学位論文

Dynamics of Peccei-Quinn Field in the Early Universe

(初期宇宙における Peccei-Quinn 場のダイナミクス)

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東京大学大学院理学系研究科
物理学専攻
瀧本 真裕

Abstract

In this thesis, we consider the cosmology of the Peccei-Quinn models paying particular attention to the dynamics of the scalar fields which have $U(1)_{PQ}$ charges. Cosmological consequences largely depend on the dynamics of such scalar fields in the early universe. If $U(1)_{PQ}$ is restored at some epoch, the $U(1)_{PQ}$ charged scalar fields start to oscillate later. Such oscillating scalar fields may dominantly decay into relativistic axion particles which become a dark radiation component of the universe, whose energy density is tightly bounded. We clarify in what circumstances the axion overproduction is avoided taking effects from the thermal plasma in the universe into account. We show that owing to the thermal dissipation effects, the axion overproduction can be avoided even when the scalar field once dominates the universe. We also consider the case where the $U(1)_{PQ}$ symmetry is always broken in the history of the universe due to field values of scalar fields. In such a case, a severe constraint is imposed on the Hubble scale during inflation. We propose two new scenarios which relax this constraint. In both scenarios, the dynamics of the $U(1)_{PQ}$ charged scalar fields play important roles.

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Chapter 1

Introduction

1.1 Overview

The standard model of the particle physics is well established after the discovery of the Higgs boson by the Large Hadron Collider experiments at CERN [4, 5]. However, we surely know that the standard model has some difficulties. The Peccei-Quinn (PQ) mechanism [6, 7] was proposed motivated by one of the most serious difficulties in the standard model: the strong CP problem [8]. If a model has the PQ mechanism, the strong CP problem is elegantly solved. In this point of view, models with the PQ mechanism (PQ model) are promising candidates of the model beyond the standard model. Therefore, it is important to test whether PQ models are consistent with present observations or not.

The PQ mechanism can be realised if a model has a global U(1) symmetry: $U(1)_{PQ}$ which is anomalously broken by the quantum chromodynamics (QCD). Then, if $U(1)_{PQ}$ is spontaneously broken by vacuum expectation values of $U(1)_{PQ}$ charged complex scalar fields (PQ field), the PQ mechanism can work. After $U(1)_{PQ}$ is spontaneously broken, one of the components in the PQ field(s) becomes a (pseudo) massless mode, which is called as "axion" [9, 10]. The properties of the axion are extensively studied in the literature (see [11] for a review).

In this thesis, we consider the physics of the PQ fields paying particular attention to their dynamics in the early universe. At the present time, the expectation values of the PQ fields are considered to settle down at the minimum of the potential and only the physics of the axion would be relevant. However, at an early epoch of the universe, the PQ fields may have field values different from that of vacuum and may show dynamical motions. In some cases, the dynamics of the PQ field drastically affects the cosmological history as we will see below and some scenarios are already excluded by the present observations [12]. In this point of view, to analyse the cosmological dynamics of the PQ field is a powerful tool to test the PQ models.

The cosmological scenario of the PQ models can be divided into two cases. One case is that the $U(1)_{PQ}$ symmetry is restored at some epoch of the universe after inflation. The other case is that the $U(1)_{PQ}$ symmetry is always broken due to the field values of the PQ fields. For both cases, the dynamics of the PQ fields affects the cosmological history.

First, let us consider scenarios where the $U(1)_{PQ}$ symmetry is restored at some epoch of the universe. The restoration of the $U(1)_{PQ}$ symmetry can be caused by the interaction between the PQ field and particles in the universe. Such interactions can stabilise the potential of the PQ field around the symmetry enhanced point, at which the PQ symmetry is restored. In fact, it is shown that in some cases the PQ field is likely to be trapped at such a point due to the interactions even if the PQ field initially exists far away from there [1, 13]. After the PQ field is trapped at the symmetry enhanced point, the PQ field stays there as long as the potential is stabilised. As the universe expands, the number densities of the particles, which stabilise the potential, decrease. As a result, the stabilisation effects become weaker and weaker. At some time, the PQ field escapes from the symmetry enhanced point and starts to roll down to the minimum of the vacuum potential. Then, the PQ field starts to oscillate around the minimum of the potential. Globally, cosmic strings are formed because the $U(1)_{PQ}$ symmetry becomes broken. Both the cosmic strings and the oscillation of the PQ field can affect the cosmological consequences as we see below.

Let us see the fate of the cosmic strings formed after the PQ symmetry breaking. When the temperature of the universe decreases to the QCD scale ($\sim$ 400 MeV), the potential of the axion is formed due to the QCD effects. As a result, cosmic strings becomes string-domain-wall networks. The fate of the string-domain-wall networks depends on the number of the minima of the axion potential, which is called as the domain wall number. If the domain wall number is greater than one, string-domain-wall networks are stable and soon over dominate the universe [14]. Thus, such a scenario is excluded. On the other hand, if the domain wall number is equal to one, string-domain-wall networks can decay into axions, which contribute to the present dark matter abundance [15] .

The oscillation of the PQ field after the PQ symmetry breaking may also affect the cosmological consequences. The oscillation of the PQ field can produce relativistic axion particles through a perturbative decay. Such axions contribute to a dark radiation component whose energy density is tightly constrained by the cosmic microwave background (CMB) observations [16]. Thus, scenarios which predict too much axion dark radiation are excluded.

So far, we have seen the case in which the PQ symmetry is restored at some epoch in the universe. Let us see the opposite case: the PQ symmetry is always broken by the PQ fields. During inflation, the axion field may have a certain value different from the vacuum expectation value. As the temperature of the universe decreases to the QCD scale and the potential for the axion is lifted up, the axion starts to oscillate around the minimum of the potential. Such an oscillating axion contributes to a part of the present cold dark matter abundance [17–19]. On the other hand, since the axion is almost massless, it acquires quantum fluctuations during inflation. Such fluctuations induce the isocurvature perturbation of the cold dark matter [20–24], which is tightly constrained by the CMB observations. This constraint is imposed on the inflation scale since the size of the quantum fluctuation is of order of the Hubble parameter during inflation. This tight constraint is incompatible with many high scale inflation models including the Higgs inflation [25], which is the most economical inflationary scenario because the Higgs field plays the role of the inflaton. There are some ways to relax the constraint on the inflation scale [26–34] and it would be worth to consider a different possibility to relax the constraint.

In this thesis, we consider the cosmology of PQ models paying particular attention to the dynamics of the PQ field, based on the author's works [2, 3, 35]. In particular, we focus on the dynamics of the PQ field oscillation after the PQ symmetry restoration in Chapter 4. We also consider the case where the PQ symmetry is always broken by the PQ fields. We propose two new scenarios in which the constraint on the inflation scale can be relaxes in Chapter 5.

In Chapter 4, we consider the dynamics after the restoration of the PQ symmetry. As the temperature of the universe decreases, the PQ scalar starts to oscillate around the minimum of the potential. In the dynamics of the PQ field oscillation, the thermal dissipation effects play important roles. This is because without the thermal dissipation effects, the PQ field often dominantly decays into axions, resulting in the axion overproduction. We study the dynamics of the PQ field taking thermal dissipation effects into account. We show that owing to the thermal dissipation effects, a wide parameter region can survive against the axion overproduction. Especially, such a parameter region includes a class of supersymmetric PQ models.

In Chapter 5, we consider scenarios in which the constraint on the inflation scale is relaxed and the PQ symmetry restoration is avoided, which is indispensable for the models with the domain wall number greater than unity. We propose two new scenarios in this direction. In the first scenario, a peculiar behaviour of scalar field motion in the expanding universe is considered. We show that if the shape of the scalar potential and the expansion rate of the universe satisfy a certain condition, a peculiar feature of the scalar field dynamics shows up, which we denote as "pseudo scaling solution". Then, we find that if the PQ field obeys the pseudo scaling solution, the constraint on the inflation scale can be relaxed without the PQ symmetry restoration. In the second scenario, the dynamics of multi PQ fields is considered. We find that if one PQ field has a larger initial value and another PQ field dominantly breaks the PQ symmetry at present day, the constraint on the inflation scale can be also relaxed without the PQ symmetry restoration. We apply this idea on the DFSZ(Dine, Fichler, Srednichki and Zhitnitskii) [36, 37] axion model, which has domain wall number greater than unity. We show this scenario can work within the framework of Higgs inflation, which has relatively large inflation scale.

1.2 Organisation of this thesis

The organisation of this thesis is as following.

In Chapter 2, we review the Peccei-Quinn mechanism and its properties. We explain the strong CP problem in Sec. 2.1. Then, we introduce the PQ mechanism which solves the strong CP problem in Sec. 2.2. In Sec. 2.3, we summarise properties of the axion field which appears in the PQ models. We introduce some PQ models in Sec. 2.4. The astrophysical and experimental constraints are summarised in Sec. 2.5.

In Chapter 3, we review the cosmology of the PQ models. We show an overview of the PQ cosmology in Sec.3.1. The cosmological consequences largely depends on whether the PQ symmetry is restored at some epoch after inflation or not. In Sec. 3.2, we review the case where the PQ symmetry is not restored in the history of the universe. In Sec. 3.3, we review the opposite case where the PQ symmetry is restored at some epoch. We discuss with what conditions the PQ symmetry is restored in Sec.3.4. Finally we review the dark radiation constraint on the axion in Sec. 3.5.

In Chapter 4 We consider the dynamics of the PQ field after the PQ symmetry restoration. In Sec. 4.1, we summarise basic ingredients of this scenario. Then, we introduce the coupled Boltzmann equation which determine the evolution of the system in Sec. 4.2. We also explain qualitative features of the dynamics. We show numerical results in Sec. 4.3. Sec. 4.4 is devoted to the conclusion of this Chapter.

In Chapter 5, we consider scenarios where the constraint on the inflation scale is relaxed by some mechanism without the PQ symmetry restoration. In Sec. 5.1, we show an overview of scenarios in this direction. In Sec. 5.2, we study the pseudo scaling solution which is a peculiar behaviour of the scalar field motion. We show that if the PQ field once obeys the pseudo scaling solution, the constraint on the Hubble parameter can be relaxed without the PQ symmetry restoration. In Sec. 5.3, we propose a new scenario in which the dynamics of multi PQ fields plays important roles. Then, we apply this mechanism to the DFSZ model with the Higgs inflation.

The Chapter 6 is devoted to the conclusion.

Chapter 2

Peccei-Quinn Mechanism and its Properties

The strong CP problem [8] is a naturalness problem related to the standard model of the particle physics. One of the most elegant ways to solve this problem is the Peccei-Quinn(PQ) mechanism [6, 7].

In this chapter, we first explain the strong CP problem. Then, we see how the PQ mechanism solves this problem. Then, we briefly review the properties of the models with the PQ mechanism. In general, models with the PQ mechanism contain an additional (pseudo) massless boson particle called "axion" [9, 10]. We list up the interactions between the axion particle and the stared model particles and summarise observational constraints on them.

2.1 Strong CP problem

The quantum chromodynamics(QCD) has a so called theta term [38] in the Lagrangian:

$$\mathcal{L}_{QCD} \supset \frac{\theta}{32\pi^2} G\tilde{G}, \tag{2.1.1}$$

where G indicates the strength tensor of the gluon field and $\tilde{G} \equiv \frac{\epsilon^{\mu\nu\rho\sigma}}{2} G_{\rho\sigma}$ is the dual tensor. The term $\tilde{G}G/32\pi^2$ takes only integer values by instanton solutions [39]. The parameter θ can be changed by the chiral phase rotation of quarks. However, the combination

$$\bar{\theta} \equiv \theta + \mathrm{Im}[\ln\det(M_D M_U)], \tag{2.1.2}$$

where $M_{D/U}$ denotes the up/down quark mass matrix, is invariant and in fact is an observable [38, 40].

Theoretical calculations indicate the neutron electric dipole moment to be $d_n \sim 10^{-16}\bar{\theta}e$ cm [41–46] with e being the absolute value of the electron electric charge. So far, there is no evidence of the neutron electric dipole moment. The current experimental limit is obtained as $|d_n| < 2.9\times10^{-26}e$cm (90% C.L.) [47]. This limit gives a bound on the value of $\bar{\theta}$:

$$|\bar{\theta}| \lesssim 10^{-10}. \tag{2.1.3}$$

On the other hand, the parameter $\bar{\theta}$ is supposed to be of $\mathcal{O}(1)$ parameter by two reasons:

- we expect the value of Im[ln det($M_D M_U$)] is of order unity because we know there exists a CP violating phase in the quark mass matrix known as CKM phase [48, 49].

- the parameter θ is an undetermined free parameter which has nothing to do with the quark mass matrix.

Thus, the smallness of the parameter $\bar{\theta} \lesssim 10^{-10}$ is considered to be unnatural. This problem is called the strong CP problem [8]. We expect that the new physics will explain the smallness of $\bar{\theta}$. One of the most elegant solutions to the strong CP problem is the Peccei-Quinn mechanism [6, 7].

2.2 Peccei-Quinn Mechanism

In Peccei-Quinn mechanism, the parameter $\bar{\theta}$ becomes a dynamical field called "axion". If the minimum of the axion potential corresponds to $\bar{\theta} = 0$, the strong CP problem is solved. First, let us consider the effective potential regarding $\bar{\theta}$ as a dynamical field. The potential energy of the Euclidian[1] QCD with some gauge changed fermions ($q_i, i = 1, 2, ..$) $V(\bar{\theta})$ can be evaluated as [8]

$$\exp\left[-\int_x V(\bar{\theta})\right] = \int \mathcal{D}A \int \prod_i \mathcal{D}\bar{q}_i \mathcal{D}q_i \, \exp\left[-\int_x \left(\frac{1}{4g_s^2}GG + \sum_i \bar{q}_i(\not{D} + m_i)q_i - \frac{i\bar{\theta}}{32\pi^2}G\tilde{G}\right)\right], \tag{2.2.1}$$

where $\mathcal{D}$ indicates the path integral, g_s denotes the gauge coupling of QCD and the masses m_i are set to be real and positive. The Dirac operator $i\not{D}$ is Hermitian and thus the eigenvalues are real. Then, it can be shown that Det[$\not{D} + m_i$] to be real and positive.

[1] The vacuum structure can be obtained by the Euclidian formalism.

By using the Schwarz inequality, we can have:

$$\begin{aligned}
\exp\left[-\int_x V(\bar\theta)\right] &= \left|\int \mathcal{D}A \prod_i \mathrm{Det}[\not{D}+m_i]\exp\left[-\int_x\left(\frac{1}{4g_s^2}GG-\frac{i\bar\theta}{32\pi^2}G\tilde G\right)\right]\right| \\
&\le \int \mathcal{D}A \left|\prod_i \mathrm{Det}[\not{D}+m_i]\exp\left[-\int_x\left(\frac{1}{4g_s^2}GG-\frac{i\bar\theta}{32\pi^2}G\tilde G\right)\right]\right| \\
&= \int \mathcal{D}A \prod_i \mathrm{Det}[\not{D}+m_i]\exp\left[-\int_x\left(\frac{1}{4g_s^2}GG\right)\right] \\
&= \exp\left[\int_x V(\bar\theta=0)\right].
\end{aligned} \tag{2.2.2}$$

This calculation indicates $V(\bar\theta = 0) \le V(\bar\theta)$ and the equality holds if and only if $\bar\theta = 0$ for $-\pi \le \bar\theta \le \pi$. In other words, if $\bar\theta$ is a dynamical field, its potential minimum exists at $\bar\theta = 0$.

Let us consider a new field a called "axion" which only has the following interaction

$$\mathcal{L}_a = \frac{1}{2}(\partial a)^2 + \frac{1}{32\pi^2}\frac{a}{F}G\tilde G, \tag{2.2.3}$$

with F being a constant. Then,

$$\langle \bar\theta + \frac{a}{F}\rangle = 0 \tag{2.2.4}$$

can be realised at vacuum, which solves the strong CP problem. The interaction Eq. (2.2.3) can be realised by the following way:

- introduce a U(1) symmetry called $\mathrm{U(1)_{PQ}}$ which is anomalously broken by QCD.
- break $\mathrm{U(1)_{PQ}}$ spontaneously by non-zero vacuum expectation values of $\mathrm{U(1)_{PQ}}$ charged scalar fields.

Then, the (pseudo) flat direction in scalar fields related to the $\mathrm{U(1)_{PQ}}$ becomes axion which has E.q. (2.2.3) type interaction. The properties of axion depend on models. Below, we briefly summarise them.

2.3 Properties of Axion

First, let us consider a rather general situation. We assume there is a set of canonically normalised complex scalars $\phi_A (A = 1, 2, ...)$ and pairs of QCD charged fermions $q_c^R,\ q_c^L$ and QCD singlet fermions $q_l^R,\ q_l^L$ where L/R indicates the projection of chirality on the

Dirac fermion. We use subscript $c(l)$ to indicate that the fermion $q_{c(l)}$ does have (does not have) a QCD charge. For convenience, we define the subscript i which runs over both c and l: $i = c, l$. We assign $\mathrm{U}(1)_{\mathrm{PQ}}$ charges for each field as $q_i^{L/R} : Q_i$ and $\phi_A : C_A$. In this notation, $\mathrm{U}(1)_{\mathrm{PQ}}$ rotation of angle β is defined as

$$q_i^\bullet \to e^{i\beta Q_i^\bullet} q_i^\bullet, \quad \phi_A \to e^{i\beta C_A}\phi_A. \tag{2.3.1}$$

We consider the following interaction which preserves the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry:

$$\mathcal{L}_{\mathrm{int}} = \left[\sum_{A,i} y_{i,A}\bar{q}_i^L q_i^R \phi_A + \text{H. c.}\right] + V(\phi_A), \tag{2.3.2}$$

where $y_{i,A}$ denotes a Yukawa coupling and $V(\phi_A)$ indicates a scalar potential. $\mathrm{U}(1)_{\mathrm{PQ}}$ invariance demands $Q_i^L - Q_i^R = C_A$ for $y_{i,A} \neq 0$. We consider the situation where PQ charged scalars obtain vacuum expectation values $\langle\phi_A\rangle = \Phi_A$ depending on the potential $V(\phi_A)$. We can define the canonically normalised "axion" field a to be

$$\phi_A = \Phi_A \exp\left[i\frac{C_A a}{F}\right], \tag{2.3.3}$$

$$F \equiv \sqrt{\sum_A \frac{1}{2}\Phi_A^2 C_A^2}, \tag{2.3.4}$$

where we do not consider other degrees of freedom in ϕ_A for simplicity. In some cases, some of ϕ_A have gauge charges and some degrees of freedoms in ϕ_A are eaten due to the Higgs mechanism. We can choose the $\mathrm{U}(1)_{\mathrm{PQ}}$ in such a way that the field a in Eq. (2.3.3) becomes a physical degree of freedom. We assume such a choice of $\mathrm{U}(1)_{\mathrm{PQ}}$ is made. In addition, the overall sizes of C_A is chosen so that the scalar configuration becomes periodic under the shift $a \to a + 2\pi F$ and no shorter period of scalar configuration exists.

After substituting Eq. (2.3.3) into the Lagrangian, we can obtain:

$$\begin{aligned}\mathcal{L}_a \supset\ & \frac{1}{2}(\partial a)^2 + \sum_i (\bar{q}_i^L i\not{D} q_i^L + \bar{q}_i^R i\not{D} q_R) \\ & - \left[\sum_{A,i} |y_{i,A}\Phi_A| \bar{q}_i^L q_i^R \exp\left(i\frac{C_i a}{F}\right) + \text{H. c.}\right].\end{aligned} \tag{2.3.5}$$

Here, $C_i \equiv C_i^L - Q_i^R$ and we remove constant phases for simplicity. We can redefine the phases of quarks like $q^L \to q^L e^{i\alpha a(x)/F}$ and $q^R \to q^R e^{-i\alpha a(x)/F}$. With the phase rotation of fermions, the Lagrangian can be generically written as [11]:

$$\begin{aligned}\mathcal{L}_a \supset &\frac{1}{2}(\partial a)^2 + \sum_i (\bar{q}_i^L i\not{D} q_i^L + \bar{q}_i^R i\not{D} q_R) \\ &+ \sum_i \left[c_{i,1}\frac{(\partial_\mu a)}{F}\bar{q}\gamma^\mu\gamma_5 q_i - \left(\bar{q}_i^L m_i q_i^R \exp[ic_{i,2}\frac{a}{F}] + \text{H.c.}\right)\right] \\ &+ c_3\frac{a}{32\pi^2 F}G\tilde{G} + c_{\text{em}}\frac{a}{32\pi^2 F}F_{\text{em}}\tilde{F}_{\text{em}},\end{aligned} \tag{2.3.6}$$

where F_{em} indicates the field strength of photon field and $m_i \equiv \sum_A |y_{i,A}\Phi_A|$. The coefficients $c_\bullet$ in Eq. (2.3.5) are read to be $c_{i,1} = c_3 = c_{\text{em}} = 0$ and $c_{i,2} = C_i$. When we rotate fermion phases as $q_i^L \to q_i^L e^{i\alpha_i a(x)/F}$ and $q^R \to q_i^R e^{-i\alpha_i a(x)/F}$, each coefficient shifts as $c_{i,1} \to c_{i,1} - \alpha_i$, $c_{i,2} \to c_{i,2} - 2\alpha_i$ and $c_3 \to c_3 + 2\sum_c \alpha_c$ [2]. Though the physics does not depend on the choice of the phases [3] , there are some convenient choices of $c_\bullet$ depending on what one would like to know.

Let us consider the potential structure of the axion field. To see this, it is convenient to set all $c_{i,2}$ to be zero by the phase rotation. In this choice, the coefficient c_3 becomes $c_3 \to c_3 + \sum_c c_{c,2}$. Note that the combination $\sum_c c_{c,2} + c_3$ is invariant under the phase rotation. We define the domain wall number N_{DW} as

$$N_{\text{DW}} \equiv \left|\sum_c c_{c,2} + c_3\right|. \tag{2.3.7}$$

Then, the interaction of the axion can be written as

$$\mathcal{L}_a = N_{\text{DW}}\frac{a}{32\pi^2 F}G\tilde{G}, \tag{2.3.8}$$

where we neglect derivative couplings and c_{em} term. As the term $G\tilde{G}/32\pi^2 F$ takes integer values, minima of the axion potential exist at $a/F = 2\pi n/N_{\text{DW}}$ with an integer n. On the other hand, the field values $a/F = 2\pi n (n = 0, \pm 1, ..)$ correspond to the same point in the scalar field configuration ϕ_A. Thus, the domain wall number N_{DW} represents the number of the minima of the scalar potential. As we will see in the next Chapter, whether $N_{\text{DW}} = 1$ or not is important for the PQ cosmology. More detailed shape of the axion potential is given in Sec. 2.3.1. For later convenience, we define the axion decay constant F_a as $F_a \equiv F/N_{\text{DW}}$. We also denote F_a as the PQ scale.

[2] Here we assume q_c belongs to the fundamental representation of the QCD for simplicity.

[3] Though properties of the field a depend on the choice of the phases , we call the field a as "axion" for simplicity.

2.3.1 Axion potential

Here, we consider the potential of the axion. We have seen the axion potential is periodic under the shift $a \to a + 2\pi F_a$ with F_a being the axion decay constant. On the other hand, the mass of the axion at vacuum is computed to be [9]:

$$m_a = \frac{f_{\pi^0} m_{\pi^0}}{F_a} \sqrt{\frac{z_d}{(1+z_d)(1+z_d+z_s)}} \simeq 6\ \mathrm{meV} \times \frac{10^9\ \mathrm{GeV}}{F_a}, \tag{2.3.9}$$

where f_{π^0} and m_π^0 are the pion decay constant and pion mass respectively and

$$z_d \equiv \frac{m_u}{m_d} \simeq 0.568 \pm 0.042, \tag{2.3.10}$$

$$z_s \equiv \frac{m_u}{m_d} \simeq 0.029 \pm 0.0003, \tag{2.3.11}$$

$$\tag{2.3.12}$$

are given in [50]. In appendix A, We derive this axion mass using the chiral perturbation theory. Noticing the periodicity of the axion and the vacuum mass, the following simple cosine form of the axion potential is widely used in cosmological studies:

$$V(a) = m_a^2 F_a^2 \left[1 - \cos(a/F_a)\right]. \tag{2.3.13}$$

In general, global structure of the axion potential can be much more complicated compared to this simple cosine form [51–56]. But the detailed form of the axion potential is beyond the scope of this thesis and we use the axion potential of Eq. (2.3.13).

When we consider the cosmological history of the axion field, it is important to know the temperature dependence of the axion mass. The temperature dependence of axion mass is estimated by the interacting instanton liquid model [57, 58] and is given by[4]

$$m_a(T) \simeq \begin{cases} 4.05 \times 10^{-4} \frac{\Lambda^2}{F_a} \left(\frac{T}{\Lambda}\right)^{-3.34} & T \gtrsim 0.26\Lambda, \\ m_a(T=0) & T \lesssim 0.26\Lambda, \end{cases} \tag{2.3.14}$$

where $\Lambda \simeq 400$ MeV denotes the QCD scale.

2.3.2 axion interaction with standard model particles

Here, we summarise interactions between the axion and other particles in the standard model. First, let us consider the physics just above the QCD scale. There exist three

[4]Now, the lattice studies on the temperature dependence of the axion mass are developing (see [59–61]).

light quarks u, d and s. The effective Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_a \supset &\frac{1}{2}(\partial a)^2 + \sum_{i=u,d,s}(\bar{q}_i^L i\not{D} q_i^L + \bar{q}_i^R i\not{D} q_i^R) \\ &+ \sum_{i=u,d,s}\left[c_{i,1}^0 \frac{(\partial_\mu a)}{F}\bar{q}_i\gamma^\mu\gamma_5 q_i - \left(\bar{q}_i^L m_i q_i^R + \text{H.c.}\right)\right] \\ &+ c_3^0 \frac{a}{32\pi^2 F} G\tilde{G} + c_{\rm em}^0 \frac{a}{32\pi^2 F} F_{\rm em}\tilde{F}_{\rm em},\end{aligned} \tag{2.3.15}$$

$$c_{i,1}^0 = -C_i/2, \tag{2.3.16}$$

$$c_{\rm em}^0 = 2\sum_i n_i C_i Q_{i,\rm em}^2, \tag{2.3.17}$$

$$c_3^0 = \sum_c C_c, \tag{2.3.18}$$

where we set all c_2 to be zero and $Q_{i,\rm em}$ denotes the electric charge of i-th PQ quark. n_i denotes the number of Dirac fermions in q_i.

When we consider axion interactions with standard model particles, it is convenient to remove the c_3 term by a chiral rotation. The following rotation removes the mass mixing between axion and pions[62, 63]:

$$q^L \to \exp(-iaQ_A/F)q^L, \;\; q^R \to \exp(iaQ_A/F)q^R, \tag{2.3.19}$$

where $q = (u, d, s)$ and

$$Q_A = c_3^0 \frac{M_q^{-1}}{2\text{tr}(M_q^{-1})}, \; M_q \equiv \text{diag}[m_u, m_d, m_s]. \tag{2.3.20}$$

After this rotation, coefficients $c_\bullet$ become model dependent values which we denote $\bar{c}_\bullet$. Below, we list up the axion coupling with photons, leptons and hadrons respectively.

Axion coupling with photon

The coefficient $\bar{c}_{\rm em}$ represents the interaction between axion particle and the photons and given by

$$\begin{aligned}\bar{c}_{\rm em} &= c_{\rm em}^0 - 6\frac{c_3^0}{\text{tr}(M_q^{-1})}\sum_{i=u,d,s} m_i^{-1} Q_{i,\rm em}^2 \\ &= c_{\rm em}^0 - e^2 c_3^0 \frac{2(4 + z_d + z_s)}{3(1 + z_d + z_s)} \simeq c_{\rm em}^0 - 1.92 e^2 c_3^0.\end{aligned} \tag{2.3.21}$$

Note that even when the all PQ charged quarks have no electric charge, the coupling $\bar{c}_{\rm em}$ have non zero value of order $\mathcal{O}(e^2)$.

Conventionally, the axion photon coupling is normalised by the following form

$$\mathcal{L}_{a\gamma} = -\frac{g_{a\gamma}}{4} a F_{\rm em} \tilde{F}_{\rm em}, \tag{2.3.22}$$

where the coupling $g_{a\gamma}$ is called axion-photon coupling and given by

$$g_{a\gamma} = \frac{\alpha_{\rm em}}{2\pi F_a} |\bar{c}_{\rm em}/N_{\rm DW} e^2|, \tag{2.3.23}$$

where $\alpha_{\rm em} = e^2/4\pi$.

