Appendix D

Singular Value Decomposition

Singular value decomposition (SVD) is a powerful set of techniques for dealing with sets of linear equations or matrices that are either singular or numerically very close to singular. In many cases where Gaussian elimination and *LU* decomposition fail to give satisfactory results, this set of techniques diagnoses what the problem is. Whereas the SVD only diagnoses the problem in some cases, in most cases it provides a useful numerical answer.

The SVD techniques are based on the following theorem of linear algebra [93]: Any square matrix $\boldsymbol{A}$ can be written as the product of a square orthogonal matrix $\boldsymbol{U}$, a square diagonal matrix $\boldsymbol{W}$ with positive or zero elements, and the transpose of a square orthogonal matrix $\boldsymbol{V}$. In other words, any square matrix $\boldsymbol{A}$ can be decomposed as follows

$$\begin{aligned} \boldsymbol{A} &= \boldsymbol{U}\boldsymbol{W}\boldsymbol{V}^{\mathrm{T}} \\ &= \boldsymbol{U}\begin{pmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_n \end{pmatrix}\boldsymbol{V}^{\mathrm{T}}, \end{aligned} \tag{D.1}$$

where the matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ have the following properties.

$$\boldsymbol{U}^{\mathrm{T}}\boldsymbol{U} = \boldsymbol{I} \tag{D.2}$$

$$\boldsymbol{V}^{\mathrm{T}}\boldsymbol{V} = \boldsymbol{I} \tag{D.3}$$

Here, $\boldsymbol{I}$ represents an identity matrix.

The inverses of $\boldsymbol{U}$, $\boldsymbol{V}$, and $\boldsymbol{W}$ are trivial to compute. Since $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal, their inverses are equal to their transposes. The inverse of the diagonal matrix $\boldsymbol{W}$ is the diagonal matrix whose elements are the reciprocals of the elements $w_k\,(k = 1, 2, \ldots, n)$.

Thus, it follows from (D.1) that the inverse of $\boldsymbol{A}$ is

$$\boldsymbol{A}^{-1} = \boldsymbol{V} \begin{pmatrix} 1/w_1 & & & \\ & 1/w_2 & & \\ & & \ddots & \\ & & & 1/w_n \end{pmatrix} \boldsymbol{U}^{\mathrm{T}}. \tag{D.4}$$

Problems arise in (D.4) when one of the w_k's becomes zero or close to zero. In the case of such problems, the matrix $\boldsymbol{A}$ becomes singular. Using (D.4), the SVD can diagnose how singular the matrix $\boldsymbol{A}$ is, first of all.

Let us define a null space and nullity, which are important concepts for singular matrices. Consider the following system of linear equations

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{D.5}$$

where $\boldsymbol{A}$ is a square matrix, and $\boldsymbol{x}$ and $\boldsymbol{b}$ are vectors. If $\boldsymbol{A}$ is singular, the subspace of $\boldsymbol{x}$ satisfies $\boldsymbol{A}\boldsymbol{x} = \mathbf{0}$. This subspace is defined to be a *null space* of $\boldsymbol{A}$ and its dimension is defined to be the *nullity* of $\boldsymbol{A}$. There is also the subspace of $\boldsymbol{b}$ that can be mapped onto by $\boldsymbol{A}$, i.e., there exists $\boldsymbol{x}$ that is actually mapped to $\boldsymbol{b}$ by $\boldsymbol{A}$. This subspace is called the *range* of $\boldsymbol{A}$ and its dimension is called the *rank* of $\boldsymbol{A}$.

Now we are ready to see that the SVD explicitly constructs the orthonormal bases for the null space and range of a matrix. In particular, the columns of $\boldsymbol{U}$ whose corresponding w_k's are nonzero are an orthonormal set of bases that span the range and the columns of $\boldsymbol{V}$ whose corresponding w_k's are zero are an orthonormal set of bases for the null space.

If $\boldsymbol{b} = \mathbf{0}$, (D.5) can be solved immediately; the solution is a linear sum of the orthonormal bases for the null space.

