博士論文

Securitization and Heterogeneous-Belief Bubbles with Collateral Constraints

（証券化および担保制約下の信念の違いによるバブルについて）

東京大学大学院経済学科研究科博士課程

前川　淳

Securitization and Heterogeneous-Belief Bubbles with Collateral Constraints

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Abstract

Miller(1977) or Harrison and Kreps(1978) show asset price is higher in heterogeneous model than common prior model. They assume no budget constraint or no limitation of financial market. Recent study explore the role of financial technology in heterogeneous belief model. In this paper, I show that some financial technology make the asset price as high as Harrison and Kreps. Key technology is securitization and especially loan backed security. I also show the asset price is put down by an insurance with short sale. These prices implies that financial technology may make bring large instability to asset markets.

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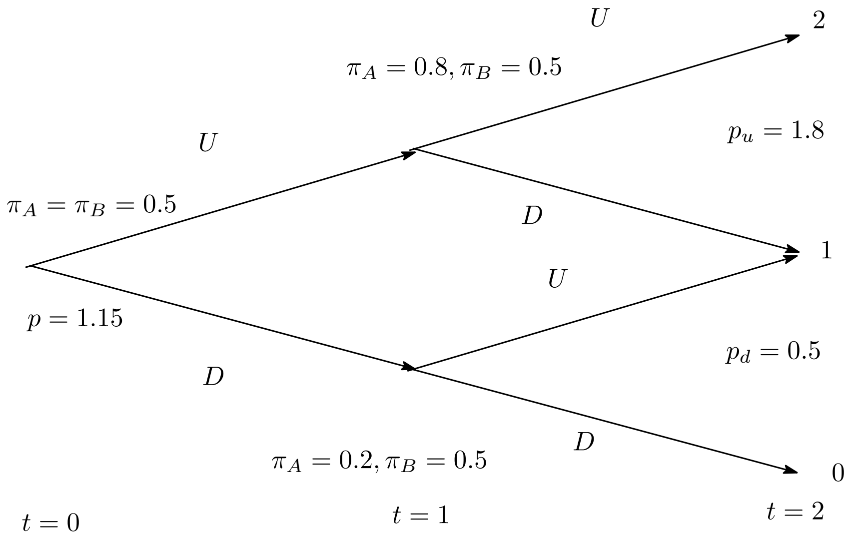


Part I
Introduction

1 Heterogeneous Belief Bubbles with Collateral Constraints

1.1 Heterogeneous Belief Bubble and Financial Technology

Heterogeneous beliefs among investors bring bubbles. In these models, investors' beliefs differ because they have different prior belief distributions.
Agents' heterogenity occur from many factors. For example, if there are over-conident about the precision of signals among investors , this leads to different prior distributions (with lower variance) about the signals' noise term. Investors without common priors can agree to disagree even after they share all their information.
In heterogeneous beliefs model with short-sale constraints, the asset price can result in bubbles. Optimistic agents buy the asset and the price rises, In short sale constraint, pessimistic traders cannot make use of the high asset prices.(Miller (1977)).
In a dynamic model, the asset price can even exceed the valuation of the most optimistic investor's expectation in the economy.
Today's optimistic traders have the option to resell the asset in the future at a high price whenever they become less optimistic. At that point other traders will be more optimistic and thus willing to buy the asset (Harrison and Kreps (1978)).

Figure provides a simple example to illustrate how heterogeneous beliefs can lead to prices that exceed even the valuation of the most optimistic agent in the economy. In the example there are two traders, A and B. with heterogeneous beliefs π_A and π_B. At each point, traders have a probability of π for good news U. Both traders have the same expected return of the asset $E_A[s] = E_B[s] = 1$ if they have to hold it until t = 2.
However, if they can resell the asset in t = 1, the asset value exceeds 1.15 for both traders. Trader B knows that he can sell the asset to A, in state U. At state U, trader A is more optimistic than trader B. Trader A expects to sell the asset to B in state D, because B is the optimist at that point. By allowing resell option, both investors are willing to pay $p = 1.15$ at time t = 0, even though their holding value is only 1. The price of the asset thus exceeds even the most optimistic agent's valuation of the asset.
A important feature of heterogeneous-beliefs bubbles is that bubbles occur with large trading volume and high price volatility(Scheinkman and Xiong (2003)). These feature is seen in recent bubbles like internet bubbles in 1990s (Ofek and Richardson(2003).
A number of papers also explore the role of credit in heterogeneous-beliefs models. If optimistic agents have limited wealth in heterogeneous belief model, they cannot buy assets and bubble prices may not be achieved. Assets are held by pessimistic agents and the asset prices decline. So, financial markets are very important for optimistic agents and for the asset prices. One important issue in financial market is collateral loan contracts. In recent crisis, loan contract with collateral is very important in financial market. This is because, for many assets, there is a class of optimists whom the asset is more valuable than it is for the rest of traders (standard economic theory, in contrast, assumes that asset prices reflect some fundamental value). These optimists are willing to pay more. If they can get their hands on more money through more highly leveraged borrowing (that is, getting a loan with less collateral), they will spend it on the assets and drive those prices up.
By collateraled loan contract, agents who have limited wealth can easily borrow money from lenders. In collateraled loan contract, "optimistic" agents can get more risky asset and "pessimistic" agents get more safe asset, like money. In subprime loan, optimistic homeowner borrow much money by loan with collateraled their house. This scheme brought serious instability in financial market. Another issue is one of financial technology, securitization.

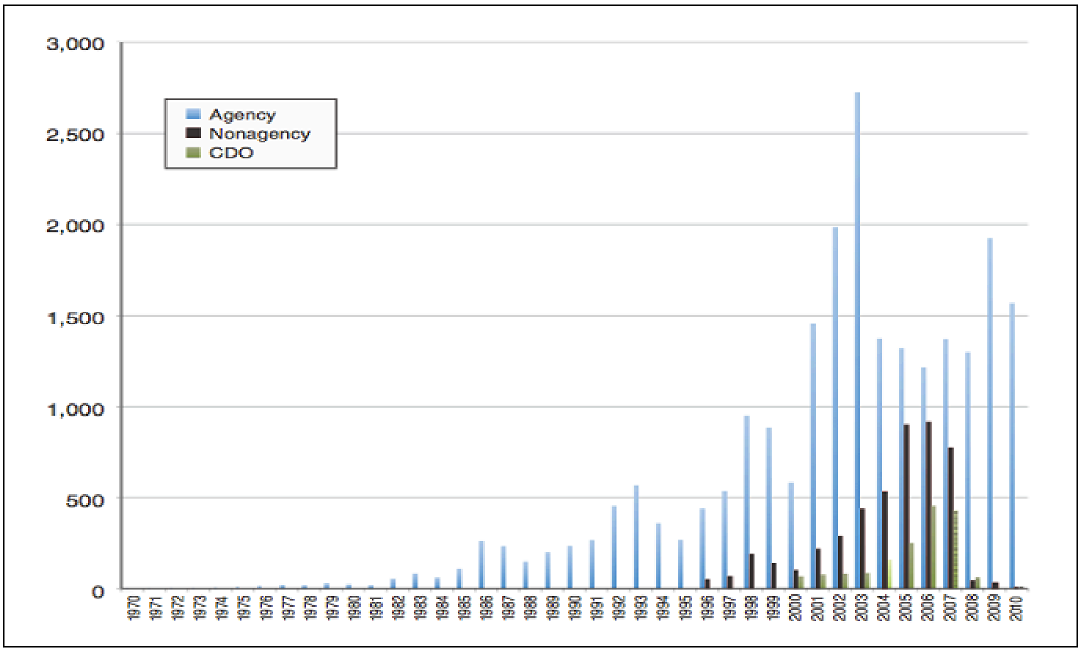
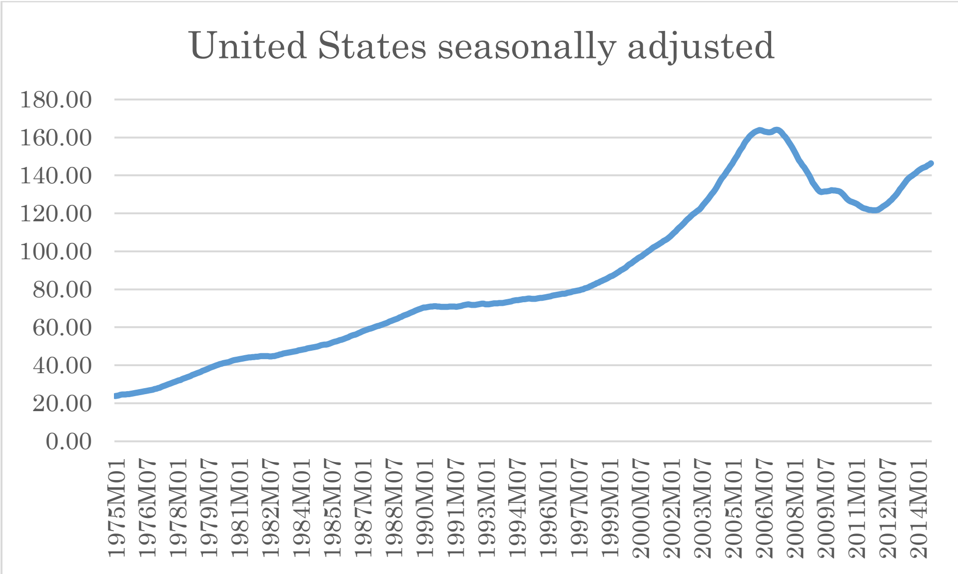


Figure (from Geanakoplos(2012)) shows that the market of the securitization and CMO tranching grew during the housing boom, which in their peak amounted to trillions of dollars a year.



Next figure shows a housing market boom and bust in recent U.S. crisis(Freddie Mac housing price index). The U.S. housing boom started around 1999. After the peak of the price in early 2006, the price declined.

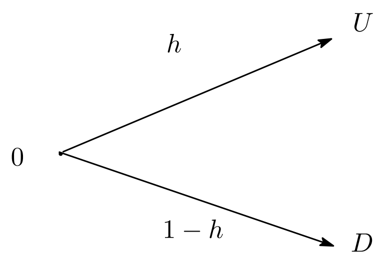
There is a close relation between financial innovations and bubbles. Many researcher said that financial innovation like leverage or MBS, increase bubbles (Shiller(2000), Brunnermeier and Oehmke(2013)). In fact, the size of financial innovation such grew in recent U.S. crisis. (see figure)

I will explain the relation between the asset price and financial technology like collateral or like sucuritization by introducing three papers.

Geanakoplos (2010) develops a model in which agents with heterogeneous beliefs have limited wealth, such that agents with optimistic views about an asset borrow funds from more pessimistic agents, with loan contracts with collateral. Because pessimistic agents have lower evaluation about the asset return, their finances to optimistic agents are not enough for optimistic belief.

In this model, optimists tend to make risky investment. When low asset return states are realized, optimists lose their wealth and more of the asset has to be held by pessimists, The asset price must be affected by pessimistic belief.

In contrast to models in the spirit of Harrison and Kreps (1978), in this setup beliefs do not change over time, such that no resell premium arises. Hence, the price of the asset always remains below the valuation of the most optimistic



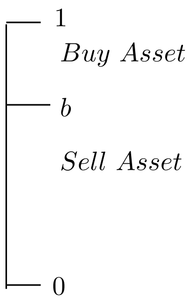
agent.
Simsek (2013) shows a similar result in their heterogeneous model. Simsek focused on the belief disagreement between optimists and pessimists. The extent to which pessimists are willing to finance asset purchases by optimists depends on the specific form of the belief disagreement. Intuitively speaking, when disagreement is mostly about the upside, pessimists are more willing to provide credit than when disagreement is about the downside. Hence, it is not just the amount of disagreement, but also the nature of disagreement among agents that matters for asset prices.
Fostel and Geanakoplos (2012) provide models in which different financial innovations affect the economic situation. Financial innovation like tranching, one of a securitization technology, raises asset prices by increasing the ability of the optimist. By tranching, optimists can take more risky positions in the asset. At the same time, in their model an unexpected introduction of credit default swaps (CDS) can lead to drastic reductions in asset prices, since it allows pessimists to more effectively bet against the bubble asset.

1.2 Heterogeneous Belief with Collateral Constraint

I will explain that the asset price is affected by loan with collateral by using model of Genakoplos(2010) and Fostel and Genakoplos(2012).
Consider a simple example with one consumption good (C), one risky asset (Y), two time periods $t = 0, 1$, and two states of nature (U and D) in next period ($t = 1$). Each unit of Y pays either 1(good) or 0.2(bad) in the two states U or D, respectively.

The asset can pays more than 0.2 at each states. 0.2 is promised by any traders who have one unit of the asset.
Let every agent own one unit of the asset at time 0 and also one unit of the consumption good at time 0. For simplicity, the consumption good is assumed to be costlessly stored in a quantity denoted by w. It is considered as money.
The agent $h \in H$ only care about the total expected consumption , no matter when they get it. They are not impatient.

The agents are distributed uniformly on a continuum, with $h \in H = [0, 1]$. The agents have heterogeneous belief about the risky asset. The agent h assigns probability h to the good outcome (1).
Storing goods and holding assets provide no difference for agents; they just



increase income in the future. Suppose the price of the asset per unit at time 0 is p, somewhere between zero and one.
The agents h who believe that the asset price is lower than his expected return:

$$h + (1 - h)0.2 > p \tag{1}$$

These agents will buy the asset.
The agent who believe that the asset price is higher than his expected return:

$$h + (1 - h)0.2 < p \tag{2}$$

These agents will sell the asset.
In equilibrium, there is a marginal buyer h_m who believe the asset price is equal to his expected return:

$$h_m + (1 - h_m)0.2 = p \tag{3}$$

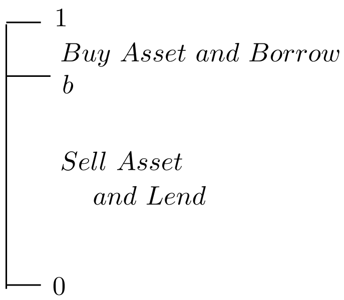
For the agent, purchasing the asset and storing the consumption good are indifferent. The asset price is determined by his expected return h_m.

The total consumption good is one and the total asset supply is one at date 0.
The agent who has higher h than the marginal buyer(h_m) sell the consumption good and buy the asset. Optimistic agents sell $1 - h_m$ units of consumption good and buy h units of the asset with price p.

$$1 - h_m = ph_m \tag{4}$$

In the equilibrium, the asset price p is about 0.68. The marginal buyer is about $h_m = 0.6$.

$$0.6 + (1 - 0.6)0.2 = 0.68 \tag{5}$$



In this economy, we introduce a loan market with collateral. This loan market is noncontingent, that is, that involve promises of the same amount of ϕ in both states.
Because agents are heterogeneous, the only way to promise the payoff is collateral. Borrowers can use the asset itself as collateral. If they default, they give the collateral to lenders. Lenders know that if the promise is ϕ in both states, they will receive: $min(\phi, 1)$ in good news and $min(\phi, 0.2)$ in bad news.
Suppose agents promise at most 0.2 units of consumption per unit of the collateral asset. That is a natural limit, since it is the biggest promise that is sure to be covered by the collateral. It also greatly simplifies our notation, because then there would be no need to worry about default.
No borrowing equilibrium is reinterpreted a tight leverage economy, where we have the constraint $\phi \leq 0$.
By using collateral to borrow, the most optimistic agents get a chance to buy the asset more. This will push up the price of the asset. But since they can borrow strictly less than the value of the collateral, optimistic wealth will still be limited. Pessimistic agents have lower evaluation about the collateral (the asset). Optimists can not borrow from the collateral loan contract.
In this economy, optimistic agents buy the asset, borrow by the collateral loan and buy the asset more. Pessimistic agents donot need cash for buying the asset, so they are willing to lend it.

The marginal buyer is indifferent to buying the asset.

$$h_m + (1 - h_m)0.2 = p \tag{6}$$

The price of Y is equal to the amount of money the agents above h_m. The numerator is then all the top group's consumption endowment, $1-h_m$ optimistic agents have $1 - h_m$ units of money. In equilibrium, they hold all the asset (one unit of the asset) and they can borrow 0.2 money per unit of the asset. They buy h units of the asset from pessimistic agents.

$$ph_m = \frac{(1 - h_m) + 0.2}{h_m} \tag{7}$$

These two equation determine the asset price p and the marginal buyer h_m. In the equilibrium, the asset price p is 0.75. The marginal buyer is $h = 0.69$.

The asset price rises from 0.68 to 0.75. By introducing the loan with collateral, the asset price gets higher. As the leverage gets higher, more optimistic agents buy the assets.
If no budget constraints model, that is, agents have enough money to trade, the asset is bought by the most optimistic agent. In this model, $h = 1$ agent buy all the asset. The asset price is simply equal to his expected return, $p = 1$.
In heterogeneous agents with limited budget model, the loan with collateral brings drastic change to the asset price. In general, the asset is held by optimistic agents in heterogeneous belief model. Optimistic agents can borrow money by using the asset as collateral. That is, buying the asset itself is the way of collecting money. Then, optimistic agents can buy the asset more and can borrow money more.
However, this effect is limited. In heterogeneous model, pessimistic agents are natural lender. They have low evaluation about the asset return. Although optimistic agents must borrow money from them, pessimists' lending is not enough from optimistic view. The asset price is higher than no borrowing model, but it is lower than usual heterogeneous belief model like Miller.
In this economy, the asset price changes with agents' belief. More concretely, with the difference of their beliefs. If pessimists have very low expectation about the return of the asset comparison with optimists, they cannot borrow enough money. As a result, they cannot raise the asset demand and the asset price gets lower. Simsek shows the linkage between their beliefs and the asset price (Simsek(2013)).