Axion coupling with leptons

The coefficient $\bar{c}_{1,i}$ represents the interaction between the i-th fermion and the axion. Here, we consider the coupling between the axion and a lepton "L". Conventionally, the coupling with leptons are normalised as

$$\mathcal{L}_{aL} = \frac{g_{aL}}{2m_i} \bar{\psi}_L \gamma_\mu \psi_L \partial^\mu a, \tag{2.3.24}$$

which gives

$$|g_{aL}| = \frac{m_L C_L}{F_a N_{\rm DW}}. \tag{2.3.25}$$

If a lepton L does not have a PQ charge, there is no tree-level interaction between the axion and the lepton L. However, an one-loop process through the $c_{\rm em}$ coupling generates the coupling g_{aL}. In such a case, the size of g_{aL} is suppressed by a factor $\mathcal{O}(\alpha_{\rm em}^2/(4\pi)^2)$ compared to the case with an order one $\bar{c}_{1,L}$ [64].

Axion coupling with hadrons

Here, we list up the couplings between the axion and hadrons. In the $\bar{c}_\bullet$ frame, the axion has the following interaction

$$\mathcal{L} \supset \sum_{i=u,d,s} \bar{c}_{i,1} \frac{(\partial_\mu a)}{F} \bar{q}_i \gamma^\mu \gamma_5 q_i. \tag{2.3.26}$$

In a energy scale below the chiral symmetry breaking, this interaction becomes the coupling between the hadrons and the axion as

$$\mathcal{L}_{aN} = \frac{\partial^\mu a}{F}\left[C_{ap}\bar{p}\gamma_\mu\gamma_5 p + C_{an}\bar{n}\gamma_\mu\gamma_5 n + iC_{a\pi N}\left(\frac{\pi^+}{f_\pi}\bar{p}\gamma_\mu n - \frac{\pi^-}{f_\pi}\bar{n}\gamma_\mu p\right)\right], \tag{2.3.27}$$

$$\mathcal{L}_{a\pi} = C_{a\pi}\frac{\partial^\mu a}{F f_\pi}(\pi^0\pi^+ + \pi^0\partial\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0), \tag{2.3.28}$$

where p, n, π denote the proton, neutron and pion respectively and $f_\pi \simeq 130$ MeV is the pion decay constant. The coefficients $C_{a(n/p)}$ are given by

$$C_{ap} = \bar{c}_{1,u}\triangle u + \bar{c}_{1,d}\triangle d + \bar{c}_{1,s}\triangle s, \tag{2.3.29}$$

$$C_{an} = \bar{c}_{1,u}\triangle d + \bar{c}_{1,d}\triangle u + \bar{c}_{1,s}\triangle s, \tag{2.3.30}$$

where $\triangle q$ are the axial vector current matrix element $\triangle q S_\mu = \langle p|\bar{q}\gamma_\mu\gamma_5 q|p\rangle$ with S_μ being the proton spin. They are given by $\triangle u = 0.84 \pm 0.02$, $\triangle d = -0.43 \pm 0.02$ and $\triangle s = -0.09 \pm 0.02$ [65]. The coefficients $C_{a\pi}$ are given by [11]

$$C_{a\pi N} = \frac{\bar{c}_{1,u} - \bar{c}_{1,d}}{\sqrt{2}}, \tag{2.3.31}$$

$$C_{a\pi} = \frac{2(\bar{c}_{1,u} - \bar{c}_{1,d})}{3}. \tag{2.3.32}$$

The couplings like C_{ap} and C_{an} are normalised as

$$\mathcal{L} \supset \frac{g_{aN}}{2m_N}\bar{N}\gamma_\mu N\partial^\mu a, \quad (N = p/n), \tag{2.3.33}$$

which gives

$$g_{aN} = 2\frac{C_{aN}m_N}{F_a N_{\rm DW}}, \quad (N = p/n). \tag{2.3.34}$$

2.4 Axion models

In this section, we review some known axion models. There are two main ingredients for the axion model.

The first ingredient of the axion model is the form of the scalar potential which generates the vacuum expectation values of the PQ charged scaler around the PQ scale F_a. In some cases, the cosmological history of the PQ field much depends on the potential of it. Thus, it is worth knowing what kind of scalar potential are probable. We introduce typical types of scalar potential of the PQ field especially within the framework of the supersymmetric models in Sec. 2.4.3.

The second ingredient of the axion model is the contents of PQ charged fields. There are two well known setups: KSVZ (Kim, Shifman, Vainshtein and Zakharov) axion model [66, 67] and DFSZ one[36, 37]. Below, we summarise them one by one.

2.4.1 KSVZ axion model

The axion model proposed by Kim [66], and by Shifman, Vainshtein and Zakharov [67] called the KSVZ model has one extra pair of PQ charged quarks. On the other hand, standard model particles do not have the PQ charge. The Lagrangian is given by

$$\mathcal{L}_{\rm KSVZ} \supset y\phi\bar{\psi}_L\psi_R + {\rm H.c.} + V_{\rm stab}(|\phi|), \tag{2.4.1}$$

where ψ denotes the QCD charged extra quark. $V_{\rm stab}$ denotes the potential of the PQ scalar ϕ, and stabilise the potential at $|\phi|/\sqrt{2} = F_a$. We assume ψ_L belongs to the fundamental representation in QCD. If we assign the PQ charge of ϕ as one, C_q also becomes one. In this model, the relevant parameters for axion interaction are given by

$$\begin{aligned}
N_{\rm DW} &= 1, && (2.4.2)\\
g_{a\gamma} &\simeq \frac{\alpha_{\rm em}}{2\pi F_a}|6Q^2_{\psi,{\rm em}}/e^2 - 1.92|, && (2.4.3)\\
g_{aL} &= \mathcal{O}\left(\frac{\alpha^2_{\rm em}}{(4\pi)^2}\frac{m_L}{F_a}\right), && (2.4.4)\\
\bar{c}_{2,u} &= \frac{1}{1+z+z_s}, && (2.4.5)\\
\bar{c}_{2,d} &= \frac{z}{1+z+z_s}, && (2.4.6)\\
\bar{c}_{2,s} &= \frac{z_s}{1+z+z_s}, && (2.4.7)\\
\bar{c}_{1,q} &= \bar{c}_{2,q}/2 \ \ (q = u, d, s), && (2.4.8)\\
& && (2.4.9)
\end{aligned}$$

Note that in this model, the domain wall number is equal to one. Thus, the axion potential has a unique minimum. Sometimes the KSVZ model is called as the Hadronic axion model.

2.4.2 DFSZ axion model

The so called DFSZ axion model is proposed by Dine, Fichler and Srednicki [36] and by Zhitnitskii [37]. This model contains the two Higgs doublet fields (H_u, H_d) and one additional complex scalar field ϕ. The potential of the scalar is given by

$$\mathcal{L}_{\rm DFSZ} \supset \lambda H_u H_d \phi^2 + V_{\rm stab}(|\phi|) \tag{2.4.10}$$

$V_{\rm stab}$ denotes the potential of the PQ scalar ϕ and stabilises the potential at $|\phi|/\sqrt{2} = F_a$. We assume that $H_{u/d}$ generates masses of the up/down type quarks. If we set the PQ charge of ϕ to be -1, the PQ charges of Higgs fields $C_{H_{u/d}}$ in our convention become

$$C_{H_u} = 2\cos^2\beta, \tag{2.4.11}$$
$$C_{H_d} = 2\sin^2\beta, \tag{2.4.12}$$

where $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$. In this model, the relevant parameters for axion interaction are given by

$$N_{\rm DW} = 6, \tag{2.4.13}$$
$$g_{a\gamma} \simeq \frac{\alpha_{\rm em}}{2\pi F_a}|8/3 - 1.92|, \tag{2.4.14}$$
$$g_{aL} = 3\sin^2\beta, \tag{2.4.15}$$
$$\bar{c}_{2,u} = \frac{6}{1+z+z_s}, \tag{2.4.16}$$
$$\bar{c}_{2,d} = \frac{6z}{1+z+z_s}, \tag{2.4.17}$$
$$\bar{c}_{2,s} = \frac{6z_s}{1+z+z_s}, \tag{2.4.18}$$
$$\bar{c}_{1,q} = \bar{c}_{2,q}/2 - C_q \quad (q = u, d, s), \tag{2.4.19}$$
$$\tag{2.4.20}$$

where we assume the leptons couple to the H_d. Noted that in this model, the domain wall number is equal to 6.Thus, the axion potential has sixth degenerate minima.

2.4.3 Models of the potential for the PQ field with supersymmetry

Since supersymmetric (SUSY) models can explain the hierarchy problem, the SUSY PQ model is well motivated. If the PQ mechanism is embedded into a SUSY framework, there appear some characteristic features. In a SUSY framework, the PQ symmetry $\phi_i \to \phi_i^{i\alpha_i}$ can be extended to a symmetry of scale transformation: $\phi_i \to \phi_i^{\alpha_i}$ due to the holomorphic property of the superpotential [68]. This scale transformation symmetry is broken by soft SUSY breaking terms. Thus, in general, a typical mass scale of the PQ scalar potential becomes soft SUSY breaking mass one.

As will be mentioned in Sec. 2.5. the PQ scale is expected to be around $F_a \sim 10^9 - 10^{12}$ GeV. This range of the PQ field is called the axion window. On the other hand, the soft SUSY breaking mass scale is expected to be not too much larger than the electroweak scale, otherwise the hierarchy problem becomes serious. Thus in most cases of interest, the PQ scale is much larger than the soft SUSY breaking mass scale. However in such a case, it seems non trivial to make the scalar potential stabilised at appropriate scale F_a. Below, we list up some examples of the PQ potential in SUSY models.

Example 1

First, we show a mechanism in which the running of the soft mass induces radiative symmetry breaking at an appropriate scale [69, 70]. Let us consider the following superpotential

$$W_{\rm PQ} = \lambda_Q \phi Q\bar{Q} + \lambda_L \phi L\bar{L}, \tag{2.4.21}$$

where ϕ denotes the PQ field, and $(\bar{Q})Q$ and $(\bar{L})L$ denote chiral multiplets in (anti-) fundamental representation of SU(5). Then, the renormalization group equations (RGEs) for the soft SUSY breaking masses are given by

$$\begin{aligned} \frac{dm_\phi^2}{dt} &= \frac{1}{8\pi^2}\left[3\lambda_Q^2(m_\phi^2 + m_Q^2 + m_{\bar{Q}}^2) + 2\lambda_L^2(m_\phi^2 + m_L^2 + m_{\bar{L}}^2)\right], \\ \frac{dm_Q^2}{dt} = \frac{dm_{\bar{Q}}^2}{dt} &= \frac{1}{8\pi^2}\left[\lambda_Q^2(m_\phi^2 + m_Q^2 + m_{\bar{Q}}^2)\right], \\ \frac{dm_L^2}{dt} = \frac{dm_{\bar{L}}^2}{dt} &= \frac{1}{8\pi^2}\left[\lambda_L^2(m_\phi^2 + m_L^2 + m_{\bar{L}}^2)\right], \end{aligned} \tag{2.4.22}$$

where $t = \log E$ (with E being the energy scale), $m_\bullet$ (with $\bullet = Q,\ \bar{Q},\ L,\ \bar{L}$) denotes the soft SUSY breaking mass. The potential of the PQ field is given by

$$V_\phi = m_\phi^2(\phi)|\phi|^2. \tag{2.4.23}$$

If m_ϕ has a positive value at high energy scale and changes the sign at a PQ scale, the PQ symmetry can be broken properly. Fig. 2.1 shows running of the soft SUSY breaking mass m_ϕ. We have solved the RGEs with the boundary condition at the ground unification scale ($M_{\rm GUT} \sim 2 \times 10^{16}$ GeV) as $m_\phi = m_Q = m_L = 10$ PeV, and $\lambda_Q = \lambda_L = 0.7$ (the red solid line) and $\lambda_Q = \lambda_L = 1$ (the green dashed line). We have taken the runnings of couplings ($\lambda_{Q,L}$ and standard model couplings) into account. Here we have ignored gaugino masses for simplicity. One can see that for $\lambda_{Q,L} \simeq 0.7$, the expectation value of the PQ field can be around $\sim 10^{10}$ GeV. Note that in this model, $\lambda_\bullet$ should be $\mathcal{O}(1)$ otherwise the running effects on the soft masses become negligible.

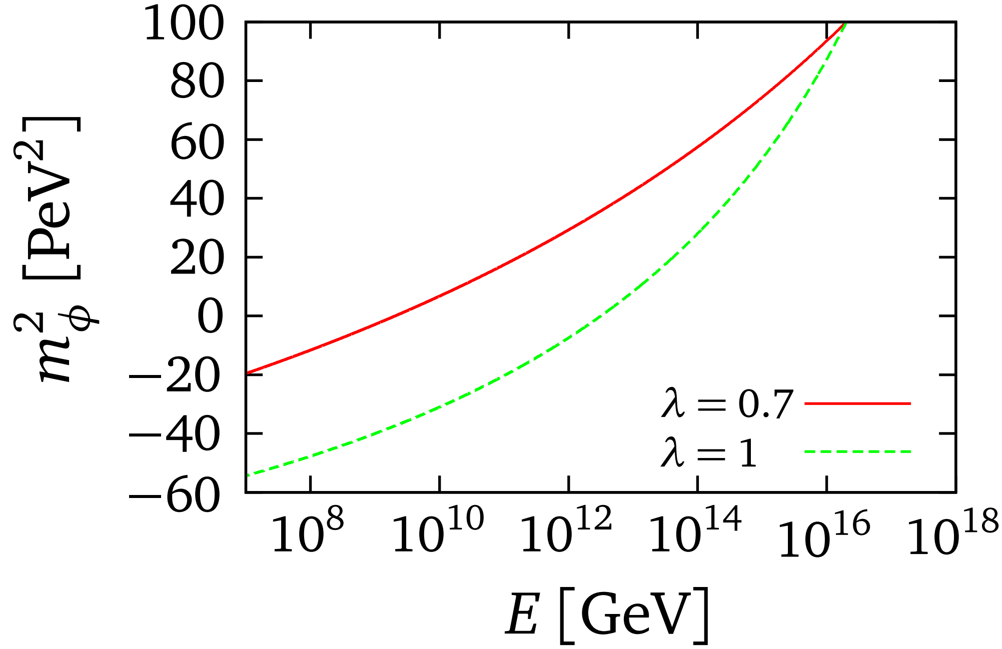

Figure 2.1: RGE evolution of the soft mass of the PQ field. Red solid and green dashed lines are the cases with $\lambda_{Q,L}(M_{\rm GUT}) = 0.7$ and 1 respectively. From [1].

Example 2

Another example of the supersymmetric axion model has the following superpotential [71]

$$W_{\rm PQ} = \frac{\gamma\phi^n S}{n M_*^{n-2}}, \tag{2.4.24}$$

where ϕ and S are the PQ field with the PQ charge +1 and $-n$ respectively. M_* denotes the cut off scale. The scalar potential is given by

$$V = -m^2|\phi|^2 + \frac{\gamma^2|\phi|^{2n}}{M_*^{2n-4}} \tag{2.4.25}$$

where $-m^2$ denotes the soft mass for ϕ and we set $S = 0$ for simplicity. If $m^2 < 0$, $n > 2$ and $M_*^2 \gg |m^2|$, the PQ field ϕ will have a vacuum expectation value much larger than the soft SUSY mass m. Then, the appropriate PQ scale can be realised.

2.5 Constraints on axion

Here we briefly summarise constraints on couplings between the axion and standard model particles which are obtained by experiments and astrophysical observations.

2.5.1 Astrophysical constraints

If couplings between the axion and standard model particles exist, axion emission process becomes an energy loss channel for stars. Such a process is limited by observed properties of stars [72–75].

The strongest astrophysical constraint on the axion-photon coupling $g_{a\gamma}$ comes from globular cluster observations in which energy loss process due to the axion-photon coupling reduces the lifetime of stars on the horizontal branch. The constraint is given by [76]

$$g_{a\gamma} \lesssim 10^{-10}\text{GeV}^{-1}. \tag{2.5.1}$$

$$\tag{2.5.2}$$

This constraint $g_{a\gamma} \lesssim 10^{-10}$ GeV^{-1} can be translated to the constraint on F_a. For the KSVZ model, this constraint becomes $F_a > 2.3 \times 10^7$ GeV and for DFSZ model it becomes 0.8×10^7 GeV.

The axion-electron coupling g_{ae} is limited by the energy loss process in globular cluster and given by [77]:

$$g_{ae} < 4.3 \times 10^{-13}. \tag{2.5.3}$$

The axion-electron coupling g_{ae} also affects the cooling process of white dwarfs. Recent studies show [78, 79] that the white dwarf cooling process can be better explained with a new energy loss channel that can be interpreted in terms of axion electron coupling $g_{ae} \simeq 2.2 \times 10^{-13}$. On the other hand, the region $g_{ae} > 5 \times 10^{-13}$ is excluded. For the DFSZ model, the constraint $g_{ae} < 5 \times 10^{-13}$ can be interpreted as $F_a \gtrsim 2 \times 10^8 \sin^2\beta$ GeV. The constraint on the KSVZ model is suppressed because there is no tree level coupling.

The axion-nucleon coupling g_{aN} is constrained by the cooling of the supernova 1987a [80]. The following range of coupling is excluded[75]:

$$3 \times 10^{-10} \lesssim g_{aN}. \tag{2.5.4}$$

This constraint is especially important for the KSVZ type models in which the bounds from g_{ae} are mild. For the KSVZ model, the following limit on F_a is obtained [75] [5]

$$F_a \gtrsim 4 \times 10^8 \text{ GeV}. \tag{2.5.5}$$

2.5.2 Experimental constraints

If the axion photon coupling $g_{a\gamma}$ exists, the Sun can be a source of axion fluxes. Produced axions can be detected at Earth through the axion photon conversion process in a macroscopic magnetic field [82]. This idea of axion detection is called "axion helioscope". The CAST (CERN Axion Solar Telescope) experiment gives most severe bound on the axion photon coupling $g_{e\gamma}$ [83]. They exclude axion models with the PQ scale around $F_a \sim 10^7 - 10^8$ FeV. The Tokyo axion helioscope also put bounds on axion models with $m_a \sim 1$ eV [84]. The future project "International Axion Observatory" (IAXO) [85] may reach the parameter range of $F_a \sim 10^9$ GeV.

Axion may contribute a significant fraction of the cold dark matter (see Chapter 3). Axions in galactic halo may be detected by using a microwave cavityy [82, 86–88]. The ADMX experiment [89] excludes the axion dark matter scenario for a certain range of the PQ scale around $F_a \sim 10^{11}$ GeV.

We have seen that constraints on three couplings g_{aN}, g_{ae} and $g_{a\gamma}$ are obtained from astrophysical observations. Constraints on the coupling $g_{a\gamma}$ are also obtained from experiments. Note that it is very unlikely that all of these couplings are accidentally suppressed. Thus, one can say that the PQ scale F_a is bounded from below as $F_a \gtrsim 10^9$ GeV. As is shown in Sec. 3.2, $F_a \gtrsim 10^{12}$ GeV requires a turning otherwise axion dark matter dominates the universe. Thus, a promising range of the PQ scale is around $10^9\text{GeV} \lesssim F_a \lesssim 10^{12}$ GeV. This range of the PQ scale is called as the axion window.

[5] If $g_{aN} \lesssim 3 \times 10^7$ GeV, axions rapidly interact and no constraint is obtained in terms of cooling. However, in such a case, thermally produced axions become a hot dark matter and this case is disfavoured by cosmological observations [81].

Chapter 3

Overview of Peccei-Quinn Cosmology

3.1 Overview

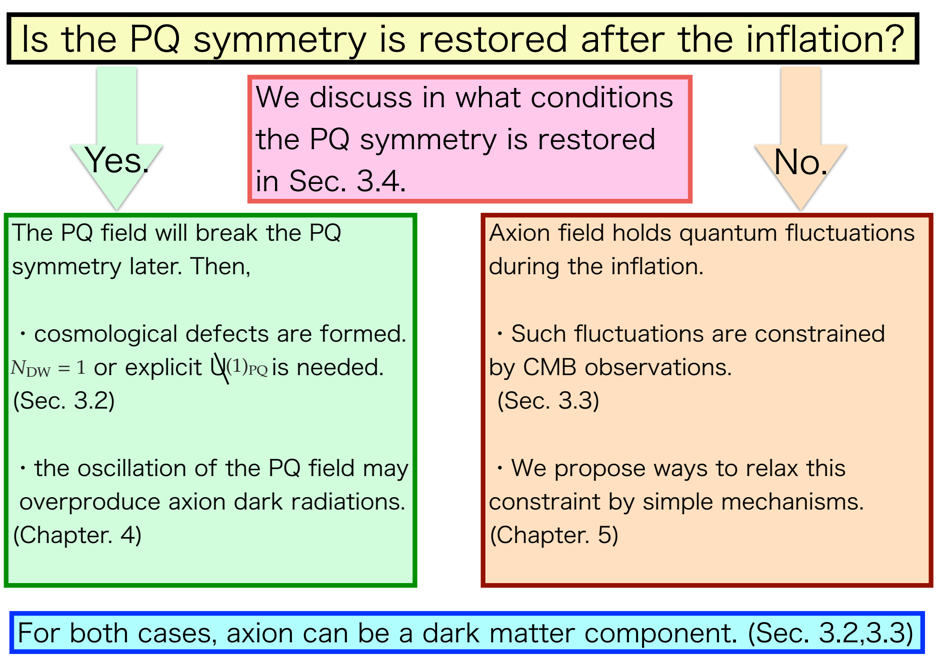

Figure 3.1: Schematic picture of the PQ cosmology and this thesis.

In this Chapter, we show an overview of the PQ cosmology paying particular attention to the dynamics of the PQ field. Fig. 3.1 shows a schematic picture of the of the PQ cosmology and this thesis.

The cosmological implications much depend on whether the PQ symmetry is restored at some epoch after the inflation or not. In general, scalar fields tend to be trapped at the symmetry enhanced point. For example, thermal potential may stabilise the scalar potential at the symmetry enhanced point. In some cases, the PQ field is trapped at the symmetry enhanced point and the PQ symmetry is restored at some epoch in the cosmological history.

If the PQ symmetry is restored at some epoch, the PQ field will go down to the minimum of the vacuum potential as the universe expands. After the breaking of the PQ symmetry, cosmic strings are formed with typical scale being the Hubble one.

When the temperature decreases to the QCD scale, the potential of the axion field is lifted up.Then, string-domain-wall networks are formed. If the domain wall number $N_{\rm ND}$ (see Eq. (2.3.7)) is greater than unity, domain walls become stable and such a scenario is excluded [14]. Thus, for $N_{\rm DW} > 1$, an explicit PQ symmetry breaking is needed. On the other hand, if $N_{\rm ND} = 1$, domain walls can decay into axion particles. Axions produced in this way can be a part of the cold dark mater component of the universe [90, 91]. We give a briefly summary of this situation in Sec. 3.3. There are another dynamics of the PQ field: the oscillation of the PQ field around the minimum of the potential. The PQ field has a sizeable decay rate to axions in general. Thus, in some cases, axion particles are overproduced and are constrained by dark radiation abundance [12]. We discuss when the axion overproduction is avoided taking thermal effects into account in Sec. 4.

The PQ symmetry may always broken after the inflation due to the non zero values of the PQ fields. In such a case, the axion field has an initial value different from the vacuum value in general. When the temperature of the universe decreases to the QCD scale, the axion mass is lifted up. Then, axion field starts to oscillate. Such oscillating axion can be a dark matter component of the universe [17–19]. If the PQ symmetry is always broken in the history of the universe, quantum fluctuations in the axion direction produced during the inflation remain until today. Such fluctuations are independent from other fluctuations in the universe and are constrained by CMB observations [16]. Since the size of fluctuations is proportional to the inflation scale, the inflation scale is bounded from above by this constraint. In Sec. 3.2 we summarise the axion dark mater abundance and constraint on the inflation scale of in this case.

As is shown above, the cosmology of the PQ field largely depends on whether the PQ symmetry is restored at some epoch or not. We discuss in what circumstances the PQ symmetry is restored in Sec. 3.4. We will see that the PQ symmetry is rather likely to be restored because thermal or non-thermal effects tend to trap a scalar field at the symmetry enhanced point.

Here, we would like to emphasise that in many cases of interest, the PQ field will be trapped at the symmetry enhanced point. From this point of view, it is important to consider the dynamics of the PQ field after the trapping. The PQ field starts to move as the universe expands. After that, there are two kinds of the dynamics of the PQ field: defect formation and the oscillation around the minimum of the potential. The dynamics of cosmological defects has been considered in the literature [15, 92–95]. However, studies on the dynamics of the oscillating PQ field is limited. For example, there are few studies which takes the thermal effects into account, that can affect the dynamics of the PQ field oscillation drastically. As we will see in Chapter 4, in the dynamics of the PQ field oscillation, the thermal effects play important roles. The study by the author [1] is the first study to discuss the dynamics of the PQ field after the trapping taking thermal effects into account. We have obtained a condition when the axion overproduction is avoided.

On the other, it is also important to consider the situation where the PQ symmetry restoration is avoided by some mechanisms. If the domain wall number $N_{\rm DW}$ is greater than one, the PQ symmetry must not be restored, otherwise stable domain walls overclose the universe. In addition, for such a case, severe constraints on the axion fluctuations are imposed. Thus, it is interesting to consider a scenario in which the PQ symmetry is not restored in the history of the universe and at the same time constraints on the axion fluctuations are relaxed. In Chapter 5 We discuss such a possibility and propose new tow scenarios in this direction.

3.2 Non Trapped PQ field

In this section, we consider the case where the PQ symmetry is always broken in the history of the universe by the field values of the PQ fields. During inflation, the quantum fluctuations are generated in the axion direction. After the inflation, the PQ field may move by a nontrivial way. We consider the dynamics of the PQ field in Sec. 3.4 and Sec. 5. In this section, we mainly consider the dynamics of the axion direction. Here, we assume that the PQ field stays at the minimum of the potential $|\phi| = F_a/\sqrt{2}$ all the time. We concentrate on the dynamics of the axion field.

In general, the axion field has a field value different from one at the minimum of the potential during inflation. When the temperature of the universe becomes the scale of QCD ($\Lambda \sim 0.1$GeV), the axion potential is lifted up and the axion starts to oscillate around the minimum of the potential. This oscillation survives until today and can be a dark matter component [17–19]. We show the dark matter abundance of the axion oscillation in Sec. 3.2.1.

Since the axion field is almost massless, it acquires quantum fluctuations during the inflation. The size of the quantum fluctuation becomes the inflation scale $\sim H_{\rm inf}/2\pi$

where $H_{\rm inf}$ denotes the Hubble parameter during the inflation. Such produced quantum fluctuation in the axion field will remain until today. On such fluctuations, severe constraints are imposed by the CMB observations [16]. In particular, the inflation scale is limited in this way because the size of the axion fluctuation is related to the inflation scale. We discuss such constraints in Sec. 3.2.2. In the review Section, we basically follow the arguments in [96].