If $\boldsymbol{b} \neq \mathbf{0}$, the important question is whether $\boldsymbol{b}$ lies in the range of $\boldsymbol{A}$ or not. If it does, the singular set of equations (D.5) does have a solution. Actually, it has more than one solution because any vector in the null space of $\boldsymbol{A}$ can be added to the solution without violating (D.5). From such solutions, we pick up the one that has the smallest length $\|\boldsymbol{x}\|$. In order to obtain the solution, $1/w_k$ is replaced by zero if w_k is zero or close to zero, and the following equation is then calculated:

$$\boldsymbol{x} = \boldsymbol{V} \begin{pmatrix} 1/w_1 & & & \\ & 1/w_2 & & \\ & & \ddots & \\ & & & 1/w_n \end{pmatrix} \boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} \tag{D.6}$$

The proof is as follows: Consider $\| \boldsymbol{x} + \boldsymbol{x}' \|$ where $\boldsymbol{x}'$ lies in the null space of $\boldsymbol{A}$. Let $\boldsymbol{W}^{-1}$ denote the modified inverse matrix of $\boldsymbol{W}$ with some elements zeroed as described above. We obtain the following equation:

$$\begin{aligned} \| \boldsymbol{x} + \boldsymbol{x}' \| &= \| \boldsymbol{V}\boldsymbol{W}^{-1}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} + \boldsymbol{x}' \| \\ &= \| \boldsymbol{V}(\boldsymbol{W}^{-1}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{x}') \| \\ &= \| \boldsymbol{W}^{-1}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{x}' \|. \end{aligned} \tag{D.7}$$

where the first equality comes from (D.6) and the second and third from the orthonormality of $\boldsymbol{V}$. Let us examine the two terms that make up the sum of the right-hand side. The first term has nonzero k-th elements only where $w_k \neq 0$, while the second one has nonzero k-th elements only where $w_k = 0$ because $\boldsymbol{x}'$ lies in the null space. Thus, the minimum length is achieved when $\boldsymbol{x}' = \boldsymbol{0}$.

If $\boldsymbol{b}$ is not contained in the range of $\boldsymbol{A}$, (D.5) has no solution. However, still in this case, (D.6) can be used to construct the best solution $\boldsymbol{x}$. This means that among the possible solutions, $\boldsymbol{x}$ will be the closest in the least square sense, which amounts to

$$\boldsymbol{x} \quad \text{which minimizes} \quad \| \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b} \| . \tag{D.8}$$

While the solution $\boldsymbol{x}$ does not satisfy (D.6) exactly, it serves as the best approximation for the solution of (D.6). The proof is similar to (D.7). Let us modify $\boldsymbol{x}$ by adding an arbitrary vector $\boldsymbol{x}'$. $\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}$ is then modified by adding $\boldsymbol{b}'(= \boldsymbol{A}\boldsymbol{x}')$. Note that $\boldsymbol{b}'$ lies in the range of $\boldsymbol{A}$. We then have

$$\begin{aligned}
\| \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b} + \boldsymbol{b}' \| &= \| (\boldsymbol{U}\boldsymbol{W}\boldsymbol{V}^{\mathrm{T}})(\boldsymbol{V}\boldsymbol{W}^{-1}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b}) - \boldsymbol{b} + \boldsymbol{b}' \| \\
&= \| (\boldsymbol{U}\boldsymbol{W}\boldsymbol{W}^{-1}\boldsymbol{U}^{\mathrm{T}} - \boldsymbol{I})\boldsymbol{b} + \boldsymbol{b}' \| \\
&= \| \boldsymbol{U}[(\boldsymbol{W}\boldsymbol{W}^{-1} - \boldsymbol{I})\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} + \boldsymbol{U}^{\mathrm{T}}\boldsymbol{b}'] \| \\
&= \| (\boldsymbol{W}\boldsymbol{W}^{-1} - \boldsymbol{I})\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b} + \boldsymbol{U}^{\mathrm{T}}\boldsymbol{b}' \| .
\end{aligned} \tag{D.9}$$

$(\boldsymbol{W}\boldsymbol{W}^{-1} - \boldsymbol{I})$ is a diagonal matrix that has nonzero k-th elements only for $w_k = 0$, while $\boldsymbol{U}^{\mathrm{T}}\boldsymbol{b}'$ has nonzero k-th elements only for $w_k \neq 0$ because $\boldsymbol{b}'$ lies in the range of $\boldsymbol{A}$. Therefore the minimum is accomplished when $\boldsymbol{b}' = \boldsymbol{0}$.

