Y_t$, so that the wage rate w_t per efficiency unit

$$w_t = (1 - \rho)\alpha^{\rho}, \tag{5}$$

which is again independent of t. We will therefore omit the time index for w_t and d_t below.

We assume that capital depreciates at some rate $0 \le \delta \le 1$ and that output each period can be split into private consumption C_t , government consumption H_t and investment X_t to capital:

$$C_t + H_t + X_t = Y_t. \tag{6}$$

The capital stock thus evolves according to

$$K_{t+1} = (1 - \delta)K_t + X_t,$$
(7)

⁶Note that we normalised the aggregate labor supply N to equal unity. Without this normalisation, we would have $a = (N/\alpha)^{1-r}$ and all calculations below still go through with the proper accounting for distinguishing individual from aggregate variables. The important point is that the constant *a* still is the aggregate output to aggregate physical capital ratio. where we allow X_t to be negative for simplicity. The total value of a unit of old capital at the beginning of period t in terms of the present consumption good is now given by

$$v = d + (1 - \delta) = \rho a + 1 - \delta.$$
 (8)

Note that v is also the total return to a purchase of a unit of capital at t-1.

Finally, we introduce the government which has to finance a given stream of expenditures H_i . Rather than fixing the level of these expenditures beforehand irrespective of the growth rate, we assume that the government wants or needs to spend a certain fraction γ of total output each period⁷:

$$H_t = \gamma Y_t. \tag{9}$$

We allow there to be three sources of government revenue: capital income taxes, taxes on labor income and government debt.

Let $\tau_{K,0}$ be the capital income tax rate in the first period t = 0 and τ_K be the tax rate for all periods after that. The distinction between the first period and all other periods will be important later for discussing grandfather clauses. Let $\tau_{L,t}$ be the tax rate on labor income, which may depend on t. Below, we will restrict ourselves to equilibria, where we need to distinguish only between tax rates $\tau_{L,0}$ for t = 0 and $\tau_L \equiv \tau_{L,t}$ for all $t \ge 1$. Capital income taxes are to be paid on the full amount of capital income, including the resale value of the capital and not just the capital gains⁸ and we assume

⁷One may imagine in some richer model, that government expenditures are another factor in producing final services and that for certain specifications of such a production function, it is optimal to keep the ratio of government services and/or government capital to private capital constant. As an example, it certainly makes sense that a richer country would want to build a better road system than a poorer country. In any case, our assumption seems to fit well with actual government behaviour, based on casual empiricism.

⁸We assume limited liability throughout. That means, that capital owners cannot be forced to pay more taxes than their capital income and likewise, workers cannot be forced to pay more taxes than their labor income. This puts some mild restrictions on γ

that all savings are financed out of after-tax labor income. Thus, there usually will be double-taxation of savings. This is actually just a matter of accounting and notation⁹: it is irrelevant for the individual agent, whether his or her savings are taxed twice or simply once at the appropriate sum of the two rates and there are many ways of writing down equivalent tax systems. All that matters for the individual is the tradeoff between consuming when young and consuming when old. With linear tax schedules, this tradeoff is constant and can be characterized by a relative price between the two relevant consumption goods, independently of the level of consumption.

Thus, the relevant relative price of the consumption good when young in terms of the consumption good when old is the private total return on capital or the after-tax interest factor on savings. It is given by

$$R = (1 - \tau_K)v = (1 - \tau_K)(\rho a + 1 - \delta)$$
(10)

and independent of t. The after-tax interest rate per period is r = R - 1.

Finally, let b be the ratio of new one-period government debt to output, which we assume to be constant for all periods. Depending on the parameters, this means that either some part of the debt is serviced and some part of the debt rolled over each period or that some new debt is issued each period. We assume that the government is not initially indebted, so that the total amount bY_0 can be used in period 0 to finance government expenditures. Payments on government debt are tax-free: this just simplifies government budget accounting, since the government would pay as well as receive any such tax. The interest rate paid on the government debt has to equal the after-tax interest rate r = R - 1 on capital.

Let

$$g_t = K_t / K_{t-1} = Y_t / Y_{t-1}$$

⁹Furthermore, even though actual tax codes seem to avoid double taxation, they are unsuccessful in doing so, since in practice, taxable capital gains are often mostly nominal gains due to inflation. Thus, our notation may not be far from describing tax practice. be the growth rate from period t-1 to period t. The government budget constraint then requires that

$$\gamma Y_0 = \tau_{K,0} v K_0 + \tau_{L,0} w K_0 / \alpha + b Y_0, \tag{11}$$

in period t = 0 and

$$\gamma Y_t = \tau_K v K_t + \tau_{L,t} w K_t / \alpha + \left(1 - \frac{R}{g_t}\right) b Y_t \tag{12}$$

in all other periods $t \ge 1$. Dividing these constraints by the capital stock and solving for the labor income tax rates $\tau_{L,t}$ gives

$$\tau_{L,0} = \frac{\gamma}{1-\rho} - \frac{b}{1-\rho} - \frac{\rho a + 1 - \delta}{(1-\rho)a} \tau_{K,0},$$
(13)

and

$$\tau_{L,t} = \frac{\gamma}{1-\rho} - \left(1 - \frac{R}{g_t}\right) \frac{b}{1-\rho} - \frac{\rho a + 1 - \delta}{(1-\rho)a} \tau_K.$$
 (14)

These two equations express the labor income tax as a function of the chosen capital income tax rates $\tau_{K,0}$, τ_{K} , the debt-output ratio b and the growth rate g_i . These equations are the key to our argument: a raise in the capital income tax rate means a fall in the labor tax rate, since we keep the fraction of government expenditure γ unchanged.

Market clearing on the capital market requires

$$bY_t + K_{t+1} = S_t,$$

where S_t is aggregate savings from period t to period t+1. Replacing aggregate savings by the appropriate expression involving wages and the savings function, the capital market clearing condition divided by K_t can be rewritten as

$$ab + g_{t+1} = \frac{1 - \tau_{L,t}}{\alpha} w \left((1 - C(R))\lambda - C(R)\frac{g_{t+1}}{R} (1 - \lambda) \right).$$
(15)

Solving this equation for g_{t+1} and making use of $w/\alpha = a(1-\rho)$ yields

$$g_{t+1} = \frac{(1 - C(R))\lambda - \frac{1}{(1 - \tau_{L,t})(1 - \rho)}}{C(R)\frac{1 - \lambda}{R} + \frac{1}{a(1 - \tau_{L,t})(1 - \rho)}}.$$
(16)

Once, $\tau_{K,0}$, τ_K and b are chosen, the after-tax interest factor on savings is given by equation (10). Thus, $\tau_{L,t}$ is determined by equation (13) for t = 0 or by equation (14) and g_{t-1} for $t \ge 1$. Given $\tau_{L,t}$, the next growth rate g_{t+1} is calculated via equation (16). Thus, solving the model means recursively determining labor tax rates and growth rates via equations (14) and (16). In particular, if b = 0, it follows that $g_t \equiv g$ for all $t \ge 2$ and $\tau_{L,t} \equiv \tau_L$ for all $t \ge 1$. Alternatively, if $\tau_{K,0}$, τ_K and b are chosen such that $\tau_{L,0} = \tau_{L,1}$, we have $g_t \equiv g$ for all $t \ge 1$ and $\tau_{L,t} \equiv \tau_L$ for all $t \ge 0$, i.e. all periods. These are the cases on which we will concentrate.

3 Higher Capital Income Taxes Mean Faster Growth: The Benchmark Case.

Consider in particular the debtless benchmark case, where b = 0, $\tau_K = \tau_{K,0}$ and where only the young earn labor income, i.e. where $\lambda = 1$. In that case, equations (13) and (14) both state

$$\tau_L = \frac{\gamma}{1-\rho} - \frac{\rho a + 1 - \delta}{(1-\rho)a} \tau_K,\tag{17}$$

and equation (16) simplifies to

$$g = a(1-\rho)(1-\tau_L)S(R;1).$$
(18)

The argument brought forward in the introduction can now formally be seen in equations (17), (10) and (18): a higher capital income tax rate leads to a lower after-tax interest factor R and a lower labor income tax τ_L . If the decrease in the labor income tax overcompensates the possible decrease in the savings S(R; 1), then a higher growth rate results.

As an example, consider the case, where the utility function for consumption is given by

$$u(c_{y,t}; c_{o,t+1}) = \log(c_{y,t}) + \beta \log(c_{o,t+1}).$$
(19)

It is easy to see that the savings function S(R; 1) is constant:

$$S(R;1)=\frac{\beta}{1+\beta}.$$

In this case, the only effect of a higher capital income tax is to lower the labor tax rate τ_c , thereby unambiguously increasing the growth rate g according to (18). In fact, the growth-rate maximizing capital income tax rate in this environment is to tax away practically all income to capital and use it to subsidize rather than tax labor income.

Likewise, if the intertemporal elasticity of substitution is some constant $\sigma < 1$ (or, equivalently, the relative risk aversion is constant at $1/\sigma > 1$), resulting in the utility function

$$u(c_{y,t}; c_{\sigma,t+1}) = \frac{c_{y,t}^{1-1/\sigma} - 1}{1 - 1/\sigma} + \beta \frac{c_{\sigma,t+1}^{1-1/\sigma} - 1}{1 - 1/\sigma},$$
(20)

it is easy to see that the savings function is given by S(R;1) = x/(1+x), where $x = \beta^{\sigma}R^{\sigma-1}$. Now, S(R;1) is decreasing in R, so that an increase in capital income taxation leads to an increase in growth even without the labor-income tradeoff, and certainly in our model as well. Let us summarize the results of these examples in the following proposition.

Proposition 1 If the overall utility is characterized by a constant intertemporal elasticity of substitution of unity or lower, $\sigma \leq 1$, then a higher capital income tax rate will unambiguously result in a higher growth rate.

It is interesting to note, that Hall (1988) has measured the intertemporal elasticity of substitution and concluded that its "value may even be zero and is probably not above .2", giving empirical credibility to the proposition above¹⁰.

 $^{10} {\rm In}$ contrast to our result, Buiter (1991) finds $\sigma \leq 0.04$ as the necessary condition for a higher capital income tax to increase growth and concludes, that this bound is too low to be satisfied. The reason for the difference to our analysis is that he considers a very

4 Three Possible Objections.

At least three objections may be raised against the result above. The first concerns the effect of positively interest elastic savings: perhaps, the logutility case is not sufficiently robust and the effect may reverse for some reasonable intertemporal elasticity of substitution $\sigma > 1$, say. Secondly, the result may just come about, because we increase the tax rate on the capital stock owned by the initially old, a nondistortionary, but time-inconsistent tax. Is the result overturned with a grandfather clause for initial capital? Finally, the old too earn labor income in the "real world" but not in the benchmark case considered above - perhaps this will undo the argument.

We examine each of these objections below. We argue that none of them matters enough and that therefore a higher capital income tax means faster growth.

4.1 Objection 1: The Interest Elasticity of Savings is Positive

Consider again the benchmark no-debt case where $\tau_{K,0} = \tau_K$ and b = 0. In general, the direction of the marginal change in the growth rate due to a marginal change in the capital income tax at a particular equilibrium will depend on the interest factor elasticity of savings

$$\eta(R) = \frac{\partial S(R;1)}{\partial R} \frac{R}{S(R;1)}$$

different continuous-time overlapping generations model with exponentially distributed lifetime: in his model, the expected, remaining lifetime is constant, regardless of age. In short, we assume, that old people know, that they have to die soon, whereas Buiter assumes, that they do not. A reconciliation of the two models and further discussion is in Bertola (1992). at the after-tax interest factor of that equilibrium. E.g., for the constant intertemporal elasticity of substitution utility function used above, we have

$$\eta(R) = \frac{\sigma - 1}{1 + \beta^{\sigma} R^{\sigma - 1}}.$$
(21)

Thus, for $\sigma \leq 1$, the elasticity is zero or negative, leading to the unambiguous result stated in the previous section. If the elasticity is positive, however, the relative strength of each effect - decreased savings due to a lower after-tax return or increased savings due to higher income when young - matters. The following result obtains.