1.3 Belief Disagreement and Collateral Constraint

The settings of Simsek(2013) are simpler than Geanakoplos(2003). It is dynamic, two period model. One risky asset and money exist in the economy. At period 0, agents trades the asset and money. The asset bring random return s in period 2. There is same loan contract with collateral as Geanakoplos. Agents can borrow money by using the asset as collateral.
There are two type optimists, optimists and pessimists. They have different belief about the asset return. There is a continuum of states at date 1, denoted by $s \in S = [0, s^{max}]$. The asset pays s dollars at state s. There are two type of traders, optimist and pessimist. Both type traders have risk neutral utility functions. But, they have different beliefs about asset return. Type j trader have a belief about asset return , distribution function $F_j(s)(j = o, p)$. Note their expectation of asset return is $E_o[.], E_p[.]$. Optimistic agents have higher expectation than pessimistic agents.

$$E_o[s] > E_p[s] \tag{8}$$

Traders know each others priors. Optimists and pessimists agree to disagree about their beliefs.
Then, optimists are natural buyer of the asset. Optimists want to buy the asset, but they have little money. Pessimists have low evaluation about the

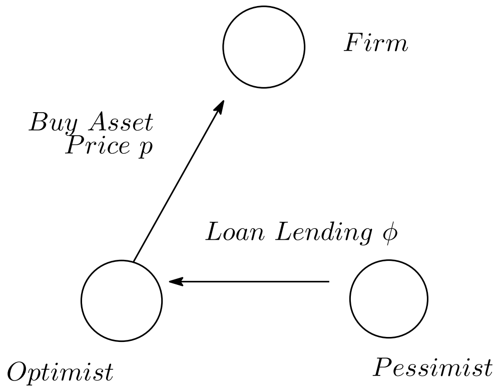


Figure 1: Simsek(2013)

asset. They can earn money by lending to optimists.
A collateral loan contract per asset at period 1 is defined as (γ, ϕ). ϕ is the amount of cash borrowing and γ is the borrowers promise payment after the revealment of s. If the borrowers cannot pay γ at date 2, they must give one unit of the asset as collateral to the lenders. The borrowers make a take-it-or-leave-it offer (γ, ϕ) to lenders.
If s is low at date 2, the loan default may occur. In this case, the borrowers must give the asset to the lenders as collateral.

- If the asset return s is high $(s > \gamma)$, the optimist can pay the promise γ to the pessimist.
- If the asset return s is low $(s \leq \gamma)$, the optimist gives the asset return s to the pessimist.

Totally, borrowers give $min(s, \gamma)$ to lenders in a loan contract (γ, ϕ).

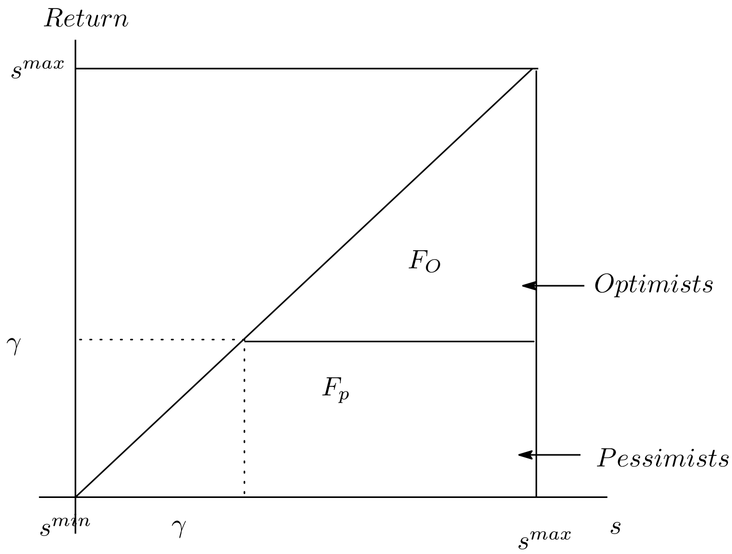
Pessimists want to lend for earning money. As the loan market is competitive, pessimists' profit is zero in equilibrium. Pessimists lend ϕ money and receive $min(s, \gamma)$ at period 2.
Their profit by the loan contract (ϕ, γ):

$$\phi - E_p[min(s, \gamma)] \tag{9}$$

Pessimists' no arbitrage condition imply:

$$\phi = E_p[min(s, \gamma)]$$

Optimists knows that the loan offer (ϕ, γ) is accepted if their no arbitrage condition is satisfied. Optimists buy the asset with price p and receive the return s at period 2. Optimists' profit by holding one unit of the asset is $E_o[s] - p$
Let a is the asset demand. They can borrow money $a\phi$ by the loan contract. They must repay $min(s, \gamma)$ to pessimists at period 2. Optimists' profit of making



the loan contract (ϕ, γ) is $\phi - E_o[min(s, \gamma)]$.
Then, optimists' problem is:

$$\begin{aligned} &\max_{a,\gamma} a(E_o[s] - p) \\ &\qquad + a(\phi - E_o[min(s, \gamma)]) \\ &\quad s.t.\ ap = n + a\phi, \\ &\qquad \phi = E_p[min(s, \gamma)] \end{aligned} \tag{10}$$

Refine the problem:

$$\begin{aligned} &\max_{a,\gamma} a(E_o[s] - E_o[min(s, \gamma)]) \\ &\quad s.t.\ ap = n + aE_p[min(s, \gamma)] \end{aligned} \tag{11}$$

FOC implies:

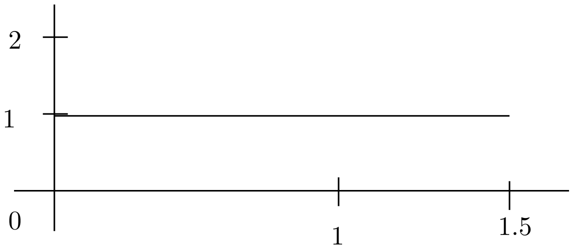
$$p = \int_{s^{min}}^{\gamma} sdF_p + \frac{1 - F_p(\gamma)}{1 - F_o(\gamma)} \int_{\gamma}^{s^{max}} sdF_o$$

The asset price is evaluated by two beliefs, optimistic and pessimistic. The asset will be held by optimists if high return sate is realized and it will be held by pessimists if low state is realized. The asset price formula imply high sate $s > \gamma$ area is evaluated by optimists and low state $s < \gamma$ is evaluated by pessimists.

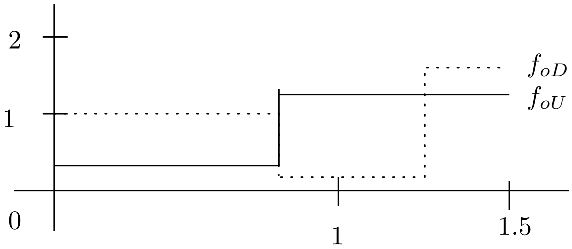
Market clearing condition $a = 1$ implies:

$$p = n + E_p[min(s, \gamma)]$$

Pessimists PDF



Optimists PDF



Optimistic agents use their own cash n and the borrowing cash $E_p[min(s, \gamma)]$ for buying the asset.
These two equations determine the equilibrium γ and the asset price p.
The following example is from Simsek(2013).
Consider the state space$S = [1/2, 3/2]$. Consider the following two cases that differ in the type of optimism. Case (i).First suppose that pessimists and optimists have the prior belief distributionsF_p and F_{oD} with density functions:

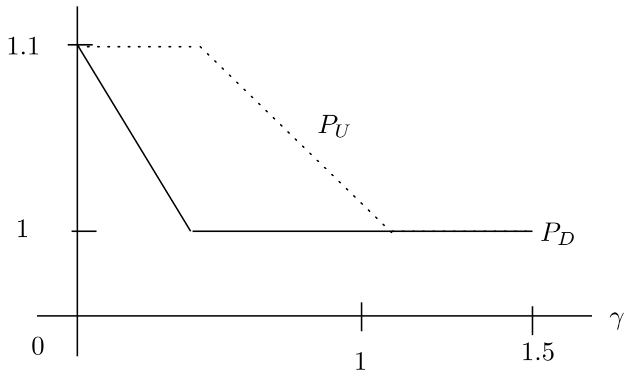
$$f_p = 1 \text{ for all } s \in S \tag{12}$$

$$f_{oD} = \begin{cases} 0.4 & \text{if s} \in [1/2, 5/6] \\ 1.3 & \text{if s} \in [5/6, 3/2] \end{cases} \tag{13}$$

Next suppose that pessimists have the same belief, but optimists ' belief is changed to the distribution F_{oU} with density function:

$$f_{oU} = \begin{cases} 1 & \text{if s} \in [1/2, 5/6] \\ 0.1 & \text{if s} \in [5/6, 7/6] \\ 1.9 & \text{if s} \in [7/6, 3/2] \end{cases} \tag{14}$$

Because $E_{oD}[s] = E_{oU}[s]$, two optimists have the same expectation about the asset return.
Figures show their beliefs.
From the equilibrium condition:



$$p = \int_{s^{min}}^{\gamma} s dF_p + \frac{1 - F_p(\gamma)}{1 - F_o(\gamma)} \int_{\gamma}^{s^{max}} s dF_o$$

The asset price is affected by their beliefs. In this example, pessimists beliefs are same. If γ gets higher, the asset price gets nearer to pessimistic expected asset return.
The asset price in f_{oU} case is higher than in f_{oD} case for all s. f_{oU} optimists have higher probability about the event of good return than f_{oD}. Because the asset price formula implies that only high return state of optimistic belief influences the asset price. Probability of low return state is not applied to the asset price.

In Simsek(2013), optimistic traders cannot have enough money in the collateral loan contract. As a result, only one part of optimistic belief is reflected in the asset price.
This is contrary to the theory of heterogeneous belief bubble like Miller(1977) or Harrison and Kreps(1978). In heterogeneous belief model, it is natural that most optimistic agents buy the asset. This implies that the asset price must be the agent's expected return(Miller(1977)). In Harrison and Kreps(1978), they show that the asset price may exceed most optimistic agents' expectation in a dynamic model that most optimistic agents changes over time.
The collateral loan influence is limited because of the disagreement of agnets' beliefs. However, Fostel and Genakoplos(2012) show this limitation can be overcome by financial technology, securitization.

1

Buy Asset and Sell Down Tranche

b_1

Hold The Duable Good

b_2

Buy Down Tranche

0

1.4 Financial Innovation and Collateral Constraint

The basic situation of Fostel and Geanakoplos(2012) is the same as Geanakoplos(2010). One consumption good (C), one risky asset (Y), two time periods $t = 0, 1$, and two states of nature (U and D) in the last period ($t = 1$), each unit of Y pays either 1 or 0.2 of the consumption good in the two states U or D, respectively.
Every agent own one unit of the asset at time 0 and also one unit of the consumption good.
The agents arranged uniformly on a continuum, with $h \in H = [0, 1]$. The agents have heterogeneous belief about the risky asset. The agent h assigns probability h to the good outcome.
In this model, a new financial technology, tranching, is introduced. Tranching is one of securitization. The holder of the asset can issue some state contingent claims. For example, they can issue some security in which they will pay some return only at good return state.
In this economy, the holder of the asset can issue Arrow D security, in which they will pay 0.2 to the holder of the asset. They can also create the Arrow D security. Notice that by buying the asset y and selling off the tranche $(0, 0.2)$, any agent can obtain the Arrow U security. They can receive the asset return only at U state.
In this case, there will be two marginal buyers h_1 and h_2 in equilibrium.
In equilibrium, all agents $h > h_1$ will buy all of y, and sell the down tranche $A = (0, 0.2)$, hence effectively holding only the Arrow U security. They believe that U will occur with high probability. These investor can recieve the asset return only at U state. Then, for optimists, this return scheme is very profitable.
Agents $h_1 > h > h_2$ will sell all their endowment asset y and buy all of the consumption good X. They have medium belief about the asset return. They believe both U and D occur with some extent probability. Then, they want risk free asset, the consumption good.
Finally, agents $h < h_2$ will sell their assets and buy the down tranche from the most optimistic investors. They believe that D wii occur with high probability. So, Arrow D security is most profitable for them.
Let π_D denote the price of the down tranche. Following equation states that total money spent on the asset should equal to the aggregate revenue from sale

of Arrow D security and optimists' endowment consumption good. The top $1-h_1$ agents are buying the asset and selling off the down tranche.
They each have wealth $1+p$ plus the revenue from optimists' sale of the tranche π_D . Finally, there is one unit of total supply of the asset.

$$(1-h_1)(1+p)+\pi_D = p \tag{15}$$

Following equation states that total money spent on the down tranche should equal to aggregate revenues from pessimists' sale of the asset p and pessimists' consumption good. The bottom h_2 agents spend all their endowments to buy all the down tranches available in the economy (which is one since there is one asset), at the price of π_D :

$$h_2(1+p) = \pi_D \tag{16}$$

Following equation states that h_1 is indifferent between buying the Arrow u security and holding the consumption good. So his expected marginal utility from buying the Arrow u security.

$$h_1 = p - \pi_D \tag{17}$$

Finally, following equation states that h_2 is indifferent between holding the down tranche and the consumption good X. So his expected marginal utility from buying the down tranche.

$$(1-h_1)0.2 = \pi_D \tag{18}$$

By these four equations, the equilibrium is determined. The price of the asset, p, the price of the down tranche π_D,; and the two marginal buyers, h_1 and h_2 are determined.
The equilibrium value is about $h_1 = 0.58$, $h_2 = 0.08$, $p = 1$, and $\pi_D = 0.17$. The asset price is much higher even than it was with collateral loan contract (0.75).
The simple reason is that the collateral loan contract is an imperfect form of tranching. When the owner of the asset y can create Arrow D security. This is more attractive for heterogeneous traders. In collateral model, agents must hold the the asset until states are realized.
Optimistic agents can take more risky position by tranching scheme. By buying assets, optimists can part with low return state security. This scheme make optimists hold a return only at high state. Pessimists can also take more pessimistic position. This makes the asset price is influenced by more extreme traders. In heterogeneous belief model, the tranching allow traders to trade goods which adapt their beliefs.

1.5 Loan Securitization and Heterogeneous Belief Bubbles

Geanakoplos show that collateral loan contracts help optimists' purchase of assets. The collateral loan contracts raise the asset prices but this effect is limited. Simsek(2013) shows that this limitation can be expressed by the belief disagreement between optimists and pessimists. Fostel and Geanakoplos(2012) show the asset price is rased again by traching tecnology.

I will show these collateral constraints vanished by introducing differnet types of financial innovation, a loan securitization. This securitization allow agents to trade the right of the loan payment. Loan borrowers can sell the future payment from lenders to other agents.

I assume one more new setting. New traders come to financial market every day. In incomplete market, today's traders cannot cooperate with tommorow's traders. If today's traders budget is small, they cannot buy enough assets by thier own cash. There are inter-temporal financial constraint in the market.

Financial innovation such as loan securitization open new markets for the traders. My paper shows that financial innovations improve the inter-temporal traders' budget constraints. By the innovation, it is possible for them to have enough cash for buying the asset. I also show that innovations increase the market instabillity, such as bubbles.

There are two original factors, loan securitizations and new optimists.

Lenders can sell loan payments to other traders by securitizations. Loan lenders can shift default risks of loan contracts to other traders. As a result, speculative lending occur in equilibrium.

New optimists come at each date in my model. These optimists buy the security which the loan lenders issues. The asset price is raised by their optimistic trades.

There are heterogeneous traders, loan contract with collateral and budget constraint. Optimistic traders comes at each date. Though new optimists have little cash, their cash is indirectly used for buying the asset by the securitization. The loan contract can be securitized and new optimists buy the secuirty. For the securitization, the lender of the loan has speculative incentive for lending cash. The borrower can have large cash by the loan and securitization.

Two factors improve traders budget constraints. Each optimistic traders have little cash. They need to make loan contract. In this economy the loan borrower need to have a collateral. This collateral constraint model is similar to Simsek(2013). Under this constraint, traders' asset demand is put down. These constraints are drastically improved by the loan securitization.

If the asset return is high, the optimists can return cash to pessimists. If asset return is low, the asset will be held by pessimists because the asset is used as collateral. The pessimistic beliefs also affect the price through collateral. Then, the asset price is lower than optimistic expectation.

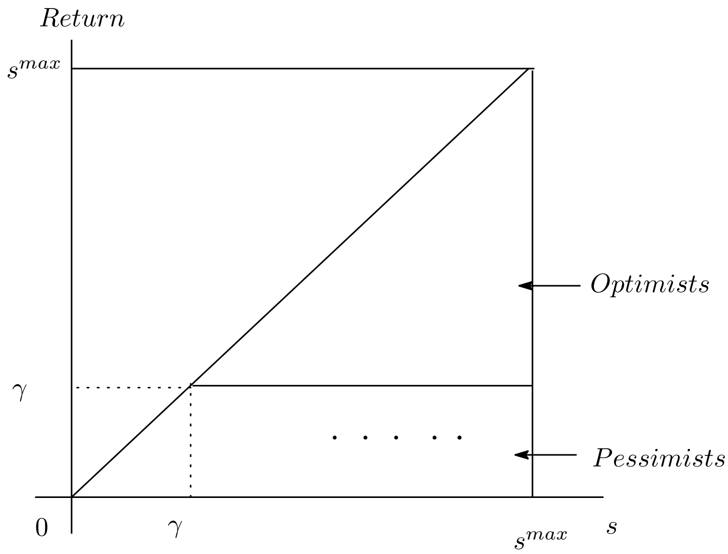
Importantly, each optimists generation cannot cooperate with each other. If securitization is not allowed, Only small cash is used for buying the asset. This price is equal to that of Simsek(2013). Optimists must rely on pessimists cash

and the asset price is lower.
Many papers also explore the role of financial innovation in heterogeneous-beliefs models.
Geanakoplos analyses heterogeneous belief model with collateral constraints. In their model, traders need some asset as collateral when they make loan contracts with other traders. Geanakoplos(1997) and Geanakoplos and Zame(1997) introduce endogenous collateral constraints into a general equilibrium framework of Arrow-Debreu. No payments in future periods cannot be promised without assets as collateral. The margin/haircuts of collateral borrowing are derived endogenously interaction with equilibrium prices.
Financial innovation increase the ability of optimistic traders. Fostel and Geanakoplos(2012) provides a model in which the asset price is raised by tranching technology. Recent U.S. housing bubble is closely connected with financial innovation like securitization(Brunnermeier(2009)). The financial progress does not only improve the market efficiency, but also amplify the market stability in heterogeneous belief model.
In Simsek(2013), there are two type traders, optimists and pessimists. Optimists have higher expectation about the asset return. Because their budgets are limited, they must borrow cash from pessimists. The loan contract need collateral assets. As pessimists have lower expectation about the asset return, the loan lending is not enough for optimists. As a result, optimists cannot raise the asset demand. If optimists have enough cash, the asset is bought by optimists and the asset price is determinded only by optimists. But, optimists must repay cash to pessimists. If low return is realized, default occurs. Optimists must give the asset to pessimists as collateral.
The asset price:

$$p = \int_0^{\gamma} s dF_p + \frac{1 - F_p(\gamma)}{1 - F_o(\gamma)} \int_{\gamma}^{s^{max}} s dF_o$$

F_p, F_o are pessimistic and optimistic beliefs. This price implies high return states are evaluated by optimists and low return states are evaluated by pessimists. In Simsek, there is no financial innovation like securitization. The loan contract among pessimists and optimists is interpreted as one of assets. If high return state is realized, the holder of the loan can receive payment. If low return state is realized, they receive the asset return.