In this way, the SVD serves as a powerful set of techniques that solve the singular system of linear equations.

Appendix E

Definition of the Surface Network

The surface network is proposed for characterizing terrain surfaces [88, 89], and is also one of the critical point graphs (CPGs). First a ridge line and a ravine line are defined [82, 83]. Let $C(t)$ denote the function $(x(t), y(t))$ of the parameter space (x, y).

Definition E.1 (Ridge and Ravine) *Suppose that a smooth surface is represented by a height function* $z = f(x, y)$. *Let* $C(t)$ *denote the function* $(C_x(t), C_y(t))$. *An ascending slope line from the point* C_0 *is a curve in* $\boldsymbol{R}$ *defined by* $C(t)$, *where* $C(t)$ *is a trajectory of the autonomous initial value problem*

$$\frac{dC}{dt}(t) = (\frac{\partial f}{\partial x}(C(t)), \frac{\partial f}{\partial y}(C(t))), \quad C(0) = C_0, \quad \textit{and} \quad 0 \leq t < \infty.$$

A descending slope line from the point C_0 *is a curve in* $\boldsymbol{R}$ *defined by* $C(t)$, *where* $C(t)$ *is a trajectory of the autonomous initial value problem*

$$\frac{dC}{dt}(t) = -(\frac{\partial f}{\partial x}(C(t)), \frac{\partial f}{\partial y}(C(t))), \quad C(0) = C_0, \quad \textit{and} \quad 0 \leq t < \infty.$$

Let P *be a pass and let* Q *be a point in a neighborhood of* P *such that the descending slope line from* Q *reaches* P. *The slope line through* Q *is then called a ridge line. Let* P *be a pass and let* Q *be a point in a neighborhood of* P *such that the ascending slope line from* Q *reaches* P. *The slope line through* Q *is then called a ravine line.*

Note that the ridge (ravine) lines cross every contour at right angles, and they go in the steepest direction through any point. In the following, a set of connected ridge (ravine) lines is also called a ridge (ravine) line.

The following is the definition of the surface network [88, 89].

Definition E.2 (Surface Network) *A surface network is a graph that satisfies the following conditions.*

(1) The vertex of the graph represents a critical point.

(2) The edge of the graph represents a ridge line from a pass to a peak or a ravine line from a pass to a pit.

Figure 5.10 illustrates the surface network with contours.

Appendix F

Notes on Continuities at Branches

Sweeping [53, 22, 19, 119] is a powerful technique for designing smooth cylindrical objects. In general, a closed curve to be swept and its trajectory is represented by B-spline functions. In this appendix, it is assumed that the control points of the closed curve move along the trajectories whose parameters are height values. This representation can be regarded as a kind of skinning [125, 139, 140], which is similar to the sweeping.

One problem that arises from this representation is that the representation suffers from the discontinuities with respect to the height value at branches. The discontinuities cause the difficulty in generating smooth surfaces around the branches.

From Morse lemma (cf. Appendix A), there exists a coordinate system that has the following quadratic representation in the neighborhood of a critical point after certain regular transformations.

$$z = f(x, y) = \begin{cases} -x^2 - y^2 & \text{for a peak} \\ \mp x^2 \pm y^2 & \text{for a pass} \\ +x^2 + y^2 & \text{for a pit} \end{cases}$$

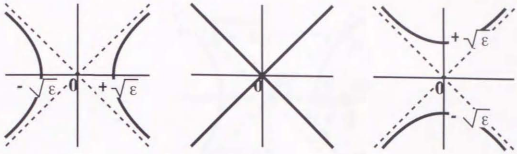
Now suppose that the surface has the following equation in the neighborhood of the pass.

$$x^2 - y^2 = C$$

Here, C represents the height value. Figure F.1 shows the changes in cross-sectional contours when we go down along the height axis from $C = +\varepsilon$ to $C = -\varepsilon$, where ε is a small positive real number. As can be seen in Figure F.1, the contours around the pass are two hyperbolas when $|\, C \,| > 0$ and two straight lines when $C = 0$.