3.2.1 Axion Dark Matter Abundance

The axion has the following potential (see Sec. 2.3.1)

$$V(a) = m_a(T)^2 F_a^2 \left[1 - \cos(a/F_a)\right], \tag{3.2.1}$$

where the temperature depending mass is given by [57, 58]

$$m_a(T) = \begin{cases} 4.05 \times 10^{-4} \frac{\Lambda^2}{F_a} \left(\frac{T}{\Lambda}\right)^{-3.34} & T > 0.26\Lambda, \\ m_a(T = 0) & T < 0.26\Lambda, \end{cases} \tag{3.2.2}$$

where T indicates the temperature of the universe and $\Lambda \simeq 400$MeV denotes the QCD scale. The axion field obeys the follow equation of motion[1] :

$$\ddot{a} + 3H\dot{a} + m_a(T)^2 a = 0. \tag{3.2.3}$$

We denote the initial value of the axion field a_i and we define the initial angle of the axion field as $\theta_i \equiv a/F_a$. When the temperature of the universe is high enough compared to the QCD scale, the axion mass is suppressed. As the temperature decreases to the QCD scale, the axion mass increases. When the axion mass becomes comparable to the Hubble parameter $m_a \simeq 3H$, the axion starts to oscillate around the minimum of the potential. By using Eq. (3.2.2), the temperature of the onset of oscillation $T_{\rm os}$ can be estimated as

$$T_{\rm os} = 0.98 \left(\frac{F_a}{10^{12}\text{GeV}}\right)^{-0.19} \left(\frac{\Lambda}{400\text{MeV}}\right) \text{GeV}. \tag{3.2.4}$$

After $T_{\rm os}$, the axion continues to oscillate until today. Such an oscillating axion becomes a cold dark matter component. The present axion dark matter density is estimated as [97]:

$$\Omega_a h^2 = 0.18\, \theta_i^2 \left(\frac{F_a}{10^{12}\text{GeV}}\right)^{1.19} \left(\frac{\Lambda}{400\text{MeV}}\right), \tag{3.2.5}$$

[1] For simplicity, we assume the case where the initial angle θ_i is not around π and the axion potential can be regarded as quadratic one.

with h being the present Hubble parameter normalised by 100km/s/Mpc. Since the axion dark matter abundance must not exceed the present cold dark matter (CDM) density [16]

$$\Omega_{\rm CDM} h^2 = 0.1199 \pm 0.0022, \tag{3.2.6}$$

the value of F_a is bounded from above:

$$F_a \leq 6.6 \times 10^{11} \theta_i^{-1.68} \text{GeV}. \tag{3.2.7}$$

This bound suggests a typical scale of F_a. If the initial angle θ_i is not so suppressed, the PQ scale should be lower than $\sim 10^{12}$ GeV. On the other hand, lower bounds of the PQ scale is obtained from astrophysical observations (see Sec. 2.5) which give $F_a \gtrsim 10^9$ GeV. Therefore, in the range 10^9 GeV $\lesssim F_a \lesssim 10^{12}$ GeV, PQ models are promising. This range of the PQ scale is called as the axion window.

During inflation, the axion field acquires quantum fluctuations of the Hubble scale during inflation: $\delta a \sim H_{\rm inf}/2\pi$ since the axion is almost massless. Such a fluctuation can become a part of the axion dark matter. The fluctuations of the axion δa can be written as

$$\langle \delta a(\vec{k}) \delta a(\vec{k}') \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{2\pi^2}{k^3} \mathcal{P}_a(k), \tag{3.2.8}$$

$$\mathcal{P}_a(k) = \frac{H_{\rm inf}^2}{4\pi^2}. \tag{3.2.9}$$

where $\langle \bullet \rangle$ denotes the vacuum expectation value. Note that squared average of axion fluctuations has a finite amplitude: $\langle \delta a^2(\vec{x}) \rangle = H_{\rm inf}^2/4\pi^2$, We can estimate the axion dark matter abundance including the fluctuation as

$$\begin{aligned} \Omega_a h^2 &= 0.18 \left(\frac{F_a}{10^{12}\text{GeV}} \right)^{-1.19} \left(\frac{\Lambda}{400\text{MeV}} \right) \times \frac{\langle (a_i + \delta a)^2 \rangle}{F_a^2} \\ &= 0.18 \left[\theta_i^2 + \frac{H_{\rm inf}^2}{4\pi^2 F_a^2} \right] \left(\frac{F_a}{10^{12}\text{GeV}} \right)^{1.19} \left(\frac{\Lambda}{400\text{MeV}} \right). \end{aligned} \tag{3.2.10}$$

By considering $\Omega_a h^2 < \Omega_{\rm CDM} h^2 \simeq 0.11$, an upper bound on the inflation scale $H_{\rm inf}$ is obtained [98]:

$$H_{\rm inf} < 5.0 \times 10^{12} \text{GeV} \left(\frac{F_a}{10^{12}\text{GeV}} \right)^{0.41}, \tag{3.2.11}$$

for a small initial angle range $\theta_i \lesssim H_{\rm inf}/2\pi F_a$. The energy fraction of axion dark matter r_a can be written as

$$r_a \equiv \frac{\Omega_a}{\Omega_{DM}} = 1.6 \times \left[\theta_i^2 + \frac{H_{\rm inf}^2}{4\pi^2 F_a^2} \right] \left(\frac{F_a}{10^{12}\text{GeV}} \right)^{1.19} \left(\frac{\Lambda}{400\text{MeV}} \right) \leq 1. \tag{3.2.12}$$

3.2.2 Constraints on the fluctuation in axion direction

Fluctuations in the axion field generated during inflation are independent from fluctuations of the background. Such fluctuations induce isocurvature fluctuations of cold dark matter [20–24] which is tightly bounded by the CMB observations [16]. Here, we summarise constraints on axion fluctuations.

The axion isocurvature perturbation S_a is defined as

$$S_a(\vec{x}) = 3(\zeta_a - \zeta), \tag{3.2.13}$$

where $\zeta(\zeta_a)$ denotes the curvature perturbations on the uniform density slice of the total matter(axion). The δN formalism [99, 100] relates the energy density of the axion $\rho_a(\vec{x})$ on the uniform density slice and the axion isocurvatue perturbation as

$$\rho_a(\vec{x}) = \bar{\rho}_a e^{S_a(\vec{x})}, \tag{3.2.14}$$

where $\bar{\bullet}$ denotes a spacial average of $\bullet$. Note that the axion energy density is proportional to the square of initial amplitude:

$$\rho_a(\vec{x}) \propto [a_i + \delta a(\vec{x})]^2. \tag{3.2.15}$$

This gives the following relation:

$$e^{S_a} = 1 + 2\frac{a_i \delta a(\vec{x})}{\tilde{a}^2} + \frac{\delta a^2(\vec{x}) - \langle \delta a^2 \rangle}{\tilde{a}^2}, \tag{3.2.16}$$

$$\tilde{a}^2 \equiv a_i^2 + \langle \delta a^2 \rangle. \tag{3.2.17}$$

The dark matter density on the uniform density surface can be written as[2]

$$\rho_{\rm CDM}(\vec{x}) = \bar{\rho}_{\rm CDM}\left[1 + r_a(e^{S_a} - 1)\right], \tag{3.2.18}$$

where r_a denotes the energy fraction of the axion defined in Eq. (3.2.12). This gives us the isocurvature density perturbations of CDM:

$$\begin{aligned} S_{\rm CDM} &= \ln\left(\frac{\rho_{\rm CDM}(\vec{x})}{\bar{\rho}_{\rm CDM}}\right) \\ &\simeq 2r_a\frac{a_i\delta a}{\tilde{a}^2} + r_a\frac{\delta a^2 - \langle\delta a^2\rangle}{\tilde{a}^2}. \end{aligned} \tag{3.2.19}$$

The power spectrum of the CDM isocurvature perturbation can be calculated as[3]

$$\langle S_{\rm CDM}(\vec{k}) S_{\rm CDM}(\vec{k}')\rangle \equiv (2\pi)^2 \delta^{(3)}(\vec{k} + \vec{k}')\frac{2\pi^3}{k^3}\mathcal{P}_{S_{\rm CDM}}(k), \tag{3.2.20}$$

$$\mathcal{P}_{S_{\rm CDM}} \simeq 4r^2\frac{(H_{\rm inf}/2\pi)^2}{a_i^2 + (H_{\rm inf}/2\pi)^2}. \tag{3.2.21}$$

[2] We assume that total system except axion field has the same origin of fluctuation.
[3] We can neglect higher correlation terms due to the smallness.

The amplitude of the CDM isocurvature perturbation is constrained by the CMB observations. The present constraint is obtained by [16] which gives

$$\left.\frac{\mathcal{P}_{S_{\rm CDM}}}{\mathcal{P}_\zeta}\right|_{k=0.002{\rm Mpc}^{-1}} < 0.038, \tag{3.2.22}$$

From this constraint and Eq. (3.2.12), we can obtain the following limit on parameters θ_i, $H_{\rm inf}$ and F_a:

$$\left[\theta_i^2 + \left(\frac{H_{\rm inf}}{2\pi F_a}\right)^2\right]\left(\frac{H_{\rm inf}}{2\pi F_a}\right)^2\left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{2.38} < 0.8\times 10^{-11}, \tag{3.2.23}$$

where we use the value of the curvature perturbation as $\mathcal{P}_\zeta(k_*) = 2.2\times 10^{-9}$ [12]. This impose a severe constraint on the inflation scale and the PQ scale. If the axion is a dominant component of the present dark matter ($F_a \sim 10^{12}$ GeV), $H_{\rm inf} \lesssim 10^7$ GeV must hold. In addition to this a constraint from a non Gaussianity of $S_{\rm CDM}$ can be imposed [101–104] especially for a small θ_i^2 region.

In this section, we have seen the case where the PQ symmetry is always broken in the history of the universe assuming that the PQ fields except axion do not play important roles. In this case, the axion can be a dark matter component. At the same time, severe constraints on the Hubble parameter at the inflation are obtained due to the Hubble size fluctuations in the axion direction. High scale inflation scenarios with $H_{\rm inf} \gtrsim 10^{7-10}$ GeV are excluded in this case. This feature seems unpleasant because a high inflation scale around $\sim 10^{13}$ GeV are within the reach of future observations [105].

There are some scenarios in which the constraints imposed on the inflation scale can be relaxed [2, 26–34]. We discuss such scenarios in Chapter 5. We also propose two new scenarios in this direction.

3.3 Trapped PQ field

In the previous section, we consider the case where the PQ symmetry is always broken. Here, we consider the opposite case: the PQ field is once trapped at the symmetry enhanced point. The trapping of the PQ field is likely to occur even if the PQ field has a large initial value during the inflation. We discuss the dynamics of the trapping in Sec. (3.4). In this review Section, we basically follow the arguments in [96].

The PQ field can be trapped at the origin by the finite density effects, which stabilise the potential of the PQ field around the symmetry enhanced point. For example, if the PQ field couples to particles which participates in the thermal bath of the universe, the thermal potential for the PQ field is formed which stabilises the potential at the symmetry enhanced point. As the universe expands, the strength of the trapping becomes weaker and weaker. At some time, the vacuum potential exceeds the effective potential generated by the finite density effects. Then, the PQ field will roll down to the minimum of the potential. Below, we consider the dynamics after the PQ field has started to move. To be concrete, we consider the following potential:

$$V = \frac{\lambda^2}{2}(|\phi|^2 - f_a^2)^2, \tag{3.3.1}$$

where λ indicates a self coupling and f_a denotes a parameter of order of the PQ scale [4] .

First, let us consider the dynamics of the radial direction of ϕ. After the end of the trapping, ϕ will oscillate around the minimum of the potential $\phi = f_a$. This oscillation will remain until a damping effect on the motion of ϕ becomes efficient. There are two mechanisms of the damping: a thermal dissipation or a perturbative decay.If thermal dissipation becomes efficient, the energy of the PQ field oscillation is converted to the thermal bath of the universe. On the other hand, if a perturbative decay damps the oscillation of ϕ, the decay products may cause a cosmological problem. For example, axion particles may be produced by the decay of the PQ field. Produced axions becomes an extra radiation component which has a negligible interaction with the standard model sector. Such a radiation component, dubbed a dark radiation, affects the evolution of the universe. The energy density of dark radiation is tightly constrained by the CMB observations [12] (see Sec. 3.5). Since the oscillation of the PQ field potentially overproduces the axions, whether the PQ field is thermally dissipated or decays drastically is a relevant question. We study the dynamics of the oscillation of the PQ field after the trapping to answer this question in Chapter 4 .

Next, let us consider the dynamics related to the phase of ϕ. At the onset of the oscillation, the PQ field has finite fluctuations in general. As a result, the phase of the field ϕ becomes spatially random after the PQ field starts to oscillate. Consequently, one dimensional topological defects are formed, which we call axion strings. After the formation of axion strings, the distribution of axion strings comes to obey a scaling solution in which the energy density of the axion string is given by

$$\rho_{\rm st} = \xi \frac{\mu}{t^2}, \tag{3.3.2}$$

where μ is the string tension of order f_a^2. ξ is called the length parameter which is constant in time. Note that ξ can be regarded as an average number of the axion strings in the Hubble volume $1/H^3$ since the Hubble parameter behaves as $H \sim 1/t$.

[4] Here and Hereafter we denote f_a as a vacuum expectation value of the complex scalar field. In general, f_a and the PQ scale F_a has a few factor difference.

The numerical simulations show $\xi \simeq \mathcal{O}(1)$ [91]. Note that if F_a is much less than the Planck scale $\sim 10^{18}$ GeV, the energy fraction of the axion strings in the universe becomes subdominant. This is because the total energy of the universe is given by $\rho_{\rm tot} \sim H^2 M_P^2 \sim M_P^2/t^2$ with M_P denoting the reduced Planck scale $M_P \simeq 2.4 \times 10^{18}$ GeV.

As axion strings evolve in time, axion particles are emitted from strings [92–94]. A typical momentum of produced axion particles becomes the Hubble scale [90, 91] and such axion particles also contribute to the dark matter density of the universe. In [91], the dark matter abundance of emitted axions is estimated by a lattice simulation and given by

$$\Omega_a^{\rm st} h^2 = 2.0 \times \xi \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{\Lambda}{400{\rm MeV}}\right), \tag{3.3.3}$$

with $\xi \simeq 0.7 - 1.0$. Note that this abundance is larger than the non-trapping case in Eq. (3.2.10).

The axion strings obey the scaling solution until the temperature of the universe becomes the QCD scale. When the temperature decreases to the QCD scale, the potential for the axion field is lifted up (see Eq. (3.2.2)). Then, the shift symmetry in the axion potential becomes discrete $Z_{N_{\rm DW}}$ symmetry with $N_{\rm DW}$ being the domain wall number. As a result, string-domain networks are formed. The domain walls separate the $N_{\rm DW}$ distinct minima of the potential. As we will see whether $N_{\rm DW} = 1$ or not crucially determines the fate of the domain walls. Below, we discuss two cases $N_{\rm DW} = 1$ and $N_{\rm DW} > 1$ in order.

3.3.1 $N_{\rm DW} = 1$ case

If $N_{\rm DW} = 1$, the minimum of the axion potential is unique, In such a case, the domain walls become disc like objects with axion string boundaries. Such discs of domain wall will soon collapse after the formation due to their tension. This fact has been numerically confirmed by [15, 106].

During the collapses of the domain walls, axion particles are produced, which contribute the amount of the dark matter. The abundance of the axions from the collapse of the domain walls are evaluated [15]:

$$\Omega_a^{\rm wall} h^2 \simeq (5.8 \pm 2.8) \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{\Lambda}{400{\rm MeV}}\right). \tag{3.3.4}$$

This amount is comparable to that of the emission from the strings in Eq. (3.3.3).

In total, the axion dark matter abundance in this scenario is given by [15]

$$\Omega_a h^2 \simeq (8.4 \pm 3.0) \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{\Lambda}{400{\rm MeV}}\right), \tag{3.3.5}$$

which gives an upper bound on the PQ scale:

$$F_a \lesssim (2.0 - 3.8) \times 10^{10}\mathrm{GeV}. \tag{3.3.6}$$

The axion becomes the dark matter with a smaller PQ scale F_a compared to the non-trapped case Eq. (3.2.10). It should be noted that the dark matter abundance in this scenario does not depend on the initial condition like the initial angle θ_i in the non-trapped case.

3.3.2 $N_{\mathrm{DW}} > 1$ case

If $N_{\mathrm{DW}} > 1$, domain walls become stable. This is because domain walls separate different potential minima and there is no way for the PQ field to settle down into one minimum. The energy density of the domain walls scales as [106]

$$\rho_{\mathrm{wall}} \propto \frac{1}{t} \tag{3.3.7}$$

Note that the total energy density of the universe scales as $\propto 1/t^2$. Such domain walls will soon dominate the universe, which is inconsistent with the present universe [14]. Thus, a scenario in which $N_{\mathrm{DW}} > 1$ and the PQ field is once trapped at the symmetry enhanced point is ruled out.

The formation of the stable domain walls occurs because the potential has a Z_{DW} symmetry. Thus, if the Z_{DW} symmetry is not exact, the domain walls become unstable. In the literature, such a possibility is considered [106].

Let us consider the explicit PQ symmetry violating term like

$$V_{\mathrm{U(1)_{PQ}}} = -CF_a^3(\phi e^{i\delta} + \mathrm{H.c.}), \tag{3.3.8}$$

where the parameter C denotes a size of the PQ symmetry violation. We assume that the strong CP problem is solved when the phase of ϕ is zero. Thus, the parameter δ denotes the gap of the potential minimum. Qualitatively, the following statements hold

- Small C and δ are required in order not to spoil the Peccei-Quinn mechanism.
- Not too small C is needed, otherwise the domain wall becomes stable again.
- The smallness of δ can be regarded as a fine-tune because there is no reason for $\delta = 0$ to be favoured.

Depending on the value of δ, C is bounded both from sides (above and below). In [106], it is shown that for $\delta \gtrsim 10^{-2}$, there is no allowed parameter region. On the other hand, for $\delta \lesssim 10^{-2}$, there exists allowed region around $\log_{10}[C] \sim -50$. The requirement of $\delta \lesssim 10^{-2}$ indicates that a kind of fine-tune is needed if $N_{\mathrm{DW}} \geq 1$ and the PQ symmetry is restored at some epoch.

3.4 Trapping of the PQ field

In this section, we consider the trapping of PQ field at the symmetry enhanced point. If the PQ field is trapped at some epoch, cosmological consequences are drastically changed compared to the opposite case. First, models with the domain wall number being greater than one ($N_{\rm DW} > 1$) are ruled out, otherwise a small explicit PQ symmetry violation is required. This is because stable domain walls are formed after the temperature becomes QCD scale which cause a cosmological disaster (see Sec. 3.3.2). Second, no constraint is imposed on the quantum fluctuations in the axion direction because such fluctuations are dissipated away after the trapping. Third, the axion dark matter abundance becomes different from that of non trapping case (see Sec. 3.3.1). Fourth, relativistic axion can be produced due to the late time decay of the PQ field oscillation. The energy density of such produced axions are bounded by the dark radiation constraint (see 3.5). Therefore, it is important to know in what circumstance the PQ field is trapped at the symmetry enhanced point. In this section, we consider the dynamics of the PQ field after inflation and discuss with what conditions the PQ field is trapped at the symmetry enhanced point.

To be specific, we consider the following interaction

$$\mathcal{L}_{\rm int} = V_\phi(|\phi|) + \sum_i y_i \phi \bar{q}_i q_i, \tag{3.4.1}$$

where V_ϕ indicates the potential of the PQ field ϕ which has a minimum at the PQ scale. q_i denotes the PQ charged quarks coupling to the PQ field. y_i is a coupling constant. We assume q_i has a QCD charge and participates in the thermal bath of the standard model sector. There are two effects to cause the trapping. One is the effect of self interaction which we will call as the non-thermal trapping. The other is interactions with particles in the thermal bath (q_i), which we denotes as the thermal trapping. The non-thermal trapping is caused by the potential term V_ϕ, while the thermal trapping is caused by the interaction term with q_i . In the KSVZ axion models, an interaction term with the extra PQ quark is indispensable and the effect of thermal trapping is important. On the other hand, models like DFSZ type, sizeable interactions with other fields is not necessary and the effect of non thermal trapping may be essential. Below, we consider both cases one by one [5] .

Before going into details, let us comment on the initial field value of the PQ field. If the PQ field has an initial value larger than F_a during inflation and if the PQ symmetry is not restored after inflation, cosmological constraints on the inflation scale can be

[5] In this thesis, we consider two effects of trapping separately. If we consider the both effects at the same time, the PQ field will be more likely to be trapped at the origin.

relaxed [26]. This mechanism is explained in Chapter 5. Bearing this point in mind, in this section, we assume the initial value of the PQ field to be much larger than the PQ scale and consider whether the trapping occurs or not.

3.4.1 Non Thermal Trapping

After inflation, the PQ field starts to oscillate around the origin of the potential. Due to the oscillation, quantum fluctuations in the PQ field are amplified by the parametric resonance [107–109]. In some cases, the PQ field comes to be trapped at the origin [110–113]. The non thermal restoration of the PQ symmetry is studied by lattice simulations in the radiation dominated background [114] and in the matter dominated one [13]. Here, we show a simple analytic discussion on this matter which is consistent with the lattice simulation [13].

To be concrete, let us consider the following potential

$$V_\phi = \frac{\lambda^2}{2M_*^{2n-4}}\left(|\phi|^2 - f_a^2\right)^n, \ (n \geq 2) \tag{3.4.2}$$

where the value of f_a is around the PQ scale, λ denotes the self coupling constant, n is an integer and M_* indicates some mass scale. We denote the initial value of ϕ as a real and positive one ϕ_i without loss of generality. We are interested in the situation $\phi_i \gg f_a$ as is mentioned before. The condition that the PQ field is almost stopped during the inflation can be written as

$$m_{\text{eff}}^2 \equiv \frac{1}{\phi_i}\frac{dV_\phi(\phi_i)}{d\phi} \simeq \lambda^2 \frac{\phi_i^{2n-2}}{M_*^{2n-4}} \lesssim H_{\text{inf}}^2, \tag{3.4.3}$$

where H_{inf} indicates the Hubble parameter during inflation. After the end of inflation, the inflaton dominated epoch begins. During this epoch, we assume the following form of the Hubble parameter: $H = p/t$. Note that $p = 2/3$ or $1/2$ corresponds to the matter or radiation dominated universe. When the Hubble parameter becomes comparable with the effective mass m_{eff}, the PQ field will start to oscillate.

In general, the PQ field initially has a quantum fluctuation $\delta\phi$. This fluctuation may be amplified by the oscillation of the ϕ due to the parametric resonance [107–109]. To see this, let us consider the fluctuation of the imaginary part $\delta\psi \equiv \text{Im}[\phi]$ since the parametric resonance is likely to occur in the imaginary part as well as the real part. The equation of motion for $\delta\psi \equiv \text{Im}[\phi]$ in the expanding universe can be written as

$$\delta\ddot{\psi}_k + 3H\delta\dot{\psi}_k + \frac{k^2}{a^2}\delta\psi_k + \lambda^2\frac{\phi(t)^{2n-2}}{M_*^{2n-4}}\delta\psi_k \simeq 0, \tag{3.4.4}$$

where k is the comoving momentum, $\delta\psi_k$ denotes the Fourier mode of $\delta\psi(x)$ and a indicates the scale factor of the universe.

In order to see the effects of the parametric resonance, let us consider the situation where the expansion of the universe is neglected. We assume that the field ϕ oscillates around the origin with an amplitude Φ. In such a case, the frequency of the oscillation becomes $\sim m_{\rm eff}$. We denote the amplitude of the oscillation of ϕ as Φ. The typical frequency of the oscillation can be given by the effective mass,

$$m_{\rm eff}(\Phi) \simeq \frac{\partial V_\phi}{\Phi \partial \phi}. \tag{3.4.5}$$

which characterise the force due to the potential term. For simplicity, we parameterise $\phi(t)$ as[6]

$$\phi(t) \simeq \Phi \cos(m_{\rm eff} t) \tag{3.4.6}$$

Then, Eq. (3.4.4) becomes the following form

$$\delta\ddot{\psi}_k + \left[\frac{k^2}{a^2} + c_1 m_{\rm eff}^2 + c_2 m_{\rm eff}^2 \cos(c_3 m_{\rm eff} t)\right]\delta\psi_k \simeq 0, \tag{3.4.7}$$

where c_1, c_2 and c_3 are $\mathcal{O}(1)$ numerical constants and we keep only typical terms for simplicity as we do not demand high accuracy in this discussion. This equation can be regarded as a so called Mathieu equation (see [108] for a review). It is known that the Mathieu equation has exponentially growing solutions in some bands along k/a. In later time $t \gg 1/m_{\rm eff}$, growing modes dominate $\delta\psi$. The growing rate $\delta\psi \propto e^{\mu t}$ can be estimated as $\mu \sim \mathcal{O}(1) \times m_{\rm eff}$. Once some modes start to grow up exponentially, the size of fluctuation $\delta\psi$ will become soon comparable to that of ϕ. In such a case, the effective mass of ϕ is lifted up by the fluctuation: $m_{\rm eff,fluc}^2 \sim \lambda^2 \delta\phi^{2n-2}/M_*^{2n-4}$. As a result, the potential of the PQ field is stabilised at the origin, which means the trapping of the PQ field. Thus, if $\phi(t)$ oscillates more than $\mathcal{O}(10^2)$ times, the PQ field is supposed to be trapped at the symmetry enhanced point [7]. We denote this critical number of oscillation as $N_{\rm cr}$ ($N_{\rm cr} \lesssim \mathcal{O}(10^2)$) : after $N_{\rm cr}$ times oscillations, the PQ field is trapped at the origin[8] .

So far, we have neglected the expansion of the universe. As the universe expands, the positions of the bands of growing modes are red shifted. If the expansion is too

[6] Strictly speaking, $\Phi(t)$ contains several oscillating modes: $\sum_N a_N \cos(2N\pi t/T)$ with T being the period of the oscillation. However, small N modes are dominant in general and we do not demand a high accuracy in this discussion.

[7] Initially, $\delta\psi$ has a fluctuation of order $H_{\rm inf}$. This fluctuation is amplified by $\sim \exp[m_{\rm eff} t]$ due to the parametric resonance. Thus, if ϕ oscillates $\mathcal{O}(10^2)$ times, the size of $\delta\psi$ is expected to be large enough to be comparable to ϕ.

[8] To obtain precise number of $N_{\rm cr}$, one may have to perform a lattice simulation.

fast, no efficient growth of $\delta\psi$ is expected. The typical band position and width in momentum space are both $\sim m_{\rm eff}$. On the other hand, the momentum is shifted by order one in one Hubble time $1/H$. Thus, the time span for fluctuations $\delta\psi$ to grow up is evaluated as one Hubble time. We come to the conclusion that if ϕ oscillates $N_{\rm cr}$ times in one Hubble time before the amplitude decreases to $\sim f_a$, fluctuations grow enough to trap the PQ field at the origin.

Now, let us derive the condition of the trapping in terms of f_a, ϕ_i and $N_{\rm cr}$. If scalar field oscillates by the potential $\propto |\Phi|^{2n}$, the amplitude scales as

$$\Phi \propto t^{-\frac{3p}{n+1}}. \tag{3.4.8}$$

The scaling of the effective mass can be obtained as

$$m_{\rm eff} \propto t^{-\frac{3p(n-1)}{n+1}}. \tag{3.4.9}$$

From these, we can obtain

$$\Phi \simeq \phi_i \left(\frac{H}{H_{\rm os}}\right)^{\frac{3p}{n+1}}, \tag{3.4.10}$$

$$m_{\rm eff} \simeq m_{\rm eff}(\phi_i) \left(\frac{H}{H_{\rm os}}\right)^{\frac{3p(n-1)}{n+1}}, \tag{3.4.11}$$

where $H_{\rm os}$ denotes the Hubble parameter at the onset of the oscillation $H_{\rm os} = m_{\rm eff}(\phi_i)$. If the amplitude decreases to f_a, the following relation holds

$$f_a \simeq \phi_i \left(\frac{H_{\rm v}}{H_{\rm os}}\right)^{\frac{3p}{n+1}}, \tag{3.4.12}$$

where $H_{\rm v}$ denotes the Hubble parameter at this time. On the other hand, if the effective mass becomes greater than $N_{\rm cr} \times H$ at $H = H_{\rm trap}$, the following relation holds:

$$N_{\rm cr} \times H_{\rm trap} \simeq m_{\rm eff}(\phi_i) \left(\frac{H_{\rm trap}}{H_{\rm os}}\right)^{\frac{3p(n-1)}{n+1}}, \tag{3.4.13}$$

$$\tag{3.4.14}$$

The condition for the PQ scalar to be trapped at the origin is given by $H_{\rm trap} \gtrsim H_{\rm v}$. This condition can be rewritten as [9]

$$\frac{\phi_i}{f_a} \gtrsim (N_{\rm cr})^{n_c}, \tag{3.4.15}$$

$$n_c \equiv \frac{3p}{n+1-3p(n-1)}. \tag{3.4.16}$$

[9] The denominator of n_c should be positive, otherwise the PQ field does not oscillate (see Sec. 5.2).

For example if $n = 2$ (quartic potential), n_c becomes 2 for the matter dominated universe and 1 for the radiation dominated universe. Considering $N_{\rm cr} \lesssim \mathcal{O}(10^2)$, we can see if the initial field value ϕ_i is much larger than v, the PQ symmetry is inevitably restored. This conclusion is consistent with the numerical lattice simulations [13, 114].