I will show that the asset price gets back to heterogeneous-belief model by introducing financial innovation, securitization.
Simsek shows another case, short sale model. In this case, pessimists have limited cash and optimists have a plenty of cash. Pessimists and optimists make short sale contract. Optimists buy the asset and pessimists borrow it. Pessimists sell the asset and repurchase it at next date. If the asset return is low, pessimists can repurchase it. But, pessimists cannot in high return states. (In heterogeneous-belief bubble model(Miller(1977), Harrison and Kreps(1978)), short sale contracts are forbidden.)
The asset price:



$$p = \frac{E_p[s]}{F_p(\gamma)\frac{\int_0^\gamma sdF_o}{\int_0^\gamma sdF_p} + 1 - F_o(\gamma)} \tag{19}$$

This price also implies the asset price is evaluated by optimists and pessimists. If the short sale contract is allowed, pessimists have incentive to buy the asset. As pessimists expect lower return, their expected return in the contract. If pessimists have enough cash, the asset price is equal to pessimistic expectation. Pessimists' limited cash is not enough for buying the asset.
I will show that the asset price is equal to pessimistic expectation by introducing one of insurance to the short sale model.
In Fostel and Geanakoplos(2012), they analyze two financial innovation, tranching and CDS. Like Simsek(2013), each trader have limited wealth. Tranching makes the collateral asset more valuable for optimists. Tranching split the asset return into pieces. The asset price is raised by increasing the ability of the optimist to take positions in the asset. This price is similar to Harrison and Kreps(1978) bubble. The bubble is defined as an asset price that is higher than any agents' expectation.
In their model, an unexpected introduction of credit default swaps can lead to reductions in asset prices. CDS allows pessimistic people to more effectively bet against the bubble asset.
Combining such heterogeneous beliefs with short-sale constraints can result in overpricing. Optimists buy the asset and pessimists cannot bet against the bubble (Miller (1977)). Ofek and Richardson (2003) link this argument to the internet bubble of the late 1990s. In a dynamic model, the asset price can even exceed the valuation of the most optimistic traders. The current holder of the asset can resell the asset in the future at a high price to future optimists. (Harrison and Kreps(1978)).

In my paper, by introducing a new financial technology, loan backed security, to heterogeneous belief model with collateral, the asset price is as high as Miller(1977) or Harrison and Kreps(1978).
The loan itself can be trade among agents. This security make the loan contract less risky for the lender. If no securitization is allowed, they must take a risk of the borrower's default in making a loan contract. By selling the loan to other agents, this risk disappears for the lenders. The default risk is now held by new agents.
Some kind of risk shifting occurs in trade of the loan security. In Simsek(2013), pessimists are natural lenders and optimists are natural borrowers. Because pessimistic lenders think the collateral is very risky, their lending is small. If they can sell the loan to other agents, they need not to take the risk of the asset. Then, the lender need not to hesitate to lend cash to optimistic traders. As a result, optimistic borrowers can get enough cash by the loan contracts. The asset demand of optimists put up the price.
In my paper, optimistic traders behave like "noise trader"(DeLong, Shleifer, Summers and Waldmann(1990)). Although there are many pessimistic traders, optimistic traders' influence is very large. Pessimists can get high return by exploiting optimistits by the a financiual technology.
My research is a part of theory that concerns the borrowing constraints on asset prices like Shleifer and Vishny(1992, 1997), Kiyotaki and Moore(1997),Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009). Financial technologies can eliminate the borrowing constraint by speculative incentive of traders. In heterogeneous belief model, borrowing constraints are important role for preventing bubble economies.
The securitized loan is traded in repo market in recent crisis(Gorton and Metrick(2011), Krishnamurthy, Negel and Orlov(2011)).
At first, I will explain the loan securitization model in part 2.
The single optimist type case (section 2) is the fundamental model in my securitization model. There is two type equilibrium in the model.
One type is small generation case. In this case, the asset price is lower than optimistic expectation. Because the number of optimistic traders is not enough, the asset price cannot be bought by optimist. The other type is large generation case. Many optimists who have little cash participate in markets. The asset price is raised up to optimistic expectation like Miller(1977).
In section 3, various optimists type model is explained. Various type optimists case is an extension of single type optimist case. There are many types of optimists who have different belief about the asset return. In this model, the asset price is higher than the one type optimists case. The asset price is similar to Harrison and Kreps(1978). Most optimistic agents buy the each date security and the asset return is evaluated by their beliefs. The asset price is higher than one type model.
In part 3, a short sale model is explained. The short sale model is an extension model of my securitization model and the asset price is symmetrical to the single optimist type model. Optimists and pessimists make short sale contract. Short sale contract put down the asset price(Miller(1977), Simsek(2013)). By

introducing an insurance, the asset price gets lower than Simsek(2013). I will show two case, one generation model and large generation model.
One generation case is same as Simsek(2013). the asset price is higher than pessimistic expectation. Because the number of pessimistic traders is not enough, high return state is not covered by short sale contract. In large generation case, many pessimists who have little cash participate in markets. The asset price falls down to pessimistic expectation. In this time, the asset price is put down by financial technology. This is similar result in CDS model in Fostel and Geanakoplos(2012).
My paper show financial technology bring largely instability to the asset market in heterogeneous belief model. The influence of the innovation must be considered in economics.

Part II
Loan Securitization Model

2 Multi Generation with Single Optimist's Type Model

2.1 Settings

In this model, we consider a symmetric equilibrium. Same action are chosen by same type traders at each date. The model is dynamic finite date model($t = 0, 1, ...T$). At date 0, one unit of risky asset is supplied by a monopolistic firm. There is a continuum of states at date 1, denoted by $s \in S = [0, s^{max}]$. The asset pays s dollars at state s. There are two type of traders, optimist and pessimist. Both type traders have risk neutral utility functions. But, they have different beliefs about asset return. Type j trader have a belief about asset return , distribution function $F_j(s)(j = o, p)$. Note their expectation of asset return is $E_o[.], E_p[.]$. Traders know each other's priors. Optimists and pessimists agree to disagree about their beliefs.
Optimists are optimistic about the asset return. Assume first order stochastic dominance. Optimist's distribution function is lower than pessimistic one for all state s.

Assumption 1 $F_p[s] \geq F_o[s]$ *for all* s

Moreover, the following assumption is necessary for the uniquness of the equilibrium.

Assumption 2 $\frac{f_o}{1-F_o} < \frac{f_p}{1-F_p}$ *for each* $s \in (0, s^{max})$

At date 0, the continuum of traders, α optimists and $1 - \alpha$ pessimists. α is small fraction. At each date t, new α optimists come to the market. All traders live up to the final date T.
Each optimist has n/α units of cash. Pessimists have no budget constraint.

The assumption implies optimists are natural buyers of the asset.
At each generation $t = 0, ...T - 1$, the continuum optimists uinformally distributed in $[0, \alpha]$ come to the market. All optimists are homogeneous. They have same belief (F_o). All traders live up to final date T.

All optimists are initially endowed with small amount of cash $n/\alpha > 0$ dollars and zero unit of the asset. Because n is small, optimists must borrow cash from pessimists.

Assumption 3 n *is small enough*

Optimists who come to the market have a chance to buy the asset or the security only at date t. Optimists buy the security or the asset and they wait

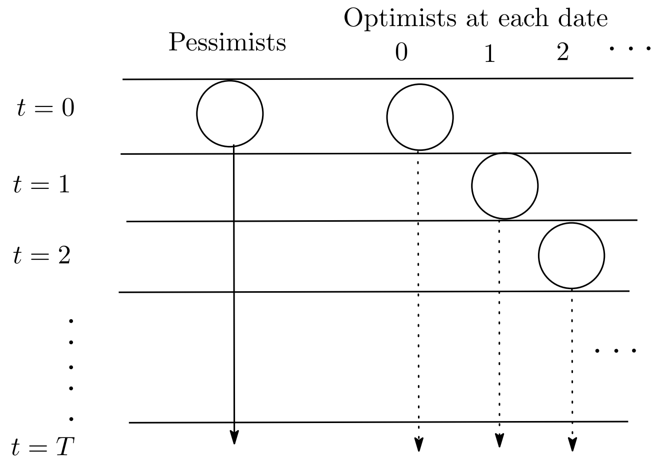


Figure 2: generation

until s is realized (date T).
Pessimists have a plenty of cash and they have no budget constraints. They are natural lenders of cash.
Optimists don't have enough cash for buying the asset. So,they must borrow some cash from pessimists. All borrowing contract in this economy is subject to a collateral constraint. Promises made by borrowers must be collateralized by assets or securities.
A collateraled loan contract at date 0 is defined as (γ_0, ϕ_0). ϕ_0 is the amount of cash borrowing and γ_0 is the borrowers promise payment after the revealment of s(date T). If the borrowers cannot pay γ_0 at date T, they must give the asset or the securities as collateral to the lenders. The cash borrowers make a take-it-or-leave-it offer (γ_0, ϕ_0) to lenders.
Because only one type of asset exists at date 0, loan contracts must be collateralized by the asset. If s is low at date T, the loan default may occur. In this case, the borrowers must give the asset to the lenders as collateral.

- If the asset return s is high ($s > \gamma_0$), the optimist can pay the promise γ_0 to the pessimist.
- If the asset return s is low ($s \leq \gamma_0$), the optimist gives the asset return s to the pessimist.

Totally, borrowers give $min(s, \gamma_0)$ to lenders in a loan contract (γ_0, ϕ_0). Then, the loan payoff is $min(s, \gamma_0)$. Pessimists evaluate the payoff as $E_p[min(s, \gamma)]$ and optimists evaluate as $E_o[min(s, \gamma_0)]$ Optimists evaluation of the payoff is higher than pessimists' for the assumption of first order stochastic dominance.:

$$E_o[min(s, \gamma_0)] > E_p[min(s, \gamma_0)]$$

There is a incentives for trading the loan contract.
The loan lender can securitize the loan contract and sell it to the other traders at next date 1. He can issue loan backed securities whose payoff is $min(s, \gamma_0)$ with price q_1. Assume the seller of securities have monopolistic power.
At date 1, traders can make loan contract with collateralized securities or assets. (In equilibrium, all assets are bought by the date 0 optimists and the date 1 loan contract is collateralled by the security..)
A security collateralized loan contract at date 1 is defined as (γ_1, ϕ_1). ϕ_1 is the amount of cash borrowing and γ_1 is the borrowers promise payment after the revealment of $min(s, \gamma_0)$. $(\gamma_0 > \gamma_1)$. If the borrowers cannot pay γ_1, they must give the securities as collateral to the lenders. The cash borrowers make a take-it-or-leave-it offer (γ_1, ϕ_1) to lenders.
The security payoff is $min(s, \gamma_0)$. If s is low at date T, the loan default may occur. In this case, the borrowers must give the security to the lenders as collateral.

- If the security return is high $(min(s, \gamma_0) > \gamma_1)$, the optimist can pay the promise γ_1 to the pessimist.
- If the security return is low $(min(s, \gamma_0) \leq \gamma_1)$, the optimist gives the security return $min(s, \gamma_0)$ to the pessimist.

That is, borrowers give $min[min(s, \gamma_0), \gamma_1] = min(s, \gamma_1)$ to lenders in a loan contract (γ_1, ϕ_1).
The same scheme can be continued until $T - 1$. At each date $t \geq 1$, the trader who lent cash at previous date $t - 1$ with loan contract $(\gamma_{t-1}, \phi_{t-1})$ can issue loan backed securities which securitize loan contracts.
The security payoff is $min(s, \gamma_{t-1})$ and the security price is q_t.
Traders can make loan contract at date t. This loan contract need the security as collateral and it is defined as (γ_t, ϕ_t). γ_t is cash repayment at date T and ϕ_t is cash borrowing at date t. For lenders, the total payment of this loan at date T is $min(s, \gamma_{t-1})$.

2.2 Equilibrium

In this paper, consider an equilibrium:

- At date 0, the asset is bought by all α optimists
- At each date $t > 0$, each security is bought by α optimists who come to the market at date t
- At each date optimists make loan contract with one pessimist
- Pessimists who make loan contract at date t securitize the loan at date $t + 1$

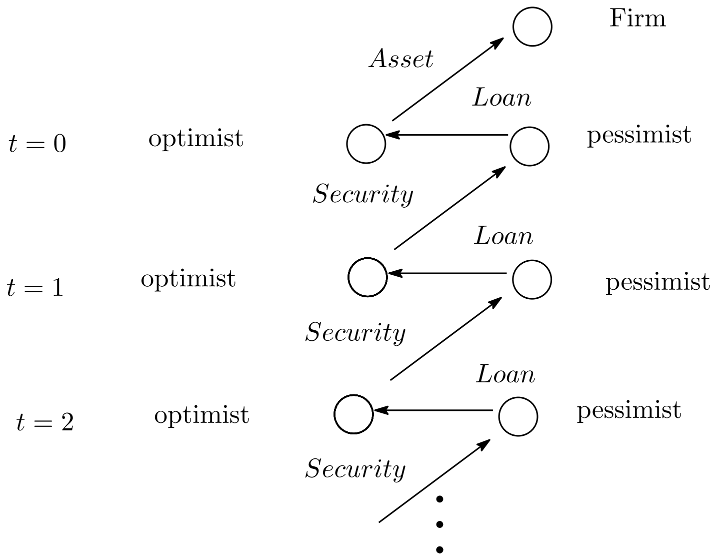


Figure 3: multi generation

2.3 Pessimists Loan Problem

Consider the equilibrium that optimists buy all assets or securities at each date. Optimists make a loan contract with the pessimist at each date.
Optimists make a take-it-or-leave-it offer (γ_t, ϕ_t) to the pessimist at date $t \leq T-2$. If accept the offer, the pessimist lends ϕ_t cash and he sell the security with price q_{t+1} at date $t+1$. Then pessimist accept the offer if $\phi_t \leq q_{t+1}$.
For pessimists, the loan contract is only way to earn cash. Because there are continuum pessimists. Their competition imply no-arbitrage condition:

$$\phi_t = q_{t+1} \tag{20}$$

At date $T-1$, the pessimist cannot sell the security at next date T. The loan payoff is $min(s, \gamma_{T-1})$. Then, pessimist competition imply:

$$\phi_{T-1} = E_p[min(s, \gamma_{T-1})] \tag{21}$$

If $\gamma_{T-1} = 0$, the security return $min(s, \gamma_{T-1}) = 0$. In this case, no loan contract is made.

$$\phi_{T-1} = 0 \tag{22}$$

Assume pessimists pay cost ϵ for making the loan contracts except one pessimist. Only one pessimists is a superior lender who need not pay cost ϵ. This pessimist can rent ϕ cash.
This pessimist make loan contract with all optimists at each date in lending competition. At date $t+1$, the date t loan lender sell the security. In this

model, there is no profit to sell the security at later date. (At each date, optimists' total cash is equal to n.) Then, the date t pessimist sell the security at date $t+1$.

2.4 Optimists Problem

At each date, the security is supplied by the pessimist who lent cash at previous date. The pessimist has monopolistic power. If the security(or the asset) price is lower than optimists' expected return, optimists buy the security(or the asset) as much as possible at each date.
Optimists offer the loan contract (γ_t, ϕ_t) to pessimists. Optimists take offers that satisfy pessimists' arbitrage condition.
Optimists have cash n/α and they borrow cash $a_t\phi_t$ from pessimists with loan contract. As the seller of the security is monopolistic power, optimists use their cash and their loan borrowing to buy the security. Then, their budget constraint is:

$$a_t q_t = n/\alpha + a_t\phi_t \tag{23}$$

The date $T-1$ optimists are price takers. The security price q_{T-1} is given by the seller. For the date $T-1$ optimists, the security return $min(s, \gamma_{T-2})$ is given by the previous date $(T-2)$. Optimists know the pessimist condition $\phi_{T-1} = E_p[min(s, \gamma_{T-1})]$.

Then, the date $T-1$ optimists problem is:

$$\begin{aligned} \max_{a_{T-1},\gamma_{T-1}} \; & a_{T-1}(E_o[min(s, \gamma_{T-2})] - q_{T-1}) \\ & + a_{T-1}(\phi_{T-1} - E_o[min(s, \gamma_{T-1})]) \\ & s.t.\; a_{T-1}q_{T-1} = n/\alpha + a_{T-1}\phi_{T-1}, \\ & \phi_{T-1} = E_p[min(s, \gamma_{T-1})] \end{aligned} \tag{24}$$

a_{T-1} is the security demand.
The first line of the utility is the profit of the security. The security has return $min(s, \gamma_{T-2})$ and it is sold with price q_{T-1}. The second line is the profit of the loan contract. Optimists borrow cash ϕ_{T-1} and they promise repayment $min(s, \gamma_{T-1})$ for each security a_{T-1}.
Rewrite the problem:

$$\begin{aligned} \max_{a_{T-1},\gamma_{T-1}} \; & a_{T-1}[E_o[min(s, \gamma_{T-2})] - E_o[min(s, \gamma_{T-1})]] \\ & s.t. a_{T-1}q_{T-1} = n/\alpha + a_{T-1}E_p[min(s, \gamma_{T-1})] \end{aligned} \tag{25}$$

q_t is the security price. Given the security price q_t and the security return $min(s, \gamma_{t-1})$, optimists maximize their utilities. The date t optimists optimization problem is:

$$
\begin{aligned}
\max_{a_t,\gamma_t}\ & a_t(E_o[min(s,\gamma_{t-1})]-q_t) \\
& +a_t(\phi_t-E_o[min(s,\gamma_t)]) \\
& s.t.\ a_tq_t=n/\alpha+a_t\phi_t, \\
& \phi_t=q_{t+1}(\gamma_t)
\end{aligned}
\tag{26}
$$

a_t is the security demand. $q_{t+1}(\gamma_t)$ is the price function of the date $t+1$ security. The date $t+1$ security depends on the date t loan contract (γ_t). This function will be explained later.
By the similar way, the date 0 optimists' problem:

$$
\begin{aligned}
\max_{a_0,\gamma_0}\ & a_0(E_o[s]-p) \\
& +a_0(\phi_0-E_o[min(s,\gamma_0)]) \\
& s.t.\ a_0p=n/\alpha+a_0\phi_0, \\
& \phi_0=q_1(\gamma_0)
\end{aligned}
\tag{27}
$$

At date 0, the asset is supplied by a monopolistic firm with price p.