The hyperbolas can be represented by quadratic *rational Bézier curves* [90, 64, 92]. Let $\boldsymbol{p}_0$, $\boldsymbol{p}_1$, and $\boldsymbol{p}_2$ denote three control points of the quadratic rational Bézier curve, and let w_0, w_1, and w_2 denote the weights associated with the control points. By setting

$$w = \frac{w_1}{\sqrt{w_0 w_2}}, \tag{F.1}$$



$C = +\varepsilon$ $\quad$ $C = 0$ $\quad$ $C = -\varepsilon$

Figure F.1: Changes in cross-sectional contours in the neighborhood of a pass

the rational Bézier curve is expressed by

$$\boldsymbol{p}(t) = \frac{(1-t)^2\boldsymbol{p}_0 + 2(1-t)tw\boldsymbol{p}_1 + t^2\boldsymbol{p}_2}{(1-t)^2 + 2(1-t)tw + t^2}, \tag{F.2}$$

where t is a parameter value. The Bézier curve $\boldsymbol{p}(t)$ can be classified by w as follows [64].

$w = 0$	straight line
$0 < w < 1$	ellipse
$w = 1$	parabola
$w > 1$	hyperbola

When $t = 1/2$, (F.2) becomes

$$\boldsymbol{q} = \boldsymbol{p}(1/2) = \frac{1}{1+w}\frac{\boldsymbol{p}_0 + \boldsymbol{p}_2}{2} + \frac{w}{1+w}\boldsymbol{p}_1. \tag{F.3}$$

Let $\boldsymbol{m}$ be the midpoint of the segment $\boldsymbol{p}_0\boldsymbol{p}_2$. Hence the above equation becomes

$$\boldsymbol{q} = (1-s)\boldsymbol{m} + s\boldsymbol{p}_1, \tag{F.4}$$

where

$$s = \frac{w}{1+w}. \tag{F.5}$$

Thus, the following equation can be obtained.

$$w = \frac{|\boldsymbol{m} - \boldsymbol{q}|}{|\boldsymbol{q} - \boldsymbol{p}_1|}. \tag{F.6}$$

Let us consider the case when $C = \varepsilon > 0$. If $\boldsymbol{p}_0$, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ have the coordinates $(1,\ -\sqrt{1-\varepsilon})$, $(0,\ 0)$, and $(1,\ \sqrt{1-\varepsilon})$ respectively as illustrated in Figure F.2, the following equation is obtained.

$$w = \frac{1 - \sqrt{\varepsilon}}{\varepsilon} \tag{F.7}$$

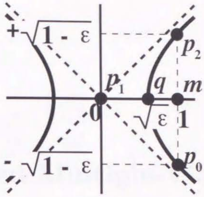


Figure F.2: The hyperbolas $x^2 - y^2 = \varepsilon$ where $\varepsilon < 0$

As the value ε approaches 0, the value w approaches ∞. This means that it is impossible to describe the changes of cross-sectional contours at the pass with the parametric curves, i.e., the quadratic Bézier curves.

This study solves this problem by introducing the local coordinates of the pass using the techniques of manifold mappings.

Appendix G

Manifold-Based Multiple-Viewpoint CAD – A Case Study of Mountain Guide-Map Generation –

There are kinds of pictures that are drawn with *multiple viewpoints*. *Mountain guide maps*, *cubism pictures* and *medical diagnosis drawings* are examples of such pictures. So far, however, they have been drawn intuitively by hand. Implementing a CAD system for such pictures requires clear modeling of their drawing processes. As a case study, this appendix presents *mountain guide-map modeling* using *manifolds* and implementation of *multiple-viewpoint CAD* based on the model. Projecting a land surface as seen from multiple viewpoints is conducted interactively using the CAD system. Finally, basic images for mountain guide maps are generated automatically.