Proposition 2 A marginally higher capital income taz leads to a marginally higher growth rate across equilibria if and only if the interest elasticity of savings is not too big:

$$\eta(R) < \frac{R}{a(1-\rho)(1-\tau_L)}.$$
 (22)

Observe, that the ratio on the right hand side of (22) equals

$$\frac{I_K}{I_L} = \frac{(1-\tau_K)vK_t}{(1-\tau_L)w(K_t/\alpha)}$$

which is simply the ratio of after-tax capital income to after-tax labor income in period t.

Proof:

Substituting (14) and (10) into (18), it follows in a straightforward manner, that $\partial g/\partial \tau_K > 0$ holds if and only if

$$\frac{S(R;1)}{(1-\rho)a} - (1-\tau_L)\frac{\partial S(R;1)}{\partial R} > 0.$$

Rewriting this inequality yields the result. •

In order to assess whether or not the claimed effect is relevant for actual economies, the theory has to become quantitative. For the purpose here, it should be enough to simply choose some rough numbers describing, say, the US economy to assess the importance of the proposition. It is important to keep in mind in this calibration exercise, that the model is about periods lasting half the life of a generation, for which we choose 30 years.

For ρ and $\tau_L,~\rho$ = .3 and τ_L = .3 may be reasonable choices, so that, roughly,

$$\eta(R) < 2R/a \tag{23}$$

is necessary and sufficient for the claimed effect. We now have to find values for R and a. We want to be somewhat conservative in these guesses, i.e. we should not overstate the interest factor R and should not understate the spillover parameter a. It is well known, that long term real rates are quite low, but positive, so that R = 1 is a good, conservatively low choice¹¹. The most difficult parameter to calibrate is the parameter a. Christiano (1988) has found, that K/Y = 10.59 or Y/K = 0.0944 on a quarterly basis. To translate that into a value for the parameter a on a 30 year or 120 quarter basis as required by equation (3), the latter number needs to be multiplied with 120, resulting in 11.33. To have a round number, we use a = 12.

Thus, if the elasticity of savings over long horizons like 30 years with respect to the after-tax interest factor R over the same horizon is less than 1/6, a higher capital income tax on these savings should lead to faster growth. E.g., for the constant intertemporal elasticity of substitution utility functions used for the benchmark example, this inequality translates into $\sigma < 1.333$ (or $1/\sigma > .75$ for the coefficient of relative risk aversion) at $\beta = 1$, R = 1 via equation (21). In order to state the required elasticity $\eta(R) < 1/6$ more intuitively, it is a good idea to annualize it: the elasticity $\eta_1(R_1)$ of "retirement" savings with respect to the yearly after-tax interest factor $R_1 = R^{1/30}$ on these savings must be less than 5 in order to get the claimed effect.

¹¹Our argument is only strengthened by considering e.g. a yearly real after-tax interest rate of 1% rather than 0%. The compounded 30-year interest factor R then computes to R = 1.35 rather than R = 1, which makes quite a difference for the right hand side of (23) in favour of our argument.

In other words, suppose the yearly interest rate on savings for retirement or long-term purposes rises from 0% to 1%. As long as that doesn't raise these savings by 5% or more, taxing these savings more will lead to faster growth as claimed.

Most of the empirical work states savings elasticities $\epsilon(r_1)$ with respect to the yearly interest rate $r_1 = R_1 - 1$ rather than the elasticity $\eta_1(R_1)$ with respect to the yearly interest factor R_1 . For some fixed $r_1 = R_1 - 1$, these elasticities translate into each other via

$$\epsilon(r_1) = \eta_1(R_1) \frac{r_1}{R_1},$$
 (24)

so that for $r_1 = .04$, say, an interest factor elasticity of 5 corresponds to an interest rate elasticity of about 0.2. Translating estimated elasticities is more problematic due to the stochastic nature of interest rates and since real yearly interest rates are notoriously low.

Empirical estimates for the interest rate elasticity range from negative, insignificant or trivially small (see e.g. Blinder (1975), (1981), Blinder and Deaton (1985), Bosworth and Burtless (1992, Hall (1988) and Skinner and Peenberg (1990)) to quite large: Boskin (1978) found the elasticity to be around 0.4 (which Summers (1981) even considers to be low on theoretical grounds). Thus, while the empirical evidence may not be as clear cut as one may desire it to be, the authors personally side with the majority of the empirical evidence pointing to low savings interest rate elasticities and conclude that this first objection of positively interest elastic savings is not a strong one.

4.2 Objection 2: Grandfather Clauses

The second objection one may raise is that the capital owned by the initial old is taxed in the equilibria considered above. Since that capital is a fixed factor, taxing it is not distortionary and thus desirable from the point of view of efficiency. It thus may not surprise some readers that increasing the tax rate on the initial capital stock can lead to faster growth and one may think that our result hinges on that (compare also to Auerbach and Kotlikoff (1983), the discussion of their paper by Stiglitz (1983 and the timeconsistency issues raised in Chari, Kehoe and Prescott (1989)). After all, taxing capital rather than labor means taxing the old rather than the young, which means a shift in the timing of government revenue receipts. If the government had to "grandfather in" rather than taxing the initial capital, it may need to issue debt in order to finance the same expenditure with a tax revenue stream shifted to the future. The higher savings of the young will then be channelled into government debt rather than capital and the overall effect may then be a decrease rather than an increase in the growth rate¹².

This argument can indeed be verified within our framework for the benchmark log-utility case:

Proposition 3 If the overall utility is given as a discounted sum of logarithmic utility functions of consumption in each period of life (see equation (19)), if only the young earn labor income, if the initial capital income taz $\tau_{K,0}$ remains fixed and if the same labor income taz $\tau_{L,0} = \tau_{L,t} \equiv \tau_L$ is chosen in all periods, then a higher capital income taz rate τ_K will unambiguously result in a lower rather than a higher growth rate.

Proof: Equation (16) implies that the constant growth rate g is given by

$$g = a(1-\rho)(1-\tau_L)\frac{\beta}{1+\beta} - ab,$$
 (25)

where τ_L is given from equation (14) by

$$\tau_L = \frac{\gamma}{1-\rho} - \left(1 - \frac{R}{g}\right) \frac{b}{1-\rho} - \frac{\rho a + 1 - \delta}{(1-\rho)a} \tau_K,\tag{26}$$

¹³This effect does not depend on whether savings are before taxes rather than after taxes, if the accounting is done right, since the deferral of the payment of the taxes can be compensated for equivalently with debt equal to the deferred taxes. I.e. beforetaxes savings are increased by an amount which simply equals the discounted deferred tax payments. and the debt level b is calculated via (13) as

$$b = \gamma - \tau_L(1-\rho) - \frac{\rho a + 1 - \delta}{a} \tau_{K,0}.$$
(27)

Substituting (27) into (25) yields

$$g = \frac{-a}{1+\beta}(1-\rho)(1-\tau_L) + a(1-\rho-\gamma) - (\rho a + 1 - \delta)\tau_{K,0}$$
(28)

and thus

$$\frac{dg}{d\tau_L} < 0$$

rather than $dg/d\tau_L > 0$ as before. Similarly , substituting (27) into (26) yields, as usual,

$$d\tau_L/d\tau_K < 0.$$

Taking these two inequalities together delivers the claim. .

Debt lowers growth rates even if it can be rolled over forever¹³. However, it is not necessary to issue debt. Alternatively, consider not lowering the labor income tax on the first young generation, but only lowering the labor income tax from the second period onwards, in which the government receives higher capital income taxes as well. The government does not "cheat" here, since the change in the tax plan is known beforehand to any generation which will be affected by it¹⁴. In contrast to the proposition above and in accordance to our general claim that increasing the capital income tax leads to faster growth, we have

Proposition 4 If the overall utility is given as a discounted sum of logarithmic utility functions of consumption in each period of life (see equation (19)), if only the young earn labor income, if the initial capital income taz $\tau_{K,0}$ and

¹³The argument is similar to the discussion of bubbles in overlapping generations models with endogeneous growth in Yanagawa and Grossman (1991)

¹⁴Note that there is no change in the resale value of the initial capital stock due to a changed saving behaviour by the first young generation according to equation (8)

the initial labor income taz $\tau_{L,0}$ remains fized and if there is no debt (b = 0), then a higher capital income taz rate τ_K will unambiguously result in a higher growth rate g_t from period $t \ge 2$ onwards.

Proof: This is a direct consequence of $S(R; 1) = \beta/(1+\beta)$ and equation (18), which yields the growth rate $g = g_t$ for $t \ge 2$.

We therefore conclude that this objection is not a serious one either. The analysis shows, however, that it is important to raise initial revenue via labor income taxes rather than debt in endogeneous growth frameworks like ours, if one is concerned about time-consistency issues and high growth at the same time.

4.3 Objection 3: The Old Work Too

Finally, let us relax the condition that it is only the young who receive labor income (cmp. Summers (1981)). Consider again the logarithmic example, where the utility function is given by (19). Unfortunately general results look rather messy. Consider the case, where the issue is whether to marginally tax capital income or to marginally subsidize capital income. We have the following result.

Proposition 5 Suppose, the utility function is given by equation (19) and b = 0. Consider the equilibrium, where $\tau_K = 0$. A marginal increase in the capital income taz rate will marginally increase the growth rate if and only if

$$\frac{1}{1+\beta}\frac{1-\lambda}{R}I_L < \frac{I_K}{I_L},\tag{29}$$

where $I_L = (1 - \rho)a - a\gamma$ is the after-taz labor income per unit of capital (or the after-taz labor share) and where $I_K = R = \rho a + 1 - \delta$ is the after-taz capital income per unit of capital. Thus, the inequality (29) compares the presently consumed fraction of future, discounted labor income (when capital is normalized to one unit) with the ratio of capital income to labor income after taxes: as long as that fraction is not too high, a higher capital income tax will still lead to faster growth.

Proof: Note, that $C(R) = 1/(1+\beta)$ is constant. Substituting equations (14) and (10) into equation (16) and some algebra reveals, that $\partial g/\partial \tau_K > 0$ if and only if

$$\frac{1}{\frac{I_L}{I_K} + \tau_K} + \frac{C(R)(1-\lambda)}{1-\tau_K}$$

has a negative derivative with respect to τ_K . It is easy to see that this is the case at $\tau_K = 0$ if and only if

$$C(R)(1-\lambda)\frac{I_L}{I_K} < \frac{I_K}{I_L}.$$

Rewriting this yields the result. •

To evaluate the issue more directly, consider the following two tables. Each entry in these tables lists firstly the derivative dg/dr_K and secondly the savings rate $S(R; \lambda)$. We chose log-utilities. For the parameters in our model we chose a = 12, $\rho = .3$, $\gamma = .2$, $\delta = .3$. For the first table we chose $\beta = 1$, whereas we chose $\beta = .5$ for the second table to evaluate the effect of a change in the discount factor. We varied both the parameter λ and the parameter τ_K in each table. Note that the parameter λ here corresponds closely to the redistribution parameter η in Jones and Manuelli (1992), section 2, since in their model wage income is negligible asymptotically. The parameter τ_K implies a value for τ_L via equation (14), which is given as well.