In the discussion above, we assume the oscillation of ϕ after the time of $H \simeq m_{\rm eff}$. This is not always correct . If the effective mass decreases faster than the Hubble parameter when the oscillation of ϕ is assumed, the resultant motion of ϕ becomes so called "scaling solution". In this case, ϕ follows an universal trajectory in which the effective mass and the Hubble parameter are always comparable. ϕ starts to oscillate when the dominant term in the potential is changed and the effective mass comes to drop down slower than the Hubble parameter. After the oscillation of ϕ, the above argument can be applied with the replacement $\phi_i \to \phi_{\rm change}$ where $\phi_{\rm change}$ denotes the field point in which the dominant term in the potential changes. The non-trivial phenomenon occurs if effective mass has the same scaling property with the Hubble parameter when we assume the oscillation of ϕ. The motion of ϕ becomes different from the oscillatory case and the scaling case, which we call a "pseudo scaling" motion. If the pseudo scaling motion occurs, the trapping of ϕ is not likely to occur. In Sec. 5.2, we study this case in details.

So far, we do not consider the interactions with the thermal bath of the universe. If we take thermal effects into account, the dynamics of the PQ field is drastically changed. But, the PQ field is also likely to be trapped at the origin due to the interactions. We consider this case (thermal trapping) in Sec. 3.4.2.

3.4.2 Thermal Trapping

Here, we consider the situation where the PQ field has interactions with particles in the thermal bath. In such a case, the dynamics of the PQ field is affected by the interaction. For example, the thermal potential may stabilise the PQ field around the origin or the thermal dissipation may damp the oscillation of the PQ field. Thus, it is important to know the effects of the interaction on the dynamics. We discuss whether the PQ field is trapped at the origin due to the interaction with particles in the thermal bath. In the KSVZ model [66, 67], an PQ charged extra quark is needed. Thus, the interactions with other fields are likely for PQ models.

To be concrete, we assume the following interaction:

$$\mathcal{L}_{\rm int} = \sum_i y_{q_i} \phi \bar{q}_i q_i, \tag{3.4.17}$$

where q_i is assumed to have a standard model gauge charge and to participate in the thermal bath of the standard model sector. Due to the interaction, the PQ field tends to be trapped at the origin. This is because $\phi = 0$ is realised at the thermal equilibrium state if the total energy of the system is large enough. If the typical time scale of the interaction is shorter than that of the expansion of the universe, trapping of the PQ field will occur. As we will see below, this is the case for most parameter regions of interest.

A study of the PQ field trapping in the presence of the thermal effects was done by the author [1]. Details are relatively complicated. Here, we briefly summarise some important points.

First, let us see the effects of the interaction Eq. (3.4.17) on the dynamics of the PQ field. Below, we list up main effects.

- Due to the couplings, a thermal potential is formed for the PQ field. The thermal potential tends to stabilise the PQ field toward the symmetry enhanced point. Even when ϕ has a large field value and q decouples from the thermal bath, a so called thermal logarithmic potential [115] is formed which stabilise the PQ field.

- The thermal dissipation effects [116–122] convert the energy in the PQ field to the thermal bath. As a result, the PQ field is likely to lose its energy and eventually to be trapped at the origin.

- If the temperature is low enough to neglect the thermal effects, non-perturbative particle production of q particles would occur as the PQ field oscillates. Such produced q particles form an effective potential for the PQ field which stabilise the PQ field at the symmetry enhanced point. Even when the decays of q is effective and particle numbers of them do not grow up, the energy in the PQ field is converted to the thermal bath through the decay process of q. As a result, the thermal bath is heated and the PQ field is likely to be trapped at the origin.

All effects above are likely to force the PQ field to be trapped at the origin. Below, we show how the PQ field is trapped by the thermal effects in some typical models. The purpose of the discussion below is to show qualitative features of the thermal trapping. Thus, we do not demand high accuracy in the discussion.

Setup

After inflation, ϕ starts to oscillate with initial field value ϕ_i when the effective mass becomes comparable to the Hubble parameter. In order to discuss the dynamics of the PQ field concretely, we first fix the situation.

We consider the following form of the zero temperature potential:

$$V_0 = \begin{cases} -m_0^2|\phi|^2 & \text{for } |\phi| \ll F_a \\ \frac{\lambda_\phi^2}{2nM_*^{2n-4}}|\phi|^{2n} \;\; (n \geq 1) & \text{for } |\phi| \gg F_a \end{cases}, \tag{3.4.18}$$

where m_0 and M_* are parameters of some mass scales and λ_ϕ indicates a coupling. We assume the minimum of the potential exists at $|\phi| = F_a/\sqrt{2}$. This potential characterise the form around the origin and far way from the origin, which are relevant in this discussion.

As we consider interactions with particles in thermal bath, we have to know the temperature of the universe. For simplicity, we assume that the inflaton behaves as matter during the inflaton-oscillation dominated era with a constant decay rate. In such a case, the energy of the radiation component $\rho_{\rm rad}$ can be written as

$$\rho_{\rm rad} \simeq \frac{H}{H_R} T_R^4 \;\; \text{for } H > H_R, \tag{3.4.19}$$

$$\rightarrow T \simeq \left(\frac{H}{H_R}\right)^{1/4} T_R, \tag{3.4.20}$$

where T_R denotes the reheating temperature and H_R denotes the Hubble parameter at the reheating.

Next, let us summarise the effects of the interactions on the dynamics of the PQ field.

Thermal Potential

As q participates in the thermal bath and its mass depends on the value of ϕ, thermal potential for the PQ field will be formed. At the one-loop level, the thermal potential is written as [123]

$$V_{\rm th}^{\rm 1-loop}(|\phi|, T) = -2\sum_i \frac{T^4}{\pi^2}\int_0^\infty dz\, z^2 \ln\left[1 + \exp\left(-\sqrt{z^2 + M_{q_i}^2(\phi)/T^2}\right)\right] \tag{3.4.21}$$

where $M_{q_i}(= y_i|\phi|)$ denotes the field value dependent mass of the particle q_i. When $M_{q_i} \lesssim T$ is satisfied, the particles q_i are populated in the thermal bath. In such a case, the one-loop thermal potential behaves quadratic one which is often called as thermal mass term. On the other hand, if $M_{q_i} \gg T$ holds, the number of the particles q_i is exponentially suppressed. In such a case, the effects of the one-loop thermal potential becomes negligible. Consequently, the behaviour of the one-loop thermal potential can be approximately written as

$$V_{\rm th}^{\rm 1-loop}(|\phi|, T) \simeq c_M \Theta(T - y|\phi|) \times y^2 T^2 |\phi|^2, \tag{3.4.22}$$

where y represent a typical value of y_{q_i}, Θ denotes the step function and c_M is an order one constant.

In the region of $M(\phi) \gg T$, higher loop effects dominate the thermal potential. Mainly, their effects come from the running of the gauge coupling. Since the mass of gauge charged field q_i depends on the field value of ϕ, the gauge coupling also depends on the value of ϕ. On the other hand, the thermal free energy of a system which have a gauge interaction contains a term proportional to g^2T^2 with g denotes the gauge coupling. Through the running of the gauge coupling, the thermal potential depends on ϕ. This kind of potential is called thermal log [115] and is given by

$$V_{\rm th}^{\rm log}(|\phi|, T) \simeq c_L \Theta(y|\phi| - T) \times \alpha^2(T) T^4 \ln\left(\frac{y^2|\phi|^2}{T^2}\right), \tag{3.4.23}$$

where c_M is an order one positive constant in general. α indicates the strength of the gauge coupling.

The effective potential for the PQ field is approximately given by the sum: $V_{\rm eff} = V_0 + V_{\rm th}$. We define the effective mass of the PQ field $m_{\rm eff}^{\phi}$ as $m_{\rm eff}^{\phi\ 2} \equiv V_{\rm eff}'/\phi$. The effective mass can be written as

$$m_{\rm eff}^{\phi}(\phi, T) \simeq \max[m_{0,\rm eff},\ yT \text{ for } y\phi < T,\ \alpha T^2/\phi \text{ for } y\phi < T], \tag{3.4.24}$$

where $m_{0,\rm eff}$ indicates the effective mass of the zero temperature potential: $m_{0,\rm eff} \equiv V_0'/\phi$. We drop off $O(1)$ constants for simplicity. The time when the PQ field starts to oscillate can be determined by

$$H_{\rm os} \simeq m_{\rm eff}^{\phi}(\phi, T). \tag{3.4.25}$$

(non) perturbative particle production at the origin

Due to the interaction terms the dispersion relations of q depends on the field value of ϕ:

$$\omega_q(t) = \sqrt{y^2|\phi|^2 + m_{{\rm eff},q}^2 + \vec{k}^2}, \tag{3.4.26}$$

where we use the subscript q to indicate particles q_i. y denotes a typical value of y_i. $\vec{k}$ is the momentum. $m_{{\rm eff},q}(\sim gT)$ denotes the effective mass of q generated by the thermal bath and g indicates a typical coupling constant of the thermal bath. When ϕ oscillates around the origin with an amplitude $\Phi \gg T/y$, the value of ω_q varies in time. Around the origin, ϕ has the following velocity:

$$|\dot{\phi}|_{\phi\sim 0} \sim m_{\rm eff}\Phi. \tag{3.4.27}$$

The dynamics of the system crucially depends on whether this velocity is fast or slow compared to the typical scale of the thermal bath

If the velocity $|\dot{\phi}|_{\phi\sim 0}$ is fast enough to break the adiabaticity of q: $|\dot{\omega}/\omega^2| \gg 1$, the non-perturbative particle production will occur [108, 124]. The condition for the non-perturbative particle production to occur is given by [10]

$$y\Phi \gg \frac{m_{\mathrm{eff},q}^2}{m_{\mathrm{eff}}^{\phi}}. \tag{3.4.28}$$

If this condition is satisfied, q particles are suddenly generated at the first crossing:

$$n_q \sim \frac{k_*^3}{4\pi^3}, \tag{3.4.29}$$

$$k_*^2 \equiv y m_{\mathrm{eff}}^{\phi}\Phi, \tag{3.4.30}$$

where n_q denotes the number density of produced q particles and k_* is a typical momentum of the produced particles. The condition Eq. (3.4.28) can be rewritten as

$$k_*^2 \gg g^2 T^2 \tag{3.4.31}$$

We denote the case where this condition holds as "fast" case. After the first crossing, the field value $|\phi|$ grows up and q becomes massive again. If q has a decay channel to lighter particles, q will decay after the production. Such a decay can occur for example, through the mixing with standard model particles. Thus, the dynamics depends on whether the mixing is large or not. Suppose that the q field has a mixing term with light particles whose coupling constant is κ, the particle q has the following decay rate:

$$\Gamma_q \sim \kappa^2 m_q \sim \kappa^2 y\Phi. \tag{3.4.32}$$

If the condition $\Gamma_q \gg m_{\mathrm{eff}}$ holds, produced q particles will soon decay at each oscillation of the PQ field. In this paper, we assume that the mixing term is not so suppressed and the condition $\Gamma_q \gg m_{\mathrm{eff}}$ holds. The case with $\Gamma_q \ll m_{\mathrm{eff}}$ is discussed in [1], where the trapping behaviour is obtained in the most parameters of interest. This is because the resonantly produced particles form an effective potential for ϕ which stabilise the potential at the origin.

Let us consider the opposite situation where the velocity of ϕ at the origin is small enough. In such a situation, particle production of q may be caused by the thermal bath.

[10] In addition, the condition $k_* > m_{\mathrm{eff}}^{\phi}$ is needed for the sudden production. As we consider the effects of the interaction, we assume the coupling y is not so suppressed and the condition $k_* > m_{\mathrm{eff}}^{\phi}$ holds.

During $y|\phi| \lesssim T$, the effective mass of q becomes lower than the temperature. The time span δt in which $y|\phi| \lesssim T$ is realised is estimated to be:

$$\delta t \simeq \frac{T}{y m_{\rm eff}^{\phi} \Phi} = \frac{T}{k_*^2}. \tag{3.4.33}$$

At the first crossing, thermal bath produces q particles by n_q:

$$n_q \sim \min[g^4 T^4 \delta t, T^3] = \min[1, g^4 \frac{T^2}{k_*^2}] T^3, \tag{3.4.34}$$

where we use the production rate being $\sim g^4 T^5$. If $k_* < g^2 T$ holds, q particles are thermalised when the PQ field exists around the origin. In such a case, we can estimate the dissipation rate of the PQ field by using knowledge of thermal field theory. We denote the case with $k_* \ll g^2 T$ as "slow" case.

We have seen that if $k_* \gg gT$ holds, non-perturbative particle production becomes effective. On the other hand, if $k_* \ll g^2 T$ is satisfied, q particles are thermalised when the PQ field passes through the origin. However, the intermediate case $gT > k_* > g^2 T$ is relatively complicated to deal with. This is because we can not rely on the criteria of adiabaticity breaking or use the thermal distribution of q particles. The purpose of this section is to show the trapping of the PQ field in some typical situations and we do not consider this intermediate case.

Figure 3.2 shows the contour plot of the temperature at the onset of the oscillation $T_{\rm os}$ as a function of the initial field value ϕ_i and the reheating temperature T_R. We take $y = 0.05$, α being the QCD one and $m_{\rm eff}^0 = 1$ GeV bearing supersymmetric models with a gauge mediation in mind [125–127]. In the region (A), the PQ field begins to oscillate with thermal log potential. In the region (B), the PQ field starts to oscillate with zero temperature mass without non perturbative particle production. In the region (C), the PQ field starts to oscillate with zero temperature mass with non perturbative particle production. As we consider the case with relatively large initial value of ϕ_i, there is no region where the PQ field starts to oscillate with the thermal mass term.

Summary So Far

So far, we have seen effects of the interaction with q particles. There are several important ingredients which determine the dynamics of the PQ field:

- What dominants the effective potential of ϕ at the onset of the oscillation.
- Whether the velocity $|\dot{\phi}|$ around the origin is fast or slow.

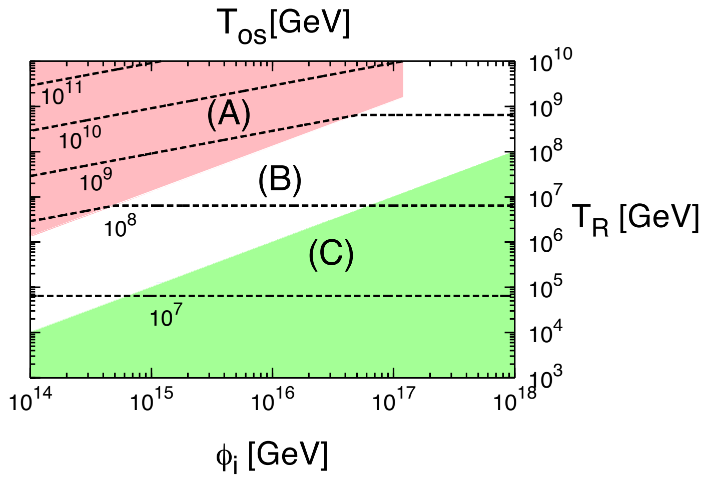

Figure 3.2: The contour of $T_{\rm os}$ in $[\phi_i,\ T_R]$ plane. In the region (A), the PQ field starts to oscillate with the thermal log term. In the region (B/C), the PQ field starts to oscillate with the zero temperature mass without/with the non perturbative particle production. From [1].

The resultant dynamics of the PQ field is much different for each situation. From now on, we consider the dynamics of the PQ field for the following cases:

- Case A : The PQ field begins to oscillate with the thermal log term.
- Case B : The PQ field begins to oscillate with the zero temperature potential, q particles are thermally produced when the PQ field passes through the origin: slow case.
- Case C: The PQ field begins to oscillate with the zero temperature potential, non-perturbative particle production is effective and the mixing between q and light particles is large: fast and large mixing case.

While the dynamics is different for each case, the conclusion is universal: for most parameter regions of interest, the PQ field is likely to be trapped at the origin.

Case A

Here, we consider the situation where the PQ field starts to oscillate with the thermal log potential. ϕ will start to oscillates with the thermal log potential if the condition

$$\phi_i < \alpha T_R \sqrt{\frac{M_{\rm PL}}{m_{0,\rm eff}(\phi_i)}}, \tag{3.4.35}$$

hold. The temperature $T_{\rm os}$ at the onset of the oscillation is given by

$$T_{\rm os} \simeq \sqrt{\frac{\alpha M_{\rm PL}}{\phi_i}} T_R. \tag{3.4.36}$$

In addition, the condition $T_{\rm os} > T_R$ should hold since otherwise the scalar oscillates with the zero temperature potential since the effective mass from the thermal log term cannot exceed the Hubble parameter during the radiation dominated era. When ϕ oscillates with the thermal log potential, the parameter k_* is given by

$$k_*^2 \sim y\alpha T^2. \tag{3.4.37}$$

When ϕ is located around the origin ($y|\phi| \lesssim T$), particles q are thermally produced ($n_q \sim T^3$) with the assumption $y < g^2$. The interactions with thermalised particles q cause the dissipation of ϕ. The oscillation averaged dissipation rate Γ_ϕ is evaluated as [128]

$$\Gamma_{\rm eff} \simeq \left(\frac{y}{g^4}\right) \alpha \frac{T^2}{\Phi}. \tag{3.4.38}$$

Assuming that the dominant potential remains the thermal log one after the oscillation [11] the scalar field scales as [121, 122]

$$\Phi \propto H^{3/2}. \tag{3.4.39}$$

To gather with Eq. (3.4.19), the dissipation rate is rewritten by

$$\Gamma_{\rm eff} \simeq \left(\frac{y}{g^4}\right) H_{\rm os} \left(\frac{H}{H_{\rm os}}\right)^{-1}, \tag{3.4.40}$$

where $H_{\rm os} \simeq \alpha T^2/\phi_i$ is the Hubble parameter at the onset of the oscillation. When $H \simeq \Gamma_{\rm eff}$ is satisfied, the oscillation of ϕ is soon damped. This condition gives:

$$\frac{H_{\rm diss}}{H_{\rm os}} \simeq \min\left[\left(\frac{y}{g^4}\right)^{1/2}, 1\right] \equiv \eta \leq 1. \tag{3.4.41}$$

where $H_{\rm diss}$ is the Hubble parameter at $H = \Gamma_{\rm eff}$.

Now, we discuss whether ϕ is trapped at the origin or not. Here, we impose the following conservative condition of the trapping: the origin is the absolute minimum

[11] If the mixing of q is negligible, q particles produced at $\phi \sim 0$ forms an effective potential which will stabilise the potential and the trapping will become more probable. Here we do not take such effects into account for simplicity.

of the potential when the dissipation effects become active $\Gamma_{\rm diss} = H$. When $\Gamma_{\rm eff} = H$ holds, the effective mass from the thermal log potential becomes

$$m_{\rm eff}^{\log}(\Phi) = \eta^{-1}\left(\frac{\alpha^2 M_{\rm PL}^2}{\phi_i^2}\right) T_R > \eta^{-1} m_{0,{\rm eff}}(\phi_i). \tag{3.4.42}$$

If this is greater than the negative mass term around the origin m_0, the trapping is expected. One sufficient condition of the trapping is written as

$$m_0 < \eta^{-1} m_{0,{\rm eff}}(\phi_i). \tag{3.4.43}$$

For most case of interest, the mass scale of the potential at the origin m_0 is at most comparable to that at far away from the origin $m_0 \lesssim m_{0,{\rm eff}}(\phi_i)$. Thus, we can conclude that for case (A), the trapping of the PQ field occurs if the negative mass around the origin m_0 is not unnaturally large $m_0 \gg m_{0,{\rm eff}}(\phi_i)$.

Case B

Here, we consider the case in which the PQ field begins to oscillate with the zero temperature potential, and q are thermally produced when the PQ field passes through the origin: $k_* < g^2 T$. In this case, the dissipation rate at the onset of the oscillation can be written as [128]

$$\Gamma_{\rm eff} \sim \frac{y}{g^4}\frac{T^2}{\Phi}\Big|_{\rm osc} = \left[\frac{y}{g^4}\left(\frac{g^2 T}{k_*}\right)\right]^2 m_{{\rm eff},0}(\phi_i)\Big|_{\rm osc}. \tag{3.4.44}$$

If the condition

$$y \gtrsim g^4 \frac{k_*}{g^2 T}, \tag{3.4.45}$$

holds, the PQ field dissipates its energy soon after the onset of oscillation because at the onset of oscillation, $H \simeq m_{{\rm eff},0}(\phi_i)$ holds. In case B, $k_* < g^2 T$ is satisfied and the condition Eq. (3.4.45) is likely to be satisfied with not so suppressed y. Here, we assume the condition Eq. (3.4.45) for simplicity.

After the energy in the PQ sector is converted to the that of thermal bath, the temperature of the thermal bath becomes

$$T_{\rm dis} \sim \max[\sqrt{m_{{\rm eff},0}(\phi_i)\phi_i}, T_{\rm os}], \tag{3.4.46}$$

where $T_{\rm os}$ denotes the temperature at the onset of the oscillation.

As before, we impose the following conservative condition of the trapping: the origin is the absolute minimum of the potential when the dissipation of the PQ field completes. To check this condition, we compare the zero temperature negative mass around the origin m_0 and the effective mass from thermal potential at $\phi = F_a$ since the zero temperature potential has the minimum at $\phi = F_a$. If $yF_a < T_{\rm diss}$, the thermal mass term dominates the thermal potential at $\phi = F_a$. On the other hand, if $yF_a < T_{\rm diss}$, the thermal log term dominates the thermal potential at $\phi = F_a$. The condition of the trapping can be written as

$$\begin{aligned} &m_0 < yT_{\rm dis} \text{ for } yF_a < T_{\rm diss}, \\ &m_0 < \alpha \tfrac{T_{\rm dis}^2}{F_a} \text{ for } yF_a > T_{\rm diss}, \end{aligned} \tag{3.4.47}$$

By noticing $T_{\rm dis}^2 > m_{\rm eff,0}(\phi_i)\phi_i$, we can obtain the following sufficient condition for the trapping:

$$\begin{aligned} &\left(\tfrac{m_0}{m_{\rm eff,0}(\phi_i)}\right)\left(\tfrac{m_0}{\phi_i}\right) < y^2, \text{ for } \ yF_a < T_{\rm dis}, \\ &\left(\tfrac{m_0}{m_{\rm eff,0}(\phi_i)}\right)\left(\tfrac{F_a}{\phi_i}\right) < \alpha, \ \ \text{ for } \ yF_a > T_{\rm dis}. \end{aligned} \tag{3.4.48}$$

In most case of interest, $m_0 \lesssim m_{\rm eff,0}(\phi_i)$ and $\phi_i \gg F_a$ will be satisfied. Thus, the trapping of the PQ field is expected with not so suppressed y.

Case *C*

Here, we consider the case where the PQ field starts to oscillate with the zero temperature potential, non-perturbative particle production is effective and the mixing between q and light particles is large: fast and large mixing case (a study of small mixing case can be found in [1]). In this case, the non-perturbative particle production occurs at the crossing of $\phi \sim 0$. As is mentioned, the number density of produced q in this way is given by k_*^3. The produced particles decay as the field value of ϕ becomes large ($\Gamma_q \sim y\kappa^2|\phi|$). Though the decay of q, the energy in the PQ field sector is converted to the thermal bath. From this, we can estimate the effective dissipation rate of the PQ field as [122]

$$\Gamma_{\rm eff} \sim \frac{y^2 m_{\rm eff,0}(\phi)}{\kappa}. \tag{3.4.49}$$

At each oscillation, the thermal bath obtains the energy density $\triangle\rho_{\rm rad} \sim k_*^4/\kappa$. This process continues until the condition of the non-perturbative production is violated:$gT_{\rm NP} \sim k_*$. The condition of the non-perturvative production is soon violated after a few Hubble time if the coupling g is at least $\mathcal{O}(0.1)$.

After the non-perturbative production stops, the system evolves keeping the relation $k_* \sim gT$. This relation $k_* \sim gT$ holds until the thermal dissipation rate becomes effective or the zero temperature mass around the origin $-m_0^2$ starts to affect the dynamics.

To be concrete, let us consider the case where the zero temperature potential far away from the origin is quadratic one ($n = 1$ in Eq. 3.4.18) bearing supersymmetric models in mind. While the relation $k_* \sim gT$ is maintained, the amplitude of the PQ field Φ and the temperature scale as

$$\Phi \propto H, \qquad T \propto H^{1/2}, \tag{3.4.50}$$

where Φ denotes the amplitude of the PQ field. The dissipation rate in this epoch can be estimated as [12]

$$\Gamma_{\rm eff} \sim y g^4 \frac{T^2}{\phi} = y^2 g^2 H_{\rm os}, \tag{3.4.51}$$

where $H_{\rm os}$ denotes the Hubble parameter at the onset of the oscillation. When $H \sim \Gamma_{\rm eff}$ is satisfied, the oscillation of the PQ field damps and the PQ field is trapped at the origin. When $H \sim \Gamma_{\rm eff}$ is realised and the oscillation of the PQ field is dissipated , the Hubble parameter becomes $H_{\rm diss}$

$$H_{\rm diss} \sim y^2 g^2 H_{\rm os}, \tag{3.4.52}$$

and the temperature becomes $T_{\rm diss}$:

$$T_{\rm diss} \sim yg\sqrt{m_{\rm eff,0}(\phi_i)\phi_i}, \tag{3.4.53}$$

because the energy density of the PQ field has evolved as $\rho_\phi \sim m^2_{\rm eff,0}(\phi_i)\phi_i^2(H/H_{\rm os})^2$.

As before, we impose the conservative condition of the trapping: the origin is the absolute minimum of the potential when the dissipation of the PQ field completes. We can obtain the following condition of the trapping:

$$\begin{aligned} &\left(\frac{m_0}{m_{\rm eff,0}(\phi_i)}\right)\left(\frac{m_0}{\phi_i}\right) < y^4 g^2, \ \ \text{for} \ \ yF_a < T_{\rm dis}, \\ &\left(\frac{m_0}{m_{\rm eff,0}(\phi_i)}\right)\left(\frac{F_a}{\phi_i}\right) < \alpha y^2 g^2, \ \text{for} \ \ yF_a > T_{\rm dis}. \end{aligned} \tag{3.4.54}$$

This condition can be satisfied if y is not so suppressed for most cases of our interest.

3.4.3 Summary of the PQ field trapping

In this section, we have seen that the PQ field is likely to be trapped at the origin even when the initial field value is much larger than the PQ scale F_a. If the PQ field has a

[12] When ϕ passes through the origin, q particles are produced $n_q \sim g^2 T^3$ (see Eq. (3.4.33)). During $\phi \sim 0$, the PQ field interacts with produced particles q. The dissipation rate due to a scattering process is estimated as $\sim y^2 g^2 T \times (n_q/T^3)$. By averaging the oscillation of ϕ, we can estimate the dissipation rate of the oscillation.

interaction with other fields, the PQ field is likely to be trapped at the origin by the thermal trapping shown as is shown Sec. 3.4.1. If the PQ field has a self interaction, the PQ also tends to be trapped by the non thermal trapping as is shown in Sec. 3.4.2. Note that if the domain wall number is greater than one, the trapping must not occur (see Sec. 3.3). Thus, some mechanisms may be needed for scenarios with $N_{\rm DM}$ and $\phi_i \gg F_a$ not to be excluded. We discuss such a mechanism in Chapter 5.

After the trapping, the PQ field starts to oscillate around the minimum of the potential when the effective mass at the origin becomes negative. We discuss the dynamics of the PQ field after the trapping in Chapter 4.

3.5 Axion as Dark Radiation

The dark radiation is an extra radiation component in the universe whose interactions with the standard model particles are negligible. The relativistic components of axion particles can be a part of the dark radiation. On the other hand, the amount of the dark radiation is tightly bounded by the CMB observations [12]. Conventionally, the amount of the dark radiation density in the early universe is parameterised by $N_{\rm eff}$, which is defined so that the total relativistic energy density of neutrons and any other dark radiation is given by

$$\rho = N_{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)\rho_\gamma, \tag{3.5.1}$$

where ρ_γ is the present photon energy density in $T \ll 1\,\mathrm{MeV}$. The standard cosmological prediction is given by $N_{\rm eff} = 3.046$. Thus, $\triangle N_{\rm eff} \equiv N_{\rm eff} - 3.046 > 0$ indicates that there exist extra radiation components other than standard model particles. The recent observation shows [12]

$$N_{\rm eff} = 3.04 \pm 0.18, \tag{3.5.2}$$

at 68% confidence level. This result is consistent with standard value $N_{\rm eff} = 3.046$, though there is still some room for an extra radiation component. However, models which predict $\triangle N_{\rm eff} \gg 1$ are of cause not consistent with the observations.

As is mentioned, the relativistic axion particles becomes a part of the dark radiation. There are two ways to produce the relativistic axion particles. One way is the thermal production in which thermal plasma produces axion particles at early universe. The other way is the non thermal production in which axion particles are produced by decay processes of heavy particles. The decay of the PQ fields can be a source non thermal production of axions.