G.1 Assumptions on Mountain Guide-Map Generation

Many ancient and medieval paintings were drawn having *multiple viewpoints*, describing objects to illustrate their points of interest. After the Renaissance time when the *perspective* view became dominant as an exact and hence scientific way of drawing, *multiple-viewpoint pictures* have been declining, and have survived only in limited cases such as *mountain guide maps*, *diagnosis drawings* of medical doctors (Figure G.1), and in some schools of art, for example *cubism*.

In our human memory, we remember scenes of our homelands as seen from various viewpoints. When we try to understand how machines are configured, we draw them as seen from different sides. In human visual cognition, multiple-viewpoint pictures are natural and there is no reason for them to be rejected. One illustrative example is our visual memory. Everybody remembers their homeland as they see it when they travel around its different locations.

This research is a step toward the science of multiple-viewpoint pictures. A case study was conducted to prove a hypothesis that there is a way to model multiple-viewpoint pictures "exactly"("exact" in a scene that we can define them without ambi-

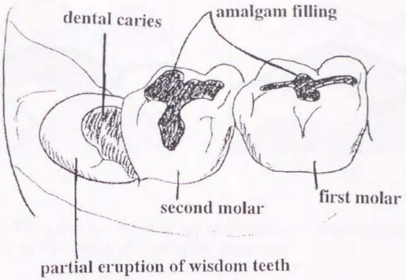


Figure G.1: A dental diagnosis drawing: the region containing the wisdom teeth has a different viewpoint from the others so that a doctor can see its dental caries.

guity and hence can automatically generate them). A popular case of *mountain guide maps* was studied.

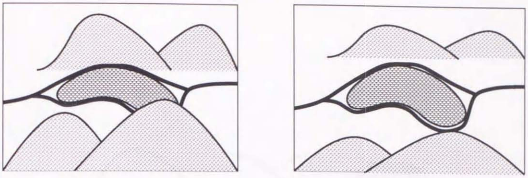
In a mountain guide map, *mountain tops*, *mountain passes*, and *lakes* are extracted to characterize land undulations. To represent such land features clearly, mountain guide maps are drawn with multiple viewpoints. For example, let us consider the difference between an ordinary *perspective picture* and a *mountain guide map* as illustrated in Figure G.2. Figure G.2(a) shows an example of an ordinary perspective picture. The lake is partially hidden by surrounding mountains while the mountain skyline is seen clearly. Figure G.2(b) shows an example of a mountain guide map where the viewpoint of the area containing the lake is changed so that we can see the whole scene of the lake from a height as well as the mountain skyline as seen from the foot of the mountain. In this way, a mountain guide map can extract as much information as possible when projecting a 3-dimensional land surface onto a 2-dimensional plane.

So far, commercially available mountain guide maps have been drawn without appropriate modeling and hence they include various ambiguous representations. The modeling of the drawing processes requires a clear understanding of the mountain guide-map generation processes. The typical drawing processes of a mountain guide map are as follows;

(1) Select an area that includes one mountain top, mountain pass, or lake.

(2) The areas are pasted together to construct the overall land surface.

(3) Each area is projected as seen from a *viewpoint*, usually a *vista point* of the area.



(a) (b)

Figure G.2: The difference between (a) an illustration of an ordinary perspective picture and (b) an illustration of a mountain guide map

In this thesis, a mountain top, a mountain pass and a lake are called *characteristic points*, and the area including one characteristic point is called a *characteristic area*. Figure G.3 illustrates the above processes. In fact, the processes can be modeled as the creation of a manifold. In the following, it is assumed that mountain guide maps are constructed through these processes.

This appendix presents *mountain guide-map modeling* using *manifolds* [59] and its implementation [121]. The method of projecting a land surface as seen from multiple viewpoints is realized using a family of blending functions. A mountain guide-map CAD system is implemented, and multiple-viewpoint images for mountain guide maps are generated.