2.5 Equilibrium Definition

Definition 1 *An equilibrium consists of the asset price p, the each date security price $\{q_t\}_{t=1,2,..,T-1}$, loan contract $\{(\gamma_t,\phi_t)\}_{t=0,1,...,T-1}$, the asset demand a_0 and the security demand $\{a_t\}_{t=1,2,..,T-1}$. They satisfy the following conditions:*

- *Given γ_{t-1} and q_t, a_t and γ_t solves optimists problem*
- *At each date t, the loan contract (γ_t,ϕ_t) satisfy pessimists arbitrage condition*
- *market clearing condition $a_t=1$ for all t*

2.6 Small T Case

The equilibrium is solved by backward induction from $T-1$. The date $T-1$ optimists are price takers. The security price q_{T-1} is given by the seller. For the date $T-1$ optimists, the security return $min(s,\gamma_{T-2})$ is given by the previous date $(T-2)$. Optimists know the pessimist condition $\phi_{T-1}=E_p[min(s,\gamma_{T-1})]$.
Then, the date $T-1$ optimists problem is:

$$
\begin{aligned}
\max_{a_{T-1},\gamma_{T-1}}\ & a_{T-1}[E_o[min(s,\gamma_{T-2})]-E_o[min(s,\gamma_{T-1})]] \\
& s.t. a_{T-1}q_{T-1}=n/\alpha+a_{T-1}E_p[min(s,\gamma_{T-1})]
\end{aligned}
\tag{28}
$$

By solving the problem, q_{T-1} and γ_{T-1} are determined.

Lemma 1 *Given γ_{T-2}, equlibrium (q_{T-1}, γ_{T-1}) is determinded by the cross-point of the following two equations:*

$$q_{T-1} = \int_0^{\gamma_T} sdF_p + \frac{1-F_p(\gamma_T)}{1-F_o(\gamma_T)} \int_{\gamma_{T-1}}^{s^{max}} min(s, \gamma_{T-1})dF_o$$

$$q_{T-1} = n + E_p[min(s, \gamma_{T-1})]$$

Proof 1 *In the equilibrium, all security is bought by optimists. Then, $a_{T-1} > 0$. From the budget constraint:*

$$a_{T-1} = \frac{n/\alpha}{q_{T-1} - E_p[min(s, \gamma_{T-1})]}$$

Substitute this the objective function:

$$\frac{n/\alpha}{q_{T-1} - E_p[min(s, \gamma_{T-1})]}[E_o[min(s, \gamma_{T-2})] - E_o[min(s, \gamma_{T-1})]] \tag{29}$$

FOC imply:

$$\frac{1-F_o(\gamma_{T-1})}{q - E_p[min(s, \gamma_{T-1})]} + (1-F_p(\gamma_{T-1}))\frac{E_o[min(s, \gamma_{T-2})] - E_o[min(s, \gamma_{T-1})]}{q - E_p[min(s, \gamma_{T-1})]} = 0$$

From the assumption ($\frac{f_o}{1-F_o} < \frac{f_p}{1-F_p}$), the second order condition is satisfied. Then,

$$q_{T-1} = \int_0^{\gamma_{T-1}} sdF_p + \frac{1-F_p(\gamma_{T-1})}{1-F_o(\gamma_{T-1})} \int_{\gamma_{T-1}}^{s^{max}} min(s, \gamma_{T-2})dF_o$$

In equilibrium, optimists buy all securities. Then $\alpha a_{T-1} = 1$. From the budget constraint:

$$q_{T-1} = n + E_p[min(s, \gamma_{T-1})]$$

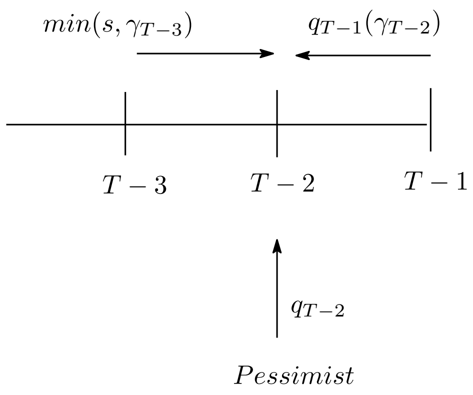
By these two equations, γ_{T-1} is uniquely determined given γ_{T-2}.//

Given γ_{T-2}, the date $T-1$ optimists determine γ_{T-1}. Let a price function $q_{T-1}(\gamma_{T-2})$ denote the equilibrium price given γ_{T-2}:

$$q_{T-1}(\gamma_{T-2}) = \int_0^{\gamma_{T-1}} sdF_p + \frac{1-F_p(\gamma_{T-1})}{1-F_o(\gamma_{T-1})} \int_{\gamma_{T-1}}^{s^{max}} min(s, \gamma_{T-2})dF_o$$

$$q_{T-1}(\gamma_{T-2}) = n + E_p[min(s, \gamma_{T-1})]$$

For the date $T-2$ optimists, security return $min(s, \gamma_{T-3})$ and the price function $q_{T-1}(\gamma_{T-2})$is given. From pessimists no arbitrage condition, $\phi_{T-2} =$



$q_{T-1}(\gamma_{T-2})$.
Then, the date $T-2$ optimist problem is written:

$$\max_{a_{T-2},\gamma_{T-2}} a_{T-2}[E_o[min(s,\gamma_{T-3})] - E_o[min(s,\gamma_{T-2})]]$$
$$s.t. a_{T-2}q_{T-2} = n/\alpha + a_{T-2}q_{T-1}(\gamma_{T-2})$$

By solving the problem, γ_{T-2} is determined.
By the similar way, the equilibrium is determined by the backward induction.

Proposition 1 *γ_t and the security price q_t are determined by these two equations in the equilibrium:*

$$\cdot\ q_t = \frac{q'_{t+1}(\gamma_t)}{1-F_o(\gamma_t)}\{E_o[min(s,\gamma_{t-1})] - E_o[min(s,\gamma_t)]\} + q_{t+1}(\gamma_t)$$

$$\cdot\ q_t = n + q_{t+1}(\gamma_t)$$

Proof 2 *The date $T-2$ optimists' problem:*

$$\max_{a_{T-2},\gamma_{T-2}} a_{T-2}[E_o[min(s,\gamma_{T-3})] - E_o[min(s,\gamma_{T-2})]]$$
$$s.t. a_{T-2}q_{T-2} = n/\alpha + a_{T-2}q_{T-1}(\gamma_{T-2})$$

By FOC imply:

$$\cdot\ q_t = \frac{q'_{t+1}(\gamma_t)}{1-F_o(\gamma_t)}\{E_o[min(s,\gamma_{t-1})] - E_o[min(s,\gamma_t)]\} + q_{t+1}(\gamma_t)$$

In equilibrium, optimists buy all securities. Then $a_{T-t} = 1/\alpha$. From the budget constraint:

$$q_{T-2} = n + q_{T-1}(\gamma_{T-2})$$

γ_{T-2} and q_{T-2} are determined by these two equations. Given γ_{T-3}, q_{T-2} is uniquely determined. Let $q_{T-2}(\gamma_{T-3})$ denote the equilibrium security price given γ_{T-3}.

By using $q_{T-2}(\gamma_{T-3})$, the date $T-3$ optimists' problem is written.

$$\max_{a_{T-3},\gamma_{T-3}} a_{T-3}[E_o[min(s,\gamma_{T-4})] - E_o[min(s,\gamma_{T-3})]]$$
$$s.t. a_{T-3}q_{T-3} = n/\alpha + a_{T-3}q_{T-2}(\gamma_{T-3})$$

By continuing the backward induction, the date t problem is written by using a security price function $q_{t+1}(\gamma_t)$.
Given γ_{t-1} and q_t, the date t optimists problem:

$$\begin{aligned} &\max_{a_t,\gamma_t} a_t(E_o[min(s,\gamma_{t-1})] - E_o[min(s,\gamma_t)]) \\ &s.t.\ a_t q_t = \frac{n}{\alpha} + a_t q_{t+1}(\gamma_t) \end{aligned} \tag{30}$$

FOC and market clearing imply:

$$\cdot\ q_t = \frac{q'_{t+1}(\gamma_t)}{1-F_o(\gamma_t)}\{E_o[min(s,\gamma_{t-1})] - E_o[min(s,\gamma_t)]\} + q_{t+1}(\gamma_t)$$

$$\cdot\ q_t = n + q_{t+1}(\gamma_t)$$

//

By the similar way, the asset price at date 0 is calculated. The date 0 optimists problem is written by using $q_1(\gamma_0)$.

$$\max_{a_0,\gamma_0} a_0\{E_o[s] - E_o[min(s,\gamma_{T-1})] - \lambda_1(\gamma_{T-1})n\}$$
$$s.t. a_0 p = n/\alpha + a_0 q_1(\gamma_0)$$

Proposition 2 *γ_0 and the asset price p are determined by these two equations in the equilibrium:*

$$\cdot\ p = \frac{q'_1(\gamma_0)}{1-F_o(\gamma_0)}\{E_o[s] - E_o[min(s,\gamma_0)]\} + q_1(\gamma_0)$$

$$\cdot\ p = n + q_1(\gamma_0)$$

Proof 3 *By FOC of the date* 0 *problem:*

$$\frac{1-F_o(\gamma_0)}{p-q_1(\gamma_0)}+q_1'(\gamma_0)\frac{E_o[s]-E_o[min(s,\gamma_0)]}{p-q_1(\gamma_0)}=0$$

FOC imply:

$$\cdot\; p = \frac{q_1'(\gamma_0)}{1-F_o(\gamma_0)}\{E_o[s]-E_o[min(s,\gamma_0)]\} \\ + q_1(\gamma_0)$$

In equilibrium, $a_0 = 1/\alpha$.

$$p = n + q_1(\gamma_0) \tag{31}$$

//

From the asset price and the security price:

$$\begin{aligned} p &= n + q_1(\gamma_0) \\ &= 2n + q_2(\gamma_2) = ... \\ &= Tn + E_p[min(s,\gamma_{T-1})] \end{aligned} \tag{32}$$

Optimists' total cash Tn raise the asset price. In this market, the highest asset price is optimistic expectation of asset return $E_o[s]$. At next subsection, this price is achieved in the large generation case.

2.7 Large T case

The equilibrium is solved by backward induction. If T is large enough, at some date t', security is bought by optimists own cash n. At date t', security payoff is $min(s,\gamma_{t'-1})$. Because the seller of the security have monopolistic power, the security price is equal to the optimistic expectation:

$$q_{t'} = E_o[min(s,\gamma_{t'-1})] \tag{33}$$

Given $\gamma_{t'-2}$, the date $t'-1$ optimist problem is:

$$\begin{aligned} \max_{a_{t'-1},\gamma_{t'-1}} \quad & a_{t'-1}\{E_o[min(s,\gamma_{t'-2})]-E_o[min(s,\gamma_{t'-1})]\} \\ s.t.\ & a_{t'-1}q_{t'-1} = n/\alpha + a_{t'-1}E_o[min(s,\gamma_{t'-1})] \end{aligned} \tag{34}$$

This is the date $T-1$ problem in small T case with $E_p[min(s,\gamma_{t'-1})] = E_o[min(s,\gamma_{t'-1})]$. By solving the problem, the seucrity price $q_{t'-1}$ is:

$$q_{t'-1} = \int_0^{\gamma_{t'-1}} s dF_o + \frac{1 - F_o(\gamma_{t'-1})}{1 - F_o(\gamma_{t'-1})} \int_{\gamma_{t'-1}}^{s^{max}} min(s, \gamma_{t'-2}) dF_o$$
$$= E_o[min(s, \gamma_{t'-2})]$$

This equation and $q_{t'-1} = n + E_o[min(s, \gamma_{t'-1})]$(From $a_{t'-1} = 1$) determine $\gamma_{t'-1}$. By backward induction, the security price at date t is calculated.

proposition 1 *In large T case, the security price at each date t is:*

$$q_t = E_o[min(s, \gamma_{t-1})]$$

The asset price is:

$$p = E_o[s]$$

Proof 4 *Assume the security price function $q_{t+1}(\gamma_t) = E_o[min(s, \gamma_t)]$ Given γ_{t-1} and q_t, the date t optimist problem is:*

$$\begin{aligned} &\max_{a_t, \gamma_t} a_t \{E_o[min(s, \gamma_{t-1})] - E_o[min(s, \gamma_t)]\} \\ &\quad s.t. \ a_t q_t = n/\alpha + a_t E_o[min(s, \gamma_t)] \end{aligned} \tag{35}$$

This problem is just same as the date $t' - 1$ problem. From this problem the security price q_t is $E_o[min(s, \gamma_{t-1})]$.
Because the security price at date t' is $q_{t'} = E_o[min(s, \gamma_{t'-1})]$, the security price at date t is $E_o[min(s, \gamma_{t-1})]$.
The date 0 problem is:

$$\begin{aligned} &\max_{a_0, \gamma_0} a_0 \{E_o[s] - E_o[min(s, \gamma_0)]\} \\ &\quad s.t. \ a_0 p = n/\alpha + a_0 E_o[min(s, \gamma_0)] \end{aligned} \tag{36}$$

Let $\gamma_{-1} = s^{max}$ and $q_0 = p$.
The date 0 problem is rewritten:

$$\begin{aligned} &\max_{a_0, \gamma_0} a_0 \{E_o[min(s, \gamma_{-1})] - E_o[min(s, \gamma_0)]\} \\ &\quad s.t. \ a_0 q_0 = n/\alpha + a_0 E_o[min(s, \gamma_0)] \end{aligned} \tag{37}$$

This is the same problem at date t. Then, the asset price is:

$$p = q_0 = E_o[min(s, \gamma_{-1})] = E_o[min(s, s^{max})] = E_o[s] \tag{38}$$

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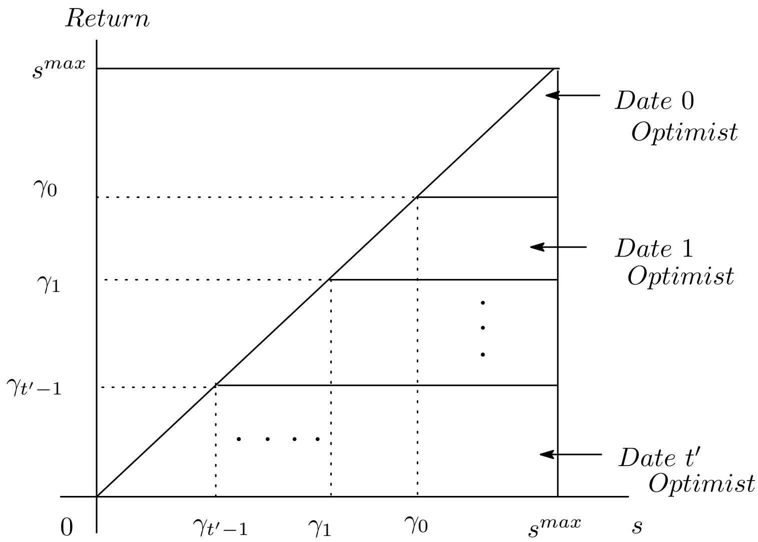


T is large enough, the asset is bought by optimists cash.($T-1 \geq t'$) The necessary number of optimists (t') is determined by the value of $E_o[s]$. From budget constraint of the date 0 optimist, budget constraint imply asset price is sum of optimist cash n.

$$p = n + n + n + ... + n + E_o[min(s, \gamma_{t'})] = (t'-1)n + E_o[min(s, \gamma_{t'})]$$

In equilibrium, asset price is $E_o[s]$. Then, the number of optimists who buy the asset or securities (that is t') is:

$$t' = \frac{E_o[s] - E_o[min(s, \gamma_{t'})]}{n}$$

t' generation optimists buy each area of asset return in equilibrium.

At date T, optimists pay the loan promise to pessimists. Pessimists give the loan return to optimists who buy the security. The date t optimists receive $min(s, \gamma_{t-1}) - min(s, \gamma_t)$.

2.8 Analysis

The equilibrium security price is determined by two equations:

- $$q_t = \frac{q'_{t+1}(\gamma_t)}{1 - F_o(\gamma_t)}\{E_o[min(s, \gamma_{t-1})] - E_o[min(s, \gamma_t)]\} + q_{t+1}(\gamma_t)$$

- $$q_t = n + q_{t+1}(\gamma_t)$$

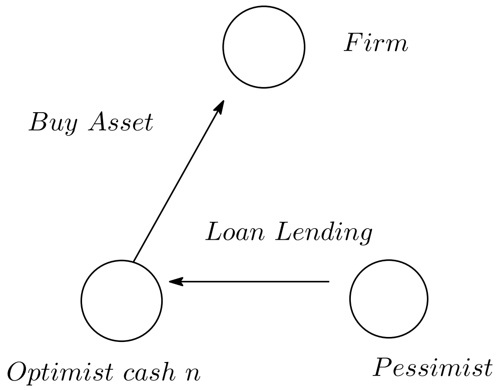


Figure 4: One Generation Case

The equilibrium asset price is also determined by two equations:

- $$p = \frac{q_1'(\gamma_0)}{1 - F_o(\gamma_0)} \{E_o[s] - E_o[min(s, \gamma_0)]\} + q_1(\gamma_0)$$

- $$p = n + q_1(\gamma_0)$$

These price equations implies:

$$n = \frac{q_{t+1}'(\gamma_t)}{1 - F_o(\gamma_t)} \{E_o[min(s, \gamma_{t-1})] - E_o[min(s, \gamma_t)]\} \tag{39}$$

$$n = \frac{q_1'(\gamma_0)}{1 - F_o(\gamma_0)} \{E_o[s] - E_o[min(s, \gamma_0)]\} \tag{40}$$

n is the total optimists' cash at each date.
At date t, optimists use their cash to receive the divided return:

$$E_o[min(s, \gamma_{t-1})] - E_o[min(s, \gamma_t)] \tag{41}$$

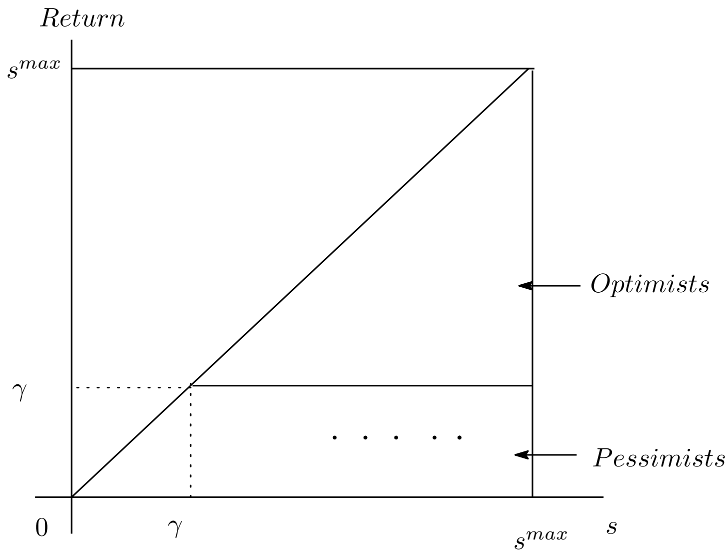
At date 0:

$$E_o[s] - E_o[min(s, \gamma_0)] \tag{42}$$

At each date, optimists buy the right to receive one part of the asset return. If securitization is not allowed, the asset price is equal to Simsek(2013). The asset price in Simsek(p^S) is equal to that of the $T = 1$ model.