First, let us consider the thermal production of axion particles. The thermal production of axion particles is estimated in [129, 130]. The abundance of thermally produce axions can be parametrised as [130]

$$\triangle N_{\rm eff} = 0.0264 \frac{Y_a}{Y_a^{\rm eq}}, \tag{3.5.3}$$

where the yield $Y_a \equiv n_a/s$ with n_a being the number density of axions and s denotes the entropy density of the universe. After the thermal production of the axion particles becomes ineffective, Y_a becomes constant. $Y_a^{\rm eq}$ denotes the yield of thermally populated axion number density. The thermal production of axions at least stops when the number density of the axion becomes thermal one. Thus, $\triangle N_{\rm eff}$ from the thermally produced axions has an upper limit 0.0264 which is far below the sensitivity of the present CMB observations.

If the PQ field mainly decays into axion particles after the energy density of the universe is dominated by the PQ field, the amount of the axion dark radiation $\triangle N_{\rm eff}$ will be inconsistent with the present observations. If the PQ field is thermally trapped at the symmetry enhanced point, the late time decay into axion particles may be harmful. This is because during the thermal trapping, the potential energy of the PQ field remains. Note that the minimum of the potential exists at the PQ scale and the symmetry enhanced point has a certain potential energy. After the trapping of the PQ field ends, such a potential energy is converted to the oscillation energy of the PQ field. In some cases, the energy density of the PQ field oscillation will soon dominate the total energy of the universe. Thus, if the PQ field is trapped at the origin, whether the axion over production is avoided or not is a relevant question. Considering that the PQ field is likely to be trapped at the symmetry enhanced point (see Sec. 3.4), it is important to investigate with what conditions, the axion over production is avoided. We deal with this issue in Chapter 4.

Chapter 4

Peccei-Quinn field Dynamics after trapping

Here, we consider the dynamics after the PQ field is thermally trapped at the symmetry enhanced point. As the universe expands and the temperature decreases, the effect of the thermal potential becomes negligible . As a result, the PQ field begins to roll down to the minimum of the potential at some time. After the PQ field escape from the thermal trapping, the PQ field starts to oscillate around the minimum of the potential. Then, the PQ field may lately decay into axion particles. The amount of produced axion particles is tightly constrained by the dark radiation observation [16]. Thus, it is important to clarify with what conditions, axion overproduction is not harmful.

In order to estimate amount of the axion dark radiation, we have to know the time evolution of the PQ field. In the dynamics of the PQ field, the thermal dissipation effects are important because they can reduce the amount of axion dark radiation. We take the dissipation effects on both the PQ field and axion particles into account and estimate the amount of axion dark radiation.

In this Chapter, we study the dynamics of the PQ field after the trapping at the symmetry enhanced point paying particular attention to the axion overproduction problem. In Sec. 4.1, we introduce our setup and summarise ingredients relevant for the dynamics of the system. In Sec. 4.2, we write down the coupled Boltzmann equations which determine the time evolution of the system. Then, we explain qualitative features of the dynamics. We show the numerical results in Sec. 4.3. The conclusion is given in Sec. 4.4

This Chapter is based on author's work [2].

4.1 Basic ingredients

4.1.1 Setup

In this thesis, we consider the hadronic axion model [66], because $N_{\rm DW} = 1$ holds [1] . We discuss the dynamics of the PQ field with the following Lagrangian

$$\mathcal{L} = |\partial\phi|^2 - (\lambda'\phi\psi\bar{\psi} + \text{h.c.}) + m_\phi^2|\phi|^2 - V_{\rm stab}(|\phi|), \tag{4.1.1}$$

where ψ and $\bar{\psi}$ are PQ quarks with (anti-)fundamental representations of color SU(3), and $V_{\rm stab}$ denotes the potential term that stabilizes the PQ field ϕ at the PQ scale $|\langle\phi\rangle| \equiv f_a = F_a/\sqrt{2}$. λ' denote a coupling constant. If we assume the supersymmetry (SUSY), ϕ will additionally couple to squarks which are the super-partners of ψ and $\bar{\psi}$[2]. We suppose the range of the PQ scale to be $f_a \sim 10^{9-10}$ GeV. The lower bound comes from the observational constraints (see Sec. 2.5). On the other hand, the upper bound comes from the condition of the axion dark matter abundance (see Sec. 3.3). In this section, we call the radial direction of the PQ field as the PQ field and the phase direction as the axion if there is no ambiguity.

We do not pay particular attention to the concrete form of $V_{\rm stab}$. The important quantities which determine the dynamics are the mass the PQ field at the origin m_ϕ ($V \supset -m_\phi^2|\phi^2|$) and at the minimum of the potential m_s and the hight of the potential at the origin $V_{\rm hight} \equiv V(0) - V(F_a)$. A typical scale of these quantities are supposed to be:

$$m_s \sim m_\phi, \tag{4.1.2}$$
$$V_{\rm hight} \sim m_\phi^2 F_a^2, \tag{4.1.3}$$

for most cases of interest. Since the details of the relation between m_ϕ, m_s, F_a and $V_{\rm hight}$ are model dependent, we assume $m_s = m_\phi$ and $V_{\rm hight} = m_\phi^2 F_a^2$ regarding m_ϕ and F_a as free parameters.

When the temperature of the universe is high enough, the PQ field is supposed to be trapped at the origin. During this epoch, the thermal potential for the PQ field is formed due to the coupling with the PQ quarks. At the origin, the effective mass of the PQ field $m_{\rm eff,\,ori}$ can be written as

$$m^2_{\rm eff,\,ori} \simeq \lambda^2 T^2 - m_\phi^2, \tag{4.1.4}$$
$$\lambda = c_M \lambda' \tag{4.1.5}$$

[1] In this Chapter, we consider the situation where the PQ symmetry is once restored. In such a case, models with $N_{\rm DW} > 1$ are excluded (see Sec. 3.3.2).

[2] In this Chapter, we study the dynamics as model independently as possible in order to see universal features of this scenario.

where c_M is an order one model dependent constant. The thermal trapping ends when the effective mass at the origin crosses zero [3] . The temperature at the end of the thermal trapping $T_{\rm end}$ is given by [4]

$$T_{\rm end} \simeq m_\phi/\lambda. \tag{4.1.6}$$

During the thermal trapping, the vacuum energy of the PQ field may dominate the energy component of the universe. The energy of the PQ field starts to dominate over that of the radiation $\rho_{\rm rad} \sim T^4$ at $T_{\rm beg}$:

$$V_{\rm hight} \gtrsim T^4, \tag{4.1.7}$$

$$\rightarrow T_{\rm beg} \sim \sqrt{m_\phi f_a}. \tag{4.1.8}$$

If $T_{\rm beg} > T_{\rm end}$, the universe experiences the vacuum energy of the PQ field dominated era. We pay a particular attention to the case $T_{\rm beg} \gg T_{\rm end}$ because in such a case, a significant energy is restored in the PQ sector initially and axion overproduction problem is severe. The condition $T_{\rm beg} \gg T_{\rm end}$ can be rewritten as

$$m_\phi \ll \lambda^2 f_a^2. \tag{4.1.9}$$

During $T_{\rm beg} > T > T_{\rm end}$, the universe expand exponentially and the ratio of the scale factor is given by $a_{\rm beg}/a_{\rm end} = T_{\rm end}/T_{\rm beg}$. Such a phenomena is called thermal inflation [131]. It should be noted that in SUSY models, the typical mass scale of the PQ field becomes the soft breaking SUSY mass scale (see Sec. 2.4.3). Thus, a plausible value of m_ϕ is considered to be much less than the PQ scale F_a 10^{9-10} GeV. In general, in SUSY PQ models, the thermal inflation is likely to occur.

After the thermal trapping, the PQ field starts to oscillate around the minimum of the potential. with the typical frequency m_s. At this stage, the PQ field can perturbatively decay into a pair of relativistic axion particles. The decay rate is given by

$$\Gamma_{\phi\rightarrow 2a} = \frac{1}{64\pi}\frac{m_s^3}{f_a^2}. \tag{4.1.10}$$

In the following, we replace m_s to m_ϕ for simplicity, otherwise stated. There are other decay channels such as a decay into a gluon pair. However, such decays are subdominant due to a loop and coupling suppression. Thus, if perturbative decay is the main

[3] The thermal trapping may end by the first order phase transition. However, As long as $V_{\rm hight}$ is much larger than the energy density of the radiation, our discussion does not change in the point of view of axion overproduction.

[4] We assume $T_{\rm end}$ is lower than the reheating temperature of the universe.

process to damp the oscillation of the PQ field, the universe would be dominated by axion particles produced in this way. They contribute to dark radiation of the universe, whose energy density is strictly bounded by CMB observations (see Sec. 3.5) [5] .

Fortunately, there is another damping process of the PQ field oscillation. The process is the thermal dissipation effect [1, 120, 121]. If thermal dissipation effect is efficient enough to convert the energy of the PQ field into the thermal bath, the axion over production is avoided. In the next section, we summarise the thermal dissipation effects on the PQ field. We also consider the thermal dissipation on the axion particles which may also reduce the amount of the axion particles.

4.1.2 Thermal Dissipation

In this section, we consider the thermal dissipation effects on the PQ field. If a scalar field couples to other particles which participate in the thermal bath, the interactions tend to damp the motion of the scalar field [116–122]. First, let us briefly summarise general aspects of the thermal dissipation.

Suppose a scalar field φ has a following interaction term:

$$\mathcal{L}_{\rm int}(x) \supset \varphi(x)\hat{O}(x), \tag{4.1.11}$$

where we assume the operator $\hat{O}$ consists of fields that are thermalised. The mean value of the scalar field obeys the following equation of motion [132]

$$0 = \Box\varphi(x) + V'_\varphi(\varphi) + \int d\tau \Pi^O_{\rm ret}(t-\tau)\varphi(\tau), \tag{4.1.12}$$

where V_φ denotes the potential of φ and

$$\Pi^O_{\rm ret}(x) \equiv -i\theta(x^0)\Pi^O_{\rm J}(x), \tag{4.1.13}$$

$$\Pi^O_{\rm J}(x) \equiv \langle[\hat{O}(x), \hat{O}(0)]\rangle, \tag{4.1.14}$$

with $\langle\bullet\rangle$ being a thermal average. The last term in Eq. (4.1.12) indicates the interaction with the thermal bath.

[5] Axions can be regarded as dark radiation if the momenta are much larger than its mass at the time of the recombination. We have checked this condition is (at least marginally) satisfied if $\triangle N_{\rm eff} > 1$ where $\triangle N_{\rm eff}$ denotes the effective number of an extra dark radiation (see Sec. 3.5). In the region $\triangle N_{\rm eff} \gg 1$, axions becomes highly relativistic because the radiation is not reheated so much.. Thus, scenarios with $\triangle N_{\rm eff} > 1$ are excluded by the constraint on the dark radiation [12] .

From the last term of Eq. (4.1.12), we can pick up the dissipation term [117]:

$$-\dot{\varphi}\Gamma_{\rm th} \subset \int d\tau \Pi^{O}_{\rm ret}(t-\tau)\varphi(\tau) \tag{4.1.15}$$

$$\Gamma_{\rm th} \equiv \frac{\Pi^{O}_{\rm J}(\omega,0)}{2\omega}, \tag{4.1.16}$$

$$\Pi^{O}_{\rm J}(\omega,\vec{p}) \equiv \int d^4x e^{i(\omega t-\vec{p}\cdot\vec{x})}\Pi^{O}_{\rm J}(t,\vec{x}), \tag{4.1.17}$$

where we assume the scalar field φ oscillates with a frequency ω. In addition, we assume the motion of φ is mild so as not to disturb the thermal correlation functions. Similar to this, the dissipation rate of φ particles can be written as

$$\Gamma^{\rm par}_{\rm th} = \frac{\Pi^{O}_{\rm J}(\omega,\vec{p})}{2\omega}, \tag{4.1.18}$$

where $(\omega,\vec{p})$ is the four momentum of the corresponding mode.

In general, a precise estimation of a thermal dissipation rate is highly complicated. This is because higher loop contributions such as ladder diagrams can also give leading contributions especially for the $\omega \ll gT$ case [133]. Fortunately, the dissipation rate of the radial direction of the PQ field is estimated completely at the leading order in the gauge coupling expansion [134–136]. On the contrary, there is no leading order calculation of the dissipation rate of axion particles. Thus, we apply a certain approximation to estimate the dissipation rate of the axion. Below, we briefly summarise the thermal dissipation rate of the PQ field and the axion.

Thermal dissipation rate of the PQ field

After the PQ field starts to oscillate around $|\phi| = f_a$, the PQ quarks ψ obtain the mass of $m_\psi = \lambda|\phi|$. Note that the condition Eq. (4.1.9) indicates that the PQ quarks are thermally decoupled in most time of the oscillation. In such a case, the dominant interaction term of the PQ field is obtained by integrating out the PQ quarks and given by

$$\mathcal{L} \supset \frac{n_\phi \alpha_s \delta\phi}{8\pi^2 f_a} G^{\mu\nu}G_{\mu\nu} \equiv \delta\phi \hat{O}_\phi, \tag{4.1.19}$$

$$\delta\phi \equiv f_a - \phi, \tag{4.1.20}$$

where α_s denotes the QCD gauge coupling strength, $G^{\mu\nu}$ denotes the gluon field strength and n_ϕ is a model dependent order one constant depending [6] . The dissipation rate of

[6] For example, if we consider SUSY models, there exists PQ squarks which affect the value of A_ϕ.

the PQ field is given by

$$\Gamma_\phi = \frac{\Pi_J^{O_\phi}(m_\phi, \vec{0})}{2m_\phi}. \tag{4.1.21}$$

The right hand side of Eq. (4.1.21) is related to the bulk viscosity of the QCD plasma and extensively studied in the literature [134–136]. The bulk viscosity can be estimated by using the effective Boltzmann equation which ensure the leading order result. The dissipation rate of the PQ field is given by

$$\Gamma_\phi^{(\mathrm{dis})} \simeq \frac{b\alpha_s^2 T^3}{f_a^2} \times \begin{cases} 1 & \text{for } m_\phi \ll g_s^4 T \\ \sqrt{g_s^4 T/m_\phi} & \text{for } g_s^4 T \ll m_\phi \ll g_s^2 T \end{cases}, \tag{4.1.22}$$

where b is a numerical constant.[7] In the case of quark gluon plasma, $b \simeq 1/(32\pi^2 \log \alpha_s^{-1})$ holds [136]. In our numerical calculation, we adopt this value of b. [8]

Thermal dissipation of the axion particles

The axion particles, which are produced by the decay of the PQ field oscillation, also interact with the thermal bath. The interaction term with the thermal bath is given by

$$\mathcal{L} \supset \frac{g^2 a}{32\pi^2 F_a} G \cdot \tilde{G}. \tag{4.1.23}$$

The production (dissipation) rate of the axion particles with momentum $p \gtrsim gT$ is estimated in [129, 130]. In our case, the typical momentum of axion particles may be much smaller than the temperature of the universe $p \ll T$. However, the dissipation rate for a soft mode $p \ll T$ may be different from a hard mode $p \gtrsim gT$ as is the case in the PQ field. In the case of the PQ field, pole contributions regulated by its thermal width give the dominant contribution . However, contrary to the PQ field case, the interaction term Eq. (4.1.23) can be rewritten as $aG^a_{\mu\nu}\tilde{G}^{a\mu\nu} = a\partial_\mu K^\mu \sim (\partial_\mu a)K^\mu$. The axion energy/momentum times temperature may be picked up in the dissipation rate. As a result, we expect a suppression factor, $p^2/(g_s^4 T^2)$, for the dissipation rate of produced axion particles compared with the PQ field dissipation rate. [9] .

[7] Here we assume the oscillation time scale of the PQ field is much slower than the interaction time scale of the thermal plasma: $m_s \ll T$, which marginally holds during the course of dynamics of interest.

[8] Strictly speaking, if the PQ quarks are also charged under other gauges like $SU(2)_L$, there is a contribution from the corresponding gauge interactions. In addition, in a class of SUSY axion models, there exist contributions from the couplings with PQ squarks and gauginos. We do not consider those contributions in the this discussion because they are model dependent.

[9] We expect the suppression factor $p^2/(g_s^4 T^2)$ by the following way. First, let us consider the dissipation rate of φ coming from the operator $y\varphi\hat{O}$ where y denotes a coupling constant and $\hat{O}$ does not contain a

In order to estimate the dissipation rate of the axion particle, we compute one loop effects with the Breit-Wigner approximation for the spectral function of gauge bosons, In appendix B, we summarise a detailed calculation of the dissipation rate of the axion particles. In fact, we observed the suppression behave of factor $p^2/(g_s^4T^2)$. Our estimation of the axion dissipation rate is given by

$$\Gamma_a^{(\mathrm{dis})} = \frac{\alpha_s^2 T^3}{32\pi^2 f_a^2}\frac{p^2}{g_s^4 T^2} f(x), \tag{4.1.24}$$

where $x = p/(g_s^4 T)$, and the definition of the function $f(x)$ is given in the Appendix [Eq. (B.0.6)]. Qualitatively, the function $f(x)$ behaves as $f \sim$ const. for $x \ll 1$ and $f \propto x^{-2}$ for $g^{-2} \ggg x \gg 1$ and takes order one values at $x \lesssim 1$. As is mentioned, a precise estimation of a thermal dissipation rate require all order resummation of relevant diagrams. In addition, the thermal dissipation rate depends on the particle contents of the model and thus, model dependent. Therefore, our estimate of the axion dissipation rate has a factor uncertainty. In order to take the uncertainty into account, we adopt the following dissipation rate in our numerical calculation:

$$\Gamma_a^{(\mathrm{dis})} = \frac{\alpha_s^2 T^3}{32\pi^2 f_a^2}\frac{p^2}{g_s^4 T^2} f(x) \times C. \tag{4.1.25}$$

where we explicitly introduce an order one parameter C which represents the uncertainty of the thermal dissipation.

Let us consider a typical momentum of the axion particles. If the decay of the PQ field into axion is negligible ($H \gg \Gamma_{\phi\to 2a}$), The energy density of axion particles are dominated by latest produced particles. In such a case, the typical momentum of the axion becomes $\simeq m_{\mathrm{phi}}/2$. On the the other, when $H \sim \Gamma_{\phi\to 2a}$ holds, the PQ field will soon decay into axion particles. After that, the energy density of the axion particles are dominated by the particles produced at the epoch $H \sim \Gamma_{\phi\to 2a}$. In this case, the typical momentum of the axion particles are written as $p \simeq (a_{\mathrm{dec}}/a(t)) \times m_\phi/2$ with a_{dec} and $a(t)$ being the scale factor at the time of $H = \Gamma_{\phi\to 2a}$ and at the time t, respectively. In our numerical analysis, we use

$$p = \frac{m_\phi}{2}\min\left(1, \frac{a_{\mathrm{dec}}}{a(t)}\right), \tag{4.1.26}$$

coupling constant. In such a case, we may expect $\Gamma_{\mathrm{th}} \sim y^2 T/\alpha_{\mathrm{damp}}$ with α_{damp} being a typical interaction rate of the thermal bath. In non Abelia gauge theory, $\alpha_{\mathrm{damp}} \sim g^4$ with g denotes the gauge coupling is expected. If the operator $\hat{O}$ has a non-trivial structure, an extra consideration of coupling counting is needed. For example, the PQ field couples to $G^{\mu\nu}G_{\mu\nu}$, which is the trace of the energy momentum tensor. If the gauge coupling g is zero, the energy momentum tensor also becomes zero. The number counting of $G^{\mu\nu}G_{\mu\nu}$ is supposed to be g^2 [134]. In the case of the axion dissipation, if the operator K does not have such a non trivial structure, the suppression of factor $p^2/(g_s^4T^2)$ is expected.

for the momentum of the axion particles. We set the initial condition as $\rho_\phi = V_{\rm hight} = m_\phi^2 f_a^2$, $\rho_a = 0$ and $T = m_\phi/\lambda$.

4.2 Boltzmann equations and its properties

In this section, we introduce coupled Boltzmann equations which determine time evolution of the system. Then, we discuss qualitative features of the dynamics.

4.2.1 Coupled Boltzmann equations

After the thermal inflation, the energy densities of the PQ field oscillation ρ_ϕ, axion ρ_a and radiation ρ_r evolve as

$$\dot{\rho}_\phi + 3H\rho_\phi = -(\Gamma_\phi^{(\rm dis)} + \Gamma_{\phi\to 2a})\rho_\phi \tag{4.2.1}$$

$$\dot{\rho}_a + 4H\rho_a = \Gamma_{\phi\to 2a}\rho_\phi - \Gamma_a^{(\rm dis)}\rho_a, \tag{4.2.2}$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma_\phi^{(\rm dis)}\rho_\phi + \Gamma_a^{(\rm dis)}\rho_a, \tag{4.2.3}$$

where H denotes the Hubble parameter. The dissipation rates and the decay rate of the PQ field are given in the previous section. We do not take thermal production effects of the axion and the PQ field into account because the amount of the axion dark radiation by the thermal production is tiny (see. 3.5).

4.2.2 Qualitative features of the dynamics

We pay particular attention to the case where the energy of the PQ field dominates the universe. In such a case, the axion will be over produced if there is no thermal dissipation effects. This is because without the thermal dissipation effects, the PQ field just decays into axion particles. Then, the energy of axion particles dominants the universe, which leads $\triangle N_{\rm eff} \gg 1$ with $\triangle N_{\rm eff}$ being the effective number of an extra dark radiation expressed in unit of one neutrino (see Sec. 3.5). $\triangle N_{\rm eff}$ is tightly bounded by the CMB observations: $\triangle N_{\rm eff} < 1$ [12]. Thus, in order not to produce too much axion dark radiation, the thermal dissipation effects must affects the dynamics in some way.

The thermal dissipation effects is proportional to $\propto T^3$. On the other hand, the Hubble parameter scales as $\propto a^{-3/2}$ in the matter dominated universe. If there is no entropy injection into the thermal bath, the thermal dissipation effects become less and less important as the universe expands because the thermal dissipation effects scale as $\propto a^{-3}$ in such a case. Here we denote "no entropy injection" as the situation where the temperature of the universe is well approximated by the scaling solution $\propto a^{-3}$. If there is no entropy injection in a few Hubble times after the onset of the PQ field oscillation, the thermal dissipation effects never affect the dynamics of the PQ field. This is because the thermal dissipation effects becomes most effective at the initial time in this case but no entropy injection means the energy density of the PQ field is not affected by the thermal dissipation effects. From these considerations, we can say that the dynamics in a few Hubble times after the onset of the PQ field oscillation is very important. In this time span, whether the axion over production is avoided or not is determined.

Now, let us consider the time span of a few Hubble time just after the onset of the PQ field oscillation. Thanks to the thermal dissipation effect of the PQ field, the thermal bath is reheated to some extent. The dynamics largely depends on whether the decay of the PQ field into axions is effective or not. In other words, whether $\Gamma_{\phi\to 2a} > H$ or $\Gamma_{\phi\to 2a} < H$ is important. This is because the entropy injection into the thermal bath in a first few Hubble time largely depends on the ratio $\Gamma_{\phi\to 2a}/H$. This ratio is given by

$$\left.\frac{\Gamma_{\phi\to 2a}}{H}\right|_{T=T_{\rm end}} \sim 10\frac{m_\phi^2 M_P}{64\pi f_a^3} \sim \left(\frac{m_\phi}{1\,\mathrm{PeV}}\right)^2\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)^3. \tag{4.2.4}$$

Below, we discuss the two cases: $\Gamma_{\phi\to 2a}\lesssim H$ and $\Gamma_{\phi\to 2a}\gtrsim H$, one by one.

The case with $\Gamma_{\phi\to 2a} \lesssim H(T = T_{\rm end})$

Here we consider the case where the decay of the PQ field into axions is negligible at the onset of the oscillation. In this case, the energy density of the thermal bath obeys the following equation:

$$\dot\rho_r + 4H\rho_r = \Gamma_\phi^{(\rm dis)}\rho_\phi. \tag{4.2.5}$$

Within a few Hubble time, the energy density ρ_r comes to obey the following solution:

$$4H\rho_r \sim \Gamma_\phi^{(\rm dis)}\rho_\phi. \tag{4.2.6}$$

Here we assume that ρ_ϕ decreases at most by a factor of a few during the first few Hubble time. From Eq. (4.2.6), we can deduce that the thermal bath is reheated to T_c:

$$\rho_r(T_c) \sim \frac{\Gamma_\phi^{(\rm dis)}(T_c)}{4H(t_{\rm end})\rho_\phi(t_{\rm end})}, \tag{4.2.7}$$

$$\to T_c \sim \left(\frac{30}{\pi^2 g_*}\right)\frac{b\alpha_s^2 m_\phi M_P}{f_a} \sim 10^7\,\mathrm{GeV}\left(\frac{200}{g_*}\right)\left(\frac{m_\phi}{1\,\mathrm{PeV}}\right)\left(\frac{10^{10}\,\mathrm{GeV}}{f_a}\right), \tag{4.2.8}$$

where we denote $t_{\rm end}$ as the time when the thermal inflation ends and the PQ field starts to oscillate. We regard this temperature T_c as a "first reheat temperature". Remember that in the derivation of T_c, we assume that ρ_ϕ decreases at most by a few factor during a first few Hubble time. If this assumption is not true, ϕ must lose most of the energy during this time span, which means that the energy density of the PQ field is well converted to the thermal bath by the thermal dissipation effects. In such a case, the axion overproduction is avoided.

Let us consider the condition for the PQ field not to be dissipated. In such a case, the temperature of the universe becomes T_c within a few Hubble time after $t_{\rm end}$. If $H \gtrsim \Gamma_\phi^{\rm (dis)}(T_c)$ holds, the thermal dissipation effects cannot damp the oscillation of the PQ field because the thermal dissipation effect becomes most effective when $T = T_c$. On the other hand, if the opposite $H \lesssim \Gamma_\phi^{\rm (dis)}(T_c)$ holds, the thermal dissipation effect damps the energy of ϕ. As the energy in the PQ sector is converted to the thermal bath, the temperature of the universe becomes higher and higher, which makes the thermal dissipation effect stronger. As a result, the whole energy of the PQ sector will be converted to the thermal bath and the axion over production is avoided. We can conclude that the axion over production is avoided if and only if $H \gtrsim \Gamma_\phi^{\rm (dis)}(T_c)$ is satisfied. This condition can be rewritten as

$$\left.\frac{\Gamma_\phi^{\rm (dis)}}{H}\right|_{T=T_c} \sim \left(\frac{30}{\pi^2 g_*}\right)^3 \frac{(b\alpha_s^2 m_\phi^{1/2})^4 M_P^4}{f_a^6} \sim \left(\frac{200}{g_*}\right)^3 \left(\frac{m_\phi}{3\,{\rm PeV}}\right)^2 \left(\frac{10^{10}\,{\rm GeV}}{f_a}\right)^6 \gtrsim 1. \tag{4.2.9}$$

The case with $\Gamma_{\phi\to 2a} \gtrsim H(T = T_{\rm end})$

Here we consider the case where the PQ field can efficiently decay into axions just after the onset of the oscillation $\Gamma_{\phi\to 2a} \gtrsim H(T = T_{\rm end})$. For simplicity, let us concentrate on the case $\Gamma_{\phi\to 2a} \gg H(T = T_{\rm end})$. In such a case, the PQ field soon decays to axion particles just after the inflation. Then, almost all energy of the PQ field is converted to the axion particles: $\rho_a = \rho_\phi(t_{\rm end})$. Then the energy density of the thermal bath obeys the following equation:

$$\dot\rho_r + 4H\rho_r = \Gamma_a^{\rm (dis)}\rho_a. \tag{4.2.10}$$

The situation becomes similar to the previous case. We can apply the same logic used in the previous case just by replacing $\phi \to a$. What we have to do first is to estimate the first reheated temperature in this case: $T_c^{(a)}$. If we parameterise $T_c^{(a)} = \delta \cdot T_c$ we can derive the condition for the axion not to remain much until today as

$$\left.\frac{\Gamma_\phi^{\rm (dis)}}{H}\right|_{T=T_c^{(a)}} \sim \delta^4 \left(\frac{200}{g_*}\right)^3 \left(\frac{m_\phi}{3\,{\rm PeV}}\right)^2 \left(\frac{10^{10}\,{\rm GeV}}{f_a}\right)^6 \gtrsim 1. \tag{4.2.11}$$

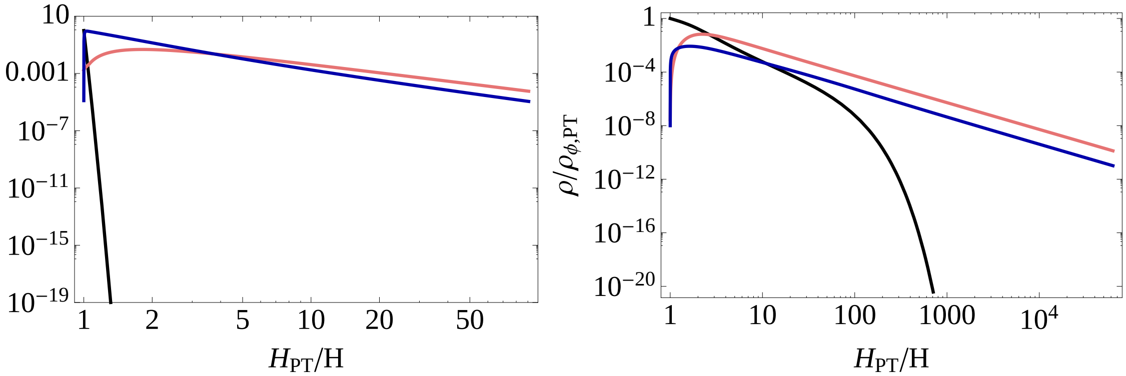

Figure 4.1: The evolutions of energy densities as a function of H normalised by the Hubble parameter at the PQ phase transition H_{PT}: $\rho_\phi/\rho_{\phi,PT}$ (black), $\rho_r/\rho_{\phi,PT}$ (pink) and $\rho_a/\rho_{\phi,PT}$ (blue) with $\rho_{\phi,PT}$ denoting the initial energy density of ϕ. Left/Right panel shows the case $[m_\phi,\ F_a] = [50\ \mathrm{PeV},\ 10^{10}\ \mathrm{GeV}]/[10\ \mathrm{TeV},\ 5\times 10^{8}\ \mathrm{GeV}]$. From [2].