G.2 Multiple-Viewpoint Projection

A mountain guide map is not an ordinary perspective image but an image with multiple viewpoints. For example, mountains are projected as seen from the foot of the mountain to show the mountain skyline clearly. Lakes are projected as seen from a height so that we can see the whole scenes of the lakes. Because of these considerations, it is necessary to implement a method of projecting a land surface as seen from multiple viewpoints.

There are several researches on *perspective projection* techniques. *Ordinary perspective projection* is explained in [13] and [86]. *Non-linear perspective projection* is discussed in [79] and [48]. *Viewpoint analysis* of multiple-viewpoint pictures is proposed in [111].

This section is devoted to the explanation of projecting a land surface from multiple viewpoints. When the viewpoint of each characteristic area is given, the system automatically generates the rendered image of a land surface with multiple viewpoints. For this purpose, smooth viewpoint interpolation, which we call *viewpoint blending*, is

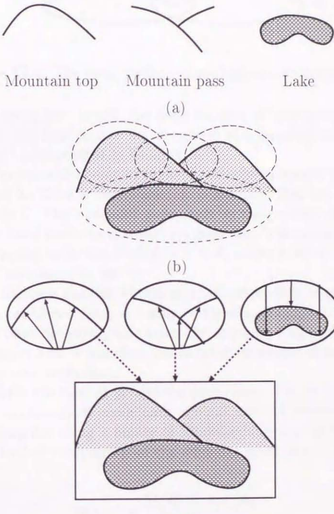


Figure G.3: Typical drawing processes of a mountain guide map

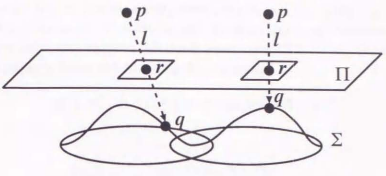


Figure G.4: The basic method of multiple-viewpoint projection

required. Before going into details, the basic method of multiple-viewpoint projection is explained. Here, the land surface is represented as a manifold constructed from a set of charts $\{(U_k, \varphi_k)\}$ as explained in Chapter 3.

Figure G.4 illustrates the basic method of multiple-viewpoint projection. Let Σ be a land surface and let Π be a *view plane*. It is assumed that the *reference point* $\boldsymbol{q}$ is on the land surface Σ. The *viewpoint* $\boldsymbol{p}$ depends on the position of the reference point $\boldsymbol{q}$. Let us call the line l connecting $\boldsymbol{p}$ and $\boldsymbol{q}$ a *view line*. The projection mapping $\mathsf{Proj} : \Sigma \rightarrow \Pi$ is the mapping such that $\mathsf{Proj}(\boldsymbol{q}) = \boldsymbol{r} \in \Pi$, where $\boldsymbol{r}$ is the intersection point of the view plane Π and the view line l.

To determine the view line, the viewpoint that corresponds to the coordinates of the reference point is calculated using a family of blending functions. The reference point is then projected onto the view plane using the mapping Proj explained above. In the following, we consider how to calculate the blended viewpoint in the case of *perspective projection* and *parallel projection*.

First we consider the case of *perspective projection*. Let us assign a set of charts $\{(U_k, \varphi_k)\}$ to the characteristic areas, each of which has the viewpoint $\boldsymbol{p}_k$. We can then find a blended viewpoint using a family of blending functions as presented in Chapter 3. The smooth blended viewpoint $\boldsymbol{p}$ of the reference point $\boldsymbol{q}(u, v, f(u, v))$ is calculated as follows.

$$\boldsymbol{p}(u, v) = \frac{\sum_k B(u_k, v_k) \cdot \boldsymbol{p}_k}{\sum_k B(u_k, v_k)} \tag{G.1}$$

Here, we use the notations of Chapter 3.