In Simsek(2013), no securitization is allowed. This is $T = 1$ case in my model. Only one generation traders participate in the market. Optimists loan borrowing is $\phi_0 = E_p[min(s, \gamma_0)]$. Their budgets is $n + E_p[min(s, \gamma_0)]$

In $T = 1$ case, the equilibrium asset price is deduced by two equations.



$$p = \int_0^{\gamma_0} sdF_p + \frac{1 - F_p(\gamma_0)}{1 - F_o(\gamma_0)} \int_{\gamma_{T-1}}^{s^{max}} sdF_o$$

and

$$p = n + E_p[min(s, \gamma_0)]$$

If high return states occur, optimists repay the loan and they still have the asset. If low return states occur, optimists must give the asset to the pessimists as collateral.

The first equation picture this situation. High return states ($s > \gamma_0$) are evaluated by optimistic belief(F_o) and low return states($s < \gamma_0$) are evaluated by pessimistic belief(F_p).
In $T = 2$ case, securitization is allowed. Optimists loan borrowing at date 0 is $\phi_0 = q_1(\gamma_0)$. Their budget is $n/\alpha + q_1(\gamma_0)$. Because $q_1(\gamma_0) > E_p[min(s, \gamma_0)]$, optimists can use bigger budget. As a result, the asset is higher.
In $T = 2$ case, the equilibrium asset price:

$$p = n + q_1(\gamma_0) = 2n + E_p[min(s, \gamma_1)]$$

The high price is sustained by new optimists cash. Consider an extreme case. Assume pessimists' evaluation of the asset return is $0(F_p(0) = 1)$.
In this case, pessimists evaluation about loan lending $E_p[min(s, \gamma)] = 0$ for any γ. So, the $T = 2$ security price at date 1 is equal to $q_1(\gamma_0) = n$. The $T - 2$ asset price is equal to $n + q_1(\gamma_0) = 2n$. This is sum of cash the date 0 optimists and the date 1 optimists.
In two generation case, the date 0 optimists can use cash from the date 1 optimists. The existence of the date 1 optimists raise the asset price.

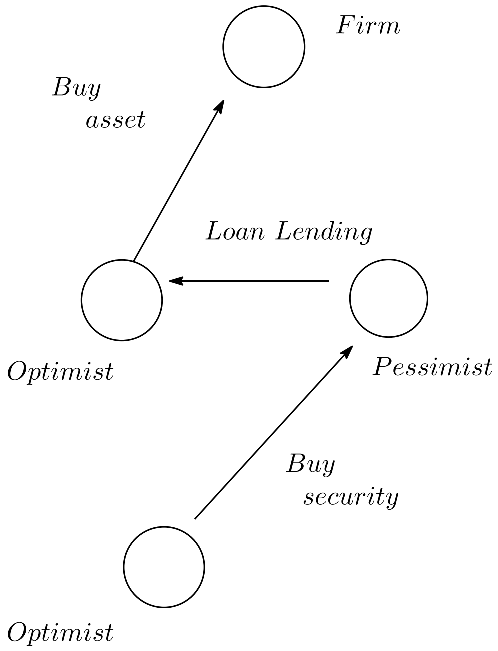


Figure 5: Two Generation Case

By making loan contract, the pessimist can sell security at next generation. That is, pessimist has speculative incentive for loan contract.

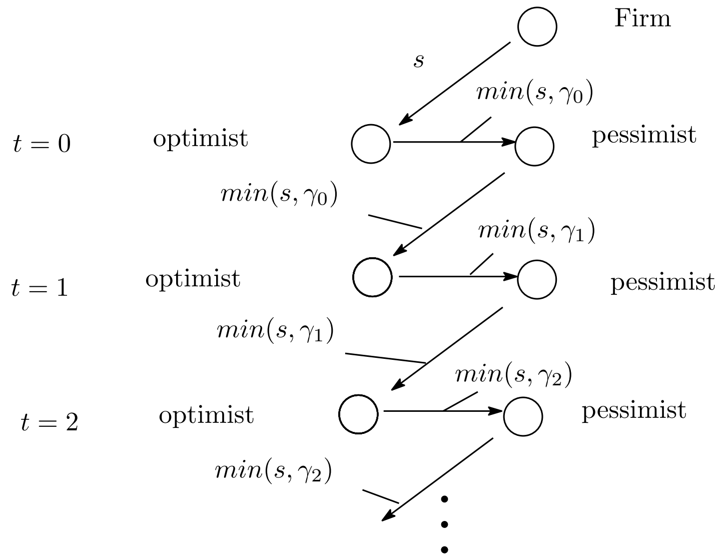
By the same logic, the asset price gets higher if T gets large. Large T implies many future optimists come to the market. Their total cash raise Tn raise the asset price.

From small T case, the asset price:

$$p = Tn + E_p[min(s, \gamma_{T-1})]$$

If pessimists' evaluation of the asset return is $0 (F_p(0) = 1)$, the asset price is simply equal to Tn.

If T is large enough, the asset price is equal to $E_o[s]$. Optimists totla return is equal to $E_o[s]$.
In Miller(1977), the asset price is equal to the optimistic expectation. In my model, Miller price is achieved by the coordination through financial technology, securitization.
Optimists have incentive to buy the asset, but they does not have enough cash. Without the security market, they have no way to cooperate with each other. Security and loan contract play role for helping their cooperation.
The loan contract causes the security and it causes the loan contract. The scheme allows the each optimist to participate in the market.
In Simsek(2013), optimists does not have enough cash, he must borrow cash from pessimist. Because loan contract is limited, he must collateralize the asset



itself. So the asset price must be influenced by pessimistic belief(F_p).
If T is large enough, pessimistic belief vanishes in the asset price. However, if there is no pessimist, the date 0 optimist have no way to bring enough cash to buy the asset. There are many pessimists in the security market.
The pessimists know that the new optimists will come to the market at the next date. So, they have strong incentive to lend their cash to the optimists. Lending cash is speculative action for the pessimist. The pessimist can sell the risk of the optimist's default to the other optimist by securitization. It is one of risk-shifting problem.(Shleifer and Vishny(1992))
As a result, the asset return is shared by the optimists and the asset price rises. This security raise the utility for both optimists and pessimists. The security technology compensate for incomplete financial market.
If there is no security market, only one optimist and one pessimist participate in the market. Many optimists can participate in economy by these securitization market.
If there are heterogeneous beliefs exists, completeness of security market make economy riskier. The advanced security market allow heterogeneous investor to act freely, so their action cause variety effect.
In Hk, optimists have a plenty of cash and they need not to borrow cash. If their budget constraints bind and no financial market, their trades may not occur. The asset price gets lower in this situation.
In Fostel and Geanakoplos(2012), a tranching technology (one of securitization) raise the asset price in heterogeneous belief model. By this technology, the asset price is influenced by more optimistic belief.

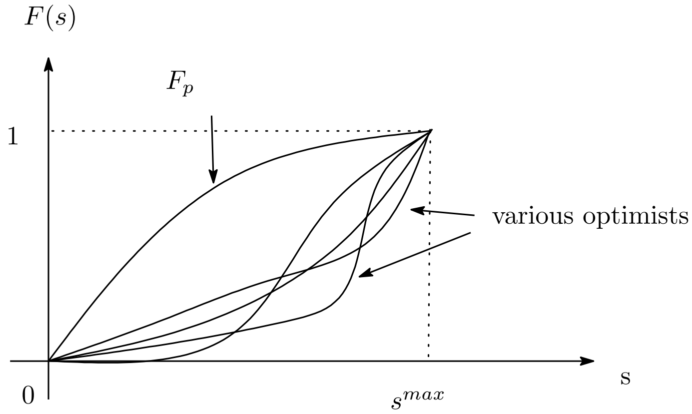
3 Multi Generation with Various Optimists' Types Model

In this section, I will show that the asset prices exceed any traders' expectations in various optimistic types case. Model settings are almost same as multi generations case.

3.1 Various Optimist Types Case

In this section, multi generation with various optimists model will be analyzed. The basic idea is the same as single optimist type case in section 3.

There are $J+1$ type of traders in this economy. J types of optimists and one type of pessimist. The optimist's type j have optimistic belief $F_{O_j}(s)$. All types of optimists have same expectation about the return of the asset ($E_{O_j}[s] = E_o[s] \forall j \in J$).



Each optimistic belief F_j first order stochastically dominates pessimistic belief F_p.

Assumption 4 $F_{O_j}[s] \geq F_p[s]$ $\forall s$ *and* $\forall j \in J$

The asset, the securities and the loan contracts is the same in section 3. At date 0, the continuum of traders, α each type optimists (There are $J\alpha$ optimists) and $1-\alpha$ pessimists. α is small fraction.
Each type optimist has n/α units of cash. Pessimists have no budget constraint. If some optimists have no cash, new optimists come to the market. Assume each type optimists' total cash is equal to n. That is, if some type optimists make loan contract with pessimists and they buy securities or assets, the same type optimists come to the market with cash.
At each generation, the pessimist raise the asset price or the security price until the highest type optimist can pay. Note j_0^* be the optimist type who can pay the highest p_j and j_t^* be the highest type at date t. Let p_j be the price which the type t optimist can pay for the asset and q_j^t be the price of type j for the t security.

$$j_0^* = argmax_j \ \ p_j$$
$$j_t^* = argmax_j \ \ q_j^t$$

Equivalently, the asset price p and the security price q_t:

$$p = p_{j^*}$$
$$q_t = q_t^{j^*}$$

At generation $t(t = 1, 2...)$, the optimist buys the security with the loan contract. In this section, for simplicity, T is assumed to be large enough. Some optimist can buy the security at t' by his own cash.
For analyzing the asset price or the security price, the type j_t^* problem at each

generation t is solved. From each generation $t(T-2 \geq t \geq 1)$, ϕ_t is solved by pessimists' arbitrage condition.

$$\phi_t = q_{t+1}$$

$q_{t'}$ is solved by t' pessimists' arbitrage condition.

$$q_{t'} = E_{o_{j^*_{t'}}}[min(s, \gamma_{t'-1})]$$

So, optimist type j^*_0 problem at generation 0:

$$\max_{a_0,\gamma_0} a_0\{E_{O_{j^*_0}}[s] - E_{O_{j^*_0}}[min(s,\gamma_0)]\}$$
$$s.t. a_0 p = n/\alpha + a_0 q_1(\gamma_0)$$

Because type j^*_0 is the highest optimists' type, they evaluate $E_{O_{j^*_0}}[s] - E_{O_{j^*_0}}[min(s,\gamma_0)]$ higher than any other types.
The type j^*_t optimist problem at generation $t(t'-1 \geq t \geq 1)$:

$$\max_{a_t,\gamma_t} a_t\{E_{O_{j^*_t}}[min(s,\gamma_{t-1})] - E_{O_{j^*_t}}[min(s,\gamma_t)]\}$$
$$s.t. a_t p = n/\alpha + a_t q_{t+1}(\gamma_t)$$

Because type j^*_t is the highest optimists' type, they evaluate $E_{O_{j^*_t}}[min(s,\gamma_{t-1})] - E_{O_{j^*_t}}[min(s,\gamma_t)]$ higher than any other types.
At generation t', the security price is simply $q_{t'} = E_{O_{j^*_{t'}}}[min(s,\gamma_{t'-1})]$. $j^*_{t'}$ is the type who has highest security price in equilibrium.

By solving each generation problem, p and q_t are calculated. Single optimist case, the asset price is equal to the optimistic ecpected return: $E_o[s]$. In this model, asset price is solved by backward induction like single optimist model. But in this case, at each generation, different type optimists buy the asset and the security. In this case, the asset price exceeds any trader's expectation about the asset return.

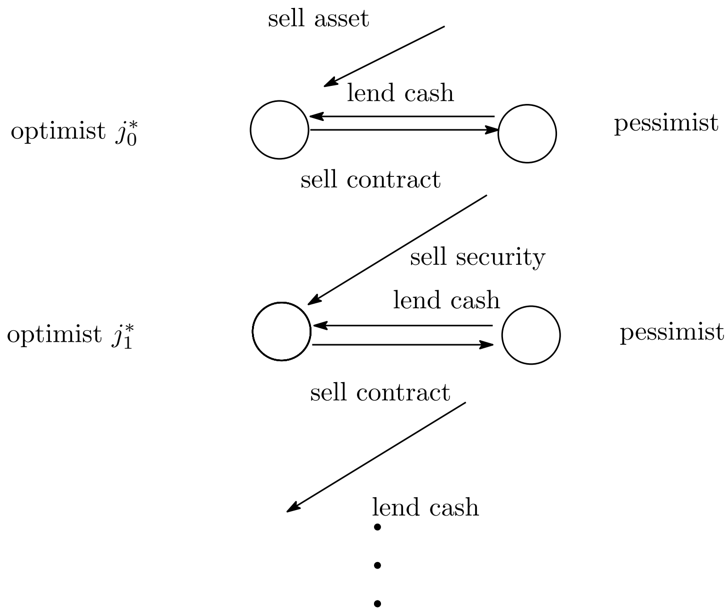
proposition 2 *In multi-generation with various type optimist model, the asset price exceeds optimistic trader's expectation of asset return:*

$$p \geq E_o[s]$$

Proof 5 *As noted above, the security price at date t' is $E_{o_{j^*_{t'}}}[min(s,\gamma_{\bar{t'}-1})]$.*

The optimist type $j^*_{t'-1}$ buys the date $t'-1$ security. The optimist problem at date $t'-1$ is:

$$\max_{a_t,\gamma_t} a_t\{E_{O_{j^*_{t'-1}}}[min(s,\gamma_{t-1})] - E_{o_{j^*_{t'-1}}}[min(s,\gamma_t)]\}$$
$$s.t. a_{t'-1} q_{t'-1} = n/\alpha + a_{t'-1} E_{o_{j^*_{t'}}}[min(s,\gamma_{\bar{t'}-1})]$$



By solving this problem,

$$q_{t'-1} = \int_0^{\gamma_{t'-1}} s dF_{O_{j^*_{t'}}} + (1 - F_{O_{j^*_{t'}}}(\gamma_{t'-1})) \int_{\gamma_{t'-1}}^{s^{max}} min(s, \gamma_{t'-2}) \frac{dF_{O_{j^*_{t'-1}}}}{1 - F_{O_{j^*_{t'-1}}}(\gamma_{t'-1})}$$

The security demand is $1/\alpha$*:*

$$q_{t'-1} = n + q_{t'}(\gamma_{t'-1}) = n + E_{o_{j^*_{t'}}}[min(s, \gamma_{\bar{t'}-1})]$$

The second equation is the same one as one type optimists case. Then, first equation is important for the security price.
If $j^*_{t'-1} = j^*_{t'}$*, that is, the same type optimists buy the securities at both dates, the date* $t'-1$ *security price is simply* $E_{O_{j^*_{t'-1}}}[min(s, \gamma_{t'-1})]$*. In the various optimists model, the optimist type who buys the security can be different at each date. As noted above, the security is bought by optimist who have the highest value. If the different type optimist buys the security at date* $t'-1$*, the date* $t'-1$ *optimist can borrow* $E_{O_{j^*{t'}}}[min(s, \gamma_{t'-1})]$ *units of cash. Because* $E_{O_{j^*{t'}}}[min(s, \gamma_{t'-1})] \geq E_{O_{j^*_{t'-1}}}[min(s, \gamma_{t'-1})]$*, the date* $t'-1$ *optimist increase his security demand by comparing with the single type optimist model. Then, the security price is higher than the single optimist model.*

Let the security price of single optimist type model given γ_{t-1} *be* $q_t^{sin}(\gamma_{t-1})$

$$q_{t'-1}(\gamma_{t'-2}) \geq E_{O_{j^*{t'-1}}}[min(s, \gamma_{t'-2})] = q^{sin}_{t'-1}(\gamma_{t'-2})$$

The security price can be exceed the optimistic expected return. The same calculation imply the each date security price exceeds the single optimist type model.
The date t optimists problem:

$$\max_{a_t,\gamma_t} a_t\{E_{O_{j_t^*}}[min(s,\gamma_{t-1})] - E_{O_{j_t^*}}[min(s,\gamma_t)]\}$$
$$s.t. a_t q_t = n/\alpha + a_t q_{t+1}(\gamma_t)$$

The security demand is $1/\alpha$*:*

$$q_t = n + q_{t+1}(\gamma_t)$$

If $q_{t+1}(\gamma_t)$ *is higher than one type model,* q_t *is higher. Because* $q_{t'-1}(\gamma_{t'-2}) \geq q_{t'-1}^{sin}(\gamma_{t'-2})$*, the each date security price is higher than single type model.*

$$q_t(\gamma_{t-1}) \geq q_t^{sin}(\gamma_{t-1}) = E_o[min(s,\gamma_t)]\ \forall t, \forall \gamma_t$$

The type j_0^* *optimist problem at date 0 is:*

$$\max_{a_0,\gamma_0} a_0\{[E_{O_{j_0^*}}[s] - E_{O_{j_0^*}}[min(s,\gamma_0)]\}$$
$$s.t. a_0 p = n/\alpha + a_0 q_1(\gamma_0)$$

Because $q_1 \geq E_o[min(s,\gamma_0)]$*, the type* j_0^* *optimist have bigger budget than single optimist type model.*
The asset demand is $1/\alpha$*:*

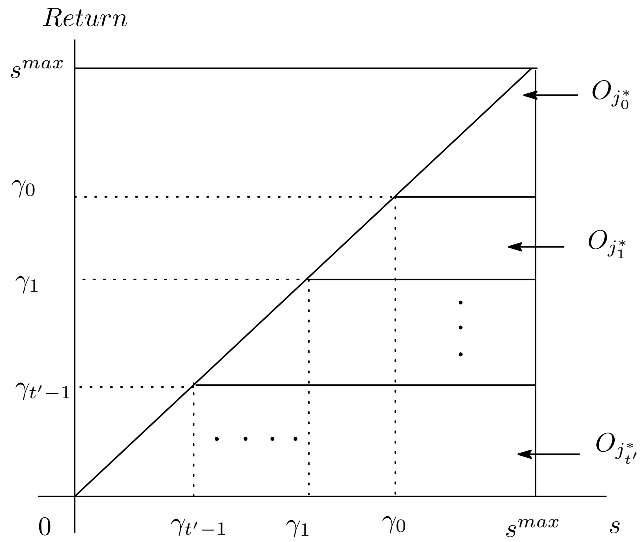
$$p = n + q_1(\gamma_0)$$

$q_1(\gamma_0)$ *is higher than one generation case. So, asset demand* a_0 *increase and asset supply is fixed, the asset price* p *exceeds the single type price* $E_o[s]$ *by both equations.*

$$p \geq E_o[s]$$

The existence of various optimist leads to the high asset price. This price is similar to Harrison and Kreps(1978). In HK, there are various type traders. They have heterogeneous beliefs about asset. At some point, trader x is optimist and trader y is pessimist. But at different point, x is pessimist and y is optimist. The holder of asset changes at each date and the most optimistic trader buys it. The asset holder knows that he can sell it to some other optimist at some future date. Then, asset price is higher than the asset holders' expectations.

