論文の内容の要旨

Study on sign problem via Lefschetz-thimble path integral
(レフシェッツスィンブル上の経路積分による符号問題の研究)

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In this thesis, I study the sign problem appearing in the path-integral approach to strongly-coupled systems. This problem appears in many interesting systems, such as finite-density quantum chromodynamics (QCD), the repulsive Hubbard model away from the half filling, frustrated spin systems, and so on. In these systems, the Euclidean classical action S takes complex values, and the oscillatory nature of the Boltzmann weight e^{-S} causes the serious technical problems in *ab initio* numerical computations. In this study, I use the recently developing technique, called the path integral on Lefschetz thimbles, in order to understand the essence and difficulties of the sign problem in the path-integral formalism.

The basic idea of the Lefschetz-thimble method is to deform the integration contours in the complexified configuration space using Cauchy integration theorem. Under this deformation, the original oscillatory integral is decomposed into the sum of nicely convergent integrals, and the sign problem becomes moderate. Each integration cycle in this decomposition is called a Lefschetz thimble. In this thesis, I first give a concise review on this method, and demonstrate how it works by computing real-time Feynman kernels for free theories.

Next, I discuss the spontaneous breaking of chiral symmetry in 0-dimensional field theories without the sign problem. Chiral symmetry breaking is one of the characteristic features of low-energy QCD spectrum, and it is important to revisit this phenomenon using Lefschetz thimbles. I developed an efficient way to compute Lefschetz thimbles when a continuous symmetry is slightly broken due to the small explicit breaking term. This study also elucidates that the Lefschetz-thimble decomposition is suitable to analyze Lee–Yang zeros in the path-integral formulation, and the phase structure of the model is discussed for complex coupling constants.

If one tries to analyze finite-density QCD in the same way, the sign problem appears even at the level of the mean-field approximation. I show that the Lefschetz-thimble decomposition respects the anti-linearly extended charge conjugation so that the effective potential becomes manifestly real, and the saddle-point analysis based on this theorem solves the sign problem appearing in the mean-field approximation. The Polyakov-loop effective model of heavy-dense QCD is studied with this method as a lucid demonstration. Since all the quarks are heavy in this model, fermion dynamics is simplified and the mean-field treatment becomes accurate.

The sign problem in finite-density QCD with light flavors is known to become too severe as the quark chemical potential goes beyond the threshold of pion mass. In order to see what happens there, I consider the one-site Fermi Hubbard model, which is exactly solvable but has the severe sign problem in the path-integral expression. In this case, the above mean-field approximation is not applicable, and I elucidate its reason by studying topological structures of Lefschetz thimbles. I also show that interference of complex phases among multiple classical solutions play a pivotal role to understand the sign problem and the correct phase structures using path integrals. I discuss this interference is also important in the finite-density QCD with light flavors if the baryon chemical potential exceeds the pion mass.

In order to give a feedback of this finding to other approaches to the sign problem, I relate this newly developing technique, the Lefschetz-thimble path integral, with a conventional approach to the sign problem, the complex Langevin method. It is known that the complex Langevin method sometimes does not give correct answers, and I analytically show that the original complex Langevin method cannot give correct results if interference of complex phases among classical solutions becomes important. This gives a simple criterion for incorrectness of the complex Langevin method without doing its numerical simulation, and suggests that the complex Langevin method is not directly applicable to the finite-density QCD. To resolve this problem, I propose to modify the complex Langevin method with some technical working hypothesis and it is numerically tested for the one-site Hubbard model. The original formulation of the complex Langevin method gives the wrong answer for this model. The modified complex Langevin method improves the result, but there exists systematic discrepancy, which requires further systematic studies on its properties.