Table 1									
$\beta = 1.0$	$\tau_K =$	-10 %	0 %	10 %	20 %	30 %			
λ	$\tau_L =$	41 %	36 %	31 %	25 %	20 %			
1.0	$dg/d\tau_K =$	2.15	2.15	2.15	2.15	2.15			
	S =	0.5	0.5	0.5	0.5	0.5			
0.8	$dg/d\tau_K =$	1.25	1.14	1.00	0.82	0.58			
	S =	0.38	0.38	0.37	0.37	0.37			
0.6	$dg/d\tau_K =$	0.69	0.56	0.42	0.23	0.01			
	S =	0.14	0.13	0.12	0.11	0.10			
0.4	$dg/d\tau_K =$	0.33	0.24	0.13	0.00	-0.15			
	S =	0.14	0.13	0.12	0.11	0.10			
0.2	$dg/d\tau_K =$	0.12	0.07	0.02	-0.05	-0.12			
	S =	0.015	0.01	-0.00	-0.02	-0.03			
0.0	$dg/d\tau_K =$	0	0	0	0	0			
	S =	-0.11	-0.12	-0.13	-0.15	-0.17			

Table 1

Table 2									
$\beta = 0.5$	$\tau_K =$	-10 %	0 %	10 %	20 %	30 %			
λ	$\tau_L =$	41 %	36 %	31 %	25 %	20 %			
1.0	$dg/d\tau_K =$	1.43	1.43	1.43	1.43	1.43			
	S =	0.33	0.33	0.33	0.33	0.33			
0.8	$dg/d\tau_K =$	0.75	0.66	0.55	0.41	0.23			
	<i>S</i> =	0.24	0.24	0.23	0.23	0.22			
0.6	$dg/d\tau_K =$	0.37	0.28	0.17	0.05	-0.11			
	S =	0.14	0.14	0.13	0.12	0.11			
0.4	$dg/d\tau_K =$	0.16	0.09	0.02	-0.06	-0.16			
	S =	0.05	0.04	0.03	0.02	0.00			
0.2	$dg/d\tau_K =$	0.05	0.02	-0.02	-0.06	-0.10			
	S =	-0.05	-0.06	-0.07	-0.09	-0.11			
0.0	$dg/d\tau_K =$	0	0	0	0	0			
	S =	-0.14	-0.16	-0.17	-0.19	-0.22			

Table 2

It is possible to find parameter combinations in these tables that look reasonable and produce a decrease in the growth rate due to an increase in the capital income tax, while at the same time keeping a positive savings rate. For example, for $\beta = 1.0$, $\lambda = .4$ and $\tau_K = .3$, the derivative has the value -0.15, while the savings rate is equal to 0.10. It is important to note, however, that the parameter ranges for which this occurs are somewhat extreme in that they require either a rather high capital-income tax to begin with¹⁵ or a rather low fraction λ of earned income when young. More importantly, perhaps, these ranges are also rather fragile in the sense that savings rates are extremely low and more often negative rather than positive for those table entries, where the derivative of the growth rate with respect to the capital income tax rate is negative.

We therefore conclude that while this objection may be the most serious of the three, the more robust result here is still the initial claim that a higher capital income tax will lead to faster growth.

5 Conclusion

We have shown that a higher capital income tax rate means faster growth in two-period overlapping generations model with endogenous growth, where government expenditures are a fixed fraction of total GNP. In this model, a higher capital income tax means a lower labor income tax, which leaves the presently young with more net income out of which to save. This in turn leads to faster growth.

We examined three objections against this argument and argue that none of these objections is serious enough. Firstly, while the effect may go the other way with sufficiently interest-elastic savings, we argue that long term savings in the US are not elastic enough for the reversal. Secondly, even

 $^{^{15} \}rm Remember that <math display="inline">\tau_K$ is the tax on the total capital income and that savings are out of after-tax labor income.

if initial capital is grandfathered in, our claimed effect holds, as long as the labor income tax is lowered only in those periods in which additional revenue is generated from higher capital income taxation, i.e. as long as lowered labor income taxes are not deficit-financed. Thirdly, while our effect can be undone, if the old earn labor income too and while it is true, that reasonable parameter values can deliver this, the range of parameters for which a reversal of our effect happens is quite fragile.

We therefore conclude that a higher capital income tax leads to faster growth. We are confident that the results can be generated also in richer models similar to those in Auerbach and Kotlikoff (1987), where the members of each generation live longer than just one period. What is apparently needed for our effect is that an increase in capital income taxation constitutes a shift in the tax burden to the relatively older agents. That this is so in practice can be seen from the calculations performed by Auerbach, Gokhale and Kotlikoff (1991).

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Chapter 3

Economic Development in a World with many Countries

1 Introduction

When we consider economic development or industrialization of a less developed country, not only domestic conditions of that country but also the economic performance of other countries is important. Since the world economy is interrelated in many respects, economic performance of a country is not independent of the performances of other countries.

For example, when we examine the economic development of Japan or Korea, economic policies of the governments or domestic economic conditions are sometimes stressed. But it is also important to know what were the economic conditions of the world economy when they industrialized. Even if the same domestic conditions can be achieved in another country as in Japan, that country may not succeed in economic development, if the world economic conditions are different.

However, the world economy is quite complicated. Effects of developed industrial countries and those of developing agricultural country are quite different. Furthermore, the effects of other agricultural countries may be affected by the performance of industrialized countries. So it is important to consider the interrelations carefully. But, in order to take into account these interrelations, traditional two country (North–South or South–South) models and small–country models are insufficient.¹

Hence, the purpose of this paper is to make clear the interrelations between the economic performances of the world economy and industrialization, by using of a multi-country model. This point is very important. As will be shown below, necessary conditions for successful industrialization are much affected by the relative position of the country and the economic structures of other countries. Even though the domestic conditions do not change, some useful policies for industrialization may become harmful policies if the economic conditions of other countries or the relative position of the country are changed.

¹There are many papers which treat the relation between the trade and economic development, see, for example, Lewis(1977), Findlay(1980) or Krueger(1984). The papers use endogenous growth models are, for example, Lucas(1988), Grossman and Helpman(1991a,b,c), or Rivera-Batiz and Romer(1991).

In this paper, we can explain the industrialization process endogenously. In the traditional two country models, it is sometimes assumed *a priori* that one country (North country) is an industrialized country and the other country (South country) is an agricultural country. But, by using of a multi-country model, we can show the process where some agricultural countries come to specialize in supplying industrial products. We assume that special technological investments or innovations (such as R&D investments) are not necessary for the industrial production. Even so, such specializations occur by the world market mechanism, and those agricultural countries face small growth rates. In order to stress this point, we use this extreme assumption.

Furthermore, in most 'endogenous growth' papers that use learning by doing as an engine of growth, the trade structure is locked-in,² since the initial positions are enhanced by dynamic scale economies. In this paper, however, the trade structure is not locked-in. Some agricultural countries come to export industrial products, even in the presence of learning by doing effects.

Even in our model, the initial positions are enhanced by the dynamic scale effects. But what determines the trade structure is the gap between the (relative) price and the (relative) productivity of a country. Although the productivity gap between the agricultural countries and the industrial countries increases, if the relative price of industrial goods also rises, some agricultural countries will succeed in industrialization.

We assume that accumulated knowledge via learning by doing partially spills across international borders. So productivity levels of a country's trade partners are important in two respects. They affect the prices through the supply of products and they affect the knowledge level of the country through the spillover effect. Hence, the structure of absolute advantage in the world economy is important for the industrialization process of agricultural countries. Both the absolute productivity conditions of the world economy and the relative position of an agricultural country in the world economy determine its development path.

²For example, Krugman(1987), Lucas(1988) and Matsuyama(1991). An exception is Young(1991). Since he has used an infinite goods model and assumed the intersectoral spillover of knowledge, the trade structure is not locked-in.

Using these insights, it is possible to study the relationship between agricultural productivity and industrialization. There is controversy about this relationship. One view is that an agricultural revolution is necessary for industrialization (Nurkse;1953, Rostow;1960). The other view is that high agricultural productivity has negative effects on industrialization, due to the focus of comparative advantage. This paper show that the answer to this problem may not be unique. The effect of agricultural productivity on industrialization is crucially dependent upon a country's relative position in the world economy.

Furthermore, we can examine the effects of less productive developed countries. If there are some industrialized countries whose industrial productivity is low, those countries may force some highly productive industrial countries to produce agricultural goods. This is because the low productivity countries depress the growth rates of industrial productivity. We find an interesting implication for infant industry protection policies. Even though some countries may succeed in industrializing due to this policy, if the industrial productivity of these countries is not very high, then other more productive industrial countries will be induced to become exporters of agricultural products.

In section 2, we present our basic model and in section 3, we present the dynamics of the trade structure and the conditions required for industrialization. In section 4, we show an example of the industrialization process, and in section 5, the effects of industrialization on economic growth rates are examined. In section 6, the relationship of agricultural productivity and industrialization is studied, and in section 7, the effects of less productive industrial countries are examined.

2 Model

In order to examine the interdependence between the economic development processes in a number of countries, we use a model of a continuum of countries. Of course there are only a finite number of countries in the real world. However, this assumption allows us to greatly simplify the arguments of this paper.³ For simplicity, there are only two types of product; product 1, an agricultural product, and product 2, an industrial product. Furthermore, labor is the only factor of production and both technologies exhibit constant returns to scale at a moment in time. In other words, we posit a standard Ricardian production structure in every country.

The countries are distributed in the range of [0,1], and indexed so that comparative advantage for product 2 is (weakly) increasing.⁴ Hence, by letting $a_i(z)$ denote the unit labor requirement for product i(i=1.2) in country z, we get.⁵

$$\frac{a_{1}(z)}{a_{2}(z)} \le \frac{a_{1}(z')}{a_{2}(z')} \quad \text{if} \quad z > z' \quad (1)$$

In order to simplify the notation, we define $1/a=a_1/a_2$ which represents relative productivity.

Next let define p_i as the price of product i (i=1,2) and call $P=p_i/p_2$, the relative price. Then, if the equilibrium relative price is P^* , by analogy with the two country Ricardian model, countries whose relative productivity (1/a) is lower than P^* , will specialize in producing agricultural goods and countries whose relative productivity is higher than P^* will specialize in produce industrial products. Let us define z^* as the country whose relative productivity is exactly equal to the relative price, P^* . Then all countries $z \in [0, z^*]$ produce agricultural products, while all countries $z \in [z^*, 1]$ produce industrial goods.⁶

For simplicity, we assume labor endowments are the same in all countries. We can normalize so that L(z)=1 for all z. Then

³Lucas(1988) also mentioned a model with a continuum of countries. But in his model, the trade structure must be locked in. So the argument of his paper is much different from that of this paper. And after the finish of this paper, kaneda(1982) also used such a kind of model.

⁴ In this paper, we mainly deal with the cases where this index is time independent. In other word, the rank of the comparative advantage structure is never changed over time. Even in those cases we can explain the industrialization, as will be explained below. But even if the index is time dependent, our argument is not affected and such cases are treated in the companion paper.

⁵ By this formation, we are allowing that the a₁/a₂ function is discontinuous for z.

⁶ Since we are using a continuum country model, it is not necessary to consider which product does z* country produce. This is the main reason why we are using this model.

$$x_1(z) = L_1(z)/a_1(z) = 1/a_1(z), \quad x_2(z) = L_2(z)/a_2(z) = 1/a_2(z).$$
 (2)

where x_i(z) is the supply of product i in country z.

Let X_i(z*) be the world supply of product i when the boundary country is z*. Then,

$$X_{1}(z^{*}) = \int_{0}^{z^{*}} x_{1}(z') dz' = \int_{0}^{z^{*}} \frac{1}{a_{1}(z')} dz'.$$
 (3)

$$X_{2}(z*) = \int_{z*}^{1} x_{2}(z') dz' = \int_{z*}^{1} \frac{1}{a_{2}(z')} dz'$$
(4)

The equilibrium relative price P* and the boundary country z* are determined by the demand equals supply condition. For the demand side, just as in Dornbusch et.al.(1977) or Krugman (1987), we assume that all countries share a common Cobb-Douglas utility function,

$$U = x_1^{\alpha} x_2^{1-\alpha}$$
. (5)

The share of spending devoted to product 1 is constant and equal to α .

From the constant expenditure share assumption and (3) and (4), we derive the following relation:

$$P* = \frac{\alpha \int_{z_*}^{1} \frac{1}{a_*(z')} dz'}{1-\alpha \int_{y_*}^{z_*} \frac{1}{a_*(z')} dz'}.$$

(6)

The equilibrium relative price P^* and the boundary country z^* satisfy (6) and⁷,

⁷If the comparative advantage function (1) is discontinuous, instead of (7) we should use $\lim_{x_{1}^{*}*} a_{1}/a_{2}(z) \le P^{*} \le \lim_{x_{1}^{*}*} a_{1}/a_{2}(z)$.

$$P* = \frac{a_1}{a_2}(z*).$$
 (7)

In other words, the boundary country z^* is determined so that the ratio of world sales of the two goods is equal to the constant ratio $\alpha/(1-\alpha)$. This is quite intuitive, because we are assuming constant expenditure shares.

