# 論文の内容の要旨
# Abstract

論文題目　Theory and Practice of Exact Algorithms for NP-Hard Problems: Branching and Reduction
(NP 困難問題に対する厳密解法の理論と応用: 探索と帰着について)

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Algorithms for computing exact solutions of NP-hard problems are widely studied both theoretically and empirically. In theoretical studies, there are two major fields: exact exponential-time algorithms and FPT algorithms. In these theoretical research fields, many interesting techniques have been developed, and faster and faster algorithms have been obtained over the years. On the other hand, in practice, these theoretical methods have been rarely used, and we often use general solvers such as integer programming solvers (e.g., CPLEX) and SAT solvers (e.g., MiniSAT) or branch-and-bound methods using problem specific lower bounds. However, for these practical methods, few theoretical analysis has been discussed. As just described, there is a gap between the theory and practice. In this thesis, we aim to close the gap by applying theoretical methods to practical problems, and conversely by giving theoretical analysis to practical methods. Especially, we focus on branching algorithms and reductions used for solving problems by SAT solvers.

The branch-and-bound method that uses an LP relaxation lower bound to prune the search space is a widely used practical technique to solve NP-hard problems. In recent years, this practical method has been rediscovered as useful for obtaining theoretically faster FPT algorithms. However, in the existing research, there is a problem on the applicability; it can be applied only to a small number of problems whose LP relaxations are known to admit some nice properties. In this thesis, we first introduce a new concept called the *discrete relaxations*. By applying the discrete relaxations instead of using the LP relaxations (actually, these two relaxations are essentially the same), we make it possible to apply the branch-and-bound methods to wider range of problems. Then, we develop an efficient method to solve the relaxation problems by exploiting the maximum flow algorithm and improve the time complexity for various problems. Finally we propose an algorithm that is obtained by combining this new theoretical algorithm with the branch-and-reduce method which are developed in the field of exact exponential-time algorithms. By conducting experiments on real-world datasets, we empirically show that the algorithm,

which are obtained from theoretical studies, is quite practical.

Then, we focus on SAT. SAT is the most basic NP-complete problem and any NP problem can be reduced to SAT in polynomial time. In theory, despite many attempts, no algorithms faster than the naive algorithm trying all the possible $2^n$ assignments are known. On the other hand, in practice, recent SAT solvers can successfully solve instances with millions of variables reduced from industrial problems, and SAT solvers are widely used to efficiently solve practical problems by reductions. Why SAT solvers can solve very huge real-world instances despite its theoretical hardness? We give an explanation to this question by considering a *width* that measures structuredness of the input. We first propose a new reduction method called the *decomposition-based reductions*. By using the decomposition-based reductions, we show that for many problems, we can reduce an instance of a problem to an instance of SAT by preserving the width and vice versa. From this theoretical result, we can say that nice structures of an original instance can be preserved through a reduction, and by using such nice structures, SAT solvers can solve the reduced instance very efficiently. Finally, by conducting experiments, we empirically confirm the power of decomposition-based reductions.