The asset price is calculated in the same manner as single optimist model. In single type case, each securities are bought by single optimist. Because single type optimist evaluate the security return, the asset price is equal to the



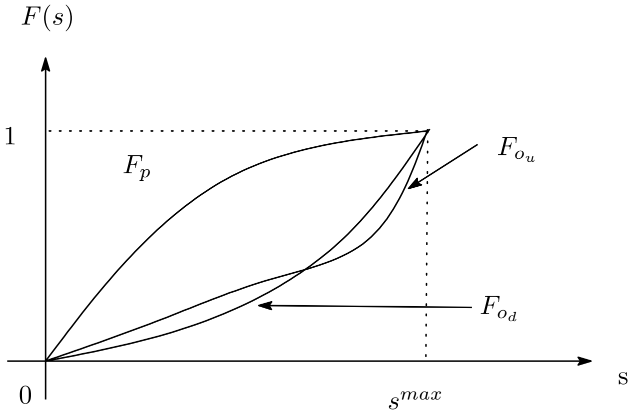
optimistic expected return. On the other hand, each return area are evaluated by the highest type optimist in various type case. In this case, each area are evaluated by the highest optimistic belief. As a result, the asset price exceeds the optimistic expectation. Next proposition show the asset return are evaluated by the most optimistic type belief.
The optimistic expectation about asset return is same:$E_o[s]$. The securitization technology divides the asset return and the each optimist evaluate the each partition.

In Simsek(2013), asset return is evaluated by the optimist and the pessimist. In equilibrium, as noted section 2, asset price:

$$p = \int_0^{\gamma_0} sdF_p + \frac{1 - F_p(\gamma_0)}{1 - F_o(\gamma_0)} \int_{\gamma_0}^{s^{max}} min(s, \gamma_0)dF_o$$

This price equation imply that optimist evaluate upper return area and pessimist evaluate lower area. Only high state part of optimistic belief is used for determining the asset price in Simsek(2013). This is the reason why asset price is lower than HK.

In this model, the most optimistic trader evaluate each area. Only his optimistic part of belief is used for evaluation of asset return. Because the asset is priced by different beliefs, it is much more optimistic than anyone's expectation. Tranching and asset pricing are analyzed by Fostel and Geanakoplos(2012) and they insist that tranching technology raise asset price in recent housing bubble. In my model, each state of asset are distributed to most optimistic trader. Then, equilibrium distribution is similar to one of equilibrium with tranching in FG(2012).



3.2 Example:Two Type Optimist Case

In this subsection, one of the various optimist type model, two optimist type model will be analyzed. There are upside optimist and downside optimist. Optimists type j have optimistic belief $F_j(s)$.($j = O_u, O_d$)(Note expectation E_{O_u},E_{O_d}) Optimists have same expectation about the return of the asset ($E_{O_u}[s] = E_{O_d}[s] = E_o[s]$). Upside optimists thinks bad event is unlikely and downside optimists think good event is unlikely. (see figure.)

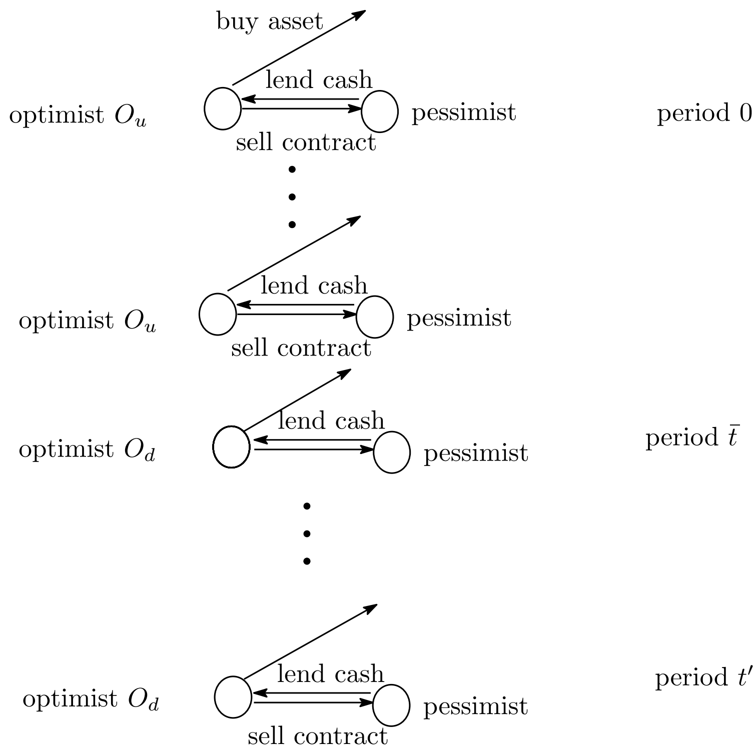
Both optimistic beliefs first order stochastic dominates pessimistic belief.

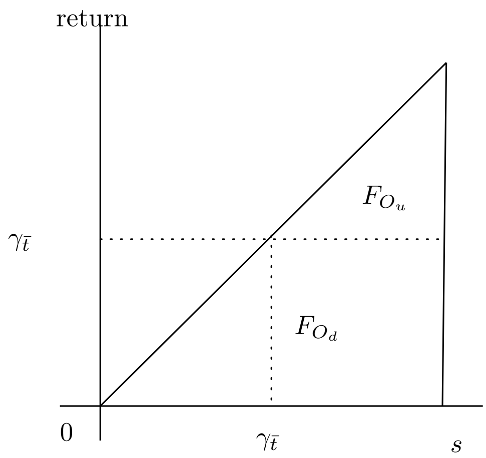
$$F_p[s] \geq F_j[s] \text{for all } s \text{ and } j = O_u, O_d$$

As seen the multi generation case, at some period t', the optimist buys the security by his cash. This security's payoff is $min(s, \gamma_{t'})$.
From the optimist type definition, $E_{O_d}[min(s, \gamma_{t'})] > E_{O_u}[min(s, \gamma_{t'})]$.
The downside optimist have higher evaluation about $min(s, \gamma_{t'})$. Then, the date t' security must be bought by the downside optimist. For the same reason, we can guess that the asset is bought by the upside optimist at date 0.
The upside optimist thinks upper return state is likely and downside optimist thinks opposite.
So, some period $\bar{t}(0 < \bar{t} < t')$ exists. For $t < \bar{t}$, the asset and securities are bought by upside optimists. For $t \geq \bar{t}$, the securities are bought by downside optimist.

Between $t \bar{+} 1 \leq t \leq T - 1$, the downside optimists buy the securities. So, the date t security price is $E_{O_d}[min(s, \gamma_{t-1})]$.
Between $0 \leq t \leq \bar{t}$, the upside optimists buy the securities.
The date $\bar{t}$ (upside) optimists problem is:

$$\begin{aligned} &\max_{a_{\bar{t}}, \gamma_{\bar{t}}} a_{\bar{t}}\{E_o[min(s, \gamma_{\bar{t}-1})] - E_o[min(s, \gamma_{\bar{t}})]\} \\ &\quad s.t.\ a_{\bar{t}} q_{\bar{t}} = n/\alpha + a_{\bar{t}} E_{O_d}[min(s, \gamma_{\bar{t}})] \end{aligned} \tag{43}$$


buy asset
lend cash
optimist O_u
pessimist
period 0
sell contract
lend cash
optimist O_u
pessimist
sell contract
lend cash
optimist O_d
pessimist
period $\bar{t}$
lend cash
optimist O_d
pessimist
period t'
sell contract



This problem is the same as $T = \bar{t}$ single optimist type model if pessimists belief and Optimists belief are F_{O_d} and F_{O_u}.
By backward induction, the asset price is calculated.

$$p = \frac{1 - F_{O_d}(\gamma_{\bar{t}})}{1 - F_{O_u}(\gamma_{\bar{t}}) + \lambda_1'(\gamma_{\bar{t}})n}\{E_{O_u}[s] - E_{O_u}[min(s, \gamma_{\bar{t}})] - \lambda_1(\gamma_{\bar{t}})n\} + E_{O_d}[min(s, \gamma_{\bar{t}})] + \bar{t}n$$

$$p = (\bar{t} + 1)n + E_{O_d}[min(s, \gamma_{\bar{t}})]$$

In multi-generation with single optimist type case, asset price is equal to the optimistic expectation. The optimists holds the asset and the securities. Because there are two type optimists, upside and downside, high return states are evaluated by upside optimist and low return states are evaluated by downside optimist.

Because the asset return is evaluated by higher type optimist, the asset price is higher than single optimist type model in section 3.

Part III
Short Sale Model

4 Short Sale and Pessimistic Asset Price Model

4.1 Settings

The short sale model is also dynamic finite date model($t = 0, 1, ...T$). There is a continuum of states at date 1, denoted by $s \in S = [0, s^{max}]$. The asset pays s dollars at state s. There are two type of traders, optimist and pessimist. Both type traders have risk neutral utility functions. But, they have different beliefs about asset return. Type j trader have a belief about asset return , distribution function $F_j(s)(j = o, p)$. Note their expectation of asset return is $E_o[.], E_p[.]$. Traders know each other's priors. Optimists and pessimists agree to disagree about their beliefs.
Optimists are optimistic about the asset return. Assume first order stochastic dominance. Optimist's distribution function is lower than pessimistic one for all state s.

Assumption 5 $F_p[s] \geq F_o[s]$ *for all* s

Moreover, the following assumption is necessary for the uniquness of the equilibrium.

Assumption 6 $\frac{f_o}{1-F_o} < \frac{f_p}{1-F_p}$ *for each* $s \in (0, s^{max})$

At date 0, the continuum of traders, α pessimists and $1 - \alpha$ pessimists. α is small fraction. At each date t, new α pessimists come to the market. All traders live up to the final date T.

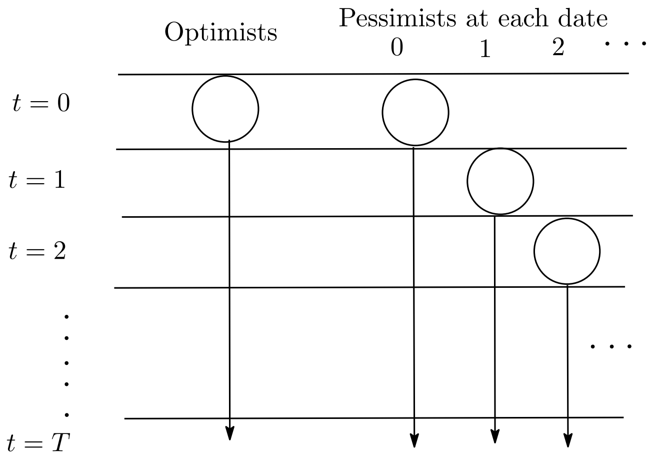
Each pessimist has n/α units of cash. The assumption implies optimists are natural buyers of the asset.
At each generation $t = 0, ...T - 1$, the continuum pessimists uniformally distributed in $[0, \alpha]$ come to the market. All pessimists are homogeneous. They have same belief (F_p). All traders live up to final date T.
All pessimists are initially endowed with small amount of cash $n/\alpha > 0$ dollars and zero unit of the asset.

Assumption 7 $n < E_p[s]$

Each optimists have n_o units of cash. n_o is large enough for buying the asset.

Assumption 8 $n > E_o[s]$

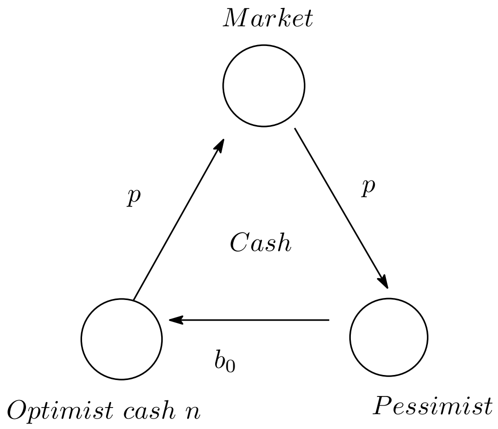
If pessimists have no cash ($n_p = 0$), optimists simply buy the asset. The asset price is equal to optimistic expectation: $p = E_o[s]$.



If pessimists have cash ($n_p > 0$), they can make short sale contracts with optimists. Optimists have high expectation about the asset return and the asset is not fascinating for pessimists. Because optimists have cash for buying the asset, the asset price exceeds pessimists expectation $E_p[s]$.
By short sale contract, pessimists lend the asset from optimists and sell it to the market at date 0. After the asset state is realized, pessimists buy back the asset and return it to optimists. Because pessimists expect that low return states occur more frequently and optimists expect high state more frequently, both traders have incentive for making the contract. Optimists can buy the asset instead of short sale contract. But, for pessimists, short sale contract is the only way to earn cash in this economy.
Short sale contracts are defined by (γ_0, b_0). In the contract γ_0, b_0, pessimists give b_0 units of cash for one unit of the asset to optimists. Pessimists borrow the asset from optimists and they repurchase the asset at date T (the asset price at date T is s). For optimists, this short sale contract imply that the cost of the buying one unit of asset is $p - b_0$. If they reject the contract (γ_0, b_0), they can get one unit of the asset by p. The short sale contract promise the buyback the asset. As pessimists have budget constraint, they cannot pay high s at date T. γ_0 is the limit of the date 1 asset price which pessimists can repurchase. Totally, pessimists repay $min(s, \gamma_0)$ to optimists at date T.
Consider optimists arbitrage condition. Optimists can buy the asset with price p. If they accept short sale contract, they pay $p - b_0$ units of cash and receive the return $min(s, \gamma_0)$ at date T.
Their arbitrage condition implies:

$$\frac{E_o[s]}{p} = \frac{E_o[min(s, \gamma_0)]}{p - b_0} \tag{44}$$

Let $q_0 = p - b_0$



$$\frac{E_o[s]}{p} = \frac{E_o[min(s,\gamma_0)]}{q_0} \tag{45}$$

Optimists buy a_o units and x units of short sale contract. Because optimists have n_o units of cash, optimists' budget constraints:

$$n_o = a_o p + x q_0 \tag{46}$$

Pessimists borrow the asset and sell it with price p at date 0. Pessimists get total $x(p - b_0)(= xq_0(\gamma_0))$ cash at date 0.

If optimists and pessimists make one short sale contract, optimists buy one unit of the asset. Pessimists borrow it and sell it to the market. That is, the total volume of the asset in the market does not change.
If no short sale contract model, the asset price is equal to optimistic expectation.

$$p = E_o[s]$$

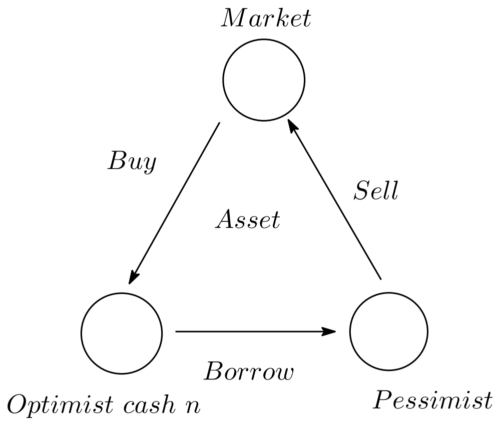
Optimists buy the asset unless the asset price exceeds $E_o[s]$. If x gets large, optimists asset damand a_o gets small. As a result, the asset price gets lower in short sale contract model.
Pessimists receive $p - b_0$ units of cash and they promise repayment $min(s, \gamma_0)$ at date T. They must hold γ_0 units of cash for the repayment.

$$x\gamma_0 = n_p + x(p - b_0) \tag{47}$$

If n_p is large, pessimists can raise x or raise γ_0, High x or high γ_0 imply low asset demand a_o from optimists budget. Pessimists cash put the asset price down.

4.2 One Generation Model

Assume no insurance is allowed in the market.
The pessimist problem:



$$\begin{aligned}\max_{x,\gamma} \;\; & x\gamma - xE_p[min(s,\gamma)] \\ s.t. \;\; & x\gamma = n/\alpha + x(p-b_0) \\ & \frac{E_o[s]}{p} = \frac{E_o[min(s,\gamma_0)]}{q_0}\end{aligned} \tag{48}$$

The first line is pessimists' expected return, which consists of their return from cash holdings net of their expected payment on short contracts. The second line is pessimists' budget constraint and optimists' condition.
x is the supply of the short sale contract. Pessimists can promise repurchase the asset up to $s = n + x(p - b_0)$. As total repayment is $min(s, \gamma_0)$, pessimists' expected utility is $x\gamma_0 - xE_p[min(s, \gamma_0)]$.
From optimists' participation condition:

$$p - b_0 = \frac{1}{\frac{E_o[s]}{p}} E_o[min(s,\gamma)] \tag{49}$$

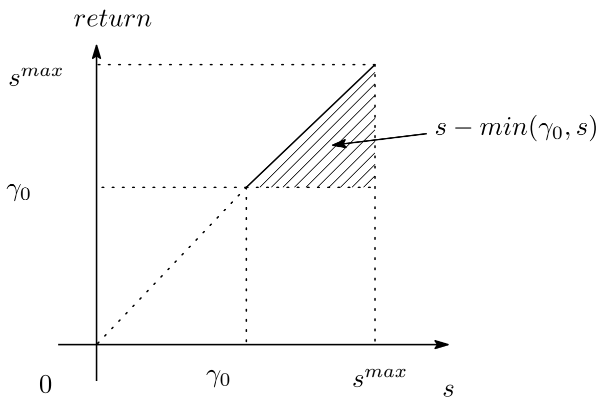
Redefine the pessimists problem:

$$\begin{aligned}\max_{x,\gamma} \;\; & x\gamma - xE_p[min(s,\gamma)] \\ s.t. \;\; & x\gamma = n/\alpha + x\frac{1}{\frac{E_o[s]}{p}} E_o[min(s,\gamma)]\end{aligned} \tag{50}$$

From budget constraint:

$$x = \frac{n}{\frac{1}{\gamma - \frac{1}{\frac{E_o[s]}{p}} E_o[min(s,\gamma)]}} \tag{51}$$

By substituting this to the objective function and first order condition, the asset price formula is calculated.



$$p = \frac{E_p[s]}{F_p(\gamma)\frac{\int_0^\gamma sdF_o}{\int_0^\gamma sdF_p} + 1 - F_o(\gamma)} \tag{52}$$

The price formula shows that the pessimistic belief is asymmetrically disciplined in equilibrium. The pessimistic belief enters the relation only through its effect on $\int_0^\gamma s\frac{dF_p}{F_p(\gamma)}$.