Next we have to specify the dynamic processes. In this paper, we use the following simple learning-by-doing formulation.⁸

$$d(\frac{1}{a_{1}(z)})/dt = \delta_{1}x_{1}(z) + \theta_{1}\int_{0}^{1}x_{1}(z')dz', \qquad d(\frac{1}{a_{2}(z)})/dt = \delta_{2}x_{2}(z) + \theta_{2}\int_{0}^{1}x_{2}(z')dz'.$$
(8)

In this specification, we are assuming that there are two types of learning effects. The first one is the own country effect. It represents a rise of knowledge by production within a country. This effect is non-exclusive within the country (that means it is external to the firm) but does not distribute over the borders. δ is the parameter of this effect. The next one is the cross-country effect. Knowledge that is useful in producing a product is increased by experience in the worldwide supply of the product. Hence this externality effect is distributed not only among the domestic firms but also among foreign firms. θ is the parameter of this effect.⁹

There are no cross-sectoral spillover effects. That is, production in one sector does not contribute to knowledge useful in producing the other. It may be natural to assume that the

⁸It is sometimes claimed that learning by doing exhibits diminishing returns to scale and cannot be an engine of growth. Even so, the argument about the dynamics of trade structure is not affected.

⁹These assumptions about the dynamics are quite similar to those of Krugman(1987), although his

model is two-country model. Krugman(1987) states, 'surely firms can learn from the experience of firms in other countries, though perhaps not as well as they can from other domestic firms.'

industrial sector has larger spillover effects than the agricultural sector, that is $\delta_1 < \delta_2$ and $\theta_1 < \theta_2$.¹⁰

Since x_i is equal to $1/a_i$ (as long as x_i is positive), we get the following dynamics for

$$-\hat{a}_{1}(z) = \delta_{1} + \theta_{1} a_{1}(z) \int_{0}^{z^{*}} \frac{1}{a_{1}(z')} dz', \quad -\hat{a}_{2}(z) = \theta_{2} a_{2}(z) \int_{z}^{1} \frac{1}{a_{2}(z')} dz', \quad z \in [0, z^{*}] \quad (9)$$

a

$$-\hat{a}_{1}(z)=\theta_{1}a_{1}(z)\int_{1}^{z^{*}}\frac{1}{a_{1}(z^{\prime})}dz^{\prime}, \quad -\hat{a}_{2}(z)=\delta_{2}+\theta_{2}a_{2}(z)\int_{1}^{1}\frac{1}{a_{2}(z^{\prime})}dz^{\prime}, \quad z\in[z^{*},1]$$
(10)

The rate of increase of the productivity may be different from one country to another. Since we are assuming that the increase of the productivity *level* due to the cross country effect is the same for all countries and independent of the production level of each country, the *rate* of increase of productivity must be a decreasing function of the level of productivity. This means that if we compare countries that supply same product, higher productivity countries achieve a lower rate of productivity growth. This characterization is consistent with some empirical results; see, for example, Mankiw et.al.(1990).¹¹

The equations (9) and (10) also show that the structure of *absolute* advantage is important for the growth of productivity. Since there are externalities due to learning by doing, a higher (absolute) productivity is better for all other countries. This means that countries with low industrial productivity should not produce industrial products if growth in worldwide industrial productivity is to be maximized. However the market allocation is determined by the mechanism of <u>comparative</u> advantage. So, even if the industrial productivity of a country is low, when its

¹⁰This is only the difference between the agricultural sector and the industrial sector in our model. The reason of this assumption is I have intended to show that even if the industrial production is not special, some countries cannot be industrialized and face low growth rates of productivity.

¹¹If they include other countries, "[t]here is no tendency for poor countries to grow faster on average than rich countries," But this observation is also consistent with our characterization, since, in our model, agricultural countries may not experience high growth rates even though their productivity is low.

comparative advantage (in the industrial product) is high, this country will become a producer of industrial goods. This point will be examined more carefully in Section 7.

3 Dynamics of the trade structure

Since the trade structure at each moment is determined by equations (6) and (7), the dynamics of the trade structure depend upon the dynamics of the relative productivity in each country and the dynamics of the relative price.

The rate of increase of the relative productivity in country z is derived from (9) and (10),

$$-\hat{a}(z) = \{ \hat{a}_1(z) - \hat{a}_2(z) \} = -\delta_1 - \theta_1 \int_0^{z*} \frac{a_1(z)}{a_1(z')} dz' + \theta_2 \int_{z*}^1 \frac{a_2(z)}{a_2(z')} dz', \quad z \in [0, z*] \quad (11)$$

$$-\hat{a}(z) = \{\hat{a}_1(z) - \hat{a}_2(z)\} = -\theta_1 \int_{0}^{z*} \frac{a_1(z)}{a_1(z')} dz' + \delta_2 + \theta_2 \int_{z}^{1} \frac{a_2(z)}{a_2(z')} dz', \quad z \in [z*,1].$$
(12)

These equations show that the rate of increase of relative productivity depends upon absolute productivity levels. More precisely, a country's rate of change in relative productivity depends both on the absolute productivity structures of the world economy and the absolute productivity level in the country itself.

The rate of increase of the relative price, when z^* is fixed (that is, when the trade structure is fixed), is derived from (6).¹²

$$\hat{P}(z*) = -\hat{X}_1(z*) + \hat{X}_2(z*) = -\delta_1 - \theta_1 z* + \delta_2 + \theta_2(1-z*).$$
(13)

In this model, the cross-country effect of agricultural sector in each country is a linear function of the world agricultural supply. So the increase of the world agricultural supply $dX_i(z^*)$ /dt is also

 $^{^{12}}$ If the trade structure is not fixed, the dynamics of the equilibrium relative price are not equal to the equation (13), since z^* changes over time, the relative price must be affected by the change. But this dynamics of the relative price is very useful to examine the changes of the trade structure.

a linear function of the world agricultural supply, $\delta_i X_i(z^*) + \theta_i z^* X_i(z^*)$. Hence, the rate of increase of the total agricultural supply is independent of the absolute advantage structure. The rate of increase of total industrial supply has the same structure. Thus, the rate of change of the relative price is independent of the absolute advantage structure, and depends only on the parameters of learning by doing and on z^* .

From (11), (12) and (13), the difference D(z) between the rate of increase of the relative productivity and the rate of change of the relative price can be written as,

$$D^{A}(z) = \theta_{1}\{z * -\int_{0}^{z*} \frac{a_{1}(z)}{a_{1}(z')} dz'\} - \delta_{2} + \theta_{2}\{\int_{z}^{1} \frac{a_{2}(z)}{a_{2}(z')} dz' - (1-z*)\}, \quad z \in [0,z*]. \quad (14)$$

$$D^{1}(z) = \delta_{1} - \theta_{1} \{ \int_{0}^{z_{*}} \frac{a_{1}(z)}{a_{1}(z')} dz' - z_{*} \} - \theta_{2} \{ (1-z_{*}) - \int_{z_{*}}^{1} \frac{a_{2}(z)}{a_{2}(z')} dz' \}, \quad z \in [z_{*}, 1].$$
(15)

Therefore, even if a country's relative productivity is lower than the relative price at the initial moment, the country may become an exporter of industrial products, when the above difference $D^{4}(z)$ is positive and large (compared to the initial difference between relative productivity and the relative price).¹³

From (14) and (15), it is clear that the parameters of the learning by doing effects and absolute productivity levels are important in determining the differences D(z). More clearly, since $\int_{z}^{1} \frac{1}{a_{z}(z')} dz'/(1-z^{*})$ is the average industrial productivity of industrial countries and $\int_{0}^{z^{*}} \frac{1}{a_{z}(z')} dz'/z^{*}$

is the average agricultural productivity of agricultural countries, one of the following conditions must be satisfied, if agricultural country is to achieve industrialization:

(1) Industrial productivity of the country is lower than the average industrial productivity of

¹³More precisely, the country z will succeed in industrialization if and only if $\left[\frac{1}{a(z)}+d(\frac{1}{a(z)})/dt\right]-[P+dP/dt]\geq 0$. This requires, $(P-\frac{1}{a(z)})(1+\hat{P})+\frac{1}{a(z)}D^{\Lambda}(z)\geq 0$.

industrial countries.

(2) Agricultural productivity of the country is higher than the average agricultural productivity of agricultural countries.

High cross-country effects enhance the prospects for industrialization.

Intuitively, if condition (1) is satisfied, the percentage increase of industrial knowledge is large due to the cross-country effect. Since the growth rate of the relative price is independent of the structure of absolute advantage, the growth rate of relative industrial productivity in this country may become higher than the growth rate of the relative industrial price.

Similarly, if condition (2) is satisfied, the percentage growth of agricultural productivity is slow due to a relatively small cross-country effect. So the growth rate of relative agricultural productivity may become lower than the growth rate of relative agricultural price, and this country may undergo industrialization.

Hence, when we consider the industrialization prospects of a country, not only the rank of the country in the domain of comparative advantage matters, but also the absolute productivity structure of the world economy and the relative position of the country in this productivity structure. It should be noted that industrialization dynamics need not come from changes in the order of comparative advantage. Even if the ranking of comparative advantage does not change over time, industrialization will occur. As long as the relative productivity of an agricultural country becomes higher than the relative price, this country can be industrialized.¹⁴

To clarify these issues, we turn now to an example.

4 Example

In this section, we will show an example in order to explain how the absolute productivity

¹⁴Even in a two country model, the agricultural country may become to produce industrial goods, when the relative price decreases sufficiently. But, in the two country model, the agricultural country also produces agricultural products. On the other hand, Krugman(1987) assumed that the number of country is two but the number of product is infinite. So, in his model, the trade structure is always locked-in. For more detail about the difference between our model and the Krugman(1987) model, see Yanagawa(1992).

structures in the world economy matter for evolution of the trade structure. We assume the initial conditions are as follows:

$$\frac{1}{a_{1}(z)} = \frac{1}{a_{1}(z')} \quad \forall z, z' \in [0, z^{*}], \qquad \frac{1}{a_{1}(z)} > \frac{1}{a_{1}(z')} \qquad z < z' \quad z, z' \in [z^{*}, 1].$$

$$\frac{d_{-}}{d} \frac{1}{dz} > 0 \qquad \forall z, \in [0, 1], \qquad (16)$$

a,(z)

That is, all agricultural countries have the same agricultural productivity but agricultural productivity in the industrial countries is a decreasing function of z. Industrial productivity is everywhere a (continuous) increasing function of z.

In this case, it is obvious from (8) that the order of comparative advantage is fixed over time. So, it is enough to check the dynamics of the boundary country, z^* , for examining the dynamics of the trade structure. To this end, we examine the difference $D^A(z^*)$, which becomes,

$$D^{A}(z*) = -\delta_{2} + \theta_{2} \{ \int_{z*}^{1} \frac{a_{2}(z*)}{a_{2}(z')} dz' - (1-z*) \}$$
(17)

Since the industrial productivities in the industrial countries are higher than that of the boundary country, the term inside of the parenthesis in (17) is positive. Hence, as long as the size of the cross-country spillovers is not too small (compared to δ_2), $D^A(z^*)$ is positive. This means that some countries can be industrialized.

At the same time, $D^{i}(z^{*})$ is positive. Therefore all industrial countries will continue to supply industrial products. This case is shown in Figure 1. Although the productivity gap between the agricultural countries and industrial countries is increased, since the growth rate of the relative price is not so high compared to the growth rates of the relative productivity, countries between $[z^{**}, z^{*}]$ can become industrialized.

We can understand several important points from this example. First, although the comparative advantage structure is enhanced by learning by doing, the trade structure can change. Second, the structure of the industrial countries is important for the industrialization of the agricultural countries. If the industrial productivites of industrial countries are lower than those of this example, the terms in parenthesis in (17) becomes negative, even though the performance of all agricultural countries is unchanged. Third, we can illustrate examples of deindustrialization: if we change the example slightly, we can show a case where some industrial countries become producers of the agricultural products. This can happen, for example, all countries have the same industrial productivity, but the agricultural productivity is a decreasing function of z. In this case, both $D^{\Lambda}(z^*)$ are negative.