The first reheat temperature in this case $T_c^{(a)}$ is determined by the following equation

$$4H\rho_r(T_c^{(a)}) \sim \Gamma_\phi^{(\mathrm{dis})}(T_c^{(a)})\rho_\phi. \tag{4.2.12}$$

Since the dissipation rate of the axion depends on a numerical function $f(x)$ (see Eq. 4.1.24) , it is difficult to write down $T_c^{(a)}$ analytically. Qualitatively, the function $f(x)$ obeys $f(x) \propto x^{n-2}$ $(0 < n < 2)$ and $f(x) \sim \mathcal{O}(1)$ for $x \lesssim 1$. Thus, the F_a dependance of the parameter $\delta \propto F_a^{n/(n+1)}$ can be bounded as $\delta \propto F_a^0$ to $F_a^{2/3}$.

If the condition Eq. (4.2.9) for $\Gamma_{\phi\to 2a} \lesssim H(t_{\mathrm{end}})$ or the condition Eq. (4.2.11) for $\Gamma_{\phi\to 2a} \gtrsim H(t_{\mathrm{end}})$ is satisfied, the temperature increases and the PQ field oscillation disappears in a few Hubble time without producing too much axion dark radiation. In the next section, we show numerical results of the Bolzmann equation.

4.3 Numerical results

In this section, we show numerical results of the Boatman equations (Eqs. (4.2.1) – (4.2.3)).

First, let us see the time evolution of the system for typical situations. Fig. 4.1 shows evolution of various quantities after the phase transition as a function of the Hubble time normalised by that at the PQ phase transition: H_{PT}/H. We set $T_{\mathrm{end}} = m_\phi$. The black, pink and blue lines represent evolution of the energy densities of the PQ field oscillation, the radiation and the axion particles produced by the decay of the PQ field respectively. These energy densities are normalised by the initial energy density of the PQ field. The left panel shows the case with $[m_\phi,\ F_a] = [50\ \text{PeV},\ 10^{10}\ \text{GeV}]$. In this case, the condition $\Gamma_{\phi\to 2a} \gtrsim H_{PT}$ is satisfied. As one can see, the PQ field soon decay to the axion particles. After that, the radiation is heated in a few Hubble times. The amount of the axion particles are dissipated and finally the energy density of the axion becomes lower than that of radiation. In this case, the effective number of the extra dark radiation becomes $\triangle N_{\text{eff}} \simeq 0.4$. On the other hand, the right panel show the case with $[m_\phi,\ F_a] = [10\ \text{TeV},\ 5\times 10^8\ \text{GeV}]$. In this case, $\Gamma_{\phi\to 2a} \lesssim H_{PT}$ is satisfied. In a few Hubble time, the radiation is heated and the PQ field is dissipated to some extent. Then, before the PQ field dominates the universe again, the PQ fields decay into axions. In this case, $\triangle N_{\text{eff}} \simeq 0.2$ holds.

Fig. 4.2 shows contour of $\triangle N_{\text{eff}} = 1$ [10] on the $[m_\phi,\ F_a]$ plane for $T_{\text{end}} = m_\phi$ (left panel) and $T_{\text{end}} = 10 m_\phi$ (right panel). In the shaded blue region, $F_a \lesssim T_{\text{end}}$ is satisfied. As is mentioned, the dissipation rate has uncertainty factor owing to the model dependence and theoretical uncertainties. To see this dependence, we vary the parameter C in Eq. (4.1.25). The upper (lower) boundary correspond to $C = 1(10)$. Above the band, $\triangle N_{\text{eff}}$ becomes smaller than one. The black dashed line represents the contour $\triangle N_{\text{eff}} = 1$ without the thermal dissipations. We can see that the region with $\triangle N_{\text{eff}} \lesssim 1$ is significantly extended by the thermal dissipation effects especially in the case of $T_{\text{end}} = m_\phi$. As we can see, the behaviour of the lines becomes changed at $[m_\phi,\ F_a] \sim [100\ \text{TeV},\ 10^9\ \text{GeV}]$ Around this point, $\Gamma_{\phi\to 2a} \sim H_{PT}$ is satisfied. If F_a is smaller than this point, the dissipation of the PQ field is important. On the other hand, if F_a is larger than this point, the dissipation of the axion becomes relevant as is discussed in Sec. 4.2.2.

4.4 Conclusion of this Section

In this section, we consider the dynamics of the PQ field after the PQ field is thermally trapped at the symmetry enhanced point. As the temperature decreases, the PQ field starts to oscillate around the minimum of the potential. The oscillation of the PQ field may cause the over production of axion particles. If a typical mass scale of the potential is smaller than the PQ scale, as is the case in a class of SUSY PQ models, the energy of the oscillation becomes larger than that of radiation. In such a case, the axion over production problem becomes serious.

[10] On the line $\triangle N_{\text{eff}} = 1$, the dissipation effects and the Hubble parameter becomes comparable at some epoch. Thus, if we change the parameter on this line slightly , the prediction of $\triangle N_{\text{eff}}$ is rather affected. In this respect, the line $\triangle N_{\text{eff}} = 1$ can be understood as the boundary whether the dark radiation is over produced or not.

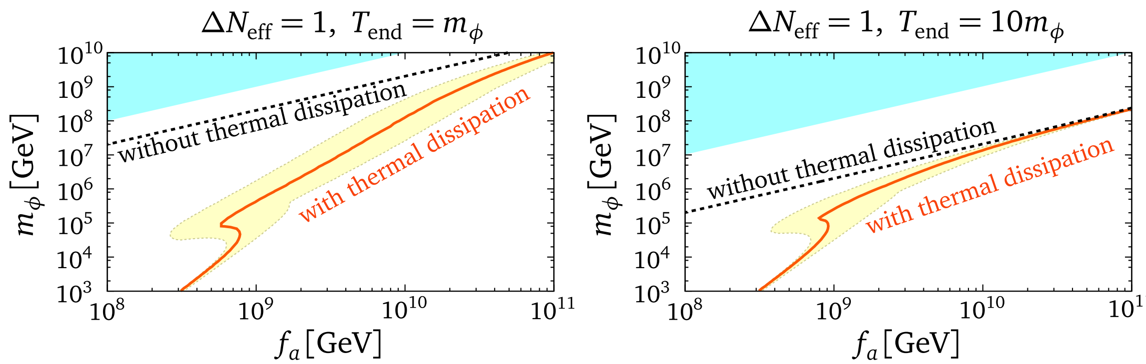

Figure 4.2: The contour of $\triangle N_{\rm eff} = 1$ on the $[m_\phi,\ F_a]$ plane for $T_{\rm end} = m_\phi$ (left panel) and $T_{\rm end} = 10m_\phi$ (right panel). In the shaded blue region, $F_a \lesssim T_{\rm end}$ holds. The upper (lower) boundary correspond to $C = 1(10)$. Above the band, $\triangle N_{\rm eff}$ becomes smaller than one. The black dashed line denotes the contour $\triangle N_{\rm eff} = 1$ without the thermal dissipations. From [2].

We take the thermal dissipation effects into account and show that the axion overproduction can be avoided even if the PQ field once dominates the universe. In the dynamics of the PQ field, the thermal dissipation effects of both the PQ field and the axion play important roles to suppress the amount of the axion dark radiation. This scenario works if the mass scale of the potential m_ϕ is larger than $\mathcal{O}(10)$ TeV - $\mathcal{O}(10)$ Pev for $F_a \sim 10^{9-10}$ GeV. Such parameter range includes some high scale supersymmetric models.

Chapter 5

Suppression of Isocurvature perturbation

5.1 Overview

In this Chapter, we consider a situation where the PQ symmetry is always broken by field values of PQ fields.

As is discussed in Sec. 3.2, in the standard cosmological scenario where the PQ symmetry is already broken during the inflation and the PQ scalar is stabilised at the minimum of the potential $|\phi| = F_a/\sqrt{2}$, the magnitude of the axion fluctuation is given by

$$\frac{\delta a}{a_i} \simeq \frac{H_{\rm inf}}{2\pi F_a \theta_i}, \tag{5.1.1}$$

where $H_{\rm inf}$ denotes the Hubble parameter during inflation, a_i indicates the initial value of the axion field and $\theta_i \equiv a_i/F_a$. In this case, the cold dark matter isogurvature perturbation is given by $S_{\rm CDM} \simeq r(2\delta a/a_i)$ with r being the axion fraction in the dark matter energy density and estimated as (see Sec. 3.2.2)

$$S_{\rm CDM} \simeq 1 \times 10^{-5} \theta_i \left(\frac{F_a}{10^{12}\,{\rm GeV}}\right)^{0.19} \left(\frac{H_{\rm inf}}{10^7\,{\rm GeV}}\right). \tag{5.1.2}$$

The Planck constraints on the this value reads $S_{\rm CDM} \lesssim 0.9 \times 10^{-5}$ [16]. For the axion window $10^9\,{\rm GeV} \lesssim f_a \lesssim 10^{12}\,{\rm GeV}$, the inflation scale is tightly bounded from above $H_{\rm inf} \lesssim 10^{7-10}$ GeV. In particular, if axion is a dominant component of the cold dark matter, the constraint reads $H_{\rm inf} \lesssim 10^7$ GeV. It excludes high-scale inflation models unless the PQ symmetry is restored after the inflation. However, if the domain wall number is greater than one as is the case in the DFSZ model [36, 37], the formation of axion domain walls leads to a cosmological disaster [14].

There are some ways to relax the constraint on the quantum fluctuation in the axion direction. For example, the value of the PQ field during inflation $|\phi_{\rm inf}|$ may be much larger than the PQ scale F_a. In this case, the isocurvature perturbation is suppressed by the ratio $F_a/|\phi_{\rm inf}|$ [26] (see also [2, 27, 28]). This is because the size of the fluctuation in the massless direction generated during the inflation is given by $\delta\phi \sim H_{\rm inf}/2\pi$ which sets the fluctuation on the angle of ϕ to be $\delta\theta = \delta\phi/|\phi_{\rm inf}|$. After the PQ field settle down at $|\phi| = F_a/\sqrt{2}$, the fluctuation of the axion can be regarded as $\delta a = \delta\theta \cdot F_a = \delta\phi F_a/|\phi_i|$, which can be suppressed if $F_a/|\phi_i| \gg 1$. Another way is to make the axion heavy during the inflation by the stronger QCD effects [28–31] or by the explicit PQ breaking term [32, 33] so as not to generate the fluctuations in the axion direction. The non-minimal kinetic term of the axion may also weaken the constraint [34].

As we discussed in Sec. 3.4, the PQ symmetry is likely to be restored by the dynamics of the PQ field after the inflation. Thus, when we consider a scenario of this direction, we have to care about the trapping of the PQ field. For example, the idea to put the PQ field far away from the PQ scale [26] may seem simple and natural. However, in such a case, the PQ symmetry is likely to be restored by the thermal or non thermal trapping (see Sec. 3.4).

We propose two new scenarios which relax the constraints on the quantum fluctuations avoiding the PQ symmetry restoration, which is inevitable for models with $N_{\rm DW} > 1$. In both cases, the dynamics of the PQ field plays important roles.

The first way uses scalar field motion in the expanding universe. If a certain condition holds, scalar motion becomes very peculiar one which is different from oscillation or a scaling solution. We call this behaviour as a "pseudo scaling". If a scalar field once obey the pseudo scaling solution, it tends not to be trapped at the symmetry enhanced point even with a large initial field value. This study is based on the work by the author [3].

In the second way, multi PQ charged scalar fields are considered. In our scenario, if one of the PQ fields has a large initial field value, the bound on the quantum fluctuations can be relaxed. At the same time, it is relatively easy for the PQ symmetry not to be restored all the time because there exist other PQ charged scalar fields. One interesting example of this set up is that the inflaton field has a PQ charge. We will show that within the framework of Higgs inflation in DFSZ axion model, the experimental constraint can be satisfied. This study is based on the work by the author [35]

5.2 Pseudo Scaling Solution

In this section, we consider a pseudo scaling solution which is a peculiar motion of a scalar field that occurs when certain conditions hold. In Sec. 5.2.1, we summarise the properties of the pseudo scaling solution. Then, we apply the pseudo scaling solution in Sec. 5.2.2. We show how the pseudo scaling solution can relax the constraint on the inflation scale without the symmetry restoration. Sec. 5.2.3 is devoted to the conclusion of this section.

5.2.1 Properties of Pseudo scaling solution

Setup

Let us consider a real scalar field theory with the Lagrangian [1]

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi), \tag{5.2.1}$$

where the scalar potential is given by the monomial one[2].

$$V(\phi) = \frac{\lambda}{n}\phi^n, \ (n > 2). \tag{5.2.2}$$

The equation of motion for ϕ in expanding universe can be written as

$$\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^{n-1} = 0, \tag{5.2.3}$$

where H denotes the Hubble parameter. We assume the following form of H:

$$H = \frac{p}{t}, \ (3p - 1 > 0). \tag{5.2.4}$$

Note that $p = 2/3(1/2)$ corresponds to the matter (radiation)-dominated (MD (RD)) universe. A more general value of p may be possible in the inflaton oscillation dominated universe. Let us parametrise ϕ and time t as

$$\psi \equiv a^{\frac{2}{p(n-2)}}\phi, \tag{5.2.5}$$

$$s \equiv \ln(t/t_i), \tag{5.2.6}$$

where $a(t)$ denotes the scale factor of the universe and t_i is the initial time. We set $a(t_i) = 1$. H_i is the initial value of the Hubble parameter. Then, the equation of motion can be rewritten as [137, 138]

$$\psi'' + \left(3p - \frac{n+2}{n-2}\right)\psi' + \mu^2\psi + \frac{\lambda p^2}{H_i^2}\psi^{n-1} = 0, \tag{5.2.7}$$

[1] In this discussion, we consider a real scalar field for simplicity. The same arguments can be applied to the complex scalar case.

[2] For n except even integer one, ϕ should be regarded as $|\phi|$.

where the prime indicates derivative with respect to s and μ^2 is given by

$$\mu^2 \equiv \frac{2(6p - 3pn + n)}{(n-2)^2}. \tag{5.2.8}$$

The equation (5.2.7) can be regarded as a motion of the rescaled scalar field ψ in the effective potential

$$V_{\rm eff} = \frac{1}{2}\mu^2\psi^2 + \frac{\lambda p^2}{nH_i^2}, \tag{5.2.9}$$

with a friction term:

$$\left(3p - \frac{n+2}{n-2}\right)\psi'. \tag{5.2.10}$$

The effective potential has a minimum at $\psi_{\rm min}$ where

$$\psi_{\rm min} = \begin{cases} 0 & \text{if } \mu^2 > 0 \\ (-\mu^2 H_i^2/\lambda p^2)^{1/(n-2)} & \text{if } \mu^2 < 0 \end{cases}. \tag{5.2.11}$$

On the other hand, there exists a critical n which gives no friction term:

$$n_c \equiv \frac{6p+2}{3p-1} \begin{cases} 4 & p = 1 \\ 6 & p = 2/3 \\ 10 & p = 1/2 \end{cases}, \tag{5.2.12}$$

$$\rightarrow p = \frac{n_c + 2}{3(n_c - 2)}. \tag{5.2.13}$$

If $n > n_c$ holds, the sign of the friction term is positive and $\mu^2 < 0$ is ensured [3]. As a result, the oscillation of ψ damps and it goes to the minimum the potential $\psi_{\rm min}$. This behaviour corresponds to the scaling solution in which the original field ϕ scales as

$$\phi(t) \propto a^{-\frac{2}{p(n-2)}} \propto t^{-\frac{2}{n-2}}. \tag{5.2.14}$$

Note that this asymptotic solution is independent from the initial condition. In such a case, the every terms in the original equation of motion Eq. (5.2.3) are comparable.

On the other hand, if $n < n_c$ holds, the friction term has a negative sign. Thus, ψ never relaxes to its potential minimum. This corresponds to the oscillating solution. This behaviour is easily understood as follows. Assuming an oscillating solution for ϕ and using the Virial theorem, we can obtain the scaling of the amplitude of ϕ : Φ

$$\Phi(t) \propto a^{-\frac{6}{n+2}} \propto t^{-\frac{6p}{n+2}}. \tag{5.2.15}$$

[3]If there exists a so called Hubble mass term, $\mu^2 < 0$ is not always satisfied.

The effective mass of ϕ scales as $m_{\rm eff} \propto \Phi^{(n-2)/2} \propto t^{-3p(n-2)/(n+2)}$. Compared to the Hubble parameter $H \sim t^{-1}$, the effective mass becomes larger and larger as time goes by. In other words, after the effective mass becomes comparable to the Hubble parameter, the potential term dominates over the Hubble friction term in the original equation of motion Eq. (5.2.3).

In the case of $n = n_c$, the scalar motion becomes peculiar one as is shown below.

Pseudo scaling solution: $n = n_c$

We have shown that a scalar field goes to a scaling solution for $n > n_c$ and oscillation solution for $n < n_c$ independently from the initial condition. However in the case of $n = n_c$, the scalar motion becomes more non trivial. If $n = n_c$, the equation of motion for ψ Eq. (5.2.7) becomes

$$\psi'' - \frac{4}{(n-2)^2}\psi + \frac{(n+2)^2}{9(n-2)^2}\frac{\lambda}{H_i^2}\psi^{n-1} = 0. \tag{5.2.16}$$

The motion of ψ is just an oscillating one without damping. Thus, resulting dynamics largely depends on the initial condition of ϕ. If the initial condition is chosen so that ψ oscillates around $\psi_{\rm min}$ and does not cross the origin $\psi = 0$, the original field ϕ does not also cross the origin $\phi = 0$. The resulting solution of the original field ϕ is not similar to the scaling solution nor the oscillating solution. We call this solution a "pseudo scaling solution".

Before considering the equation of motion for ψ, let us specify the initial condition. One cosmologicaly motivated initial time t_i is just after the inflation. In order for ϕ not to move down the potential during inflation,

$$m_{\rm eff}^2(\phi(t_i)) \equiv \lambda\phi(t_i)^{n-2} \lesssim H_i^2, \tag{5.2.17}$$

must hold. Then, the initial condition can be written as

$$\phi(t_i) = \phi_i, \tag{5.2.18}$$

$$\dot{\phi}(t_i) \simeq -\frac{\partial_\phi V(\phi_i)}{3H_i}, \tag{5.2.19}$$

where we assume $\phi_i > 0$ without loss of generality. The condition (5.2.17) indicates $\psi_i \lesssim \psi_{\rm min}$. In particular, we assume the case $\psi_i \ll \psi_{\rm min}$ in which the Hubble parameter is much larger than the effective mass at the initial time.

The initial velocity of ψ can be given by

$$\psi' = t_i\left(\dot{\phi}_i + \frac{6}{n+2}H_i\phi_i\right). \tag{5.2.20}$$

The conserved energy density of ψ can be written as

$$\rho_\psi = \frac{1}{2}\psi_i'^2 + V_{\rm eff}(\psi_i) \simeq -\frac{n+2}{9n(n-2)}\frac{\lambda\phi_i^n}{H_i^2}, \tag{5.2.21}$$

where we have used the approximation $\psi_i \ll \psi_{\rm min}$ or $|\dot{\phi}_i| \ll |H_i\phi_i|$. Note that the energy density ρ_ψ is negative. This indicates that ψ never crosses the origin. As a result, the original field ϕ does not oscillate around the origin $\phi = 0$. The rescaled field just oscillates around the potential minimum $\psi_{\rm min}$ [4] .

In terms of ψ, the dynamics seems to be simple. However, the "pseudo scaling" solution looks like non trivial in terms of ϕ. Fig. 5.1 shows typical behaviours of the pseudo scaling solution. We have numerically solved the equation of motion for ϕ with an initial condition $H_i = 1$, $\phi_i = 0.2\psi_{\rm min}$, $\dot{\phi}_i = -V'(\phi_i)/3H_i$ and we have taken $\lambda = 1$ in Planck unit. The left(right) panel is $p = 1/2(2/3)$, $n = 6(10)$ case which corresponds to the MD (RD) background. The red solid line represents a numerical solution, while the green dashed line is a scaling solution ($\phi \propto t^{-2/(n-2)}$). We can see that a "psuedo scaling" solution is actually different from the scaling solution and shows a kind of periodic behaviour. This behaviour can be understood by considering the equation of motion Eq. (5.2.7). Below, let us understand main features of the pseudo scaling solution and how the periodicity depends on the initial condition of ϕ.

First, note that the dynamics of ψ is dominated by the region $\psi \ll \psi_{\rm min}$ because $|\rho_\psi| \ll |V(\psi_{\rm min})|$ holds in our case of interest [5]. If we consider the dynamics in the region $\psi \ll \psi_{\rm min}$, we can neglect the second term in Eq. (5.2.16). In the $\psi \ll \psi_{\rm min}$, The equation of motion is approximated as

$$\psi'' - \frac{4}{(n-2)^2}\psi = 0. \tag{5.2.22}$$

This equation of motion has a general solution which is a sum of a growing and decaying mode:

$$\psi(s) = A\exp\left(\frac{2s}{n-2}\right) + B\exp\left(\frac{-2s}{n-2}\right), \tag{5.2.23}$$

where the coefficients A and B are determined by initial conditions. In terms of the original field ϕ, this solution reads

$$\phi(t) = A + B\left(\frac{t_i}{t}\right)^{4/(n-2)}. \tag{5.2.24}$$

[4] This behaviour depends on the assumption of the initial condition $3H_i\dot{\phi} \simeq -V'$. However, as long as $\dot{\phi}_i \sim \mathcal{O}(V'/H)$, the dynamics is almost the same.

[5] This statement holds as long as $\dot{\phi}_i \sim \mathcal{O}(V'/H_i)$.

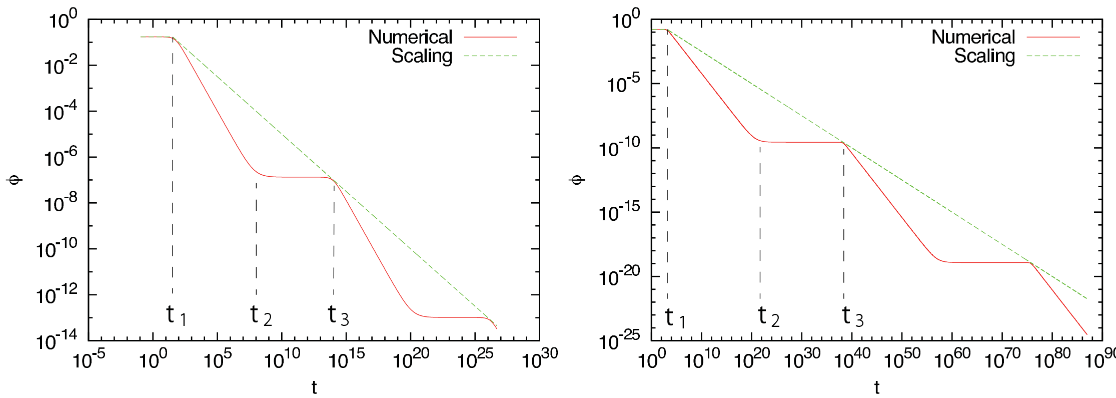

Figure 5.1: The time evolution of the original field $\phi(t)$ for $n = n_c$ with initial condition $H_i = 1$, $\phi_i = 0.2\psi_{\rm min}$, $\dot{\phi}_i = -V'(\phi_i)/3H_i$ and we have taken $\lambda = 1$ in Planck unit. The left panel is a case for for $p = 2/3\,(n = 6)$ and the right panel is for $p = 1/2\,(n = 10)$. The red solid line shows a numerical solution, which corresponds to a "pseudo scaling" solution, while the green dashed line represents a hypothetical scaling solution for representative purpose. From [3].

Let us divide the time into three phase: the first phase $t_i < t < t_1$, the second phase $t_1 < t < t_2$ and the third phase $t_2 < t < t_3$. We define t_1 as the time when the motion of ϕ starts. t_2 is defined as the time when the motion of ϕ stops. After $t = t_2$, ϕ starts to move again at t_3. Fig. 5.1 shows t_1, t_2 and t_3 of example cases. The second term of Eq. (5.2.16) affects the dynamics only for some short spans around t_1, t_2 and t_3 as one can see below.

First phase
In the time span $t_i < t_1$, the coefficients A and B are determined by the initial condition of $\phi(\psi)$ as

$$A = \frac{1}{2}\left(\psi_i + \frac{n-2}{2}\psi_i'\right), \tag{5.2.25}$$

$$B = \frac{1}{2}\left(\psi_i - \frac{n-2}{2}\psi_i'\right). \tag{5.2.26}$$

This solution breaks down when ψ becomes close to $\psi_{\rm min}$ at $t \simeq t_1 (s = s_1)$. After that, ψ passes though $\psi_{\rm min}$ and goes back to $\psi_{\rm min}$ again within an order one unit time in s. The time $s_1(t_1)$ can be estimated as

$$s_1 \simeq \frac{n-2}{2}\ln\left(\frac{\psi_{\rm min}}{\psi_i}\right), \tag{5.2.27}$$

$$\rightarrow t_1 \simeq t_i\left(\frac{\psi_{\rm min}}{\psi_i}\right)^{(n-2)/2} \simeq \left(\frac{1}{\lambda\psi_i^{n-2}}\right)^{1/2} = \frac{1}{m_{\rm eff}(\phi_i)}. \tag{5.2.28}$$

Note that the time t_1 is the epoch when the Hubble parameter becomes comparable to the effective mass of ϕ. This indicates that ϕ starts to move when the Hubble parameter becomes equal to the effective mass.

Second phase

Here, we consider the time span $t_1 < t < t_2$. As is mentioned, after t_1, ψ passes though $\psi_{\rm min}$ and goes back to $\psi_{\rm min}$ again within a few time in terms of s. After that, ψ can be approximated by the form of Eq. (5.2.23) again. What we have to know is the coefficients A and B in this situation. We define the position ψ_j somewhere between ψ_i and $\psi_{\rm min}$ like $\psi_j \sim 0.1\psi_{\rm min}$. Suppose that ψ passes through ψ_j at s_j after t_1 with a velocity ψ'_j. In the time span $s_1 < s < s_j$, the potential term $\propto \psi^n$ affects the dynamics and $s_j - s_1 \sim \mathcal{O}(1)$ holds. Since the dynamics is dominated by the region $\psi \ll \psi_{\rm min}$ as is noted before, we do not discuss the dynamics during $s_1 < s < s_j$.