From optimists' budget constraints and the asset supply is one:

$$n_o = p + xq(\gamma) \tag{53}$$

Optimists arbitrage condition:

$$q(\gamma) = \frac{1}{\frac{E_o[s]}{p}} E_o[min(s, \gamma)] \tag{54}$$

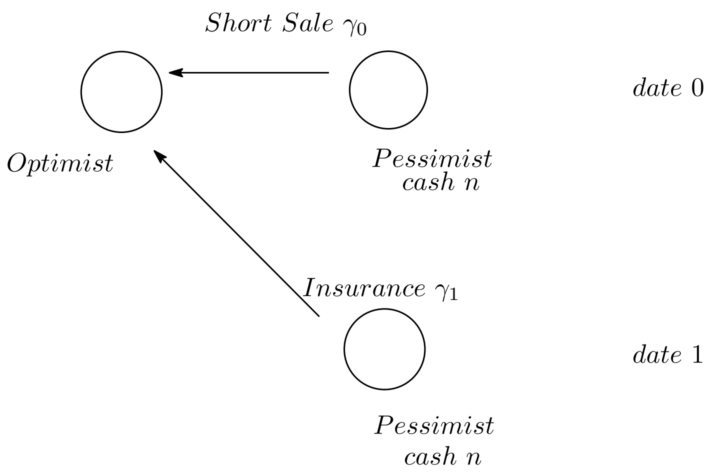
These three equation determine γ_0, the asset price p and $q_0(\gamma_0)$.

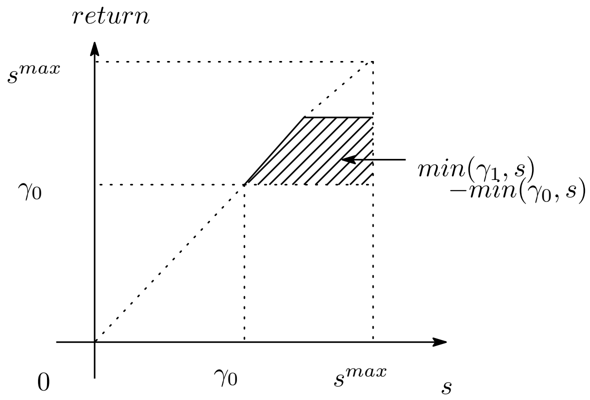
4.3 Two Generation Model

Because pessimists have small cash n, the short sale contract cannot cover high return state($s > \gamma_0$). Optimists evaluation of the payoff is higher than pessimists' for the assumption of first order stochastic dominance.:

$$E_o[s - min(s, \gamma_0)] > E_p[s - min(s, \gamma_0)]$$

There is a incentives for making an insurance contract.
Pessimists at date 1 can sell an insurance contract defined by γ_1. The buyer of the contracts pays $q_1(\gamma_1)$ to the seller. The seller promise to payment $s - \gamma_0$ at date T (the asset price at date T is s) if state s exceeds γ_0. The short sale contracts need to collateralized by $\gamma_1 - \gamma_0$ units of cash.
If asset return s is low($s < \gamma_1$), the seller can pay the promise. However, if asset return exceed $s > \gamma_1$, he cannot. So, he give γ_1 units of cash to the buyer as collateral. Totally, the seller pay $min(s, \gamma_0) - min(s, \gamma_1)$ to the buyer.





Optimists can buy an insurance contract at date 1 with $q_1(\gamma_1)$. Assume the seller of securities have monopolistic power.

If traders buy the assets with price p and sell the short sale contract with price q_0, they can receive the return $s - min(s, \gamma_0)$ at date T.
If optimists buy the insurance, they can receive $min(s, \gamma_1)$ units of cash at date T.
Optimists arbitrage condition implies:

$$\frac{E_o[s - min(s, \gamma_0)]}{p - q_0} = \frac{E_o[min(s, \gamma_1)]}{q_0 + q_1} \tag{55}$$

Given γ_0,p, q_0 and q_1, the date 1 pessimists problem:

$$\max_{x,\gamma_1} \; x(\gamma_1-\gamma_0) - E_p[x(min(s,\gamma_1) - min(s,\gamma_0))]$$
$$s.t. \;\; x(\gamma_1-\gamma_0) = n/\alpha + xq_1 \tag{56}$$
$$\frac{E_o[s - min(s,\gamma_0)]}{p - q_0} = \frac{E_o[min(s,\gamma_1)]}{q_0+q_1}$$

Pessimists prepare cash $\gamma_1-\gamma_0$. This cash is supported by their own cash and the insurance sales. They promise to pay $min(s,\gamma_1) - min(s,\gamma_0)$ at date T.
Optimists buy x_0 units of the short sale contract at date 0. Then, optimists also buy x_0 units of the insurance at date 1.

$$x = x_0 \tag{57}$$

Because γ_0 is given for the date 1 pessimists, the objective function is:

$$x(\gamma_1-\gamma_0) - [min(s,\gamma_1) - min(s,\gamma_0)] = x[\gamma_1 - E_p[min(s,\gamma_1)]] - xA$$

If optimists make the short sale contract and buy the insurance, they can receive payoff $E_o[min(s,\gamma_1)]$ at date 2. Optimists' expected asset return is $E_o[s]$. Optimists arbitrage condition implies:

$$\frac{E_o[s]}{p} = \frac{E_o[min(s,\gamma_1)]}{q_0+q_1} \tag{58}$$

From optimists' budget constraints:

$$n_o = a_0 p + x(q_0+q_1) \tag{59}$$

These one problem and two equations determine γ_0, p, q_0, x_0
From the date 1 pessimists budget constraint:

$$x(\gamma_1-\gamma_0) = n/\alpha + xq_1 \tag{60}$$

From the date 0 pessimists budget constraint:

$$x\gamma_0 = n/\alpha + xq_0 \tag{61}$$

These two equations imply:

$$x\gamma_1 = 2n/\alpha + x(q_0+q_1) \tag{62}$$

Given γ_0, p, q_0, x_0, these three equations determine γ_1, q_1
By backward induction, γ_0, p, q_0, x_0 is solved. The date 0 pessimists problem:

$$\max_{x,\gamma_1} \; x[\gamma_0 - min(s,\gamma_0)]$$
$$s.t. \;\; x\gamma_0 = n/\alpha + xq_0 \tag{63}$$

Optimists arbitrage condition implies:

$$\frac{E_o[s]}{p} = \frac{E_o[min(s,\gamma_1)]}{q_0 + q_1} \tag{64}$$

From optimists' budget constraints:

$$n_o = a_0 p + x(q_0 + q_1) \tag{65}$$

These one problem and two equations determine γ_0, p, q_0, x_0
From the date 1 pessimists budget constraint:

$$x(\gamma_1 - \gamma_0) = n/\alpha + xq_1 \tag{66}$$

From the date 0 pessimists budget constraint:

$$x\gamma_0 = n/\alpha + xq_0 \tag{67}$$

These two equations imply:

$$x\gamma_1 = (n/\alpha + n/\alpha) + x(q_0 + q_1) \tag{68}$$

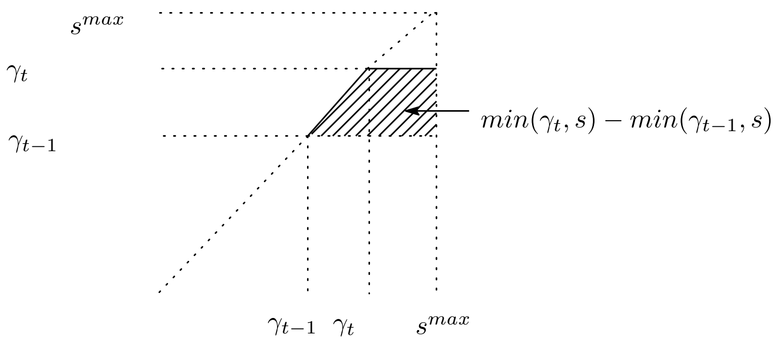
The date 0 pessimists and the date 1 pessimists using $2n/\alpha$ units of cash and guarantee $min(s,\gamma_1)$ payment at date T.
Apparently, γ_1 in this case is higher than γ_0 in no insurance model. If not so, the date 1 pessimists can raise γ_1 to earn more profit. (The date 1 pessimists have n/α units of cash.)
Pessimists can pay higher return by short sale and insurance model.
Let $q_0 + q_1 = q^1$ and let q^0 be q_0 in one gneration model.
From optimists condition:

$$\frac{E_o[s]}{p} = \frac{E_o[min(s,\gamma_1)]}{q^1}$$

From optimists condition in one generation model:

$$\frac{p}{q^0} = \frac{E_o[min(s,\gamma_0)]}{q^0}$$

Then, $\frac{p}{q^1}$ is smaller than $\frac{p}{q^0}$. In two generation case, optimists can get return $E_o[min(s,\gamma_1)]$ with price q^1. $E_o[min(s,\gamma_1)]$ is higher than $E_o[min(s,\gamma_0)]$ in one generation model. Because the asset return is $E_o[s]$, the short sale contract and the insurance return gets closer to the asset return. Then, the price q^1 also gets closer to the asset price p.



4.4 Large Generation Model

This scheme continue to $T-1$. At date t, pessimists sell an insurance defined by γ_t. Optimists buy $q_t(\gamma_t)$ to the seller. Pessimists promise to payment $s - \gamma_{t-1}$ at date T (the asset price at date T is s) if state s exceeds γ_{t-1}. The short sale contracts need to collateraled by $\gamma_t - \gamma_{t-1}$ units of cash.
If asset return s is low($s < \gamma_t$), the seller can pay the promise. However, if asset return is high ($s > \gamma_t$), he cannot. So, he give γ_t units of cash to the buyer as collateral. Totally, the seller pay $min(s, \gamma_t) - min(s, \gamma_{t-1})$ to the buyer.
Optimists can buy an insurance contract at date t with $q_t(\gamma_t)$. Assume the seller of securities have monopolistic power.

These scheme is extended to large generation. The date t insurance is defined by γ_t. The seller of the insurance promises payment $s - min(s, \gamma_{t-1})$ if possible. The insurance needs to be collateralized by $\gamma_t - \gamma_{t-1}$ units of cash. Like the short sale contract, the seller totally pay $min(\gamma_t, s) - min(s, \gamma_{t-1})$ at date T.

If T is large enough, pessimists can insure all asset return s. In this case, the asset price p is easily calculated by each pessimists problem. Let t' denote the date when the insurance supports $s - min(s, \gamma_{t'-1})$. The date t' pessimists can afford to insure the payment $s - min(s, \gamma_{t'-1})$.

$$n > x(s - min(s, \gamma_{t'-1})) \tag{69}$$

Optimists have monopolistic power and pessimists promise $s - min(s, \gamma_{t'-1})$ at date T. The insurance price at date t' is determined by pessimists' zero profit condition.

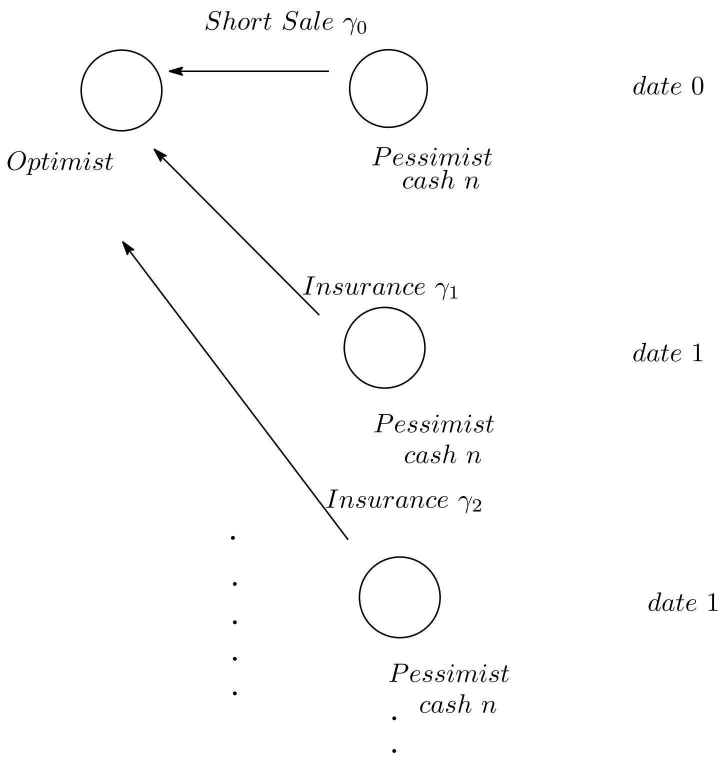
$$q_{t'} = E_p[s - min(s, \gamma_{t'-1})] \tag{70}$$

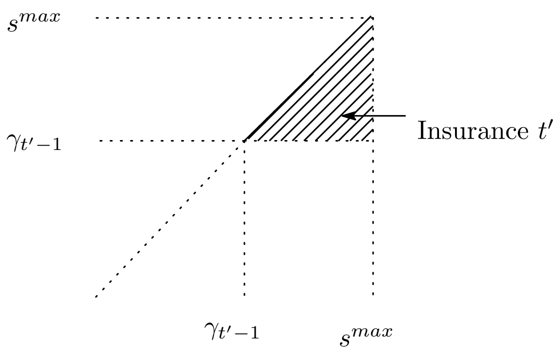
Given $\gamma_{t'-1}$, from optimists' participation condition at date t':

$$q_{t'} = E_p[s - min(s, \gamma_{t'-1})] \tag{71}$$

Given $\gamma_{t'-2}$, the date $t'-1$ pessimist problem:

$$\begin{aligned} \max_{x, \gamma_{t'-1}} \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) - [min(s, \gamma_{t'-1}) - min(s, \gamma_{t'-2})] \\ s.t. \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) = n/\alpha + x q_{t'-1}(\gamma_{t'-1}) \end{aligned} \tag{72}$$


Short Sale γ_0
date 0
Optimist
Pessimist
cash n
Insurance γ_1
date 1
Pessimist
cash n
Insurance γ_2
date 1
Pessimist
cash n


s^{max}
$\gamma_{t'-1}$
Insurance t'
$\gamma_{t'-1}$
s^{max}

$(q_{t'-1}(\gamma_{t'}) = E_p[s - min(s, \gamma_{t'-1})])$
If optimists make the short sale contract and buy each date insurances, they receive $E_o[s]$ Consider optimists arbitrage condition:

$$\frac{E_o[s]}{p} = \frac{E_o[s]}{q_1 + q_2 + ... + q_{t'-1} + q_{t'}} \tag{73}$$

The asset price p is equal to $q_1 + q_2 + ... + q_{t'-1} + q_{t'}$.

$$p = q_1 + q_2 + ... + q_{t'-1} + q_{t'} \tag{74}$$

Let $p_{t'-1}$ denote $p-(q_1+q_2+...+q_{t'-1})$. Because $q_{t'} = E_p[s-min(s, \gamma_{t'-1})]$, $q_{t'-1}$ is equal to $p_{t'-1} - E_p[s - min(s, \gamma_{t'-1})]$.
The date $t' - 1$ pessimist problem:

$$\begin{aligned} \max_{x, \gamma_{t'-1}} \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) - E_p[min(s, \gamma_{t'-1}) - min(s, \gamma_{t'-2})] \\ s.t. \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) = n + x(p_{t'-1} - E_p[s - min(\gamma_{t'-1}, s)]) \end{aligned} \tag{75}$$

Proposition 3

$$p_{t'-1} = E_p[s - min(\gamma_{t'-2}, s)] \tag{76}$$

Proof 6 *The date $t' - 1$ pessimist problem:*

$$\begin{aligned} \max_{x, \gamma_{t'-1}} \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) - E_p[min(s, \gamma_{t'-1}) - min(s, \gamma_{t'-2})]) \\ s.t. \quad & x(\gamma_{t'-1} - \gamma_{t'-2}) = n + x(p_{t'-1} - E_p[s - min(\gamma_{t'-1}, s)]) \end{aligned} \tag{77}$$

From budget constraint:

$$x = \frac{n}{\gamma_{t'-1} - \gamma_{t'-2} - p_{t'-1} + E_p[s - min(\gamma_{t'-1}, s)]} \tag{78}$$

By substituting x in utilities:

$$\frac{n}{\gamma_{t'-1} - \gamma_{t'-2} - p_{t'-1}(\gamma_{t'-1}) + E_p[s - min(\gamma_{t'-1}, s)]}(\gamma_{t'-1} - \gamma_{t'-2}) - E_p[min(s, \gamma_{t'-1}) - min(s, \gamma_{t'-2})]) \tag{79}$$

By differentiating at $\gamma_{t'-1}$:

$$F_p(\gamma_{t'-1})\frac{n}{\gamma_{t'-1} - \gamma_{t'-2} - p_{t'-1} + E_p[s - min(\gamma_{t'-1}, s)]}\{p_{t'-1} - E_p[min(s, \gamma_{t'-2})]\} \tag{80}$$

By FOC:

$$p_{t'-1} = E_p[min(s, \gamma_{t'-2})] \tag{81}$$

By backward induction from this problem to the date 0 pessimist problem, the asset price p is calculated. Next proposition show the asset price is as low as the pessimistic expectation.
Let $p_t \equiv p - (q_1 + q_2 + ... + q_{t-1})$.