5 Dynamics of economic growth rates

In this section, we consider the dynamics of growth rates. Since the growth rate of nominal wages is dependent on the choice of numeraire, we choose the growth rate of utility as the measure of economic growth. The growth rate of utility is easily derived from the utility function (3), by choosing the agricultural price as the numeraire.

$$\hat{U}(z) = \hat{w}(z) + (1 - \alpha)\hat{P}.$$
 (18)

The growth in wage rate is given as follows:

$$\hat{\mathbf{w}}(z) = \begin{cases} -\hat{a}_1(z), & z \in [0, z^*] \\ -\hat{a}_2(z) - \hat{P}, & z \in [z^*, 1] \end{cases}$$
(19)

It should be emphasized that the above growth rate of the relative price is different form that given in (13). The growth rate of the relative price in (13) assumes that z^* is fixed. When z^* decreases (increases), the growth rate of the relative price becomes higher (lower) than in (13), since the supply of the industrial product increases (decreases).

The effects of industrialization on economic growth in the world economy are ambiguous. When the number of industrial countries expands, the growth rates of industrial productivity are increased via the spillover effect. But, simultaneously, the growth rate of the relative price increases and this has a negative impact on the growth rate of utility in the industrial countries. So, the total effect depends upon parameter values. The economic growth rates of the agricultural countries are similarly determined.

Next, we will examine the effects of industrialization on steady-state growth rates. By definition, in the steady state, the trade structure is fixed. and the agricultural (industrial) productivity levels in agricultural (industrial) countries converge to a common level, since the growth rate of productivity is negatively related to a country's level of the productivity. The growth rate of (nominal) wage becomes,

$$\hat{w}(z) = \{ \delta_1 + \theta_1 z *, z \in [0, z *] \\ \delta_2 + \theta_2 (1 - z *) - [-\delta_1 - \theta_1 z * + \delta_2 + \theta_2 (1 - z *)] = \delta_1 + \theta_1 z *. z \in [z *, 1]$$

$$(20)$$

This means that, even if the industrial countries experience higher growth rate of productivity compared to the agricultural countries, the growth rates of nominal wages are equalized, since the growth rate of the relative price perfectly offsets the higher growth rate of productivity. So all countries experience the same growth rate of utility in the steady state,

$$\hat{U}(z) = \alpha [\delta_1 + \theta_1 z^*] + (1 - \alpha) [\delta_2 + \theta_2 (1 - z^*)]. \quad z \in [0, 1] \quad (21)$$

Since the cross country effect depends on the number of countries producing a good, a decrease in z* increases the growth rate of industrial output, and decreases the growth rate of the agricultural product. But those growth effects should be weighted by the share of those products and parameter values of spillover. Hence, if

$$\alpha \theta_1 < (1-\alpha)\theta_2,$$
 (22)

that is, if the expenditure share for the agricultural product and the cross country effect θ_1 are small, a decrease in z^* (i.e. more widespread industrialization) increases every country's growth rate of utility.

Take off

Next, we consider economic growth rates during the industrialization process. Before industrialization, the growth rate of the wage (measured by the agricultural price) of an agricultural
country is determined by the growth rate of agricultural productivity. But, during industrialization, the growth rate of the wage becomes,

$$\frac{\frac{1}{Pa_2} + d(\frac{1}{Pa_2})/dt - \frac{1}{a_1}}{1/a_1} = -\hat{a}_1 + \frac{\left[\frac{1}{Pa_2} + d(\frac{1}{Pa_2})/dt\right] - \left[\frac{1}{a_1} + d(\frac{1}{a_1})/dt\right]}{1/a_1}.$$
(23)

The most right-hand side term is an additional benefit due to the changing of specialization from agriculture to industry. Since the relative price P becomes lower than the relative productivity a_i/a_2 of the countries succeeded in industrialization, this term is always positive. Furthermore, since the growth rate of the relative price is determined by the worldwide economic condition, the size of this benefit depends upon the world economic structure.¹⁵ So if this benefit is large, an agricultural country experiences high growth rate of the wage during the industrialization process. This offers an explanation of the phenomenon known as 'take-off'.

6 Industrialization and Agricultural Productivity

By using the model in the previous section, we can explain the relation between the agricultural productivity and industrialization. There is a controversy about this relationship, as we have discussed in the introduction.¹⁶

As explained in the section 2, a country with an initially high level of agricultural productivity experiences a lower rate of growth in agricultural productivity (compared to other agricultural countries). But, the lower growth rate of the agricultural productivity means a higher growth rate of the relative productivity in industry. So a higher agricultural productivity has a positive effect on the prospects for industrialization. However, the relative productivity at the initial moment also is important for industrialization prospects. Obviously, a higher agricultural

¹⁵Since the share of each country is very small, the industrialization of this country does not affect the growth rate of the relative price.

¹⁶ See Matsuyama(1991) on this issue. He also treats this problem by using a model of learning-by-doing, but he models a small country and uses a much different approach from mine.

productivity means a lower relative productivity at the initial moment. So, in this sense, a higher initial agricultural productivity also has a negative effect on industrialization prospects.

Whether a high agricultural productivity contributes to industrialization or not hinges on a comparison of these two opposite effects. But, the former (growth) effect depends on the absolute productivity structure in the world economy and the relative position of the country in that structure, as explained in the section 3. Whereas, the letter (impact) effect depends on the industrial productivity of the country in question. Hence the world economic structure and the relative position of the country are both important.

This means that we cannot answer the controversy about the role of agricultural productivity in development by only considering the domestic conditions. Conditions in the outside world help to determine the implication for growth of a productive agricultural sector.

From the discussion in the section 3, it is clear that if industrial countries have high industrial productivity, then the growth effect becomes large. So, in this case, a productive agricultural sector may contribute to industrialization. On the other hand, if other agricultural countries have sufficiently high industrial productivity, the impact effect which reduces the prospects for industrialization becomes large. Hence the growth effect may be offset and a high agricultural productivity may inhibit industrialization.

It is even possible that high agricultural productivity countries and low agricultural countries will succeed in industrialization, while only the middle agricultural productivity countries fail to take-off. This is because the high agricultural countries get the benefit of the growth effect while the low agricultural productivity countries get the benefit of the impact effect, whereas both effects may be small for the intermediate countries.

Next, we examine the effects of an exogenous shock to agricultural productivity. First, suppose that all agricultural countries experience an increase in the growth rate of agricultural productivity, d.

$$-\hat{a}(z) = -\delta_1 - \theta_1 \int_{1}^{z^*} \frac{a_1(z)}{a_1(z')} dz' + \theta_2 \int_{z^*}^{1} \frac{a_2(z)}{a_2(z')} dz' - d. \qquad z \in [0, z^*]$$
(24)

In this case, the growth rate of total agricultural supply also increases by d, and so does the growth rate of the relative price equally, that is,

$\hat{P}(z*) = -\delta_1 - \theta_1 z* + \delta_2 + \theta_2 (1-z*) - d.$ (25)

Therefore, this shock does not affect the distance between the relative productivity of any agricultural country and the relative price. So, this shock does not contribute to industrialization in any country. This conclusion can be derived for any shock to agricultural productivity that affects the growth rate of all agricultural countries similarly.¹⁷ This point has an important policy implication. If all agricultural countries adopt some policy which raises (or decreases) the growth rate of agricultural productivity, such policy does not affect the prospects for industrialization or take–off. Furthermore, some industrial countries may revert to agricultural countries as a result of this productivity shock.

Lastly, let us consider the case in which only a subset of agricultural countries experience a rise in the growth rate of their agricultural productivity. The effect of this shock also depends on the structure of the world economy. Obviously, the rise in the growth rate of the agricultural productivity has an immediate negative effect on industrialization, since it decreases relative industrial productivity and increases the distance between the relative productivity and the relative price.

However, the shock may contribute to the <u>future</u> industrialization of affected countries, if it is only temporary and if the industrial productivity of the countries is sufficiently high. The reason is as follows: After the shock, the agricultural productivity of those countries will become

¹⁷ However, this conclusion is dependent on the assumption about the utility function. If the share of agricultural products depends on the income of world economy, those shocks may affect industrialization, because the relative price is affected by the changes of the income of agricultural countries.

high. But, this means, as explained above, those countries will experience high growth rate of the relative productivity after the shock. On the other hand, if the industrial productivity of those countries are sufficiently high, the ranking of comparative advantage of those countries will not be decreased so much by the shock. Therefore, in the future, the growth effect may become larger than the impact effect, and those countries may be industrialized.

Effects of this shock on other agricultural countries are obvious. Since the shock increases the supply of the agricultural products, it decrease the relative price. Therefore some agricultural countries whose comparative disadvantage in industry is sufficiently low, will be led to industrialize.

7 Effects of Less Productive Developed Countries

In this section, we examine the effects of less productive industrial countries. In section 2, we have shown that if the industrial productivity of some industrial countries is low, other industrial countries experience low growth rates of industrial productivity. In this section, we will show that those low industrial productivity countries also impede the industrialization of agricultural countries, and moreover, in some cases, they force some industrial countries to deindustrialize.

Let us suppose that the industrial productivity of a subset of industrial countries became very low (compared to other countries) by an exogenous shock. Since those industrial countries decrease the spillover of knowledge for industrial production, the growth rates of industrial productivity in the agricultural countries are reduced. Therefore, the prospects for industrialization of the agricultural countries are diminished. This effect is obvious from (14). Since the average industrial productivity becomes low by the low industrial productivity of those countries, $D^A(z)$ is decreased.

However, those low industrial productivities also decrease $D^{i}(z)$ of the industrial countries. Hence, some industrial countries whose relative industrial productivities are not so high, may become producers of the agricultural product. This is a quite intuitive phenomenon. The reduction in the size of the spillover also decreases the growth rate of industrial productivity of all industrial countries. But the growth rate of the relative price is independent of the absolute productivity structure. Hence the relative productivity of some countries becomes smaller than the relative price, and those countries become producers of agricultural products.

It should be noted that the countries that deindustrialize may have absolutely higher industrial productivity than other countries that remain active in industry. From the viewpoint of the world economy, the most productive industrial countries should produce the industrial goods, because they generate the largest externalities. However, the market allocation is determined by comparative advantage. Even though the industrial productivity is low, if the relative productivity is high, such countries produce industrial products, and high productive countries experience the deindustrialization. From this result, we note an important implication for infant industry protection policy. If the protected countries' industrial productivities remain lower than those in other industrialization problem just discussed will occur. So, for example, even if N countries succeed in industrialization by the infant industry protection policy, more than N industrial countries may revert to producing agricultural products. In this manner, infant industry protection in some countries may slow growth in the world economy.

8 Conclusion

In this paper, we have shown that industrialization or take-off of an agricultural country is dependent upon the conditions of the world economy and the relative position of the specific country. This observation is, I believe, important to consider economic development problems in less developed countries. We have also shown that whether high agricultural productivity is beneficial for industrialization or not depends on the structure of the world economy. For a given set of domestic conditions, for different external conditions, we find different implications of high agricultural productivity for the industrialization prospects.

Furthermore, we have examined the effects of negative productivity shocks in some

industrial countries. We have shown that if industrial productivities of some industrialized countries are low, other high productive countries may have to be importers of industrial products.

In this paper, homothetic preference have been assumed. If we assume non-homothetic preferences, the relative price is affected by the income. So there may be another effect that enhances prospects for industrialization. We have also assumed that there are no distortions in the world economy. It may be interesting to consider cases in which domestic markets in some countries are not perfectly competitive. Examination of trade policies also is an important subject for future research.

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Chapter 4

Network Externality and Trade Policies

Network Externality and Trade Policies*

by

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October 1989

This version

June 1992

Abstract

Many tradable products have a "network externality" property, which means the utility which comes from the consumption of the product is affected by the number of consumers of the product.

