論文の内容の要旨

論文題目　Long-time asymptotic states of periodically driven open quantum systems

(時間周期駆動量子開放系の定常状態)

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Surrounded by a heat bath, the system with a time independent Hamiltonian relaxes into an equilibrium state [1]. When the system couples to the heat bath with infinitesimal strength, the state is described by a canonical Gibbs distribution, $\rho \propto e^{-\beta H}$, where H is the Hamiltonian of the system and β is the inverse temperature of the heat bath [2], The closed form solution allows us to evaluate the properties of the equilibrium state without details of the heat bath except for its temperature. The mechanism behind the universal closed-form solution and its emergence from the complicated dynamics remain in the focus of active studies for a long time [3, 4, 5, 6].

When a system is subjected to a periodic driving field, no universal closed form expression is known for the long-time asymptotic state, and hence we have to explicitly take the heat bath into account and analyze the dissipative dynamics numerically or analytically. In a coherent quantum dynamics, a system presents a variety of non-trivial dynamical properties. We review here a driven two-level system, which shows peculiar phenomena, e.g., Rabi oscillations [7] and suppression of quantum tunneling [8]. In cold atom optics, recent experimental progress enables us to control macroscopic properties by using off-resonant and strong oscillating field. A truncated Floquet Hamiltonian obtained from time evolution operator over one period and a perturbative method is an efficient tool to engineer these properties theoretically, which is called

“Floquet engineering”. This Hamiltonian depends on the amplitude and frequency of the external field, and new phases or topological structures, absent in equilibrium, have been found; see the recent review by Bukov *et al.* [9], references therein. We here discuss the utility of the truncated Floquet Hamiltonian through the review of the superfluid-to-Mott-insulator phase transition [10], and its applicability.

In Chap. 2, we explain a master equation formalism, which is one of the conventional methods to treat the dissipation effects. In this framework, it is known that the long-time asymptotic state of a system under a periodic driving field generally does not have a closed form solution due to the lack of the conserved energy owing to the breakdown of the time-translational symmetry [11]. However, it does not always deny the existence of the closed form solution, and then we may ask a question: Which system can acquire the closed form solution?

Indeed, a closed form solution arises in some systems: a single particle subjected to a modulated harmonic oscillator [12, 13] and systems where the time dependence can be eliminated through a unitary transformation [14]. In a non-integrable system we are tempting to use the Floquet basis and see whether its diagonal elements of the asymptotic density matrix are the Boltzmann distribution or not. But before doing so, we have to determine what quantity should take place of the energy E. One of the natural ideas that this role could be played by is an averaged energy of the Floquet state, i.e., the expectation value of the system Hamiltonian averaged over one period of the driving. This idea has been tested in [12, 15]. They demonstrate that in some region the Boltzmann factor with an “effective temperature”, which is different from the actual temperature of the heat bath, appears, while in other region so-called infinite temperature states appear. This separation of two regions is related to coexistence of semi-classically chaotic and regular Floquet states.

These previous studies imply the existence of parameter regimes where the long-time asymptotic states can be described in a closed-form solution. The aim of this thesis is to find the conditions without limiting to some specific model. In order to give a basis for the analysis, in Chap. 2, we introduce two types of quantum mater equations: Floquet Lindblad equation [16] and Redfield equation [17]. The Lindblad equation has a much simpler structure than the Redfield equation, but via the derivation it needs the rotating wave approximation (RWA) or secular approximation. We discuss the range of the applicability of these master equations, and explicitly show that the asymptotic state in the Lindblad formalism is independent of the dissipation strength. Thus, the Lindblad equation is inappropriate for the investigation of the effects of the dissipation strength on the asymptotic state.

In Chap. 3, we study the conditions in the framework of the Lindblad equation. The simple structure of the Lindblad equation and the Kubo-Martin-Schwinger relation [18], which holds for the correlation functions of the bath operators, allow us to introduce the notion of the Floquet-Gibbs state, i.e., a state whose density matrix is diagonal in the basis of the Floquet state with diagonal elements given by the Boltzmann distribution of the quasienergies. Thus, the Floquet states and quasienergies take place of the eigenenergy states and eigenenergies in the conventional canonical Gibbs state. Here, we found that even when the driving field is strong, the asymptotic state is expressed by the Floquet-Gibbs state under the following three conditions:

(i) the driving frequency is much larger than the spectral width of the system Hamiltonian,

(ii) the driving Hamiltonians commute with itself at different instants of time,

(iii) the driving Hamiltonian and the system-bath interaction Hamiltonian should commute.

The condition (i) restricts the system with a relatively small Hilbert space and the condition (iii) requires a fine tuning of the system-bath coupling.

The conditions obtained in Chap. 3 thus strictly restrict the class of systems attaining the Floquet-Gibbs state. We ask a following question: Is the long-time asymptotic state independent of the microscopic details of the coupling to the heat bath? The answer for the system with a time independent Hamiltonian is "*yes*", because the asymptotic properties of the system only depend on the temperature and/or chemical potential of the heat bath. However it is "*no*" for the system subjected to a periodic driving field. We find that the conditions (i) and/or (iii), which severely restrict the emergence of the Floquet-Gibbs state, can be lifted by imposing conditions on timescales of the three constituents, the system, heat bath, driving field. Namely,

(i)' the relaxation rate of the system (controlled by the dissipation strength) is higher than the heating rate (controlled by the frequency of the driving field);

(iii)' the period of the driving field is shorter than the timescale of the bath dynamics (controlled by the cutoff frequency of the spectral density).

The condition (i)' suppresses the resonance transition between the eigenenergy state of the effective (truncated) Floquet Hamiltonian, which allows us to circumvent the condition (i). When the condition (iii)' is satisfied, energy excitation of the bath due to the periodic driving is suppressed, in which the condition (iii) is no more necessary. We illustrate these scenarios with aid of a spin chain model by using the Redfield equation, and demonstrate that the Gibbs state of the effective Floquet Hamiltonian can be realized without imposing strict conditions on the system Hamiltonian. This result clearly indicates that the long-time asymptotic states subjected to a periodic driving field strongly depend on the detailed property of the coupling to the heat bath in contrast to the system with time-independent Hamiltonian.

In Chap. 5, we study a cooperative phenomenon in a driven cavity system by combining the mean field approach and the Lindblad formalism. We adopt the Dicke model in which multiple two-level systems interact with a single quantized mode of photon field. Owing to the effects of the atom-photon coupling and the dynamical effects induced by a laser field, we find a novel type of dynamical phase transition subjected to a strong driving field. Through the study of an effective spin model, we found that this phenomenon is originated from the synergistic effects of the CDT, which is observed in the driven two-level systems and the interaction between atoms. We also study the asymptotic system under a fast and strong driving field by using the Floquet Lindblad equation. When the system meets the conditions for the Floquet-Gibbs state, we demonstrate that this cooperative phenomenon can be understood in terms of the Floquet-Gibbs state of the truncated Floquet Hamiltonian

Finally, in Chap. 6, we give a conclusion and future prospect.

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