Proposition 4 *If T is large enough in multi generation model, the asset price is equal to the pessimistic expected asset return.*

$$p = E_p[s] \tag{82}$$

Proof 7 *Assume $p_{t+1} = E_p[min(s, \gamma_t)]$*

The date t pessimists problem:

$$\begin{aligned} \max_{x,\gamma_t} \; & x\{(\gamma_t - \gamma_{t-1}) - E_p[min(s,\gamma_t) - min(s,\gamma_{t-1})]\} \\ s.t. \; & x(\gamma_t - \gamma_{t-1}) = n + xq_t(\gamma_t) \end{aligned} \tag{83}$$

Optimists arbitrage condition:

$$p = q_1 + q_2 + ... + q_{t'-1} + q_{t'} \tag{84}$$

As $p_{t+1} = E_p[s - min(s, \gamma_t)]$, this condition implies p_t condition.

$$p_t = q_t + p_{t+1} = q_t + E_p[s - min(s, \gamma_t)] \tag{85}$$

Rewrite pessimists problem:

$$\begin{aligned} \max_{x,\gamma_t} \; & x\{(\gamma_t - \gamma_{t-1}) - E_p[min(s,\gamma_t) - min(s,\gamma_{t-1})]\} \\ s.t. \; & x(\gamma_t - \gamma_{t-1}) = n + x(p_t - E_p[s - min(s,\gamma_{t-1})]) \end{aligned} \tag{86}$$

We can solve the problem by similar way as the date $t' - 1$ problem:

$$p_t = E_p[min(s, \gamma_{t-1})] \tag{87}$$

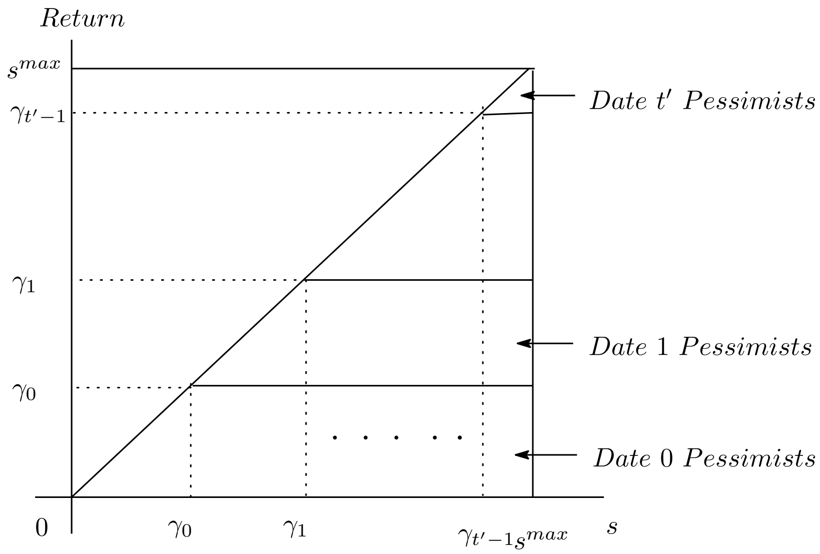
Because $p_{t'-1} = E_p[min(s, \gamma_{t'-1})]$, mathmatical induction imply $p_t = E_p[min(s, \gamma_{t-1})]$. By backward induction, the date 0 pessimists problem:

$$\begin{aligned} \max_{x,\gamma_t} \; & x\{\gamma_0 - E_p[min(s,\gamma_0)]\} \\ s.t. \; & x\gamma_0 = n + xq_0(\gamma_0) \end{aligned} \tag{88}$$

Optimists arbitrage condition:

$$q_0 = p - E_p[s - min(s, \gamma_0)] \tag{89}$$

Rewrite the date 0 pessimists problem:



$$\max_{x,\gamma_t} \ x\{\gamma_0 - E_p[min(s,\gamma_0)]\} \\ s.t. \ \ x\gamma_0 = n + x(p - E_p[s - min(s,\gamma_0)]) \tag{90}$$

From budget constraint:

$$x = \frac{n}{\gamma_0 - p + E_p[s - min(s,\gamma_0)]} \tag{91}$$

By substituting x in utilities:

$$\frac{n}{\gamma_0 - p + E_p[s - min(\gamma_0,s)]}(\gamma_0 - E_p[min(s,\gamma_0)]) \tag{92}$$

By differentiating at γ_0:

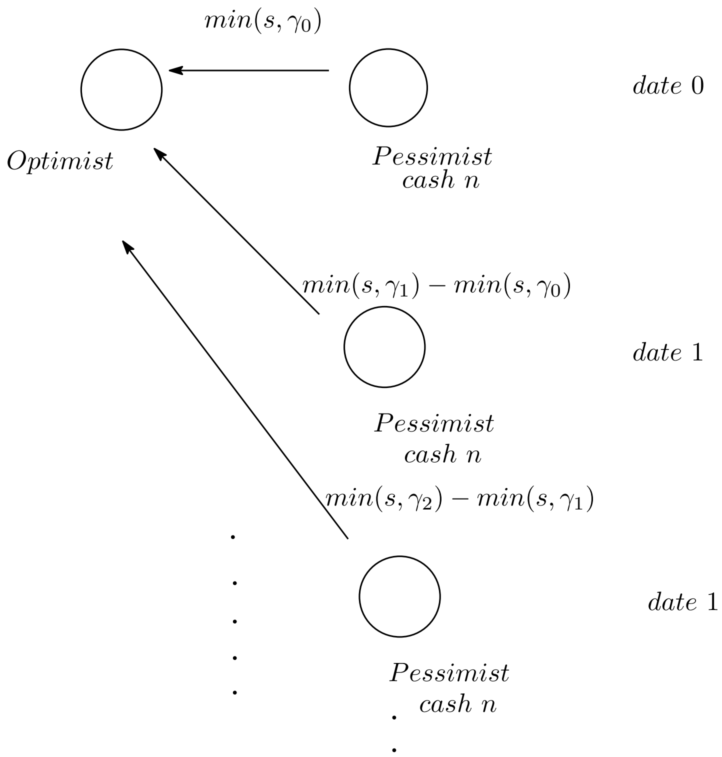
$$F_p(\gamma_0)\frac{n}{\gamma_0 - p + E_p[s - min(\gamma_0,s)]}\{p - E_p[s]\} \tag{93}$$

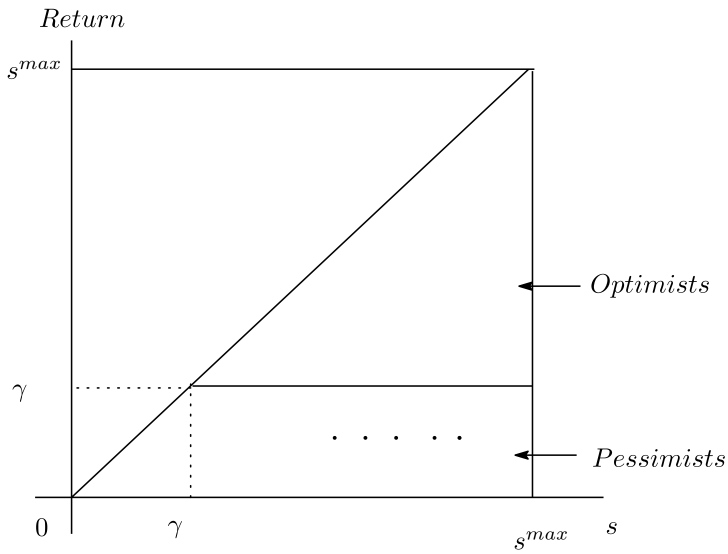
By FOC:

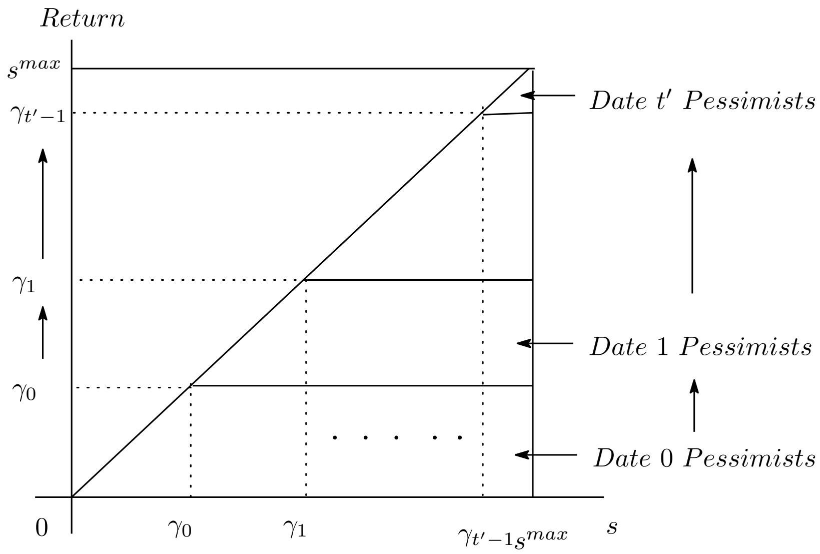
$$p = E_p[s] \tag{94}$$

In equilibrium, the asset return is covered by pessimists' insurances
The asset price is influenced by only pessimistic belief. In two generation model, pessimists of two generation cover all state s. Even if each pessimist have little cash, large generation of pessimists can sell the insurances and cover all state s.

In short sale model at Simsek(2013), pessimists can make short contract with optimists. Because pessimists have limited cash, they cannot insure all


$min(s, \gamma_0)$
$date$ 0
$Optimist$
$Pessimist$
$cash\ n$
$min(s, \gamma_1) - min(s, \gamma_0)$
$date$ 1
$Pessimist$
$cash\ n$
$min(s, \gamma_2) - min(s, \gamma_1)$
$date$ 1
$Pessimist$
$cash\ n$


$Return$
s^{max}
$Optimists$
γ
$Pessimists$
0
γ
s^{max}
s



asset return. So the asset price must be influenced by pessimistic belief and optimistic belief.

In this paper, pessimists are distributed among generations. They have incentive to sell insurances, but they does not have enough cash. Without insurance market, they have no way to cooperate with each other. Insurance technologies make them cooperating.

From one generation model, the price formula:

$$p = \frac{E_p[s]}{F_p(\gamma_0)\frac{\int_0^{\gamma_0} sdF_o}{\int_0^{\gamma_0} sdF_p} + 1 - F_o(\gamma_0)} \tag{95}$$

If $\gamma_0 = s^{max}$:

$$\begin{aligned} p &= \frac{E_p[s]}{F_p(s^{max})\frac{\int_0^{s^{max}} sdF_o}{\int_0^{s^{max}} sdF_p} + 1 - F_o(s^{max})} \\ &= \frac{E_p[s]}{\frac{E_o[s]}{E_p[s]}} \\ &= E_p[s] \end{aligned} \tag{96}$$

So, the asset price in the multi generation model equilibrium is equal to that of one generation (or Simsek(2013)) in which pessimists have enough cash for covering all the asset return.

This price can be interpreted by optimists cash. From optimists' budget constraint:

$$n_o = ap + x(q_0 + q_1 + ... + q_{t'}) \tag{97}$$

In large generation case, the asset and the short sale contract(with insurance) is indifferent for optimists. The price is $E_p[s]$ and the return is $E_o[s]$. From arbitrage condition:

$$p = q_0 + q_1 + ... + q_{t'} = E_p[s]$$

Then,

$$n_o = E_p[s](a + x)$$

Because total asset supply is one, the short sale demand is determined.

$$x = \frac{n_o}{E_p[s]} - 1$$

The short sale contract make a new insurance and the insurance make a new insurance too. The scheme allows the each pessimist to participate in markets. Optimistic belief vanishes in the asset price. However, if there is no optimist, no trader have incentive to buy the asset.
Optimists know that pessimists will come to the market at the next date. Because they agree to disagree about their beliefs, the short sale contract can be made in equilibrium. Optimists have incentive for making short contract and for buying insurances. Their inter-mediation is essential for pessimistic asset price.
In securitization model in the previous section, the asset price is equal to optimistic expectation ($E_o[s]$). If there are financial frictions in heterogeneous belief model, the asset price lie between pessimistic expectation and optimistic expectation. Financial technologies, like securitization or like insurance, allow traders to cooperate among generations. As a result, the asset price is by most optimistic or most pessimistic belief. Financial innovations make market instabilities in both model.
Fostel and Geanakplos(2012) analyses the asset price movement and the timing of introducing new financial technologies. The financial technology improve financial frictions. In their model, the asset price change with the new technology. By tranching, one of securitization technology, the asset price rises and by CDS one of insurance technology, the asset price falls. This paper and Maekawa(2013) can support their result by the theoretical framework.
In heterogeneous economy, new financial technologies can be dangerous for economy. Essential heterogeneity may appear in market and the asset price is influenced by extreme beliefs.

Part IV
Conclusion

In my paper, by introducing the securitization and the simple dynamic setting to Simsek(2013), the asset price will be as high as Harrison and Kreps.
In homogeneous belief model, securitizations improve market efficiency. On the other hand, in heterogeneous belief model, securitizations allow many optimists to participate in markets and asset prices will be raised.
In heterogeneous belief model, financial frictions can make the asset price lower level. Like Harrison and Kreps(1978) or Miller(1977), optimists have heavy influence on pricing. Financial frictions like borrowing constraints can prevent these trader to participate in market. In these situations, financial technologies, like securitizations, loose this friction and the asset price is raised by noisy trader who have little cash.
In my model, securitizations are very important. By securitizations, loan contracts are appealing for cash lender. Because of speculative reason, pessimists lend cash to optimists very easily.
By the dynamic securitization scheme, the asset return is distributed to many partitions. Each partition is very small, but this smallness allow various optimist to receive payoff. Most optimistic traders evaluate their partitions and the total asset evaluation is higher than any trader's expected return of asset. Heterogeneous Belief Bubbles occur in this model.
This is very similar to Bubble in HK(1978). In their model, traders' beliefs change in each date and the most optimistic trader get the asset at each date. Then, the asset price is raised. In my paper, traders' beliefs are different about the frequency of state and expected return is same among optimists. But only by the heterogeneity about frequency, the asset price exceeds any traders' expectation in my model.
Heterogeneous beliefs are said to be eliminated in markets in economics. Noisy traders may lose cash and they may stay out from markets in future.
In short sale model, the asset price is put down by the insurance. The asset price moves very drastically with financial technologies. This implies that the asset market gets very unstable by introducing these schemes.
Financial technologies allow many traders to participate in market before the asset return realizes. Before the asset return realizes, there are little incentive to correct their beliefs. If there are heterogeneity in markets, financial technologies can amplify it.
Fostel and Geanakplos(2012) analyses the asset price movement and the timing of introducing new financial technologies. The financial technology improve financial frictions. In their model, the asset price change with the new technology. By tranching, one of securitization technology, the asset price rises and by CDS one of insurance technology, the asset price falls.
In heterogeneous economy, new financial technologies can be dangerous for economy. Essential heterogeneity may appear in market and the asset price is influ-

enced by extreme beliefs.
In short sale contract model, insurances allow many pessimists to participate in markets and asset prices will be down.
In heterogeneous belief model, financial friction can make asset price stabilized. Like Harrison and Kreps(1978) or Miller(1977), optimistic traders raise the asset price. Pessimistic traders have a great influence in short sale model.
The asset price is equal to the most pessimistic expectation. In Miller or Harrison and Kreps, short sale constraint is assumed. Short selling and insurances put down the asset price in heterogeneous model.
If there are many heterogeneous traders in markets, introducing new financial technologies can be a risky policy. These technologies may change the asset prices drastically.
Before asset return realize, there are littele incentive to collect their belief for them. If there are heterogeneity in markets, financial technology can amplify it.

5 Reference

Brunnermeier, M. (2009), "Deciphering the Liquidity and Credit Crunch 2007-2008," Journal of Economic Perspectives, 23(1), pp. 77-100.

Brunnermeier, Markus K. and Martin Oehmke, 2013, The Maturity Rat Race, Journal of Finance 68(2), 483-521.

DeLong J. B., A. Shleifer, L. H.Summers and R. J. Waldmann(1990) "Noise Tradr Risk in Financial Markets" Jouranal of Political Economy, 98(4)703-738

Fostel, A., and Geanakoplos, J.(2012): "Tranching, CDS, and asset prices: How financial innovation can cause bubbles and crashes," AEJ:Macroeconomics, 4(1)190-225.

Geanakoplos, J. (1997), "Promises, Promises," in The Economy as an Evolving Complex System II, ed. by Arthur, B., S. Durlauf, and D. Lane, 285-320. Addison-Wesley.

Geanakoplos, J.(2003): "Liquidity, Default and Crashes: Endogenous Contracts in General Equi- librium,"in Advances in Economics and Econometrics: Theory and Applications II, Econometric Society Monographs: Eighth World Congress, ed. by Dewatripont, M., L. P. Hansen, and S. J. Turnovsky, Cambridge University Press, vol.2 170-205.

Genakoplos, J.(2010) "The Leverage Cycle" in NBER Macroeconomics Annual 2009, vol. 24 University of Chicago Press.

Gorton, G., and A. Metrick (2011): "Securitized banking and the run on repo," Journal of Financial Economics, 104(3), 425-451.

Gromb, D. and D. Vayanos (2002), "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs,"Journal of Financial Economics, (66)361-407.

Harrison.J.M., and D. Kreps(1978):"Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations" Quarterly Journal of Economics, (89) 323-326

Kiyotaki, N. and J. Moore (1997), "Credit Cycles," Journal of Political Economy,Vol.105 (2), p.211-248.

Krishnamurthy, A., S. Nagel, and D. Orlov (2011): "Sizing Up Repo," Working Paper, Stanford University.

Miller, E. M.(1977):"Risk, Uncertainty, and Divergence of Opinion," Journal of finance, 32(4), 1151-1168.

Ofek, E., and M. Richardson (2003): "DotCom Mania: The Rise and Fall of Internet Stock Prices," Journal of Finance, 58(3), 1113-1137.

Sheinkman, J. and W. Xiong(2003) "Overconfidence and Speculative Bubbles," Journal of Political Economy, 111(6), 1183-1219

Shleifer, A. and R. Vishny (1992), "Liquidation Values and Debt Capacity: A Market Equilibrium Approach,"Journal of Finance, 47, p.1343-1366.

Shleifer, A. and R. Vishny(1997), "The Limits of Arbitrage",Journal of Finance, 52, p.35-55.

Shiller, R. J. (2000): Irrational Exuberance. Princeton University Press.

Simsek, A.(2013): "Belief Disagreements and Collateral Constraints" Econometrica, 81(1), 1-53