This paper attempts to examine the trade equilibrium and the desired trade policy where network externality exists. It shows that trade equilibrium is comprised of multiple equilibria and that tariff policy is useful in eliminating "bad" equilibria. By the tariff policy, however, welfare of "good" equilibria may be sacrificed.

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¹¹ an very grateful to Professors Gene Grossman, Jonathan Eaton, Motoshige Itoh, Masahiro Okuno-Fujiwara, and Yoshitugu Kanemoto for valuable comments. I also wish to thank Kiminori Matsuyama for his suggestions.

1 Introduction

High tech products, such as computers or video players, are very actively traded worldwide. Many of these high tech products, however, have a special property: the utility which comes from the consumption of the product is affected by the number of other consumers of the product.

For example, the number of persons who use a computer or similar hardware is very important, since this affects the variety of computer software that is made available.

The size of the computer communication or data network is also important, and depends upon the number of users. If many agents use a computer, or other computers which enter the same network, network size is enlarged and the utility which comes from the purchase of the computer is also increased.

Similar demand side externalities, however, can be found in many other tradable products carrying great weight in international trade, such as automobiles, cars and photocopying machines, for example. Because the quality of maintenance or other post-purchase services also depends on the number of users, or cumulative buyers. When there are too few users, the service network will be thin and services may be insufficient. These demand side positive externalities have been examined under the name of "network externalities" by Katz and Shapiro(1985). Many other papers have recently dealt with this phenomenon (for example, Farrell and Saloner 1985, 1986).

This paper, therefore, attempts to consider international trade theory in the light of the property of network externality.¹ This examination is meaningful for a number of reasons.

First, as explained above, many tradable products have the property of network externality.

Second, if network externality exists, the international trade equilibrium is very different from that

¹ Of course, there is literature about consumption externality and international trade (see, for example, Bhagwai et.al.[1971]). But this paper examines network externality and effects of tariffs, so much different from the literature. Furthermore, Krishna(1988) also examined trade policies where network externalities exist, but with the main focus on the problem of multimarket interactions.

presented in traditional theories. Because consumers must form expectations about network size, the international trade equilibrium becomes greatly dependent on expectations, and hence there are usually multiple equilibria. Therefore, it may happen that only foreign products are supplied even if cost conditions are exactly the same, should consumers expect that.²

Third, the desired trade policies thus differ considerably from the traditional arguments where network externalities exist. As tariff policy can affect the structure of international trade, by changing the uniff rate, expectations are also affected. The equilibria that are attainable are, in turn, also affected. The domestic government, therefore, can eliminate some of the "bad" equilibria (for example, where only foreign products are supplied) by choosing the appropriate rate, although it is not able to choose an equilibrium directly.

Trade policy of this sort differs substantially from traditional trade theory, such as, for example, the rent shifting tariff policy. All previous trade theories have examined the welfare of *an* equilibrium, but the tariff policy examined in this paper is useful in *choosing or eliminating* the equilibria.

Such a "bad equilibrium elimination" policy has recently come under consideration in other conomic environments. For example, Diamond and Dybvig (1983) examined deposit insurance policy which eliminates bank run equilibrium. Another example is Grandmont(1985), who examined policies which eliminate the chaotic business cycle equilibria (see also Matsuyama[1988]). This paper is a first attempt to examine such policy in terms of the international trade environment.

This paper also considers problems of 'compatibility' decisions. Where there are network externalities, the supplying firms are able to exert control over network size to some extent by compatibility decisions. As mentioned above, for example, network size will be enhanced if two computer systems become compatible. If two video recorder systems (for example, β and VHS) are standardized,

² Frank Graham's old argument states that increasing returns also generate multiple equilibria (see, for example, Ethier[1982]). But Graham's tariff policy does not aim to change the equilibrium structure, and differs from the argument employed in this paper.

the software network is enlarged, and, since the same parts can be used, the maintenance network is also expanded.³

On the other hand, for the products to be compatible, firms must bear compatibility costs. Hence private incentives alone may not lead to optimal compatibility decisions. Tariff policies may be useful in solving the incentive problem, since the tariff rate affects the profits of the Noncompatibility equilibrium and the Compatibility equilibrium. But, in order to provide the incentives, the tariff rate may deviate from the first best tariff level.

This paper is organized as follows. Section 2 explains the model of tradable goods subject to network externality. Section 3 considers cases where there is no compatibility choice. It shows that there are multiple equilibria and indicates which tariff policy is useful in eliminating bad equilibria for domestic welfare. Furthermore, it is shown that this tariff rate may be higher than the usual rent shifting tariff rate. This means that to eliminate the bad equilibria, the domestic welfare of the good equilibria may have to be sacrificed. Section 4 considers the possibility of compatibility, and the tariff policy that is useful in solving the compatibility incentive problem. Section 5 then presents some conclusions and further ideas.

2 Model

There are two firms, domestic and foreign, which supply their products to domestic consumers. We will not take the foreign market into consideration for the sake of simplicity.⁴

Suppose that those products are homogenous, and if there are no network externality effects, the (inverse) demand function can be written as linear:

(1) $P(x_d, x_f) = H - (x_d + x_f).$

³ This paper does not distinguish explicitly between the "compatibility decision" and the "standardized decision" and just uses the former.

⁴ The two country model is treated in a companion paper. But it is doubtful whether the network externality effects of the foreign market are effective for domestic consumers.

 x_t and x_t are the demand quantities for the domestic and foreign products respectively.

If network externality effects exist, the domestic consumers get additional benefits and the (inverse) demand function shifts up. So if we suppose that the valuation function for these benefits is the same among domestic consumers, the (inverse) demand function becomes:

(2) $p_d = P(x_d, x_f) + u(D)$, and $P_f = P(x_d, x_f) + u(F)$.⁵

D is the (expected) network size of the domestic product; F is the (expected) network size of the foreign product. The determination of D and F is explained later. u is the valuation function of the (expected) network size. Assume that:

<u>Assumption 1</u> u(0)=0, u'>0, u''<0.

(A) Profit function

Therefore, from (2), the profit function of the domestic firm, π_a , and the profit function of the foreign firm, π_r , can be written as,

(3) $\pi_d = (p_d - c)x_d = (H - x_d - x_f + u(D) - c)x_d$.

(4) $\pi_{f} = (p_{f} - c - t)x_{f} = (H - x_{d} - x_{f} + u(F) - c - t)x_{f}$

in the domain of $x_d \ge 0$ and $x_l \ge 0$. c is the constant marginal cost and the cost functions are the same for

⁵ Such a formulation is justified by considering the following situation. Willingness to pay for the homogenous product, h is uniformly distributed from $\rightarrow o$ to H with density one. Hence a consumer whose willingness to pay is h₁ wants to buy domestic product if $p_{4^{-1}}(D) \leq p_{1^{-1}}(F)$ and $h \geq p_{4^{-1}}(D)$. Hence, if $p_{4^{-1}}(D) < p_{1^{-1}}(F)$, $h < a \geq p_{4^{-1}}(F)$, $x_{4^{-1}} = p_{4^{-1}}(F)$.

the domestic and foreign firms.⁶ t is the specific tariff rate.

We assume Cournot competition. Hence the first-order conditions for profit maximization are:

(5)
$$\frac{\partial \pi_d}{\partial x_d} = H - x_d - x_f + u(D) - c - x_d \le 0, \quad x_d[H - x_d - x_f + u(D) - c - x_d] = 0.$$

(6)
$$\frac{\partial \pi_t}{\partial x_t} = H - x_d - x_t + u(F) - c - t - x_t \le 0$$
, $x_t [H - x_d - x_t + u(F) - c - t - x_t] = 0$.

By this formulation, we are implicitly assuming that the tariff rate and the expected network sizes D and F are determined before the producers determine the supply quantities. This means the firms cannot affect expectation of the consumers nor cannot commit themselves beforehand.

(B) (Expected) network size

Now we should specify the expected network sizes, D and F, but distinguish between the noncompatibility and compatibility cases. In the non-compatibility case, the network size of the domestic (foreign) firm only depends on the supply of the domestic (foreign) product, $x_d(x_t)$. Hence by assuming that the consumers correctly anticipate the network size (rational expectation), we get D=D(x_d), D'>0 and F=F(x_d), F'>0. Then, for simplicity, we assume that D= x_a and F= x_c :

(7) Non-compatibility case: $D=x_d$, $F=x_f$.

On the other hand, in the compatibility case, a consumer of a domestic (or foreign) product can enjoy both domestic product and foreign product networks. Thus it is natural to assume that $D=x_a+x_f$ and $F=x_a+x_f$, by assuming the rational expectation also:

⁶Even if $c_d \neq c_n$, the argument in this paper can be applied directly, by using $c_r - c_d + t$ instead of t.

(8) Compatibility case: $D=x_d+x_f$, $F=x_d+x_f$.

(C) Equilibrium

Therefore, by using (7) or (8), we can rewrite the first-order conditions (5) and (6):

Non-compatibility case

(9)
$$H - x_d - x_f + u(x_d) - c - x_d \le 0$$
, $x_d [H - x_d - x_f + u(x_d) - c - x_d] = 0$.

(10)
$$H - x_d - x_f + u(x_f) - c - t - x_f \le 0$$
, $x_f [H - x_d - x_f + u(x_f) - c - t - x_f] = 0$

Compatibility case

(11)
$$H-x_{d}-x_{t}+u(x_{d}+x_{t})-c-x_{d}\leq 0$$
, $x_{d}[H-x_{d}-x_{t}+u(x_{d}+x_{t})-c-x_{d}]=0$.
(12) $H-x_{d}-x_{t}+u(x_{d}+x_{t})-c-t-x_{s}\leq 0$, $x_{s}[H-x_{d}-x_{s}+u(x_{d}+x_{s})-c-t-x_{s}]=0$.

These reaction correspondences characterize the economy in each cases.

In this model, therefore, the tariff rate is first determined, expectations are then formed (given the tariff rate), and lastly, Cournot competition occurs (given the tariff rate and expectations). We define the equilibrium in each case as follows:

Definition 1

Non-compatibility Equilibrium is the pair of x_d and x_f which satisfies (9) and (10).

Definition 2

Compatibility Equilibrium is the pair of x_d and x_f which satisfies (11) and (12).

(D)Welfare of the domestic country

If the domestic firm supplies x_d and the foreign firm supplies x_p , the welfare of the domestic country in the domain of $x_k \ge 0$, $x_k \ge 0$, can be formalized as follows:

Consumers' surplus

 $\int_{H \to (x_d + x_f)}^{H} (\beta + x_d + x_f - H) d\beta = (x_d + x_f)^2/2.$

Profit of the domestic firm

 $(p_d - c)x_d = (H - x_d - x_f + u(D) - c)x_d = x_d^2$.

Tariff revenue

txf

Then the domestic welfare is

$$W = \frac{(x_{d} + x_{f})^{2}}{2} + x_{d}^{2} + tx_{f}.$$

3 Non-compatibility case

As described in the previous section, the non-compatibility equilibrium is defined by (9) and (10). An example (where the tariff rate is zero) of this non-compatibility equilibrium is depicted in Figure 1.

the DD'D" curve is the domestic firm's reaction correspondence (9), and the FF'F" curve is the foreign firm's reaction correspondence (10). Because of the network externality effects, those curves are not linear as Figure 1. The distance between the DD'D" (FF'F') curve and the dot line D'd' (F'f') represents the network externality effect, u. There are multiple equilibria even if we exclude the unstable equilibria (points B and G).⁷

At point A, only foreign products are supplied; this equilibrium can be called the Foreign Monopoly Equilibrium. At point E, only domestic products are supplied; this we can term the Domestic Monopoly Equilibrium. Lastly, the equilibrium where both firms provide products, such as point C, we

⁷The definition of the stability is the traditional one; the (absolute) slope of the domestic reaction function is steeper than that of the foreign reaction function.

term the Duopoly Equilibrium.