We can also assign *weight* parameters to the viewpoints of the charts. Let $w_k{}'$ be the weight parameter of the chart (U_k, φ_k) of the perspective projection. The equation of the blended viewpoint is modified as

$$\boldsymbol{p}(u, v) = \frac{\sum_k w_k{}' \cdot B(u_k, v_k) \cdot \boldsymbol{p}_k}{\sum_k w_k{}' \cdot B(u_k, v_k)}. \tag{G.2}$$

In the case of *parallel projection*, the mapping Proj can be determined if the view direction $\boldsymbol{m}$ is given. The view line l is then expressed as $t\boldsymbol{m} + \boldsymbol{q}$ where t is a parameter value. Assume that the chart (U_k, φ_k) has its own view direction $\boldsymbol{m}_k$, which is expressed by the unit vector $\boldsymbol{m}_k = (\cos\Theta_k \cos\Phi_k, \sin\Theta_k \cos\Phi_k, \sin\Phi_k)$ with two parameters Θ_k and Φ_k. Here Θ_k is an *angle of rotation* and Φ_k is an *angle of elevation*.

We can now find the smooth blended view direction $\boldsymbol{m}$ of the reference point $\boldsymbol{q}(u, v, f(u, v))$ using a family of blending functions as,

$$\boldsymbol{m}(u, v) = (\cos\Theta \cos\Phi, \sin\Theta \cos\Phi, \sin\Phi), \tag{G.3}$$

where

$$\Theta(u, v) = \frac{\sum_k B(u_k, v_k) \cdot \Theta_k}{\sum_k B(u_k, v_k)}, \quad \text{and} \tag{G.4}$$

$$\Phi(u, v) = \frac{\sum_k B(u_k, v_k) \cdot \Phi_k}{\sum_k B(u_k, v_k)}. \tag{G.5}$$

Let w_k'' be the *weight* parameter of the chart (U_k, φ_k) of the parallel projection. The angles Θ and Φ are represented with the weight parameters as,

$$\Theta(u, v) = \frac{\sum_k w_k'' \cdot B(u_k, v_k) \cdot \Theta_k}{\sum_k w_k'' B(u_k, v_k)}, \quad \text{and} \tag{G.6}$$

$$\Phi(u, v) = \frac{\sum_k w_k'' \cdot B(u_k, v_k) \cdot \Phi_k}{\sum_k w_k'' \cdot B(u_k, v_k)} \tag{G.7}$$

We can thus find the smooth viewpoint or direction blended among the charts, and the land surface is projected with multiple viewpoints or directions.

G.3 Results

A prototype system of a mountain guide-map CAD is implemented based on the presented model. With the system, multiple-viewpoint images can be generated. This section provides examples of the basic images[1] for mountain guide maps.

Using the prototype system, the view parameters of the charts such as viewpoints or view directions are specified. View parameter setting with chart assignment is shown in Figure G.5. With these given parameters, the prototype system automatically generates a basic image for a mountain guide map. Modification of these parameters is also conducted easily and interactively using the CAD system.

The following images are generated in the prototype CAD system. Figure G.6 and Figure G.7 are the basic images for the mountain guide maps around Lake Ashinoko, which is a famous tourist area with a scenic crater lake in Japan. Here, we see the influences on the images when the viewpoint or view direction of the area including the lake is changed. Figure G.6(a) is the image of the perspective projection with one viewpoint. This is the same as that of ordinary perspective projection. Figure G.6(b) is the image of the perspective projection with multiple viewpoints. The viewpoint of

[1] Here, a map without land marks such as stations, bus stops, and hotels is called a *basic image* for a mountain guide map.

the area including the lake is changed to one from a height to avoid the surrounding mountains. Figure G.7(a) is the image of the parallel projection with one view direction. This is the same as that of ordinary parallel projection. Figure G.7(b) is the image of the parallel projection with multiple view directions. The whole scene of the lake can be seen because the area including the lake is as seen from a different view direction from the others. Figure G.8 is a pair of basic images for the mountain guide map around Mt. Fuji, which is the highest mountain in Japan. Figure G.8(b) is designed using the prototype system so that the scenic features of the area can be seen better in this figure than in Figure G.8(a). These results demonstrate the capability of the prototype system.