By using the fact that the total energy ρ_ψ is conserved, we can estimate ψ'_j as

$$\psi'_j = -\left(2\rho_\psi + \frac{4}{(n-2)^2}\psi_j^2\right)^{1/2} \simeq \frac{-2}{n-2}\psi_j(1-\epsilon), \tag{5.2.29}$$

$$\epsilon \equiv \frac{n^2-4}{36n}\frac{\lambda\psi_i^n}{H_i^2\psi_j^2} = \frac{n-2}{n(n+2)}\frac{\psi_i^n}{\psi_j^2\psi_{\rm min}^{n-2}} \ll 1. \tag{5.2.30}$$

Then, we can estimate the coefficients A and B of Eq. (5.2.23) after s_j as

$$A_j \simeq \frac{1}{2}\psi_j\epsilon, \;\; B_j \simeq \psi_j \tag{5.2.31}$$

Note that $A_j \ll B_j$ holds. The end of the second period $t = t_2$ $(s = s_2)$ is the time when the decaying mode $\propto B_j$ becomes equal to the growing mode $\propto A_j$. The time $(t_2)s_2$ is estimated as

$$s_2 - s_j \simeq \frac{n-2}{4}\ln\frac{2}{\epsilon}, \tag{5.2.32}$$

$$\rightarrow t_2 \simeq c\, t_1\left(\frac{\psi_{\rm min}}{\psi_i}\right)^{n(n-2)/4}, \tag{5.2.33}$$

where c represents uncertainty in $s_j - s_1$ and is an $\mathcal{O}(1-10)$ constant . We have numerically checked that the ψ_i dependence on t_2 is given by Eq. (5.2.33) if $m_{\rm eff}(\phi_i) \ll H_i$ is satisfied. Note that t_2 is much longer than the inverse mass scale of the scalar field $1/m_{\rm eff}(\phi_i) = t_1$. From Eq. (5.2.24), we can see that the original field scales as $\propto t^{-4/(n-2)}$ during this time span. $\phi(t)$ is given by

$$\phi(t) \simeq \phi_i \left(\frac{t_1}{t}\right)^{4/(n-2)} \quad \text{for } t_1 < t < t_2, \tag{5.2.34}$$

and at t_2, it becomes

$$\phi(t_2) \sim \phi_i \left(\frac{\psi_i}{\psi_{\rm min}}\right)^n. \tag{5.2.35}$$

Note that in this time span, ϕ satisfies $\ddot{\phi} + 3H\dot{\phi} \simeq 0$. In other words, the potential term becomes irrelevant soon after t_1. In fact, the effective mass $m_{\rm eff}(\phi)$ becomes smaller and smaller compared to the Hubble parameter in this time span. During this time span, the energy density of ϕ is dominated by the kinetic term and scales as $\rho_\phi \propto a^{-6}$.

Third phase Finally, let us consider the third phase $t_2 < t < t_3$. This phase is just a continuation of the second phase which ends when the growing mode dominates over the decaying one. Thus, in the third phase, ψ can be given by

$$\psi(s) \simeq \frac{1}{2}\psi_j \epsilon \exp\left(\frac{2}{n-2}(s - s_j)\right). \tag{5.2.36}$$

This behaviour is valid until ψ becomes close to $\psi_{\rm min}$. Thus, $s_3(t_3)$ can be estimated as

$$s_3 - s_2 \simeq \frac{n-2}{4} \ln \frac{2}{\epsilon}, \tag{5.2.37}$$

$$\rightarrow \quad t_3 \simeq c\, t_2 \left(\frac{\varphi_{\rm min}}{\varphi_i}\right)^{n(n-2)/4} \simeq \frac{1}{m_{\rm eff}(\phi(t_2))}. \tag{5.2.38}$$

It can be seen that the ratio of the time t_3/t_2 is equal to t_2/t_1. During this time span, $\phi(t)$ remains almost constant: $\phi(t) \simeq \phi(t_2)$.

After $t = t_3$, we have exactly the same dynamics: the sequence of the second and the third phase. This is because ψ does not have a damping term. One period of "pseudo oscillation" consists of the second and third phase. In one period of pseudo oscillation, the original field ϕ reduces its field value by an amount of $\epsilon \simeq (\psi_i/\psi_{\rm min})^n$. The scaling of ϕ consists of two regime: $\phi = $ const. and $\phi \propto t^{-4/(n-2)}$. If we average the pseudo oscillation, the scaling solution is obtained. However, the period of the pseudo oscillation is much longer than the Hubble time scale and one can not approximate a pseudo solution as a scaling solution in many cases of interest.

Finally, we show qualitative differences between the scaling solution and the pseudo scaling solution. In the case of the scaling solution, the all terms in the equation of motion (the kinetic term, the Hubble friction term and the potential term) are comparable. If the potential is not monomial, the motion of ϕ deviates from the scaling solution as soon as the dominant term of the potential changes. On the other hand, in the case of the pseudo scaling solution, the kinetic and the Hubble friction term dominate over the potential term in most time. Thus, even after the dominant term in the potential changes the motion still follows the pseudo scaling solution until the potential term becomes comparable to the kinetic or the Hubble friction term.

The case with Hubble mass

Here we give brief comments on the case where the Hubble mass term exists in the potential:

$$V = \frac{c_h}{2}H^2\phi^2 + \frac{\lambda}{n}\phi^n, \tag{5.2.39}$$

with c_h being a constant. The equation of motion for ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + c_h H^2\phi + \lambda\phi^{n-1} = 0. \tag{5.2.40}$$

In this case, the equation of motion for the rescaled field ψ is the same with Eq. (5.2.7) except the constant parameter μ^2. Now, μ^2 is given by

$$\mu^2 \equiv \frac{2(6p - 3pn + n)}{(n-2)^2} + c_h p^2. \tag{5.2.41}$$

Note that in the case of $n = n_c$, the equation of motion becomes

$$\psi'' + \left(\frac{-36 + c_h(n+2)^2}{9(n-2)^2}\right)\psi + \frac{(n+2)^2}{9(n-2)^2}\frac{\lambda}{H_i^2}\psi^{n-1} = 0. \tag{5.2.42}$$

If $|c_h| \ll 1$, the Hubble mass term does not qualitatively affect the dynamics of pseudo scaling solution. On the other hand If $|c_h| \gtrsim 1$, the initial position of ϕ cannot be taken freely because the position of ϕ is fixed during the inflation. In particular, if $c_h \gtrsim 1$, we have $\phi_i = 0$ and no dynamics is expected.

Let us consider the negative Hubble mass case $c_h \lesssim -1$. In such a case, the initial position of ϕ is given by the minimum of the potential:

$$\phi_H = \left(\frac{-c_h H^2}{\lambda}\right)^{1/(n-2)}. \tag{5.2.43}$$

After the inflation, ψ oscillates around $\psi_{\rm min}$. The initial condition $\phi_i \sim \phi_H$ leads a consequence that ψ does not pass around the origin. The oscillation time scale of ψ becomes an order one in terms of $s = \ln(t/t_i)$. Thus, the original field ϕ follows the temporal minimum of the potential $\phi_H(t) \propto t^{-2/(n-2)}$ with oscillation around it . The time scale of this oscillation is close to the Hubble time scale. One can see that the case $c_h \lesssim -1$ and $|c_h| \ll 1$ are qualitatively different.

Let us consider the dynamics more closely. The initial amplitude of oscillation $\triangle\phi_H$ around the temporal minimum of the potential ϕ_H depends on the origin of the Hubble mass term. One example of the Hubble mass term comes from the Planck suppressed interactions

$$-\mathcal{L} \supset \frac{\phi^2}{6M_P^2}\left(\frac{c_k}{2}\dot{I}^2 + c_v V_{\rm inf}(I)\right) \tag{5.2.44}$$

where $M_{\rm PL}$ denotes the reduced Planck mass and c_k, c_v are constants of order unity. I and $V_{\rm inf}$ indicate the inflaton and the inflaton potential respectively. During the inflation, we have $c_h = c_v$. After inflation, the inflaton starts to oscillate and both the kinetic term and the potential term have comparable energies. If the inflaton oscillates harmonically, $c_h = (c_k + c_v)/2$ holds. The shift of the coefficient c_h from the inflation ere to the inflaton oscillating era occurs in time scale of the inflaton mass. Thus, in general ϕ cannot track the temporal minimum just after the inflation [139]. The oscillation amplitude $\triangle\phi_H$ around the temporal minimum of the potential ϕ is expected to be $\sim \mathcal{O}(\phi_H)$ [6]. Another example of the Hubble mass term comes from the coupling to the Ricci scalar R

$$-\mathcal{L} = \frac{c_R}{24}\phi^2 R. \tag{5.2.45}$$

During inflation we have $c_h = c_R$, and during the inflaton oscillating era we have $c_h = c_R/4$ in average [7] if the inflaton harmonically oscillates. In this case, the oscillation amplitude $\triangle\phi_H$ is also expected to be $\sim \mathcal{O}(\phi_H)$.

The amplitude $\triangle\phi_H$ decreases as the universe expands:

$$H(\triangle\phi)^2 \propto a^{-3}, \tag{5.2.46}$$

$$\to \triangle\phi \propto t^{(1-3p)/2}, \tag{5.2.47}$$

where we use the number density (frequency $\times$ amplitude^2) conservation. From this, we can obtain the following scaling for the ratio of the oscillation amplitude $\triangle\phi$ and the temporal minimum:

$$\frac{\triangle\phi}{\phi_H} \propto t^{2(n_c-n)/(n-2)(n_c-2)}. \tag{5.2.48}$$

[6] Even if $c_k = c_v$, ϕ cannot trace the temporal minimum [139].

[7] R oscillates between positive and negative values.

For $n_c > n$, the oscillation amplitude decreases compared to the temporal mimimum. In such a case, ϕ safely relaxes to the temporal minimum of the potential. On the other hand, for $n_c < n$, the amplitude increases compared to the temporal minimum. In such a case, the oscillation of ϕ does not vanish. In the case of $n = n_c$, ψ oscillates around the temporal minimum and never passes through the origin if $\Delta\phi \lesssim \phi_H$ initially.This fact is important for the PQ symmetry restoration [138] because if the PQ field does not oscillate around the origin, the effects of non thermal trapping becomes negligible.

5.2.2 Implications of Pseudo Scaling Solution on the PQ Cosmology

So far, we have seen general properties of the pseudo scaling solution. Here, we apply the pseudo scaling solution to the PQ cosmology. There are two advantages of the pseudo scaling solution to the PQ cosmology.

First advantage of the pseudo scaling solution is that even if the PQ field has an initial value ϕ_i much larger than the PQ scale F_a, the PQ field does not oscillate around the origin. As a result, the trapping of the PQ field at the origin can be avoided. If the PQ field has a large initial value $\phi_i \gg F_a$. the cold dark matter isocurvature S_{CDM} becomes suppressed by the ratio F_a/ϕ_i as is mentioned before. Then, the constraint on the inflation scale becomes relaxed. This advantage is also provided by the case of the scaling solution and the case of the negative Hubble mass. However, the second advantage below is unique to the pseudo scaling solution.

Second advantage is that even if the potential have lower power terms $\phi^n (n < n_c)$, the PQ field follows the pseudo scaling solution for a long time compared to the cases where the potential term is always effective in the equation of motion. In the case of the scaling solution and the case of the negative Hubble mass, the potential term is comparable to the kinetic or the Hubble friction term in the equation of motion. As a result, when the dominant term of the potential changes to lower power of n, the PQ field starts to oscillate. The oscillation of the PQ field causes the non thermal trapping of the PQ field at the origin which results in a cosmological disaster if the domain wall number is greater than one.

Below, we summarise the second advantage. Then, we consider the constraint on the inflation scale in a simple setup.

The delay of the oscillation in the pseudo scaling solution

Here we show that the pseudo scaling solution can be more powerful in solving the domain wall problem than the case of the scaling solution and the negative Hubble mass. The advantage of the pseudo scaling solution is that even if the potential has additional terms, the PQ field tends not to oscillate once the PQ field follows the pseudo scaling solution. The oscillation of the PQ field may cause the trapping of the PQ field at the origin which is disfavoured if $N_{\rm DW} > 1$. To see this advantage, let us consider the following potential as an example

$$V = \frac{\lambda_l}{l}\phi^l + \frac{\lambda}{n}\phi^n, \tag{5.2.49}$$

where we assume $n,\ n_c > l$ and $l > 2$. The dominant term of the potential changes at $\phi_c \equiv (\lambda_l/\lambda)^{1/(n-l)}$. In the region $\phi \gg \phi_c$, the potential term ϕ^n dominates over the ϕ^l term. On the other hand, in the region $\phi \ll \phi_c$, the opposite holds. First, let us consider the scaling solution case $n > n_c$. If the initial value of ϕ is larger than ϕ_c, the field ϕ obeys the scaling solution at first. When ϕ crosses ϕ_c, the dominant term of the potential becomes ϕ^l. Then, ϕ start to oscillate around the origin with an amplitude $\sim \phi_c$. This is also the case with the negative Hubble mass. If $\phi_c \gg F_a$ holds, the non trapping trapping will occur (see sec. 3.2).

On the other hand, in the case of the pseudo scaling solution $n = n_c$, the dynamics becomes dominated by the ϕ^l term when the Hubble parameter becomes H_l:

$$\frac{H_l}{\sqrt{\lambda}} \equiv \left(\frac{\lambda_l}{\lambda}\right)^{\frac{n-2}{2(n-l)}} = \phi_c^{\frac{n-2}{2}}. \tag{5.2.50}$$

We can derive this as follows. In terms of the rescaled field ψ, the effective potential can be written as

$$V_{\rm eff}(\varphi) = \frac{\mu^2}{2}\psi^2 + \frac{\lambda_l p^2}{lH_i^2}\left(\frac{H_i}{H}\right)^{\frac{2(n-l)}{n-2}}\psi^l + \frac{\lambda p^2}{nH_i^2}\psi^n. \tag{5.2.51}$$

The amplitude to ψ is given by $\simeq \psi_{\rm min}$. The potential term ψ^l becomes effective once the condition

$$\frac{\lambda_l}{H_i^2}\left(\frac{H_i}{H}\right)^{\frac{2(n-l)}{n-2}}\psi_{\rm min}^2 \sim \mathcal{O}(1), \tag{5.2.52}$$

holds. This condition is equivalent to Eq. (5.2.50). Suppose, for example, the ϕ^l term starts to dominate the dynamics in the second phase $t_1 < t < t_2$, the field value of the PQ field at $H = H_l$ is estimated as

$$\phi_C \equiv \phi\Big|_{H=H_l} \simeq \frac{\phi_c}{\phi_i}\phi_c. \tag{5.2.53}$$

After $H = H_l$, the PQ field starts to oscillate with an amplitude ϕ_C. Note that the amplitude ϕ_C is suppressed by the factor ϕ_l/ϕ_i compared to the scaling solution case ϕ_l. If $\phi_C < F_a$, the PQ field will oscillate around F_a not the origin. Thus, in this respect, the symmetry restoration is less likely to occur in the pseudo scaling case compared to the scaling solution and the negative Hubble mass case.

Constraints on the inflation scale

We have seen that if the PQ field obeys the pseudo scaling solution, the PQ symmetry restoration is likely to be avoided even if the initial field value ϕ_i is much larger than the PQ scale F_a. Here, we derive constraints on the inflation scale in the case of the pseudo scaling solution. To be concrete, let us consider the case where the inflation oscillates harmonically during the inflaton dominated ere (p=2/3). Then the critical value of the potential power becomes $n_c = 6$. We consider the following potential

$$V \supset \frac{g^4}{3M_*^2}|\phi|^6, \tag{5.2.54}$$

where g denotes a coupling constant and M_* is a cut off scale. During inflation, the effective mass of ϕ must not be larger than the Hubble parameter otherwise, the PQ field moves during the inflation. This condition is given by

$$\phi_i < \frac{\sqrt{M_* H_{\rm inf}}}{g}. \tag{5.2.55}$$

We parameterise the initial field value of ϕ as

$$\phi_i \equiv \zeta \frac{\sqrt{M_* H_{\rm inf}}}{g}, \tag{5.2.56}$$

with ζ being a parameter not greater than one. In this case, the isocurvature perturbation of the cold dark matter is suppressed by a factor ϕ_i/F_a compared to Eq. (5.1.2) and given by

$$S_{\rm CDM} \simeq 10^{-5}\theta_i \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{M_*}{10^{19}{\rm GeV}}\right)^{-1/2} \left(\frac{H_{\rm inf}}{10^{9}{\rm GeV}}\right)^{1/2} \times \frac{g}{\zeta} \lesssim 0.9 \times 10^{-5}. \tag{5.2.57}$$

where the right hand side denotes the observational constraint [16]. If g/ζ or F_a is small, the constraint on the inflation scale becomes relaxed compared the ordinary case Eq. (5.1.2) though this relaxation is also expected in the case of the scaling solution.

Next, let us consider the constraint on the additional potential terms. To be concrete, let us consider the following potential

$$V = g^4 \left[\kappa^2 \left(|\phi|^2 - v^2\right)^2 + \frac{|\phi|^6}{3M_*^2}\right], \tag{5.2.58}$$

where κ is a constant parameter and v denotes a mass scale around F_a. The dominant term in the potential changes at $\phi_c \sim \kappa M_*$. In the case of the scaling solution or the negative Hubble mass, the PQ field oscillates around the origin after $\phi = \phi_c$ and the non thermal restoration of the PQ symmetry will occur provided $\phi_c \gg F_a$,. On the other hand, in the case of the pseudo scaling solution, if the condition

$$\phi_C \equiv \frac{\phi_c}{\phi_i}\phi_c = \frac{\kappa^2 M_*^2}{\phi_i} \lesssim F_a \tag{5.2.59}$$

is satisfied, the PQ field will not oscillate around the origin and the symmetry restoration can be avoided [8] . We can see that the the symmetry restoration is less likely to occur compared to the case of the scaling solution or the negative Hubble mass ($\kappa M_* < F_a$).

5.2.3 Conclusion of the pseudo scaling solution

In this section, we have seen the properties of the pseudo scaling solution and how this solution can relax the constraint on the inflation scale in PQ cosmology.

If the PQ field has a large initial value compared to the PQ scale F_a, the constraint on the inflation scale is relaxed. However, in such a case, the PQ field tends to oscillate around the origin with a large initial amplitude. Then, the PQ symmetry will be non thermally restored. If the PQ field obeys the pseudo scaling solution, this is not the case. If a scalar field obeys the pseudo scaling solution, the amplitude of a scalar field decreases without oscillating around the origin of the potential. Even when the dominant term of the potential changes, the scalar field continues to obey the pseudo scaling solution. This feature is pleasant for the PQ field motion because the PQ field can have a large initial value without the non thermal trapping.

5.3 Multi fields solution

In this section, we consider a variant type of the scenario that suppresses the quantum fluctuation in the axion direction. We introduce multi PQ fields that play different roles. We assume that a PQ field has a large field value compared to the PQ scale during the inflation. The PQ symmetry is broken at present by another PQ field whose phase direction corresponds to the axion field. As a result, the quantum fluctuation in the axion direction can be suppressed as we see below. Since the PQ symmetry is likely restored by the dynamics of the PQ field (see Sec. 3.4). we also discuss whether the PQ symmetry restoration can be avoided.

We apply this mechanism to the DFSZ axion model in which there are two Higgs doublets with PQ changes. In particular, we consider the Higgs inflation scenario [25] in which the Higgs boson plays the roll of the inflaton. The DFSZ model has a domain wall

[8] Here, we assume that the PQ field goes to the region $\phi < F_a$ in time interval $t_1 < t < t_2$ for simplicity.

number greater than one. In such a case, the PQ symmetry must not be restored after the inflation otherwise axion domain walls lead a cosmological disaster (see Sec. 3.2). If the PQ symmetry is not restored, the inflation scale is bounded from above, which usually excludes high scale inflation models including the Higgs inflation. In our scenario, the PQ symmetry is mainly broken by the Higgs fields during the inflation. After the inflation, the PQ field which dominantly breaks the PQ symmetry becomes different from the Higgs fields. As a result, the isocurvatur perturbation of the axion cold dark matter is suppressed.

We first explain the mechanism by using a simple toy model in Sec. 5.3.1. Then, we consider the Higgs inflation scenario in Sec. 5.3.2.

5.3.1 Toy model analysis

Here, we explain our idea using a toy model. Let us consider the model with two PQ fields ϕ and S that are gauge singlets and have opposite PQ charges. Then we assume the following Lagrangian

$$\mathcal{L} = |\partial_\mu \phi|^2 + |\partial_\mu S|^2 - V - \lambda(\phi^2 S^2 + \text{h.c.}), \tag{5.3.1}$$

where the coupling λ can be taken real and positive without loss of generality, and

$$V = \lambda_\phi \left(|\phi|^2 - \frac{v_a^2}{2} \right)^2 + m_S^2 |S|^2 + \lambda_S |S|^4 + \lambda_{\phi S} |\phi|^2 |S|^2. \tag{5.3.2}$$

Here, we assume coupling constants λ_ϕ, λ_S and m_S^2 are real and positive. We also assume $\lambda_{\phi S} < 2\lambda$ for simplicity. The parameter v_a is assumed to be of order of the PQ scale. The stability of the potential at large $|S|$ and $|\phi|$ is ensured by the condition

$$\lambda_\phi \lambda_S > (\lambda - \lambda_{\phi S}/2)^2. \tag{5.3.3}$$

A term promotional to ϕS can be avoided by a Z_2 symmetry under which only S transforms as $S \to -S$. The minimum of the potential exists at $S = 0$ and $|\phi| = v_a/\sqrt{2}$. The PQ scale F_a and v_a are related as $F_a = v_a/N_{\rm DW}$ with $N_{\rm DW}$ being the domain wall number. At the minimum of the potential, the massless mode, axion, is exactly the phase direction of ϕ:

$$\phi = \frac{v_a}{\sqrt{2}} \exp\left(i \frac{a}{v_a} \right), \tag{5.3.4}$$

where a denotes the axion.

However, the role of ϕ and S are inverted is S has a large field value compared to that of ϕ. In fact S can have a large field value during the inflation. For examle, the value of S can be as large as $|S| \sim H_{\rm inf}/\sqrt{\lambda_S}$ during the inflation. S can be the inflaton by, for example, adding non-minimal couplings to gravity [25]. Let us parameterise the phases of ϕ and S as

$$\phi = \frac{v_\phi}{\sqrt{2}} \exp\left(i\frac{a_\phi}{v_\phi}\right), \tag{5.3.5}$$

$$S = \frac{v_S}{\sqrt{2}} \exp\left(i\frac{a_S}{v_S}\right), \tag{5.3.6}$$

where $v_\bullet$ is assumed to be real and $a_\bullet$ parameterise the phase of the corresponding field. At the vacuum where the potential takes the minimum value, $v_\phi = v_a$ and $v_S = 0$ hold. However, v_ϕ and v_S may not be the same with the vacuum values in early epoch of the universe. We assume that initially S has a non zero field value v_S which is much larger than v_ϕ. In such a case v_ϕ is given by

$$v_\phi \simeq \max\left[v_a, \sqrt{\frac{2\lambda - \lambda_{\phi S}}{2\lambda_\phi}} v_S\right]. \tag{5.3.7}$$

The massless mode, which we denote by a consists of the combination of a_ϕ and a_S and is given by

$$a \simeq a_S - \frac{v_\phi}{v_S} a_\phi, \tag{5.3.8}$$

which is dominated by the phase of S. The field "a" acquires quantum fluctuations $\sim H_{\rm inf}/2\pi$ during inflation. On the other hand the orthogonal component, which is dominated by a_ϕ, has a mass of

$$m_{a_\phi}^2 \simeq \lambda v_S^2. \tag{5.3.9}$$

Therefore, if m_{a_ϕ} is larger than the Hubble scale during inflation, the massive mode mainly consisting of a_ϕ does not develop quantum fluctuations during the inflation. This condition is given by

$$\lambda \gtrsim \left(\frac{H_{\rm inf}}{v_S}\right)^2. \tag{5.3.10}$$

a_ϕ acquires quantum fluctuations through the small mixing with a which is given by

$$\frac{\delta a_\phi}{a_\phi}\Big|_{\rm inflation} \simeq \frac{H_{\rm inf}}{2\pi}\frac{v_\phi}{v_S}. \tag{5.3.11}$$

The axion isocavature fluctuation after v_ϕ and v_S settle down at the vacuum value can be estimated as

$$\left(\frac{\delta a_\phi}{a_\phi}\right) \simeq \frac{H_{\rm inf}}{2\pi v_S \theta_i}. \tag{5.3.12}$$

Compared to the standard scenario in Eq. 5.1.1, the isocurvature perturbation is suppressed by the factor $\sim F_a/v_S \ll 1$. Since we do not introduce explicit PQ symmetry breaking terms, there is always a massless mode if the PQ symmetry is spontaneously broken. However, the massless mode during the inflation does not have to be the same as that in the present universe. In our scenario, the axion at present was massive during the inflation, while the massive mode at present was massless during the inflation. Then, the cold dark matter isocurvature perturbation is given by

$$S_{\rm CDM} \simeq 1 \times 10^{-5} \theta_i \left(\frac{F_a}{10^{12}\,{\rm GeV}}\right)^{0.19} \left(\frac{H_{\rm inf}}{10^{7}\,{\rm GeV}}\right) \left(\frac{F_a}{v_S}\right) \lesssim 0.9 \times 10^{-5}. \tag{5.3.13}$$

where the right hand side indicates the observational constraint [16]. Thus, the inflation scale as large as $H_{\rm inf} \sim 10^{13}$ GeV can be allowed with $v_S \sim 10^{18}$ GeV even if the axion is the dominant component of the cold dark matter.

In this scenario, S has a large initial field value and starts to oscillate after the inflation, which must decay into standard model particles. The oscillation of S potentially cause resonant particle production of axions [108] or may induce the dynamical motion of ϕ. Thus, we have to check whether this scenario can be consistent with the present universe or not taking the dynamics of S and ϕ after inflation into account. We denote the frequency of S after the onset of the oscillation as $m_S^{\rm osc}$.

First, we consider the axion production due to the oscillation of S. If the condition

$$\frac{\sqrt{\lambda} v_S}{m_S^{\rm osc}} < 1, \tag{5.3.14}$$

is satisfied, the axion production of the broad resonance type is avoided even just after the onset of the oscillation of S [9]. Even if this condition is satisfied, the narrow resonance would harmfully produce axion particles. However, if the decay rate of S is sizeable, the narrow resonance is ineffective [140].

Second, let us consider the motion of ϕ induced by the oscillation of S. The potential of ϕ varies as S oscillates and ϕ would also oscillate around the origin. In such a case the non thermal trapping of ϕ may (see Sec. 3.4). If both S and ϕ are trapped at the symmetry enhanced point, the PQ symmetry is restored and our scenario does not work. The motion of ϕ becomes most violent if S decays just after the onset of the oscillation. After the decay of S, ϕ will start to oscillate around the origin. This oscillation may cause the non-thermal trapping. As is discussed in Sec. 3.4, the criteria for ϕ not to be trapped at the origin due to the non thermal trapping is that the number of ϕ oscillation

[9] This condition is derived from the adiabaticity of the a_ϕ. The particles of a_S may be resonantly produced and S may be trapped at the origin. However in such a case, the axion field is given by a_ϕ and trapping of S becomes irrelevant for the axion cosmology.

within one Hubble time does not exceed $N_{\rm cr} \sim \mathcal{O}(10^2)$ until the oscillation amplitude of ϕ reduces to $\sim F_a$. This leads to a constraint

$$\left(\frac{H_{\rm inf}}{\sqrt{\lambda_\phi} f_a}\right)^n \lesssim \mathcal{O}(10^2), \tag{5.3.15}$$

where $n = 1/2(1)$ for the matter (radiation) dominated back ground evolution and we assume the condition Eq. (5.3.10) is marginally satisfied: $\sqrt{\lambda} v_S \sim H_{\rm inf}$.

Finally, let us consider the thermal effects on ϕ. Here, we give a conservative criteria for the PQ symmetry not to be restored by the thermal effects: the minimum of the potential for ϕ does not exist at the origin after S decays assuming that S is in the thermal bath due to the interactions with the standard model sector [10] . In such a case, the potential of ϕ is stabilised by the interaction with S. The criteria denoted above can be written as

$$\lambda_{\phi S} \lesssim \lambda_\phi \left(\frac{F_a}{T_{\rm max}}\right)^2, \tag{5.3.16}$$

where $T_{\rm max}$ denotes the maximum temperature after the inflation.

In order for the PQ symmetry not to be restored, the conditions Eq. (5.3.10), Eq. (5.3.14), Eq. (5.3.15) and Eq. (5.3.16) are required. Note that these conditions can be satisfied for reasonable choice of parameters. This is partially because the field ϕ does not necessarily have a large field value during the inflation which weaken the effects of the trapping. This feature is one advantage over the single PQ field case.