Denote x_a^D , x_f^D as an equilibrium pair of the Duopoly Equilibrium, x_a^{DM} , x_f^{DM} as an equilibrium pair of the Domestic Monopoly Equilibrium (hence $x_t^{DM}=0$ by definition) and x_a^{FM} , x_f^{FM} as an equilibrium pair of the Foreign Monopoly Equilibrium (hence $x_a^{FM}=0$ by definition). Of course, x_a^{\dagger} , x_f^{\dagger} , i=D, DM, FM, is dependent on the tariff rate t.

The expectations of the consumers determine which equilibrium will actually occur. But domestic welfare varies greatly with the equilibrium.

Comparison of the domestic welfare

In the usual Cournot models, a duopoly equilibrium is better than a monopoly equilibrium in terms of economic welfare. But when network externality effects exist, this is not always true, because, if the monopolist supplies a sufficiently large number of products (which means a large network), this network externality effect may overcome the inefficiency of the monopoly.

In order to explain this, a critical network size, x*, is introduced, and defined as:

u'(x*)=1

If the network size is smaller than x^* , u^* becomes larger than 1. But, since the marginal network benefit, u^* , affects the slope of the marginal revenue curve, u^* also affects the slope of the reaction correspondence. Hence if u^* is very large, the reaction correspondence is greatly affected and a monopoly equilibrium may become better than a duopoly equilibrium. So we get the following lemma:

Lemma 1

As long as $x_t^{D} \ge x^*$, x_t^{FM} is smaller than $(x_d^{D} + x_t^{D})$. As long as $x_d^{D} \ge x^*$, x_d^{DM} is smaller than $(x_d^{D} + x_t^{D})$. Proof

(13)
$$-dx_f = \frac{1}{2 - u'(x_f)} dx_d.$$

By differentiating equation (10) with a fixed tariff rate, we get: Because u"<0, if $x_t^{FM} > x_t^D$ (if $x_t^{FM} \le x_t^D$, obviously $x_t^{FM} < (x_t^D + x_t^D)$),

(14)
$$-(x_t^{D}-x_t^{FM}) < \frac{1}{2-u'(x_t^{D})}(x_d^{D}-0)$$

Hence $-(x_f^{D}-x_f^{PM})$ is smaller than $x_d^{D}-0$, as long as $u'(x_f^{D}) \leq 1$. From the same method, as long as $u'(x_q^{D}) \leq 1$. From the same method, as long as $u'(x_q^{D}) \leq 1$.

Q.E.D.

Hence from this Lemma we can easily derive the following proposition:

Proposition 1

Under any positive or negative tariff rate, the Duopoly Equilibrium is better than the Foreign Monopoly Equilibrium for domestic welfare as long as $x_{\mu}^{D} > x^{*}$.

Proof

See Appendix A.

Although we only deal with cases where, under free trade, $x_t^{p} > x^*$ is satisfied, it should be noted that if the network size is small, a monopoly equilibrium may be desirable for the domestic government.

As long as the condition of Proposition 1 is satisfied, therefore, one big problem arises from the viewpoint of the domestic government; if the Foreign Monopoly Equilibrium is selected, the realized welfare becomes lower,⁸ but the domestic government cannot choose an equilibrium directly and optimal welfare directly. This problem does not arise in the traditional international trade model.

But it is not always the case that the three types of equilibria exist. Under some tariff rates, one or two types are unable to become the Non-compatibility Equilibrium.

Hence, although the domestic government cannot choose an equilibrium directly, it can change the equilibrium structure and eliminate the "bad" equilibria by using appropriate tariff policies, since the tariff shifts the FF'F" curve in Figure 1.⁹ Such trade policies have not been considered in the standard literature, and their consideration here represents a significant contribution by this paper.

The "bad" equilibrium elimination tariff

Since the tariff rate affects the marginal cost of the foreign firm, a rise of the tariff rate shifts the reaction curve FF'F" to the inside. Therefore, if the tariff rate is sufficiently high, the Foreign Monopoly Equilibrium will disappear. On the other hand, if the tariff rate is sufficiently low (or negative), the Domestic Monopoly Equilibrium will disappear.

Lemma 2

(a)When H-c≥x*

The Non-compatibility Equilibrium cannot include the Foreign Monopoly Equilibrium, if and only if u(H-c)<H-c+t, and the Non-compatibility Equilibrium cannot include the Domestic Monopoly Equilibrium, if and only if u(H-c-t)<H-c-2t.

^bThe main reason that domestic consumers do not choose the desirable equilibrium is the coordination failure among the consumers. It is actually quite difficult to coordinate expectations explicitly.

⁹In this paper, we concentrate to examine tariff policies, although they may be second-best policies.

(b) When H-c<x*

The Non-compatibility Equilibrium cannot include the Foreign Monopoly Equilibrium, if and only if $u(x^*) < x^*+t$, and the Non-compatibility Equilibrium cannot include the Domestic Monopoly Equilibrium, if and only if $u(x^*) < x^*-t$.

Proof

In order for the Non-compatibility Equilibrium to include the Foreign Monopoly Equilibrium, from (9) and (10), the following two conditions must be satisfied:

(15) $H-c+u(0)-0=H-c\leq x_{f}$

(16) $H-c-t+u(x_f)=2x_f$

Those mean that $u(x_t)$ - $x_t \ge 1$. When $H_c \ge x^*$, however, $u(x_t)$ - x_t must be smaller than $u(H_c)$ - H_c from (15). Hence, when $H_c \ge x^*$, $u(H_c)$ - H_c -t is sufficient to exclude the Foreign Monopoly Equilibrium.

Furthermore, if $u(H-c)\ge H-c+t$, $H-c\le \{u(H-c)-u(x_t)\}-\{(H-c)-x_t\}+x_t$ from (16). But, when $H-c\ge x^*$,

we can find x, which satisfies this inequality and (15). Hence u(H-c)<H-c+t is also the necessary condition.

Since u(x)-x is maximized at x^* , $u(x^*)<x^*+t$ is obviously sufficient to exclude the Foreign Monopoly Equilibrium. But, when H-c< x^* , this is also the necessary condition, because if $u(x^*)\ge x^*+t$, H-c+ $\{(x^*-x_t)-\{u(x^*)-x_t\}\le x_t$ from (16), and we can find x_t which satisfy this inequality and (15).

By the same argument, we can easily show that when $H-c\ge x^*$, u(H-c-t)< H-c-2t is the necessary and sufficient condition for excluding the Domestic Monopoly Equilibrium. And, when $H-c< x^*$, $u(x^*)< x^*$ t is the necessary and sufficient condition for excluding the Domestic Monopoly Equilibrium.

Q.E.D.

An implication of this lemma is that the existence condition is dependent on the tariff rate which generates the cost difference between the foreign and domestic firms. Hence the tariff rate is important for multiplicity. When the marginal cost is sufficiently high by the high tariff rate, even if the network externality effect exists, the firm cannot take all the market share.

From this lemma, we can easily derive the minimum tariff rate, <u>t</u>, in order to eliminate the Foreign Monopoly Equilibrium.

Proposition 2

The tariff rate must be higher than $\underline{t}=u(H-c)-(H-c)$ (when $H-c \ge x^*$), $u(x^*)-x^*$ (when $H-c < x^*$) to eliminate the Foreign Monopoly Equilibrium.

Proof

Obvious, from Lemma 2.

Q.E.D.

The intuition of Proposition 2 is as follows. If the tariff rate is sufficiently high, it is irrational for consumers to expect the realization of the Foreign Monopoly Equilibrium. Because even if consumers expect a large network by monopolization, the price of the foreign products become too high to monopolize because of the high marginal cost. Hence consumers do not expect the Foreign Monopoly Equilibrium and thus this equilibrium is actually never realized.

The new trade-off

Even if the Foreign Monopoly Equilibrium is climinated, the foreign firm may be able to supply as a duopolist from (9) and (10).¹⁰ Hence, although the tariff policy to eliminate the Foreign Monopoly

¹⁰If the foreign firm is not able to supply even as a duopolist, the problem becomes simple. Since the domestic welfare in the Domestic Monopoly Equilibrium is independent of tariff rates, no trade-off problems exist for the domestic government.

Equilibrium is derived, this tariff rate also affects domestic welfare in the Duopoly Equilibrium (but not in the Domestic Monopoly Equilibrium). Therefore the tariff rate may be welfare decreasing in the context of the Duopoly Equilibrium. As the domestic government cannot choose the equilibrium regime (for example, the Foreign Monopoly Equilibrium or the Duopoly Equilibrium), it may face a trade-off between the benefit of the elimination of the Foreign Monopoly Equilibrium and the cost of the welfare decreasing effects of the Duopoly Equilibrium.

The effects of the tariff rate on domestic welfare under the Duopoly Equilibrium are derived by the following Proposition 3.

Proposition 3

As long as the Duopoly Equilibrium is selected, the optimal tariff rate is,

$$t *= x_{f}^{D} [1 - u'(x_{f}^{D})] + x_{d}^{D} \frac{1 + u'(x_{d}^{D})}{2 - u'(x_{d}^{D})}$$

Proof

See Appendix B.

In this model, if there are no network externality effects, the optimal tariff rate is always positive, since the demand function becomes linear.¹¹ But, from the network externality effects, the optimal tariff rate is not always positive. If the network size of the foreign firm is small, subsidization of the foreign firm may be beneficial for the domestic government.

It is difficult to compare tariff rate t and t* unless the u function is specified. But it is clear that

¹¹See, for example, Brander and Spencer(1984).

if (*<<u>c</u>], and the domestic government decides to eliminate the Foreign Monopoly Equilibrium, domestic welfare under the Duopoly Equilibrium must be reduced.

This situation is illustrated in Figure 2. The domestic welfare under the Duopoly Equilibrium is figured by the DE curve, and maximized at t*. On the other hand, the FME curve represents the domestic welfare under the Foreign Monopoly Equilibrium, and does not exist under the tariff rates higher than g(>t*). So, if the government chooses the tariff rate higher than t, the domestic welfare under the Duopoly Equilibrium is not maximized.

Therefore, if $t^* < J$, the domestic government has to weigh the gains from the elimination of the Foreign Monopoly Equilibrium against the welfare loss under the Duopoly Equilibrium. But in order to do so, it must forecast the probability that the Foreign Monopoly Equilibrium is selected, or choose an equilibrium by some refinement techniques.

However, there exists no sufficient theoretical explanation of this problem, and thus two (insufficient) alternative views are presented here.

One view is that the domestic government puts the prior probability on each equilibria, and decides a policy. In this view, the choice of policy is crucially dependent on the policymaker's belief.

The other view is that the policymaker can correctly expect the equilibrium choice made by the domestic consumers under each tariff rate. So if domestic consumers choose the Duopoly Equilibrium under the tariff rate t*, the domestic government can choose t*. And if it is expected that the Foreign Monopoly Equilibrium will be chosen under the t*, the government chooses higher than t, as long as the welfare of the Duopoly Equilibrium under t is larger than that of the Foreign Monopoly Equilibrium under t*.

Tariff rate eliminating the Domestic Monopoly Equilibrium

Next, we consider tariff policy that eliminates the Domestic Monopoly Equilibrium. Although we did not examine domestic welfare under the Domestic Monopoly Equilibrium explicitly, it may be higher than that under the Duopoly Equilibrium, since with the network externality effect, total supply under the Domestic Monopoly Equilibrium may be larger than that under the Duopoly Equilibrium.

But if the Domestic Monopoly Equilibrium is better than the Duopoly Equilibrium for the domestic government, the desirable policy is simple. The tariff rate level will be chosen to attain the Domestic Monopoly Equilibrium. This policy almost amounts to a shutting down of the market against foreign firms.

Thus we will consider only those cases where welfare under the Domestic Monopoly Equilibrium is lower than that under the Duopoly Equilibrium. In those cases, it is worthwhile to consider the tariff policy which eliminates the Domestic Monopoly Equilibrium. But, unfortunately, as shown in Proposition 4, such a tariff rate is not consistent with the rate that eliminates the Foreign Monopoly Equilibrium.

Proposition 4

As long as the Domestic Monopoly Equilibrium and the Foreign Monopoly Equilibrium constitute the Non-compatibility Equilibrium under free trade, it is impossible to eliminate both the Domestic Monopoly Equilibrium and the Foreign Monopoly Equilibrium.