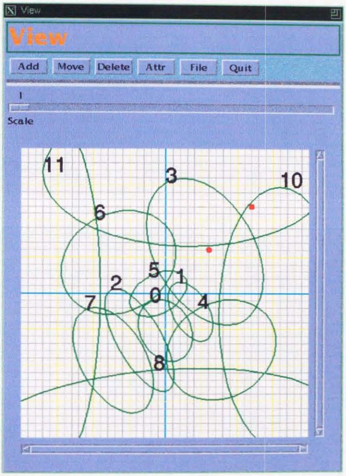
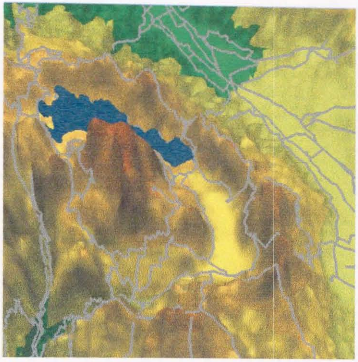


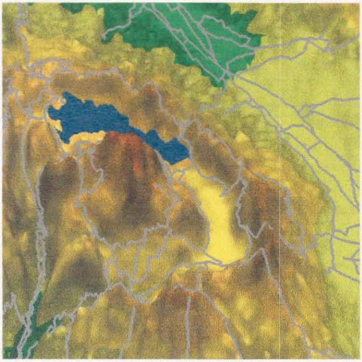
Figure G.5: A display example of view parameter setting with chart assignment



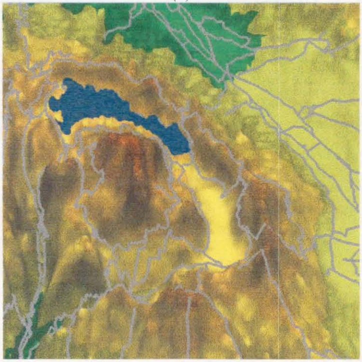
(a)

(b)

Figure G.6: The image of perspective projection with (a) one viewpoint and (b) multiple viewpoints

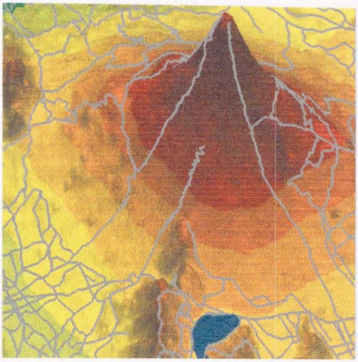


(a)

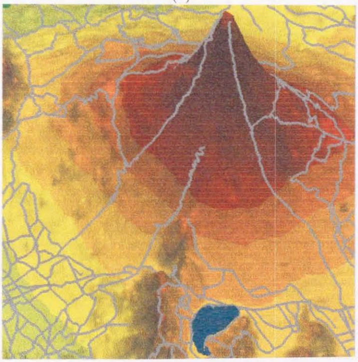


(b)

Figure G.7: The image of parallel projection with (a) one view direction and (b) multiple view directions



(a)



(b)

Figure G.8: The basic image for the mountain guide map around Mt. Fuji of (a) ordinary projection and (b) multiple-viewpoint projection

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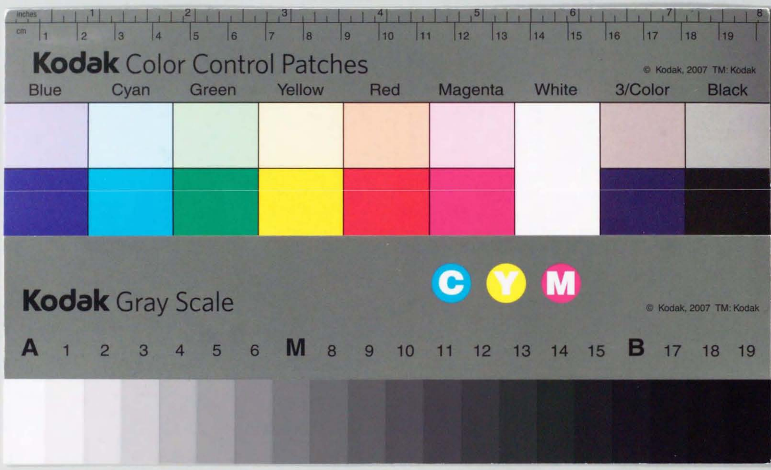

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