5.3.2 DFSZ axion model with Higgs inflation

Here, we apply our mechanism to relax the constraint on the inflation scale to the Higgs inflation scenario which demands a high inflation scale $H_{\rm inf} \sim 10^{13}$ GeV within the DFSZ axion model which has the domain wall number $N_{\rm DW} = 6$. As is discussed, if the domain wall number is greater than one, high scale inflation scenarios are usually excluded by the isogurvature constraint Eq. (5.1.1).

The DFSZ axion model [36, 37] contains the PQ charged two Higgs doublets (see Sec. 2.4.2). In this model, S^2 in the toy model Eq. (5.3.1) would be identified the gauge invariant combination of the Higgs doubled $H_u H_d$. Higgs inflation models requires non-minimal couplings to the gravity. The action in the Jordan frame is given by

$$S = \int d^4x \sqrt{-g_J} \left(\mathcal{L}_g + \mathcal{L}_J - V_J + \mathcal{L}_J^{(\rm SM)}\right), \tag{5.3.17}$$

[10] Note that initially the energy density of S dominates over that of ϕ. Just after S decays, the energy density of the radiation is expected to be much larger than ϕ. Even if the total sector reaches the thermal equilibrium state, the PQ symmetry will be broken by the field value of ϕ if the criteria is satisfied.

where

$$\mathcal{L}_g = \left(\frac{M_P^2}{2} + \xi_u|H_u|^2 + \xi_d|H_d|^2\right) R_J, \tag{5.3.18}$$

$$\mathcal{L}_J = |\mathcal{D}_\mu H_u|^2 + |\mathcal{D}_\mu H_d|^2 + |\partial_\mu \phi|^2, \tag{5.3.19}$$

$$\begin{aligned} V_J = & \, m_u^2|H_u|^2 + m_d^2|H_d|^2 + (\lambda\phi^2 H_u H_d + \text{h.c.}) \\ & + \lambda_u|H_u|^4 + \lambda_d|H_d|^4 \\ & + \lambda_{ud}|H_u|^2|H_d|^2 + \lambda'_{ud}|H_u H_d|^2 \\ & + \lambda_{u\phi}|H_u|^2|\phi|^2 + \lambda_{d\phi}|H_d|^2|\phi|^2 + V(|\phi|), \end{aligned} \tag{5.3.20}$$

$$V(|\phi|) = \lambda_\phi\left(|\phi|^2 - \frac{v_a^2}{2}\right)^2, \tag{5.3.21}$$

where $\mathcal{L}^{(\mathrm{SM})}$ denotes the standard model Lagrangian except the Higgs self interactions. R_J indicates the Ricci scalar and the objects with subscript J represents those in the Jordan frame. The $\xi_\bullet$ denotes a coupling constant. Here we assume $H_{(u/d)}$ couples to (up/down)-type quarks. We assign the PQ charge of ϕ to be one. The coupling $(\lambda\phi^2 H_u H_d + \text{h.c.})$ mediates the PQ charge to the standard model. In this model, the domain wall number N_{DW} is equal to 6. Thus, the PQ symmetry must not be restored, otherwise stable domain walls lately dominate the universe (see Sec. 3.3).

In the present vacuum, the PQfield ϕ obtains a vacuum expectation value $|\phi| = v_a/\sqrt{2}$ and the two Higgs doublets obtains electroweak scale vacuum expectation values. We assume v_a is of order of the PQ scale. In such a case, the axion consists of a combination of the phases of H_u, H_d and ϕ:

$$a \simeq a_\phi - \frac{\sin 2\beta}{v_a}\left(v_d a_u + v_u a_d\right), \tag{5.3.22}$$

where we define

$$H_u^0 = \frac{v_u}{\sqrt{2}}\exp\left(i\frac{a_u}{v_u}\right),\ H_d^0 = \frac{v_d}{\sqrt{2}}\exp\left(i\frac{a_d}{v_d}\right),\ \phi = \frac{v_\phi}{\sqrt{2}}\exp\left(i\frac{a_\phi}{v_\phi}\right), \tag{5.3.23}$$

and $\tan\beta \equiv v_u/v_d$. Note that the axion mainly consists of a_ϕ and the mixing is suppressed by the ratio $\sim v_{u/d}/F_a$. Another massless mode, $a_G \equiv \cos\beta a_d - \sin\beta a_u$ is eaten by the Z boson and the other orthogonal mode in the phases becomes massive, which is identified as the pseudo scalar Higgs that we denote as a_h.

Let us consider a case where $v_{u/d}$ takes a value much larger than F_a. In such a case, the axion can be identified as

$$a \simeq \cos\beta a_u + \sin\beta a_d - \frac{v_\phi}{v\sin 2\beta}a_\phi, \tag{5.3.24}$$

where $v \equiv \sqrt{v_u^2 + v_d^2}$. In fact, in the Higgs inflation scenario $v_{u/d}$ can take a large value during the inflation. As a result, the axion isocurvature perturbation can be suppressed as is the case with the toy model in the previous subsection.

From now on, we briefly review the Higgs inflation scenario within the two Higgs doubled model. A detailed study of the Higgs inflation with the two Higgs doublets can be found in Ref. [141]. After the conform transformation $g^E_{\mu\nu} = \Omega^2 g^J_{\mu\nu}$ where

$$\Omega^2 = 1 + \frac{2\xi_u |H_u|^2}{M_P^2} + \frac{2\xi_d |H_d|^2}{M_P^2}, \tag{5.3.25}$$

and the subscript E represents the Einstein frame, the acion can be rewritten as

$$S = \int d^4x \sqrt{-g_E}\left(\frac{M_P^2}{2} R_E + \mathcal{L}_E - V_E + \mathcal{L}_E^{(\mathrm{SM})}\right). \tag{5.3.26}$$

The scalar potential in the Einstein frame is given by

$$V_E(H_u, H_d, \phi) = \frac{V_J}{\Omega^4}. \tag{5.3.27}$$

During inflation, the quartic terms of H_u and H_d are relevant and that can be written as

$$\frac{V_E}{M_P^4} = \frac{\lambda_u v_u^4 + \lambda_d v_d^4 + \bar{\lambda}_{ud} v_u^2 v_d^2}{4(1 + \xi_u v_u^2 + \xi_d v_d^2)^2}. \tag{5.3.28}$$

with $\bar{\lambda}_{ud} \equiv \lambda_{ud} + \lambda'_{ud}$. We can see that if $v_{u/d} \gg M_P/\sqrt{\xi_{u/d}}$, the potential becomes almost flat. As we can see later, terms involving ϕ do not affect the dynamics of the inflation. A stable path of the inflation exists along

$$\frac{v_u^2}{v_d^2} = \frac{2\lambda_d \xi_u - \bar{\lambda}_{ud}\xi_d}{2\lambda_u \xi_d - \bar{\lambda}_{ud}\xi_u}, \tag{5.3.29}$$

if $2\lambda_d \xi_u - \bar{\lambda}_{ud}\xi_d > 0$ and $2\lambda_u \xi_d - \bar{\lambda}_{ud}\xi_u > 0$. The potential energy for the inflaton can be given by

$$\frac{V_E}{M_P^4} = \frac{\lambda_u \lambda_d - \bar{\lambda}_{ud}^2/4}{4(\lambda_u \xi_d^2 + \lambda_d \xi_u^2 - \bar{\lambda}_{ud}\xi_u \xi_d)}\left(1 - e^{-2\chi/\sqrt{6}M_P}\right)^2, \tag{5.3.30}$$

where χ denotes the canonically normalised inflaton field. In the large field limit ($v \gg M_P/\xi$), ξ is given by

$$\chi = \sqrt{\frac{3}{2}} M_P \log\left(1 + \frac{\xi_u v_u^2}{M_P^2} + \frac{\xi_d v_d^2}{M_P^2}\right). \tag{5.3.31}$$

Note that the Higgs inflation scenario in the two Higgs doublet model is essentially the same to the case with a single Higgs double case. For simplicity, we assume $\lambda_u \sim \lambda_d \sim \lambda_{ud} \sim \lambda'_{ud}$ and $\xi_u \sim \xi_d$. We denote $\lambda_{\rm eff}$ and $\xi_{\rm eff}$ as typical values of them. The typical values of $v_{u/d}$ during the inflation is given by $v_h \sim 10 M_P/\sqrt{\xi}$. To reproduce the density perturbation obtained from the CMB observation [16],

$$\xi \sim 5 \times 10^4 \sqrt{\lambda_{\rm eff}}, \tag{5.3.32}$$

must hold. We expect $\lambda_{\rm eff}$ to be $\mathcal{O}(1)$ to explain the Higgs boson mass of 125 GeV. However, the running effects may reduce the $\lambda_{\rm eff}$ at high scale [142, 143].

Next, let us consider the behaviour of the PQ field ϕ during the inflation. The dynamics of the PQ field ϕ is qualitatively the same to the toy model case. During the inflation, ϕ obtains a field value:

$$v_\phi \simeq \max\left[v_a, \sqrt{\frac{\lambda}{\lambda_\phi}} v_h\right]. \tag{5.3.33}$$

a_ϕ obtains a mass of

$$m_{a_\phi}^2 \simeq \frac{\lambda v_h^2}{\Omega^2} \simeq \frac{\lambda M_P^2}{\xi}. \tag{5.3.34}$$

To suppress the quantum fluctuation of a_ϕ during inflation, the condition

$$\lambda \gtrsim \frac{\xi H_{\rm inf}^2}{M_P^2}, \tag{5.3.35}$$

must be satisfied. The condition for the PQ symmetry not to be restored due to the thermal trapping, the following condition is imposed (see Eq. (5.3.16))

$$\lambda_{u\phi}, \lambda_{d\phi} \lesssim \lambda_\phi \left(\frac{F_a}{T_{\rm max}}\right)^2, \tag{5.3.36}$$

where the maximum temperature of the universe $T_{\rm max}$ can be as high as $\sim 10^{13}$ GeV in Higgs inflation case [121, 144, 145].

Finally let us consider the quantum fluctuation of the axion. During inflation, the massless mode is dominated by the pseudo scalar Higgs a_h, which develops quantum fluctuations. It should be noted that the canonically normalised field is $\tilde{a}_h \equiv a_h M_P/\sqrt{\xi} v_h$. Thus, the quantum fluctuation is develops as $\delta\tilde{a}_h \simeq H_{\rm inf}/2\pi$. On the other hand, the mixing between a_ϕ and a_h is given by $\sim v_\phi/v_h$. Then, we can estimate the effective fluctuation of a_ϕ to be $\delta a_\phi/v_\phi \sim \sqrt{\xi}H_{\text{inf}}/(2\pi M_P)$. This gives the axion isocurvature fluctuation as

$$\frac{\delta a_\phi}{a_\phi} \sim \frac{\sqrt{\xi}H_{\text{inf}}}{2\pi M_P \theta_i}. \tag{5.3.37}$$

The cold dark matter iso curvature perturbation S_{CDM} is given by (Eq. 5.3.13) with a replacement $v_S \to M_P/\sqrt{\xi}$. Conditions for the PQ symmetry not to be restored can be also rewritten by this replacement $v_S \to M_P/\sqrt{\xi}$. We can see that the constraint is significantly relaxed. If F_a is smaller than 10^{12} GeV or θ_i is smaller than $O(1)$, the cosmological constraint can be satisfied.

5.3.3 Summary of the Multi fields solution

In this section, we have shown a way to suppress the axion isocurvature perturbation based on the dynamics of the multi PQ fields. If a PQ field has a large field value during inflation and another PQ fields breaks the PQ symmetry at present time, the axion isocurveture can be suppressed. In this mechanism, the PQ symmetry is less likely to be restored because some PQ fields can have relatively small field values which tends to avoid the non thermal trapping. Especially, this mechanism can work in the Higgs inflation scenario within the DFSZ model. High scale inflation scenarios such as the Higgs inflation scenario are within a reach of proposed future CMB observation [105] . On the other hand, the DFSZ model is one simple possibility of the PQ models. Thus, it seems interesting that the Higgs inflation within the DFSZ model can work without the PQ symmetry restoration.

So far, we have considered the quartic interaction term between the PQ fields. There are other possibilities. For example, let us consider the case with the cubic term $V = \mu\phi H_u H_d$ with μ being a some mass scale. During inflation, ϕ settles around $\phi \sim \mu/\lambda_{H\phi}$ and the phase direction of ϕ obtains a mass of $\sim \lambda_{H\phi}v_h^2$ where we denote $\lambda_{H\phi}$ as a typical quartic coupling between the Higgs and ϕ. If $\mu/\lambda_{H\phi} \lesssim F_a$, ϕ will not pass through the origin after inflation. As a result the PQ symmetry restoration due to the non thermal trapping can be avoided. The higher dimensional teams may also affect the dynamics of the PQ field.

Chapter 6

Conclusion

In this thesis, we have considered the cosmology of the Peccei-Quinn models. We have paid particular attention to the dynamics of the PQ field.

First, we have studied the dynamics of the PQ field after the PQ symmetry is restored at some epoch in the universe. When the thermal mass of the PQ field becomes comparable to the negative mass at the symmetry enhanced point, the PQ field starts to oscillate around the minimum of the potential. The oscillation of the PQ field often dominantly decays into relativistic axion particles. Such produced axions become a part of the dark radiation component whose energy density is tightly constrained by the CMB observations. We have found that the constraint on the axion overproduction can be relaxed by taking thermal dissipation effects into account. We have shown even if the PQ field oscillation once dominates the universe, the energy of the PQ field can be converted to the thermal plasma in the universe owing to the thermal dissipation effects and amount of axion dark radiation can by suppressed. This scenario works if the potential mass scale around the symmetry enhanced point is larger than $\mathcal{O}(10)$ TeV - $\mathcal{O}(10)$ PeV for $F_a \sim 10^9$ GeV - 10^{10} GeV. This parameter region includes a class of high scale supersymmetric PQ models.

Second, we have considered scenarios in which the PQ symmetry is always broken by field values of the PQ fields, which is indispensable for models with the domain wall number greater than unity. This is because otherwise stable domain walls overclose the universe. In such a scenario, the axion field acquires quantum fluctuations of order of the inflation scale during inflation since the axion is almost massless. Such quantum fluctuations induce the cold dark matter isocurvature perturbation, which is tightly bounded by the CMB observations. Since the size of quantum fluctuations is of order of the inflation scale, the bound is imposed on the inflation scale: $\lesssim 10^{7-10}$ GeV. We have proposed two new scenarios in which the constraint on the inflation scale is relaxed without the PQ symmetry restoration.

The first scenario is based on the pseudo scaling solution, which is a peculiar behaviour of the scalar motion different from the scaling solution or oscillation solution. We have summarised the properties of the pseudo scaling solution. Then, we have applied this mechanism to the PQ cosmology. We have shown that if the PQ field once obeys the pseudo scaling solution, the constraint on the inflation scale is likely to be relaxed and the PQ symmetry restoration is not likely to occur.

In the second scenario, we have considered the dynamics of multi PQ fields. We have found that if one PQ field had a large field value during inflation and another PQ field dominantly breaks the PQ symmetry at the present time, the constraint on the inflation scale can be relaxed avoiding the PQ symmetry restoration. We have applied this mechanism to the DFSZ axion model, whose domain wall number is greater than unity. We have shown that this scenario can work within the framework of the Higgs inflation, whose inflation scale is high $\sim 10^{13}$ GeV.

We have seen variety of the dynamics of the PQ field. We hope that continuous efforts to investigate such possibilities will lead us to the more fundamental theory of the nature.

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Appendix A

Axion mass

Here, we derive the axion mass in Eq. (2.3.9). In the energy scale lower than the QCD scale, the strong gauge coupling constant becomes non-perturbative. In such a region, the QCD sector can be described by an effective theory called as the chiral perturbation theory (see [146] for a review). The properties of the axion can also be described by the framework of the chiral perturbation theory.

In the following, we briefly review the chiral perturbation theory. Then, we derive the axion mass.

A.1 Chiral perturbation theory

The chiral perturbation theory(ChPT) makes use of the symmetry in the QCD sector. Thus, we first consider the symmetry in the QCD sector. Then, we introduce the ChPT.

Just above the QCD scale, the strong sector contains the gluon field and three light quarks (up, down and strange). If we neglect the masses of these quarks due to their smallness (this procedure is called as "chiral limit") , the Lagrangian can be written as

$$\mathcal{L} = \sum_{i=u,d,s} (\bar{q}_i^R i\not{D} q_i^R + \bar{q}_i^L i\not{D} q_i^L) - \frac{1}{4} GG, \tag{A.1.1}$$

where u, d, s denote up, down and strange quarks respectively and G denotes the field strength tensor of the gluon. L/R indicates a left/right handed quark field. This Lagrangian has a classical global $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ symmetry in which the Left/Right handed quarks are rotated by $\mathrm{U}(3)_{L/R}$ in the flavour space:

$$\begin{pmatrix} u^{L/R} \\ d^{L/R} \\ s^{L/R} \end{pmatrix} \to U_{L/R} \begin{pmatrix} u^{L/R} \\ d^{L/R} \\ s^{L/R} \end{pmatrix}, \tag{A.1.2}$$

with $U_{L/R}$ being the U(3) matrix. We can define the (axial vector) vector symmetry $\mathrm{U}(3)_{(A)V}$ in which U_L and U_R are chosen as the (opposite) same rotation. The classical global symmetry in the Lagrangian can be decomposed into $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A$. In quantum level, $\mathrm{U}(1)_A$ is broken by the anomaly. Thus, $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V$ is the symmetry in the theory.

It was shown that in the chiral limit, $\mathrm{SU}_V(3) \times \mathrm{U}(1)_V$ cannot be broken [147]. On the other hand, if $\mathrm{SU}(3)_A$ is not broken, the states are doubly degenerated: if the generator in $SU(3)_A$ is applied to an arbitrary state, one would have a degenerate state with an opposite parity. However, such a doubling in states is not observed. Thus, $\mathrm{SU}(3)_A$ symmetry should be broken in the present vacuum and associated Numbu-Goldstone modes are mesons.

There is a method which successfully describes the low energy QCD dynamics based on the symmetry arguments: the chiral perturbation theory [148]. The effective Lagrangian in the chiral perturbation theory is constructed based on the symmetry $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V$ taking account of spontaneous symmetry breaking. One building block of the ChPT is the meson field $U(x)$:

$$U(x) = \exp\left[i\frac{\phi(x)}{f_\pi}\right], \tag{A.1.3}$$

$$\phi(x) = \sum_{a=1}^{8} \lambda_a \phi_a \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{3}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}, \tag{A.1.4}$$

where λ_a denotes the generator of SU(3), f_π is a pion decay constant and π, η, K denotes pion, eta meson and kaon respectively. We define the flavour rotation Under $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ on $U(x)$ as

$$U(x) \to U_R U(x) U_L^\dagger. \tag{A.1.5}$$

The lowest order Lagrangian which is invariant under $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V$ can be written as

$$\mathcal{L}_{\mathrm{kin}} \equiv \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger]. \tag{A.1.6}$$

So far, we have neglected quark mass matrix in the QCD Lagrangian:

$$-\mathcal{L}_{\mathrm{mass}} = \bar{q}_R M q_L + \mathrm{H.c.}. \tag{A.1.7}$$

If there exists a quark masses, the symmetry $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V$ is explicitly broken. However, the Lagrangian would be invariant if the mass matrix M transformed under the flavour rotation as

$$M \to U_R M U_L^\dagger. \tag{A.1.8}$$

Then, one can construct the Lagrangian $\mathcal{L}(U, M)$ which is invariant under Eq. (A.1.5,A.1.8). The lowest order term with respect to M can be written as

$$\mathcal{L}_M = \frac{f_\pi^2 B}{2}\,\mathrm{tr}(MU^\dagger + UM^\dagger). \tag{A.1.9}$$

with a constant B [1] . It should be noted that the Lagrangian $\mathcal{L}_{\mathrm{kin}} + \mathcal{L}_M$ can reproduce the results of current algebra [148].

We can include the anomolously broken symmetry $U(1)_A$ in ChPT. The meson field matrix is extended to

$$U(x) = \exp\left[i\frac{\phi(x)}{f_\pi} + i\frac{\sqrt{2}\eta'}{\sqrt{3}f_\pi}\right], \tag{A.1.10}$$

where η' denotes the η' meson. Including instanton effects, the lowest order effective Lagrangian can be written as [149, 150]:

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial U \partial U^\dagger] + \frac{f_\pi^2 B}{2}\,\mathrm{tr}(MU^\dagger + UM^\dagger) - \frac{A f_\pi^2}{4N_c}[\theta - i\log\det U]^2, \tag{A.1.11}$$

where $N_c = 3$ is the number of the color, θ denotes the vacuum angle and A is a constant parameter. The last term gives a mass to η'.

A.2 Axion mass

The chiral perturbation theory can also describe the PQ model. Just above the QCD scale, the Lagrangian with axion field can be written as (see Sec. 2.3)

$$\begin{aligned}\mathcal{L}_a \supset\ & \frac{1}{2}(\partial a)^2 + \sum_{i=u,d,s}(\bar{q}_i^L i\not{D} q_i^L + \bar{q}_i^R i\not{D} q_R) \\ & + \sum_i\left[c_{i,1}\frac{(\partial_\mu a)}{F}J^\mu_{A,i} - \left(\bar{q}_i^L m_i q_i^R \exp[i c_{i,2}\frac{a}{F}] + \mathrm{H.c.}\right)\right] \\ & + c_3\frac{a}{32\pi^2 F}G\tilde{G},\end{aligned} \tag{A.2.1}$$

[1]The combination $\mathrm{tr}(MU^\dagger - UM^\dagger)$ gives a wrong behaviour under parity.

where $J_{i,A}$ denotes the axial current. The replacement:

$$M \to \mathrm{diag}[m_i e^{-ic_{i,2}\frac{a}{F}}], \tag{A.2.2}$$
$$\theta \to \theta + c_3 \frac{a}{F}, \tag{A.2.3}$$

and inclusion of an additional term:

$$\mathcal{L}_{\partial a} = +\sum_i \left[c_{i,1} \frac{(\partial_\mu a)}{F} J^\mu_{A,i} \right], \tag{A.2.4}$$

in Eq. (A.1.11) can describe the axion-meson system. In general, the fields a, π, η, K are not in mass eigen states. Thus, in order to obtain physical particle states, one has to diagnose the mass matrix. As is mentioned in Sec. 2.3, the following rotation in Eq. (A.2.1) removes the mass mixing between axion and pions[62, 63] [2] :

$$q^L \to \exp(-iaQ_A/F) q^L, \;\; q^R \to \exp(iaQ_A/F) q^R, \tag{A.2.5}$$

where $q = (u, d, s)$ and

$$Q_A = c_3^0 \frac{M_q^{-1}}{2\mathrm{tr}(M_q^{-1})}, \; M_q \equiv \mathrm{diag}[m_u, m_d, m_s]. \tag{A.2.6}$$

Then, one can obtain physical axion mass:

$$m_a = \frac{f_{\pi^0} m_{\pi^0}}{F_a} \sqrt{\frac{z_d}{(1+z_d)(1+z_d+z_s)}} \simeq 6\ \mathrm{meV} \times \frac{10^9\ \mathrm{GeV}}{F_a}, \tag{A.2.7}$$

where f_{π^0} and m_π^0 are the pion decay constant and pion mass respectively and

$$z_d \equiv \frac{m_u}{m_d} \simeq 0.568 \pm 0.042, \tag{A.2.8}$$
$$z_s \equiv \frac{m_u}{m_d} \simeq 0.029 \pm 0.0003. \tag{A.2.9}$$

[2]Here, we neglect the mixing with η' because it is rather heavy $\sim$ 1GeV.

Appendix B

Estimation of axion dissipation rate

In this appendix, we estimate the thermal dissipation rate of relativistic axions with soft momenta $p \ll g_s^2 T$ at the one-loop order. The following estimation may be made more precise by the resummation of many diagrams, but we postpone this task as a future work.

The dissipation rate of relativistic axion is given by

$$\Gamma_a^{(\mathrm{dis})} = \left.\frac{\Pi_{\mathrm{J}}(P)}{2p_0}\right|_{p_0=p}. \tag{B.0.1}$$

where

$$\Pi_{\mathrm{J}}(P) = \frac{\alpha_s^2}{128\pi^2 f_a^2}\int_P e^{iP\cdot x}\left\langle\left[\{F^{a\mu\nu}\tilde{F}^a_{\mu\nu}\}(x)\{F^{a\mu\nu}\tilde{F}^a_{\mu\nu}\}(0)\right]\right\rangle, \tag{B.0.2}$$

where $\langle\bullet\rangle \equiv \mathrm{tr}[e^{-\beta\hat{H}}\bullet]/\mathrm{tr}[e^{-\beta\hat{H}}]$ denotes the thermal average. By performing angular integrations, one can obtain [130]

$$\begin{aligned}\Pi_{\mathrm{J}}(P) =& f_{\mathrm{B}}^{-1}(p_0)\frac{d_a}{8}\frac{\alpha_s^2}{64\pi^5 f_a^2}\frac{1}{p}\int_0^\infty dk\int_{-\infty}^{\infty}dk_0\int_{|k-p|}^{k+p}dq\, kq f_{\mathrm{B}}(k_0)f_{\mathrm{B}}(p_0-k_0)\\ &\Big\{\left(\rho_L(K)\rho_T(Q)+\rho_T(K)\rho_L(Q)\right)\left((k+q)^2-p^2\right)\left(p^2-(k-q)^2\right)+\\ &\rho_T(K)\rho_T(Q)\left(\left(\frac{k_0^2}{k^2}+\frac{q_0^2}{q^2}\right)\left((k^2-p^2+q^2)^2+4k^2q^2\right)\right)+8k_0q_0(k^2+q^2-p^2)\Big\},\end{aligned} \tag{B.0.3}$$

where $q_0 = p_0 - k_0$, $d_a = N_c^2 - 1 = 8$, f_{B} is the Bose-Einstein distribution function, and $\rho_{T/L}$ indicates the spectral function for the transverse/longitudinal mode of gauge boson in the thermal bath.

To calculate this integral, we approximate the spectral density for the transverse mode by the Breit-Wigner form:

$$\rho_T(P) = \frac{2p_0\Gamma_p}{[p_0^2 - \omega_p^2]^2 + [p_0\Gamma_p]^2}; \ \ \omega_p \equiv \sqrt{p^2 + m_\infty^2}, \tag{B.0.4}$$

where m_∞ indicates the asymptotic mass which is a effective mass of the hard momentum modes $\sim T$. In the case of non-abelian gauge theory with N_F flavor, the asymptotic mass is given by $m_\infty^2 = [C(\mathrm{ad}) + N_F T(\mathrm{F})]g_s^2 T^2/6$ with $C(\mathrm{ad})$ being the adjoint Casimir and T(F) being the normalisation index. Here we use the rate of large angle scatterings, $\Gamma_p \sim g_s^4 T^3/p^2$ as the the thermal width Γ_p [135] . We omit contributions from longitudinal mode ρ_L because the loop-integral is typically dominated by a hard momentum $\sim T$. After some calculations, one can obtain the dissipation rate of the axion:

$$\Gamma_a^{(\mathrm{dis})} = \frac{\alpha_s^2 T^3}{32\pi^2 f_a^2} \times C \frac{p_0^2}{g_s^4 T^2} f(x), \tag{B.0.5}$$

where $x = p_0/g_s^4 T$, and the function f can be approximated as

$$f(x) \simeq \frac{d_a}{\pi^2}\int_a dy \frac{e^y}{(e^y-1)^2}\frac{(y^2-a^2)^{3/2}}{y} \frac{1}{2}\int_{-1}^{1} \frac{d\epsilon}{1 + g_s^4 \epsilon \sqrt{1 - \frac{a^2}{y^2}}\frac{x}{y}} \frac{y^2 - a^2 + y^2\epsilon^2 + 2\sqrt{y^2-a^2}\,y\epsilon}{x^2\left(1 + \sqrt{1-\frac{a^2}{y^2}}\epsilon\right)^2 (y^2-a^2)^2 + 1}, \tag{B.0.6}$$

with

$$a = m_\infty/T, \tag{B.0.7}$$

and C denotes an uncertain factor. Remember that our estimation of the dissipation rate is supposed to be an order-of-estimation because it is based on a simplified calculation. Thus, we introduce the parameter C to take account of the uncertainty. Qualitatively, $f(x)$ behaves as $f(x) \sim$ const. for $x \ll 1$ and $f(x) \propto x^{-2}$ for $x \gg 1$. The approximation of the estimation of f is valid for $p < g^2 T$.

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