論 文 の 内 容 の 要 旨

論文題目

On Parameterized Inapproximability of Several Optimization Problems
（幾つかの最適化問題のパラメータ化近似不可能性）

氏　　名　　林　冰　凱
Lin Bing kai

In this thesis, we study the parameterized inapproximability of several optimization problems. Approximation algorithm and parameterized complexity are two powerful methods to deal with hard computational problems. Approximation algorithm finds a solution that is close to the optimum in polynomial time. While in the area of parameterized complexity we consider problems with a parameter k and design algorithms to find the exact solutions in $f(k)\cdot|x|^{O(1)}$-time on input an instance x. There exist problems that do not admit polynomial time approximation algorithms within certain ratios if NP ≠ P. The PCP theorem is the main technique to prove hardness results of polynomial time approximation. However, it does not rule out the existence of parameterized approximation algorithms. For example, it has been an open question whether there are any $f(\mathrm{k})\cdot n^{O(1)}$-time algorithms to find a dominating set with $2k$ vertices of G given that G has a dominating set with k vertices. One of the main contributions of this thesis is to refute the existence of such algorithm under FPT ≠ W[1], a standard hypothesis in parameterized complexity. Our starting point is the parameterized inapproximability of Maximum k-Subset Intersection Problem.

Given a collection F of subsets over a finite set with m elements, the goal of Maximum k-Subset Intersection is to select k distinct subsets from F such that their intersection size is as large as possible. The decision version of this problem is related to the k-Biclique problem, which asks if an input graph contains a complete bipartite subgraph with k vertices in each side. A longstanding open question in parameterized complexity is whether there exist any $f(k)\cdot n^{O(1)}$-time algorithms for k-Biclique. This thesis also gives a negative answer to this question assuming FPT ≠ W[1].

The core result in this thesis is to provide a gap-producing reduction from k-Clique to Max k-Subset Intersection. More precisely, we construct a family F of subsets on input a graph G and a small positive integer k in polynomial time such that if G contains a k-clique then there exist $s := k(k\text{-}1)/2$ subsets in F with intersection size no less than $n^{\Theta(1/k)}$, otherwise every s subsets

in F have intersection size at most $O(k!)$.

We then derive the parameterized inapproximability of minimum dominating set problem based on the hardness approximation result of Maximum k-Subset Intersection. Significantly, our reduction does not rely on the PCP-theorem.

Finally, we consider the problem of finding the maximum clique whose edges use at most k distinct colors on input a multigraph with colored edges, which we call the Maximum k-Edge-Color Clique. We show that if the input graph has unbounded number of multi-edges between every two vertices, then this problem does not admit fpt-approximation algorithms within any computable ratio function r(k) assuming FPT $\neq$ W[1]. We also point out that the fpt-inapproximability of Maximum k-Edge-Color Clique Problem on simple graphs implies the fpt-inapproximability of k-Clique Problem.