Proof

When H-c<x*, this problem is obvious. So we should consider only the case of H-c≥x*. From Lemma 2, if the Domestic Monopoly Equilibrium and the Foreign Monopoly Equilibrium constitute Non-compatibility Equilibrium under free trade, u(H-c)≥(H-c) must be satisfied. From Proposition 2, therefore, to eliminate the Foreign Monopoly Equilibrium, t must be positive. Hence (H-c-t)-(H-c-t) must be positive from u"<0. From Lemma 2, however, in order to eliminate the Domestic Monopoly Equilibrium, t<(H-c-t)-u(H-c-t) must be satisfied. Obviously, this is impossible.

Q.E.D.

This proposition is clear from Figure 1. Tariff policies can shift only the FF'F" curve. But it is impossible to eliminate both points A and E by shifting just the FF'F" curve.

4 Compatibility case

As explained in section 2, when two firms make their products compatible, network size will be enlarged from the supply size of a product $u(x_i)$ i=d,f to the supply size of both products $u(x_d+x_i)$. Then the reaction correspondences becomes (11) and (12). From those, it can be easily shown that the Compatibility Equilibrium is unique, by contrast to the Non-compatibility Equilibrium.

For the products to be compatible, firms may have to bear compatibility costs. So if the domestic and foreign firms can decide whether their products are compatible or not,¹² the compatibility decision may not be desirable for the domestic government, because firms decide without considering the benefits to domestic consumers.

In order to consider the compatibility decisions, we must specify the assumptions about agreements of the compatibility decisions and side payments. To simplify the argument, we make the following assumptions.

Assumption 2

(a)Only when both firms are in agreement, compatibility is realized.

(b)To be compatible, both firms must incur fixed cost F.

(c)The transfer of profits between the firms (side payments) is impossible.

¹²Since multiple equilibria exist in the Noncompatibility equilibrium, the firms have to forcast which equilibrium will be chosed when they do not make their products compatible. Then there is a theoretical difficulty as in the section 3. So, we will only check the incentives in the cases where the governmet and the firms can forcast the equilibrium selection perfectly. But this analysis can be extended easily to more general expectation cases.

Under these assumptions, the private compatibility decisions are insufficient for the domestic soverment 13

Proposition 5

As long as the tariff rate is nonnegative, the private incentives for the compatibility are smaller than the government's incentive.

Proof

See Appendix C.

The intuitive reason of this proposition is quite simple. The firms make the compatibility decisions without considering the consumers' surplus and the tariff revenue, even though they are increased by the realization of compatibility. So the private incentives for the compatibility is insufficient from the viewpoint of the domestic government. When Noncompatibility Equilibrium is the Foreign Monopoly Equilibrium or the Domestic Monopoly Equilibrium, more strong result is obtained.

Proposition 6

When Noncompatibility Equilibrium is the Foreign Monopoly Equilibrium or the Domestic Monopoly Equilibrium, the compatibility is not realized, even if the fixed cost F=0, as long as $x_t^{PM} > x^*$ (F.M.E.) or $x_t^{DM} > x^*$ (D.M.E.).

Proof

See Appendix C.

¹⁹This result is crucially dependent upon the assumption 2. Under different assumptions, excessive private incentives may exist. But, in general, the governmet intervention is necessarly.

Hence the domestic government has an incentive to intervene in compatibility decisions. Tariff policies are useful for such intervention, because tariffs affect the profits of the Non-compatibility Equilibrium and Compatibility Equilibrium. In order to realized the Compatibility Equilibrium, the tariff rate must satisfy the following conditions.

$(\mathbf{x_d}^{CO})^2 - \mathbf{F} \ge (\mathbf{x_d}^*)^2$	a=D, or DM, or FM.
$(x_t^{CO})^2 - F \ge (x_t^*)^2$	a=D, or DM, or FM.14

As in the non-compatibility case, it is crucial whether the network size $(x_4^{CO}+x_t^{CO})$ is larger than x^* or not. If $(x_4^{CO}+x_t^{CO})<x^*$, the situation becomes quite different form the normal situation; an decrease of the tariff rate increases the profit of the domestic firm, since the network externality effect overcomes the strategic effect. Hence, the negative tariff rate may be necessary to give the incentive of the compatibility.

Even if $(x_a^{CO}+x_t^{CO})>x^*$, there is no guarantee that the first best tariff rate under the Compatibility Equilibrium satisfies the above conditions. So the domestic welfare may not be maximized in order to achieve the Compatibility Equilibrium.¹⁵

5 Conclusion and Further Ideas

This paper has examined international trade and trade theory in the context of network externality. It has been shown that the optimal tariff rate differs considerably from the traditional rent shifting tariff

¹⁶This means that if the government know that the Domestic Monopoly Equilibrium is selected, a=DM. If the government do not know which equilibrium is selected in the noncompatibility case, these conditions have to be satisfied for any a.

¹⁵In some cases, combinations of the tariff and other policy tools (taxes, for example) are necessary to achive the Compatibility Equilibrium. For more detail, see Yanagawa(1991).

policies.

Although, as mentioned in the introduction, network externality effects are very important in the international environment, the phenomena dealt with in this paper are highly restricted ones. So it is possible to consider various extensions.

(1) One natural extension is a consideration of two country models. In a two country model, the supply sizes of the foreign market also affect the domestic consumers' network externality effects. Hence the strategic behavior between the domestic firm and the foreign firm becomes more complicated. Furthermore, the conflicts between the domestic government and the foreign government also become important.

(2) It may be possible to consider exchange rate policies which achieve similar goals as with this tariff policy, and implications of such policies may be interesting for considering the optimal interventions.

(3) Consideration of import quota policies is another extension. Because the quota policies can directly restrict the network size of the foreign firm, the effect of the quota policies differ considerably from the tariff policies explained above. Comparison between tariff policy and quota policy is of value in terms of future research.

(4) International standardization problems constitute applications of the two country model. But if the governments directly join the standardization negotiation process, this game becomes very different from the simple two country model, because the governments have power to decide both the tariff rate and compatibility simultaneously.

Furthermore, if we consider multicountry models, we can deal with situations such as those where some countries create standards and the others do not.

With respect to the problems of intellectual property rights, those situations may be very important. Because, if some countries do not enter the standardization, the standardized group may incur serious losses.

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(5) Furthermore, the network externality problem can be applied to direct investment problems. As explained in this paper's introductory section, network externality effects are crucially reliant on maintenance or service networks. However, direct investment may make it possible to supply higher quality maintenance service compared to simple exporting, since plants are built in the host country. So even if the network size is the same, a product supplied by direct investment has greater network externality effects compared to a product simply imported. This view may enliven the theory of direct investment with new insights.

Appendix A

Proof of Proposition 1. In this Appendix we prove that if $x_r^{PM} < (x_a^{D} + x_r^{D})$ and the welfare of F.M.E. is lower than that of D.E.

(the welfare of F.M.E.)-(the welfare of D.E.)

$$= (x_t^{PM})^2/2 + tx_t^{PM} - (x_a^{D})^2 - (x_a^{D} + x_t^{D})^2/2 - tx_t = (x_t^{PM})^2/2 - (x_a^{D} + x_t^{D})^2/2 + t(x_t^{PM} - x_t^{D}) - (x_a^{D})^2$$

$$< (x_t^{PM})^2/2 - (x_t^{D} + x_t^{D})^2/2 + tx_t^{D} - (x_t^{D})^2$$

Hence if $t\leq 0$ this welfare difference is negative. We can show, however, that this is negative even if t>0. From (9) and (10),

$$t-x_{d}^{D}=H+u(x_{f}^{D})-c-x_{d}^{D}-2x_{f}^{D}-[H+u(x_{d}^{D})-c-x_{d}^{D}-x_{f}^{D}]=u(x_{f}^{D})-u(x_{d}^{D})-x_{f}^{D}.$$

But, $u(x_f^{D}) < u(x_d^{D})$, because,

(9)-(10)
$$(x_d^{D}-x_f^{D})-\{u(x_d^{D})-u(x_f^{D})\} = t.$$

and $x_f^D > x^*$.

Therefore t- x_d^{D} <0, and the welfare of F.M.E. is lower than that of D.E.

Q.E.D.

Appendix B

Proof of Proposition 3. The problem for the domestic government is to maximize:

$$\frac{(x_d^{D} + x_f^{D})^2}{2} + (x_d^{D})^2 + tx_f^{D}$$

with respect to t. Therefore, the first order condition is:

$$(x_d^{D} + x_f^{D}) \frac{d(x_d^{D} + x_f^{D})}{dt} + 2x_d^{D} \frac{dx_d^{D}}{dt} + x_f^{D} + t \frac{dx_f^{D}}{dt} = 0$$

Then, to derive dx_d^{D}/dt and dx_f^{D}/dt , we differentiate (9) and (10) with respect to t:

$$\{2-u'(x_d^{D})\}dx_d^{D}/dt+dx_f^{D}/dt = 0$$

$$dx_d / dt + \{2 - u'(x_f)\} dx_f / dt = -1.$$

By defining D as $D = \{2 - u'(x_d^D)\}\{2 - u'(x_f^D)\} - 1$, we get,

 $dx_d^{D}/dt=1/D, dx_f^{D}/dt=-\{2-u'(x_d^{D})\}/D.$

Therefore, the first order condition is:

$$(x_d^{D}+x_f^{D}) = \frac{u'(x_d^{D})-1}{D} + \frac{2x_d^{D}}{D} + x_f^{D} + t \frac{u'(x_d^{D})-2}{D} = 0$$

and from this, we get:

$$t *= x_f^{D} [1 - u'(x_f^{D})] + x_d^{D} \frac{1 + u'(x_d^{D})}{2 - u'(x_d^{D})}.$$

Q.E.D.

Appendix C

Proof of Proposition 5

We prove this proposition in two parts.

(a) If
$$(x_d^{CO})^2 - F \ge (x_d^{i})^2$$
 and $(x_f^{CO})^2 - F \ge (x_f^{i})^2$, $x_d^{CO} > x_d^{i}$ and $x_f^{CO} > x_f^{i}$ for each i=D, DM, FM. Hence,

$$(x_d^{CO}+x_f^{CO})^2/2-(x_d^i+x_f^i)^2/2+t(x_f^{CO}-x_f^i)+(x_d^{CO})^2-x_d^i)^2-F$$

is obviously positive as long as t≥0.

(b) Even if

$$(x_d^{CO}+x_f^{CO})^2/2-(x_d^i+x_f^i)^2/2+t(x_f^{CO}-x_f^i)+(x_d^{CO})^2-x_d^i)^2-F$$

is positive, $(x_a^{CO_3^2}$ -F may be smaller than $(x_a^*)^2$, as long as the differences of the consumers' surplus and the tariff revenue are sufficiently large.

Q.E.D.

Proof of Proposition 6

In order to realize the Compatibility Equilibrium by the private incentives, $x_a^{CO}_{ai}$ and $x_t^{CO}_{x_t}$, i=DM, or FM, are necessary. So, suppose $x_s^{CO}_{x_t}$ ^{FM} and $x_a^{CO}_{x_t}$ ^{FM=0}. From the equation (10), (12) and u"<0,

$$\{u(x_{f}^{CO}+x_{d}^{CO})-u(x_{f}^{FM})\} < u'(x_{f}^{FM})(x_{f}^{CO}+x_{d}^{CO}-x_{f}^{FM}).$$

Therefore,

$$(2-u'(x_f^{FM}))(x_f^{CO}-x_f^{FM}) = \{1-u'(x_f^{FM})\}x_d^{CO} < 0.$$

But the left-hand side of the above inequality is positive as long as $u^*(x_t^{PM})<1$. This is a contradiction, then. The same argument is possible in the case of $x_d^{CO} > x_a^{DM}$, $x_t^{CO} > x_t^{DM} = 0$. This cases is also a contradiction if $u^*(x_d^{DM})<1$. Hence the compatibility equilibrium is never realized by the private incentives, even if F=0.

Q.E.D.

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